Event-by-Event and Critical Fluctuations in Light-Nuclei Production

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INT workshop Chirality and Criticality in Heavy-Ion Collisions, Aug 21-25

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1. Introduction
2. Light Nuclei Yield in A Single Event
3. Light Nuclei Yield with E-by-E Fluctuations
4. Light-Nuclei Yield with Critical Fluctuations

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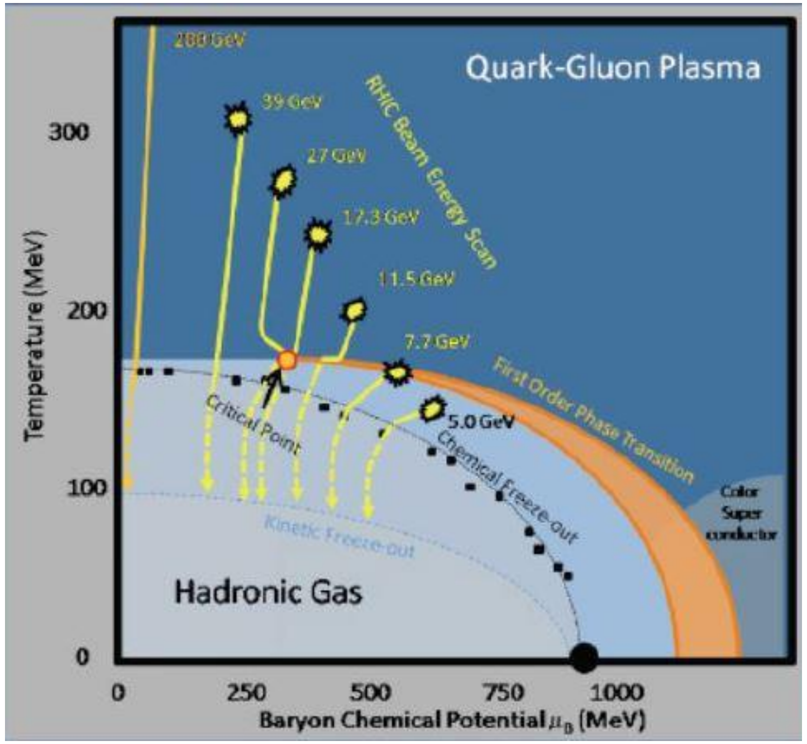
QCD phase diagram

- **Lattice QCD** (small μ_B finite T):
  - Crossover
- **Effective models** (large μ_B)
  - 1st order phase trans.

→ Critical point

- Lattice QCD: sign problem at large μ_B
- Effective models: parameters dependent

→ **Heavy-ion collisions :**

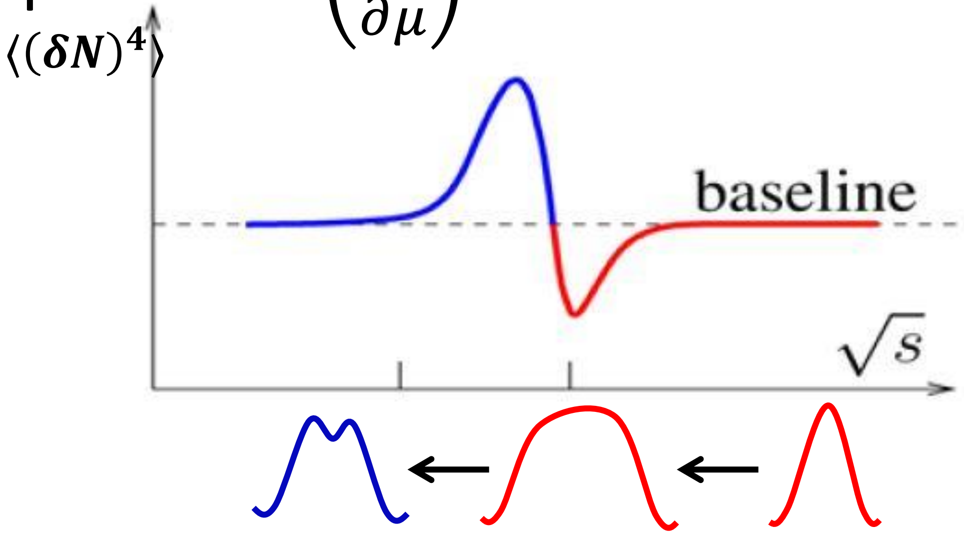


- tuning $\sqrt{s_{NN}}$,mapping $T-\mu$ phase diagram: RHIC(BES),NICA,FAIR,J_PARC….

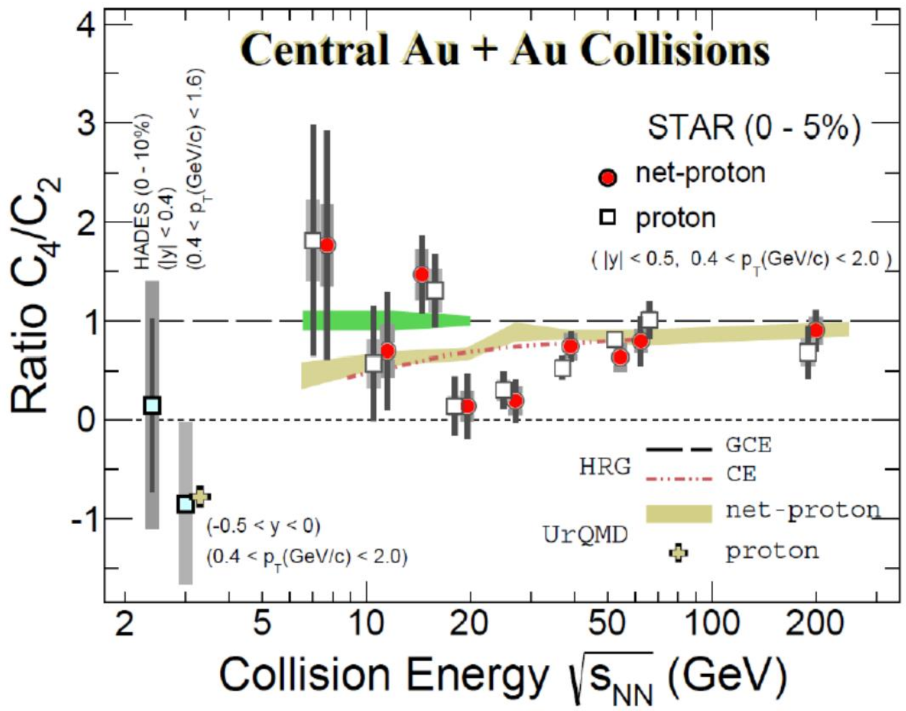
Net-proton fluctuations near critical point

- Characteristic feature of critical point:
  - long range correlation
  - large fluctuations
- **Non-monotonicity** of Net-Proton Cumulant

$\sim$ susceptibilities $\left(\frac{\partial}{\partial\mu}\right)^n \ln Z$

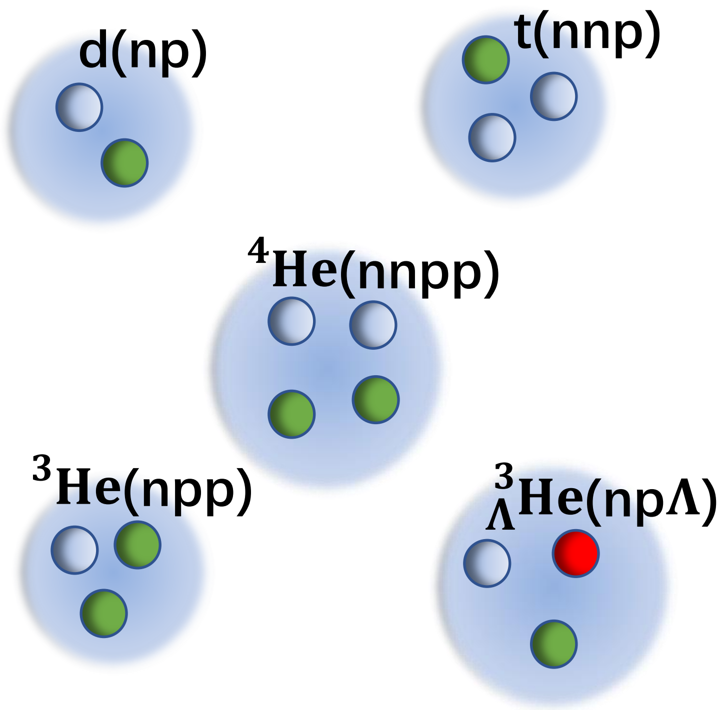


M.Stephanov, PRL 107,052301

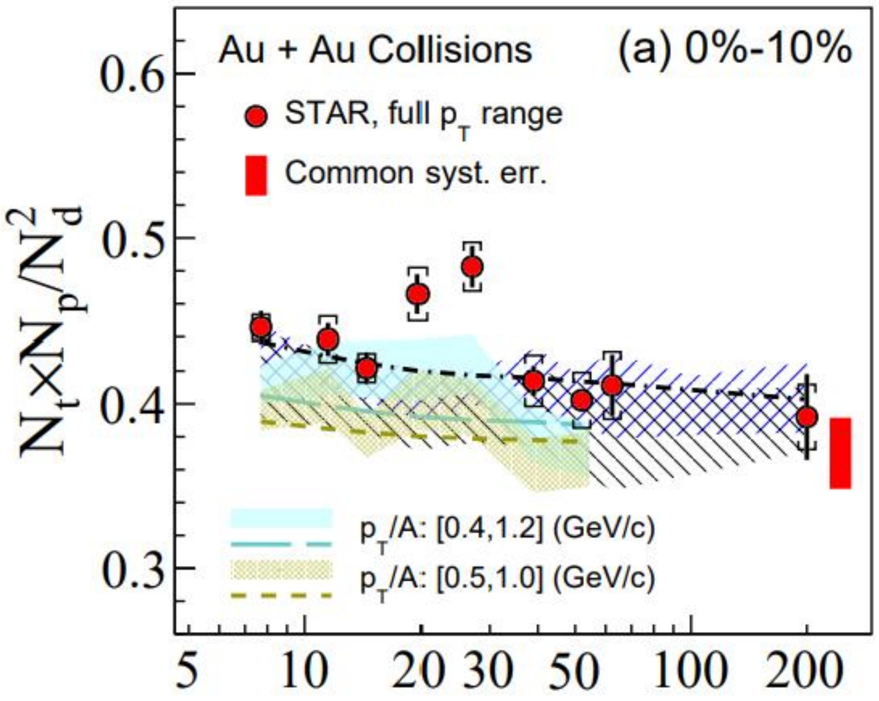


STAR, PRL 126,092301
STAR,PRL 128,202303

Light Nuclei Production

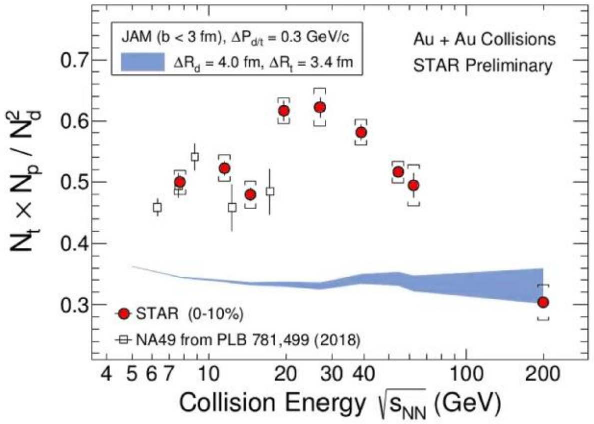


STAR Collaboration, PRL 130.202301

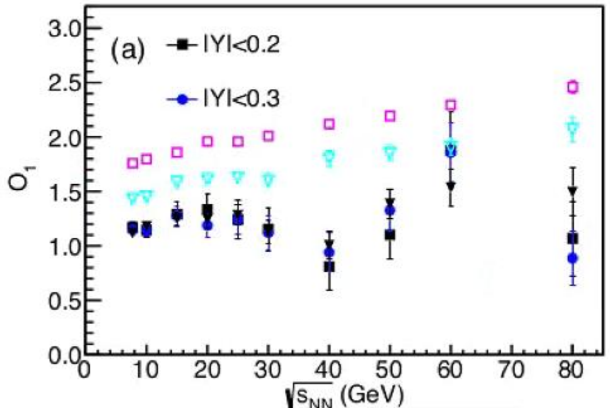


- **Non-monotonic behavior** also been observed
- Light nuclei produced at late stage of heavy-ion collisions

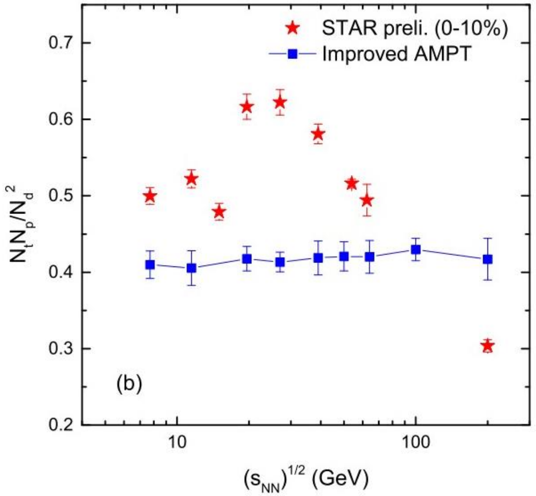
Dynamical models on Light-Nuclei



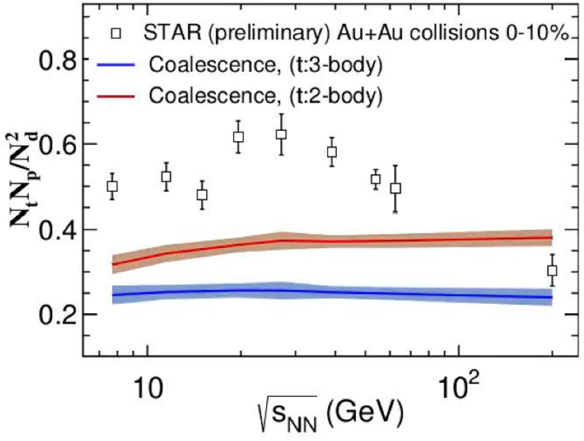
Hui Liu et al., PLB (2020)



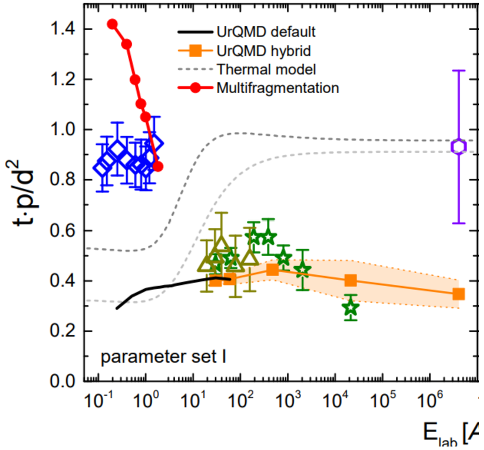
X.Deng et al., PLB (2020)



K-J.Sun et al., PRC (2021) ….
K-J.Sun et al., Phys. Lett. B, 781:499–504(2018)
K-J.Sun et al., Phys. Lett. B,774:103–107(2017)



W. Zhao et al., PRC (2018)



P.Hillmann et al., 2109.05972

And others….

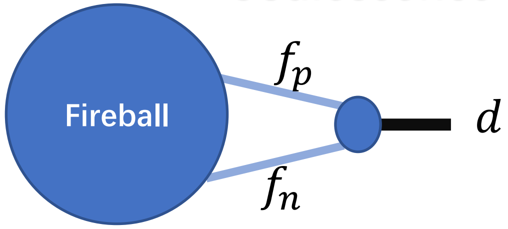
Can light nuclei detect critical effects?

- **Light-nuclei production:**

  phase-space, nucleons interaction

  Fireball size R, homogeneity length l

Coalescence

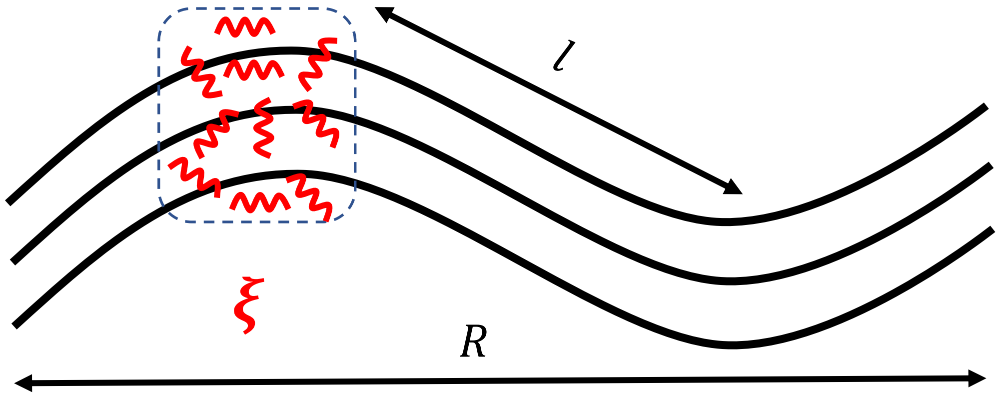


- **Homogeneity:**

  Nucleons close to each other in r space have similar momentum p

  =>Homogeneity length $l \sim 1/\partial_\mu u^\mu$

  R.Scheibl, U.Heinz, PRC 59, 1585



R : Fireball size
l:homogeneity length
ξ: correlation length

- **When not so close to critical point:**
  - Fireball size R, homogeneity length $l \gg \xi$
  - **Background is large**, comparing critical signal

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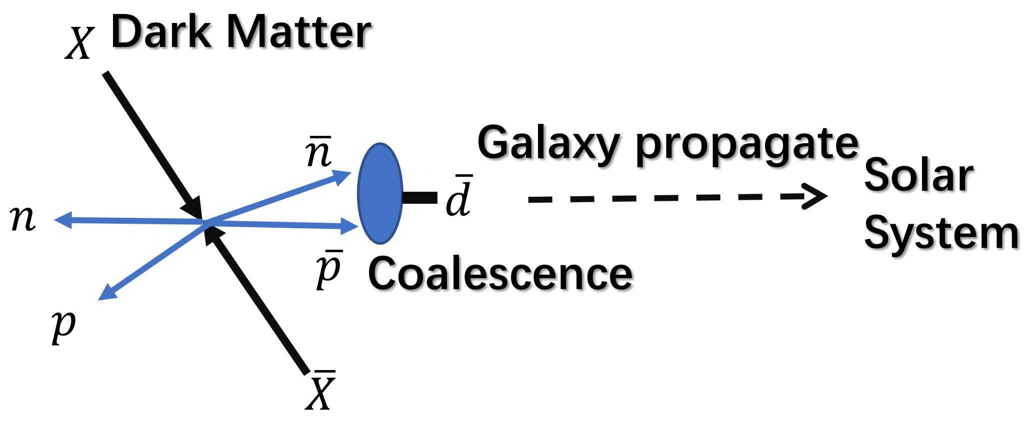
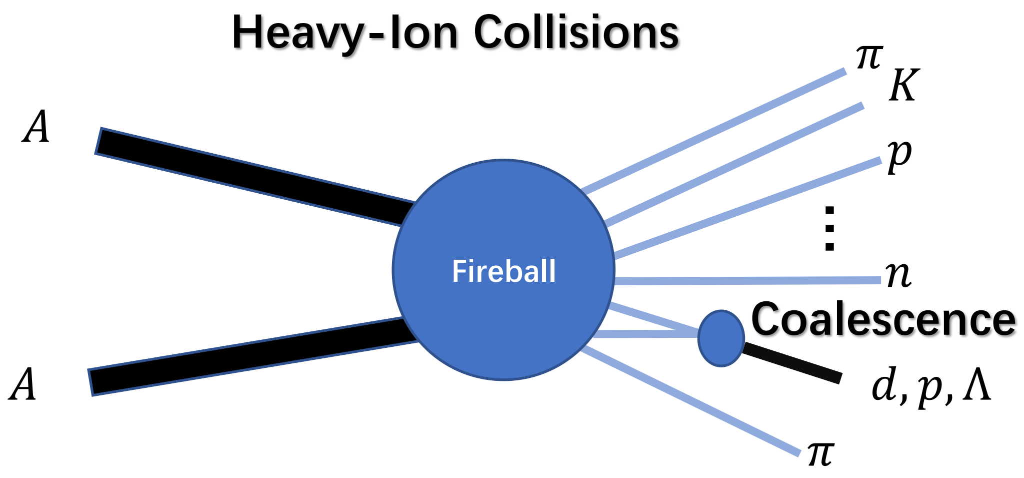
Coalescence is widely used model





Anti Light nuclei as Indirect detection of Dark Matter

See N.Fornengo et al., JCAP 09 (2013) 031 for review

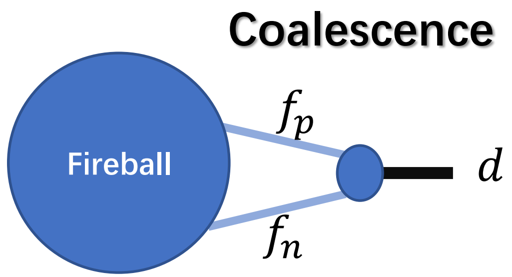
Coalescence in Heavy-Ion Collisions

- quark + quark -> hardon
- S quark -> Lambda polarization
- nucleon + nucleon -> light nuclei

R.J.Fries et al., PRC 68.044902
L.-W.Chen et al., PRC 68.017601
X.-L. Sheng et al., PRD 102. 056013

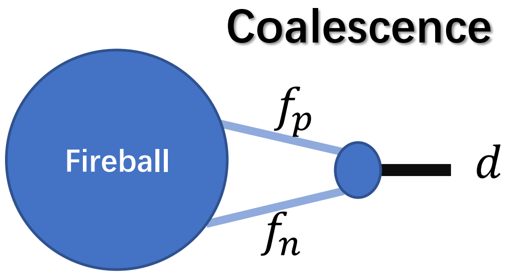
Coalescence model

$$N_A = g_A \int \left[\prod_{i}^{A} d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i, \boldsymbol{p}_i) \right] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^{A})$$



Coalescence model

$N_A = g_A \int$ [Phase-space distribution] [Wigner function]



- Two ingredients in Coalescence model:
  - Constituent particle distribution
  - Wigner function (probability to produce the light nuclei):
    Only depends on the relative distance in phase space $x_p - x_n$
    NOT $(x_p + x_n)/2$

Phase-space distribution in coalescence

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

Example 1: Gaussian

$$f(\boldsymbol{r},\boldsymbol{p}) = \frac{\rho_0}{(2\pi mT)^{3/2}} \exp\left(-\frac{\boldsymbol{r}^2}{2R_s^2}\right) \exp\left(-\frac{\boldsymbol{p}^2}{2mT}\right)$$

$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i,\boldsymbol{p}_i)\right] W_A(\{\boldsymbol{r}_i,\boldsymbol{p}_i\}_{i=1}^A)$$

$$N_d = g_d N_p^2 \left[\left(R_s^2 + \frac{\sigma_d^2}{2}\right)\left(mT + \frac{1}{2\sigma_d^2}\right)\right]^{-\frac{3}{2}}$$

$$N_t = g_t N_p^3 \left[\left(R_s^2 + \frac{\sigma_t^2}{2}\right)\left(mT + \frac{1}{2\sigma_t^2}\right)\right]^{-3}.$$

If the size difference is negligible ($\sigma_d = 2.26, \sigma_t = 1.59$fm):

$$\frac{N_t N_p}{N_d^2} = \frac{g_t}{g_d^2} = \frac{4}{9}$$

The effects from Gaussian profile exactly cancels

Phase-space distribution in coalescence

Example 1: Gaussian

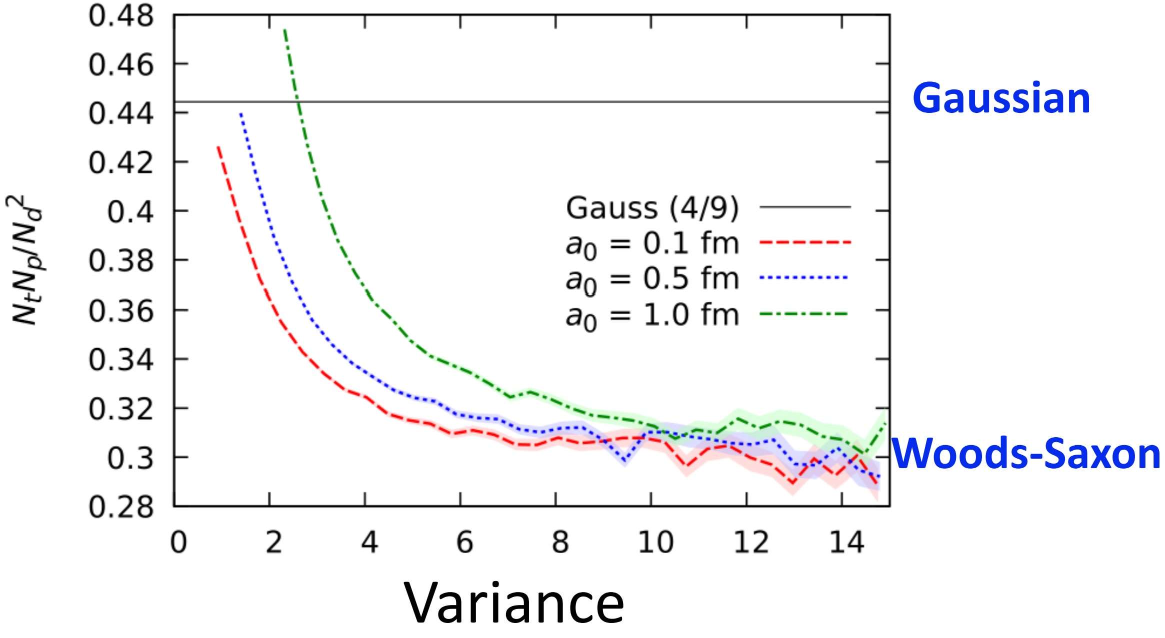
$$f(\boldsymbol{r},\boldsymbol{p}) = \frac{\rho_0}{(2\pi mT)^{3/2}} \exp\left(-\frac{\boldsymbol{r}^2}{2R_s^2}\right) \exp\left(-\frac{\boldsymbol{p}^2}{2mT}\right)$$

$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i,\boldsymbol{p}_i)\right] W_A(\{\boldsymbol{r}_i,\boldsymbol{p}_i\}_{i=1}^A)$$

Example 2: Woods-Saxon

$$f(\boldsymbol{r},\boldsymbol{p}) = \frac{\rho_{\mathrm{WS}}}{1+\exp\frac{r-R_0}{a_0}} \cdot \frac{1}{(2\pi mT)^{3/2}} \exp\left(-\frac{\boldsymbol{p}^2}{2mT}\right)$$

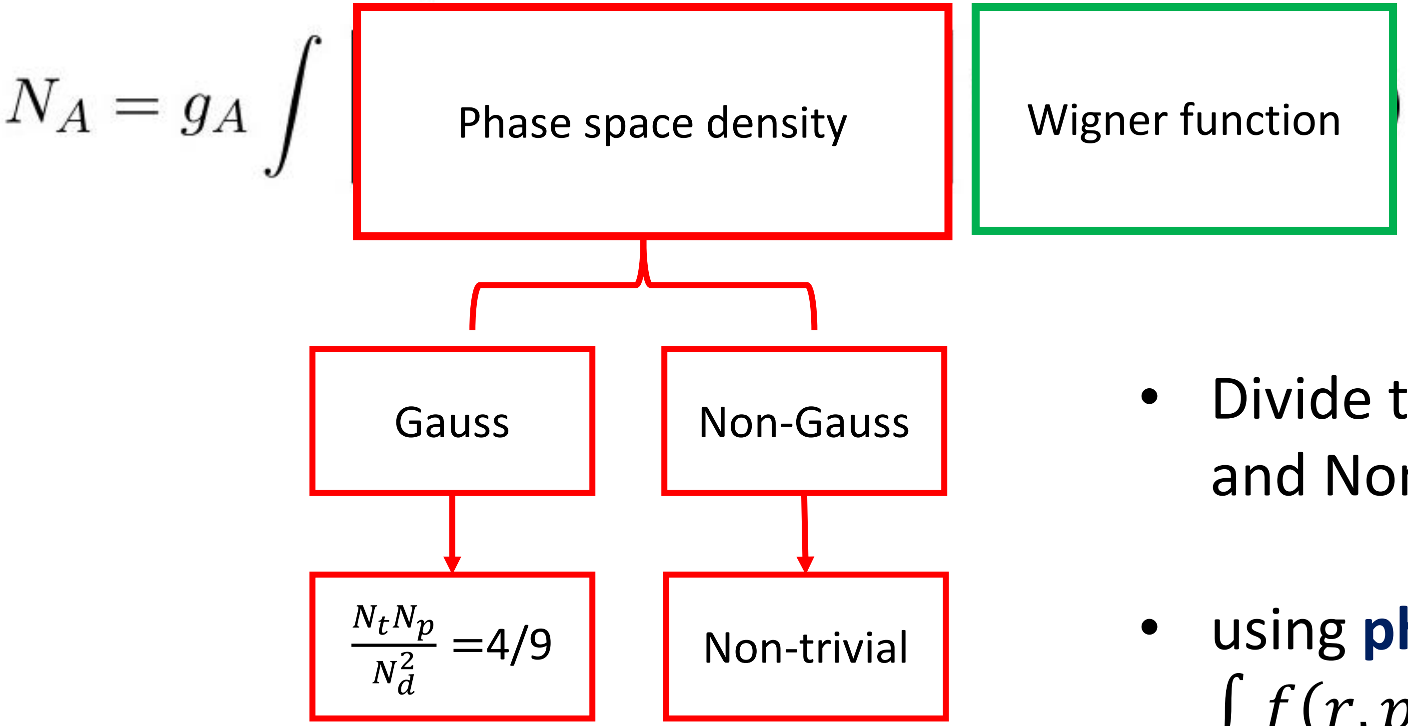
SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905



Non-trivial effects from the non-Gaussian component

Light-nuclei yield

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905



- Divide the distribution into Gaussian and Non-Gaussian component

- using **phase-space cumulant** $\langle r^n p^m \rangle \sim \int f(r,p) r^n p^m$

$f(r,p)$ expressed with characteristic function

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i, \boldsymbol{p}_i) \right] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$

$N_d: A = 2$
$N_t: A = 3$
$N_{4He}: A = 4$

$$\frac{f(\boldsymbol{z}_i)}{N_p} = \rho(\boldsymbol{z}_i) = \int \frac{d^6\boldsymbol{t}_i}{(2\pi)^6} e^{-\mathrm{i}\boldsymbol{t}_i\cdot\boldsymbol{z}_i} \exp\left[\sum_{\boldsymbol{\alpha}\in\mathbb{N}_0^6} \frac{\mathcal{C}_{\boldsymbol{\alpha}}}{\boldsymbol{\alpha}!} (\mathrm{i}\boldsymbol{t}_i)^{\boldsymbol{\alpha}} \right],$$

$\boldsymbol{z} \sim (\boldsymbol{r}, \boldsymbol{p})$

Phase-space cumulant

$$C_\alpha \sim \langle r^n p^m \rangle \sim \int f(r,p) r^n p^m$$

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(C_2^{hs} + cosnt.)}} \right]^{A-1} [1 + O(\{C_\alpha\}_{|\alpha|\geq 3})]$$

- This arises from the fact that Wigner function only depends on relative distance, and the Gaussian terms can be diagonalized.

N_d, N_t, N_{4He} have similar behavior in case of Gaussian phase-space density

Similar result with: R.Scheibl, U.Heinz, PRC 59, 1585; K.Blum, M.Takimoto,PRC 99,044913

Phase-space cumulant in light nuclei

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(C_2^{hs} + cosnt.)}}\right]^{A-1} [1 + O(\{C_\alpha\}_{|\alpha|\geq 3})]$$

2nd phase-space cumulant

$$C_2 = 2\begin{pmatrix} \#\langle \boldsymbol{r}\boldsymbol{r}^T\rangle & \langle \boldsymbol{r}\boldsymbol{p}^T\rangle \\ \langle \boldsymbol{p}\boldsymbol{r}^T\rangle & \#\langle \boldsymbol{p}\boldsymbol{p}^T\rangle \end{pmatrix}$$

$$\sim 2\begin{pmatrix} \#R_{fireball}^2 & \#l_{homoge} \\ \#l_{homoge} & \#T_{fo} \end{pmatrix}$$

Phase-space cumulant $\langle r^n p^m\rangle \sim \int f(\boldsymbol{r},\boldsymbol{p}) r^n p^m$

$\langle \boldsymbol{r}\boldsymbol{r}^T\rangle$: Geometric property of the profile

$\langle \boldsymbol{p}\boldsymbol{p}^T\rangle$: Thermal property of the profile

$\langle \boldsymbol{r}\boldsymbol{p}^T\rangle$: Dynamical property from the expansion

Relevant scales in light-nuclei yield N_A : Fireball size $R_{fireball}$, homogeneity length l_{homoge} and freeze-out temperature T_{fo}

Example: Anisotropic flow (Blast-Wave)

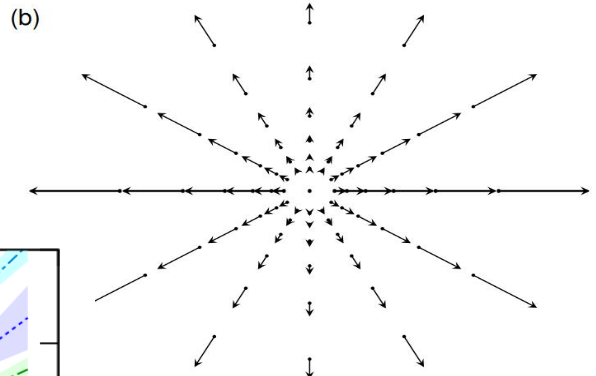
SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

phase-space distribution

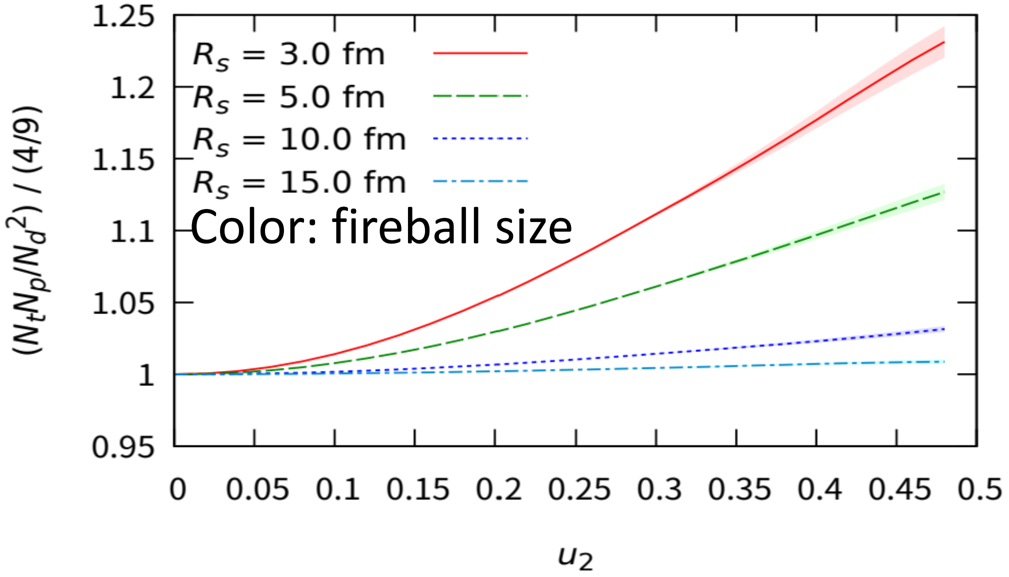
Blast-wave flow P.Huovinen et al, PLB 503, 58(2001)

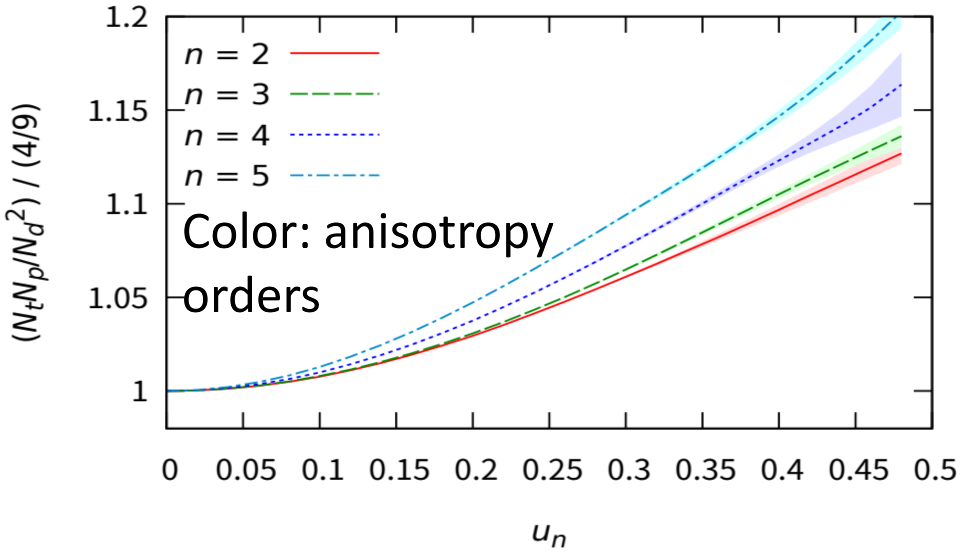
Anisotropy param.

$$f(\boldsymbol{r},\boldsymbol{p}) = \frac{\rho_0}{(2\pi m T)^{3/2}} e^{-\frac{r^2}{2R_s^2}} \exp\left(-\frac{m}{2T}\left[\frac{\boldsymbol{p}}{m} - \boldsymbol{v}(\boldsymbol{r})\right]^2\right) \qquad \boldsymbol{v}(\boldsymbol{r}) = \frac{1}{R_s}(r_x, r_y, 0)^{\mathrm{T}}(1 + 2u_2 \cos 2\phi_s)$$



Profile for n=2





Anisotropic effects is negligible when fireball size is large

Momentum anisotropy increase the ratio

Light-nuclei yield in a single event

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i, \boldsymbol{p}_i) \right] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(C_2^{hs} + cosnt.)}} \right]^{A-1} [1 + O(\{C_\alpha\}_{|\alpha|\geq 3})]$$

N_d, N_t, N_{4He} have similar behavior in case of Gaussian phase-space density

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Light-Nuclei Yield with E-by-E Fluctuations

K.Murase, **SW,** In preparation

Single Event

- Non-trivial background distribution in a single event

$$f(\boldsymbol{r},\boldsymbol{p}) = f_0(\boldsymbol{r},\boldsymbol{p})$$

Assumption: No(Critical) fluctuations

Event-by-Event

$$f^{(1)}(\boldsymbol{r},\boldsymbol{p}), f^{(2)}(\boldsymbol{r},\boldsymbol{p}), f^{(3)}(\boldsymbol{r},\boldsymbol{p}) \cdots$$

$$N_A^{(1)}, \quad N_A^{(2)}, \quad N_A^{(3)} \cdots \rightarrow \langle N_A \rangle$$

Event average

- Initial fluctuations
- Hydro fluctuations
- Fluctuations induced by hard process
- etc..

Light-Nuclei Yield with E-by-E Fluctuations

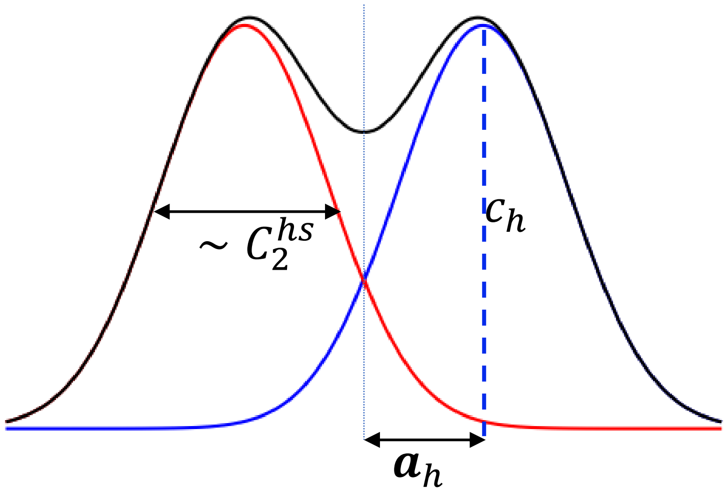
K.Murase, **SW**, In preparation

Light Nuclei in Single Event

$$N_A^f(\{c_h, \boldsymbol{a}_h\}) = g_A \int [\prod_i^A d^6 \boldsymbol{z}_i f(\boldsymbol{z}_i)_{\{c_{h_i}, \boldsymbol{a}_{h_i}\}}] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$

Fluctuating Gaussian Profile

$$f(\boldsymbol{z})_{\{c_h, \boldsymbol{a}_h\}} = \frac{1}{n}\sum_{h=1}^{n} c_h \frac{1}{\sqrt{\det(2\pi \mathcal{C}_2^{hs})}} \exp[-\frac{1}{2}(\boldsymbol{z}-\boldsymbol{a}_h)^T (\mathcal{C}_2^{hs})^{-1}(\boldsymbol{z}-\boldsymbol{a}_h)]$$



Light Nuclei in Event-Averaged

$$N_A = \int [\prod_{h=1}^{n} dc_h d^6 \boldsymbol{a}_h] \Pr(\{c_h, \boldsymbol{a}_h\}) N_A^f(\{c_h, \boldsymbol{a}_h\})$$

Distribution of Hot Spots

$$\Pr(\{c_h, \boldsymbol{a}_h\}) = \prod_{h=1}^{n} p(c_h) \frac{\exp[-\frac{1}{2}\boldsymbol{a}_h^T (\mathcal{C}_2^{hc})^{-1} \boldsymbol{a}_h]}{\sqrt{\det(2\pi \mathcal{C}_2^{hc})}}$$

$p(c_h)$: Distribution of hot spot magnitude c_h

C_2^{hc}: Covariance of hot-spot centers

Result: Light-Nuclei Yield

K.Murase, **SW,** In preparation

Yield(Event averaged)

$$N_A = g_A 8^{A-1} \det(C_2^{\mathrm{hs}} + 1)^{-(A-1)/2} \times \sum_{\substack{m_1 \geq \cdots \geq m_n \geq 0 \\ \sum_{h=1}^{n} m_h = A}} \frac{n! A!}{n^A S_{m_1,\ldots,m_n}} \frac{I_{m_1} \cdots I_{m_n}}{\prod_\lambda \left(\sum_{h=1}^{n} \frac{m_h}{A} \frac{1}{1+m_h \lambda} \right)^{1/2}},$$

$I_m \sim \int dc\, p(c) c^m$: Moment of profile $f(\boldsymbol{r}, \boldsymbol{p})$ magnitude

λ: eigenvalues of $C_2^{hc}\left(C_2^{hs} + 1\right)^{-1}$ for profile center

$S_{m_1,\cdots m_n}$: symmetry factor

Result: Light-Nuclei Yield

K.Murase, **SW,** In preparation

Yield Ratio [A=2(deuteron)&A=3(triton)]

$$\frac{N_t N_p}{N_d^2} = \frac{g_t}{g_d^2} \frac{\langle c^3 \rangle + \frac{3(n-1)\langle c^2 \rangle \langle c \rangle^2}{\prod_\lambda (1+\frac{4}{3}\lambda)^{1/2}} + \frac{(n-1)(n-2)\langle c \rangle^3}{\prod_\lambda (1+\lambda)}}{\left[\langle c^2 \rangle + \frac{(n-1)\langle c \rangle^2}{\prod_\lambda (1+\lambda)^{1/2}}\right]^2}.$$

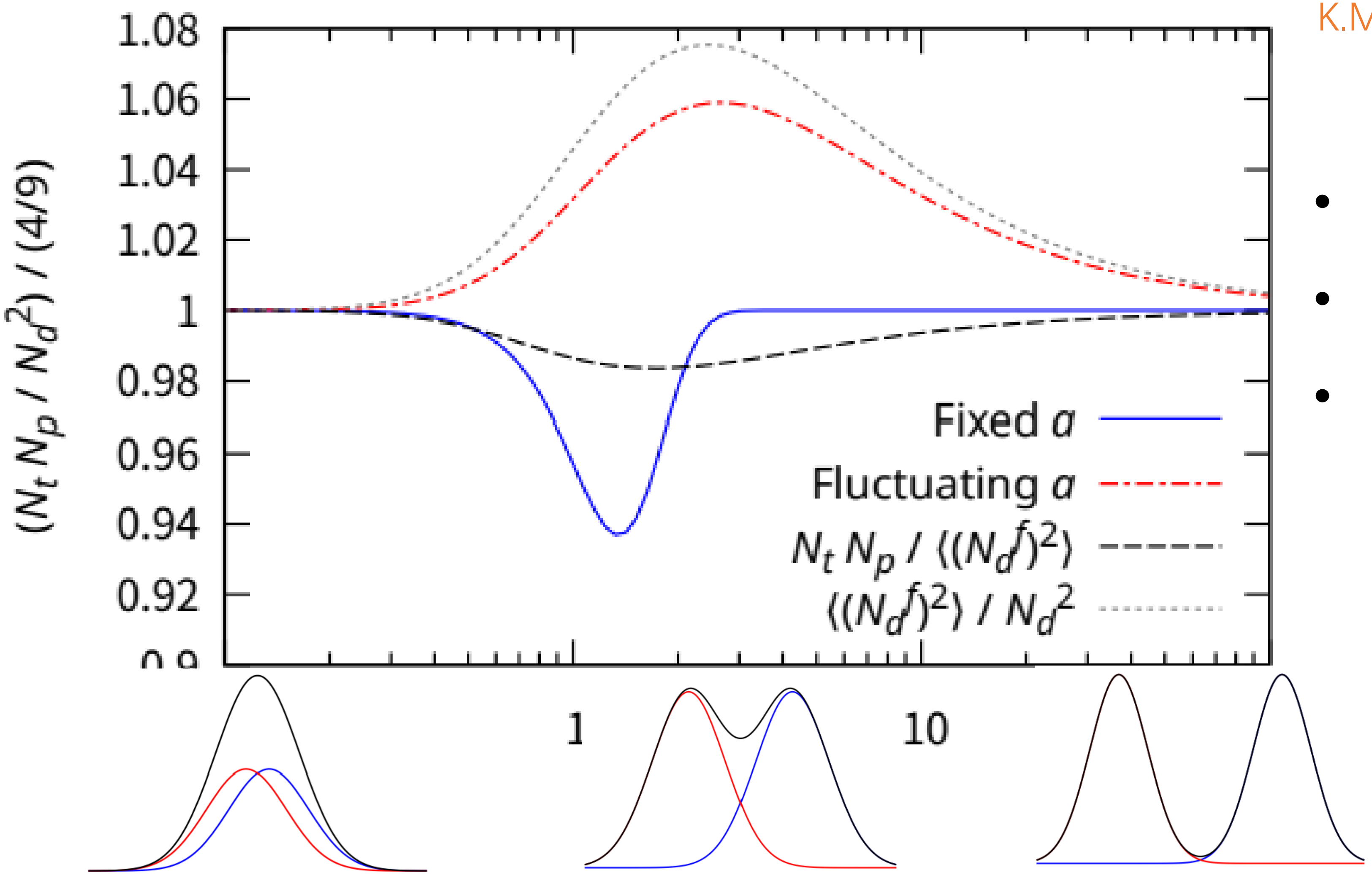
$\langle c^n \rangle \sim \int dc\, p(c) c^n$: Moment of profile $f(\boldsymbol{r}, \boldsymbol{p})$ magnitude

λ: eigenvalues of $C_2^{hc}\left(C_2^{hs} + 1\right)^{-1}$ profile center

g_t, g_d: statistic factor

Double-Gaussian(n=2) in 1d

K.Murase, **SW,** In preparation



- Small a ~ 1 Gaussian
- Large a ~ sum of Gaussian
- Intermediate a ⇒ Non-Gaussian

$$\frac{N_t N_p}{N_d^2} = \frac{N_t N_p}{\langle (N_d^f)^2 \rangle_f} \frac{\langle (N_d^f)^2 \rangle_f}{N_d^2}$$

$$= \frac{N_t N_p}{\langle (N_d^f)^2 \rangle_f} \left[1 + \frac{\langle (N_d^f - N_d)^2 \rangle_f}{N_d^2} \right]$$

Position fluctuations increase the ratio

Double-Gaussian(n=2) in 1d

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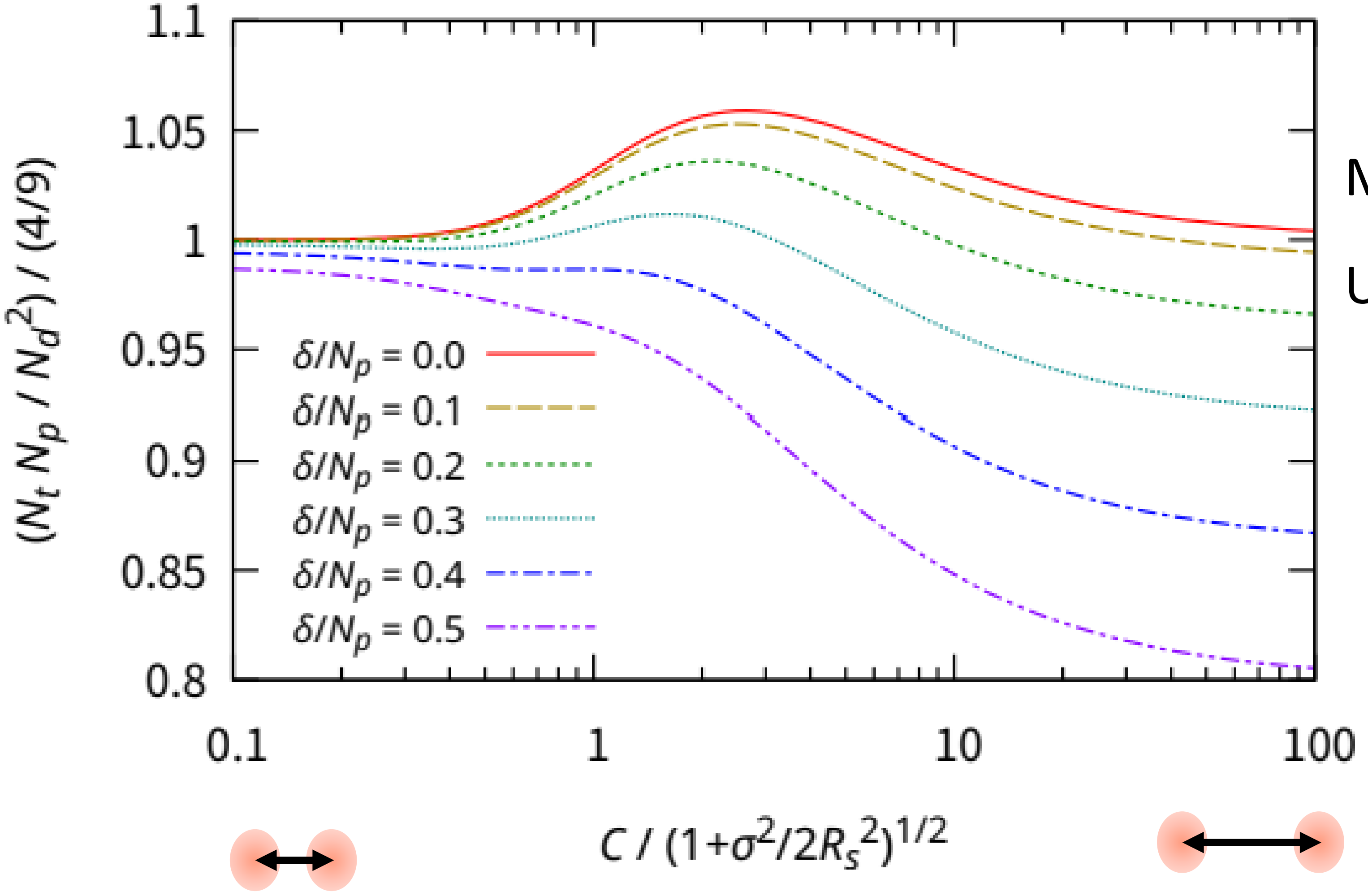
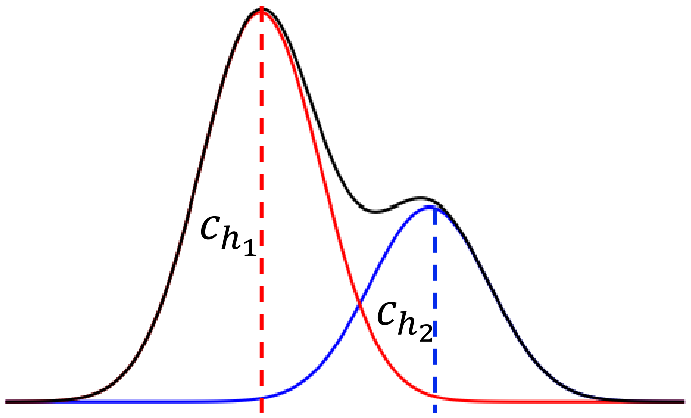


Magnitude fluctuations with c_h:

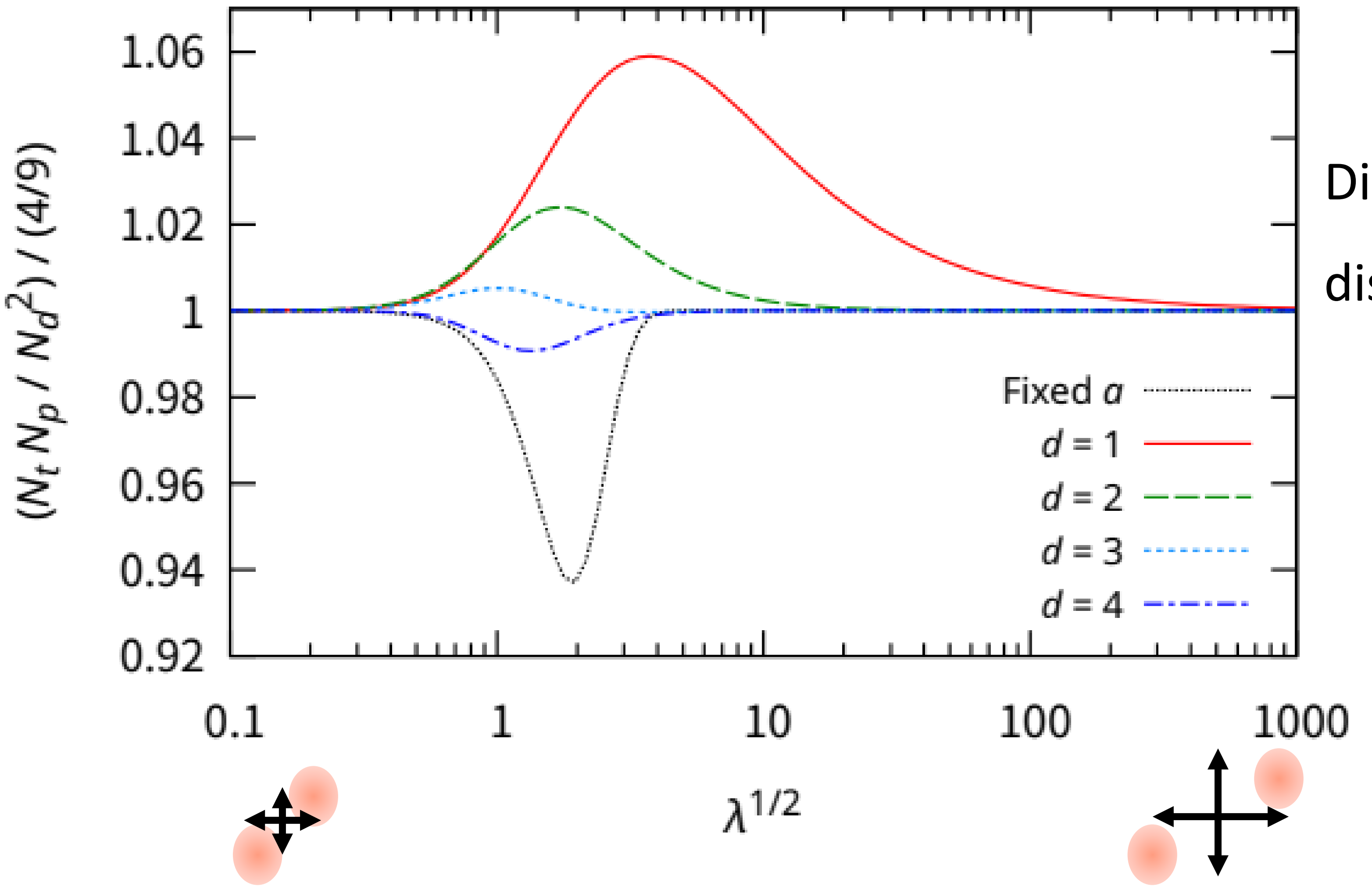
Uniform distribution:$[N_p - \delta, N_p + \delta]$



Magnitude fluctuations decrease the ratio

Double-Gaussian(n=2) in multi-dimensional

K.Murase, **SW,** In preparation

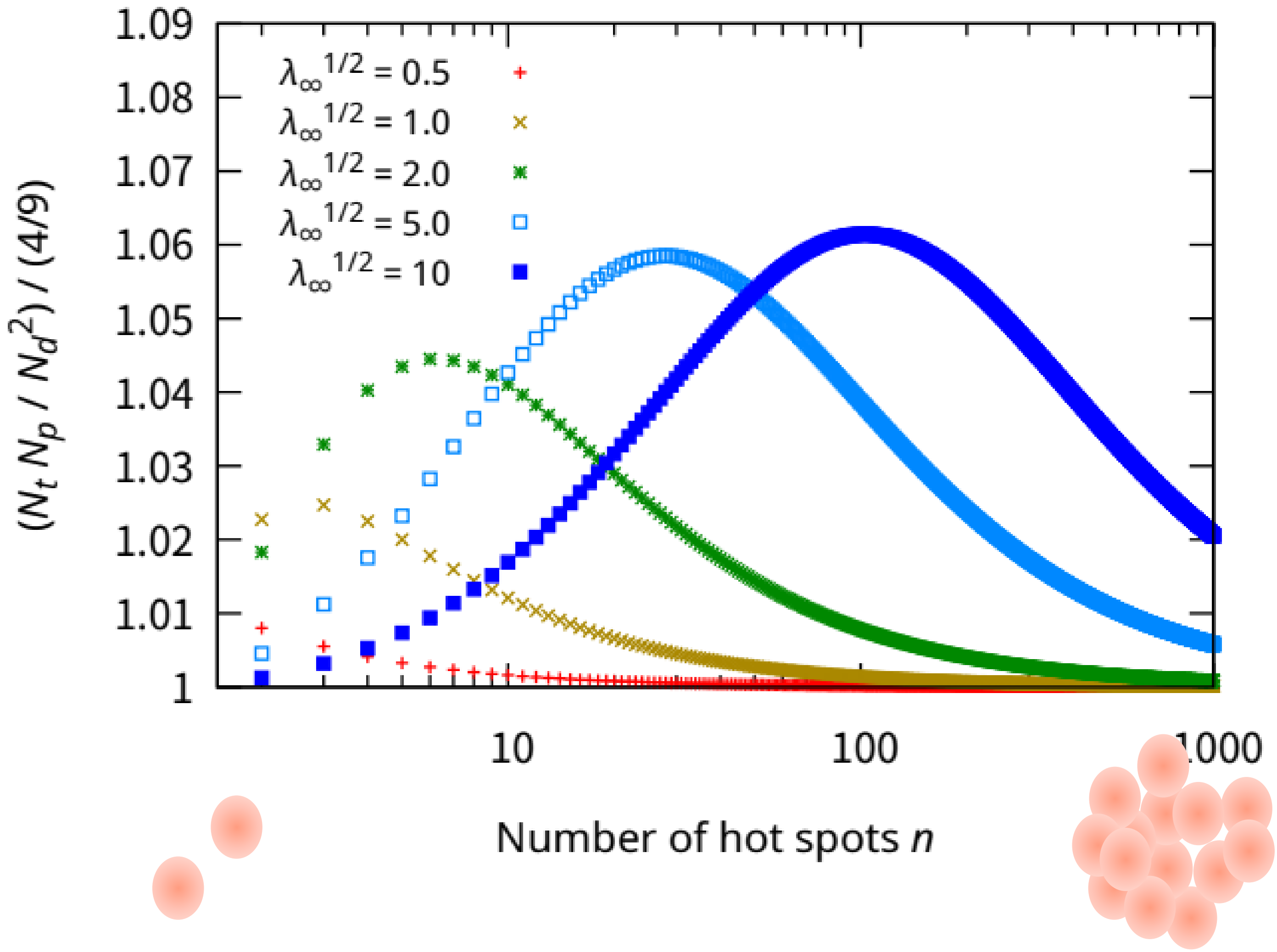


Dimensionality of the fluctuating distance: a

The increase from the fluctuating distance depends on the dimensionality

$N_t N_p / N_d^2$ with n Gaussian

K.Murase, **SW,** In preparation



Large Number of hot spots ⇒

Fluctuations are smeared out

A peak at a certain hot-spot number “n”

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Critical fluctuations δf in light nuclei

SW, K.Murase, S.Zhao, H.Song, in preparation

Introduce critical fluctuations δf

$$N_A \sim \langle (f_0 + \delta f)^A \rangle_\sigma \sim f_0^A + \langle (\delta f)^2 \rangle_\sigma^{\beta_2} + \langle (\delta f)^3 \rangle_\sigma^{\beta_3} + \langle (\delta f)^4 \rangle_\sigma^{\beta_4} + \cdots$$

Bkg | 2-point correlator | 3-point correlator | 4-point correlator

- N_A: includes contribution from $\mathbf{2, 3}, \ldots \boldsymbol{A}$**-point critical correlator**
- **Contribution hierarchy:** $f_0^A \gg \langle (\delta f)^2 \rangle_\sigma^{\beta_2} \gg \langle (\delta f)^3 \rangle_\sigma^{\beta_3} \gg \cdots \gg \langle (\delta f)^A \rangle_\sigma^{\beta_A}$

Light nuclei yield with critical fluctuations

SW, K.Murase, S.Zhao, H.Song, in preparation

$$N_A = g_A \int$$ [Phase space density: Background+Critical] [Wigner func]

Scales of R, l

Scale of ξ

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(C_2 + cosnt.)}}\right]^{A-1} [1 + \#(2pt.)^{\#} + \#(3pt.)^{\#} + \#(4pt.)^{\#}]$$

Bkg **Cri**

N_A share a analogous structure $N_A \propto [\ldots]^{A-1}[Bkg + Cri]$ => Construct ratios of N_A suppress Bkg and highlight Cri

$\widetilde{\boldsymbol{R}}(\boldsymbol{A}, \boldsymbol{B})$ =Ratio(N_t, N_d)-statistical factor $\sim \mathcal{O}(\xi)$

$\widetilde{\boldsymbol{R}}(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$ =Ratio(N_t, N_d)-#Ratio(N_t, N_d, N_{4He}) $\sim \mathcal{O}(\xi)$

Example: near critical regime

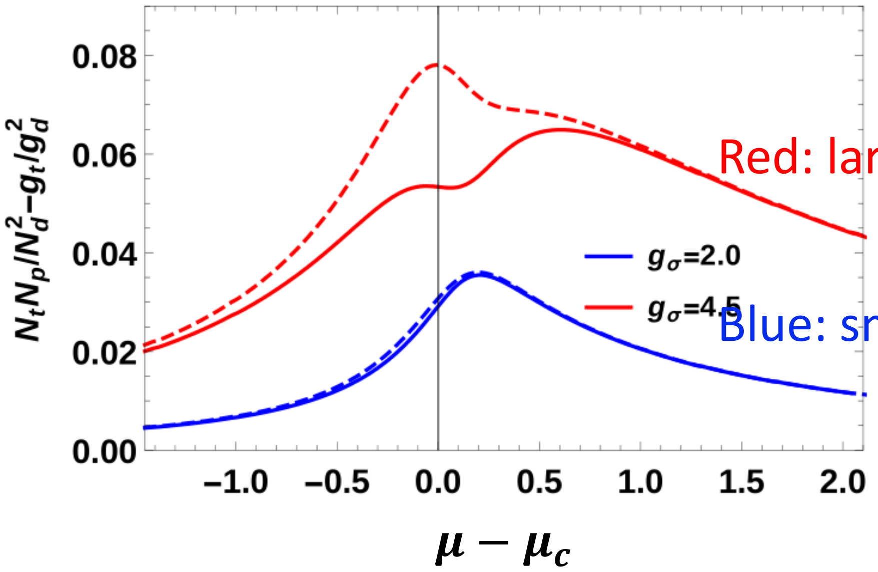
SW, K.Murase, S.Zhao, H.Song, in preparation

Ratio 1

Ratio(N_t, N_d)-statistical factor
~2pt - 3pt - (2pt)^2

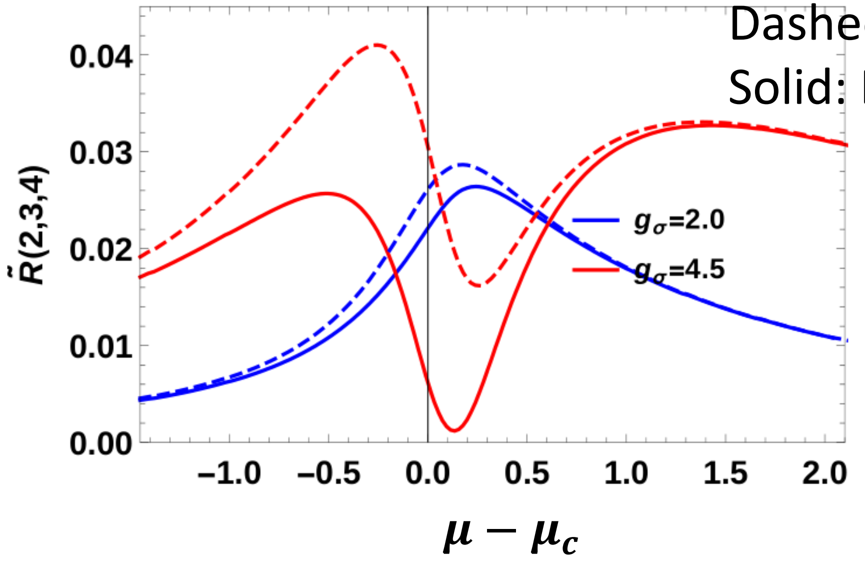
Ratio 2

Ratio(N_t, N_d)-Ratio(N_t, N_d, N_{4He})
~2pt -4 (2pt)^2



Red: large critical signal

Blue: small critical signal



Dashed: Above T_c
Solid: Below T_c

Light nuclei ratios have a peak near critical point μ_c, also have double peak because of (2pt.)^2. when the critical effect is large

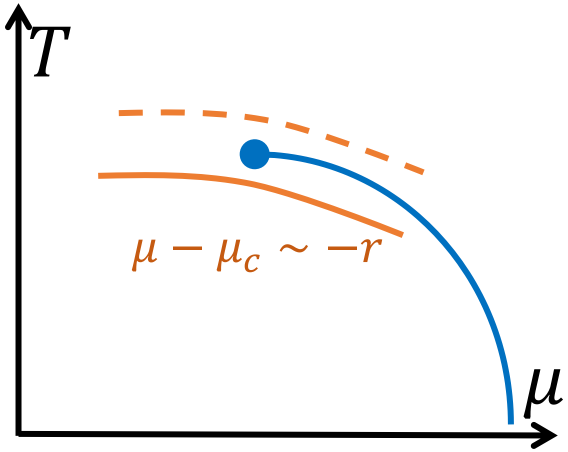
Conclusion and Outlook

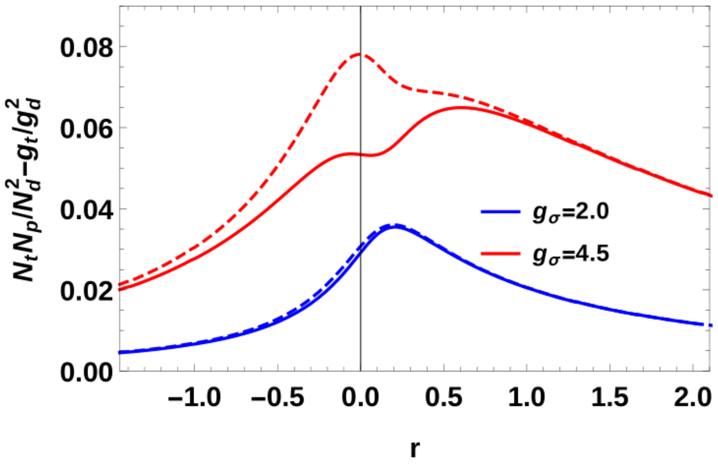
- N_d, N_t, N_{4He} depends on fireball size, homogeneity length, freeze temperature in analogous way when nucleon distribution close to Gaussian, because Wigner function depends on relative distance
- Construct the ratios to suppress the background effects
- Event-by-event profile fluctuations increase the yield ratio
- Long range correlation results a peak, and the square of 2-point correlation induces a double peak

Backup

Example: in the Ising critical regime

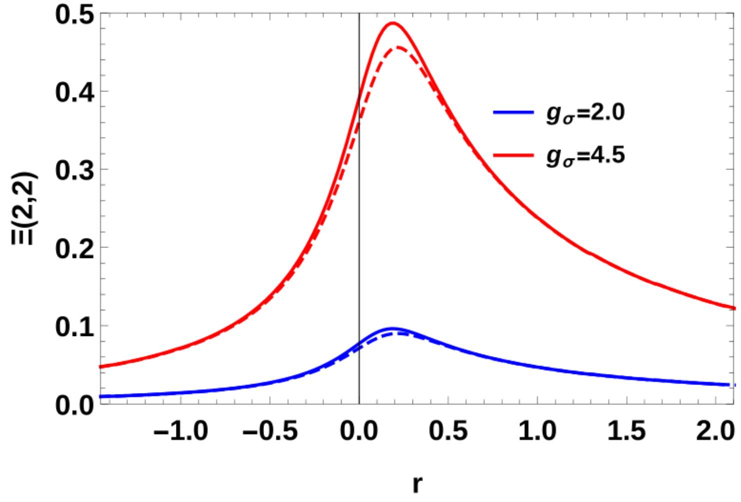
SW, K.Murase, S.Zhao, H.Song, in preparation

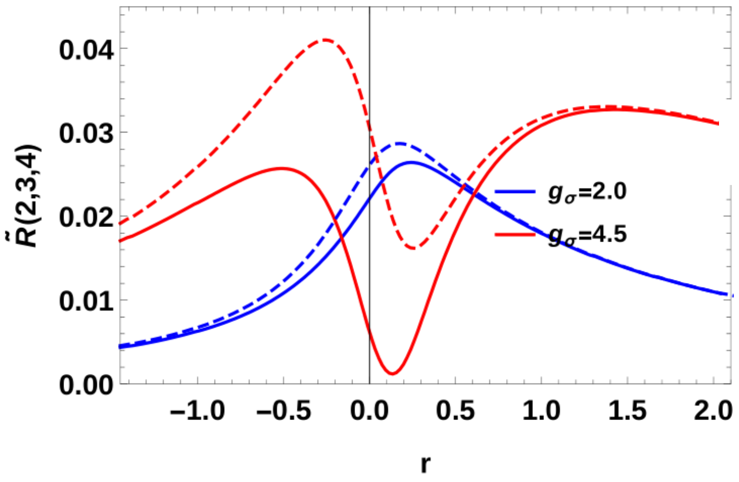




Ratio(N_t, N_d)-statistical factor

~2pt - 3pt - (2pt)^2





Ratio(N_t, N_d)-Ratio(N_t, N_d, N_{4He})

~2pt -4 (2pt)^2