

# INTERPLAY BETWEEN QUANTUM AND CLASSICAL CORRECTIONS TO $\hat{q}$

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BASED ON [2207.08842](#) WITH JACOPO GHIGLIERI



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INT WORKSHOP: PROBING QCD AT HIGH ENERGY AND DENSITY WITH JETS



1 Introduction

2 Classical Corrections

3 Quantum Corrections – Background

4 Quantum Corrections in a weakly coupled QGP

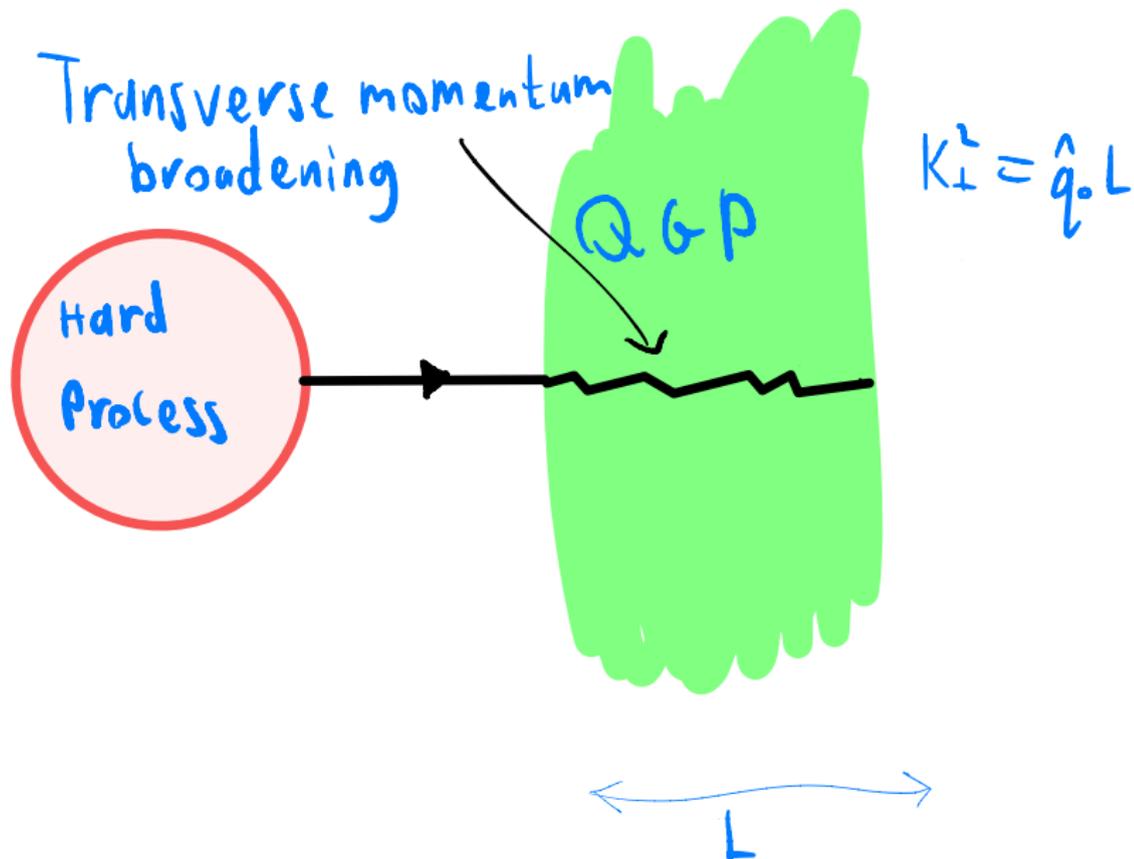
1 Introduction

2 Classical Corrections

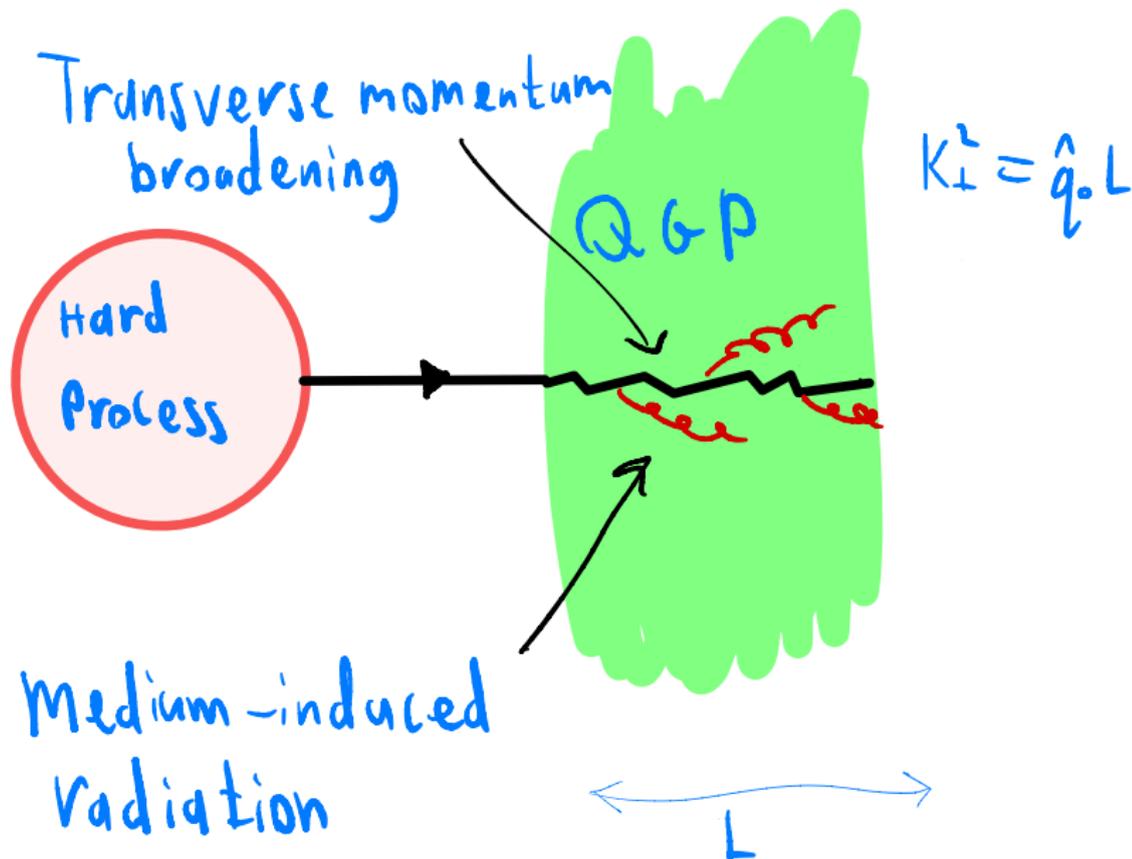
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# JET ENERGY LOSS



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- $\hat{q}$  has been extracted from:
  - ▶ Experimental data [Burke et al., 2014, Han et al., 2022]
  - ▶ Bayesian analysis [Cao et al., 2021, Xie et al., 2023]

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- Plays role of transport coefficient in kinetic description
- Controls in-medium shower in multiple scattering regime

# DEFINING $\hat{q}(\mu)$

- Can be related to the **transverse scattering rate,  $\mathcal{C}(k_{\perp})$**

$$\hat{q}(\mu) = \int^{\mu} \frac{d^2 k_{\perp}}{(2\pi)^2} k_{\perp}^2 \mathcal{C}(k_{\perp})$$

$$\lim_{L \rightarrow \infty} \langle W(x_{\perp}) \rangle = \exp(-\mathcal{C}(x_{\perp})L)$$

- $W(x_{\perp})$  is a Wilson loop defined in the  $(x^+, x_{\perp})$  plane

[Casalderrey-Solana and Teaney, 2007, D'Eramo et al., 2011, Benzke et al., 2013]

- Can compute perturbatively for the case of a **weakly coupled QGP**



[Ghiglieri and Teaney, 2015]

# THERMAL SCALES IN A WEAKLY COUPLED QGP

- $T$ , **hard scale** associated with energy of individual particles  
⇒ hard-hard interactions can be described perturbatively
- $gT$ , **soft scale** associated with energy of collective excitations  
⇒ soft-soft interactions can also be described perturbatively
- $g^2T$ , **ultrasoft scale** is associated with nonperturbative physics  
⇒ loops can be added at no extra cost (Linde problem)  
⇒ cannot use perturbation theory

# HTL EFFECTIVE THEORY

- For hard-soft interactions, we are not so lucky either...  
Turns out that one can add loops for free  
 $\implies$  perturbative expansion breaks down
- Hard Thermal Loop (HTL) effective theory comes to the rescue, allowing us to resum these loops



# SUMMARY OF PERTURBATIVE EXPANSION

At leading order,  $\hat{q}_0 \sim g^4 T^3$ :

- Contribution from **hard scale** [Arnold and Xiao, 2008]
- Contribution from **soft scale** [Aurenche et al., 2002]

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- **soft scale**
- **ultrasoft scale**
- Logarithmically enhanced **quantum corrections**

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## CLASSICAL CORRECTIONS – BACKGROUND

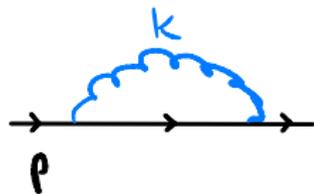
- Corrections coming from exchange of gluons between medium and parton that are  $\lesssim gT$

# CLASSICAL CORRECTIONS – BACKGROUND

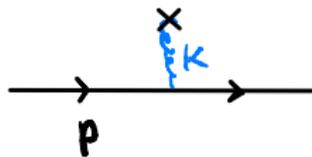
- Corrections coming from exchange of gluons between medium and parton that are  $\lesssim gT$

$$\Rightarrow n_B(\omega) \equiv \frac{1}{\exp\left(\frac{\omega}{T}\right) - 1} \gg 1$$

$\Rightarrow$  corrections are enhanced!



$$p \gg T$$

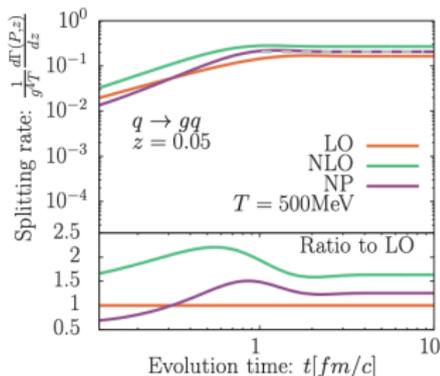


$$k \sim gT \quad k = (\omega, \vec{k})$$

- Can compute some of these classical corrections using **Hard Thermal Loop** (HTL) effective theory, but analytically difficult in practice
- Thanks to observation from [Caron-Huot, 2009], these classical corrections can be computed in **Electrostatic QCD** (EQCD)
- EQCD is a 3 dimensional theory of static modes
  - ⇒ Can be studied on the **lattice!**
  - ⇒ Paved way for **non-perturbative** (NP) determination of classical corrections to  $C(k_{\perp})!$

# NON-PERTURBATIVE MOMENTUM BROADENING

- Series of papers [Panero et al., 2014, Moore et al., 2021, Schlichting and Soudi, 2021], culminated with NP determination of in-medium splitting rate for medium of finite size
- Difference between rate from LO kernel and NP kernel can be up to 50%!



# OUTLINE

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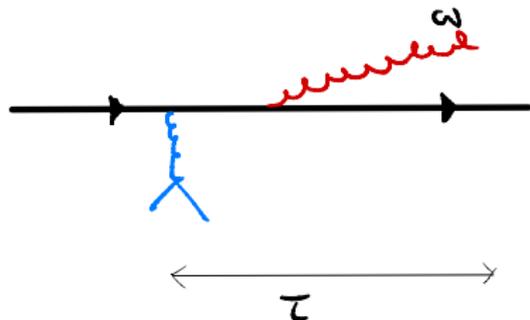
4 Quantum Corrections in a weakly coupled QGP

- $\mathcal{O}(g^2)$  corrections found to have double logarithmic  $\sim \ln^2(L/\tau_{\min})$  and single logarithmic enhancements by [Liou et al., 2013](LMW) and separately by [Blaizot et al., 2014](BDIM) for a medium with **static scattering centers**
- These are **radiative, quantum corrections**, coming from keeping track of the recoil during the medium-induced emission of a gluon

# PHYSICAL PICTURE

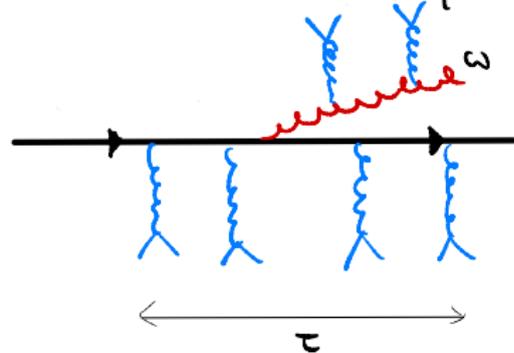
- LMW and BDIM argued that these **quantum corrections** come from the **single-scattering regime**
- formation time  $\tau = \omega/k_{\perp}^2$

single scattering



multiple scattering

$$k_{\perp}^2 \sim \hat{q} \cdot \tau$$
$$\Rightarrow \tau \sim \sqrt{\frac{\omega}{\hat{q}^2}}$$



- Resummation of double logs performed recently

[Caucal and Mehtar-Tani, 2022]

⇒ effectively renormalising  $\hat{q}(\mu)$

- Same double logs found in calculation of double gluon emission with overlapping formation times [Arnold et al., 2021]

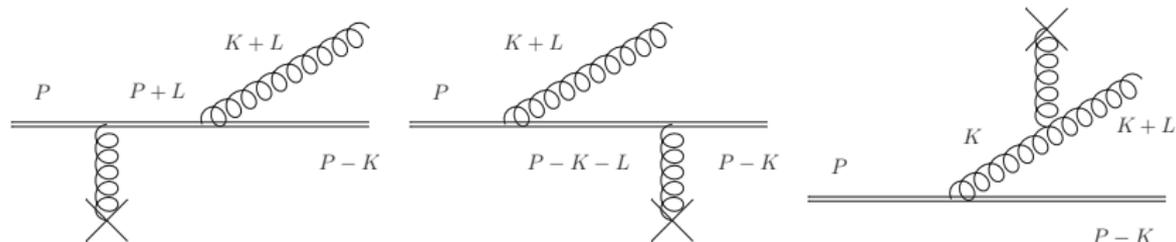
⇒ Universality

- Both **LMW** and **BDIM** calculations used the **Harmonic Oscillator Approximation (HOA)**, which is more well-suited to **multiple scattering regime**
- Both calculations also assume medium to be composed of **static scattering centers**
  - Not clear how phase spaces of classical  $\mathcal{O}(g)$  and quantum  $\mathcal{O}(g^2)$  corrections are connected

Which is larger:  $K\mathcal{O}(g)$  or  $\ln^2(\#)\mathcal{O}(g^2)$ ?

Hard to say... But can definitely make a start by revisiting computation of quantum corrections

# DOUBLE LOGS FROM THE LITERATURE



$N = 1$  term in opacity expansion emerges from dipole picture

$$\delta C(k_{\perp}, \rho)_{\text{LMW}} = 4\alpha_s C_R \int \frac{d\omega}{\omega} \int^{\rho} \frac{d^2 l_{\perp}}{(2\pi)^2} C_0(l_{\perp}) \frac{l_{\perp}^2}{k_{\perp}^2 (\mathbf{k}_{\perp} + \mathbf{l}_{\perp})^2} \quad (1)$$

## DOUBLE LOGS FROM THE LITERATURE

$$\delta C(\mathbf{k}_\perp, \rho)_{\text{LMW}} = 4\alpha_s C_R \int \frac{d\omega}{\omega} \int^\rho \frac{d^2 l_\perp}{(2\pi)^2} C_0(l_\perp) \frac{l_\perp^2}{k_\perp^2 (\mathbf{k}_\perp + \mathbf{l}_\perp)^2}$$

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  $|\mathbf{k}_{\perp} + \mathbf{l}_{\perp}| \gg l_{\perp} \Rightarrow$  Single Scattering

$$\delta C(k_{\perp}, \rho)_{\text{LMW}} = 4\alpha_s C_R \int \frac{d\omega}{\omega} \int^{\rho} \frac{d^2 l_{\perp}}{(2\pi)^2} C_0(l_{\perp}) \frac{l_{\perp}^2}{k_{\perp}^4}$$

## DOUBLE LOGS FROM THE LITERATURE

$$\delta\mathcal{C}(\mathbf{k}_\perp, \rho)_{\text{LMW}} = 4\alpha_s C_R \int \frac{d\omega}{\omega} \int^\rho \frac{d^2 l_\perp}{(2\pi)^2} C_0(l_\perp) \frac{l_\perp^2}{k_\perp^2 (\mathbf{k}_\perp + \mathbf{l}_\perp)^2}$$

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↓  $\hat{q}_0(\rho) \rightarrow \hat{q}_0 \Rightarrow$  HOA

$$\delta\mathcal{C}(\mathbf{k}_\perp)_{\text{LMW}} = 4\alpha_s C_R \hat{q}_0 \frac{1}{k_\perp^4} \int \frac{d\omega}{\omega}$$

Reminder:  $\hat{q}(\mu) = \int^\mu \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{C}(k_\perp)$

## DOUBLE LOGS FROM THE LITERATURE

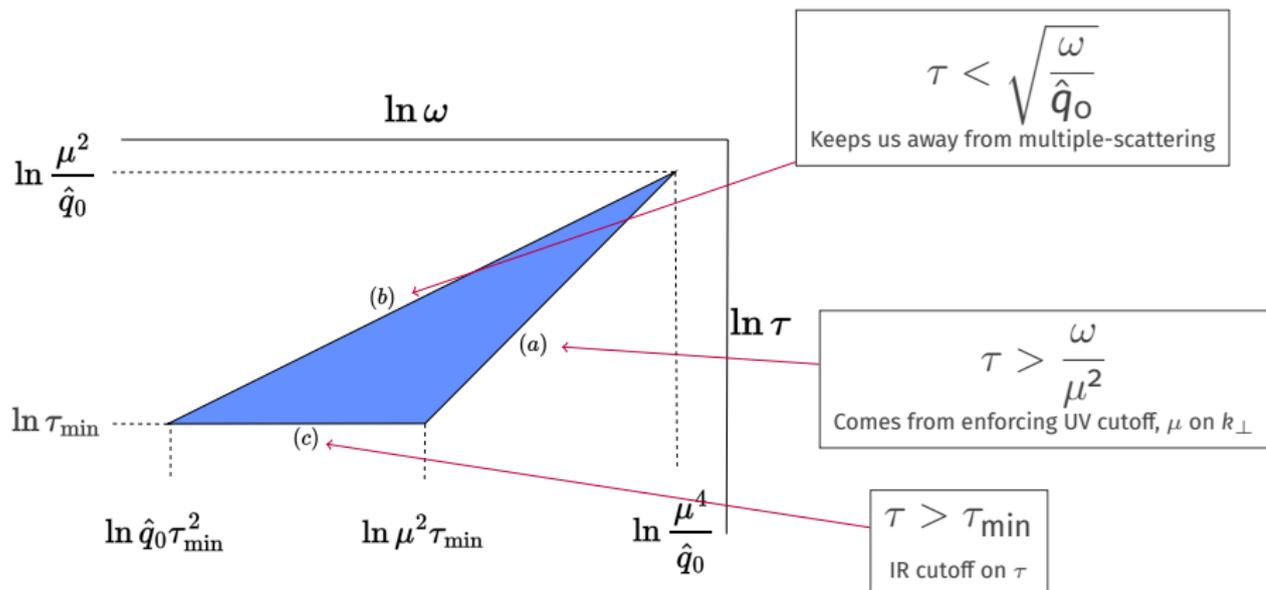
Convenient to work instead with formation time,  $\tau = \frac{\omega}{k_{\perp}^2}$

$$\delta \hat{q}_{\text{LMW}}(\mu) = 4\alpha_s C_R \hat{q}_0 \int^{\mu} \frac{d^2 k_{\perp}}{(2\pi)^2 k_{\perp}^2} \int \frac{d\omega}{\omega} = \frac{\alpha_s C_R}{\pi} \hat{q}_0 \int^{f(\mu)} \frac{d\tau}{\tau} \int \frac{d\omega}{\omega}$$

But what should limits be???

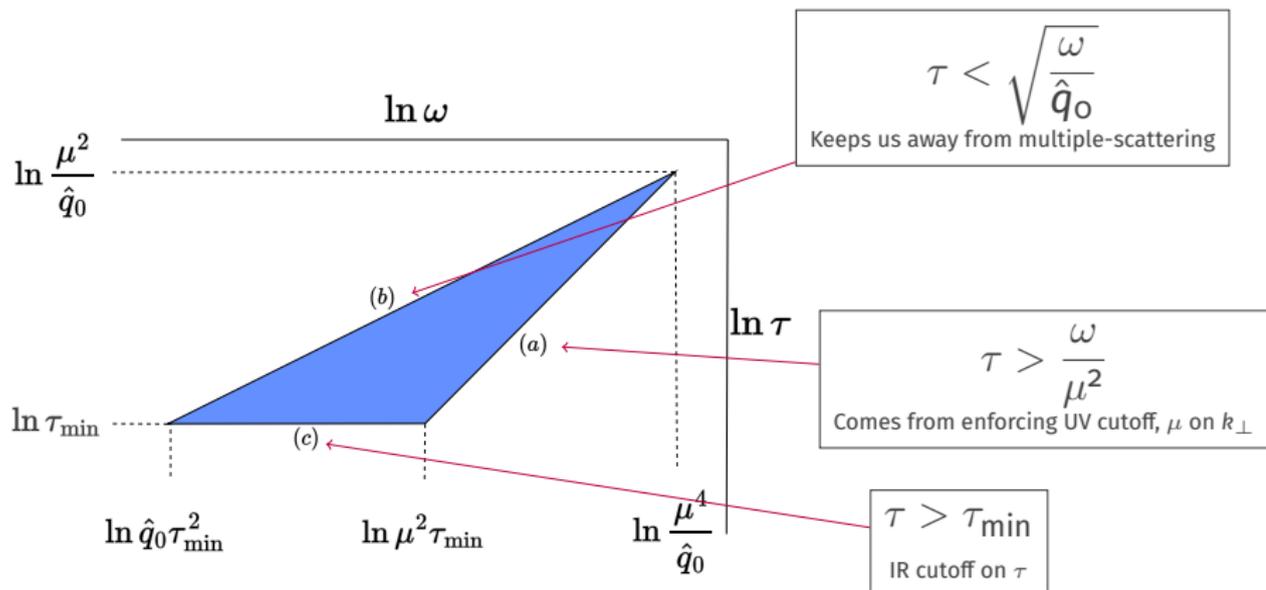
Will of course dictate size of logs!

# DOUBLE LOGS FROM THE LITERATURE



$$\delta \hat{q}_{\text{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi} \hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0 \tau^2}^{\mu^2 \tau} \frac{d\omega}{\omega} = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \ln^2 \frac{\mu^2}{\hat{q}_0 \tau_{\min}}$$

# DOUBLE LOGS FROM THE LITERATURE



$$\delta \hat{q}_{\text{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi} \hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0 \tau^2}^{\mu^2 \tau} \frac{d\omega}{\omega} \stackrel{\mu^2 = \hat{q}_0 L}{=} \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \ln^2 \frac{L}{\tau_{\min}}$$

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# FROM STATIC SCATTERING TO A WEAKLY COUPLED QGP

How to adapt BDIM/LMW result to weakly coupled QGP?

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$$\Rightarrow n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1} \text{ can be ignored iff } \omega_{\min} = \hat{q}_0 \tau_{\min}^2 \gg T$$

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But is this consistent with single scattering?

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$\hat{q}_0 \sim g^4 T^3$

Need to demand  $g^4 T^3 \tau_{\min}^2 \gg T$

$$\Rightarrow \tau_{\min} \text{ should be } \gg \frac{1}{g^2 T}$$

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Need to demand  $g^4 T^3 \tau_{\min}^2 \gg T$

$$\Rightarrow \tau_{\min} \text{ should be } \gg \frac{1}{g^2 T}$$

But  $\frac{1}{g^2 T}$  is the mean free time between **multiple scatterings!**

$\Rightarrow$  Would lead us away from **single scattering regime!**

# FROM STATIC SCATTERING TO A WEAKLY COUPLED QGP

⇒ In order to stay away from multiple scattering regime, must account for thermal effects

$$\delta \hat{q}_{\text{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi} \hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0 \tau^2}^{\mu^2 \tau} \frac{d\omega}{\omega}$$

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Introduce intermediate regulator

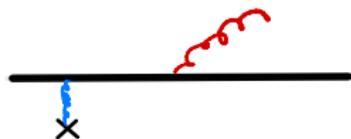
$$\tau_{\text{int}} \ll 1/g^2 T$$

$$\begin{aligned} \delta \hat{q}_{1+2}(\mu) &= \frac{\alpha_S C_R}{\pi} \hat{q}_0 \int_{\tau_{\text{int}}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0 \tau^2}^{\mu^2 \tau} \frac{d\omega}{\omega} (1 + 2n_B(\omega)) \\ &= \frac{\alpha_S C_R}{2\pi} \hat{q}_0 \left\{ \ln^2 \frac{\mu^2}{\hat{q}_0 \tau_{\text{int}}} - \frac{1}{2} \ln^2 \frac{\omega_T}{\hat{q}_0 \tau_{\text{int}}^2} \right\} \end{aligned}$$

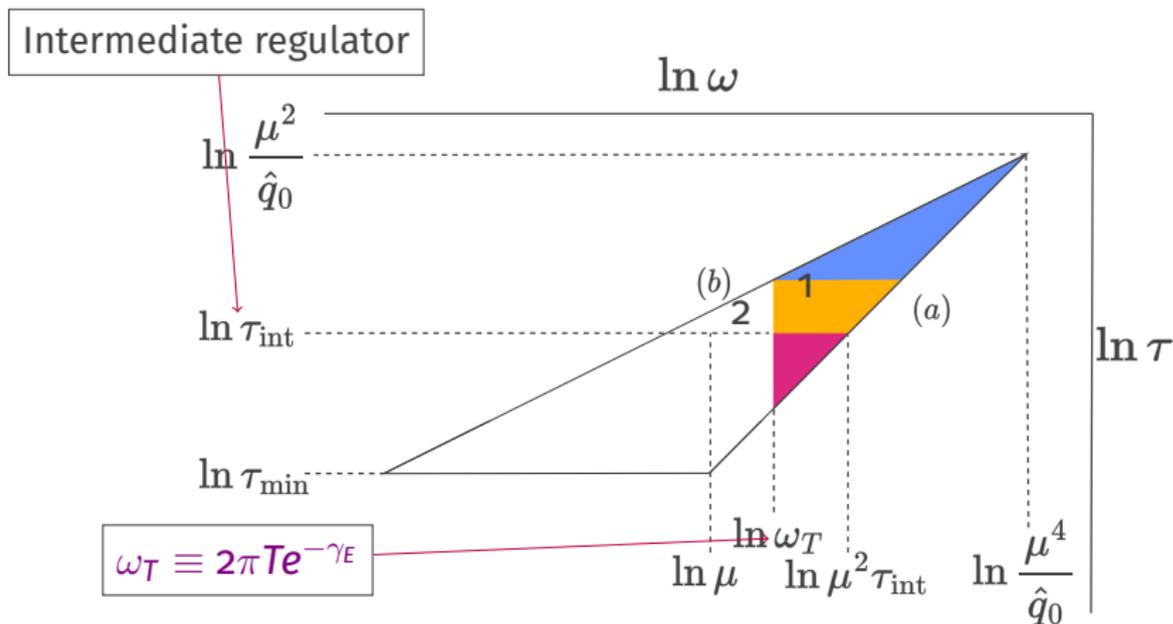
$$\omega_T \equiv 2\pi T e^{-\gamma_E}$$

# FROM STATIC SCATTERING TO A WEAKLY COUPLED QGP

$2n_B(\omega)$  accounts for stimulated emission and absorption of thermal gluons



# DOUBLE LOGS IN A WEAKLY COUPLED QGP



$$\delta \hat{q}_{1+2}(\mu) = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \left\{ \ln^2 \frac{\mu^2}{\hat{q}_0 \tau_{\text{int}}} - \frac{1}{2} \ln^2 \frac{\omega_T}{\hat{q}_0 \tau_{\text{int}}^2} \right\}$$

# STRICT SINGLE SCATTERING

- Investigate which logs are produced by *soft, collinear* modes through a *semi-collinear* process associated with formation time  $\tau_{\text{semi}} \sim 1/gT$  [Ghiglieri et al., 2013, Ghiglieri et al., 2016]

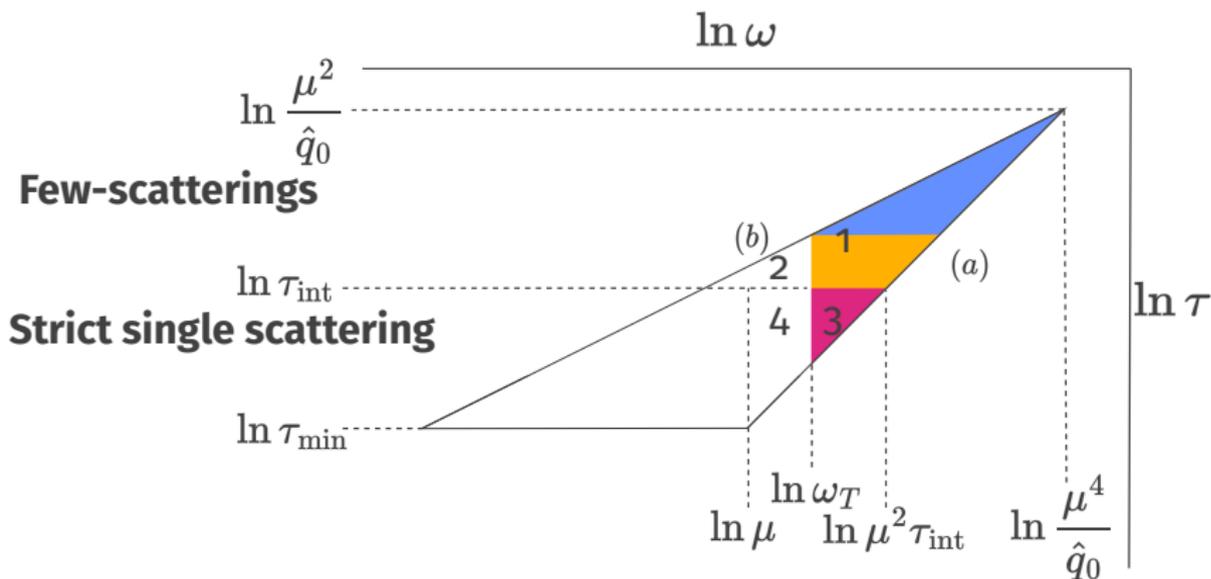


Only spacelike interactions with medium

Now timelike interactions are allowed too

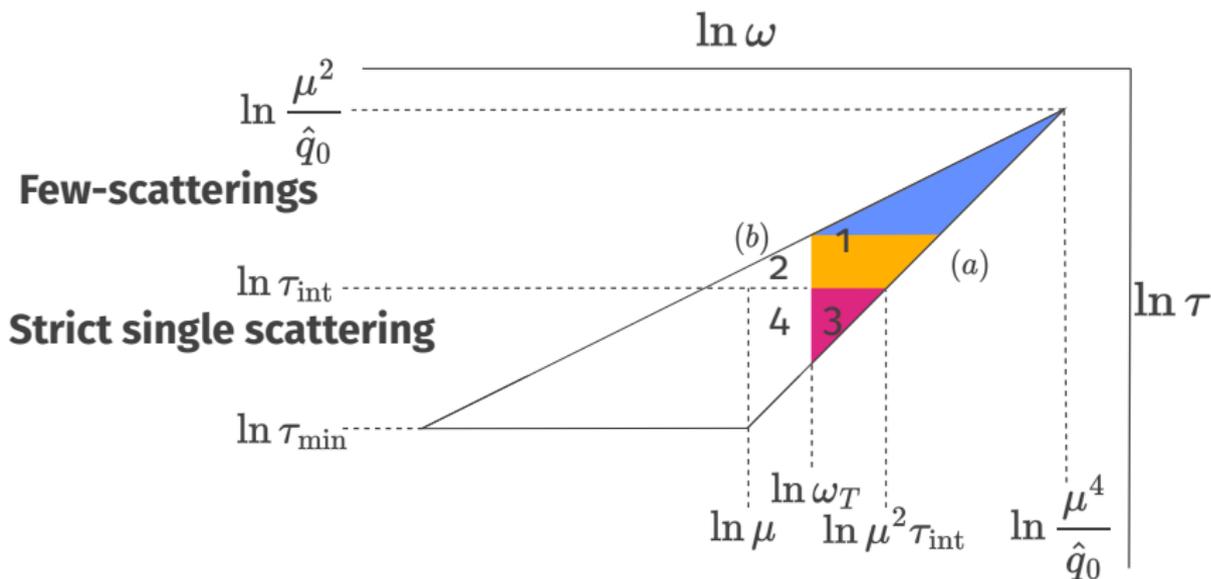
⇒ Going beyond instantaneous approximation!

# DOUBLE LOG WITH SINGLE SCATTERING



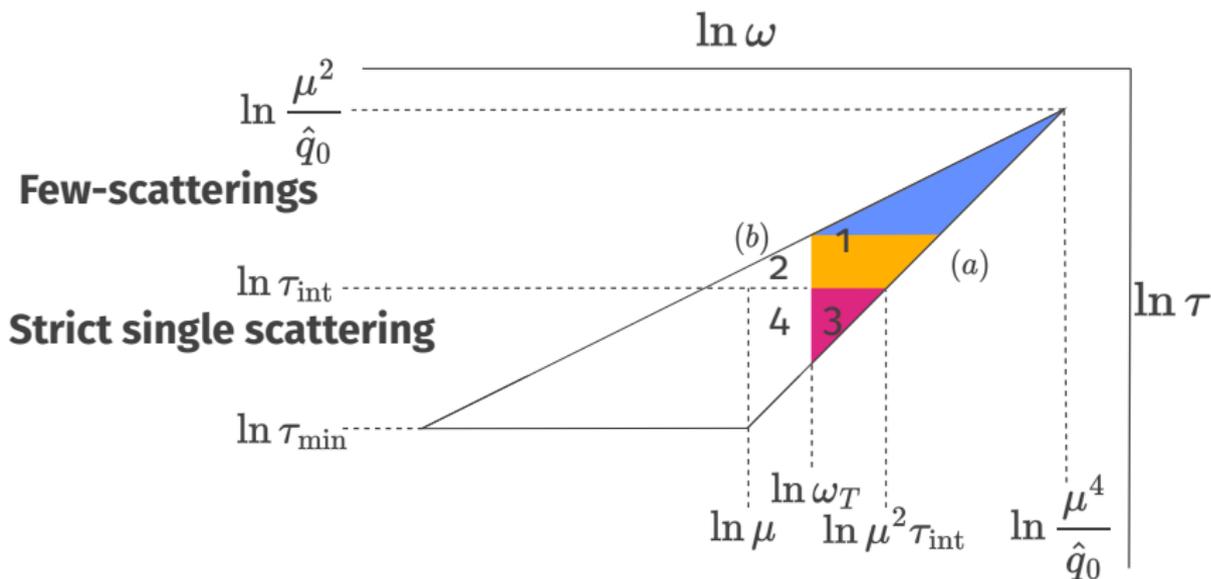
$$\delta \hat{q}_{3+4}(\mu) = \frac{\alpha_S C_R}{2\pi} \hat{q}_0 \ln^2 \frac{\mu^2 \tau_{\text{int}}}{\omega_T} + \text{subleading logs}$$

# DOUBLE LOG WITH SINGLE SCATTERING



$$\delta \hat{q}_{\text{GW}}(\mu) = \delta \hat{q}_{1+2}(\mu) + \delta \hat{q}_{3+4}(\mu) = \frac{\alpha_S C_R}{4\pi} \hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0 \omega_T}$$

# DOUBLE LOG WITH SINGLE SCATTERING



Why is it that region 2 and 4 do not contribute to the double Logs?

# VACUUM AND QUANTUM CORRECTION CANCELLATION

First, note that

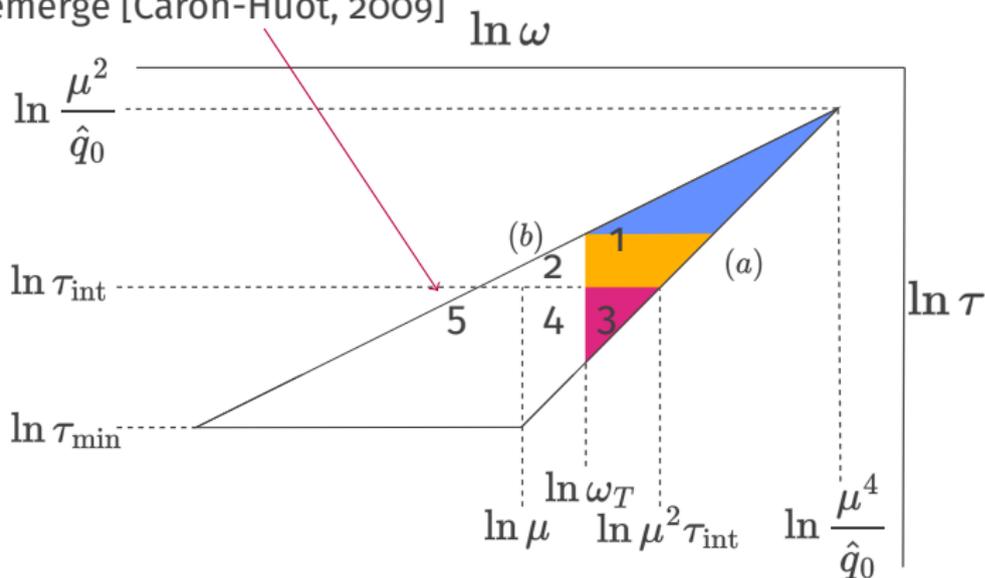
$$\lim_{\frac{\omega}{T} \rightarrow 0} \left( 1 + 2n_B(\omega) \right) = 1 + \frac{2T}{\omega} - 1 \quad (2)$$

The absence of the IR scale in any logarithms can then be seen by looking at the following integral, with  $\nu_{IR} \ll T \ll \nu_{UV}$

$$\begin{aligned} \int_{\nu_{IR}}^{\nu_{UV}} \frac{d\omega}{\omega} \left( \underbrace{1}_{\text{vacuum}} + \underbrace{2n_B(\omega)}_{\text{thermal}} \right) &= \underbrace{\ln \frac{\nu_{UV}}{\nu_{IR}}}_{\text{vacuum}} + \underbrace{\frac{2T}{\nu_{IR}} - \ln \frac{2\pi T}{\nu_{IR} e^{\gamma_E}}}_{\text{thermal}} + \dots \\ &= \frac{2T}{\nu_{IR}} + \ln \frac{\nu_{UV} e^{\gamma_E}}{2\pi T} + \dots \end{aligned} \quad (3)$$

# RELATION TO CLASSICAL CORRECTIONS

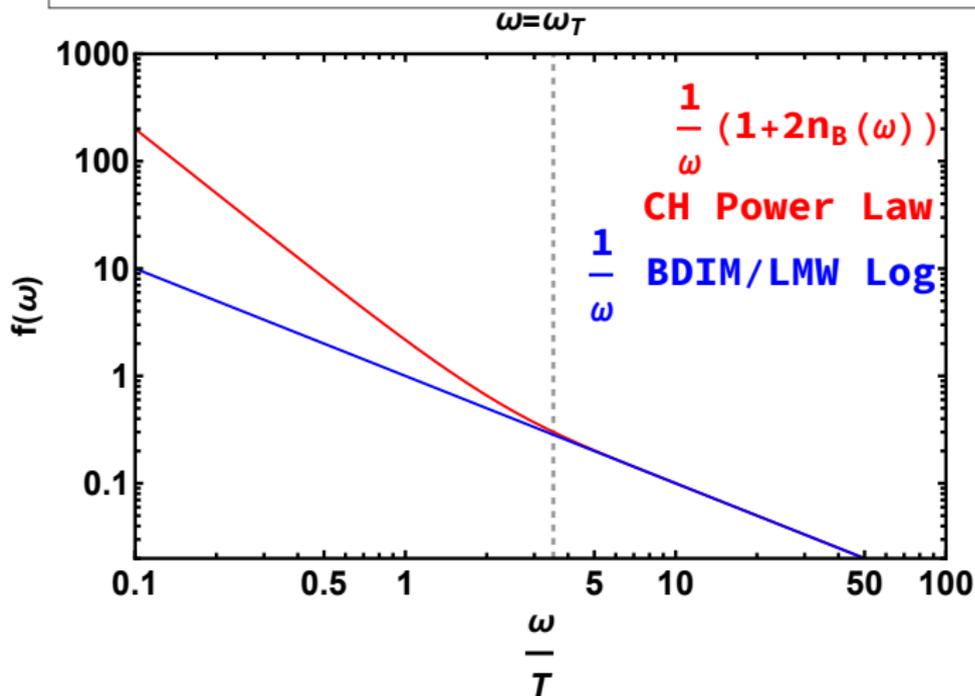
Region of phase space from which classical  $\mathcal{O}(g)$  corrections emerge [Caron-Huot, 2009]



How can we understand the transition to power law enhancement in regions 2 and 4?

# RELATION TO CLASSICAL CORRECTIONS

Can understand transition by looking at  $\omega$  integrand,  $f(\omega)$

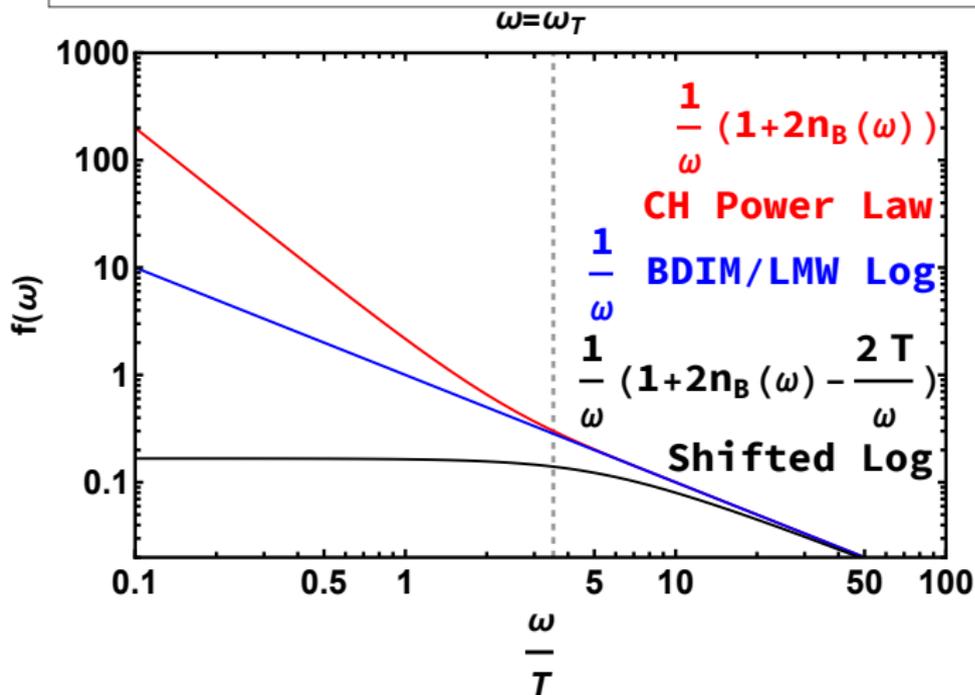


$$\omega_T \equiv 2\pi T e^{-\gamma E}$$

$$n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}$$

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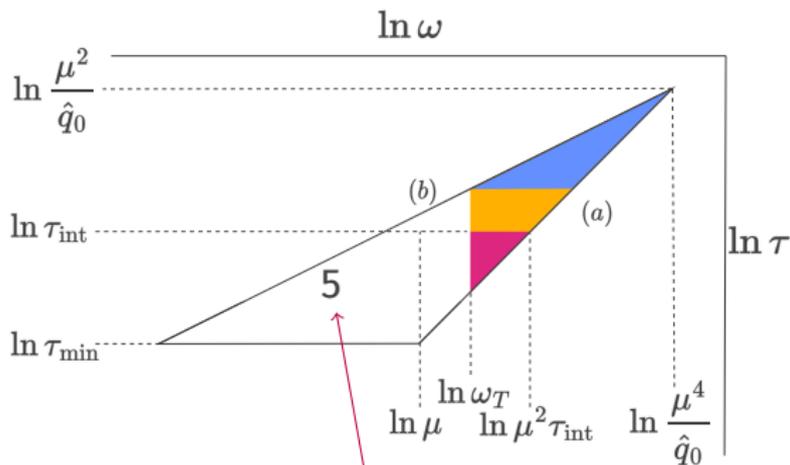


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$$n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}$$

# RELATION TO CLASSICAL CORRECTIONS

Our results include power law corrections depending on our IR cutoff



They cancel against cutoff-dependent corrections computed from [Caron-Huot, 2009]  $\Rightarrow$  Non-trivial check!

# GOING BEYOND HARMONIC OSCILLATOR APPROXIMATION

HOA not well-suited to [single-scattering](#)

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So how can we go beyond it?

$$\delta \hat{q}_{\text{GW}}(\mu) = \frac{\alpha_S C_R}{4\pi} \hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0 \omega_T} \longrightarrow \frac{\alpha_S C_R}{4\pi} \hat{q}(\rho) \ln^2 \frac{\#}{\omega_T}$$

where  $\hat{q}(\rho) \propto \ln \frac{\rho^2}{m_D^2}$

$\rho$  separates us from neighbouring region  
with simultaneously single-scattering and multiple scatterings

Appearance of  $\hat{q}_0$  in double log signifies lack of understanding of  
transition between single scattering and multiple scattering regimes

# GOING BEYOND HARMONIC OSCILLATOR APPROXIMATION

HOA not well-suited to [single-scattering](#)  
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$$\delta \hat{q}_{\text{GW}}(\mu) = \frac{\alpha_s C_R}{4\pi} \hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0 \omega_T} \longrightarrow \frac{\alpha_s C_R}{4\pi} \hat{q}(\rho) \ln^2 \frac{\#}{\omega_T}$$

where  $\hat{q}(\rho) \propto \ln \frac{\rho^2}{m_D^2}$

Need to solve transverse momentum-dependent LPM equation without **HOA** [Ghiglieri and Weitz, 2022] in order to shed light on how these issues could be addressed

# SUMMARY/OUTLOOK

- Have computed double logarithmic corrections to  $\hat{q}$ , showing how they are connected to classical corrections  
⇒ Still need to go beyond **HOA** to quantitatively determine argument of logs
- Try to better understand transition from single scattering to multiple scattering using Improved Opacity expansion  
[Barata et al., 2021]
- Would similar calculation of logs in calculation of double gluon emission [Arnold et al., 2021] alter conclusion?
- Incorporate our findings for  $\omega \sim T$  and smaller  $\tau$  regions into resummation equations [Caucal and Mehtar-Tani, 2022]

**THANKS FOR LISTENING!**

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# WHAT IS EQCD?

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- Arrive at **3 dimensional gluon effective field theory** for momenta,  $gT \sim m_D$ , inverse screening length for chromoelectric fields
- EQCD Lagrangian

$$\mathcal{L}_{\text{EQCD}} = \frac{1}{2g_{3d}^2} \text{Tr} F_{ij}F_{ij} + \text{Tr} D_i\Phi D_i\Phi + m_D^2 \text{Tr} \Phi^2 + \lambda (\text{Tr} \Phi^2)^2$$

where  $\Phi$  is adjoint scalar,  $i, j = 1, 2, 3$

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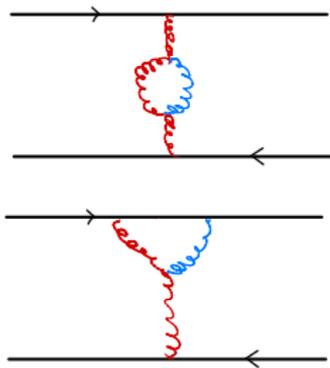
$$\mathcal{L}_{\text{EQCD}} = \frac{1}{2g_{3d}^2} \text{Tr} F_{ij} F_{ij} + \text{Tr} D_i \Phi D_i \Phi + m_D^2 \text{Tr} \Phi^2 + \lambda (\text{Tr} \Phi^2)^2$$

where  $\Phi$  is adjoint scalar,  $i, j = 1, 2, 3$

- $\Rightarrow$  “**Dimensional Reduction**”

# CONTRIBUTING DIAGRAMS

- Can think of sticking together amplitude and conjugate amplitude to get diagrams on the right
- **Black lines** represent hard parton in the amplitude and conjugate amplitude
- **Red gluons** are bremsstrahlung, represented by thermal propagators
- **Blue gluons** are those that are exchanged with the medium and are represented by **HTL** propagators



# WHERE DO THESE DIAGRAMS COME FROM?

