Topological phases with average symmetries

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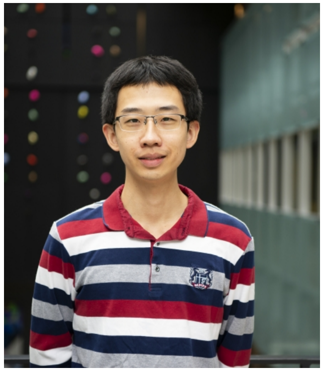
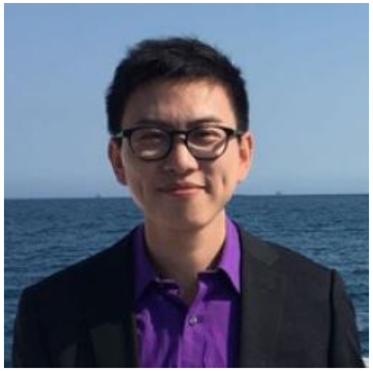


Ruochen Ma
Perimeter

Jianhao Zhang
Penn State



Zhen Bi
Penn State

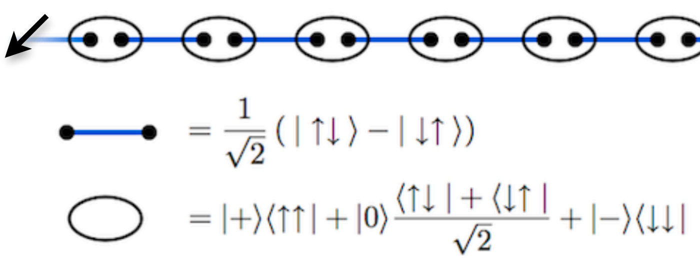
Meng Cheng
Yale

Ma, CW, 2209.02723

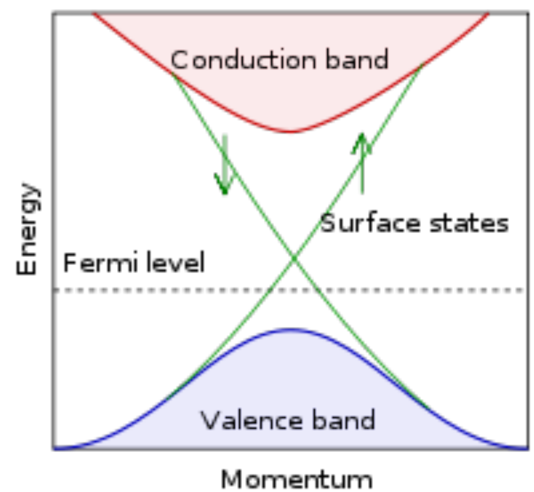
Ma, Zhang, Bi, Cheng, CW, 23??.?????

Symmetry-protected topological (SPT) phases

- Gapped & unique ground state, short-range correlations
- Short-range entangled (SRE): adiabatically connected to product state (e.g. atomic insulator), but adiabatic path must break symmetry
- Nontrivial boundary states characterized by t'Hooft anomaly



Haldane-AKLT chain



Topological insulator

This talk: *average* SPT and *average* anomaly

- Q: what if "protecting symmetry" broken by quenched disorder, but restored upon averaging? Does SPT make sense for "*average symmetry*"?

- Does quantum anomaly make sense for average symmetry?

- Example: lattice symmetries are always average symmetries in reality

- Another motivation: disordered quantum critical phenomena (quantum Hall transitions, superconductor-insulator transition, critical spin liquids…)

Defining average SPT

- Ensemble of local Hamiltonians $H = H_{\text{clean}} + \Sigma_i v_i \mathcal{O}_i$
- Short-range correlated disorder $\overline{v_i v_j} \sim e^{-|i-j|/\xi}$
- Exact symmetry: $[K, H] = 0$ for any disorder realization
- Average symmetry: v transforms nontrivially under G, but probability distribution $P[v]$ is invariant under G
- Simplest case: total symmetry $\mathcal{G} = K \times G$
- Symmetric ensemble: $\mathcal{G}$ is not spontaneously broken

Defining average SPT (cont.)

- Short-range entangled (SRE) ensemble: for each disorder realization I, $|\Psi_I\rangle$ is SRE, with correlation length ξ_I upper-bounded in the ensemble

- Continuous deformation: deforming H_{clean}, $\mathcal{O}_i$, $P[v]$ while preserving all the symmetry & SRE conditions

- Average SPT (ASPT) $\equiv$ equivalence classes under deformation

Digression: disordered vs. decohered ASPT

- We call the previous definition "disordered ASPT"
- Another physical context for ASPT: mixed quantum state in open system
- Exact symmetry $K\rho = e^{i\alpha}\rho$
- Average symmetry $G\rho G^{-1} = \rho$
- Deformation: symmetric finite-depth quantum channels
- Physically very different from disordered ASPT, although mathematical classification similar

(de Groot, Turzillo, Schuch; Ma, CW; Zhang, Qi, Bi; Lee, You, Xu)

Topological response

- Topological response of SPT ≈ symmetry flux traps symmetry charge (e.g. IQHE = $U(1)$ flux traps $U(1)$ charge)

- Average symmetry: charge not well-defined, but flux is

- Example: $\mathbb{Z}_2^{ave}$ can have anti-periodic b.c.
  $$\phi(x = L) = -\phi(x = 0) \qquad\qquad v(x = L) = -v(x = 0)$$

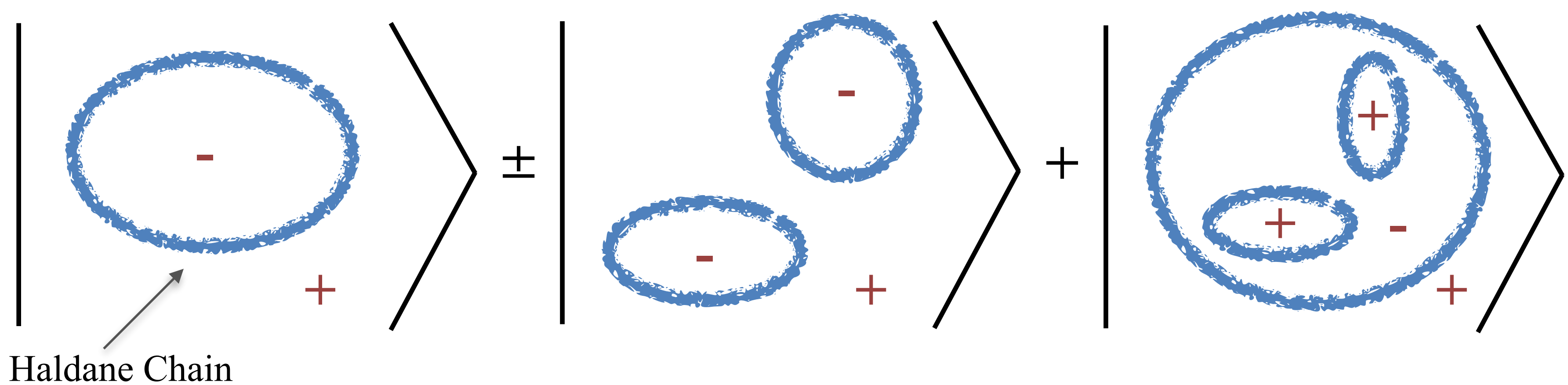
- *Average topological response ≈ average flux traps exact charge (more generally, invertible states)*

Decorated domain wall (standard version)

- Start from G-breaking state, make domain walls (symmetry defects) that are decorated with lower-dim SPT

- Form quantum superposition of ("condense") such domain walls

- Example: $SO(3) \times \mathbb{Z}_2$ decorate $\mathbb{Z}_2$ domain walls with $SO(3)$ Haldane chain



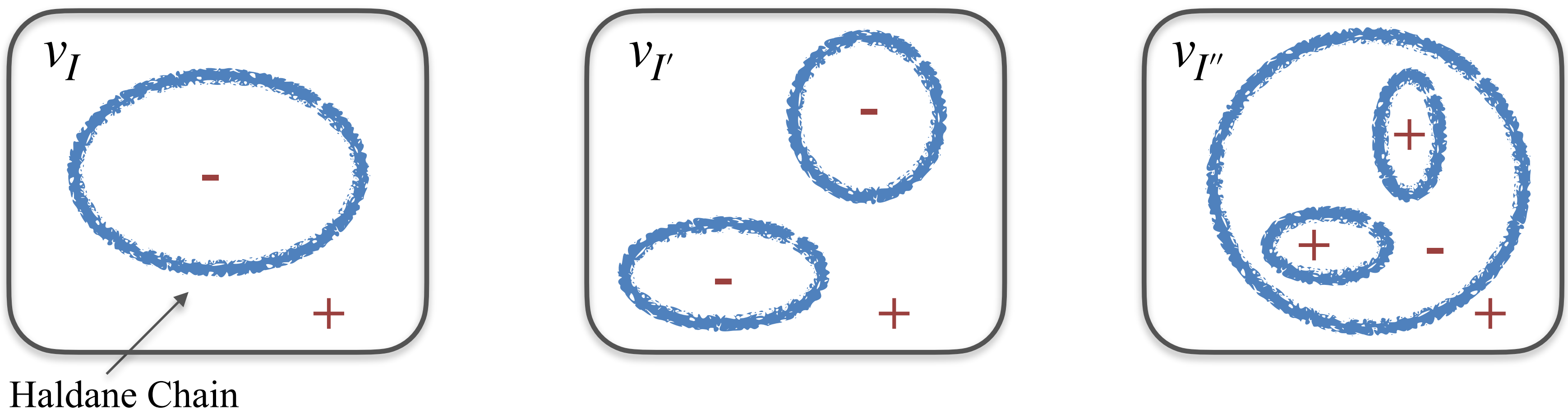
(Chen, Lu, Vishwanath)

Decorated domain wall for ASPT

- Very similar picture, except domain walls do not move (pinned by local disorders)

- Domain walls proliferate, not as a quantum superposition, but probabilistically in disordered ensemble

- Example: $SO(3) \times \mathbb{Z}_2^{(ave)}$ decorate $\mathbb{Z}_2^{(ave)}$ domain walls with $SO(3)$ Haldane chain



Crucial differences

- In ASPT, domain walls proliferate probabilistically — no analogue of superposition phase factor

- Example: the Levin-Gu state for $\mathbb{Z}_2$ symmetry has no ASPT analogue

$$\sum_{Z_i=\pm 1} (-1)^{\#\text{ of Domains}} \otimes_i |Z_i\rangle$$

- A subtler point: decorating a $(0+1)d$ G-defect with a K-charge does not produce a nontrivial ASPT

- Helpful analogy: atomic insulators with different integer $U(1)$ charge per unit cell can be smoothly interpolated to each other via Anderson localized insulators

General classification

- Within group-cohomology, bosonic ASPT with $\tilde{G} = G \times K$ are classified by

$$\sum_{p=2}^{d+1} H^{d+1-p}(G, H^p(K, U(1)))$$

- More generally

$$1 \to K \to \tilde{G} \to G \to 1$$

- Classification involves the Atiyah-Hirzebruch spectral sequence, with

$$E_2^{p,q} = H^p(G, h^q(K))$$

$h^{q>1}(K)$ classifies K-symmetric invertible phases in q spacetime dimensions, and $h^{0,1}(K) = 0$

Intrinsically disordered ASPT

- Example: $(3+1)d$ fermions with
$$\tilde{G} = \mathbb{Z}_4^{ave} \times \mathbb{Z}_2^{ave} \times \mathbb{Z}_2^F$$

- On each intersection of $\mathbb{Z}_4^{ave}$ and $\mathbb{Z}_2^{ave}$ domain wall, decorate a Kitaev chain

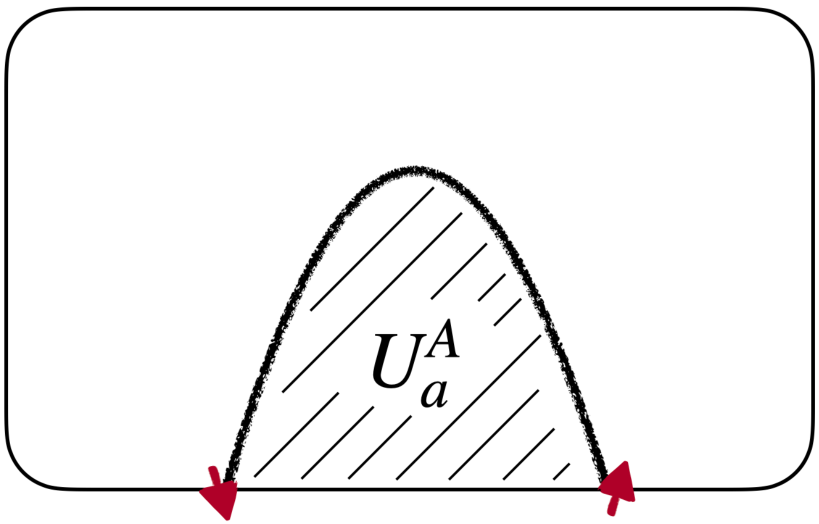
- This ASPT is forbidden in clean limit: an obstruction $\in H^5(\mathbb{Z}_4 \times \mathbb{Z}_2, U(1))$

- But the obstruction becomes trivial for average symmetries

- "Less is more!"

Long-range entanglement from anomaly

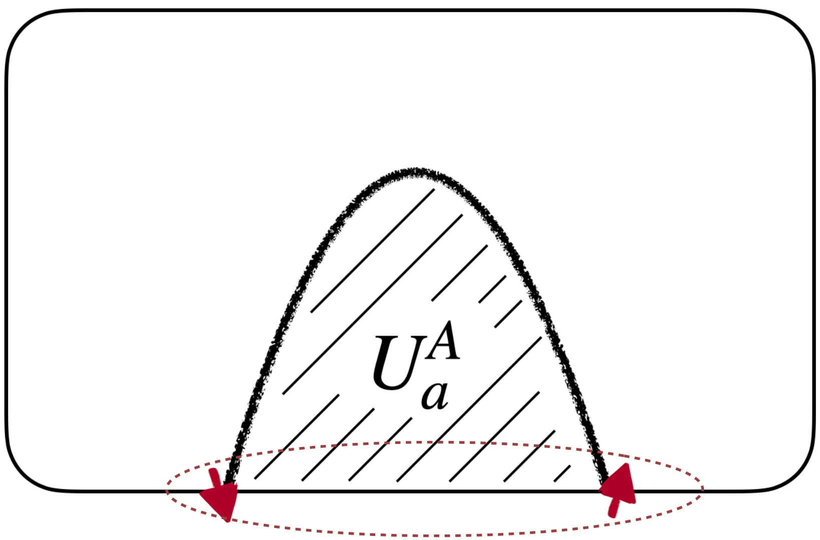
- Use the previous $SO(3) \times \mathbb{Z}_2^{(ave)}$ SPT as example
- Suppose: a short-range entangled boundary for some disorder realization



- Can create a domain wall by changing disorder (flipping random field)
- Domain wall creates two spin-$\frac{1}{2}$ on boundary

Long-range entanglement from anomaly

- SRE of original state $\Longrightarrow$ regions far away from domain wall do not couple to the spin-$\frac{1}{2}$ moments
- To keep $SO(3)$ symmetry, the two spin-$\frac{1}{2}$ must form singlet
- Infinite many ways to create such long-range singlets with domain walls, with essentially the same probability as the original state



Conclusion:

SRE boundary state can appear at most with vanishing probability

Application: average LSM

- Lattice systems with average translation symmetry and spin-$\frac{1}{2}$ per unit cell (or any fractional symmetry representation $\in H^2(K, U(1))$) have mixed anomaly between $SO(3)$ (K) and average translation

- Dynamics of such systems must be nontrivial: long-range entanglement

(Kimchi, Nahum, Senthil)

- Classic example: random singlet state in random bond spin-$\frac{1}{2}$ chain

Summary

- SPT protected by average (and exact) symmetries
- Topological response and decorated domain wall picture
- Intrinsically disordered ASPT
- Average anomaly and long-range entanglement
- Average LSM constraints

Much more to explore…

- What about average SET (symmetry-enriched topological orders)?

  (On-going work)

- Use average anomaly to study disordered critical systems (quantum Hall plateau transition, disordered DQCP, disordered spin liquids, Weyl/Dirac semimetals, etc.)?

- Even compressible states (dirty non-Fermi liquids)???

- Non-equilibrium setting?