To bind or not to bind? A tale of two nucleons

INT-24-1: Fundamental Physics with Radioactive Molecules
4th March — 12th April, 2024

André Walker-Loud
BERKELEY LAB

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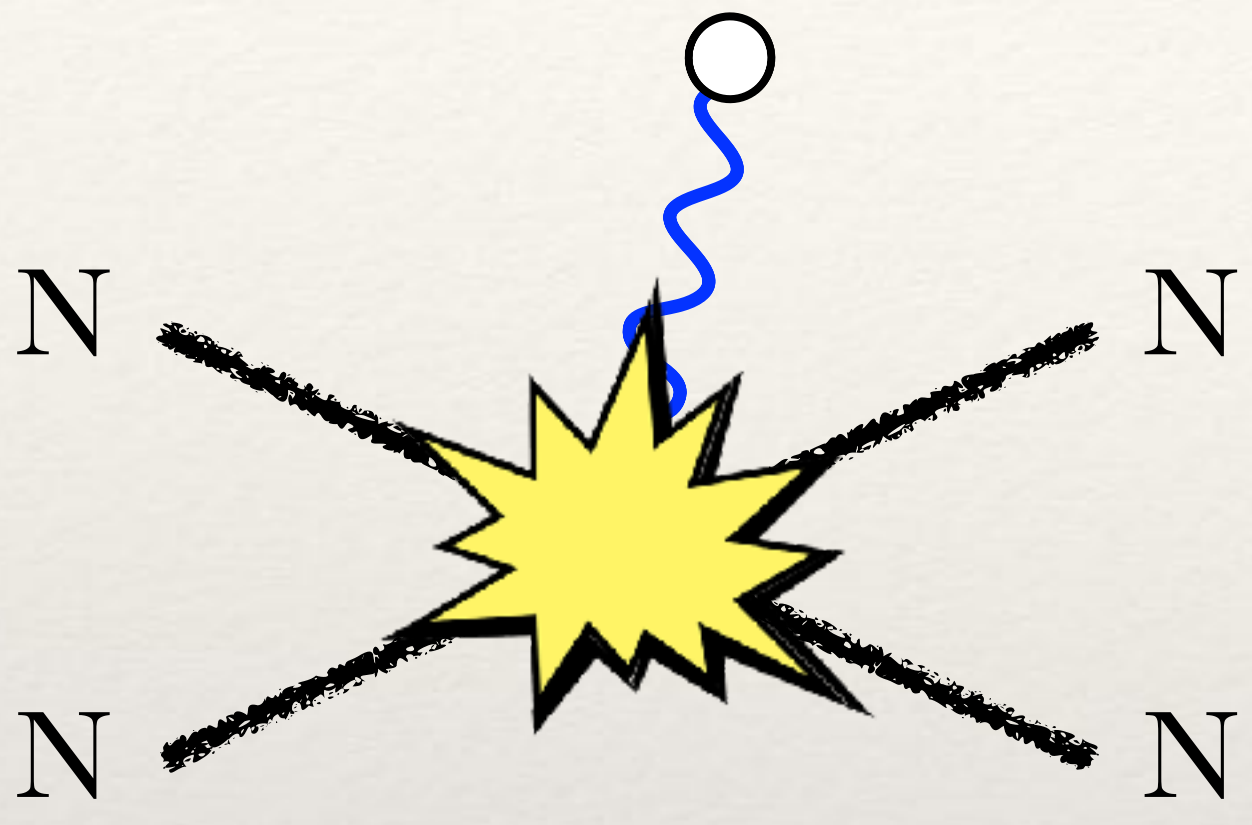
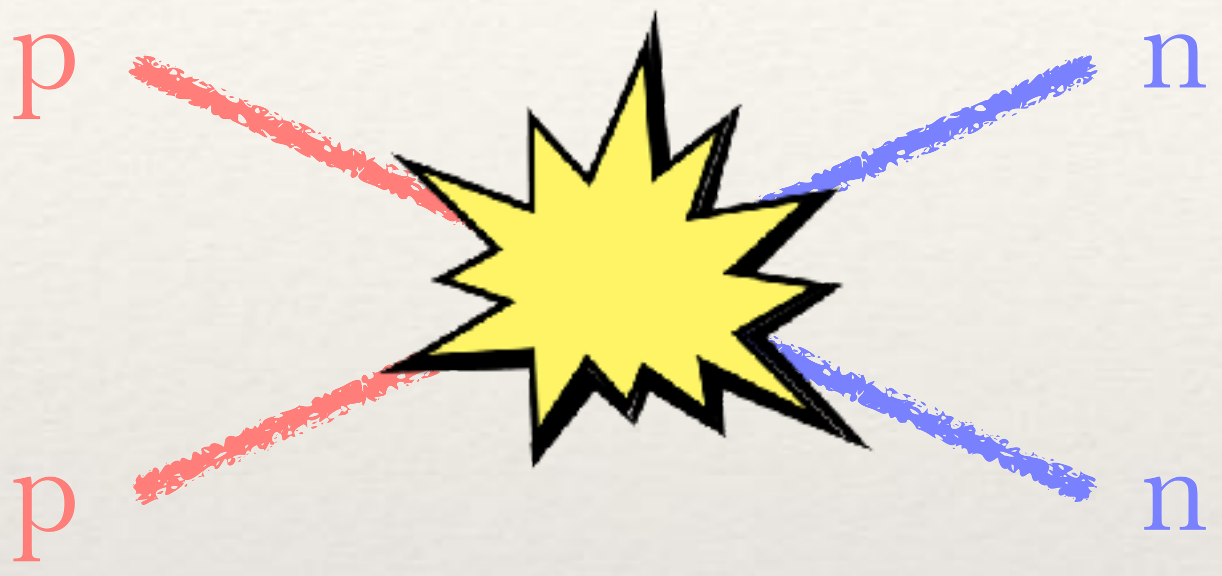
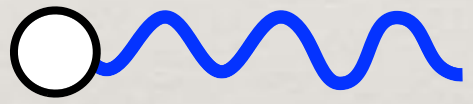
Lattice QCD for Fundamental Physics in Nuclei

- What would I like to be talking with you about today?





neutrinoless double β-decay
- long-range current
- short-range 4-quark operator



- electroweak current
  - axial-vector
  - scalar
  - …

- 4-quark operator
  - parity violating
  - CP violating
  - …

Lattice QCD for Fundamental Physics in Nuclei

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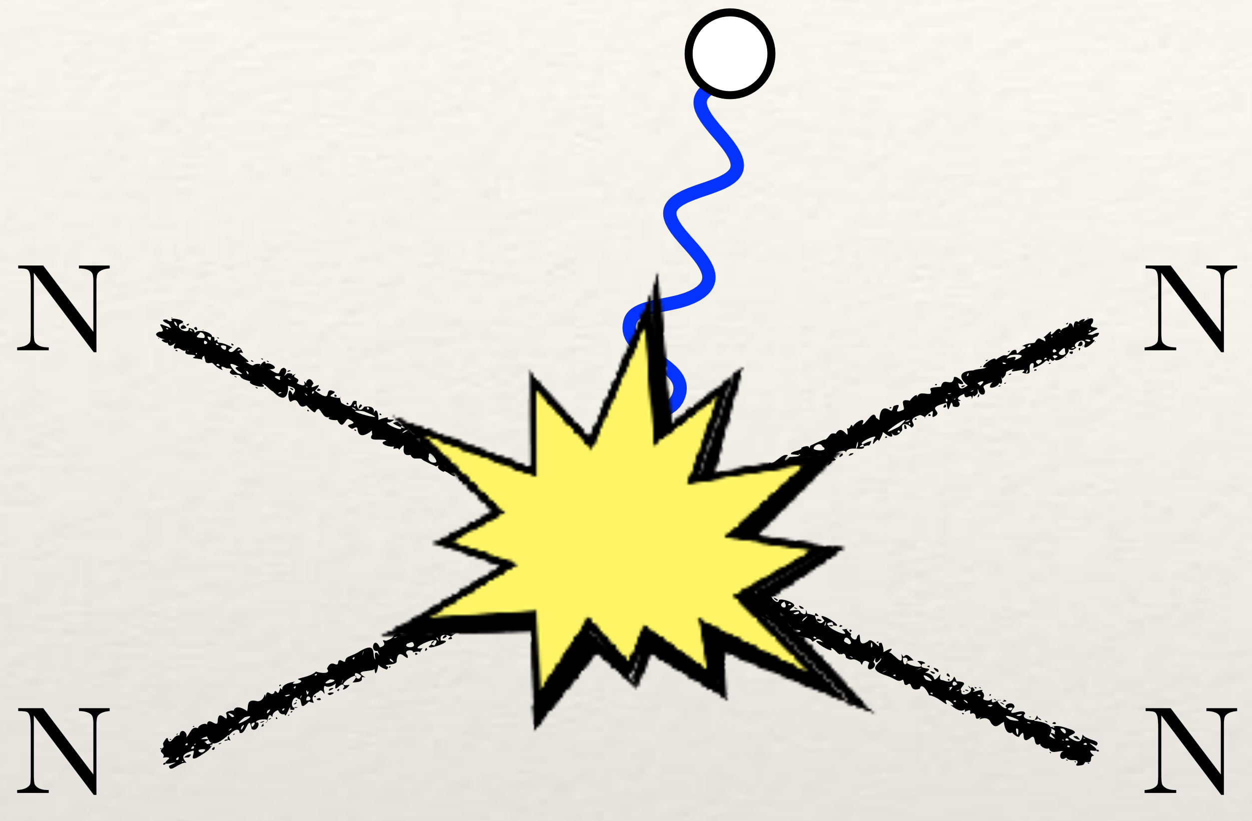

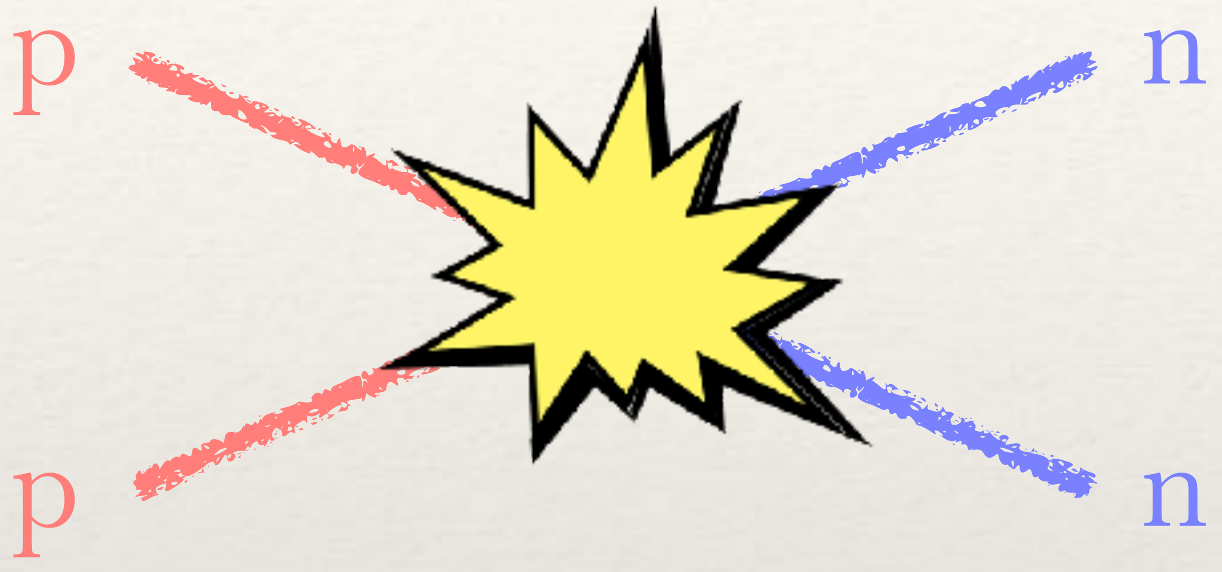

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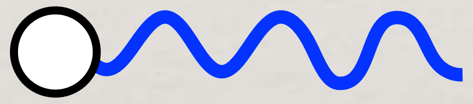

- electroweak current
  - axial-vector
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  - …
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- For such processes, LQCD is (the only) tool that provides fully quantified theoretical uncertainties — at least for SM-rare or BSM matrix elements
- LQCD can provide, in principle, One, Two, (maybe Three) body matrix elements
- These results can be coupled with many-body nuclear EFT to make predictions for nuclear matrix elements / reactions
- Such LQCD calculations are very expensive and challenging

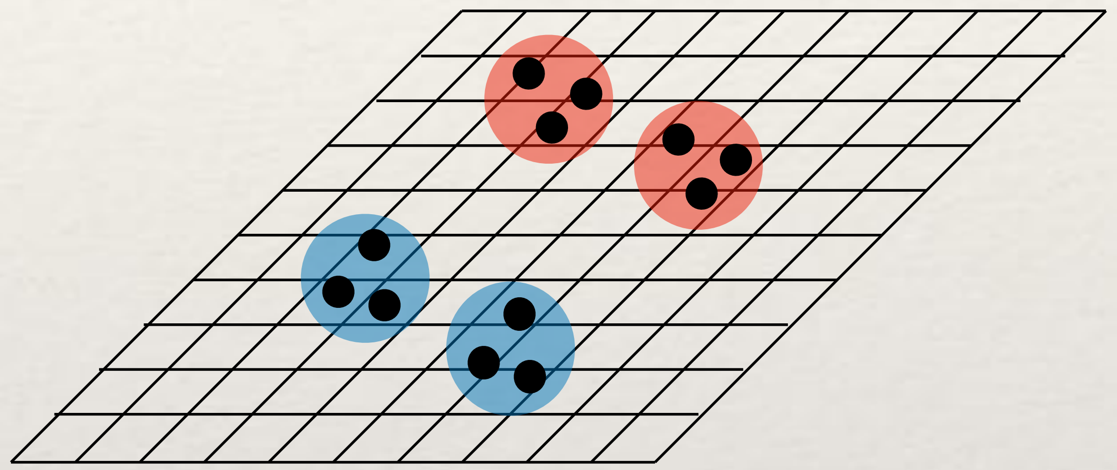
Lattice QCD for Fundamental Physics in Nuclei

- Today:
  - Describe how LQCD might fit in this fundamental physics research program
  - Describe current status of two-nucleon calculations and future prospects
  - Give a selective summary of some state-of-the-art results and future prospects

Lattice QCD for Fundamental Physics in Nuclei

- Lattice QCD is pre-*ab initio*
  - pre: comes before
  - pre: preliminary (we haven't computed anything quantitatively relevant to $A \geq 2$ systems yet 😂)

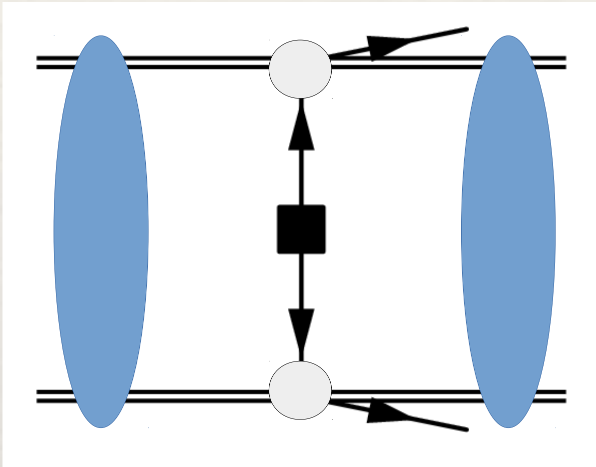
- Lattice QCD is QCD formulated in Euclidean spacetime on a discrete grid



  - Maximally predictive:
    - Set the scale with a hadron mass, e.g. m_Ω
    - Determine $m_{u/d}$ with the pion mass and m_s with the kaon mass (add isospin breaking if desired/needed)
    - Everything else is a prediction!
  - Control several systematic uncertainties, predict select physical observables directly from quarks & gluons
    - Isolate S-matrix of interest (can be more challenging that it sounds)
    - Extrapolate/Interpolate numerical results to physical quark mass limits
    - Extrapolate to continuum limit with $N_a \geq 3$ lattice spacings
    - Extrapolate to infinite volume limit

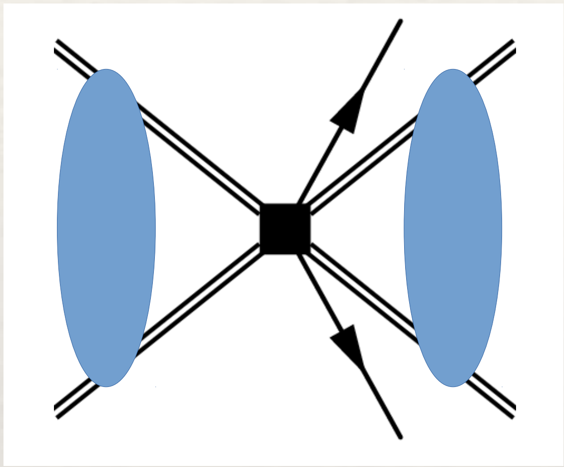
LQCD for Fundamental Physics in Nuclei: $0\nu\beta\beta$

- Neutrinoless Double β-decay could be mediated by light Majorana neutrinos (long range), or by heavy neutrinos (short range) mimicked by 4-quark/2-electron operators
- In either case, there are short-distance (on the nuclear scale) operators whose matrix elements are required to predict the nuclear decay rate



known (predictive)

+



unknown coupling (LEC)

Cirigliano et al. PRC 97 (2018) [1710.01729]
Cirigliano et al. PRL 120 (2018) [1802.10097]
Cirigliano et al. PRC 100 (2019) [1907.11254]

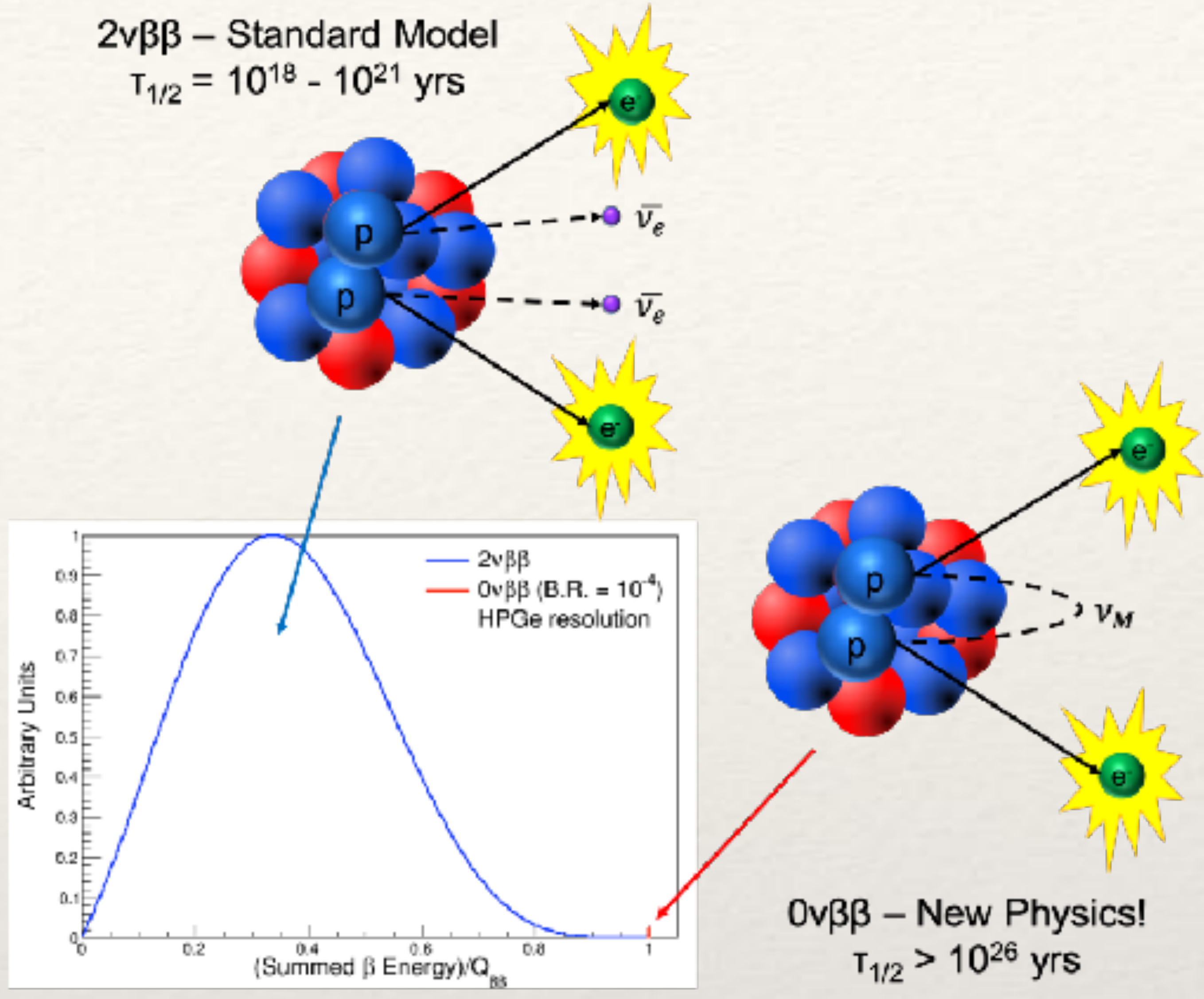
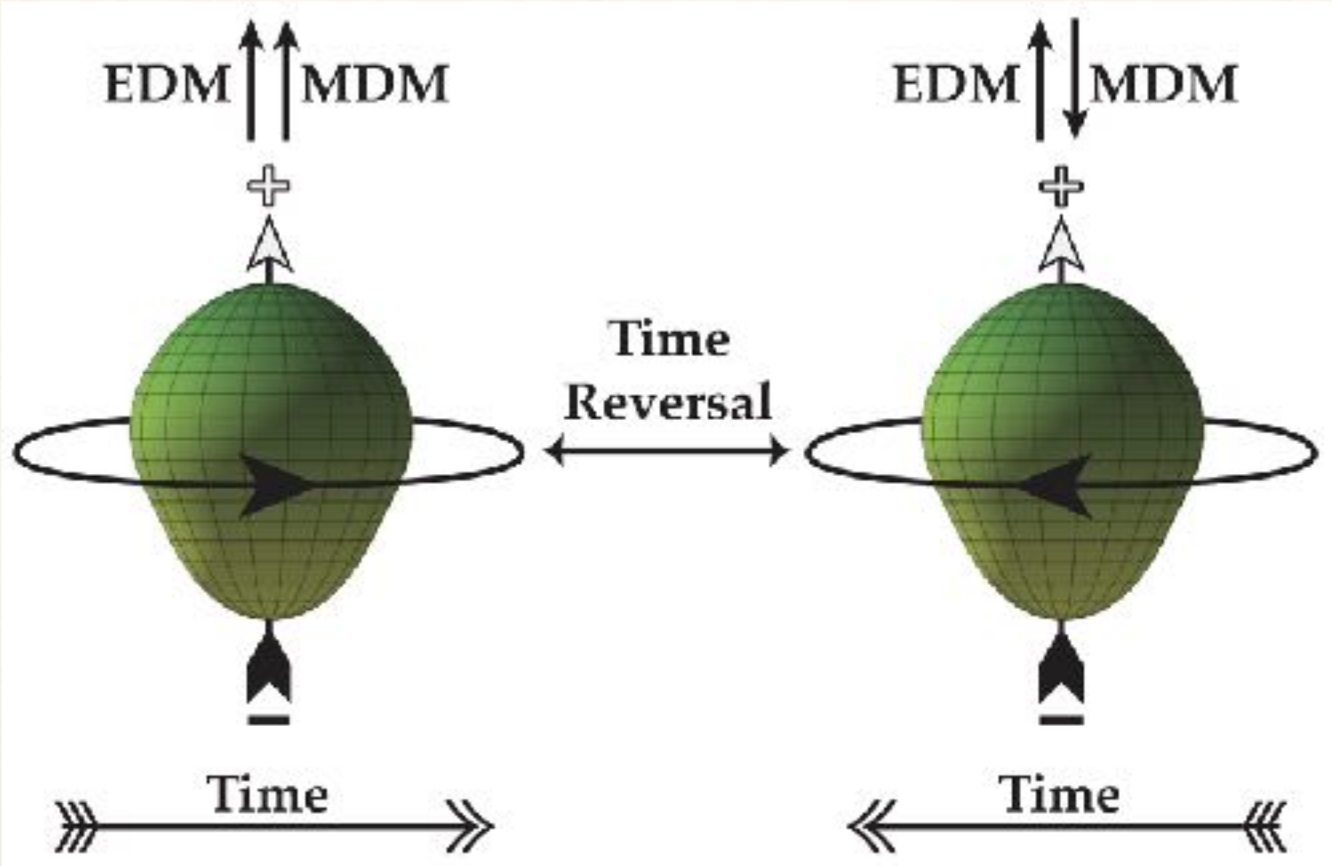


image from https://jgruszko.web.unc.edu/physics/

- In principle — perfect problem for LQCD: compute the $nn \to pp$ amplitude and then match to EFT
  - Short-distance 4-quark operators: tractable problem over the next few years
  - Long-distance with light Majorana neutrino: more like a 5-10 year effort on exaScale computers
    - Already two independent pheno estimates — agree on sign and magnitude
      Cirigliano et al, JHEP 05 (2021); Richardson et al PRC 103 (2021);
      Prior to non-zero experimental measurement, worth the human/computing resources required?

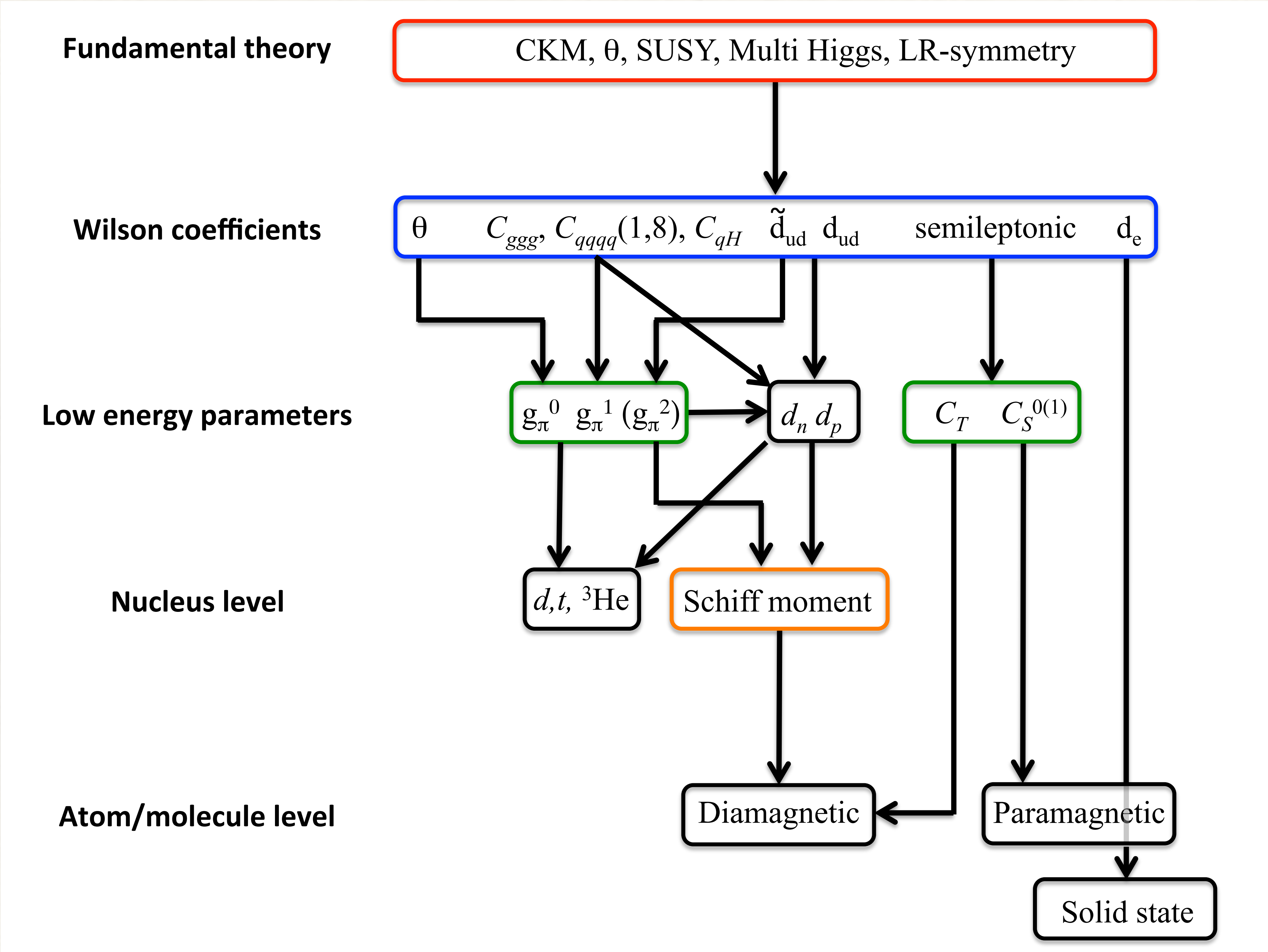
LQCD for Fundamental Physics in Nuclei: EDMs

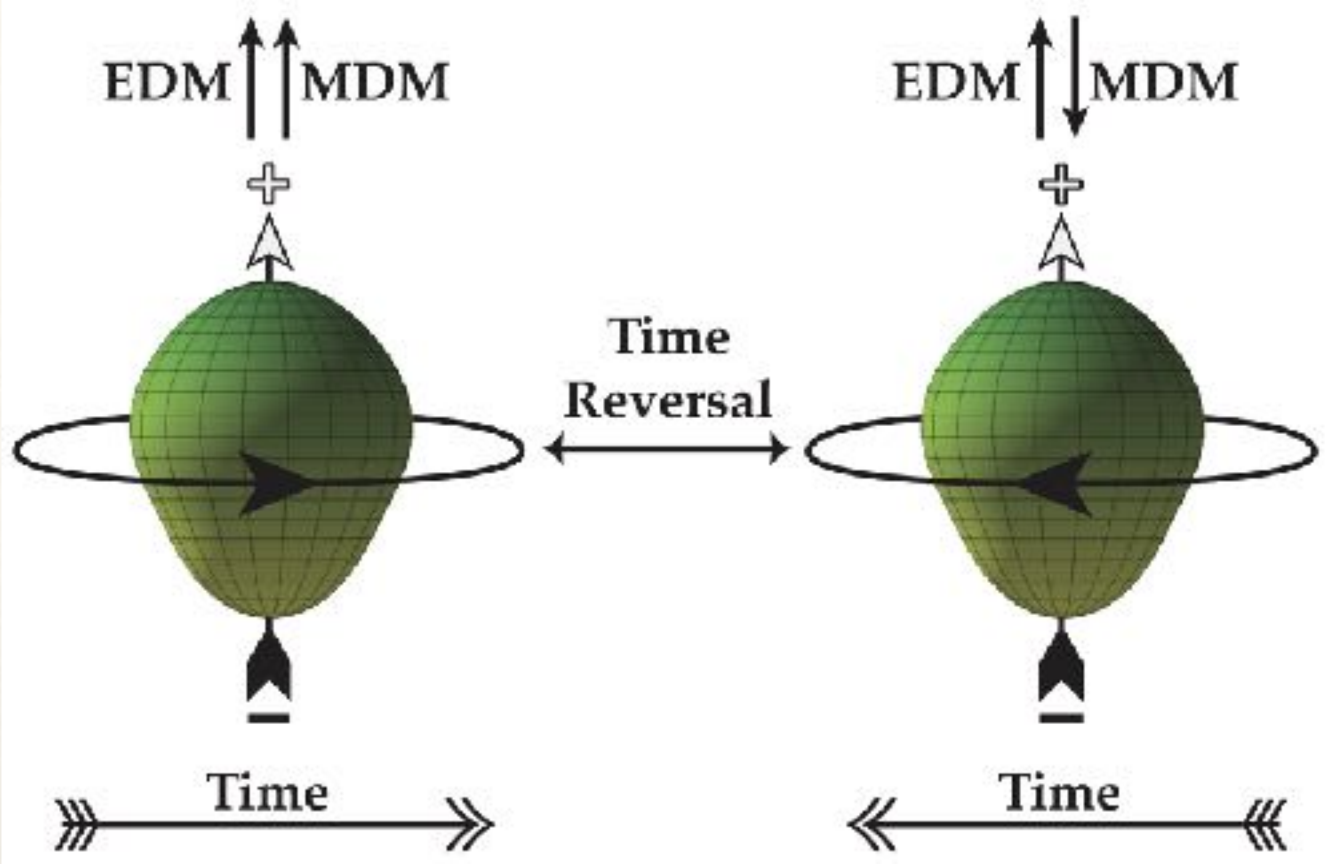
- Motivated by matter/anti-matter asymmetry
  $\eta \equiv \frac{N_B}{N_\gamma} \approx 6 \times 10^{-10}$
- SM (Standard Model) CP violation orders of magnitude too small to explain η
- Why is $\theta_{\text{QCD}} \ll 1$ ($\lesssim 10^{-10}$)?
  - Even if $\theta_{\text{QCD}} \approx 10^{-11}$, too small to explain η
- Strong motivation for BSM (Beyond the SM) CP violation



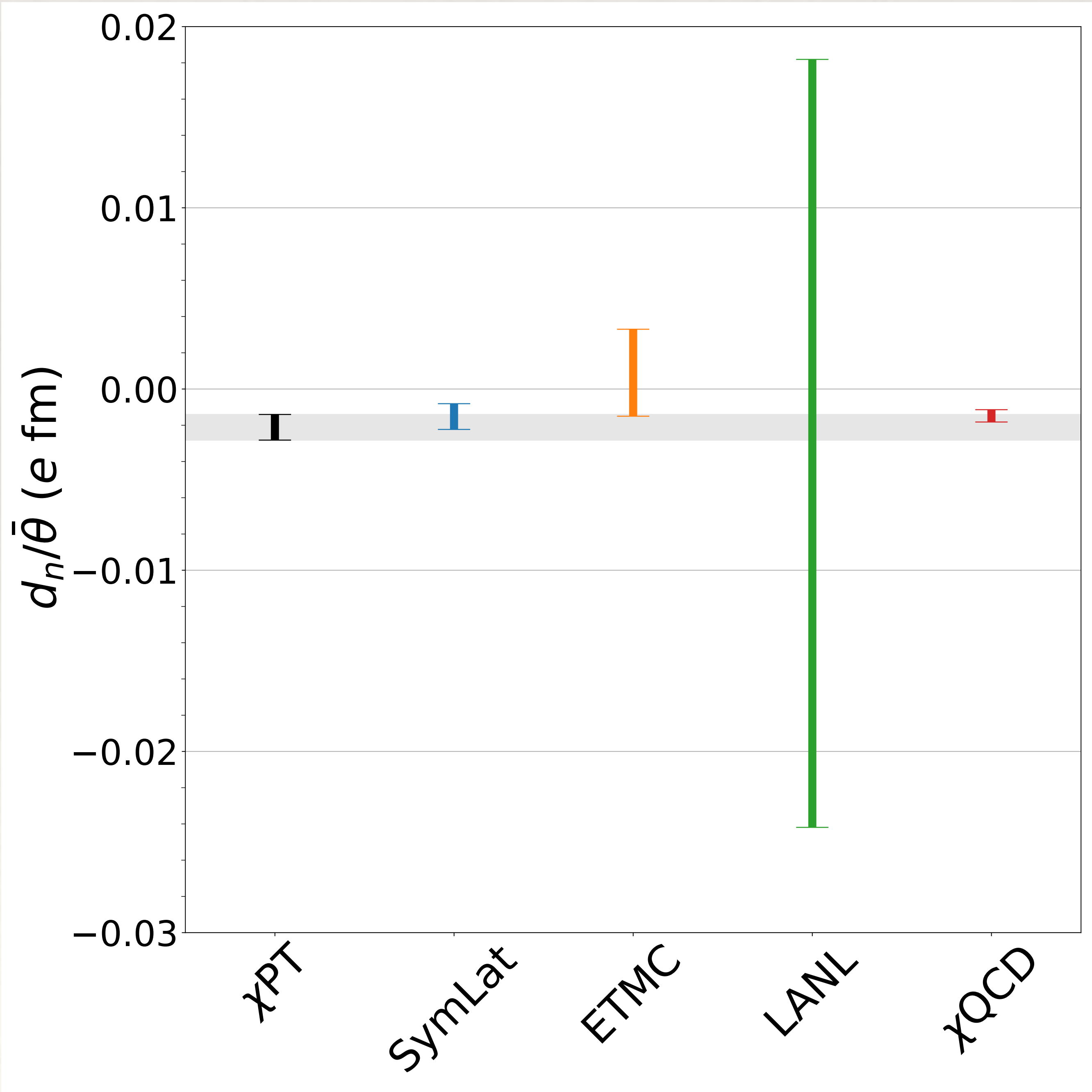
LQCD for Fundamental Physics in Nuclei: EDMs

- Suppose BSM CP violating physics occurs at heavy scale
- We can use EFT to parameterize this new physics in terms of higher-dimensional operators constructed with SM fields





- Computing neutron EDM from θ_{QCD} alone is very challenging
- Adding new BSM operators (which all mix under renormalization) is significantly more challenging



figures from LRP FSNN White Paper: [2304.03451](2304.03451)

LQCD for Fundamental Physics in Nuclei: EDMs

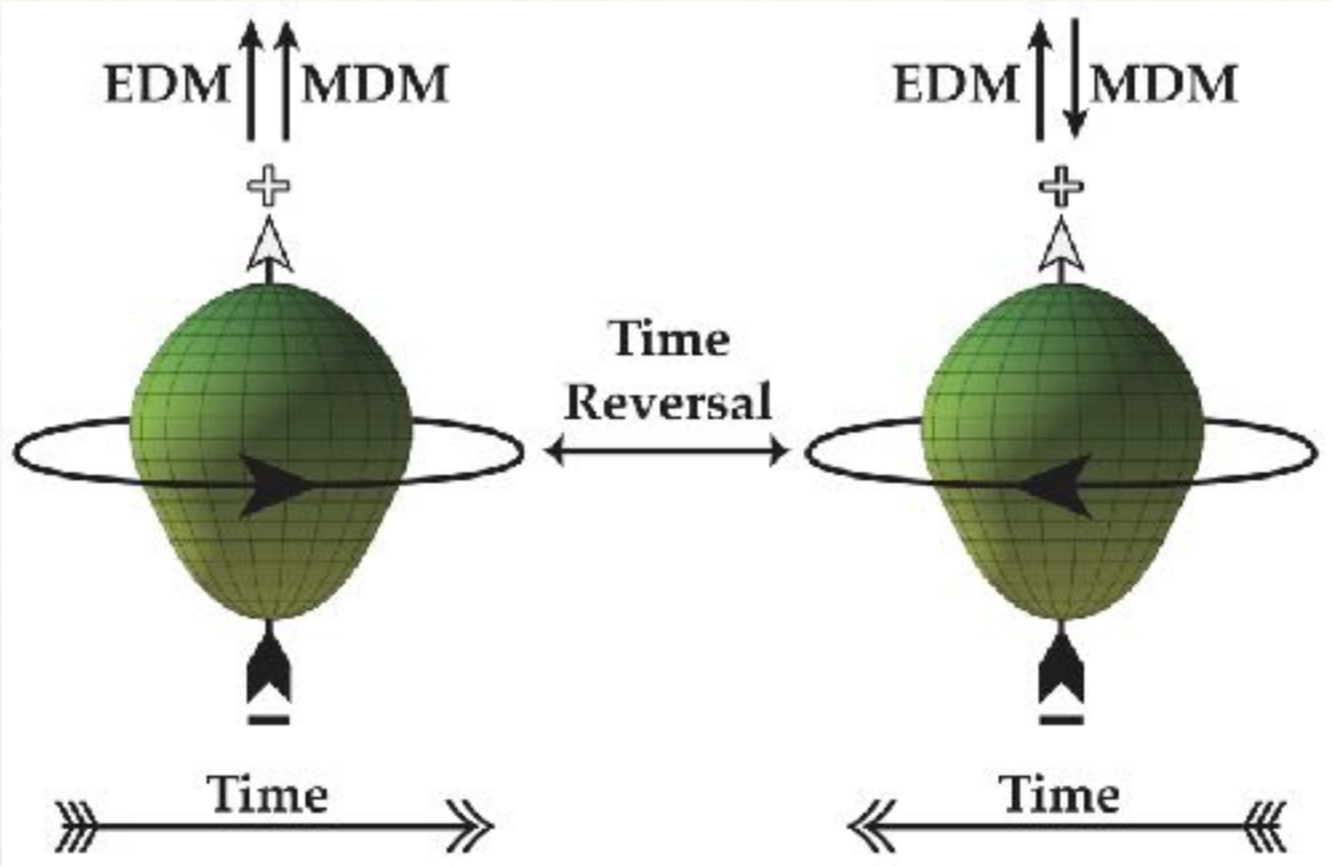
- neutron EDM:
  - obtaining experimental signal is daunting
  - theoretically "clean" (still very challenging and noisy)
- nuclear EDM:
  - prospect for significantly enhanced signals with radioactive nuclei
  - theoretically challenging (requires modeling)
  - How does LQCD fit into this strategy?
    - Recall - for $\theta_{\rm QCD}$, the CP-odd $\pi - N$ coupling $\bar{g}_0$ can be related to $\delta M_{n-p}^{m_d-m_u}$

      Crewther, Vecchia, Veneziano, Witten PLB 88 (1979)

$$\bar{g}_0 = \frac{\delta M_{n-p}^{m_d-m_u}}{m_d - m_u} \frac{2 m_u m_d}{m_u + m_d} \bar{\theta}_{\rm QCD}$$

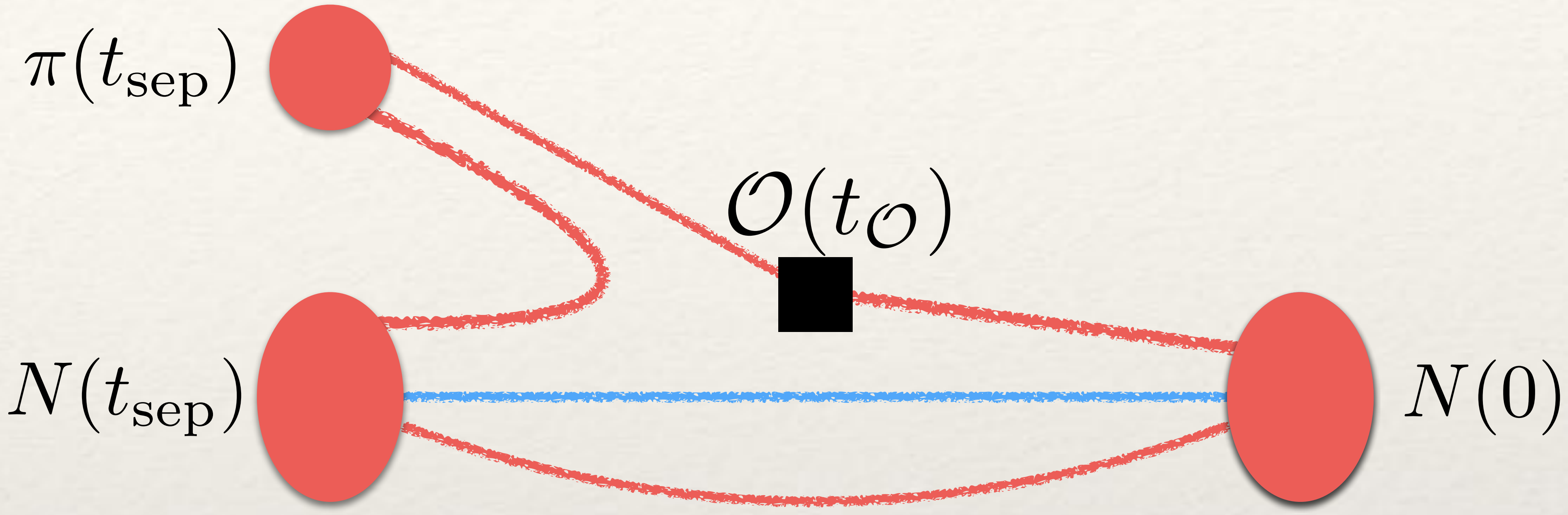
    - One can use the same symmetry to related BSM CP-violating operators to CP-conserving ones and compute shifts in the hadronic spectrum induced by such CP-even operators

      de Vries, Mereghetti, Seng, Walker-Loud, PLB 766 (2017)
    - Suffers from same renormalization challenge

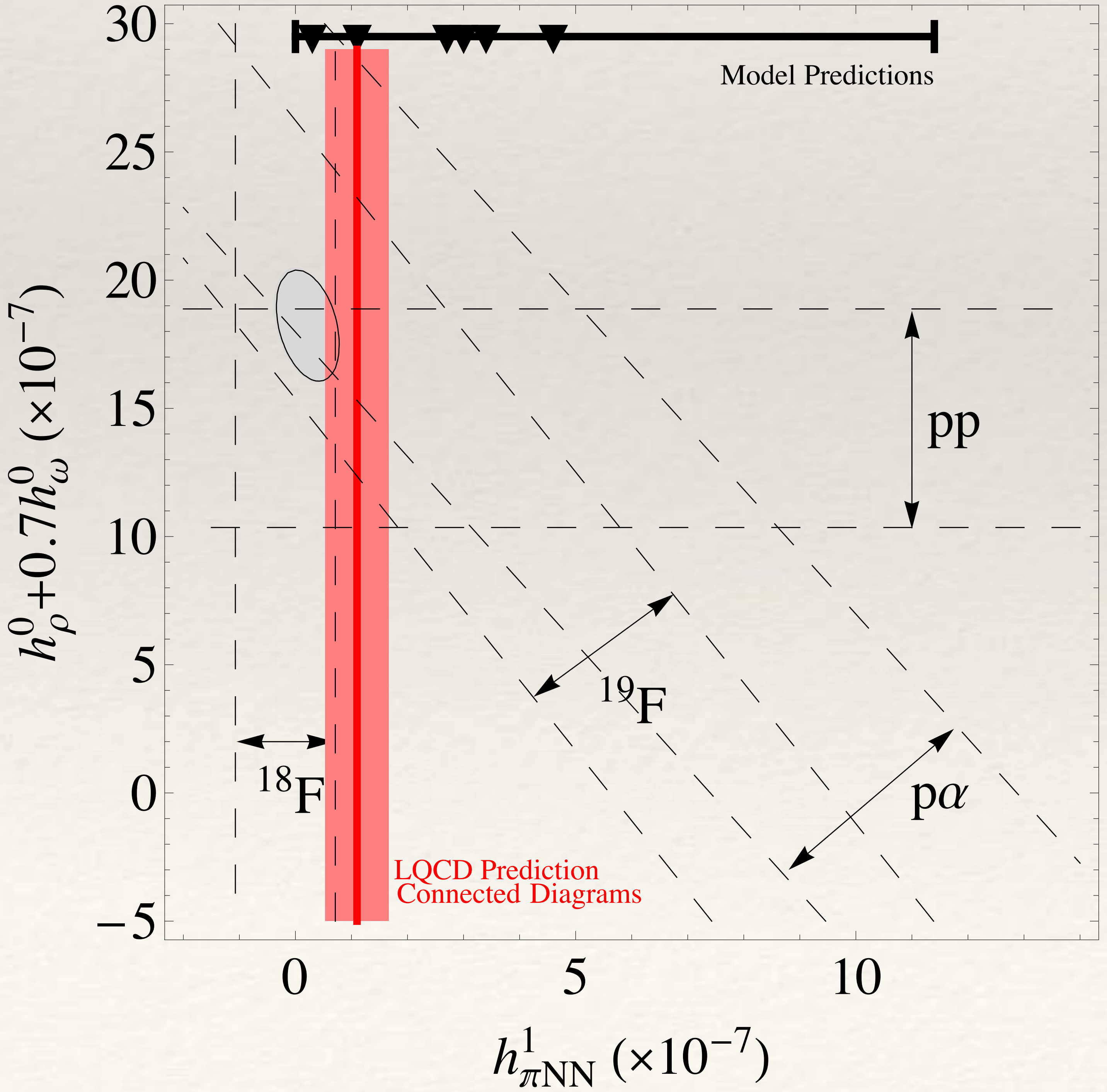


LQCD for Fundamental Physics in Nuclei: Parity Violation

- Parity violating hadronic matrix elements involve multi-hadrons in the initial and/or final state
  - introduces additional LQCD challenges to relate finite-volume matrix element to that in infinite volume these conversion factors (Lellouch-Lüshcer) can be O(100%)



Wasem, PRC 85 (2012) [1108.1151]
(tour de force - one new result since then
Petschlies, Schlage, Sen, Urbach, EPJA 60 (2024))



- ✓ First LQCD calculation
- ✓ Numerical result
- ✘ use of good operator basis
- ✘ control over renormalization
- ✘ control over finite-V to inf-V
- ✘ control over disconnected diagram
- ✘ control of excited states
- ✘ control over chiral extrapolation

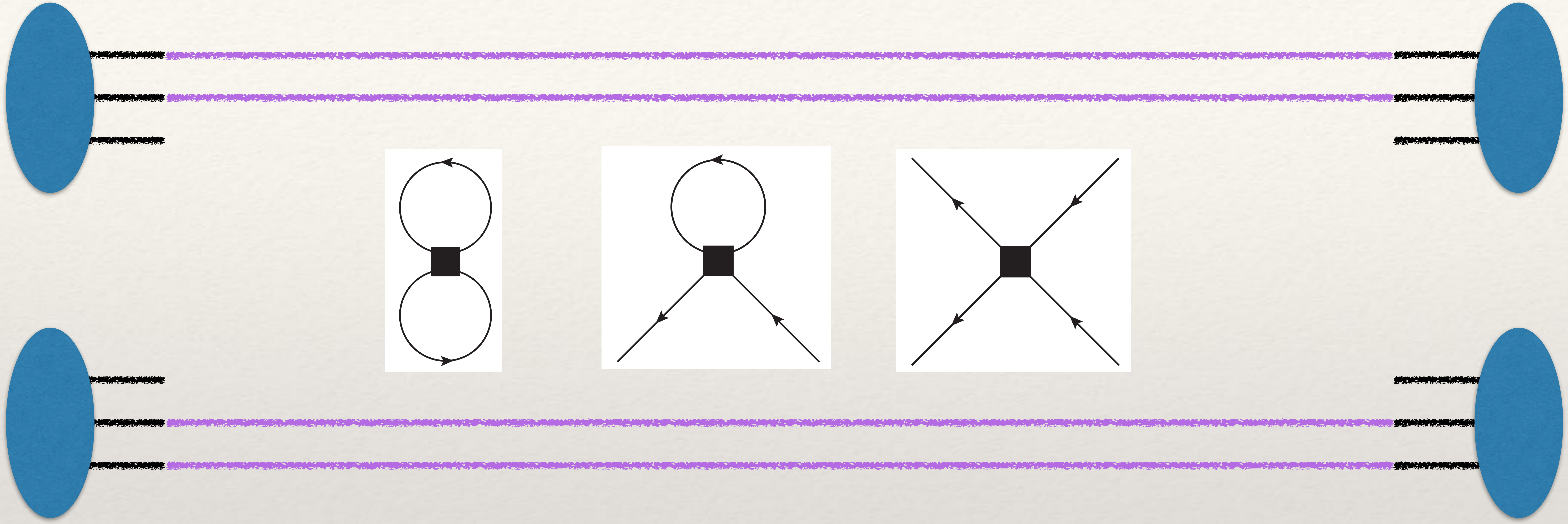
New LQCD calculation
$h^1_\pi = 8.1 \pm 1.0 \times 10^{-7}$

NPDGamma, PRL 121 (2018)
$h^1_\pi = 2.6 \pm 1.2 \pm 0.2 \times 10^{-7}$

LQCD systematics not finalized by any means

LQCD for Fundamental Physics in Nuclei: Parity Violation

- What about $\langle NN | O_W | NN \rangle$?



LQCD for Fundamental Physics in Nuclei: Parity Violation

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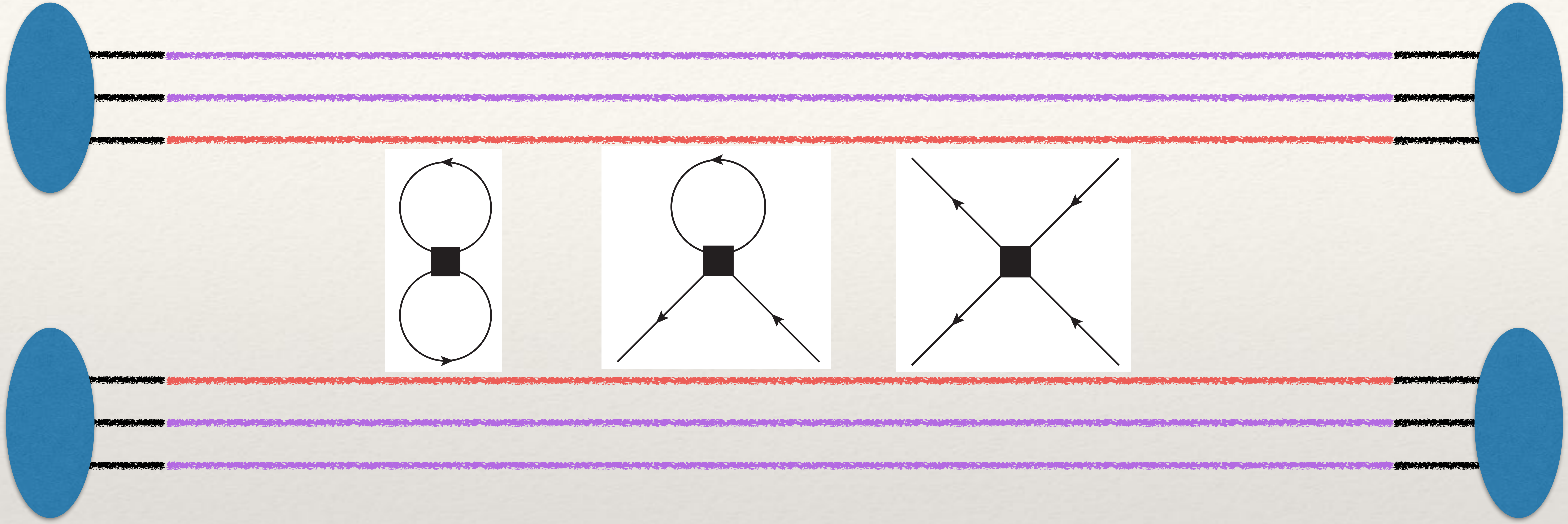

ΔI=0

LQCD for Fundamental Physics in Nuclei: Parity Violation

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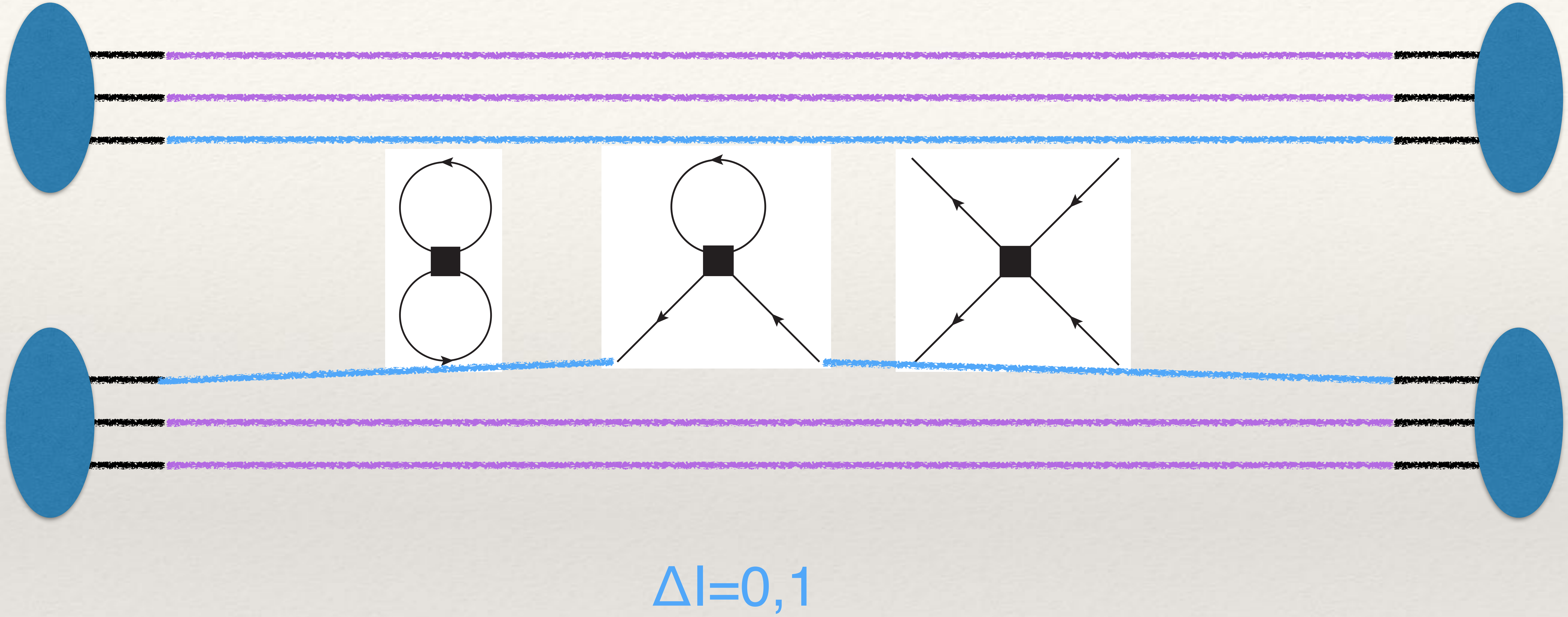

LQCD for Fundamental Physics in Nuclei: Parity Violation

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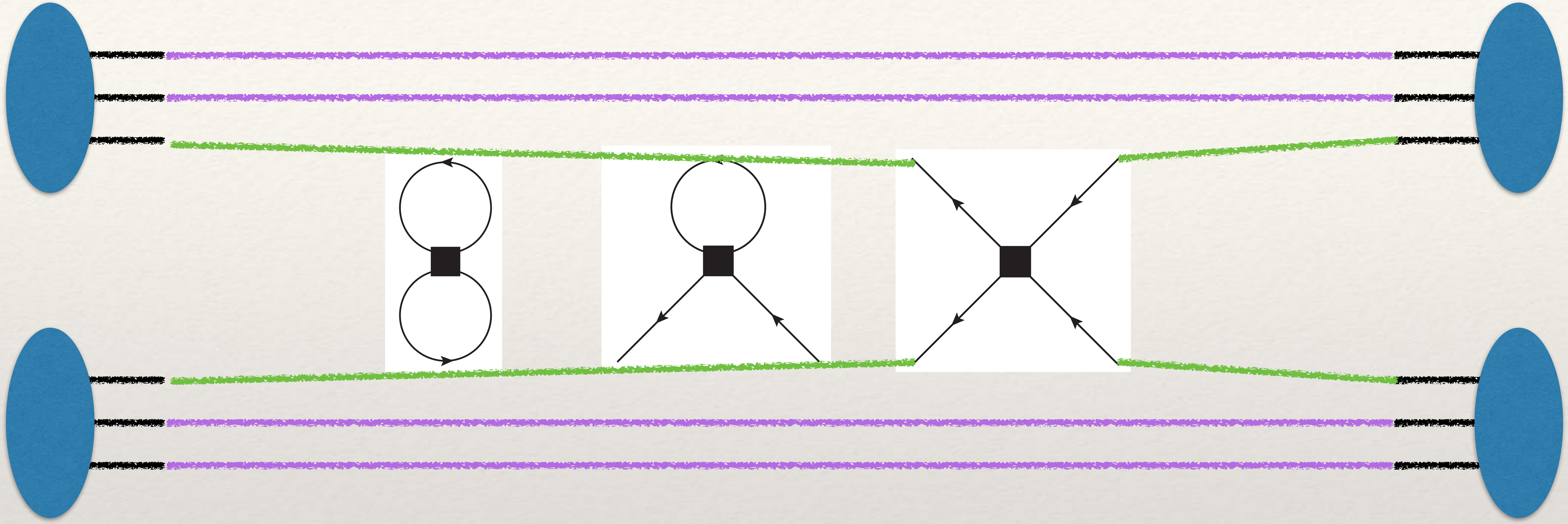

LQCD for Fundamental Physics in Nuclei: Parity Violation

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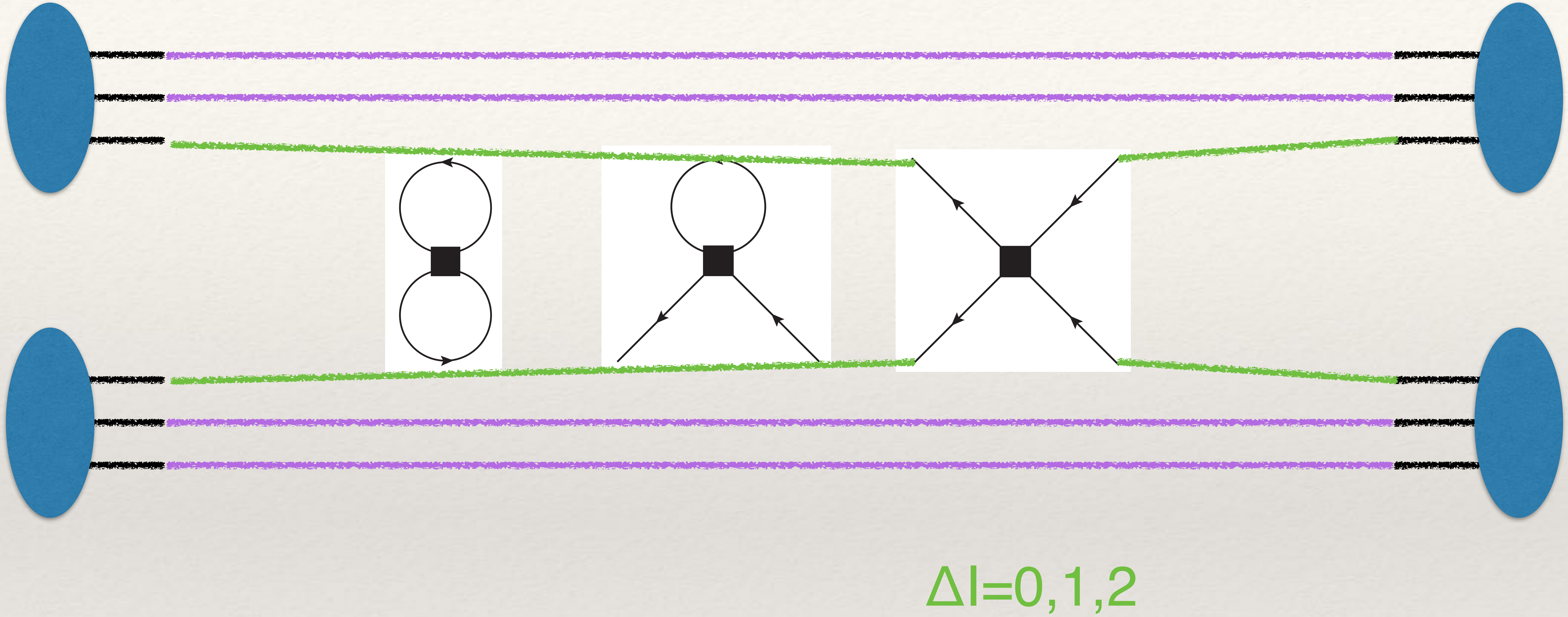
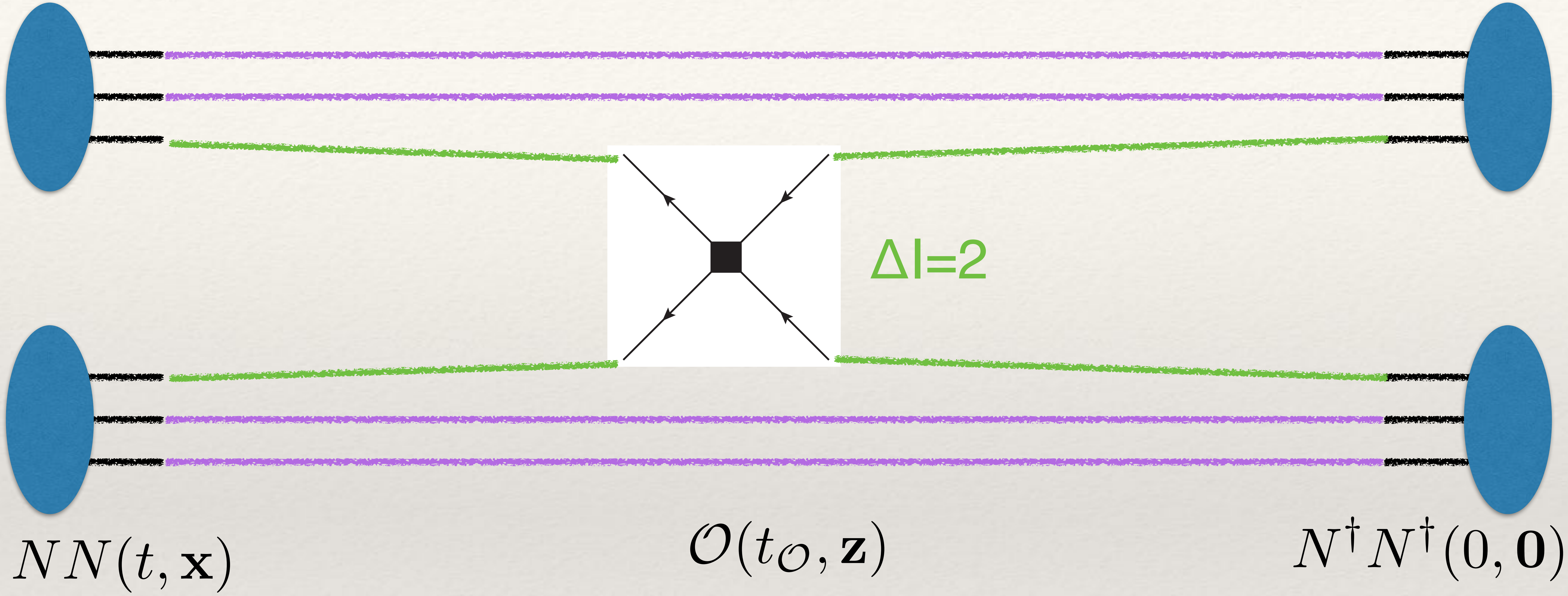


- The "disconnected" quark loops are numerically more expensive, and stochastically noisier
- The non-perturbative renormalization becomes more challenging also

LQCD for Fundamental Physics in Nuclei: Parity Violation

- What about $\langle NN | O_W | NN \rangle$?



- To project the operator, O, onto definite momentum, and to project the final NN state onto definite momentum, we need all-to-all propagators (expensive): $\sum_{\mathbf{x}}, \sum_{\mathbf{z}}$
- Not possible with (old) standard NN calculations with local creation operators and momentum space annihilation operators

LQCD for Fundamental Physics in Nuclei: Parity Violation

- What about $\langle NN | O_W | NN \rangle$?
  - We started the $\Delta I=2$, NN calculation in 2015 Kurth et al., 1511.02260
  - Ultimately, we decided that the growing concern regarding the NN bound-state controversy, combined with the challenge of performing the 4-quark matrix element calculation, were too severe to proceed
  - So we went back to basics to improve our NN calculations before trying to tackle the parity violating matrix element calculations

Lattice QCD for Fundamental Physics in Nuclei

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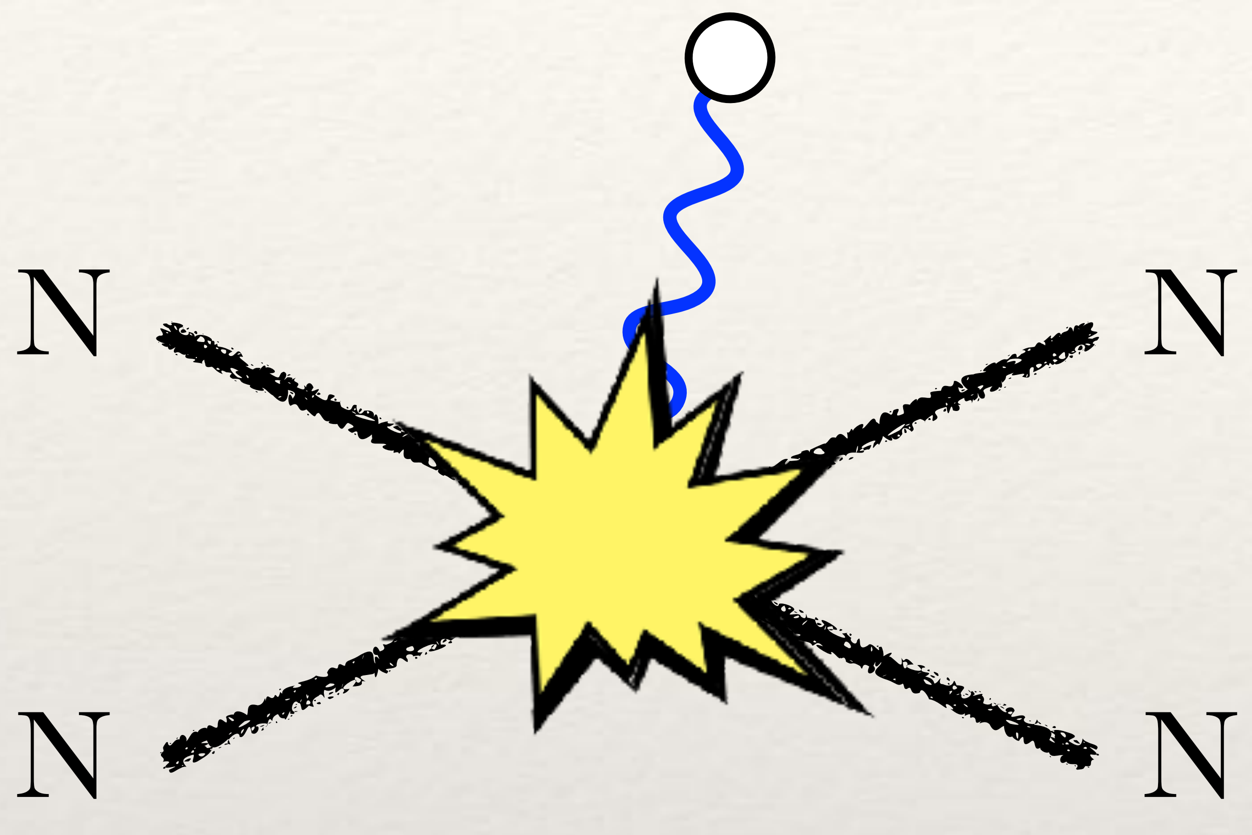
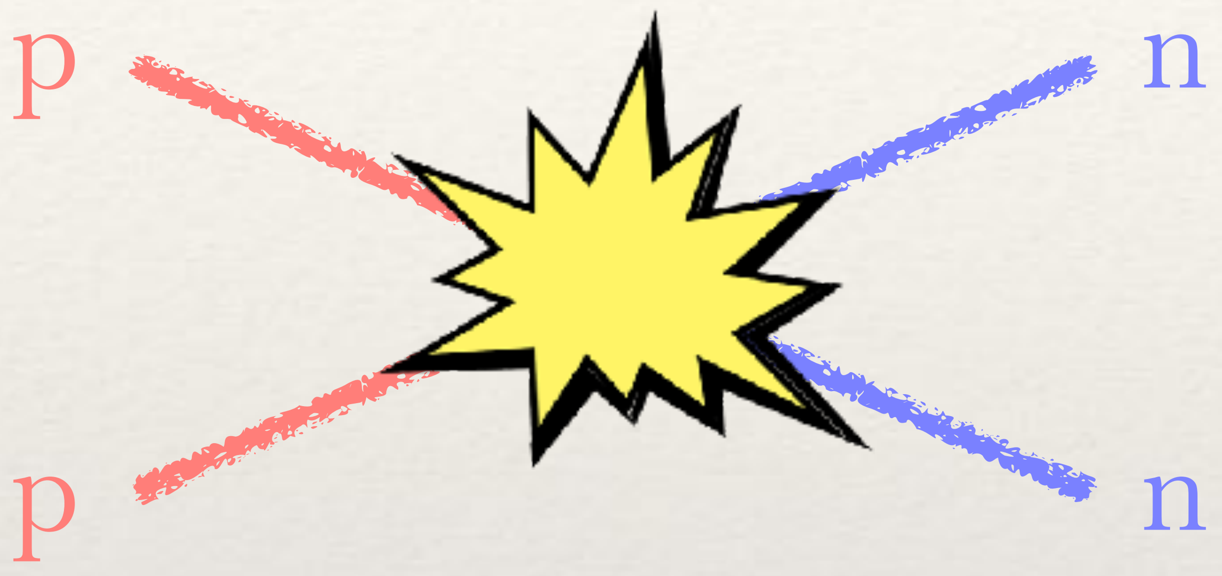




neutrinoless double β-decay
- long-range current
- short-range 4-quark operator

- I presented definite ways that LQCD can provide important and/or critical input to fundamental physics research with radioactive (and stable) nuclei

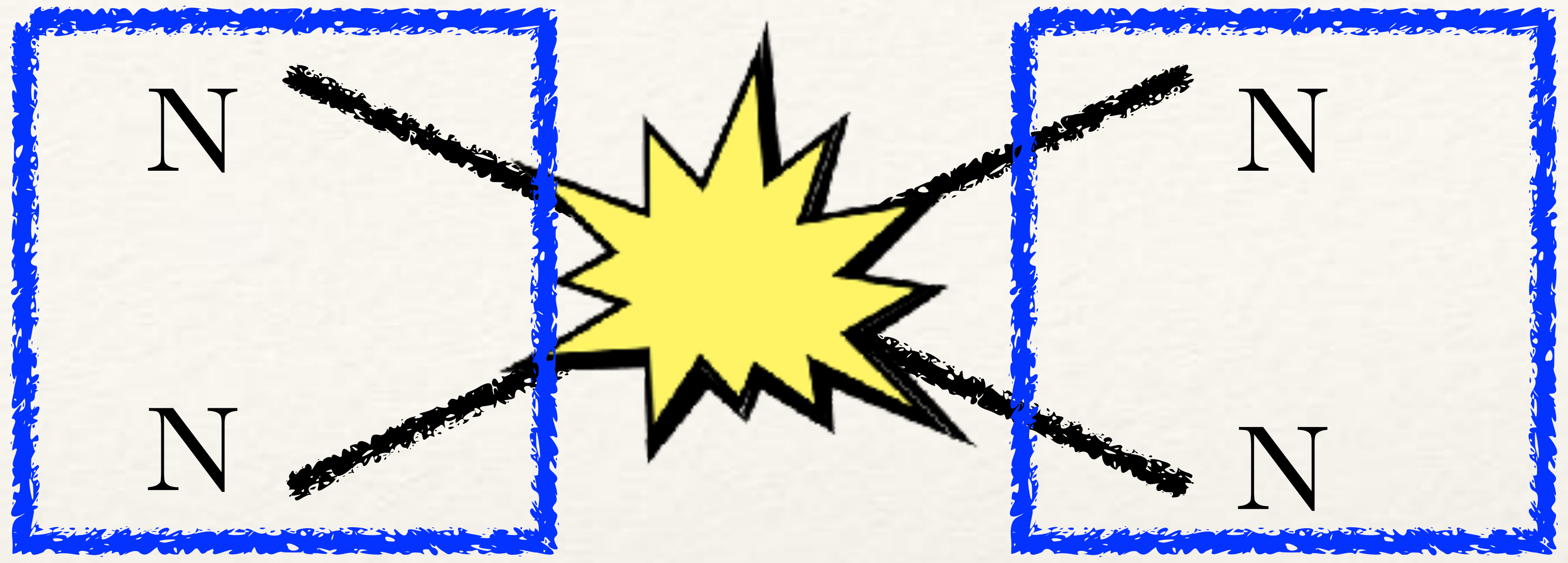
- I described several challenges that we are facing in such an endeavor

- My colleagues and I decided to focus on the NN interactions/spectroscopy before tackling matrix element calculations
  - Why not just go directly for the matrix elements (which are more phenomenologically relevant)?

  - If the NN spectrum is not correct - the matrix elements can be arbitrarily “wrong”

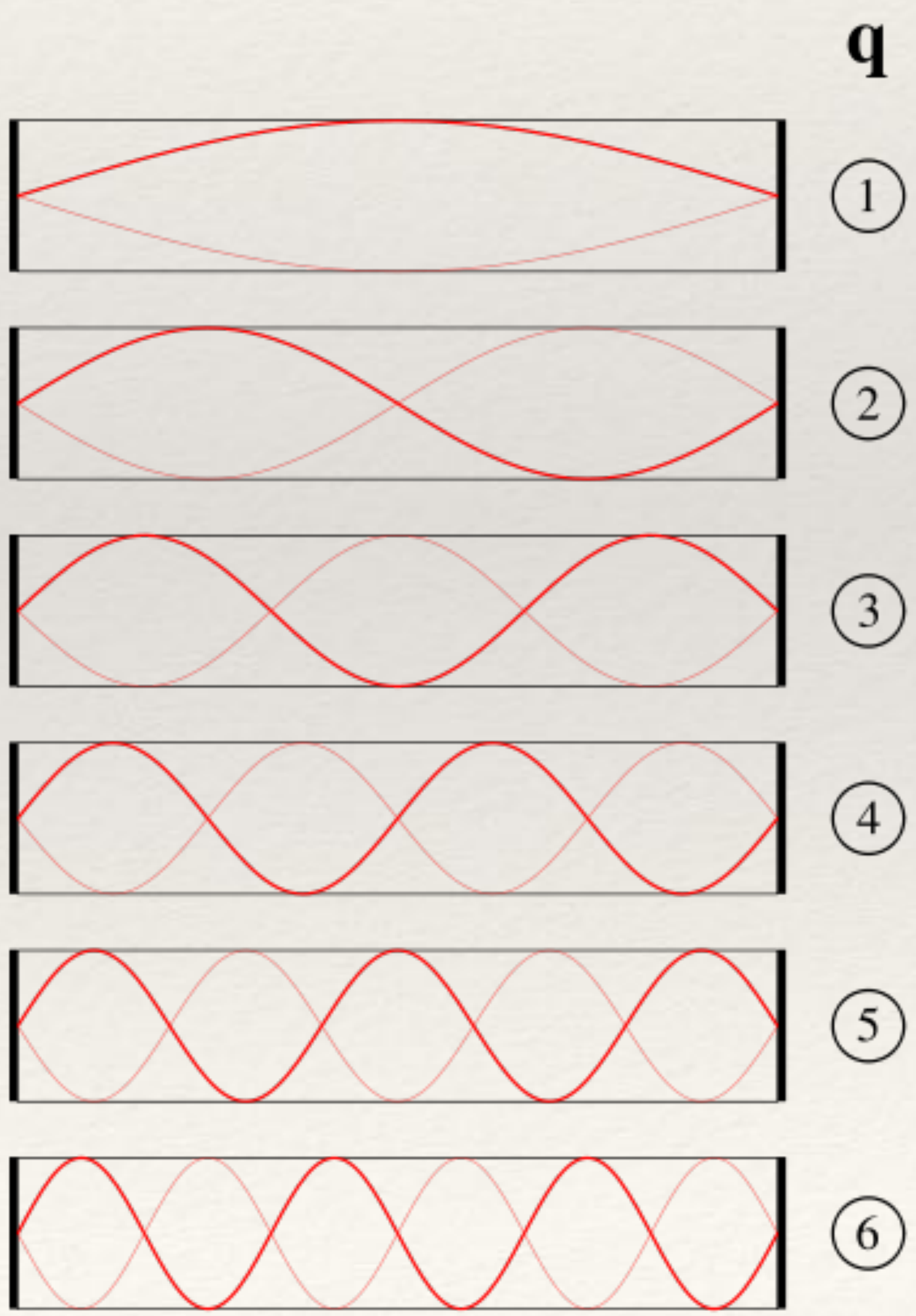
To bind or not to bind? A tale of two nucleons

- Computing two-nucleon interactions with lattice QCD



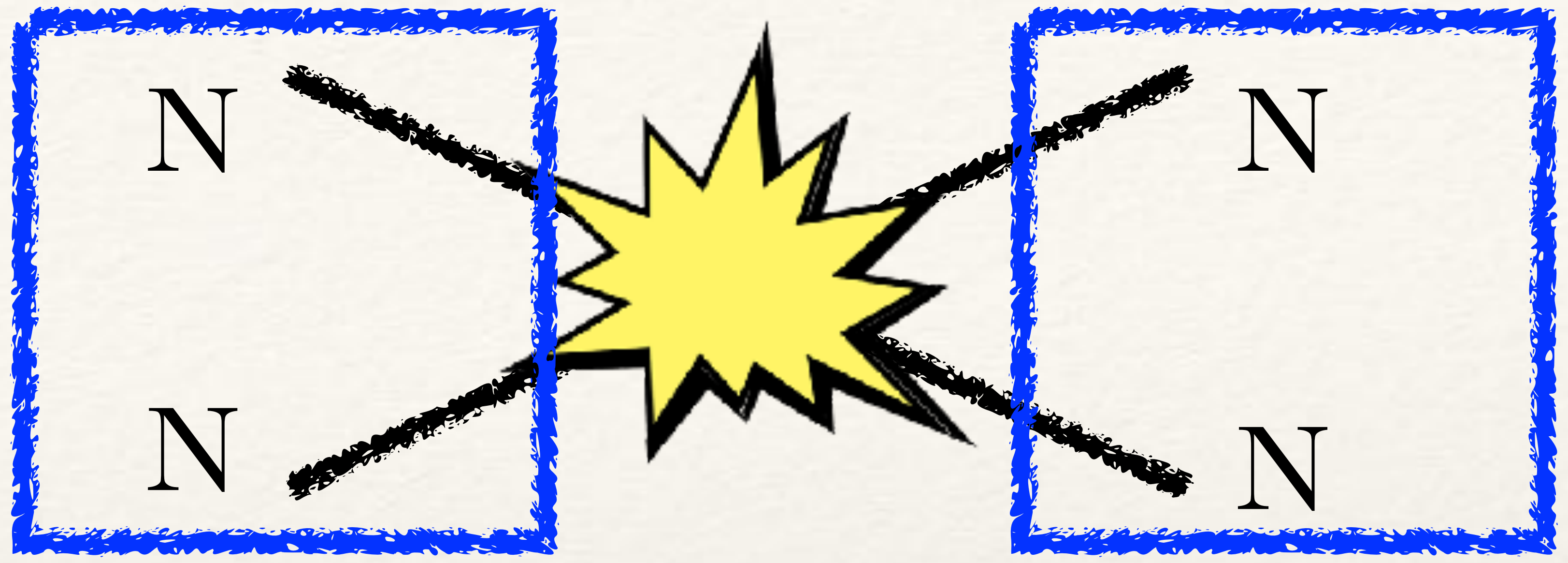
- LQCD calculations are performed in finite, periodic volumes of size $L \sim 3 - 6\text{fm}$
- There is no scattering in LQCD calculations
  - no asymptotic states
  - Euclidean spacetime
- Relate the finite volume spectrum to infinite volume scattering amplitude (Lüscher Quantization Condition)
  - free hadrons: $E_n = \sqrt{m^2 + p_n^2}, \quad p_n = \frac{2\pi}{L}n$
  - interacting hadrons: $E_q = \sqrt{m^2 + q^2}$,

    $q \neq p_n \longrightarrow T(q) \propto e^{2i\delta(q)} - 1 \propto \frac{1}{q\cot\delta(q) - iq}$ (single channel approx)



To bind or not to bind? A tale of two nucleons

- How do we determine the energy, E_q?



$$\begin{aligned} C(t) &= \sum_{\mathbf{x}} \langle \Omega | O(t,\mathbf{x}) O^\dagger(0,\mathbf{0}) | \Omega \rangle \\ &= \sum_{\mathbf{x}} \langle \Omega | e^{\hat{H}t} O(0,\mathbf{x}) e^{-\hat{H}t} O^\dagger(0,\mathbf{0}) | \Omega \rangle \\ &= \sum_n \sum_{\mathbf{x}} \langle \Omega | e^{\hat{H}t} O(0,\mathbf{x}) e^{-\hat{H}t} | n \rangle \langle n | O^\dagger(0,\mathbf{0}) | \Omega \rangle \\ &= \sum_n e^{-E_n t} \sum_{\mathbf{x}} \langle \Omega | O(0,\mathbf{x}) | n \rangle \langle n | O^\dagger(0,\mathbf{0}) | \Omega \rangle \\ &= \sum_n e^{-E_n(\mathbf{p}=0)t} \langle \Omega | O(0) | n, \mathbf{p}=0 \rangle \langle n, \mathbf{p}=0 | O^\dagger(0) | \Omega \rangle \\ &= \sum_n e^{-E_n t} z_n z_n^\dagger \end{aligned}$$

focus on 0-momentum

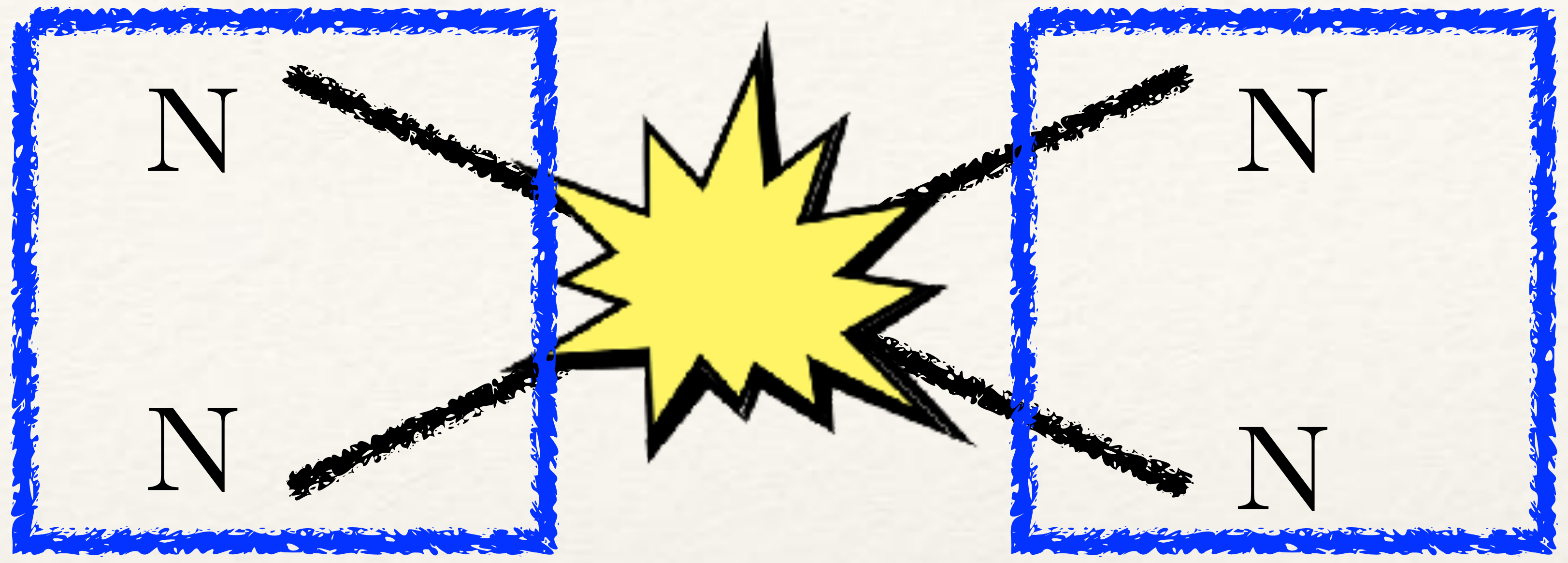
time-evolve operator

multiply by 1, $1 = \sum_n |n\rangle\langle n|$

define vacuum to have 0-energy

sum of exponentials

To bind or not to bind? A tale of two nucleons



- How do we determine the energy, E_q?
  $C(t) = \sum_n e^{-E_n t} z_n z_n^\dagger$

- Exponential decay of signal with respect to the variance
  - $\frac{S}{N}(t) \approx \frac{1}{\sqrt{N}} e^{-A(M_N - \frac{3}{2} m_\pi)t}$

- Physics of interest (interaction energies) are at the per-mille level of the total energy
  Deuteron: $B_D \approx 2.2$ MeV, $E_{NN} \approx 2$ GeV

- The excited state energy gap is set by kinetic energy of nucleons, much smaller than the typical inelastic excited state energy

- pion production threshold becomes very close to $2M_N$ at m_π^{phys}

- short-time is polluted by excited states (as can be intermediate times) while late times are too noisy to resolve signals - and we must precisely determine a per-mille contribution to the total energy

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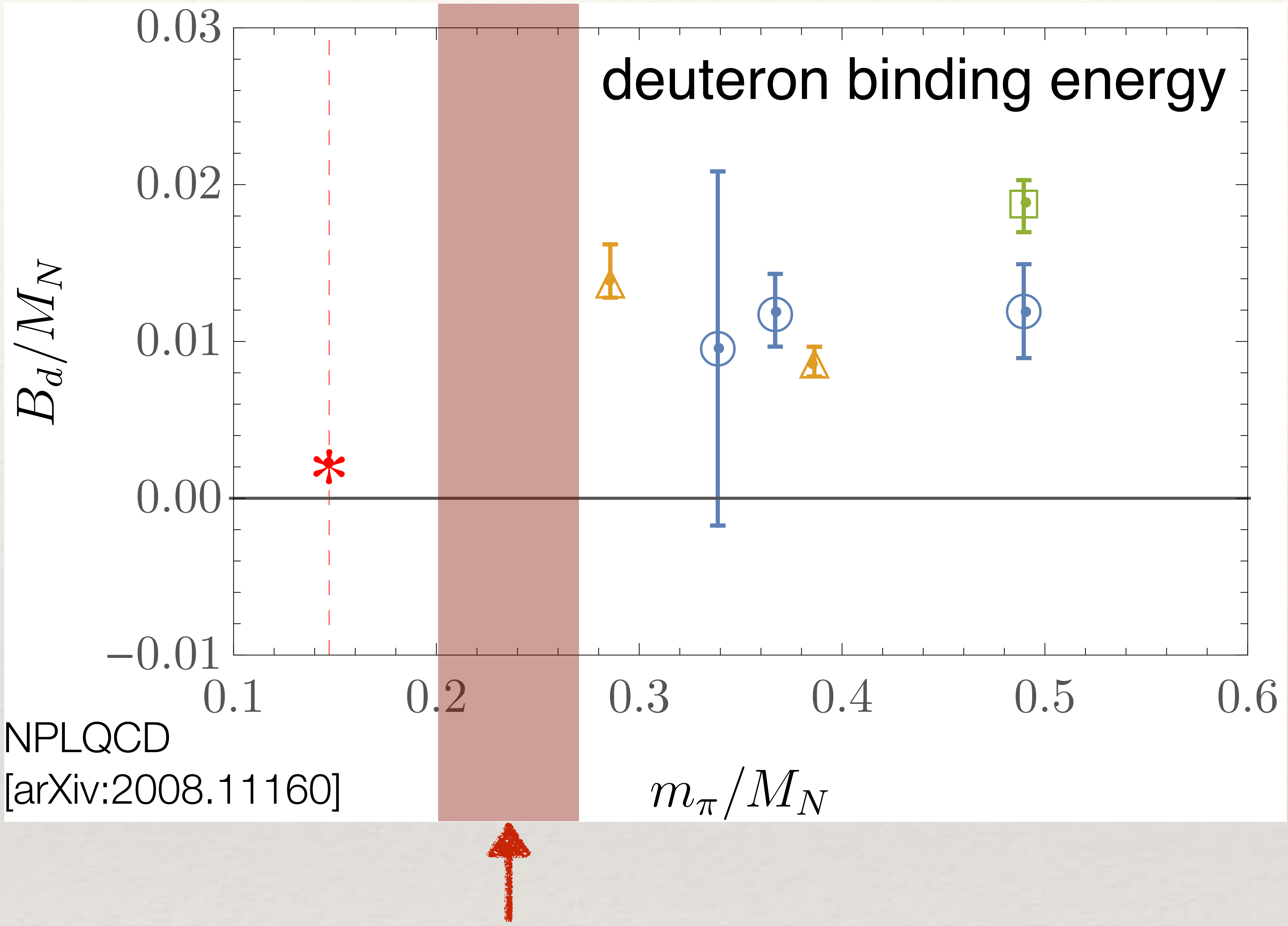
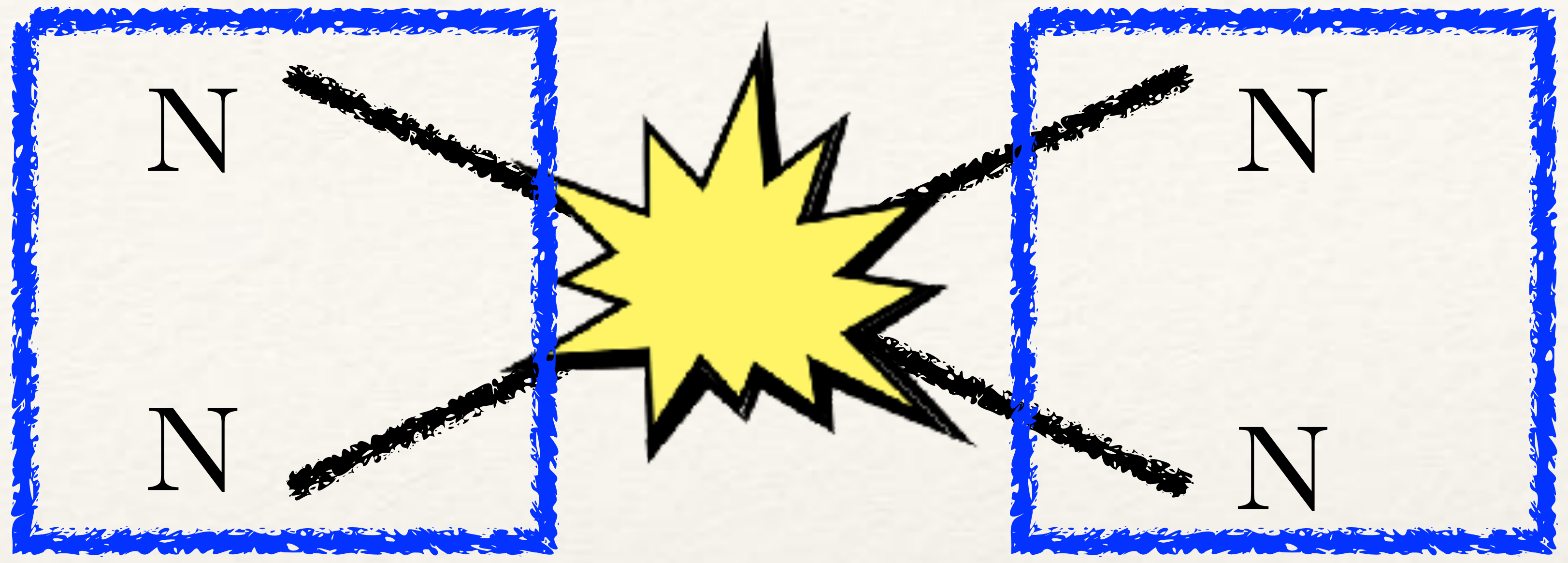


Estimated upper range of validity of NN EFT



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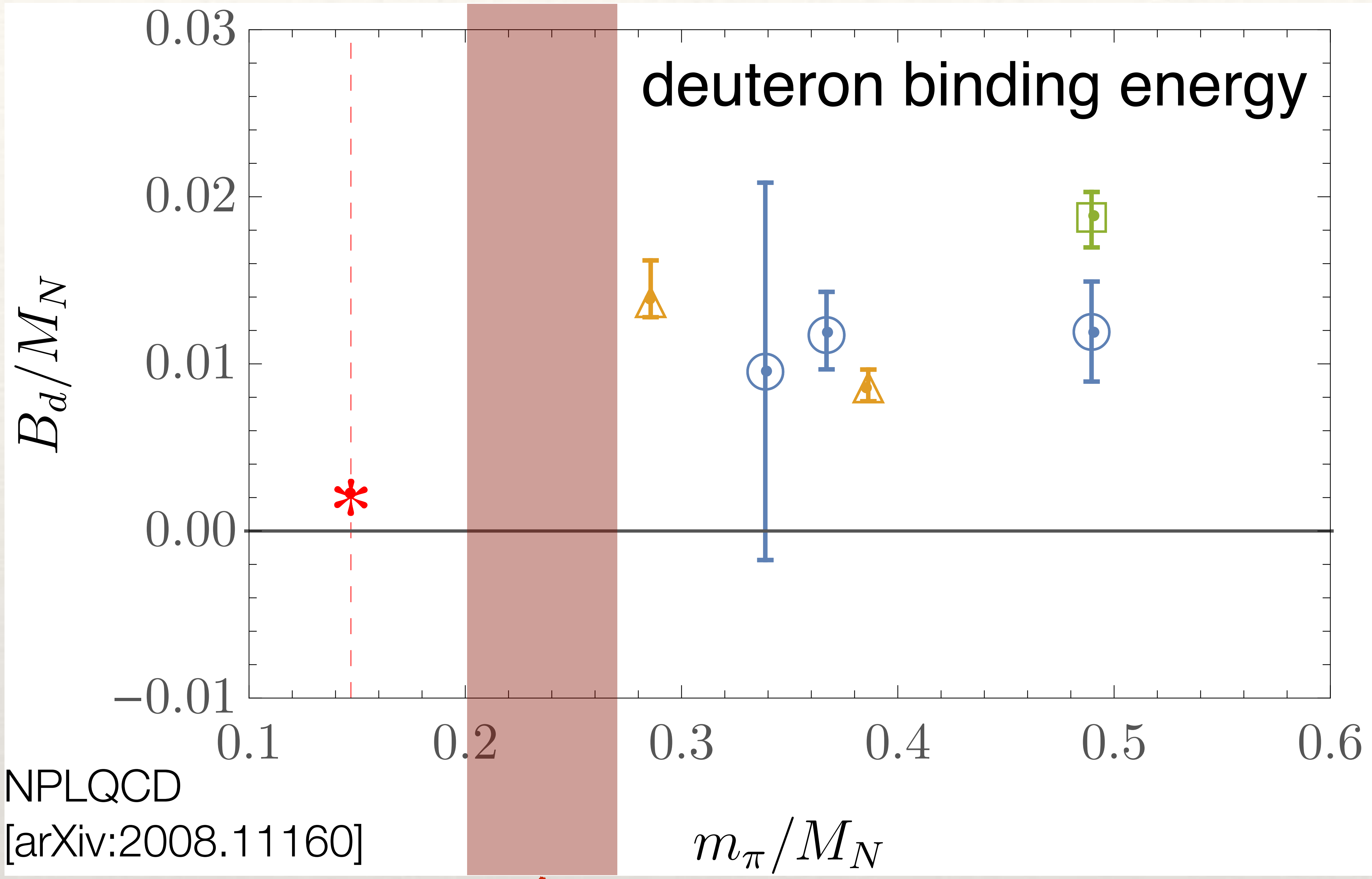
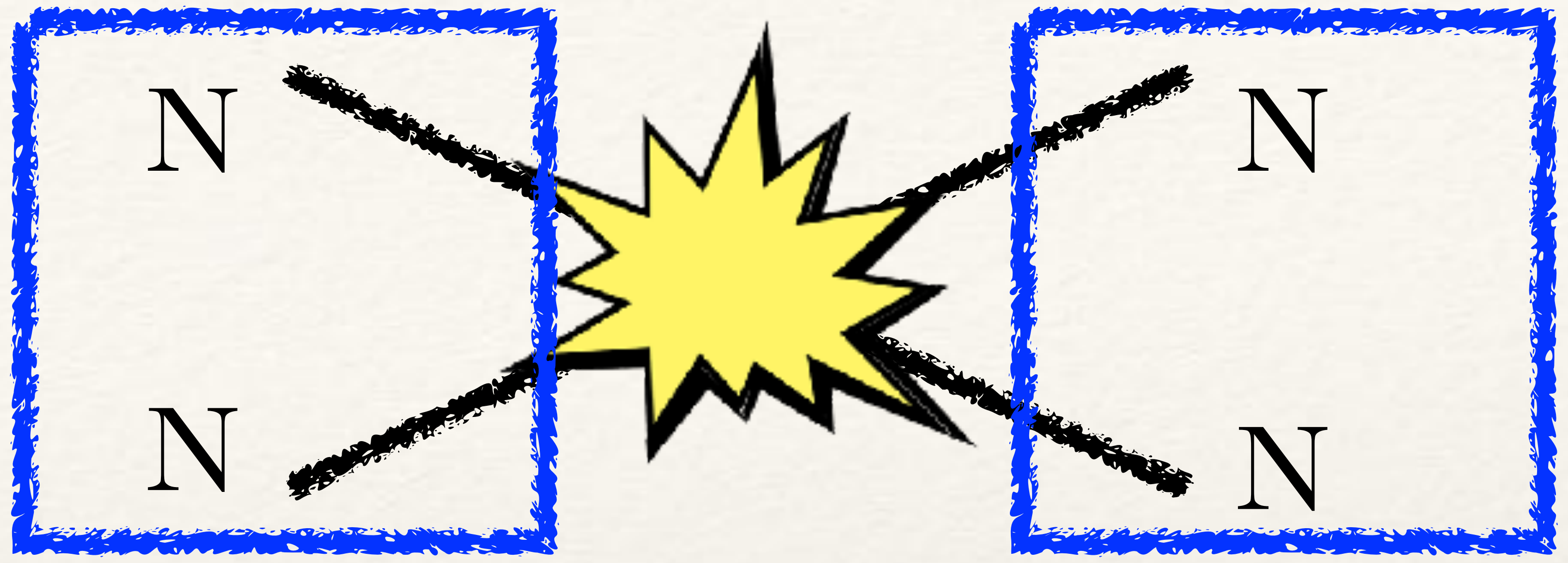


NPLQCD
[arXiv:2008.11160]

Estimated upper range of validity of NN EFT



2006 NPLQCD - first dynamical LQCD calculations of NN

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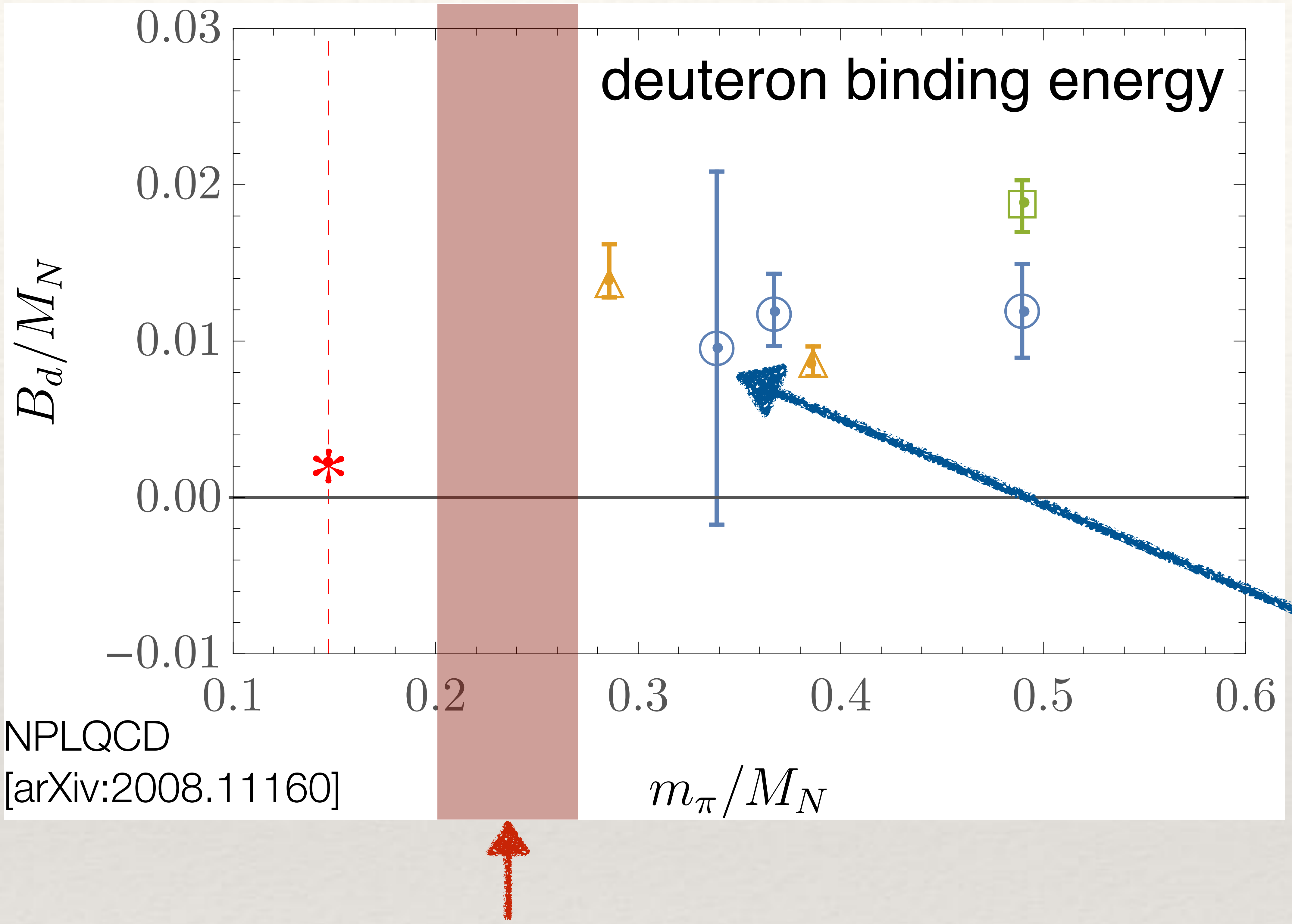

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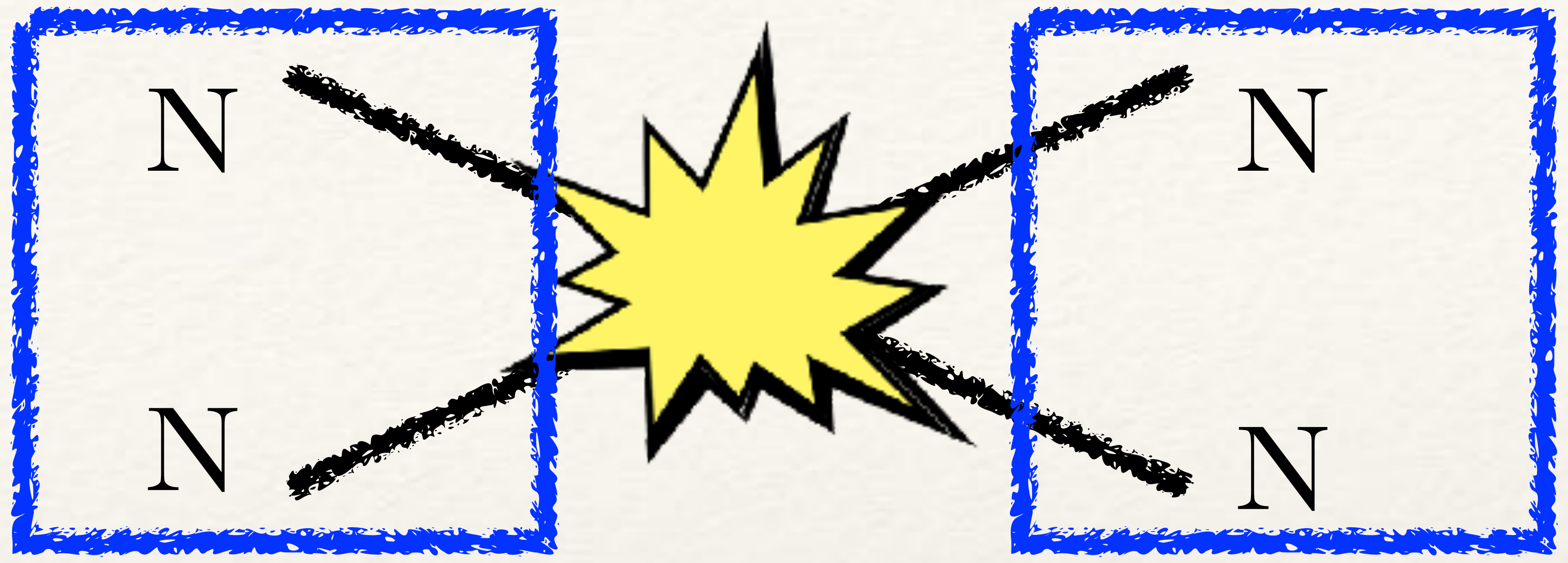

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2011 NPLQCD $M\pi \simeq 390$ MeV

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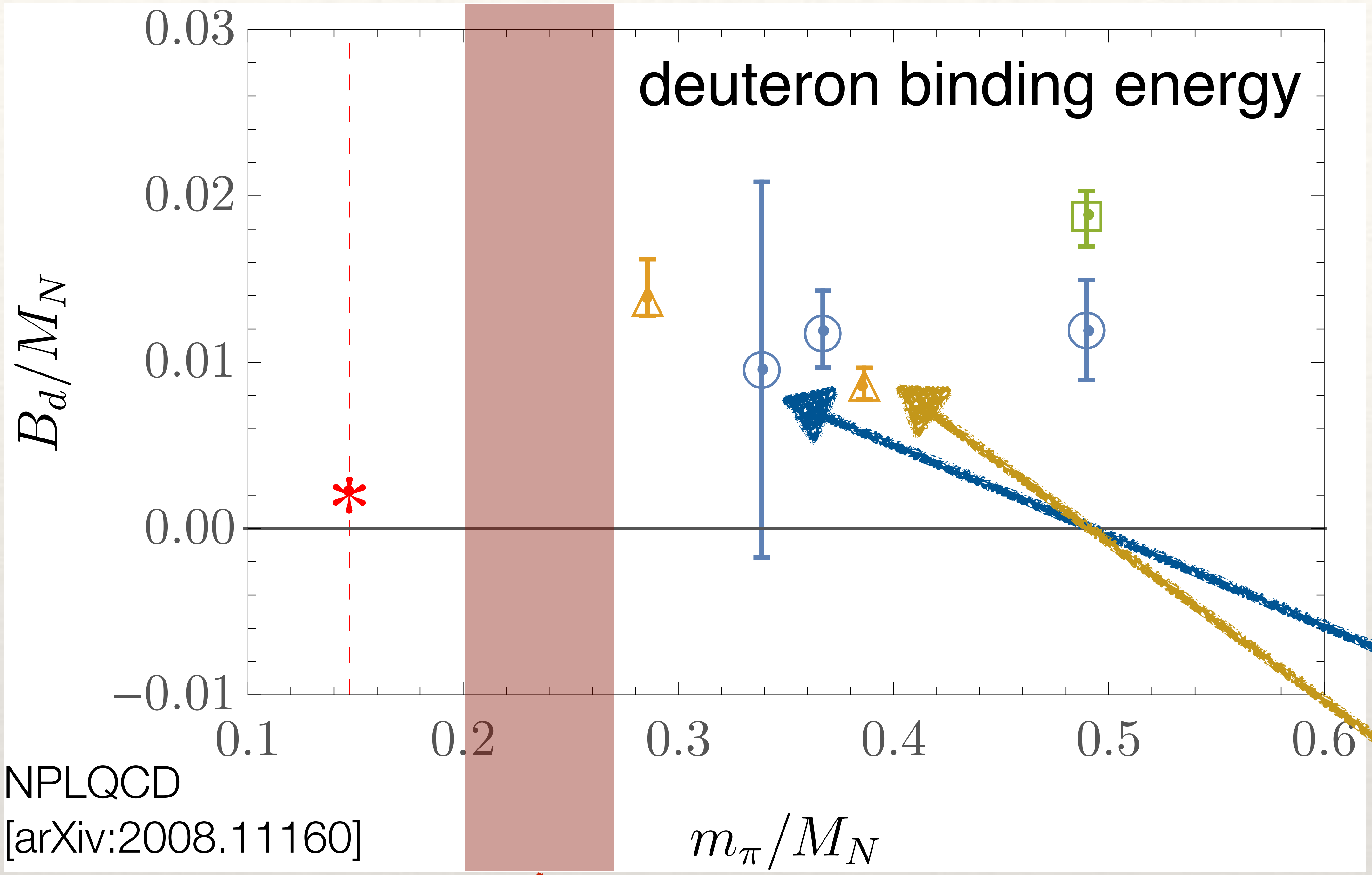

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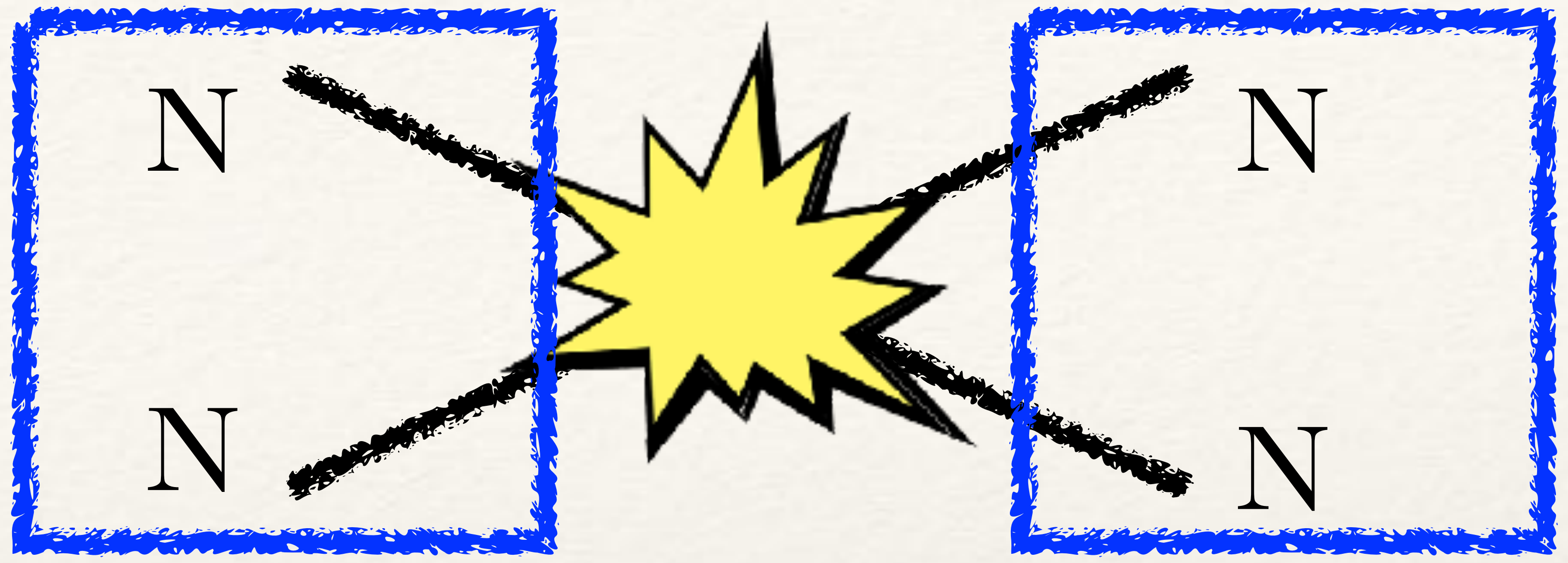

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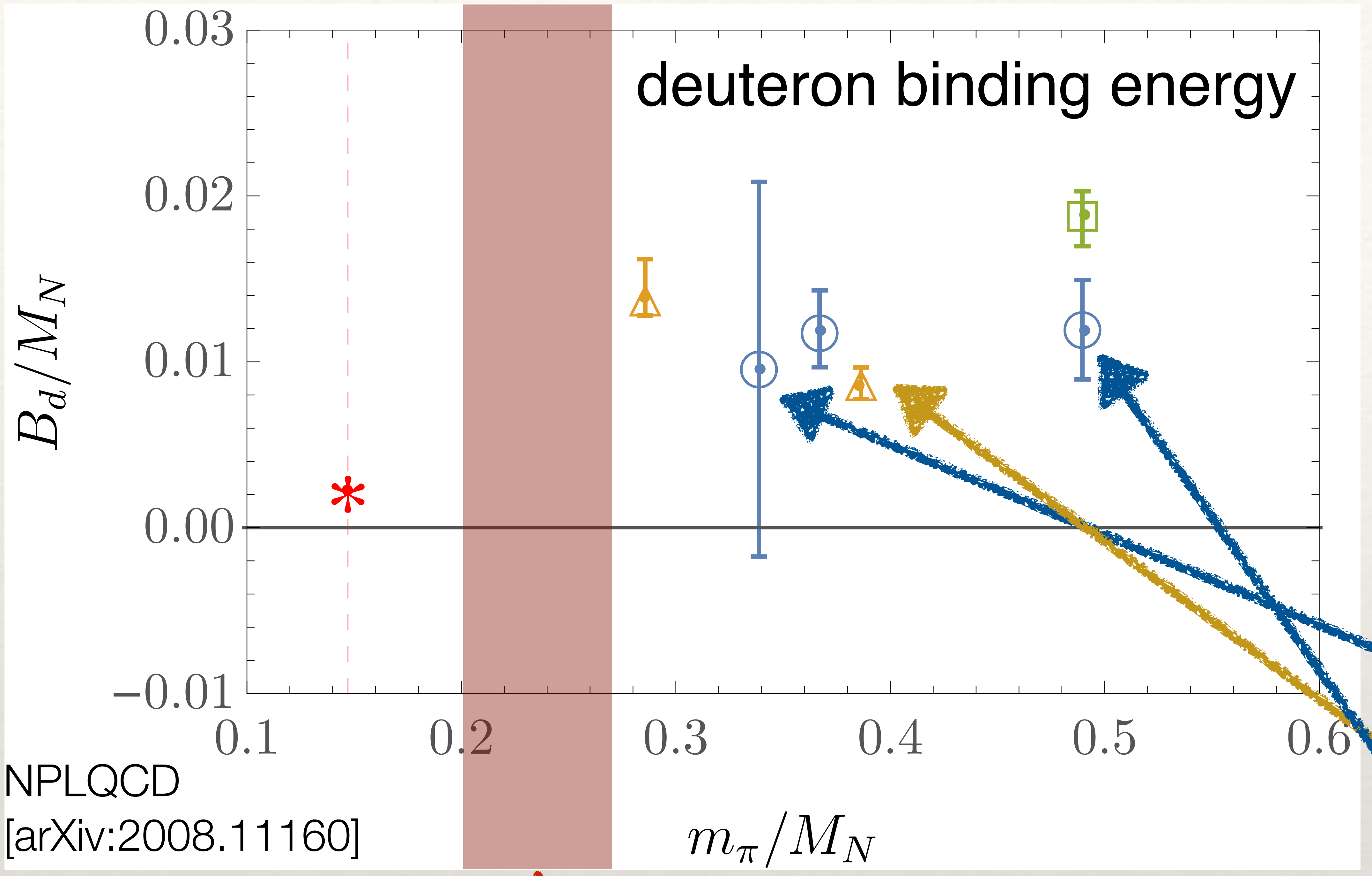

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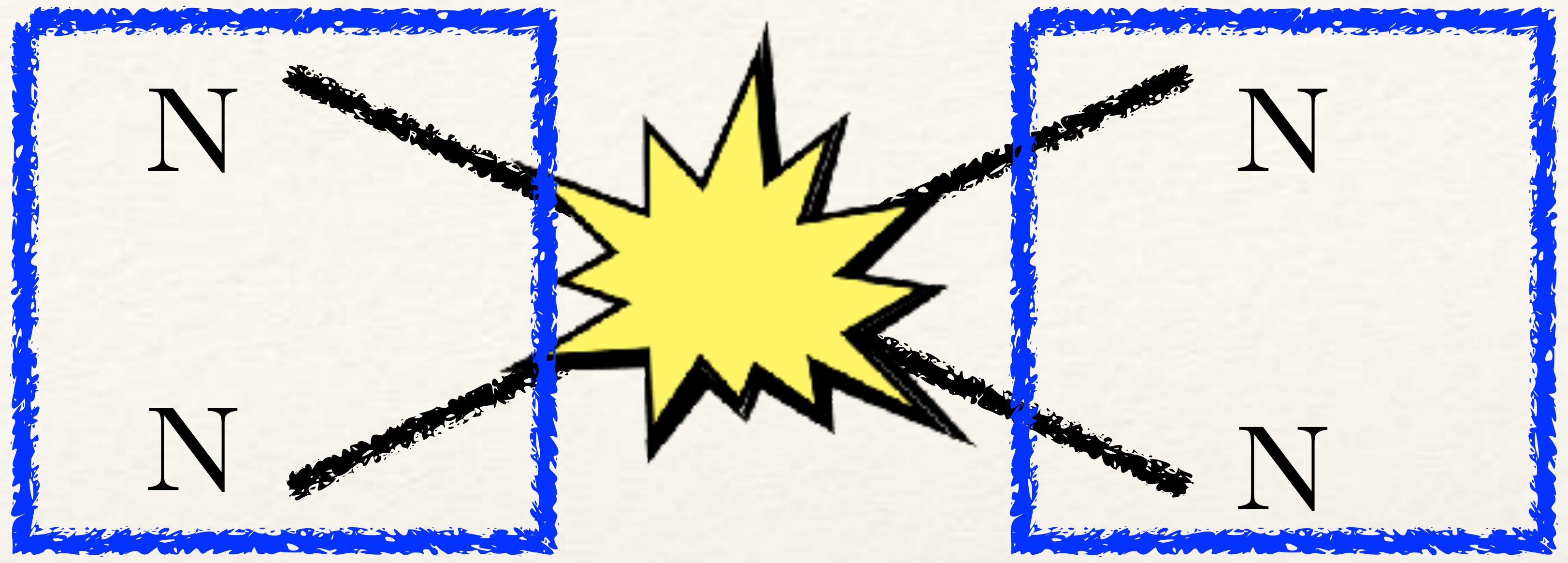

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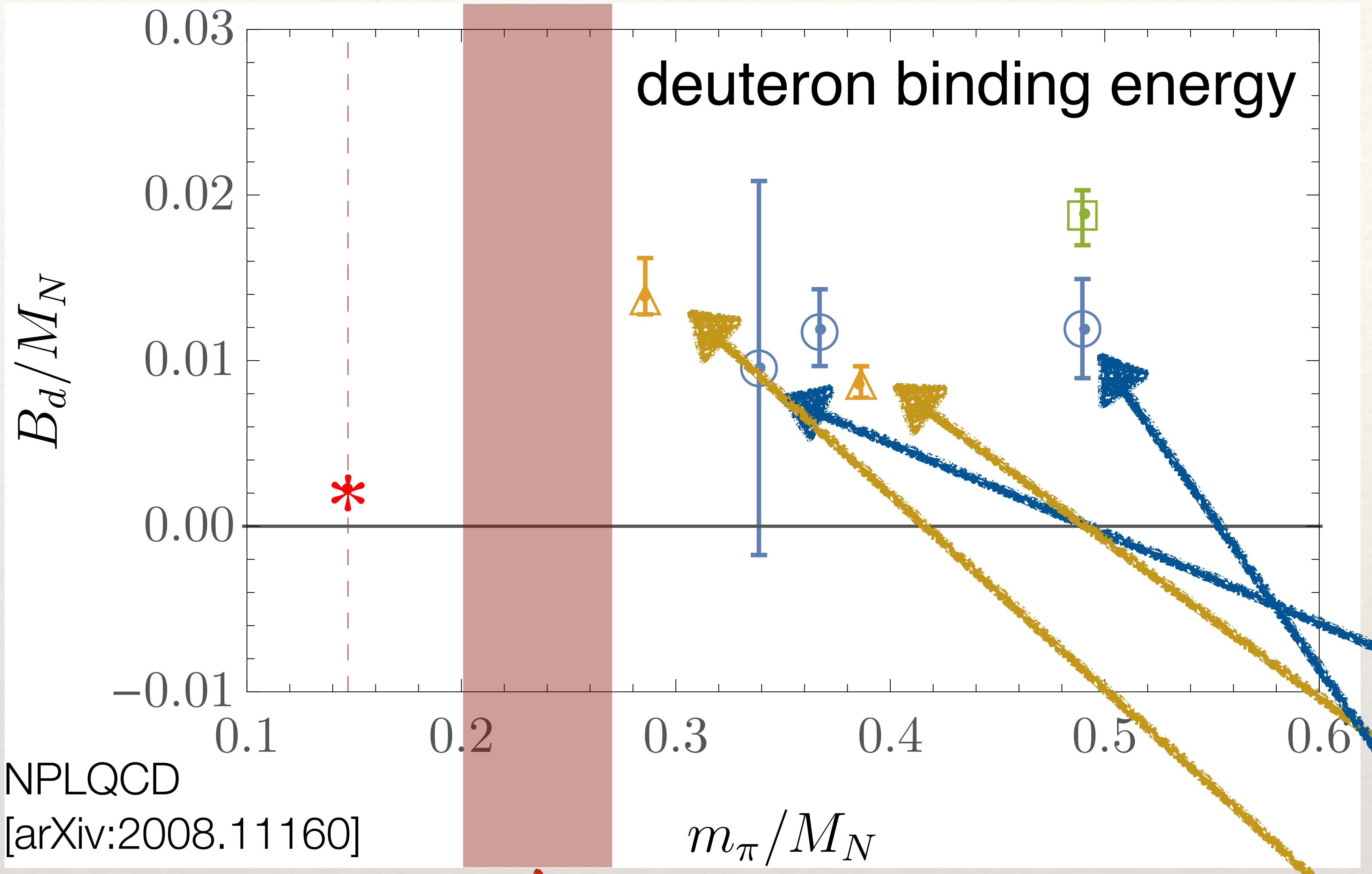

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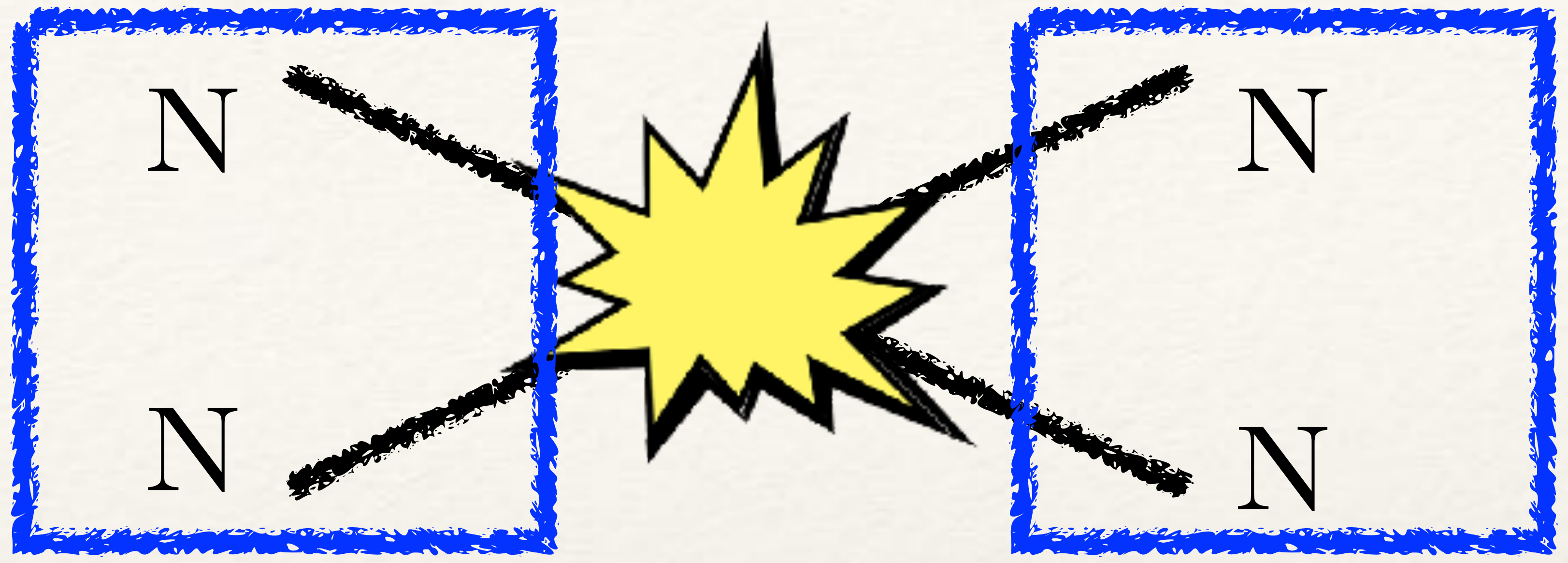

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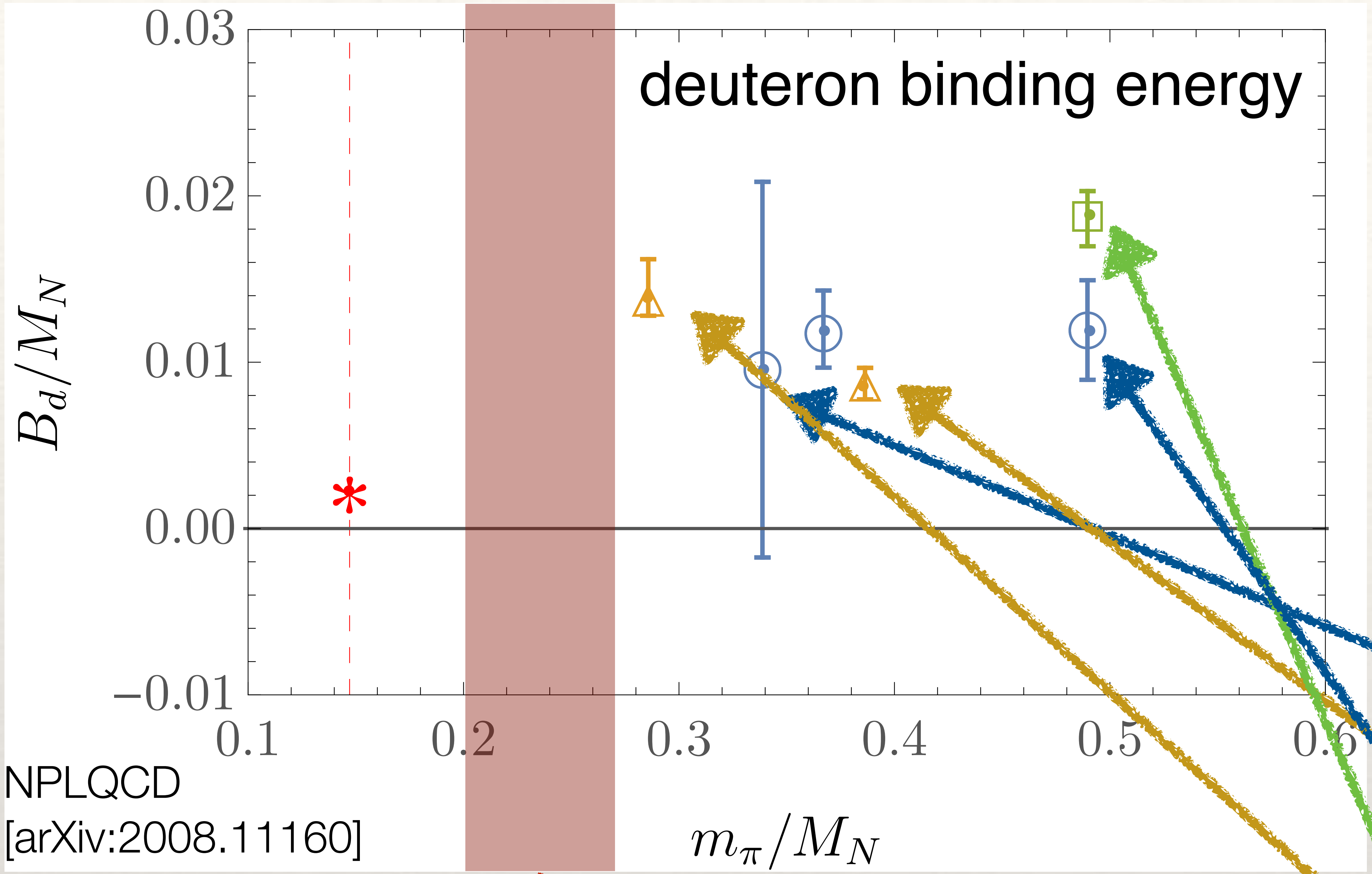
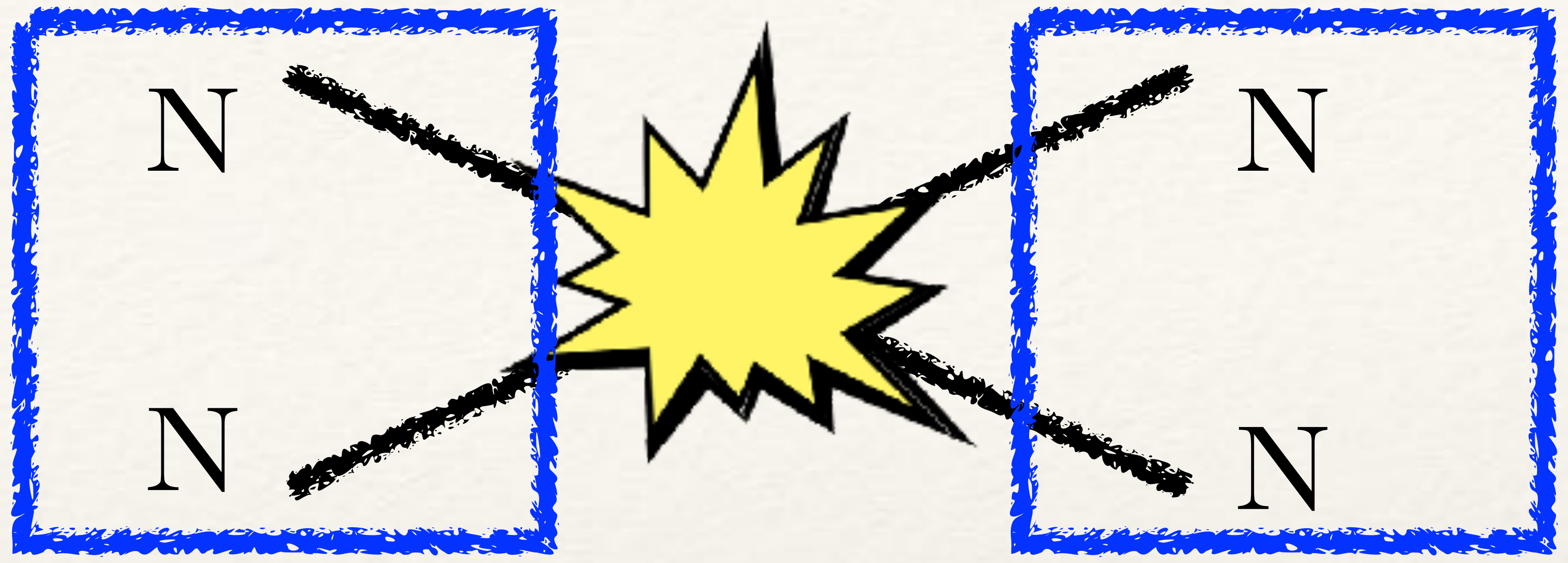
2015 Yamazaki et al. $M\pi \simeq$ 310 MeV

To bind or not to bind? A tale of two nucleons



Estimated upper range of
validity of NN EFT



2006 NPLQCD - first dynamical LQCD calculations of NN

2011 NPLQCD $M\pi \simeq$ 390 MeV

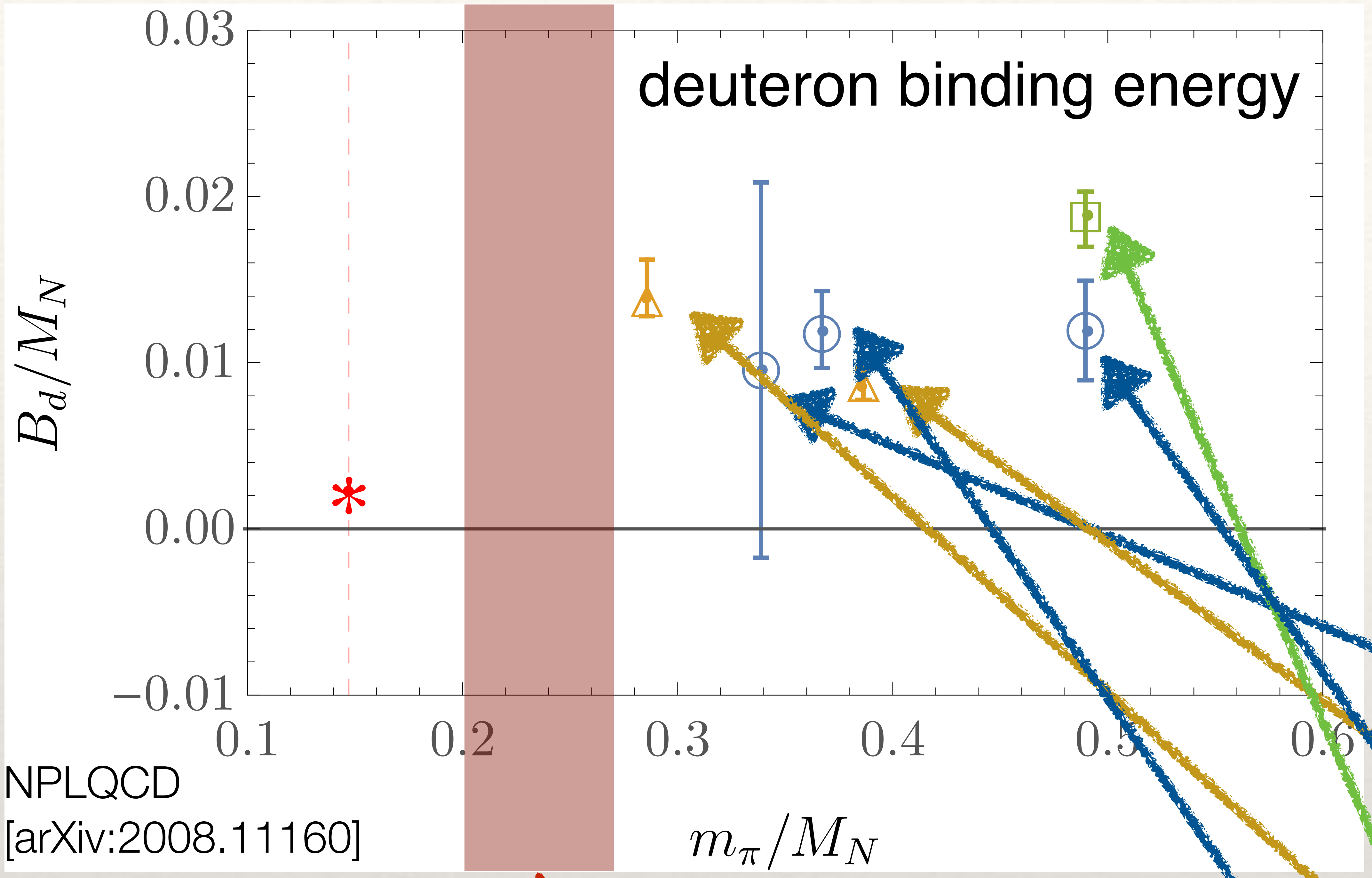
2012 Yamazaki et al. $M\pi \simeq$ 510 MeV

2012 NPLQCD $M\pi \simeq$ 800 MeV

2015 Yamazaki et al. $M\pi \simeq$ 310 MeV

2015 CalLat $M\pi \simeq$ 800 MeV + P,D,F waves

To bind or not to bind? A tale of two nucleons



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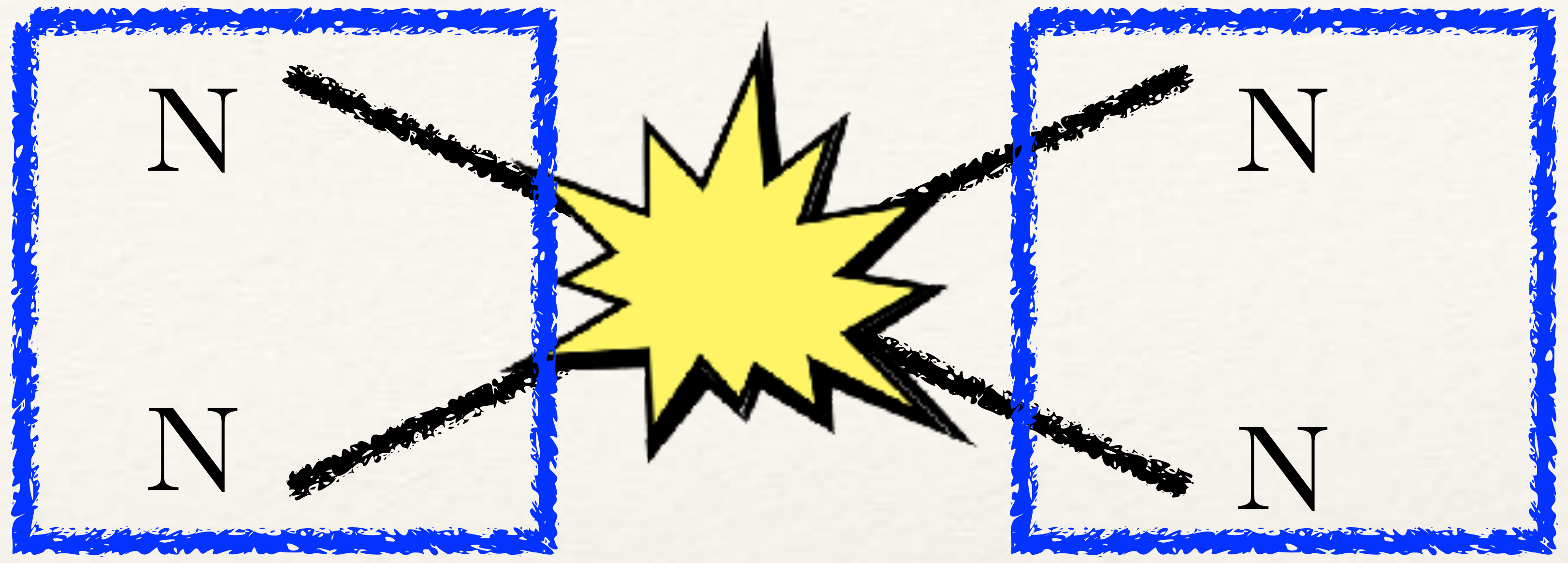

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2015 CalLat $M\pi \simeq 800$ MeV + P,D,F waves

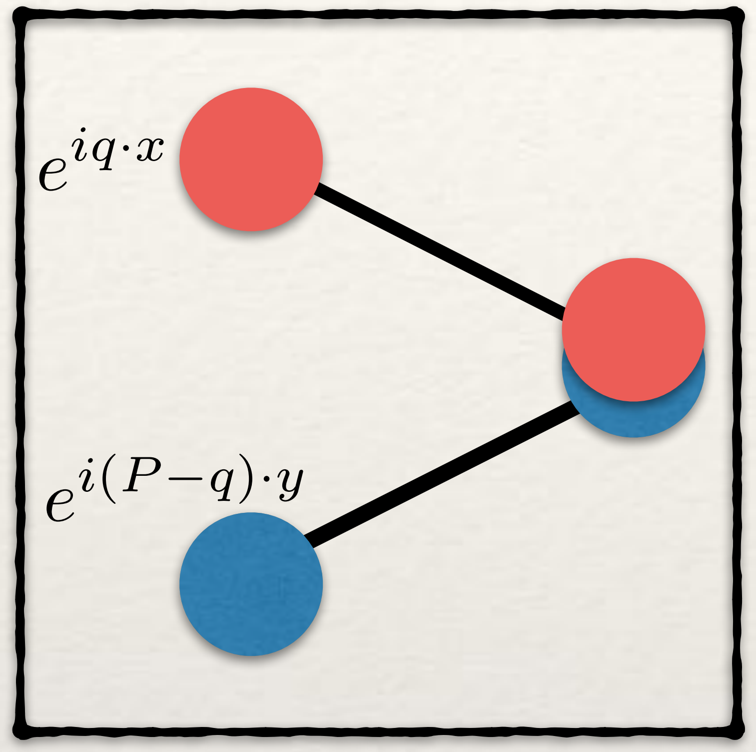
2015 NPLQCD $M\pi \simeq 450$ MeV

2020 NPLQCD $M\pi \simeq 450$ MeV

(blue = work I was involved in)

Do di-nucleons bind @ heavy pion mass?

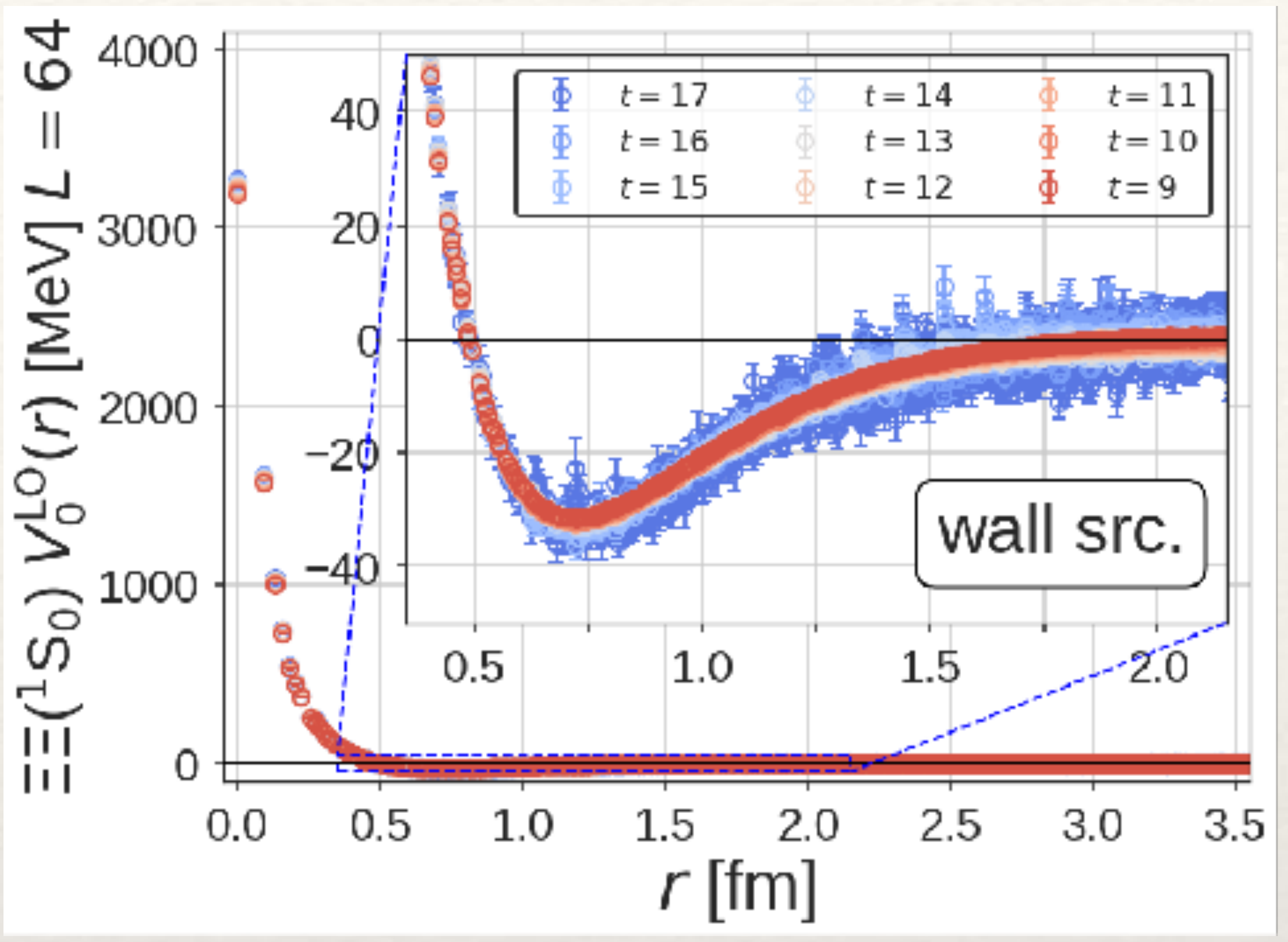
NPLQCD,
Yamazaki et al.,
CalLat (2015)



Compact, hexa-quark
creation operator

Deep bound di-nucleons

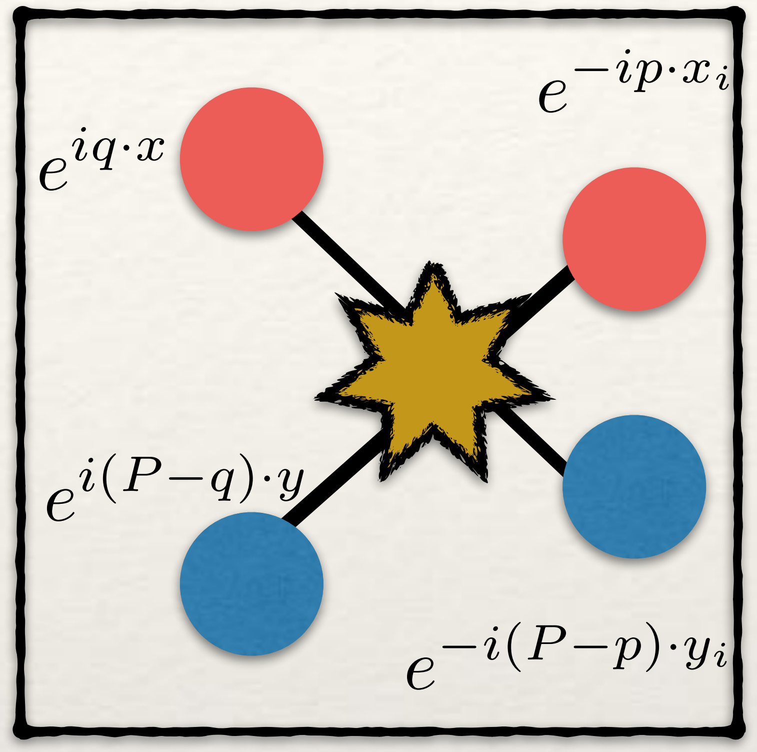
HAL QCD Potential



diffuse - wall source

no bound state

"Mainz" (Distillation)
CoSMoN (stochastic LapH
NPLQCD (sparsened momentum)



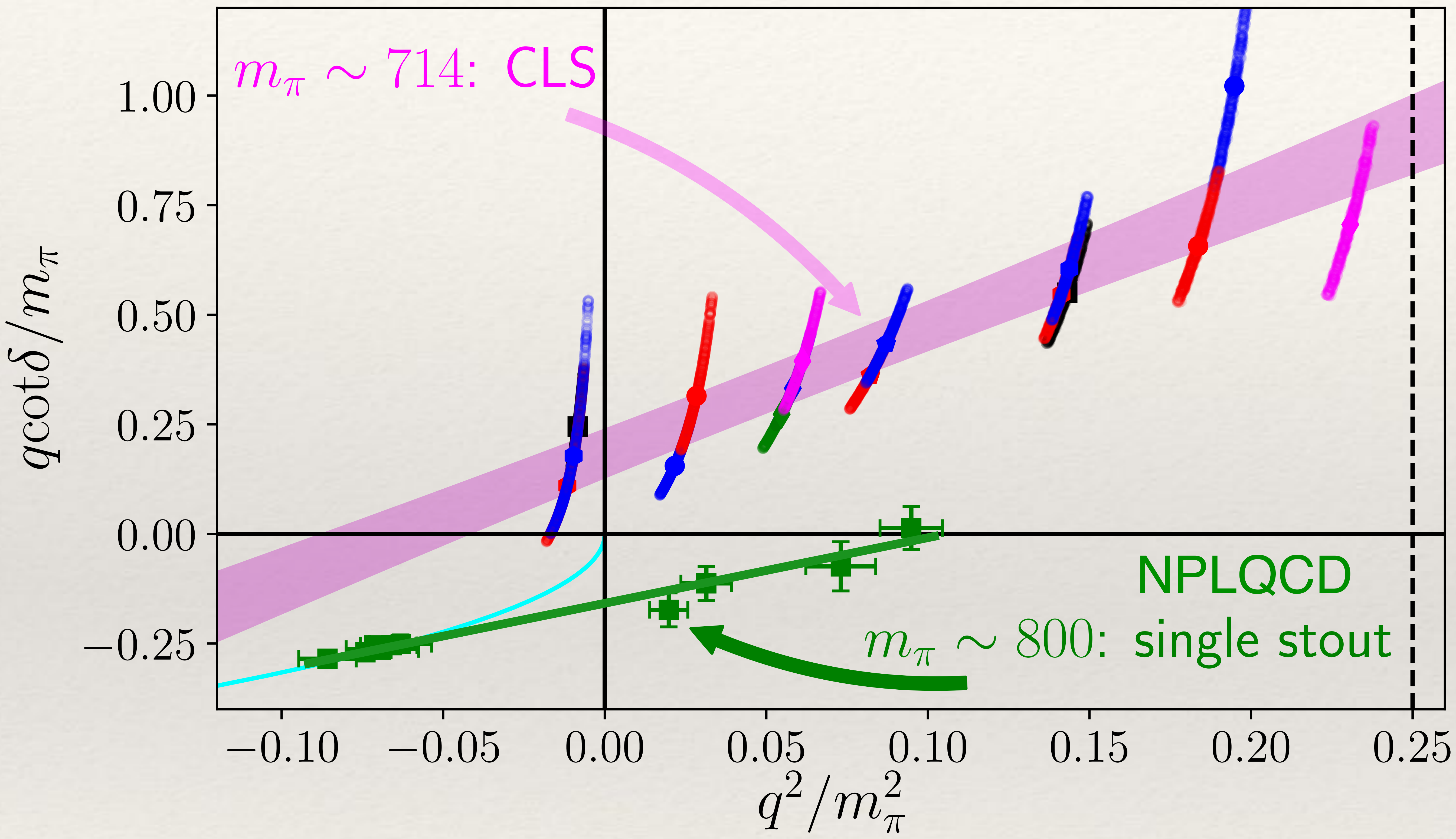
momentum-space
creation & annihilation
positive-definite correlation matrix

no bound state

The methods lead to different spectrum!
But, the spectrum can not depend upon the creation/annihilation operators!
At least one method must be wrong!

- ❑ To investigate the discrepancy - compute all methods on the same gauge configurations
- ❑ work at $m_u = m_d = m_s \approx m_s^{\rm phys}$ to match previous work and reduce resource requirements

- 16 energy levels with (expected) negligible overlap with non S-wave



- We find a virtual bound state (like dineutron) - a purely imaginary solution with negative sign

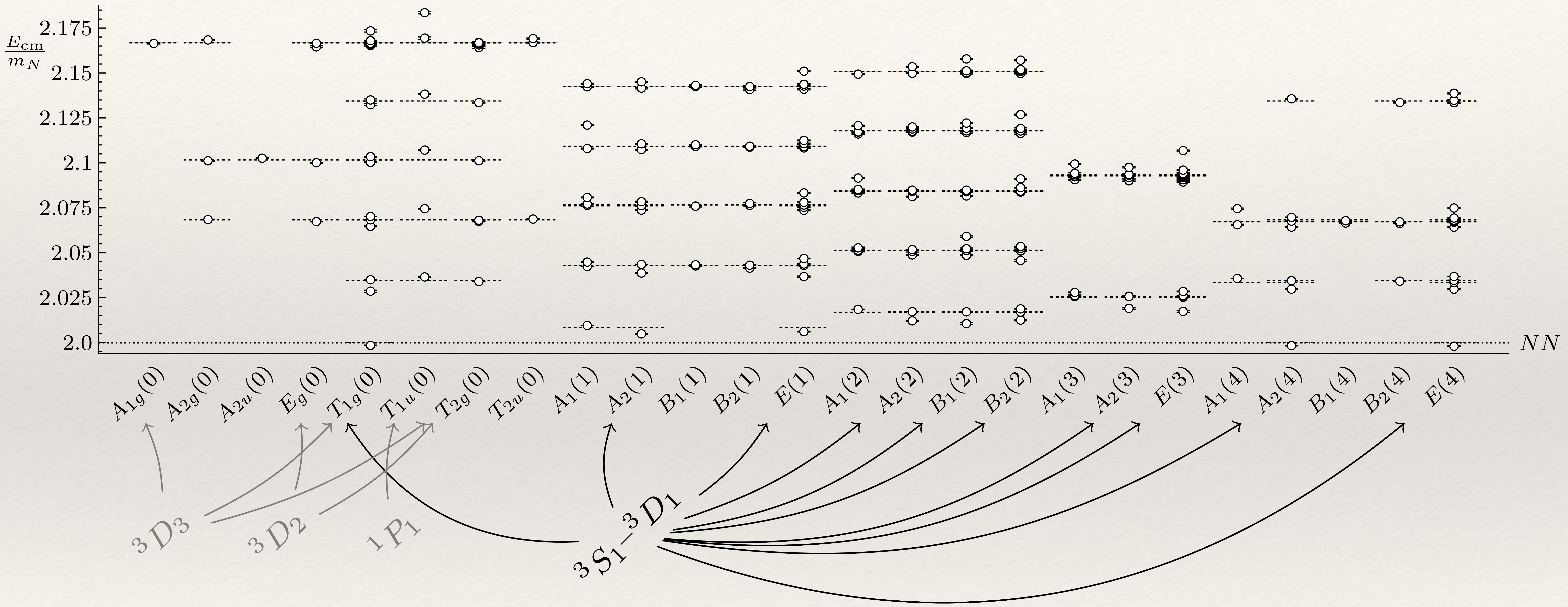
$$\frac{q_-^{\rm deut}}{m_\pi} = -\,i0.132(32)$$

- We can infer the size of the potential from causality and unitarity: Wigner PRD 98 (1955), Phillips and Cohen PLB 390 (1997)

$$r_0 \leq 2\left[R - \frac{R^2}{a} + \frac{R^3}{3a^2}\right]\,, \quad m_\pi R \gtrsim 2.0\,, \quad R \gtrsim 0.55 \text{ fm}$$

More costly – but MANY more energy levels

- arXiv:2009.11825

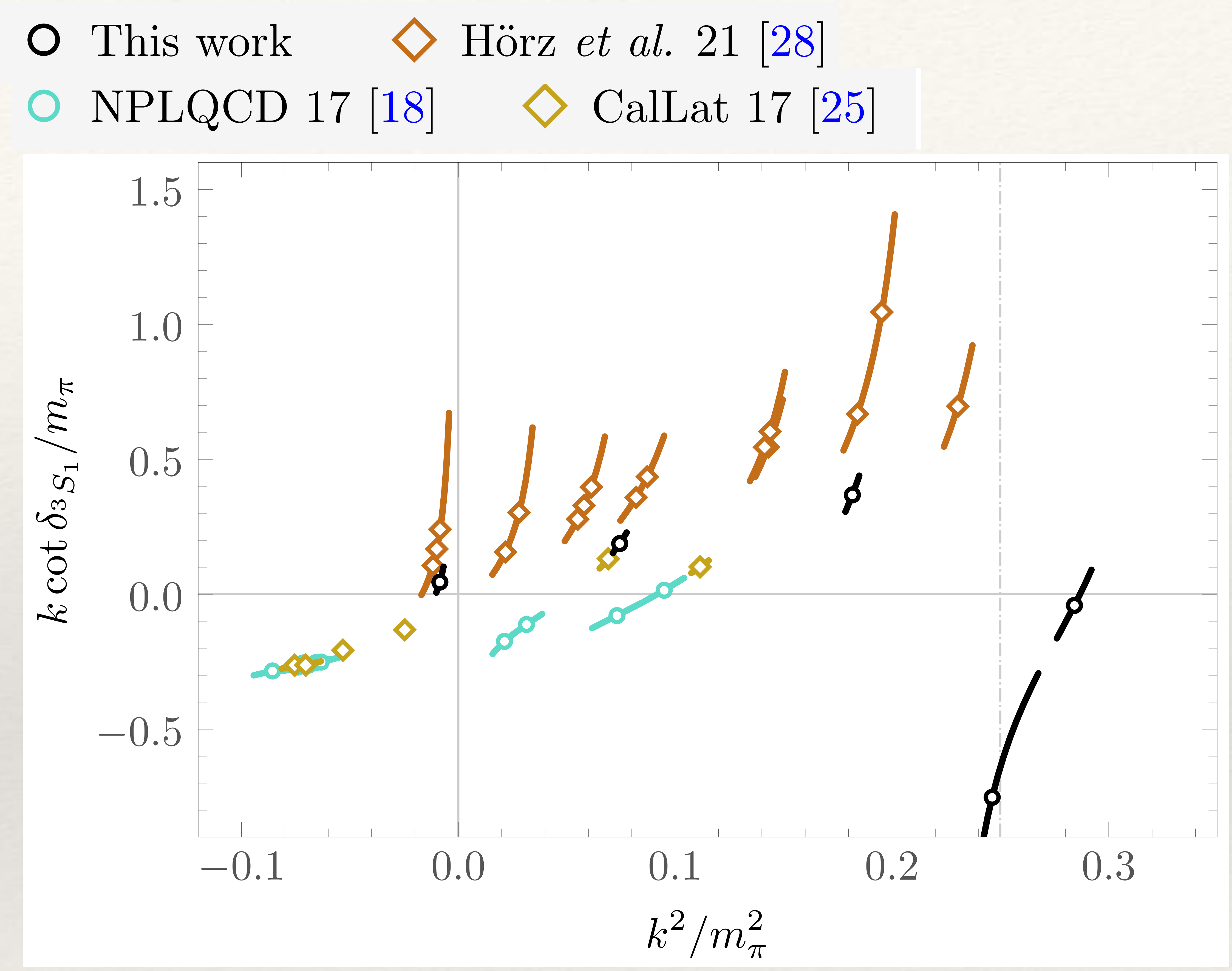


(only shown for total zero momentum)

(in the following: assume negligible $S-D$ mixing)

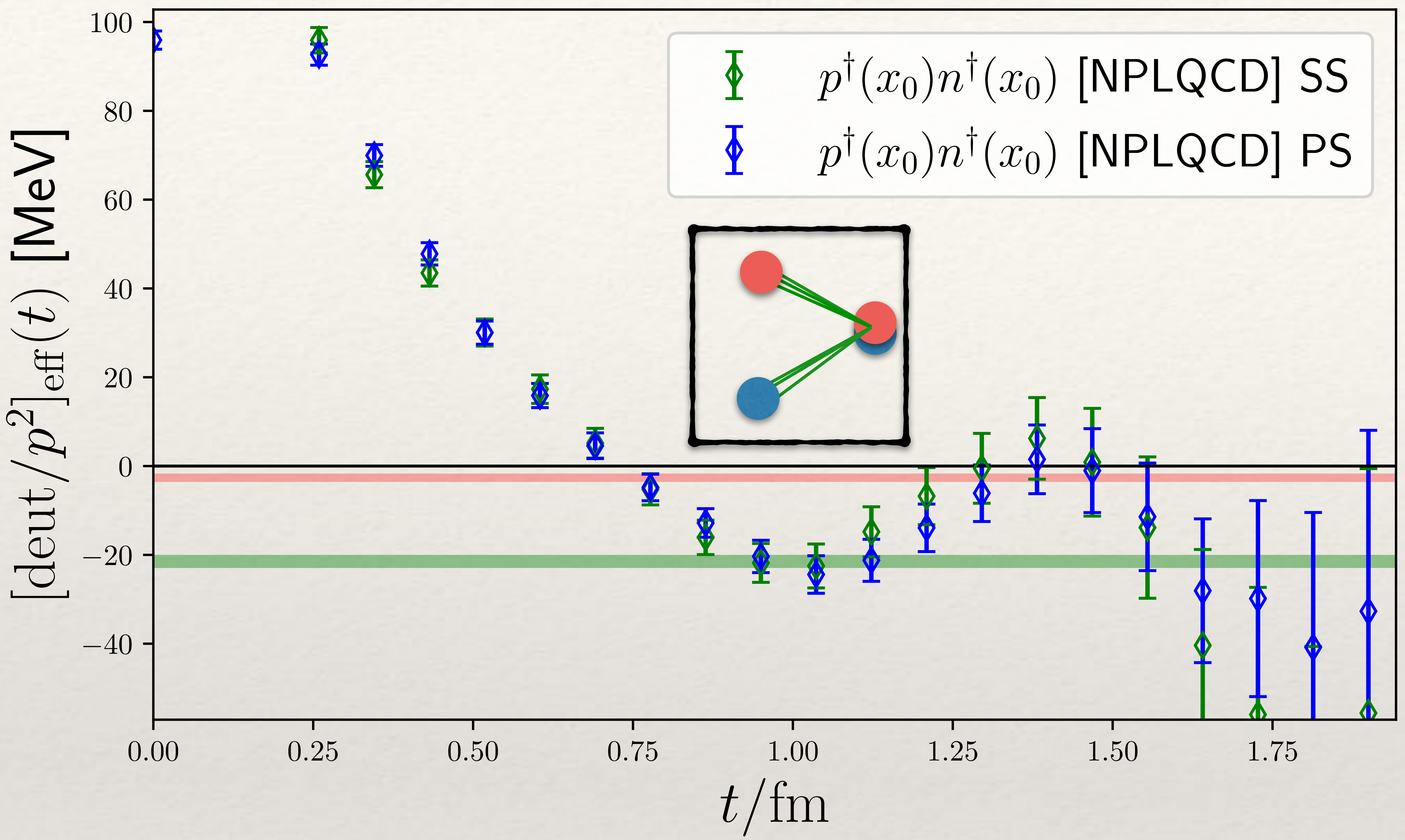
NPLQCD update with momentum-space

- NPLQCD Collaboration used an alternative momentum-space method and repeated their calculation @ $m_\pi \approx 800$ MeV Amarasinghe et al. 2108.10835
- Their new results are qualitatively consistent with other momentum-space methods
- Their new results are not consistent with their old results provided they have momentum-space sources in the basis
- They have not concluded the old methods are wrong

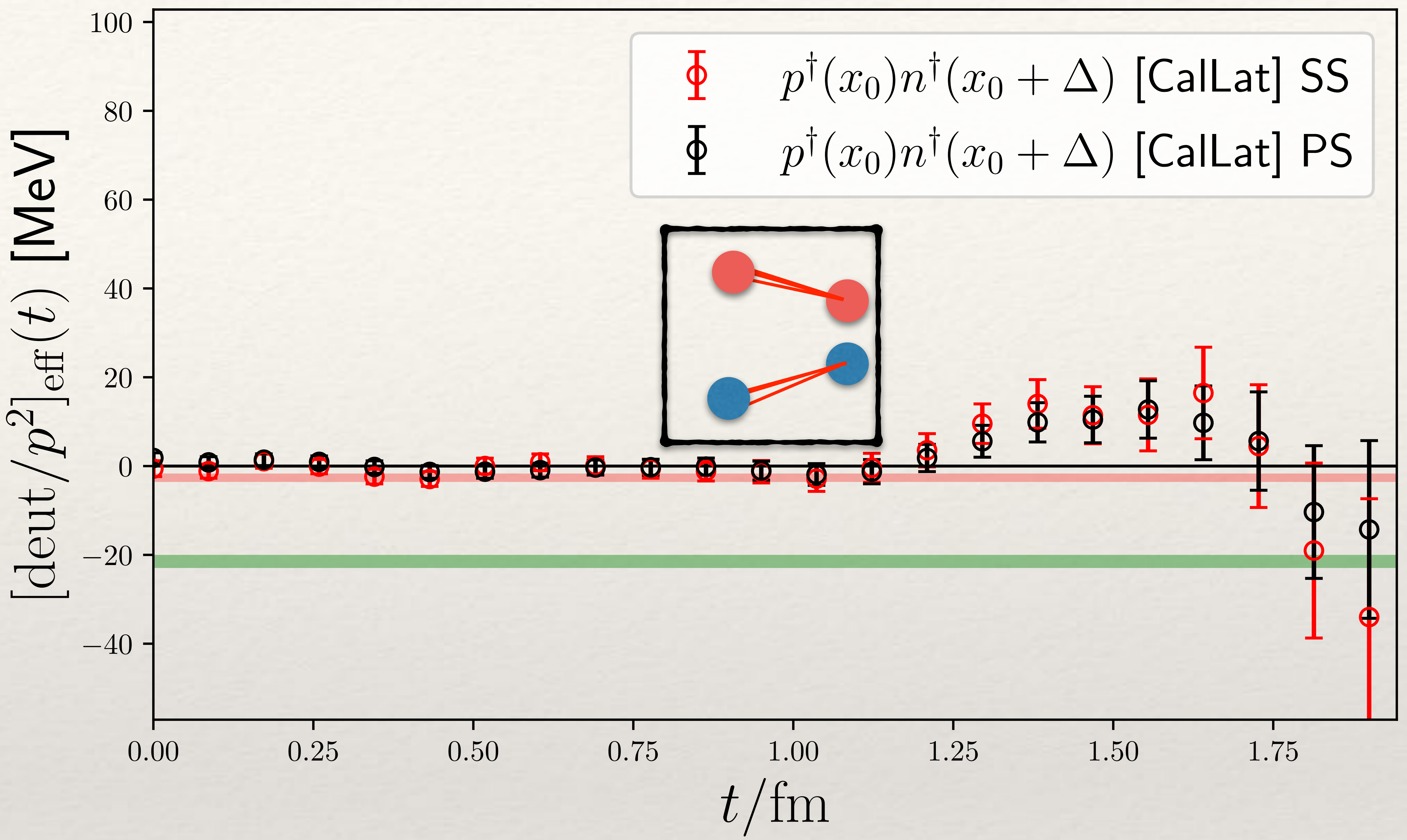


Updates since 2009.11825 — compare with local/displaced NN source

Local HexaQuark creation operator



displaced nucleon creation operator



sLapH g.s. energy in T_{1g} from 2009.11825

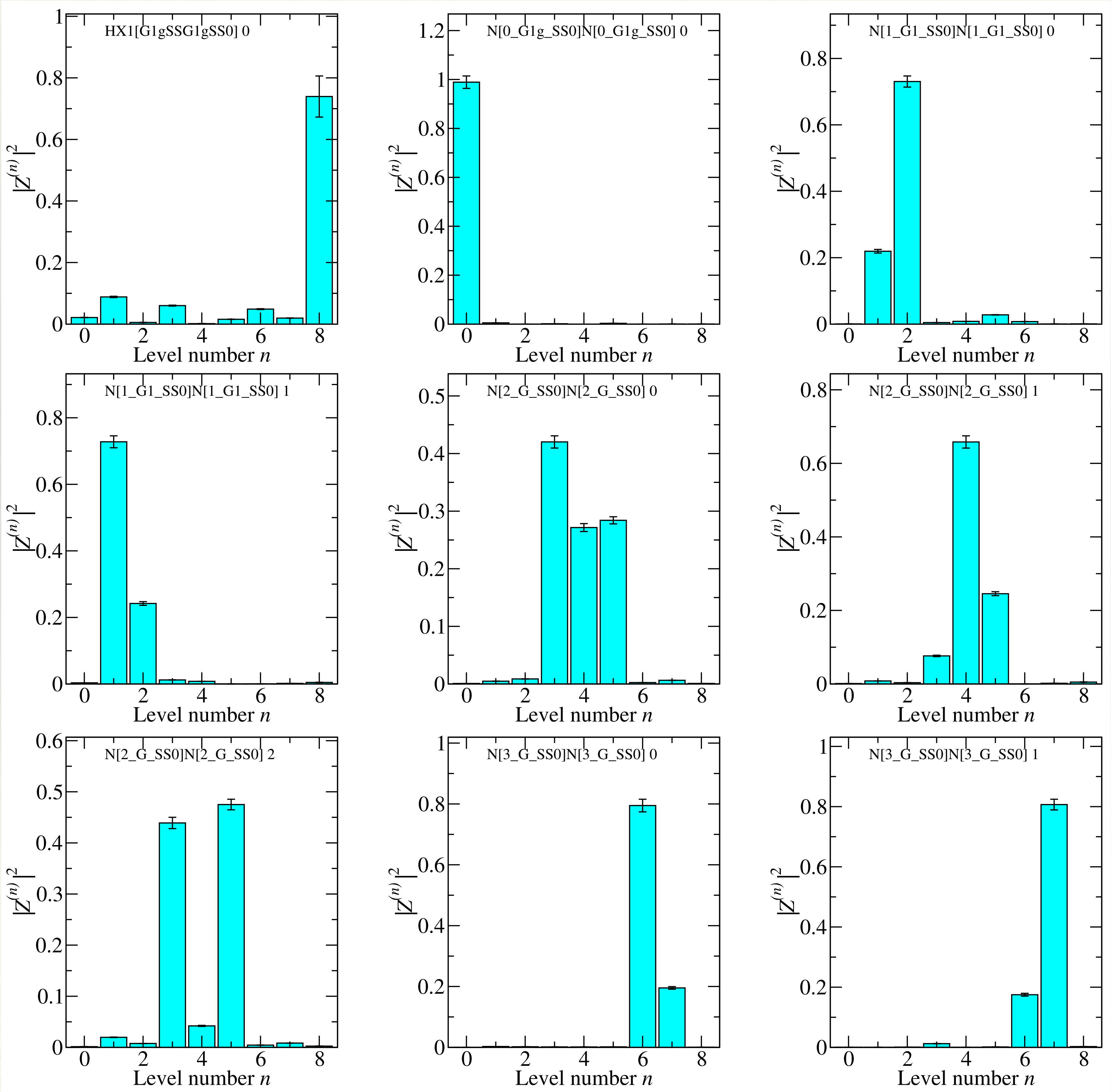
NPLQCD (2012, 2017) / CalLat (2015) g.s. energy from local NN creation operator

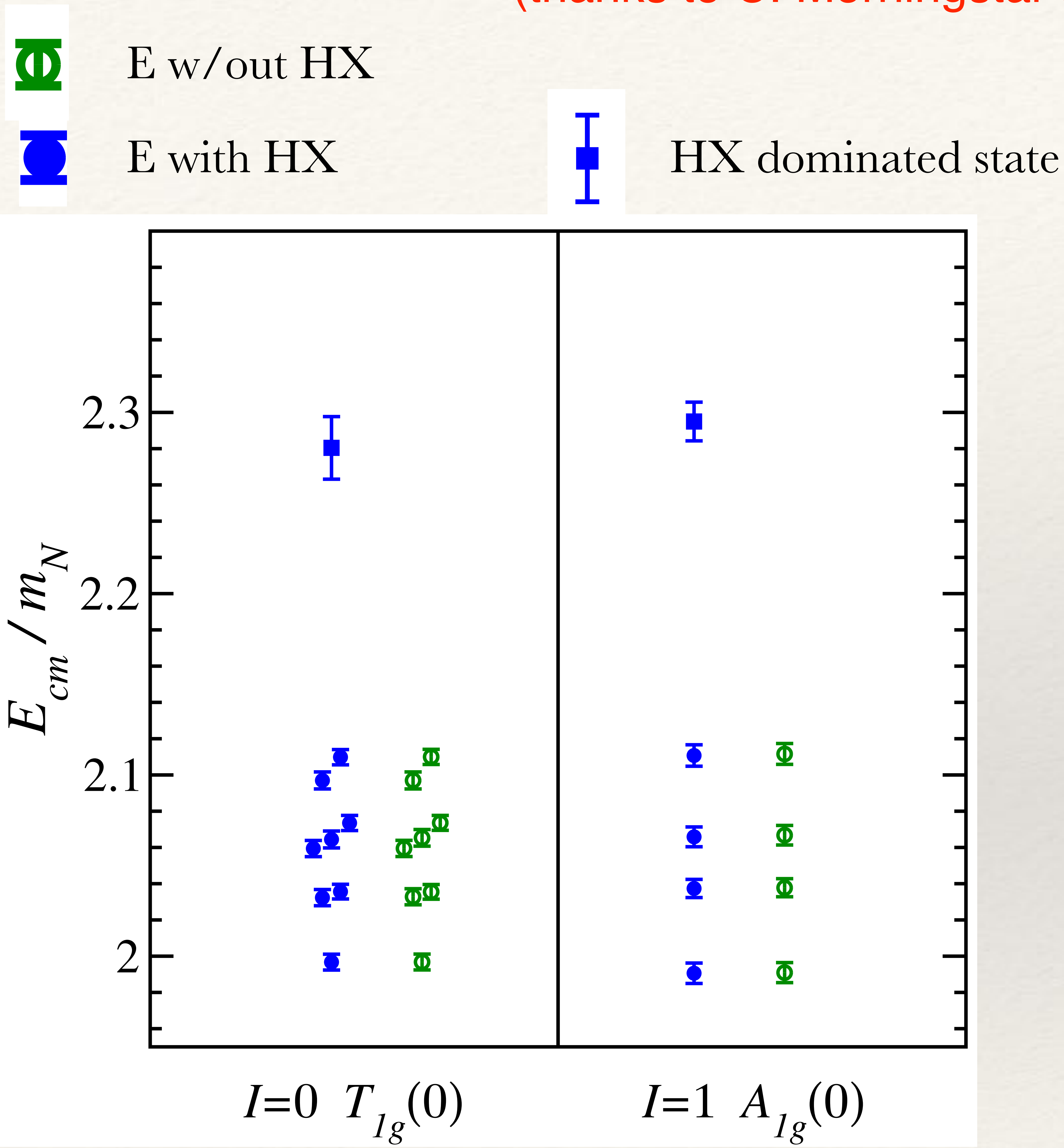
- pulling $p^\dagger(x_0)n^\dagger(x_0+\Delta)$ apart at creation leads to significantly different excited state contamination
- extracting stable ΔE is challenging
- local $p^\dagger(x_0)n^\dagger(x_0)$ strongly couples to NN-inelastic states that are unique to NN (not N on its own) e.g. $\Delta\Delta$

Updates since 2009.11825 — add hexaquark to basis

(thanks to C. Morningstar and S. Skinner)

- hexaquark (HX) operator strongly overlaps with highest state in the spectrum (top left)
- N(p)N(p) operators mostly overlap onto a single state, with some mixing (except with highest state)

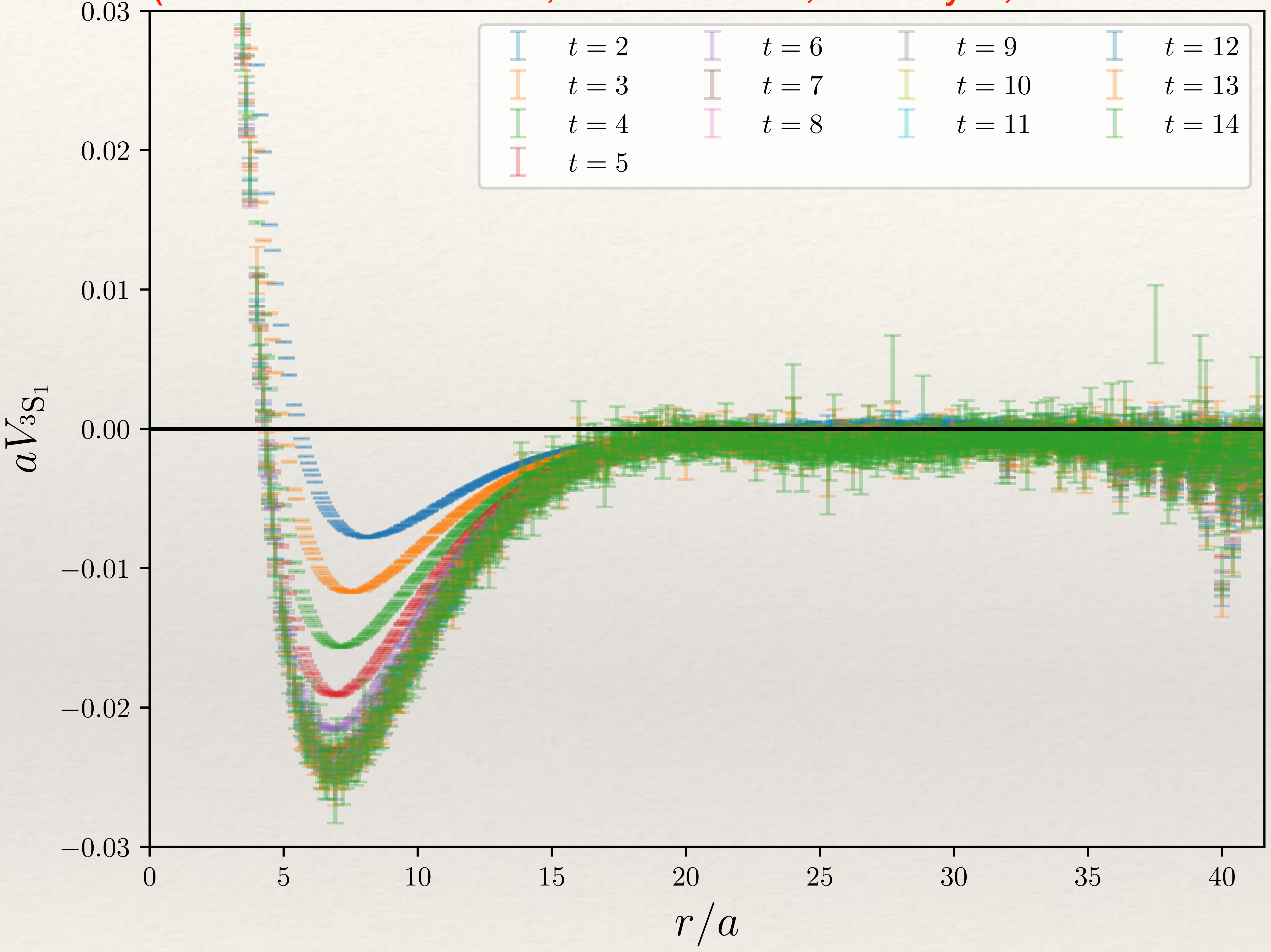




- we find the HX operator is NOT needed to determine the low-lying NN spectrum

Updates since 2009.11825 — HAL QCD potential

(thanks to C. Körber, K. McElvain, A. Meyer, A. Nicholson)



PRELIMINARY

- Potential "saturates" at $t \sim 8$
- Can we perform a $t \to \infty$ extrapolation of V(r)?
- Insensitivity to various functional forms of V(r)

$$V(r) = \sum_n b_n e^{-r^2/2\sigma_n^2}$$

$$V(r) = A_\pi \frac{e^{-m_\pi r}}{r}\left(1 - e^{-r^2/r_0^2}\right)^n + \frac{w_0 + w_1 r + w_2 r^2}{1 + e^{(r-r_0)/a}}$$

regulated OPE + Woods-Saxon

Wiringa, Stoks, Schiavilla PRC 51 (1995)

$$V(r) = A_\pi \frac{e^{-m_\pi r}}{r}\left(1 - e^{-r^2/r_0^2}\right)^n + H.O.\ basis$$

- $m_u = m_d = m_s \approx m_s^{\text{phys}} \longrightarrow m_\pi \approx 714$ MeV
  $a \approx 0.086$ fm, $V = 48^3 \times 96$

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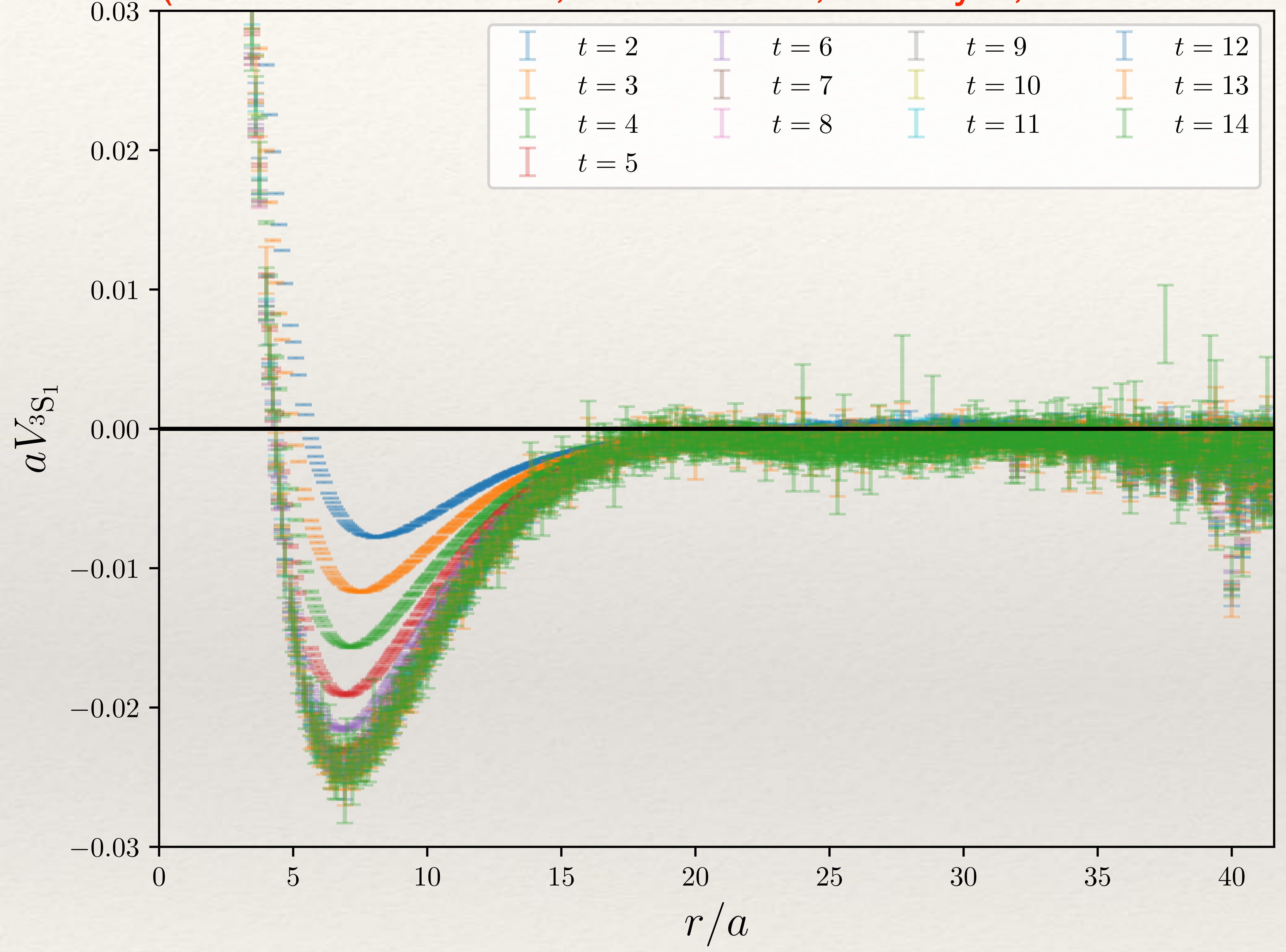
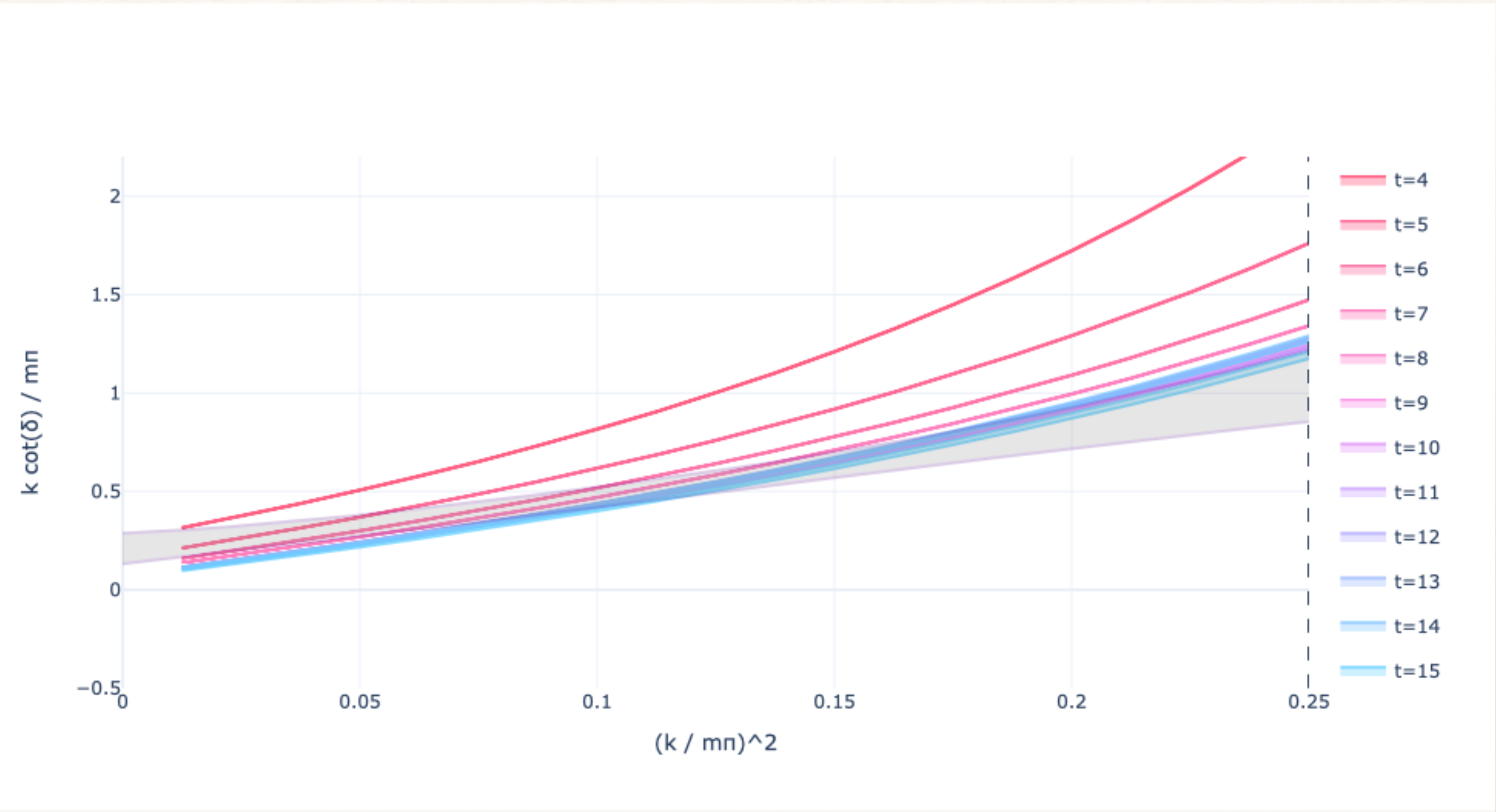


PRELIMINARY



- ☐ gray band - our Lüscher (standard) results
- ☐ HAL QCD potential is consistent at large t

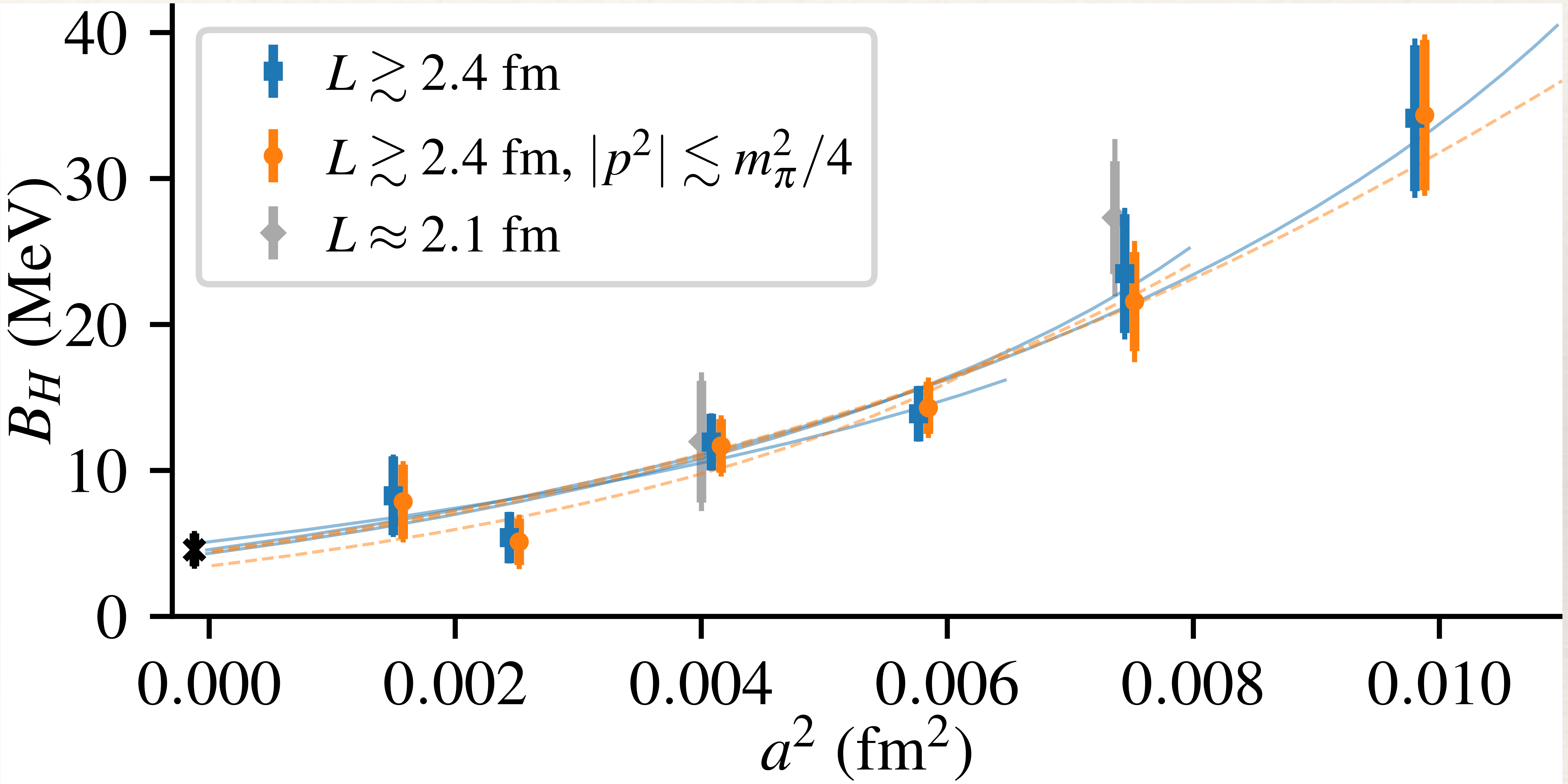
- ☐ $m_u = m_d = m_s \approx m_s^{\rm phys} \longrightarrow m_\pi \approx 714$ MeV
  $a \approx 0.086$ fm, $V = 48^3 \times 96$

To bind or not to bind?

- This is a question that is unfortunately not one we can absolutely answer - we can only find numerical evidence
- We (the community) often rely upon Lüscher quantization condition analysis of spectrum to detect inconsistent energy levels — in the case of old NPLQCD & CalLat results (at least at $m_\pi \approx 800$ MeV), the observed spectrum did not show signs of sickness
- However, we are observing a preponderance of evidence that the older methods with present statistics, are yielding qualitatively incorrect spectrum —
  I believe the old results are wrong (including those I was involved with)
  I believe the di-nucleon system unbinds at pion masses heavier than physical
- The newer (at least newly applied to two-nucleon) methods are more expensive
  but, they are more robust and they yield a much richer spectrum (many more energy levels obtained in the same calculation)
- The path forward is clear — we need to apply these methods @ lighter pion masses where they have a chance of having an impact on our understanding of NN interactions
  - To have an impact, we must have $m_\pi \lesssim 200$ MeV (underway!)

Discretization effects in di-baryon systems?

- A new'ish result also showed surprisingly large discretization effects - O(1000%)
  use of non-perturbative, O(a)-improved clover-Wilson action (CLS)
  [Green, Hanlon, Junnarkar, Wittig, PRL 127 - 2103.01054]



Discretization effects in di-baryon systems?

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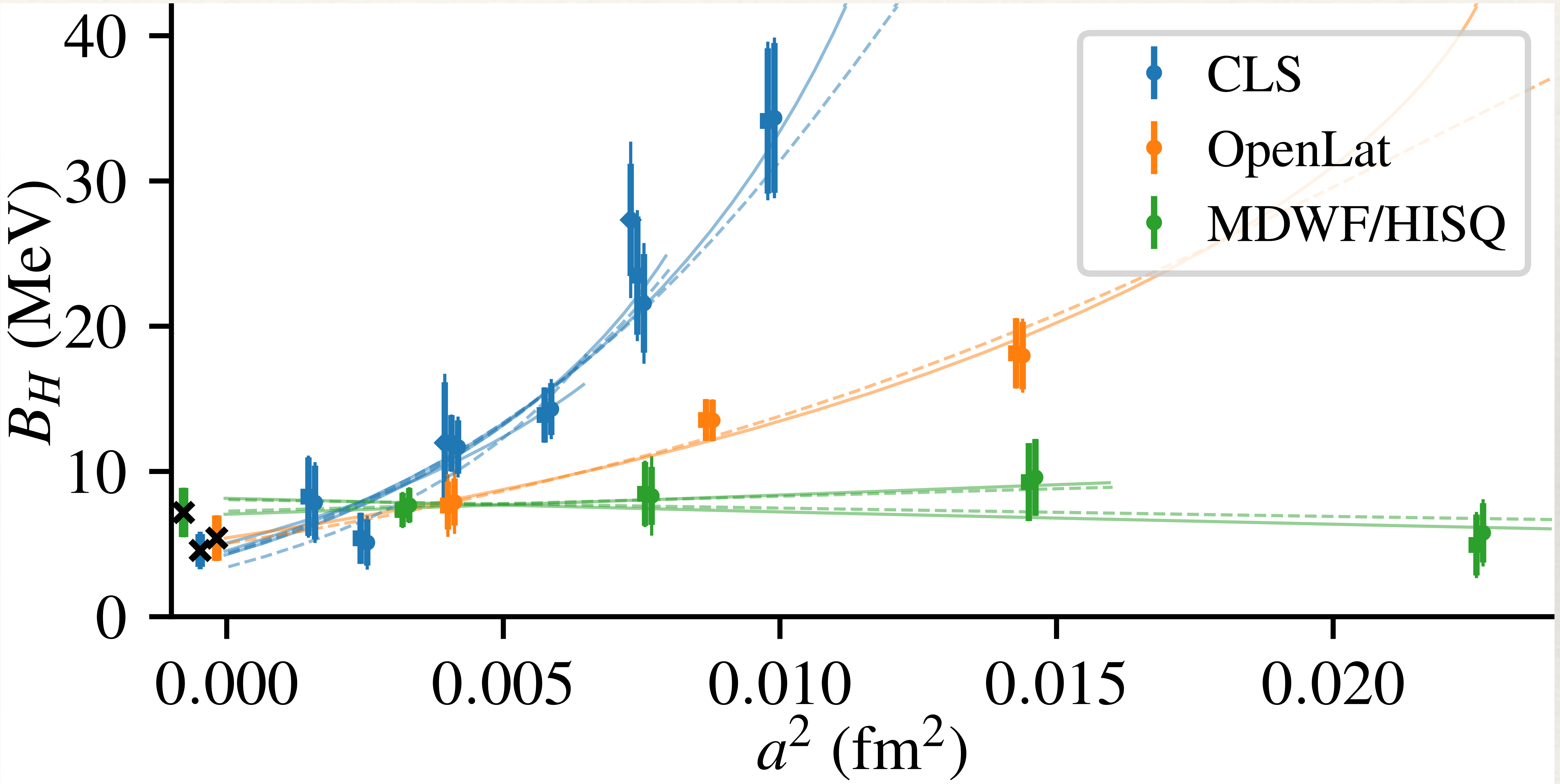
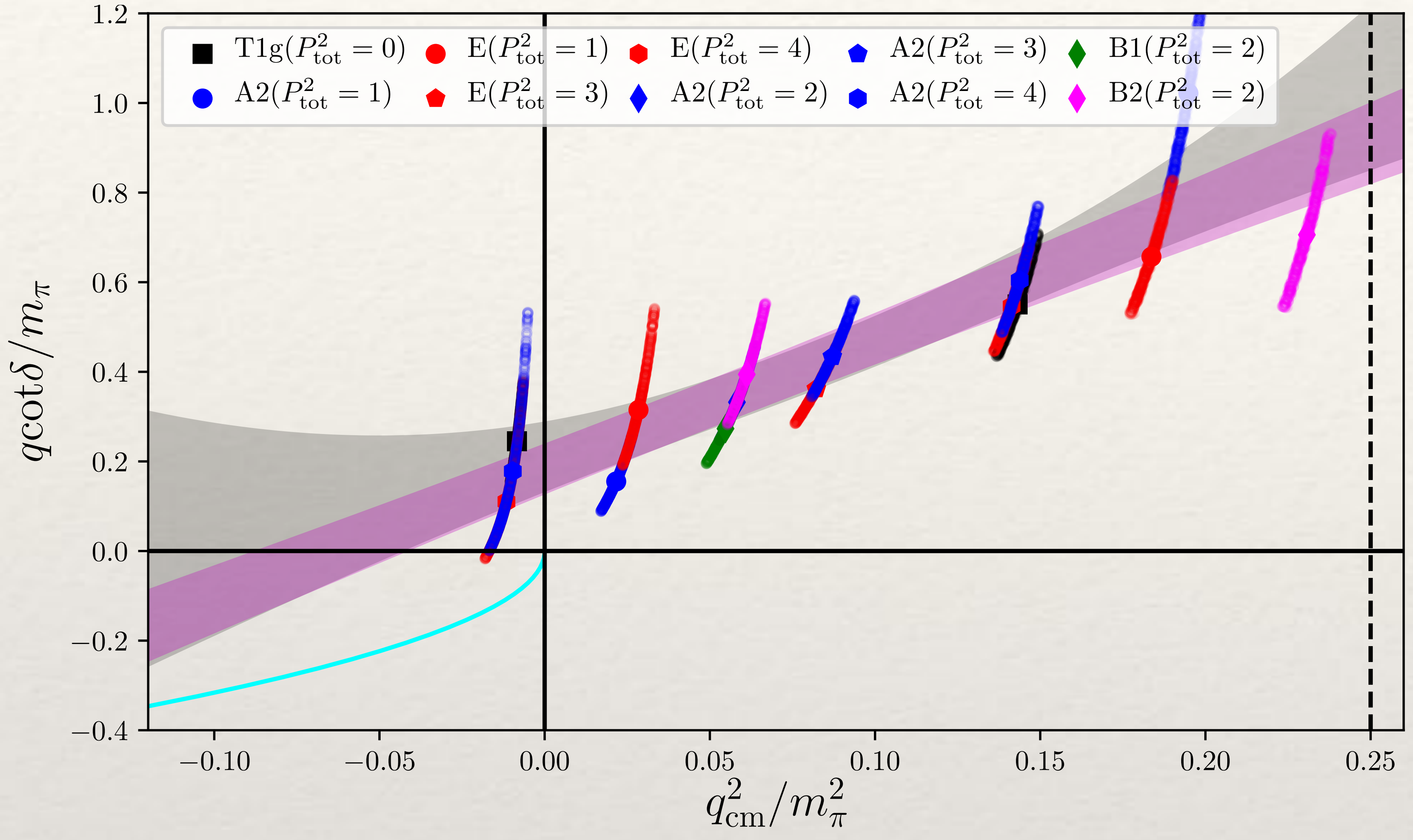
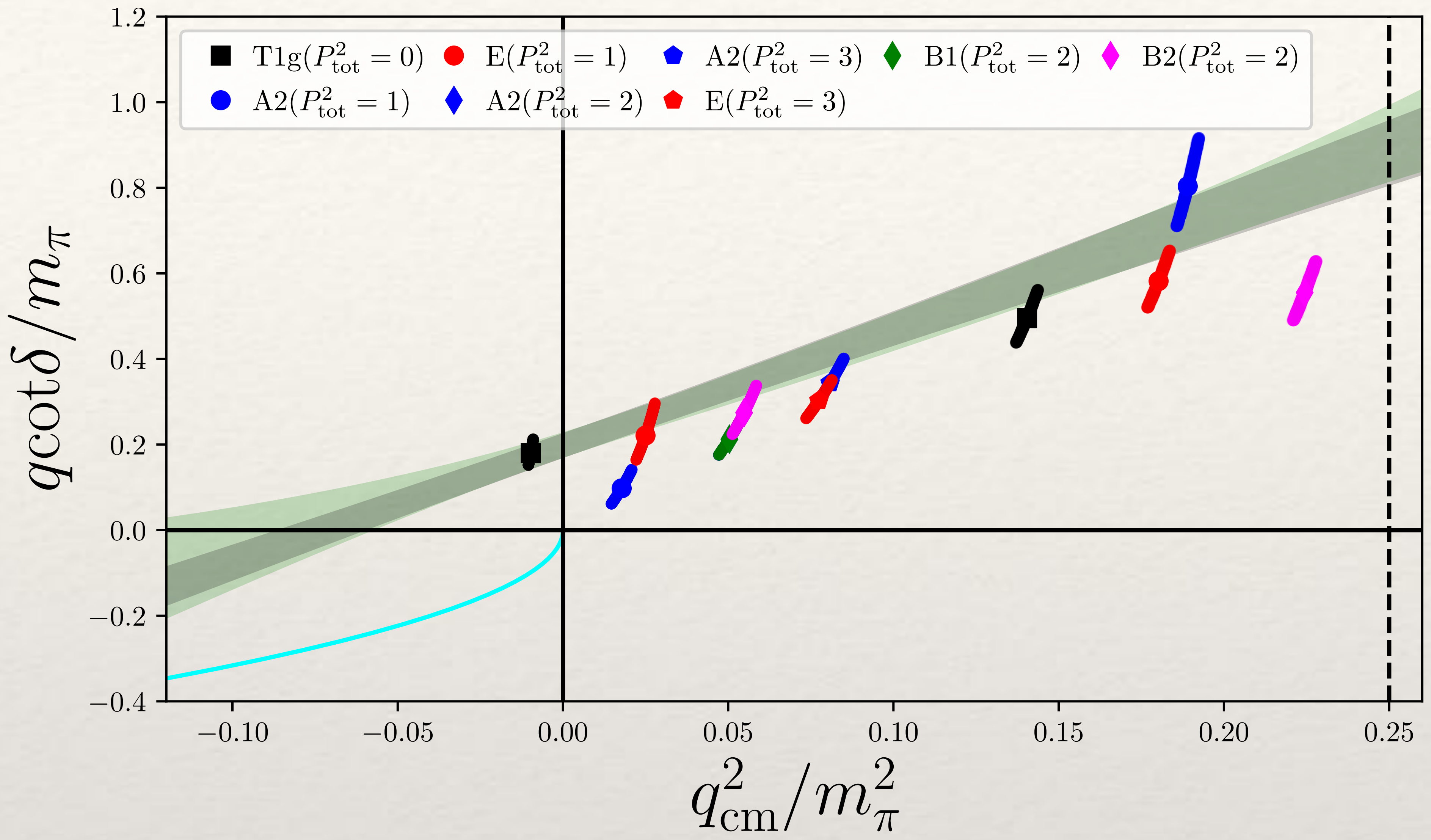
- We are performing a study to understand how large discretization effects are with different lattice actions
  - $t_{\rm MC} \approx \dfrac{1}{a^6}$
- OpenLat: exponentiated clover
- MDWF / HISQ: mixed action with chiral valence fermions



UPDATE of [2009.11825] - 2x higher statistics

- 16 energy levels with (expected) negligible overlap with non S-wave





- Tension in the phase shift analysis - why?
- 2020: results were imprecise enough, we could ignore box-mixing

$P_{tot} = 0$ $\quad P_{tot} = \frac{2\pi}{L}$

- It is understood how the leading partial wave mixing is induced by the cubic-box (Lüscher quantization condition)
- Briceno, Davoudi, Luu, Savage, PRD88 (2013) remove leading physical S-D wave mixing sensitivity:

$$\frac{1}{3}\left(E_{A_2} + 2E_E\right), \ \frac{1}{3}\left(E_{A_2} + E_{B_1} + E_{B_2}\right)$$

$\vec{n}_{tot} = (0,0,n)$ $\quad \vec{n}_{tot} = (0,n,n)$

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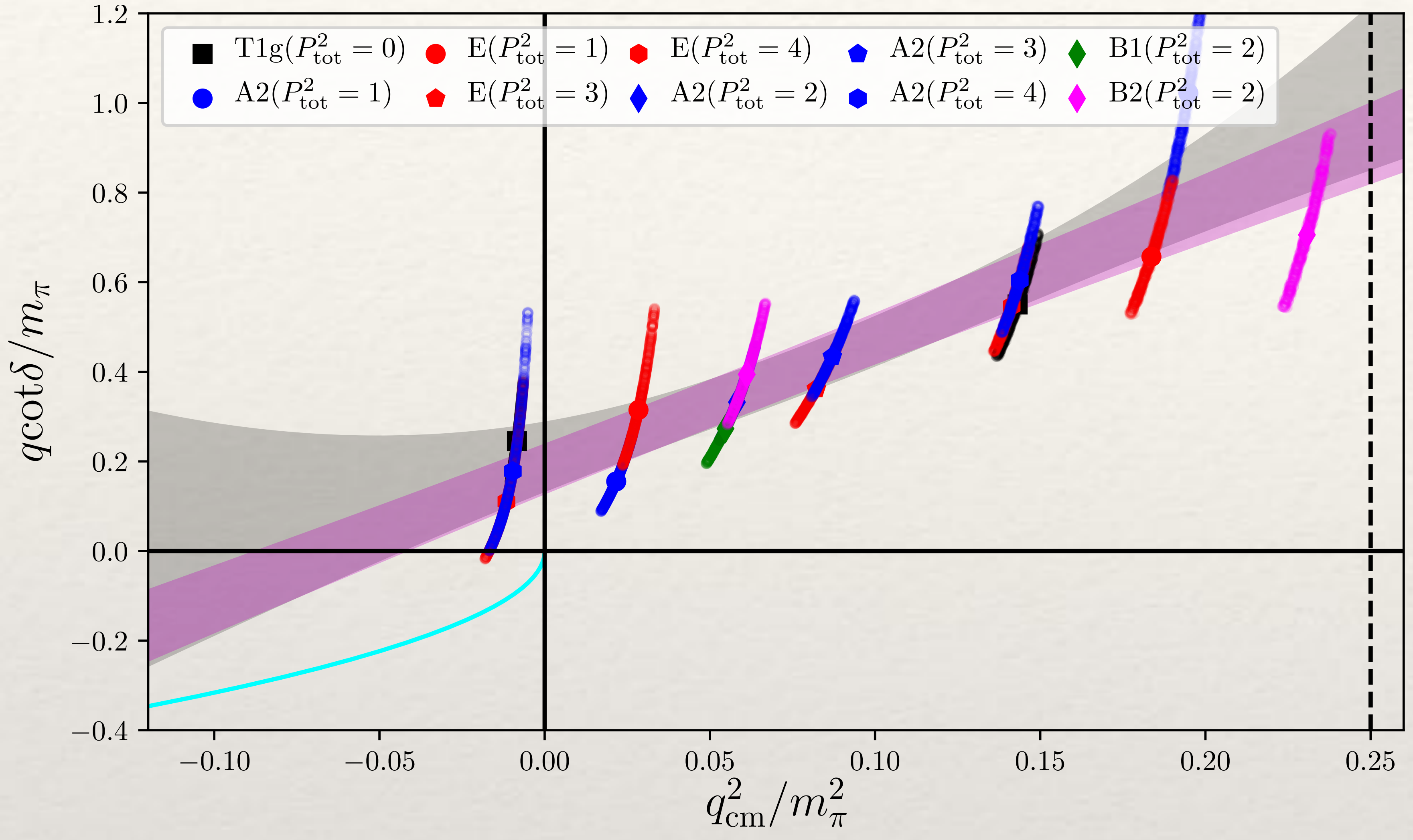

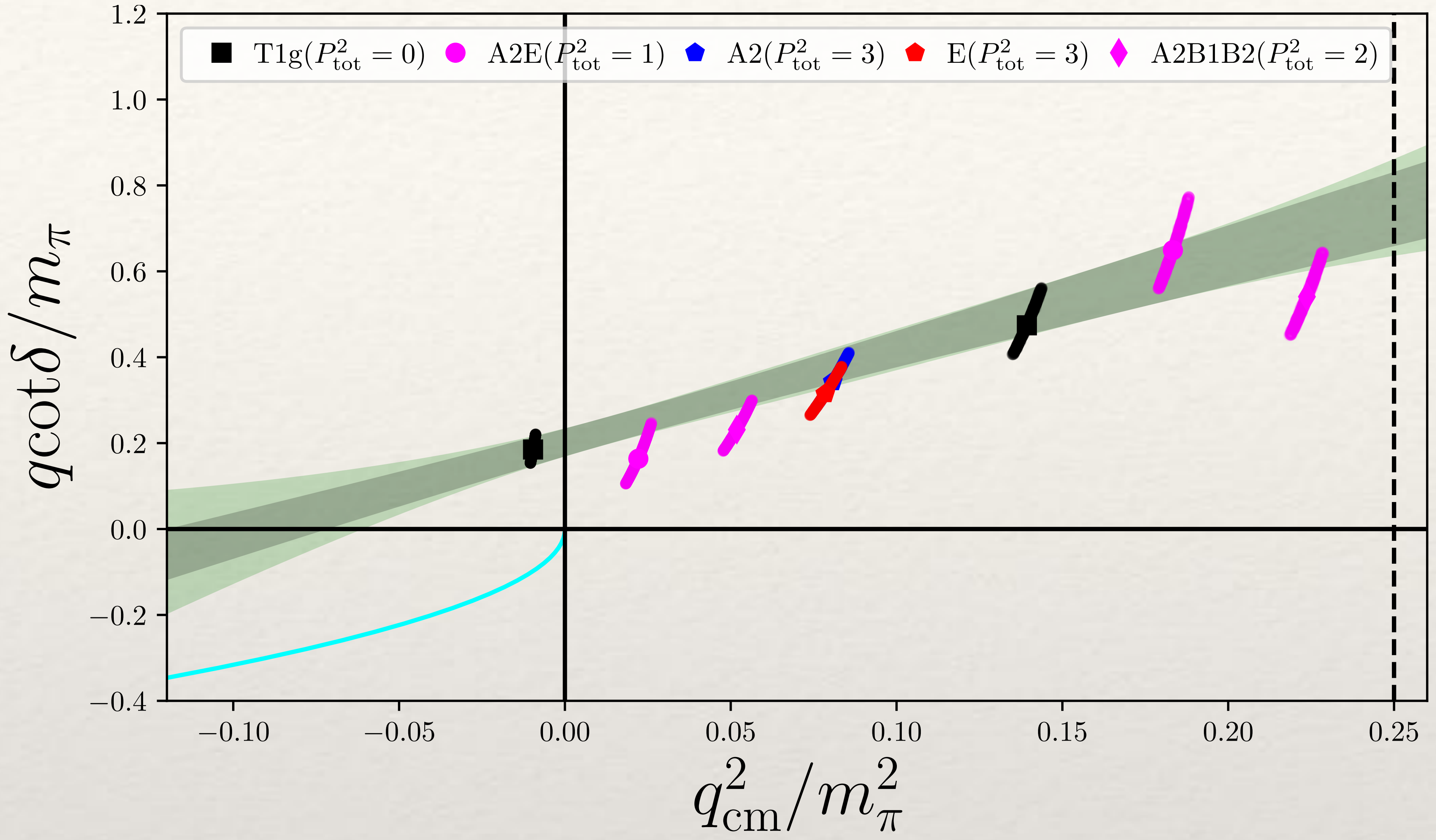

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$\vec{n}_{\rm tot} = (0,0,n)$ $\qquad \vec{n}_{\rm tot} = (0,n,n)$

Do di-nucleons bind @ heavy pion mass?

- Lessons learned:
  - In order to determine the correct spectrum — it is important to use momentum space sources
    - The spectrum and matrix elements determined with spatially local creation operators suffer from unquantified systematic uncertainties
  - The HAL QCD potential results are consistent with those determined via Lüscher at the 1σ-level over a large range of energy
  - There may be large discretization corrections in the spectrum
    - What about matrix elements?
    - This finding is very sensitive to the choice of lattice action (discretization scheme)

LQCD: Select Highlights

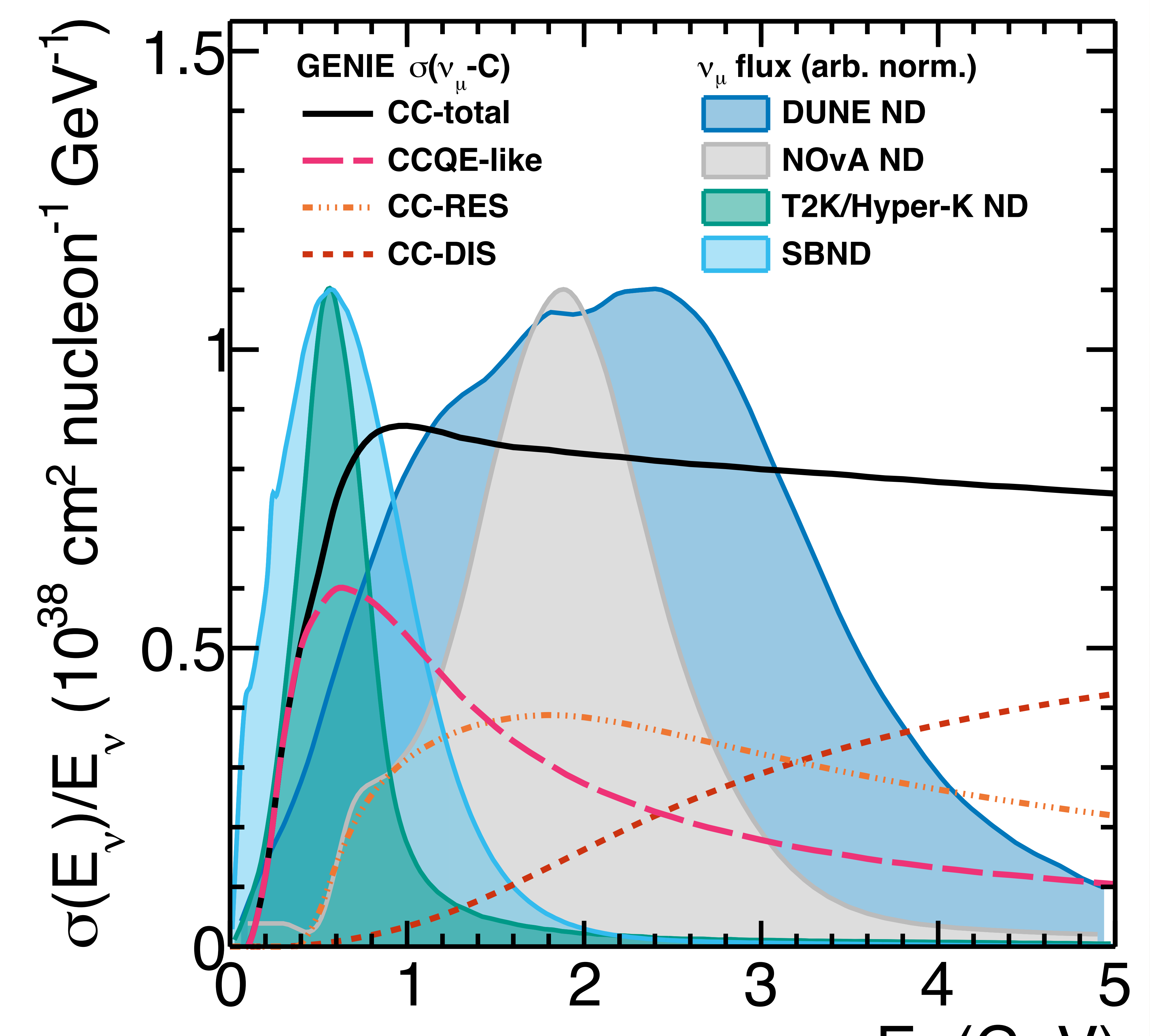
- $\nu - N$ scattering
- $\pi - N$ scattering
- χPT convergence

$\nu - A$ scattering for neutrino oscillation parameters

- Theoretical prediction of ν-A cross sections
  from the Standard Model
  with full uncertainty quantification

- Very challenging to achieve this goal:
  Most likely, it is impossible to have a unified theoretical description of ν-A cross sections over the range of ν-energy of interest
  - Lattice QCD: single nucleon, resonance region, …
  - Effective Field Theory (EFT): Low-energy, small-A
  - high energy: DIS and Regge (model)

- The problem demands a description of medium-A
  - over broad range of energy
  - pion production, resonance region
  - final state interactions
  - …



(C. Wilkinson)

What is possible? A "realist" perspective

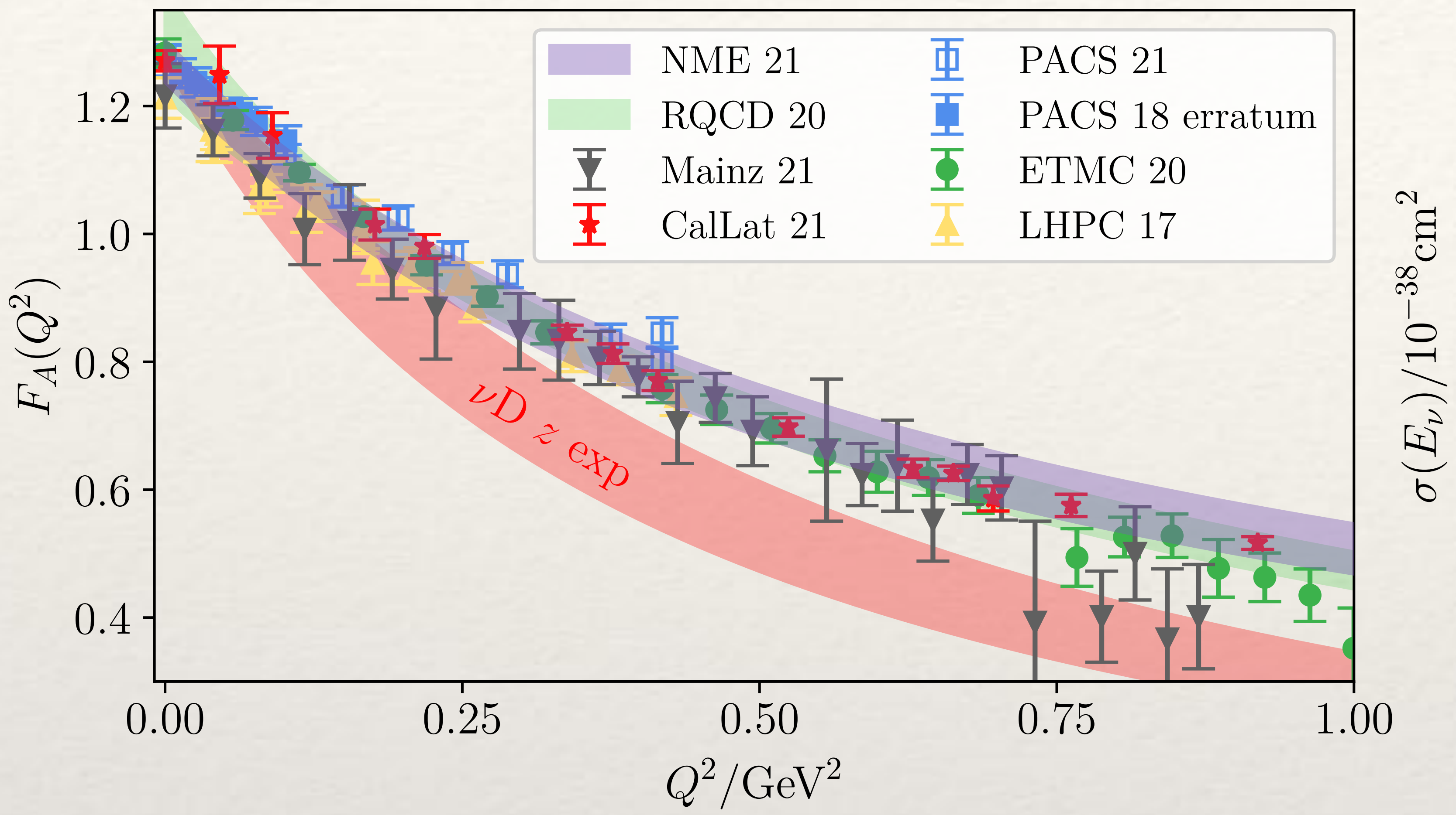
- Lattice QCD (LQCD) can determine single nucleon
  - quasi-elastic
  - resonance region, pion production
  - DIS
  - two-nucleon cross section (corrections)
  - maybe, maybe, light nuclear cross sections

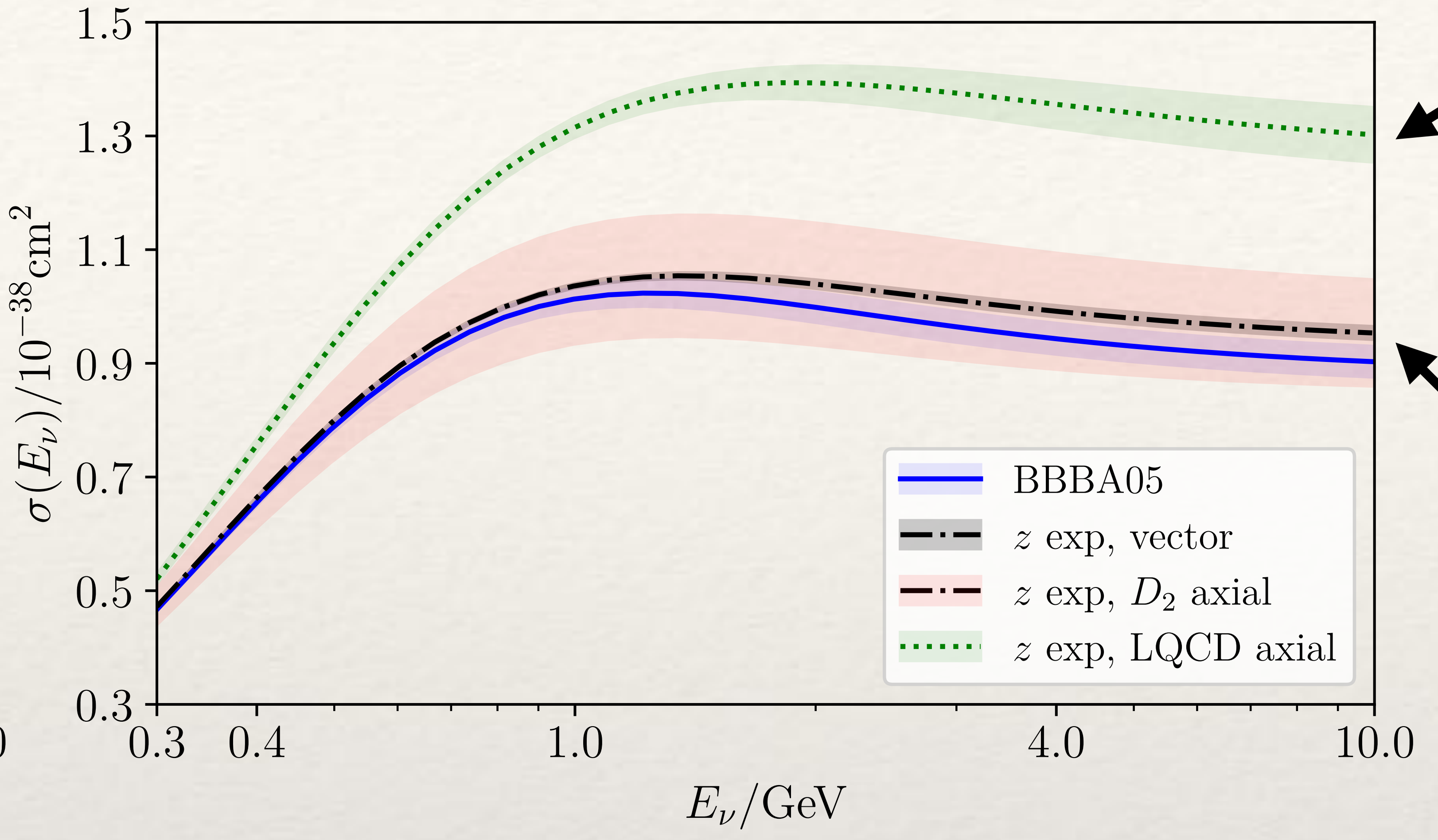
Even, even if we could compute ν-^{12}C, it almost certainly will not be the most economical way to propagate QCD results to nuclear cross sections

- No EFT that can describe ν-A reaction over entire range of E_ν
  - Use EFT, with LQCD input, to describe region of parameter space against which nuclear models can be constrained
    - in this region at least, rigorous, systematically improvable uncertainty rooted in the SM
    - This will allow for calibration of nuclear model uncertainty

ν-N cross section

Meyer, Walker-Loud, Wilkinson
Ann. Rev. Nucl. Part. Sci. 72 (2022)





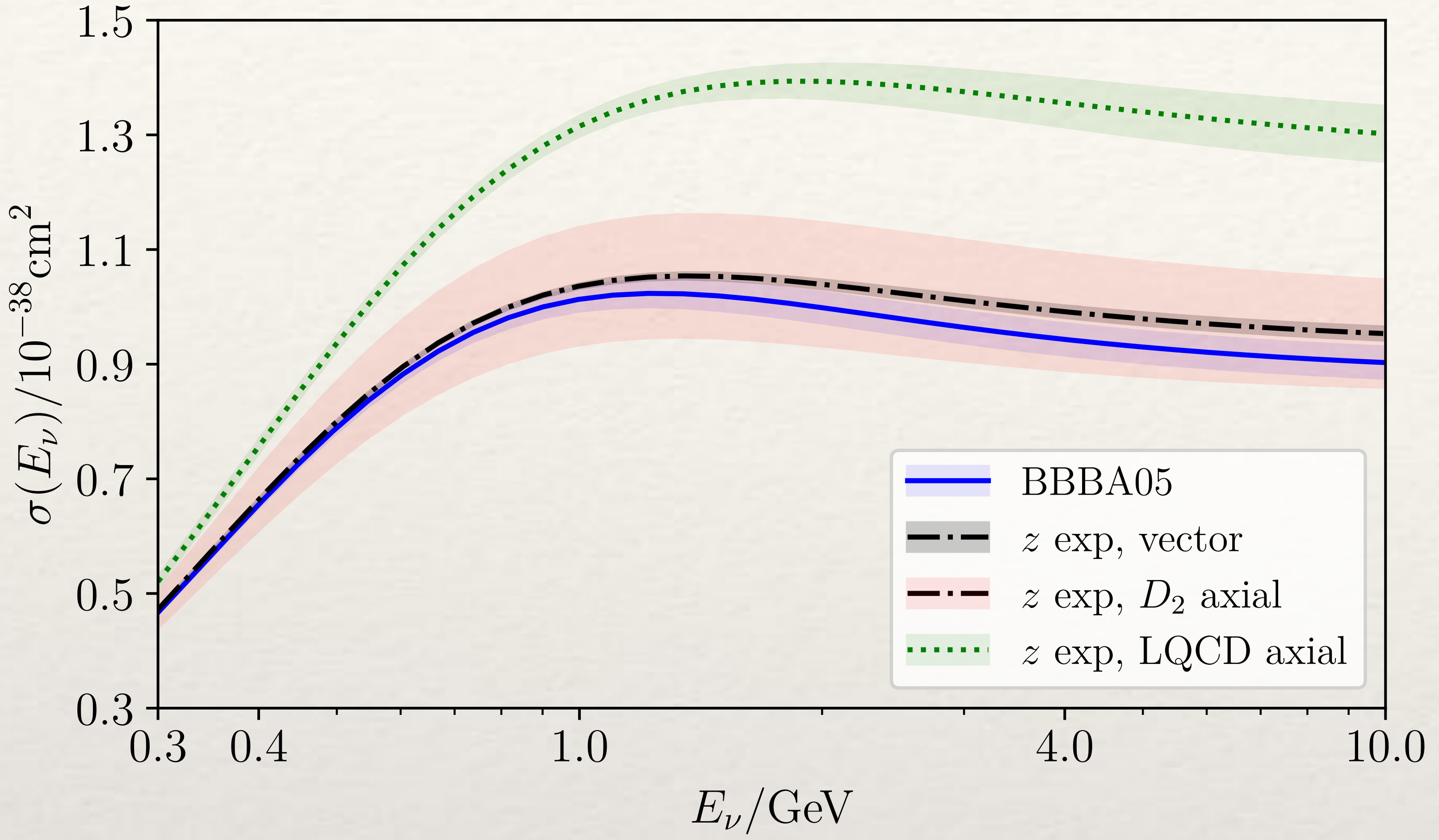
This discrepancy impacts the ability to interpret neutrino oscillation parameters

Some of this uncertainty comes from nucleon electromagnetic form factors

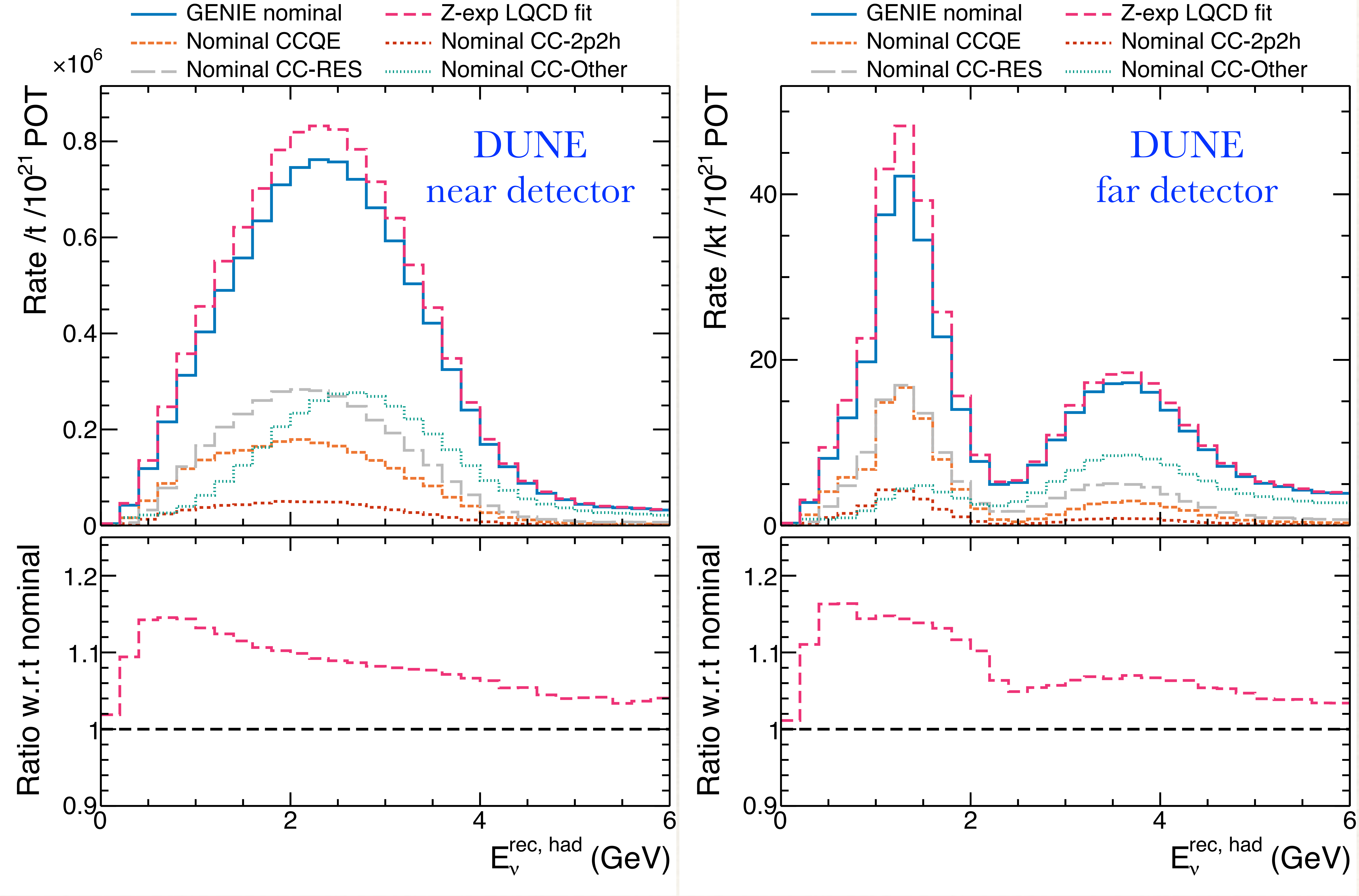
- Lattice QCD determination of $F_A(Q^2)$ is inconsistent with older phenomenological extraction
- results in 30% increase in ν-N cross section

ν-N cross section

Meyer, Walker-Loud, Wilkinson
Ann. Rev. Nucl. Part. Sci. 72 (2022)



- Lattice QCD determination of $F_A(Q^2)$ is inconsistent with older phenomenological extraction
- results in 30% increase in ν-N cross section
- Energy dependent change



ν-N cross section

Meyer, Walker-Loud, Wilkinson
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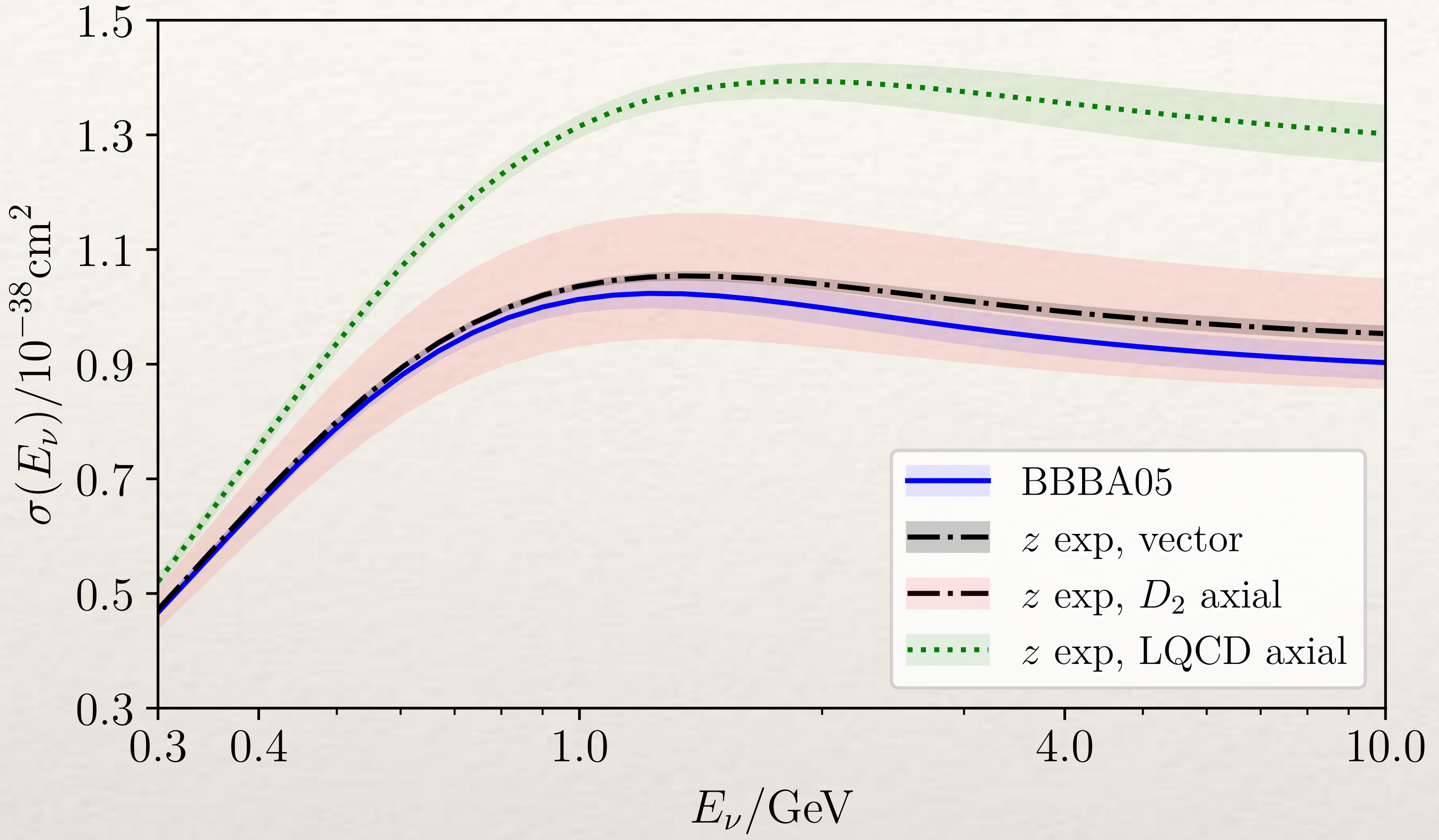

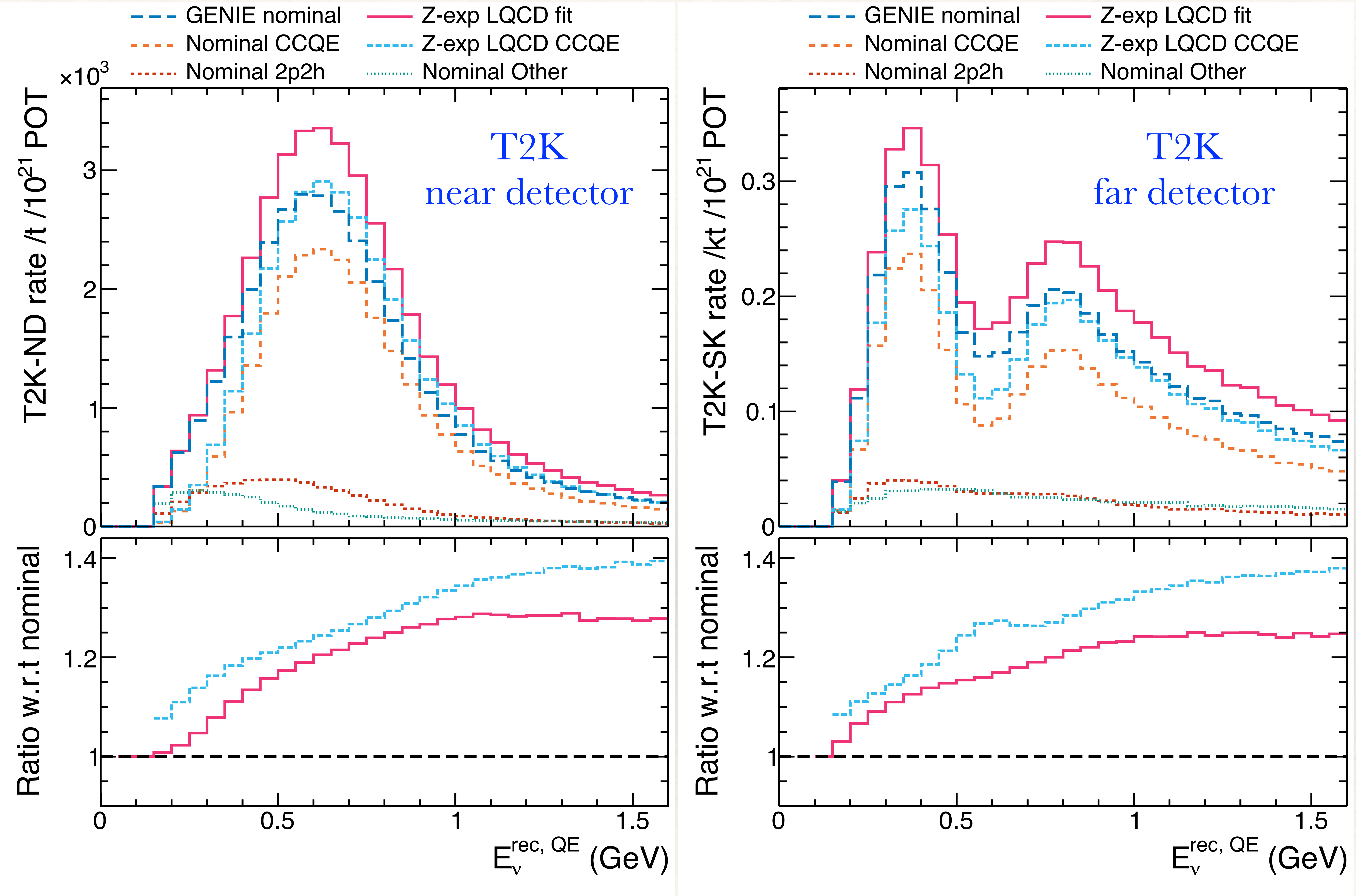

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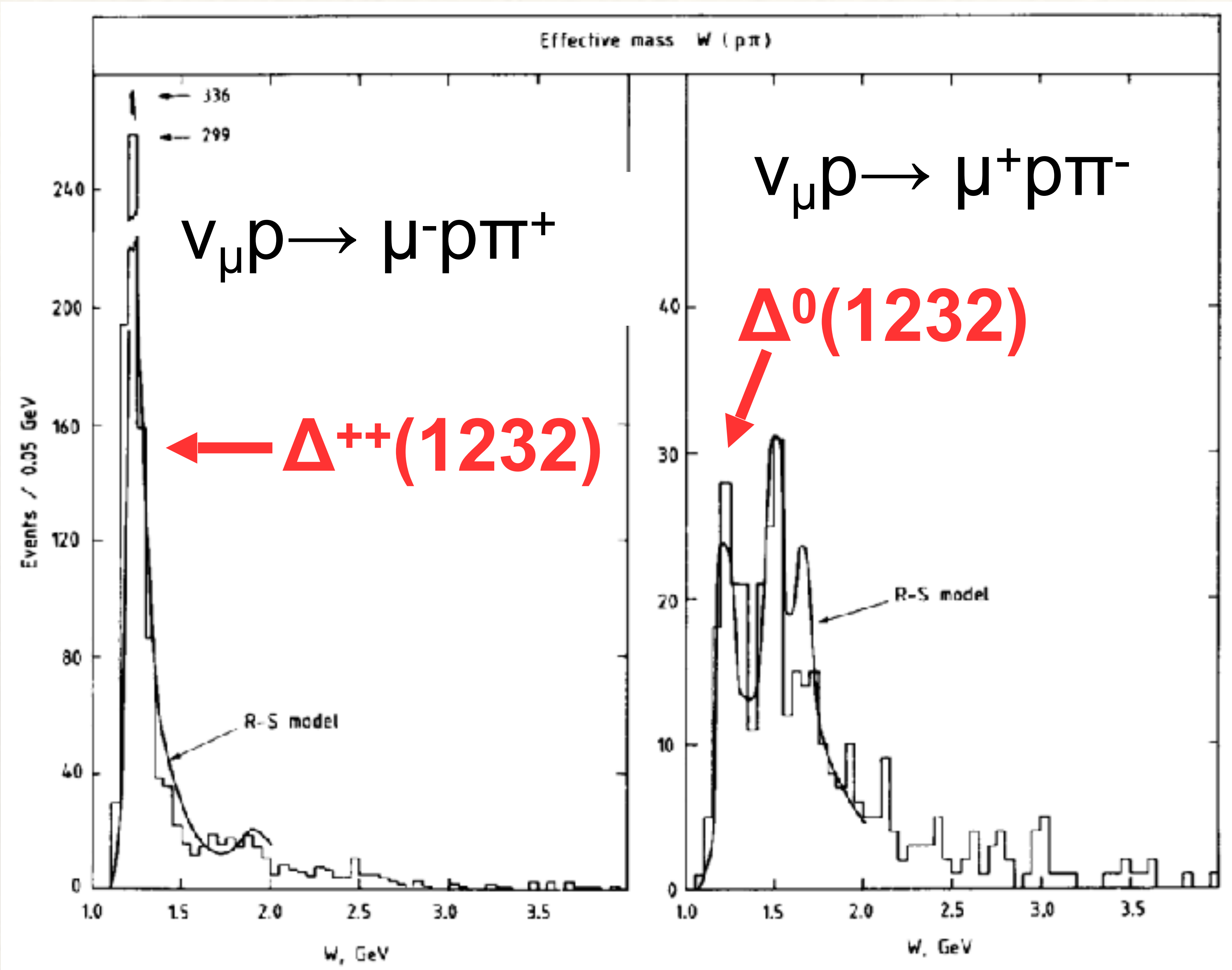
Future directions

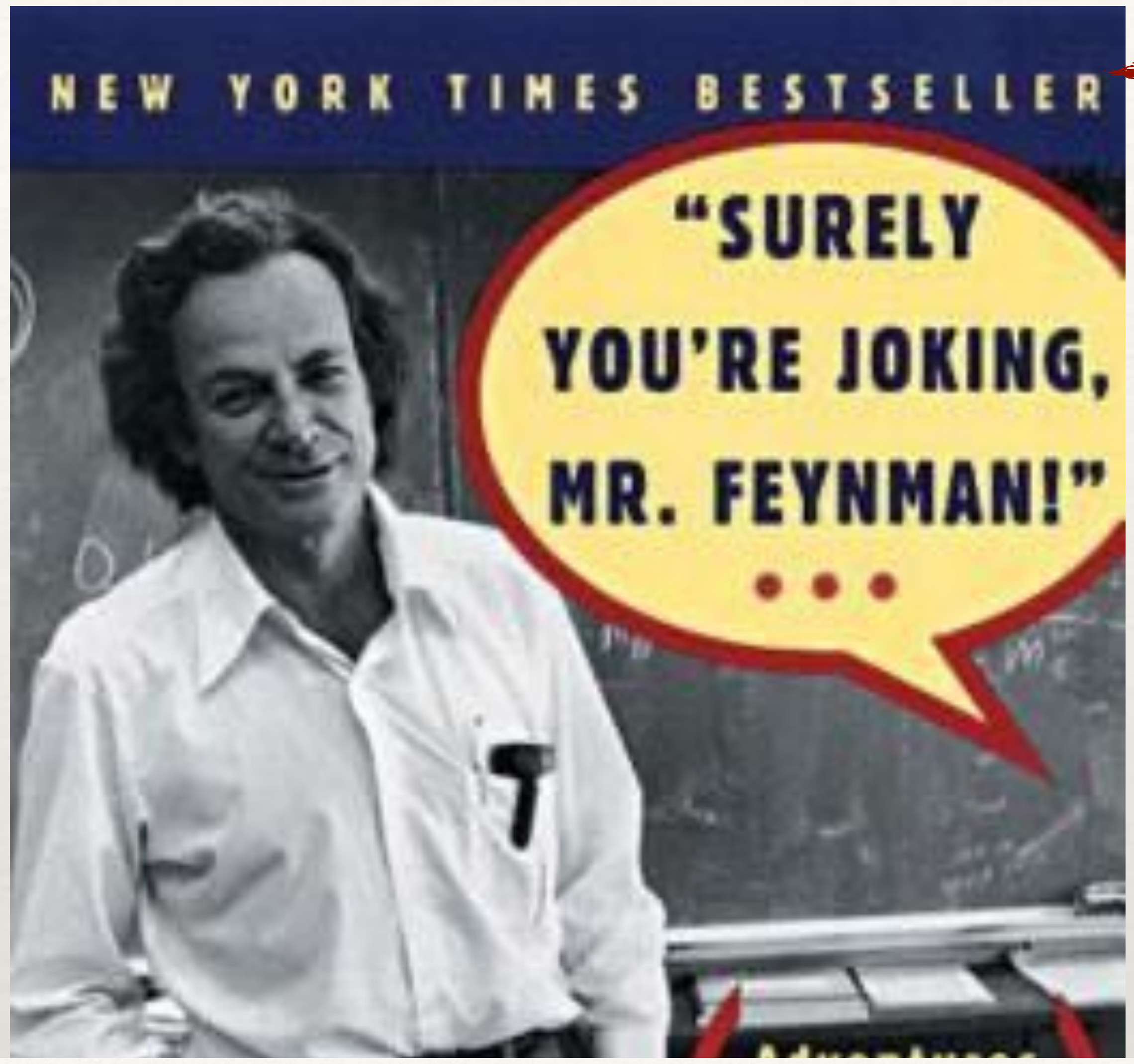
State of the Field

Nucl. Phys. B264 221 (1986)



$$W^2 = (\Sigma E)^2 - |\Sigma \mathbf{p}|^2$$

Need LQCD calculations of $\pi - N$!



Indeed not!

Our pion production model uses a description of resonance production that is "naive and obviously wrong in its simplicity" [F.K.R. PRD3 (1971)]

I trust some bright motivated physicists will fix this soon

- ❑ Current models are unsatisfactory:
  - ❑ Simplistic description of neutrino-nucleon interaction
  - ❑ Unsophisticated description of the nucleus
- ❑ Heavy reliance on old data (experiments shut down)
- ❑ ~10% uncertainties on effective parameters at best

arXiv > hep-lat > arXiv:2208.03867

High Energy Physics – Lattice

[Submitted on 8 Aug 2022 (v1), last revised 7 Feb 2023 (this version, v3)]

Elastic nucleon–pion scattering at $m_\pi = 200$ MeV from lattice QCD

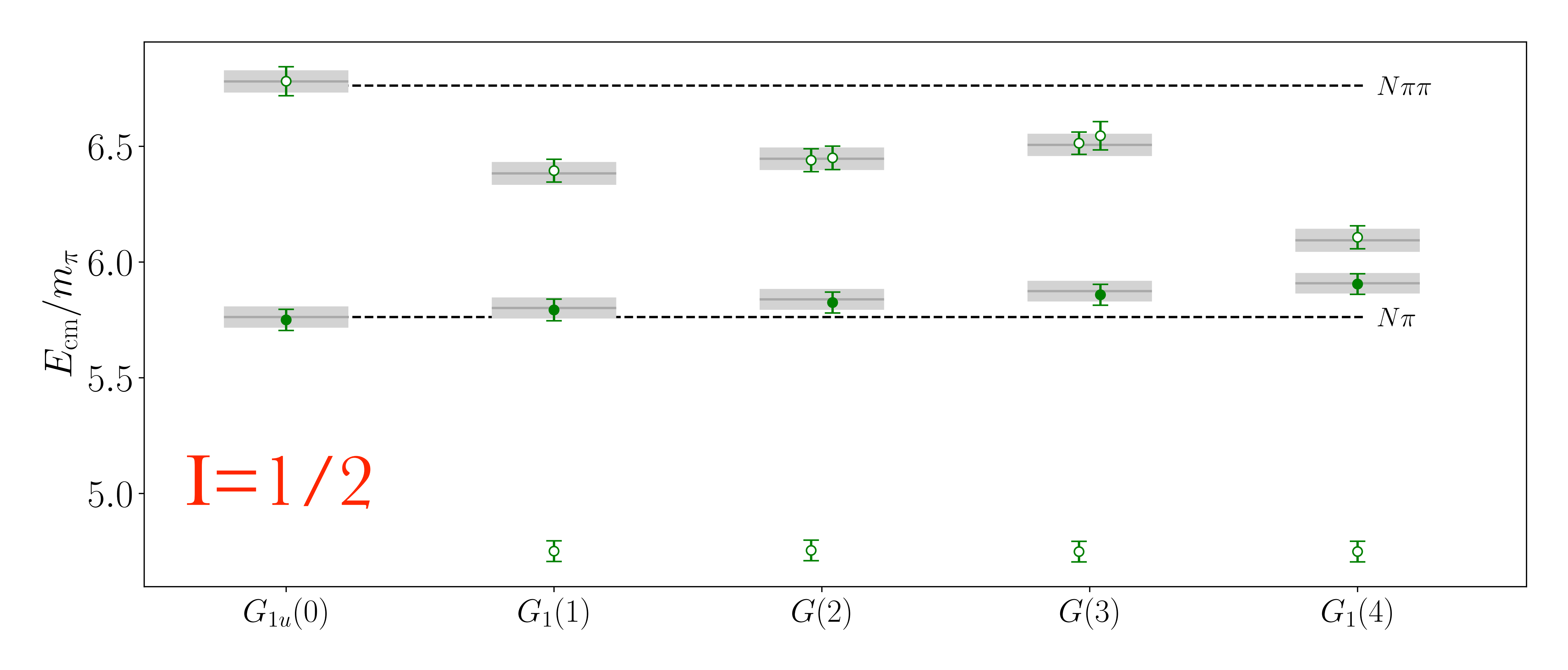
John Bulava, Andrew Hanlon, Ben Hörz, Colin Morningstar, Amy Nicholson,
Fernando Romero-López, Sarah Skinner, Pavlos Vranas, André Walker-Loud
Nucl. Phys. B 987 (2023) 116105

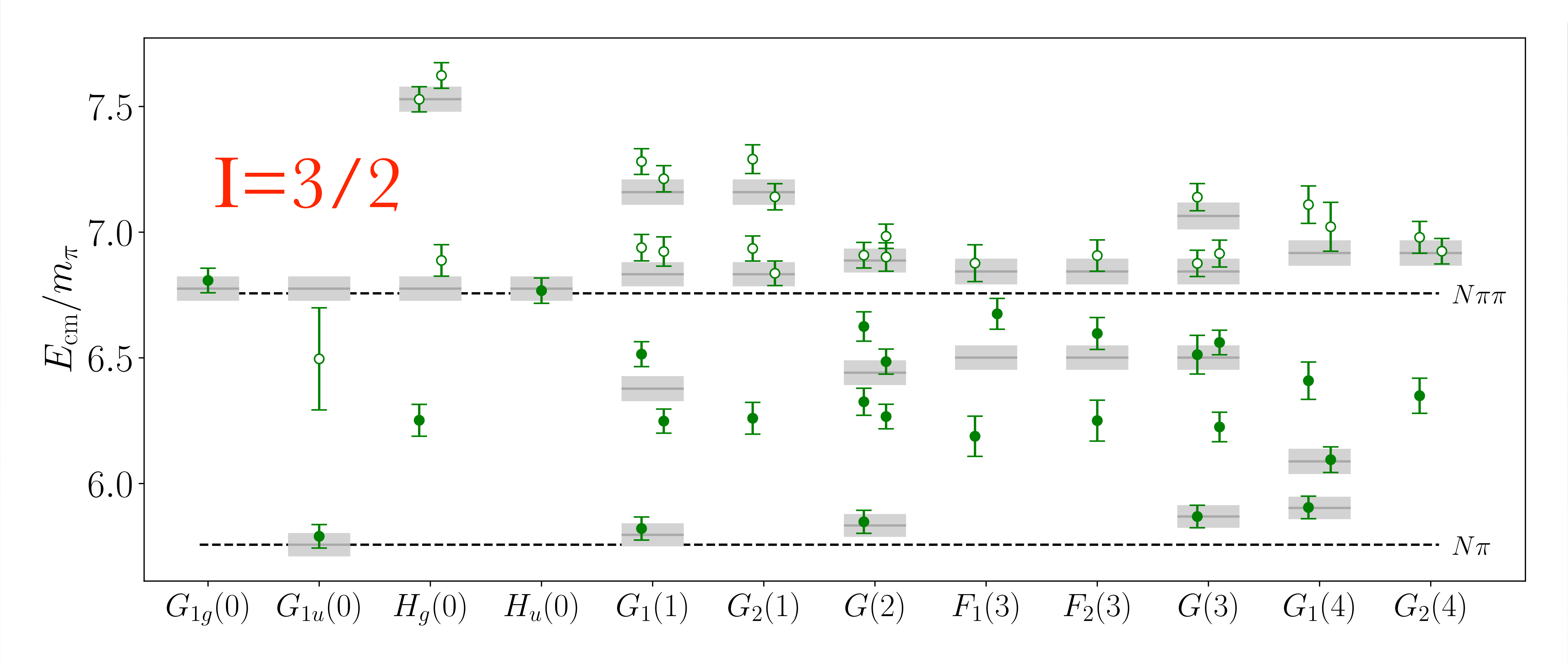
- Exciting in its own right
- Stepping stone towards NN (at this light pion mass)
- m_π is light enough that
  - the Δ is unstable
  - optimistic that EFT could be convergent-ish

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

☐ Various irreps used to determine the spectrum

$\boldsymbol{d}$	Λ	dim.	contributing $(2J,\ell)^{n_{\mathrm{occ}}}$ for $\ell_{\max}=2$
$(0,0,0)$	$G_{1\mathrm{u}}$	2	$(1,0)$
	$G_{1\mathrm{g}}$	2	$(1,1)$
	H_{g}	4	$(3,1),\ (5,2)$
	H_{u}	4	$(3,2),\ 5,2)$
	$G_{2\mathrm{g}}$	2	$(5,2)$
$(0,0,n)$	G_1	2	$(1,0),\ (1,1),\ (3,1),\ (3,2),\ (5,2)$
	G_2	2	$(3,1),\ (3,2),\ (5,2)^2$
$(0,n,n)$	G	2	$(1,0),\ (1,1),\ (3,1)^2,\ (3,2)^2,\ (5,2)^3$
(n,n,n)	G	2	$(1,0),\ (1,1),\ (3,1),\ (3,2),\ (5,2)^2$
	F_1	1	$(3,1),\ (3,2),\ (5,2)$
	F_2	1	$(3,1),\ (3,2),\ (5,2)$

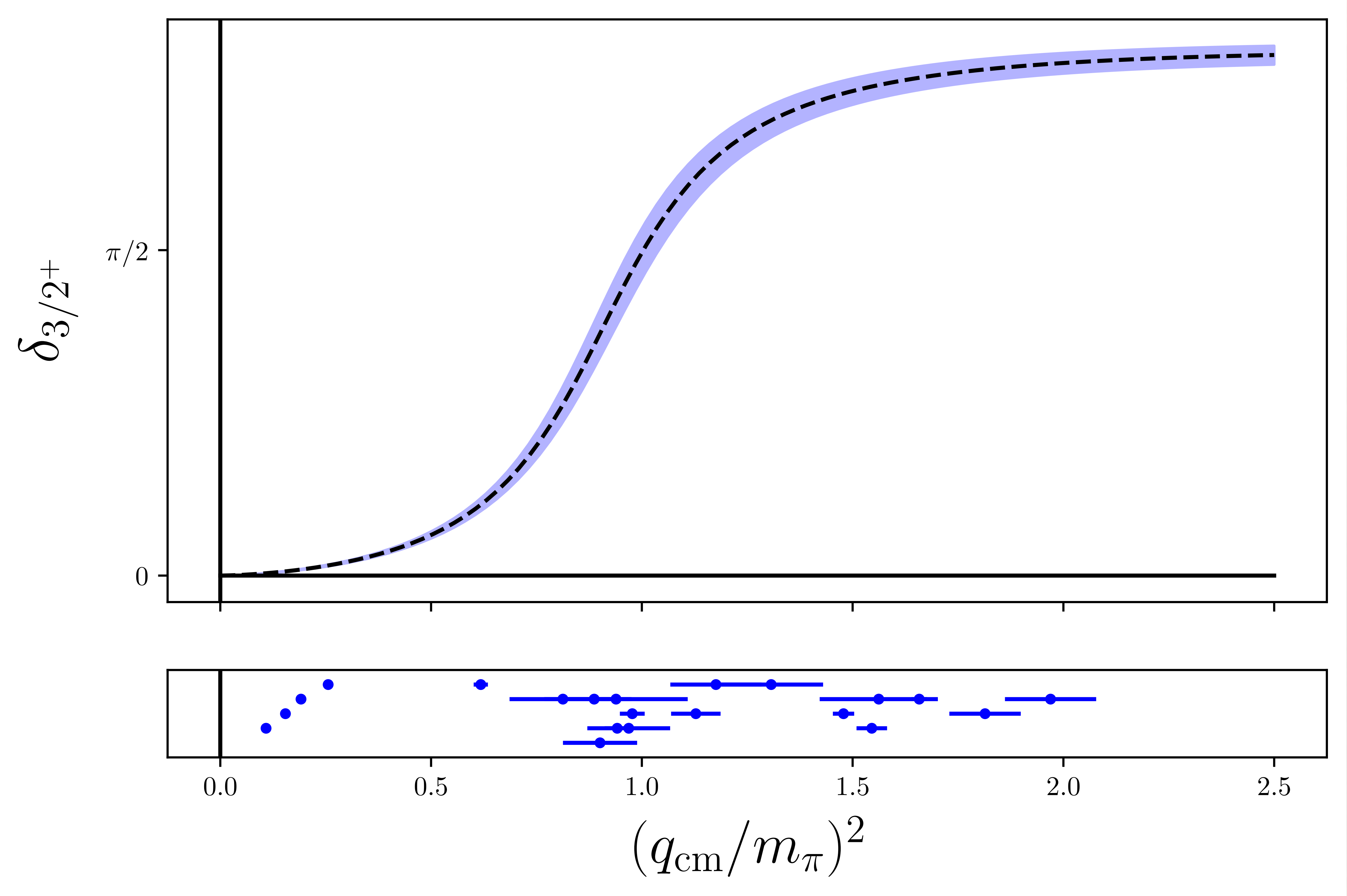


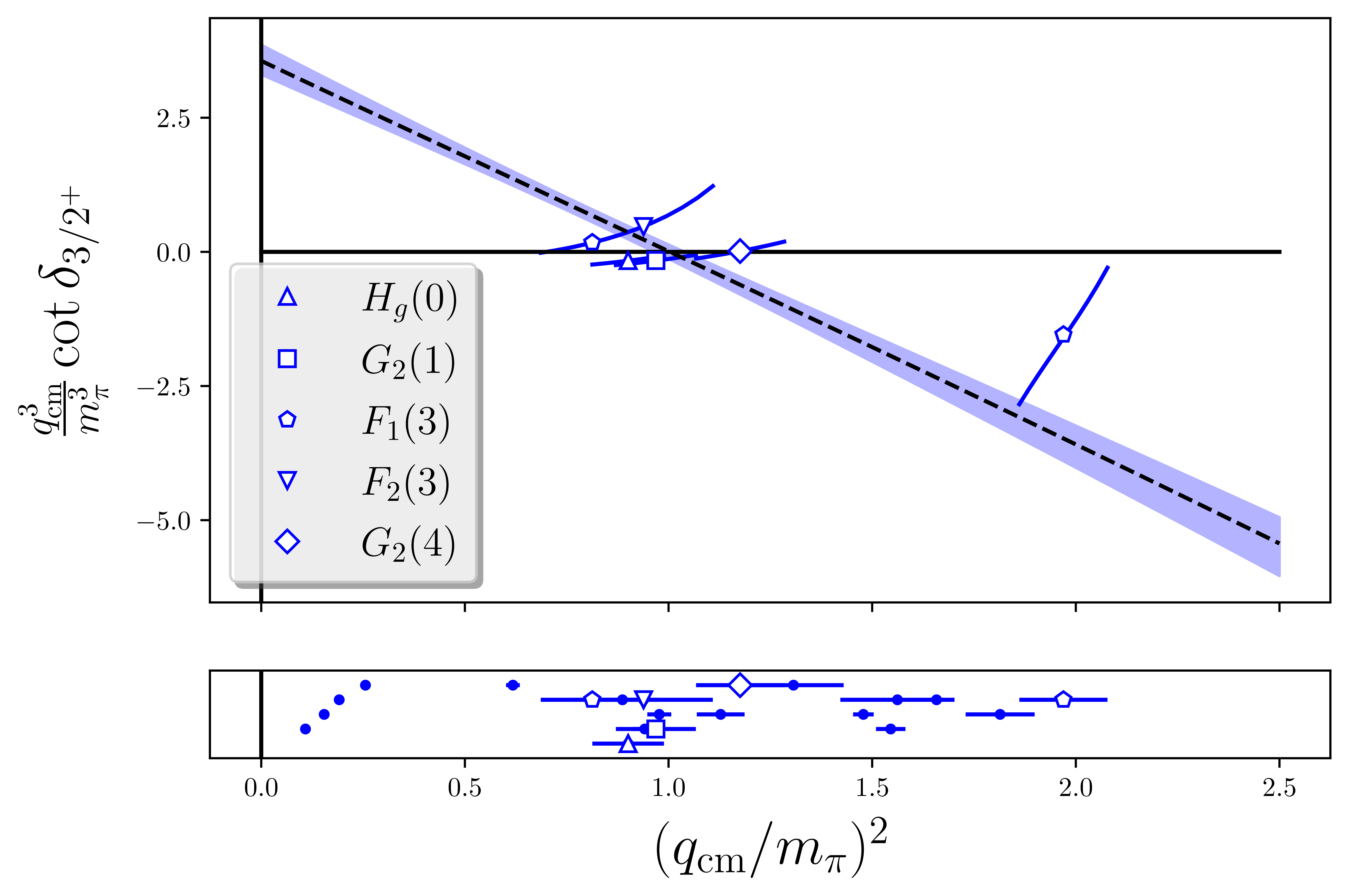


Note: the gray bands and green energy levels are correlated, which is not reflected visually in the plots

Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

- FV Spectrum to Scattering Amplitudes - spectrum method comparison - resulting amplitude





$$\frac{q^3_{\rm cm}}{m^3_\pi}\cot\delta_{3/2^+} = \frac{6\pi\sqrt{s}}{m^3_\pi g^2_{\Delta,\rm BW}}(m^2_\Delta - s)$$

open symbol: contributes to single partial wave

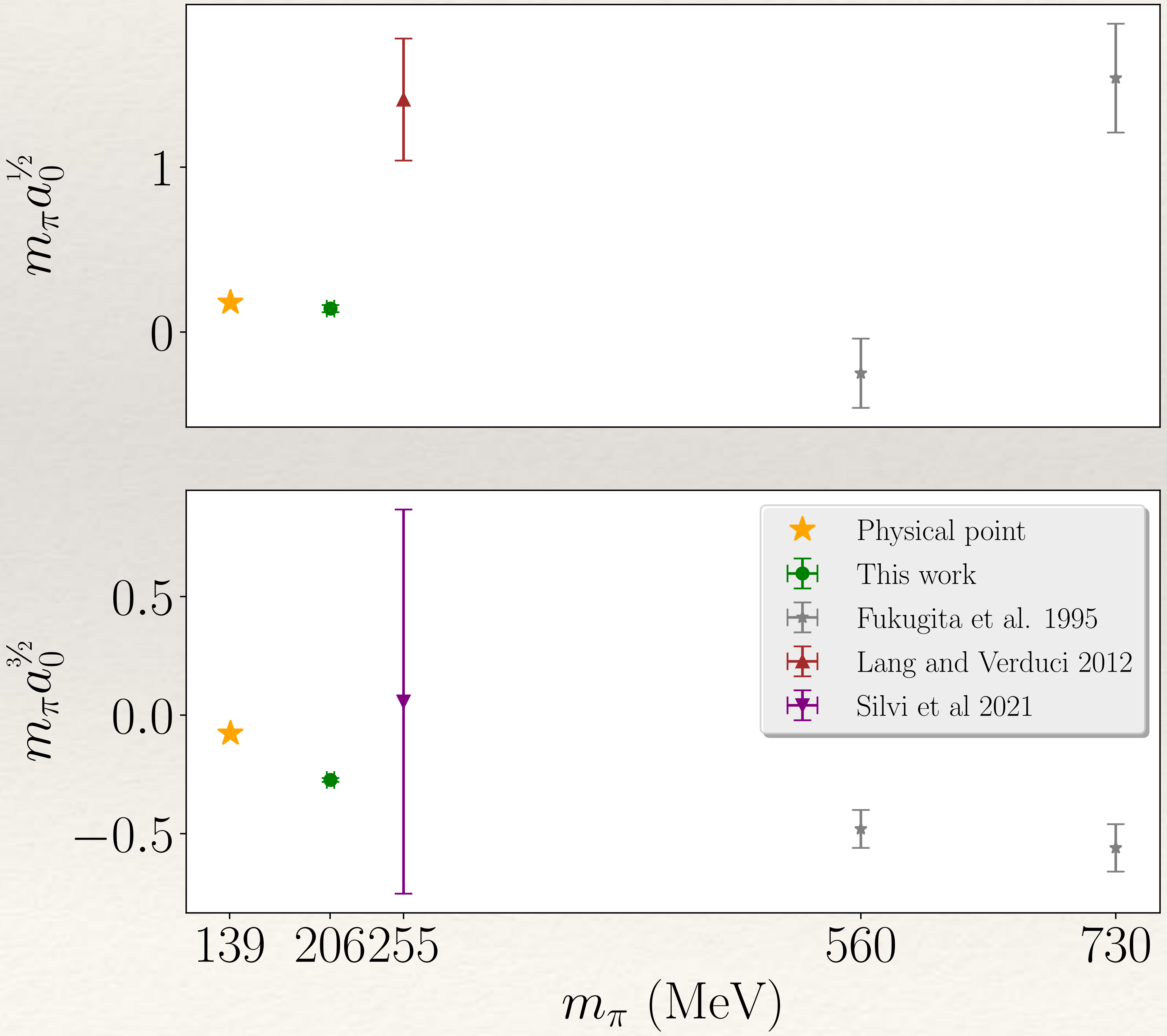
closed symbol: contributes to both partial waves

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

Results for scattering lengths and effective Delta-resonance parameters

$$m_\Delta = 1268(17)\ \mathrm{MeV} \qquad \frac{m_\Delta}{m_\pi} = 6.257(35), \qquad g_{\Delta N\pi} = 14.41(53)$$

$$m_\pi a_0^{3/2} = -0.2735(81)\,, \qquad m_\pi a_0^{1/2} = 0.142(22),$$



	m_π (MeV)	$m_\pi a_0^{1/2}$	$m_\pi a_0^{3/2}$
Pheno. (isospin limit)[27]	140	0.1788(38)	−0.0775(35)
This work	200	0.142(22)	−0.2735(81)

[27] Hoferichter, Ruiz de Elvira, Kubis, Meissner, PLB 760 (2016)

- These results present a puzzle for SU(2) baryon χPT
  - the magnitude changes so dramatically from $m_\pi \approx 140$ MeV
  - not expected
  - convergence issue for SU(2) baryon χPT?

Convergence of SU(2) baryon χPT

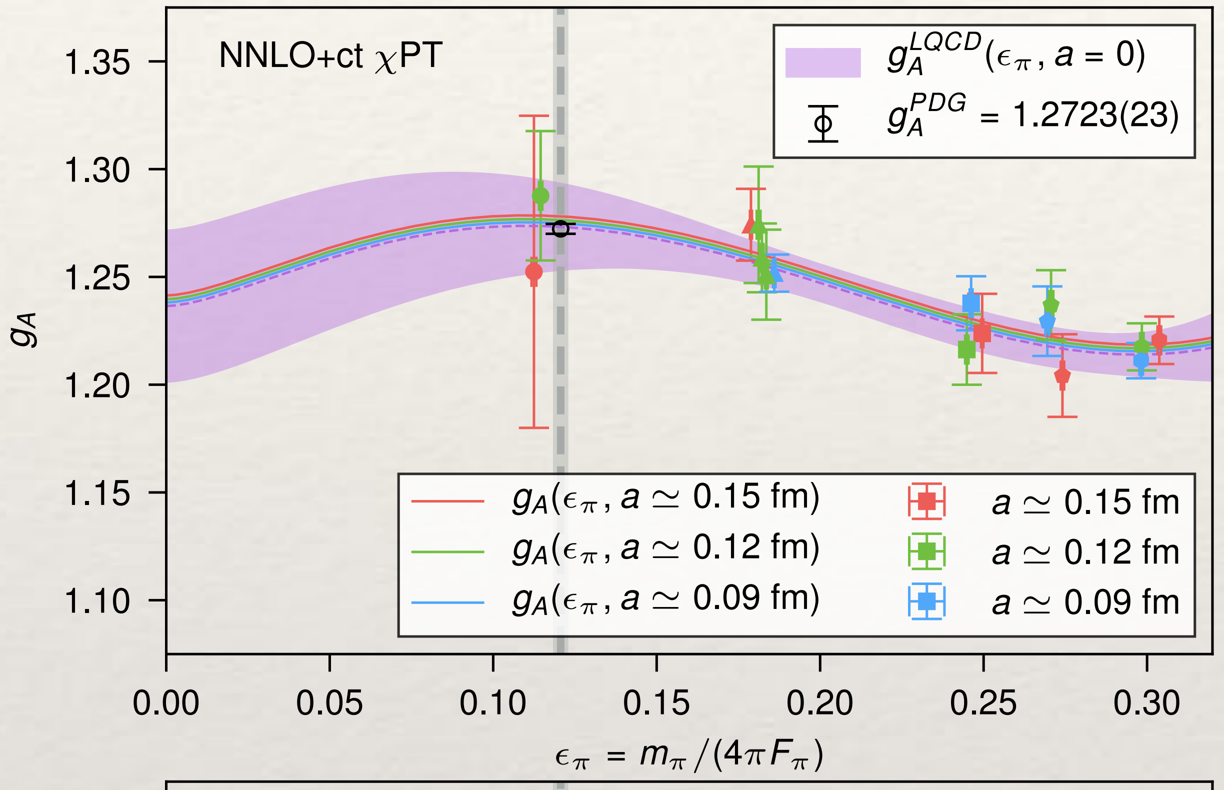
- **Is the fine-tuning that is present in the low-energy NN scattering persistent as the up/down quark masses are changed from their physical values?**
  - Academic: understanding our universe in terms of SM parameters
  - Practical: for the foreseeable future, LQCD calculations of NN interactions will require extrapolations from $m_\pi^{LQCD} \rightarrow m_\pi^{\rm phys}$

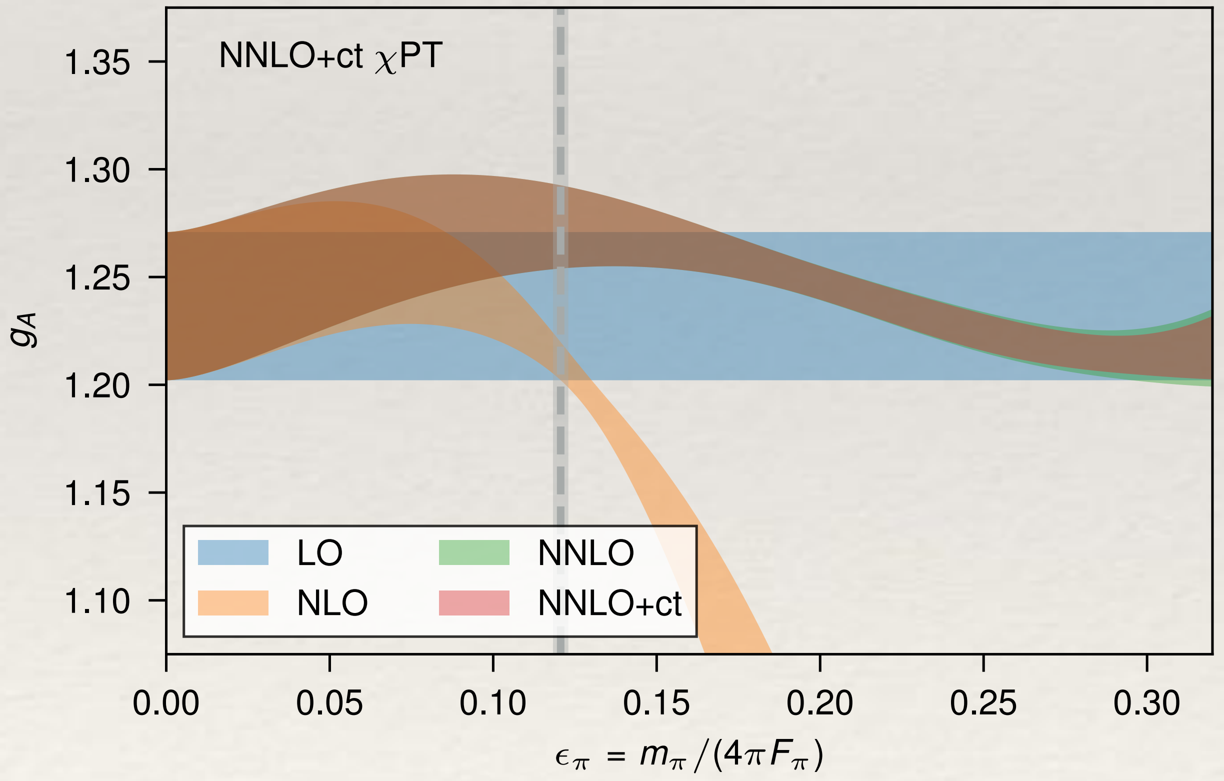
    As the pion mass is changes, the appropriate EFT (power counting) might change

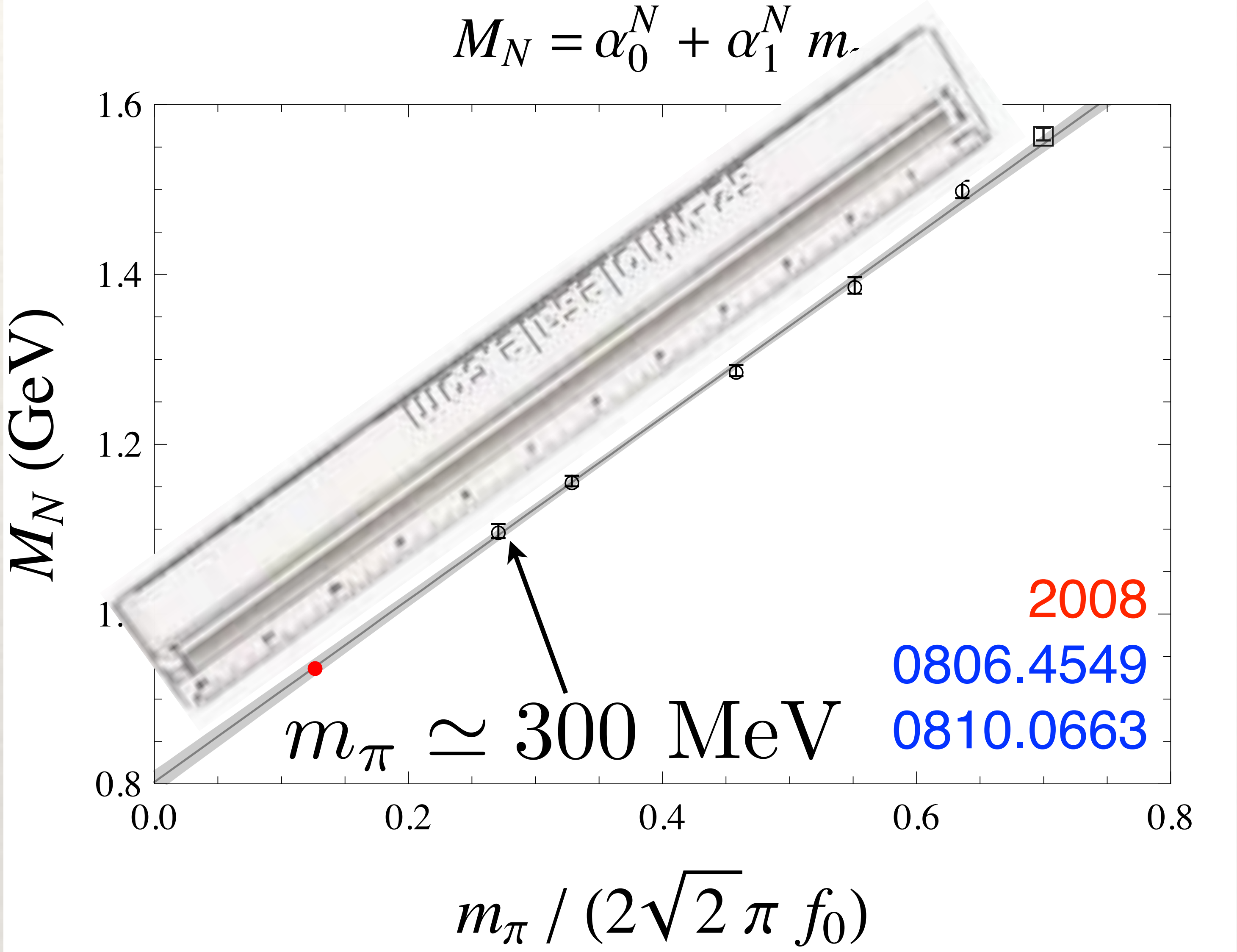
- **EFT provides us with predicted pion mass dependence for observables**
  - Do we observe this expected pion mass dependence in LQCD results?
  - If no or yes, what does it teach us about the efficacy of the EFT?

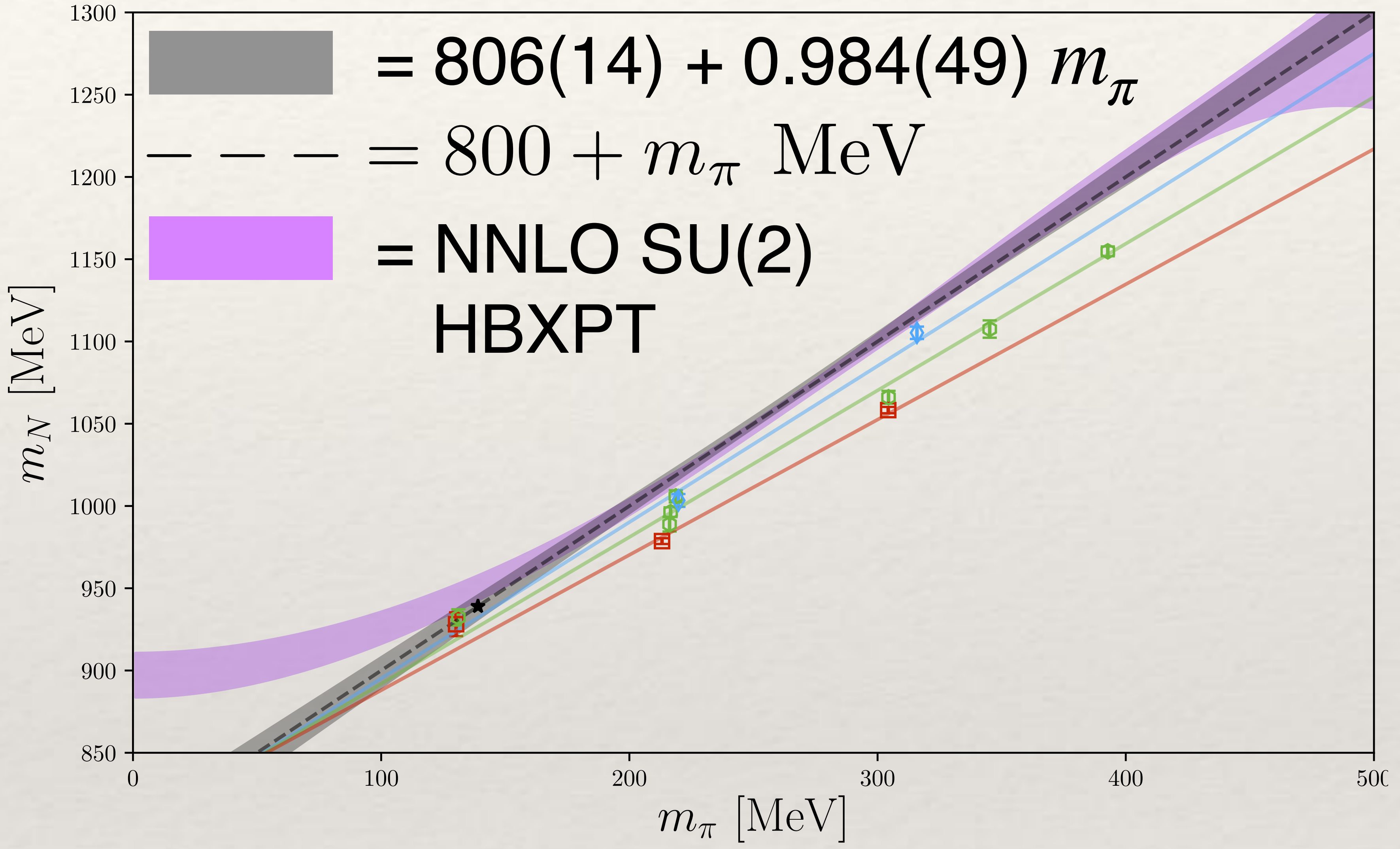
Convergence of SU(2) baryon χPT

- **Can we map out the convergence pattern of our EFTs versus m_π?**
  - LQCD results for M_N and g_A suggest that SU(2) baryon XPT w/out Δ is non-convergent









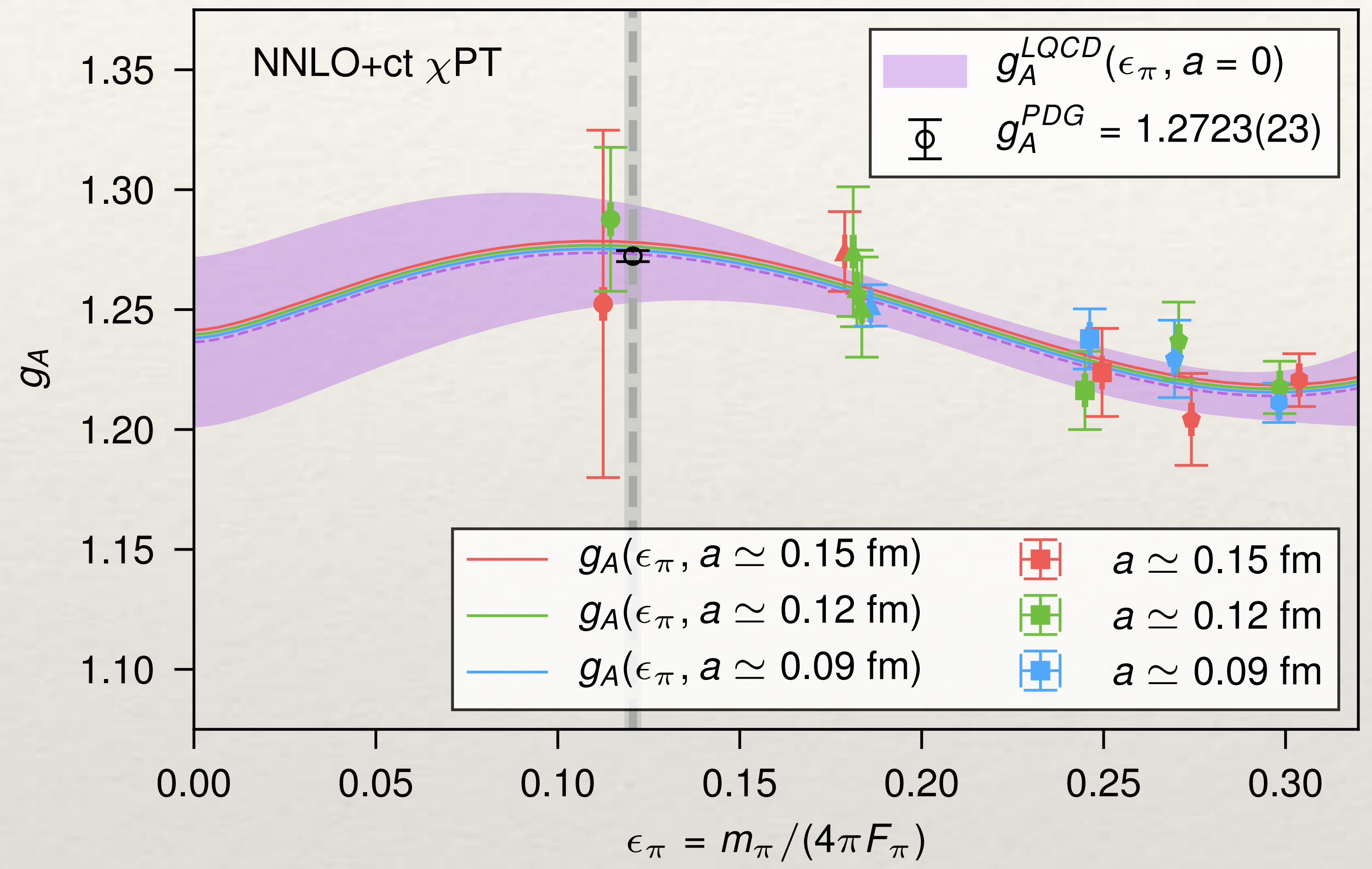
$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + c_4\epsilon_\pi^4$$

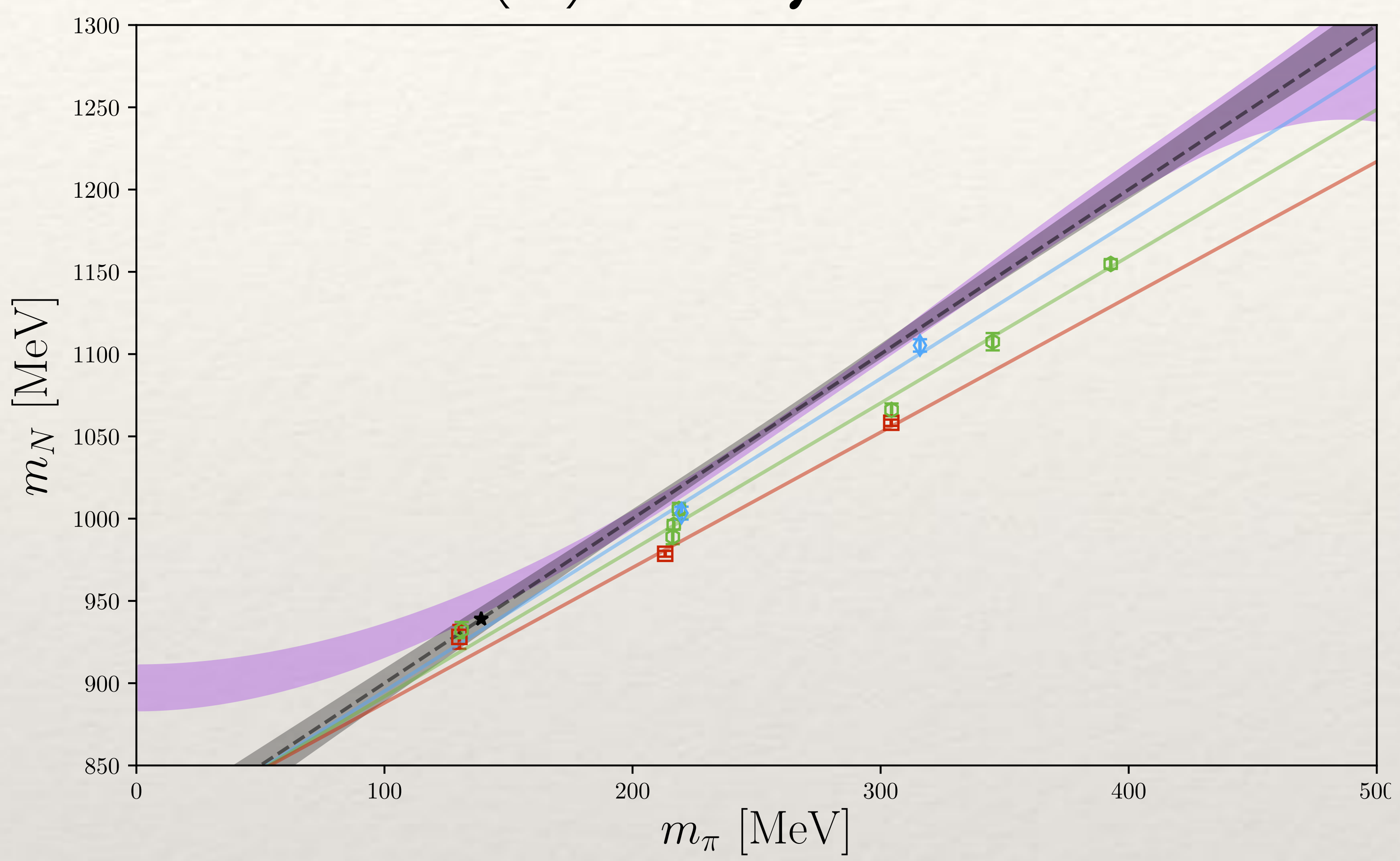
N^nLO	LO	NLO	N^2LO	N^3LO
N^2LO	1.237(34)	-0.026(30)	0.062(14)	–
N^3LO	1.296(76)	-0.19(12)	0.045(63)	0.117(66)

@ m_π^{phys}

Convergence of SU(2) baryon χPT

- **Can we map out the convergence pattern of our EFTs versus m_π?**
  - LQCD results for M_N and g_A suggest that SU(2) baryon XPT w/out Δ is non-convergent





  - The flat (g_A) and linear (M_N) pion mass dependence indicates strong cancellations between orders - a sign of breakdown
  - Adding explicit Δ will improve convergence of g_A (large-Nc) but make M_N worse
  - Adding Δ to LQCD requires *Nπ* scattering to determine all LECs
  - Adding Δ to SU(2) χPT requires adding it to NN EFT…

Outlook

- Still significant effort required for LQCD to become relevant to fundamental physics of radioactive nuclei
  - There are clear matrix elements where LQCD can provide important contributions
  - We should consider the resource requirements (human and computing) when deciding what to compute

- Obtaining the NN spectrum, and hence scattering amplitudes, it is important to use momentum-space creation operators
  - Preponderance of evidence that NN controversy is resolved — local operators give the wrong spectrum with available computing resources
  - In order to be relevant, we need to perform calculations @ $m_\pi \lesssim 200$ MeV
  - There are potentially significant discretization effects present in NN systems which must be accounted for

- In the single nucleon sector, we see that LQCD is having an important impact
  - Nucleon axial form factor from LQCD in tension with old pheno extraction
  - LQCD $F_A(Q^2)$ leads to $\approx 30\,\%$ enhancement of $\nu - N$ cross section
  - Further progress requires new methods that will enable $N \to N\pi(\Delta)$
  - LQCD is being used to stress-test SU(2) baryon χPT
  - LQCD can be used to determine larger set of LECs in Δ-full EFT

Thank You

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U.S. DEPARTMENT OF ENERGY Office of Science
NNSA National Nuclear Security Administration
NSF National Science Foundation