Pion-nucleon scattering with lattice QCD at M$\pi \approx$ 200 MeV

Accessing and Understanding the QCD Spectra
INT 20r-2c, March 2023

André Walker-Loud

BERKELEY LAB

Lattice QCD and Effective Field Theory for "Nuclear Physics"

- ❑ What are our long term goals?
- ❑ What questions will we try and answer in support of these goals?

✴Nuclear Physics:
low-energy processes involving one, two, … nucleons, pions and currents

What are our long-term goals?

- Quantitatively understand the emergence of nuclear physics from the Standard Model (SM)
  - to possibly contribute to our understanding of nuclear physics in extreme environments, such as neutron stars
    - NNN, YN, YNN, … interactions
  - to predict nuclear reactions that contribute to low-energy precision tests of the SM
    - neutron β-decay (test V-A structure)
    - $0\nu\beta\beta$
    - ν-A scattering
    - permanent EDMs in nucleons and nuclei

What are our long-term goals?

- Quantitatively understand the emergence of nuclear physics from the Standard Model (SM)

Many Body Nuclear Methods — GFMC, IMSRG, coupled-cluster, HOBET, Shell Model, …

↑

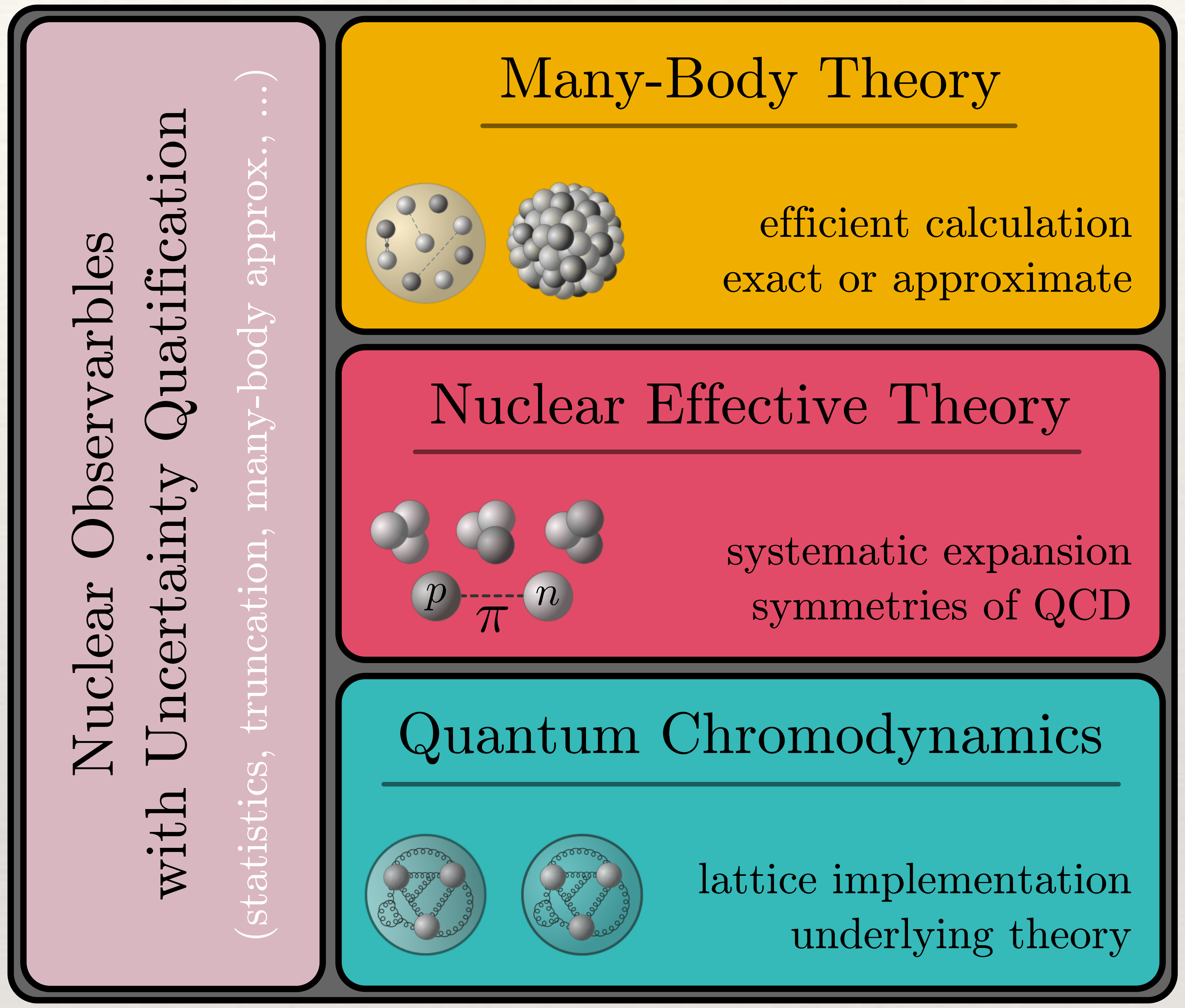
NN (YN) EFT — two-nucleons (pions)

↑

Hadronic EFT — pions, kaons, nucleons, hyperons delta (decuplet)

↑

QCD — quarks, gluons and lattices (M. Creutz)



(fig: C. Drescher)

What questions will we try and answer to support these goals?

- **Is the fine-tuning that is present in the low-energy NN scattering persistent as the up/down quark masses are changed from their physical values?**
  - Academic: understanding our universe in terms of SM parameters
  - Practical: for the foreseeable future, LQCD calculations of NN interactions will require extrapolations from $m_\pi^{LQCD} \rightarrow m_\pi^{\mathrm{phys}}$
    As the pion mass is changes, the appropriate EFT (power counting) might change

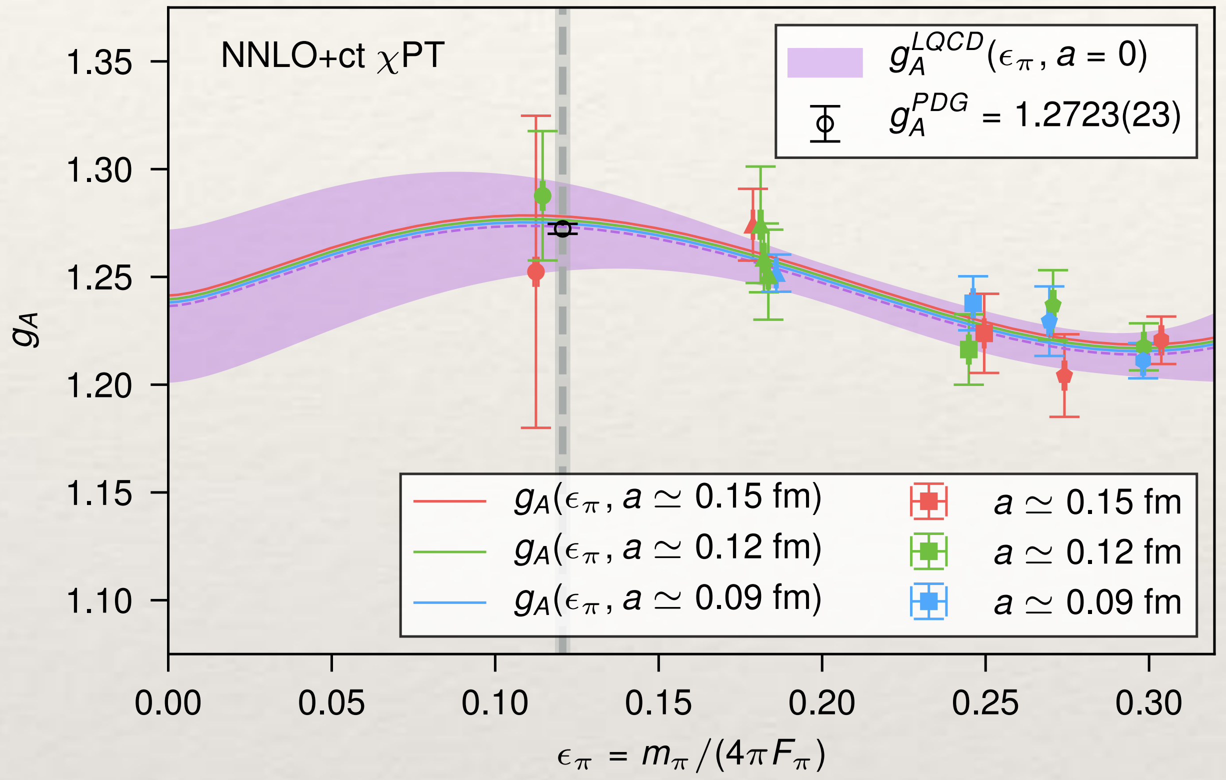
- **EFT provides us with predicted pion mass dependence for observables**
  - Do we observe this expected pion mass dependence in LQCD results?
  - If no or yes, what does it teach us about the efficacy of the EFT?

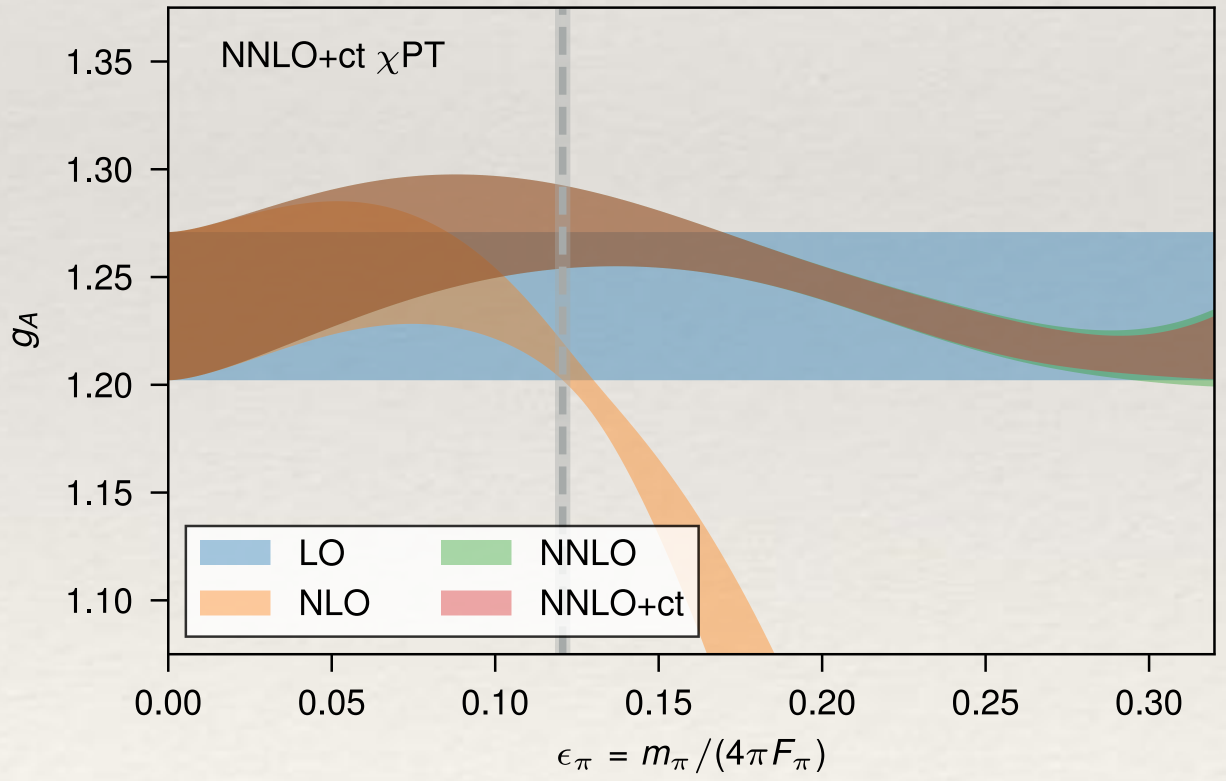
What questions will we try and answer to support these goals?

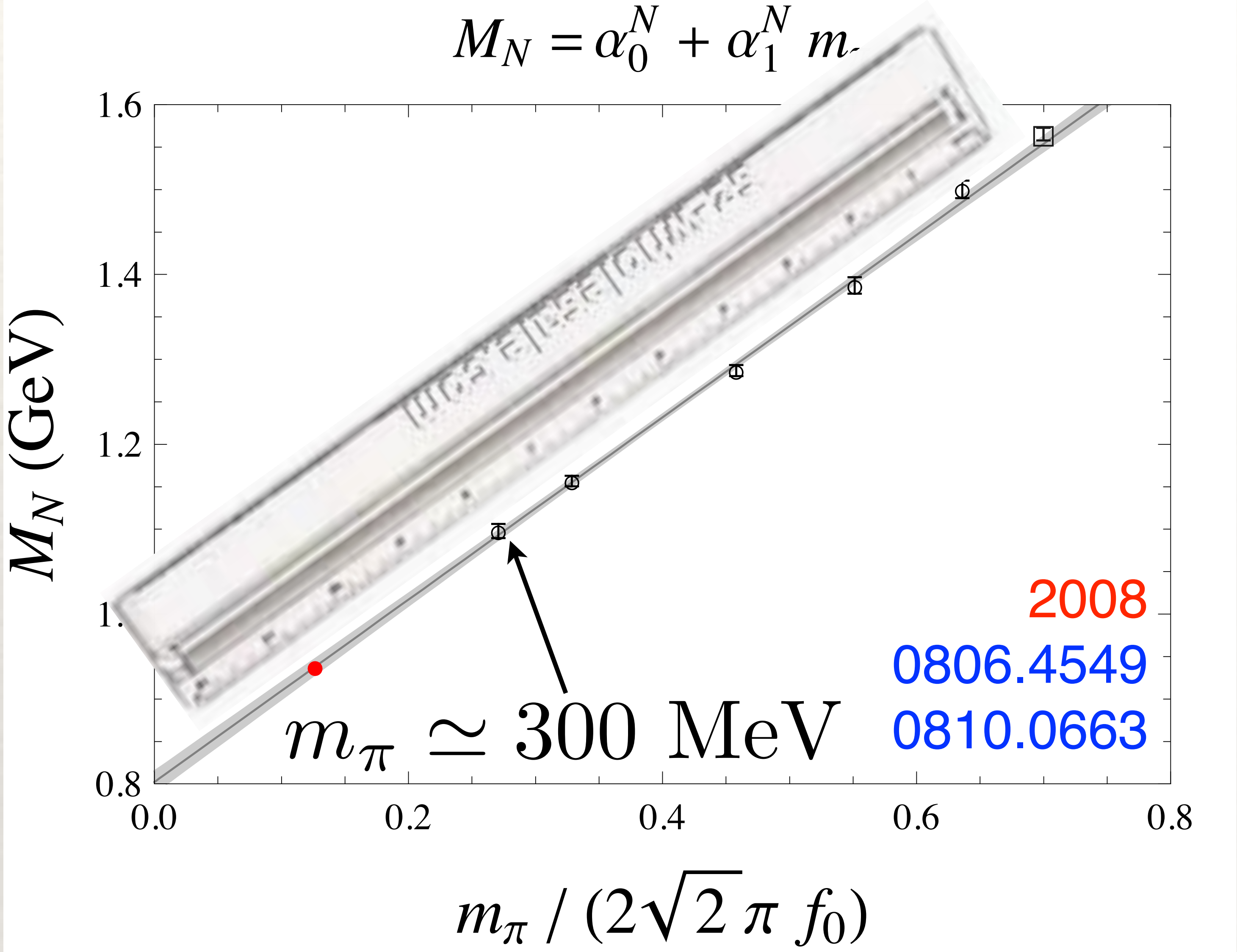
- **Can we map out the convergence patter of our EFTs versus m_π?**
  - m_π, m_K, F_π, F_K: MILC Collaboration has demonstrated that SU(3) XPT provides a qualitative, but not a precise quantitative description at $m_s \approx m_s^{\text{phys}}$
    C. Bernard, CD2015 [1510.02180]
  - M_B: SU(3) heavy baryon XPT (HBXPT) is not a convergent expansion @ m_s^{phys}
    LHP Collaboration [0806.4549], PACS-CS Collaboration [0905.0962]
    NPLQCD Collaboration [0912.4243]
    - $\rightarrow$ YN SU(3) EFT is a model (theoretical uncertainty is not controlled)

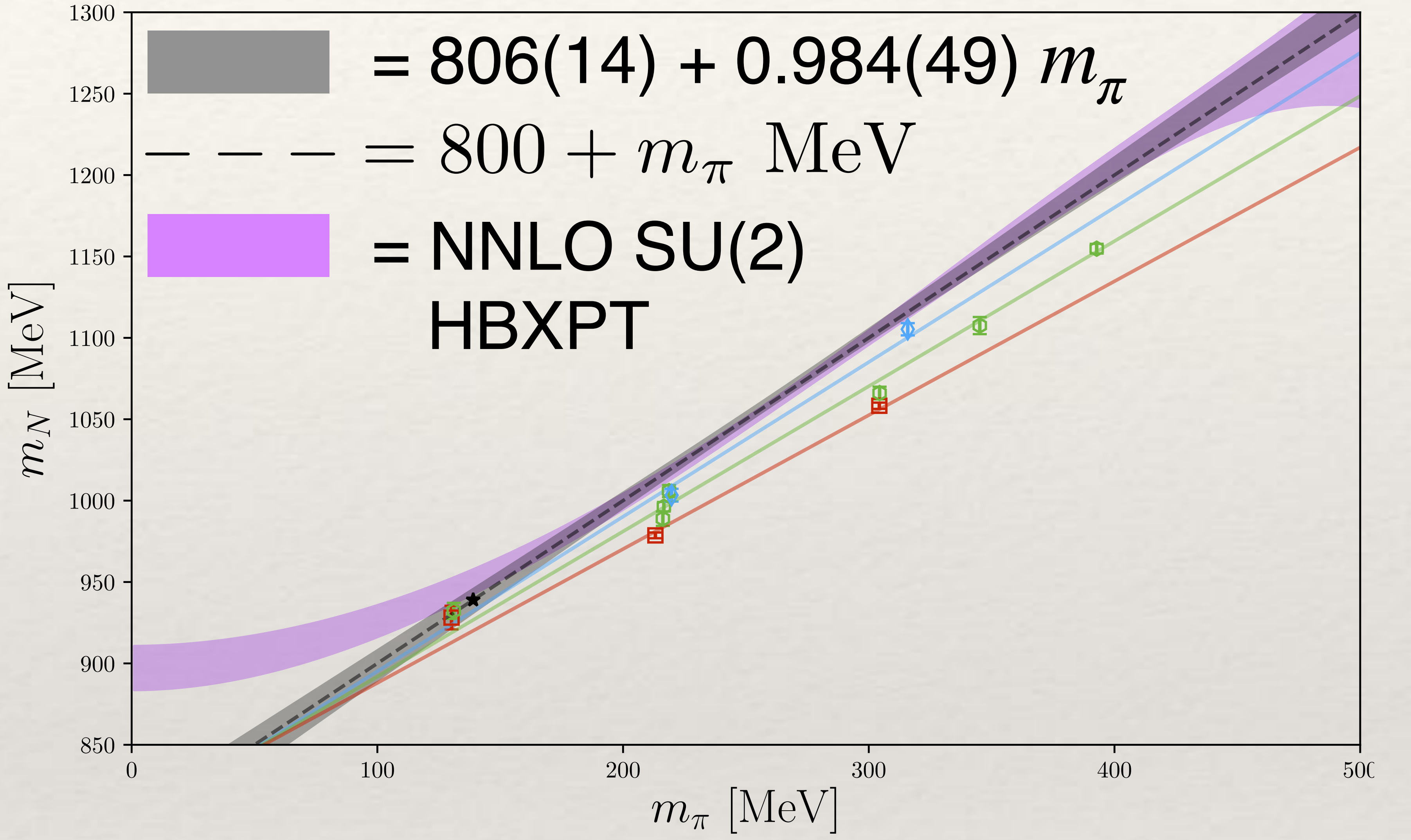
What questions will we try and answer to support these goals?

- **Can we map out the convergence patter of our EFTs versus m_π?**
  - LQCD results for M_N and g_A suggest that SU(2) baryon XPT w/out Δ is non-convergent









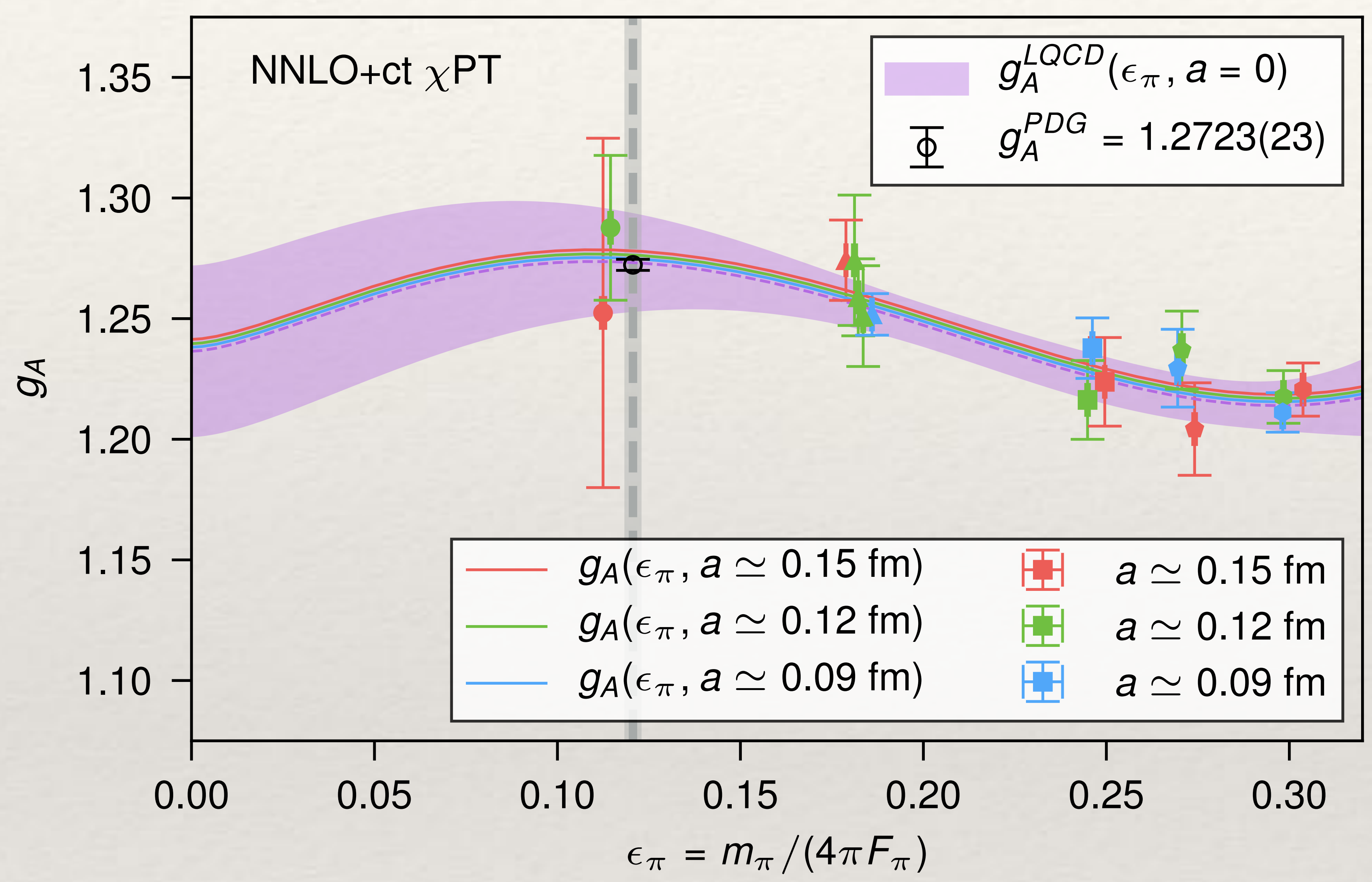
$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + c_4\epsilon_\pi^4$$

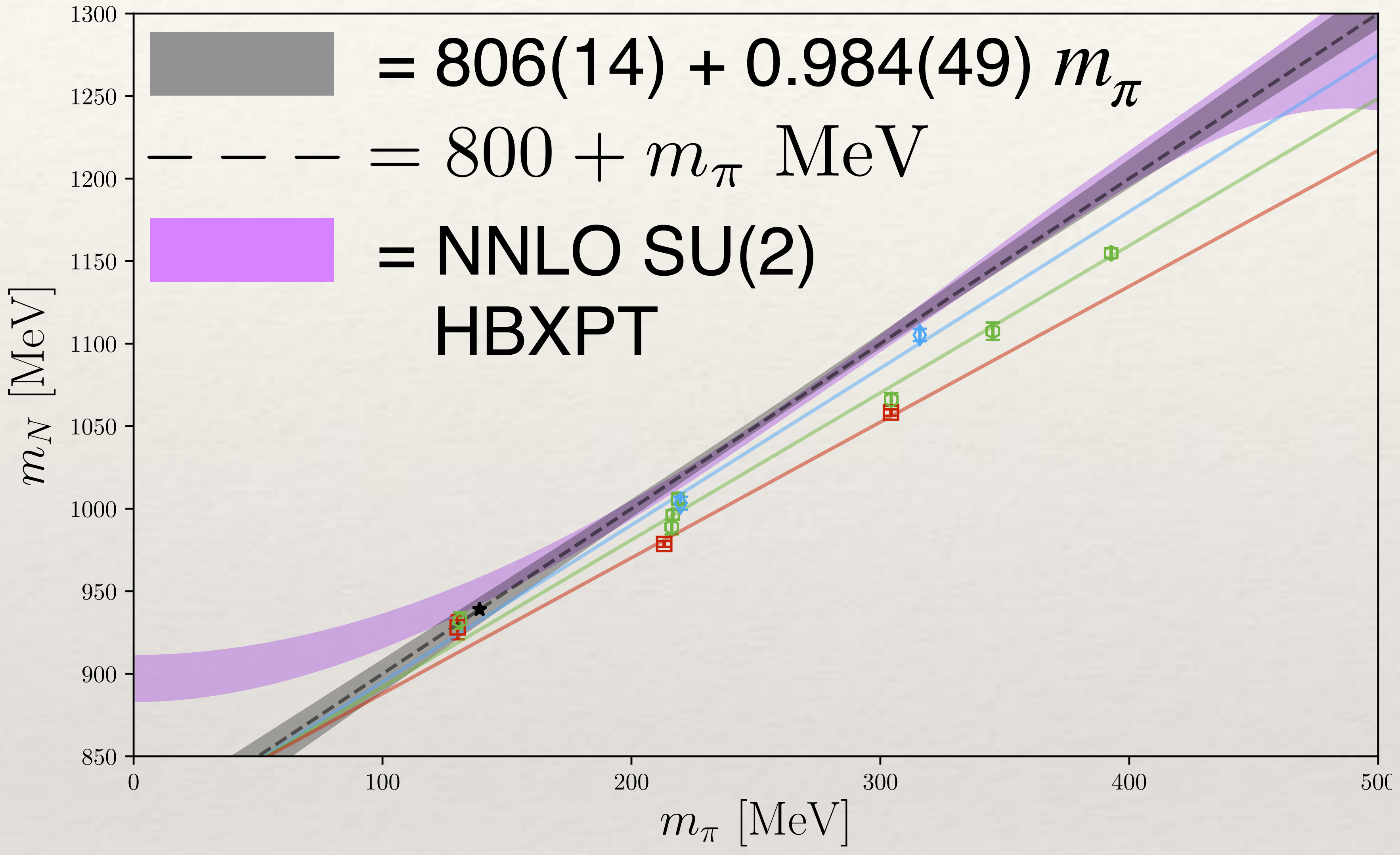
N^nLO	LO	NLO	N^2LO	N^3LO
N^2LO	1.237(34)	-0.026(30)	0.062(14)	–
N^3LO	1.296(76)	-0.19(12)	0.045(63)	0.117(66)

@ m_π^{phys}

What questions will we try and answer to support these goals?

- **Can we map out the convergence patter of our EFTs versus m_π?**
  - LQCD results for M_N and g_A suggest that SU(2) baryon XPT w/out Δ is non-convergent





  - The flat (g_A) and linear (M_N) pion mass dependence indicates strong cancellations between orders - a sign of breakdown
  - Adding explicit Δ will improve convergence of g_A (large-Nc) but make M_N worse
  - Adding Δ to LQCD requires *Nπ* scattering

arXiv > hep-lat > arXiv:2208.03867

High Energy Physics – Lattice

[Submitted on 8 Aug 2022 (v1), last revised 7 Feb 2023 (this version, v3)]

Elastic nucleon–pion scattering at $m_\pi = 200$ MeV from lattice QCD

John Bulava, Andrew Hanlon, Ben Hörz, Colin Morningstar, Amy Nicholson, Fernando Romero-López, Sarah Skinner, Pavlos Vranas, André Walker-Loud

Nucl. Phys. B 987 (2023) 116105

- Exciting in its own right
- Stepping stone towards NN (at this light pion mass)
- m_π is light enough that
  - the Δ is unstable
  - optimistic that EFT could be convergent-ish

arXiv > hep-lat > arXiv:2208.03867

High Energy Physics – Lattice

[Submitted on 8 Aug 2022 (v1), last revised 7 Feb 2023 (this version, v3)]

Elastic nucleon–pion scattering at m_π = 200 MeV from lattice QCD

John Bulava, Andrew Hanlon, Ben Hörz, Colin Morningstar, Amy Nicholson,
Fernando Romero-López, Sarah Skinner, Pavlos Vranas, André Walker-Loud

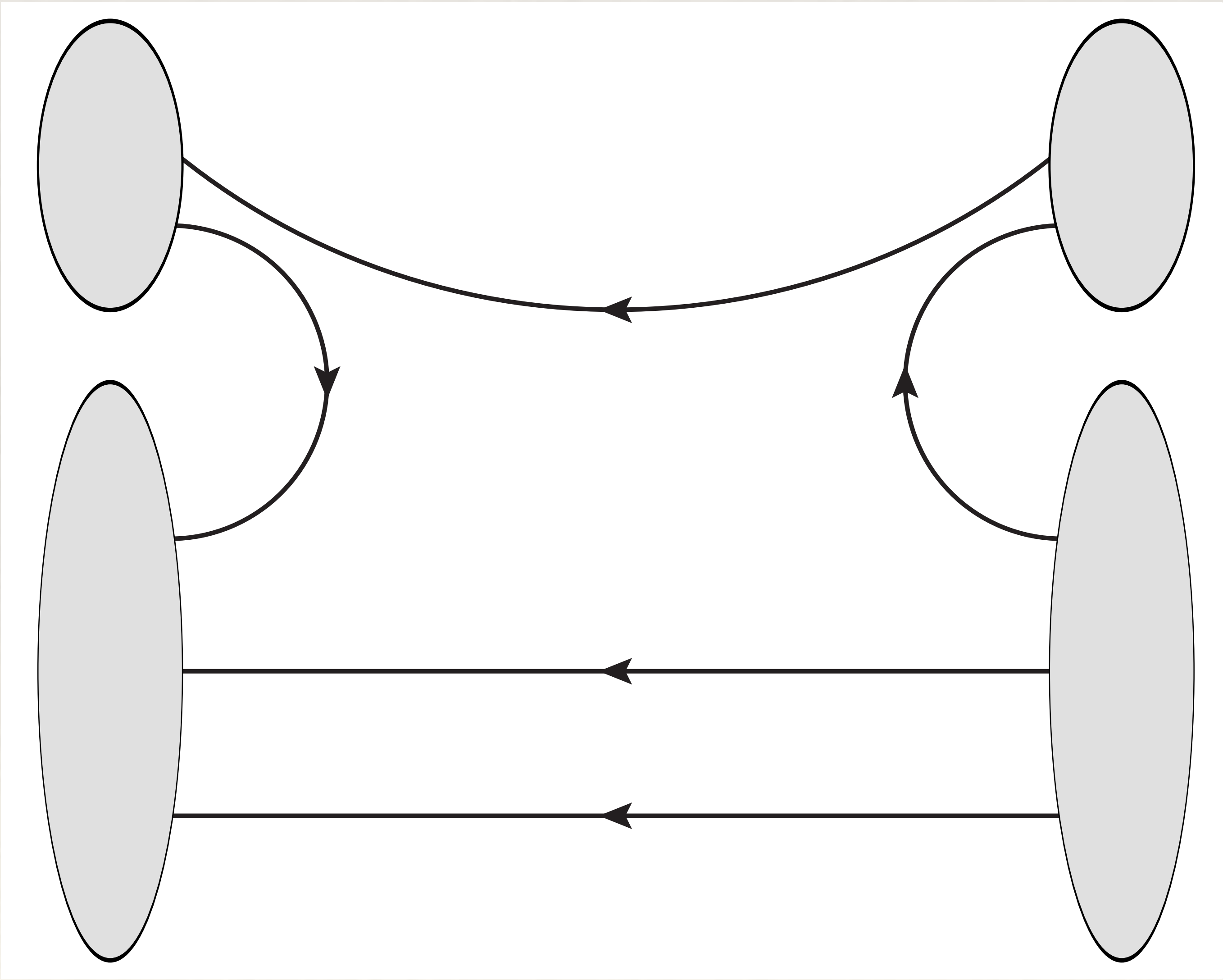
Nucl. Phys. B 987 (2023) 116105

- Single CLS ensemble (D200)
  - a ≈ 0.063 fm, V=64^3x128, N_{cfg}=2000, $m_\pi \approx 200$ MeV, $m_\pi L \approx 4.2$
  - $\mathrm{tr}\left(M_q\right) = \mathrm{tr}\left(M_q^{\mathrm{phys}}\right)$
    - choice made by CLS (not us) when generating configs - simplifies renormalization
    - $m_K \approx 480$ MeV

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

- What were the goals of this calculation?
  - Is the sLapH method capable — with reasonable statistics/resources — in achieving precise estimates of the interaction energies/scattering amplitudes at light pion masses?
  - It seems the answer is yes, fortunately
  - To carry out the study, we used

N_{D}	(ρ, n_ρ)	N_{ev}	$N_{\mathrm{R}}^{\mathrm{fix}}$	$N_{\mathrm{R}}^{\mathrm{rel}}$	Noise dilution	N_{t_0}
~~2560~~ 2176	(0.1,36)	448	6	2	$(\mathrm{TF,SF,LI16})_{\mathrm{fix}}(\mathrm{TI8,SF,LI16})_{\mathrm{rel}}$	4

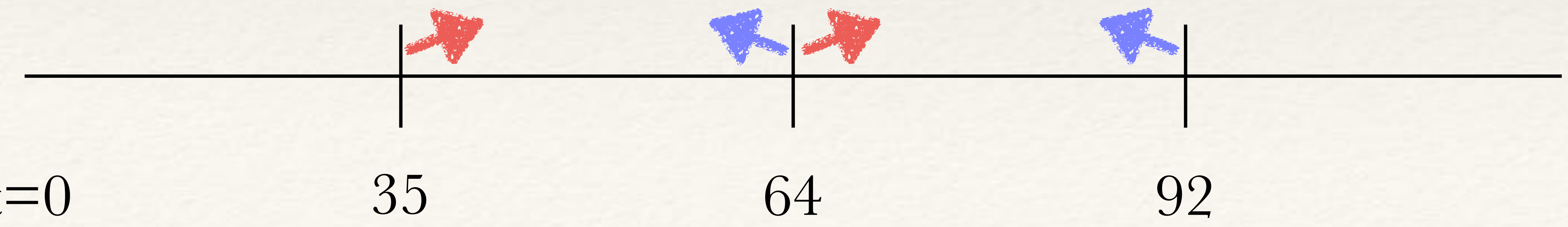


N_{D} Dirac inversions per config

N_{ev} No. of eigenvectors of 3D Laplacian

$N_{\mathrm{R}}^{\mathrm{fix}}$ No. of noise sources for fixed lines

$N_{\mathrm{R}}^{\mathrm{rel}}$ No. of noise sources for relative lines



Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

Results

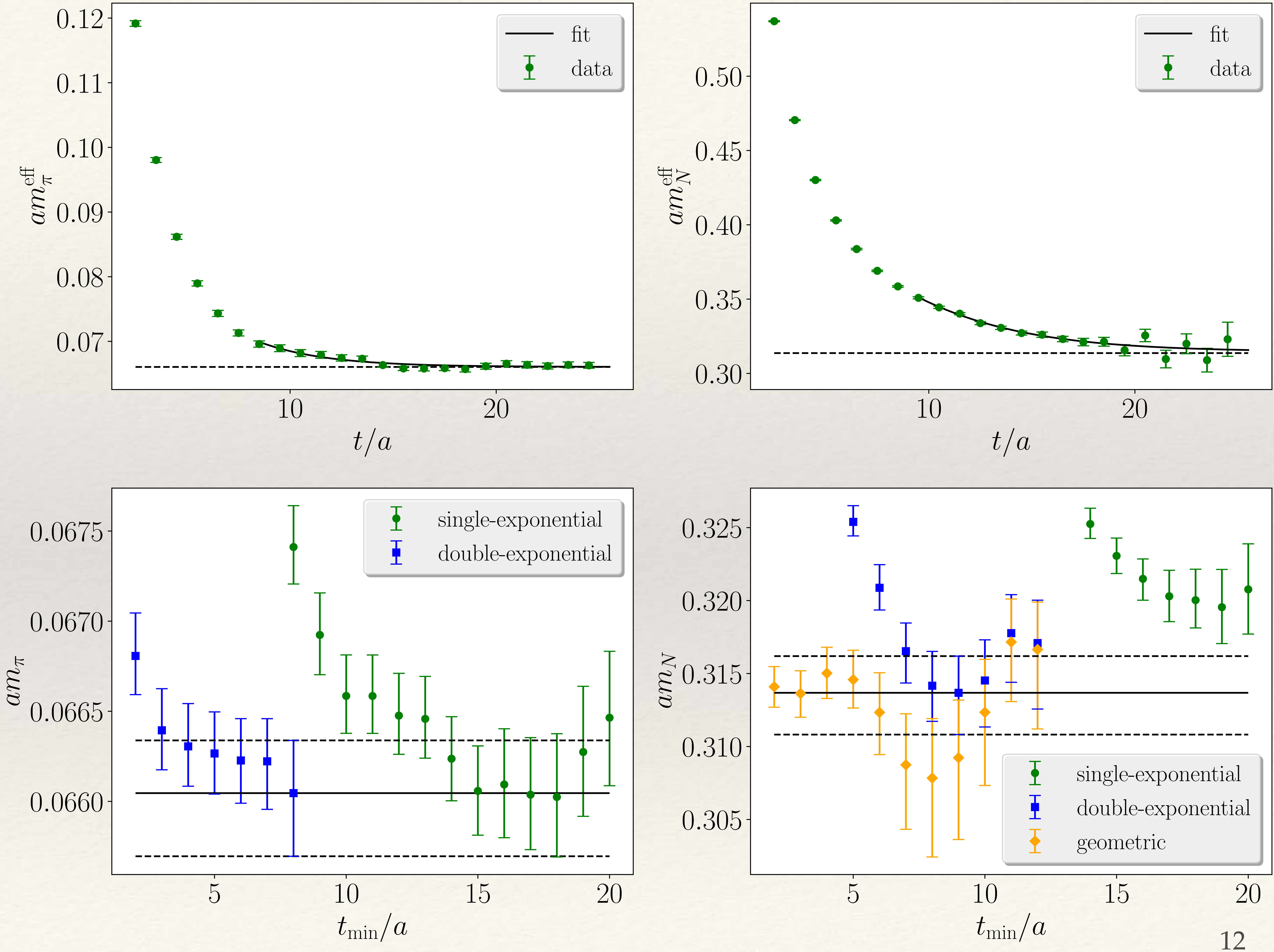
- We explored multi-exponential fits as well as a "geometric series" fit

$$C_{\text{geom.}}(t) = \frac{Ae^{-E_0 t}}{1 - Be^{-\Delta E t}}$$

- This GS fit does quite well
- Our interest is quantifying uncertainty on ground state

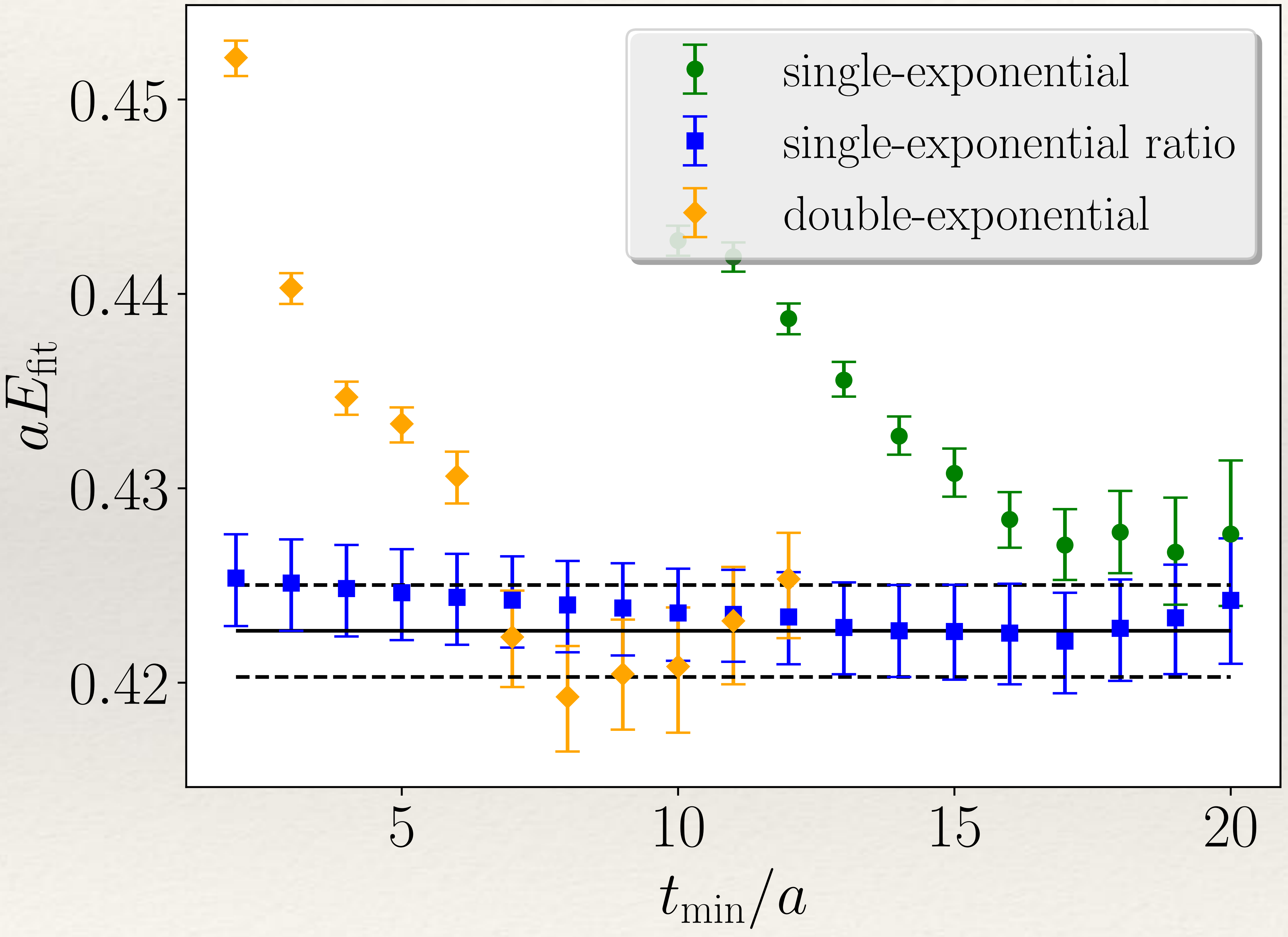
- We also tried a multi-state version of the GS ansatze

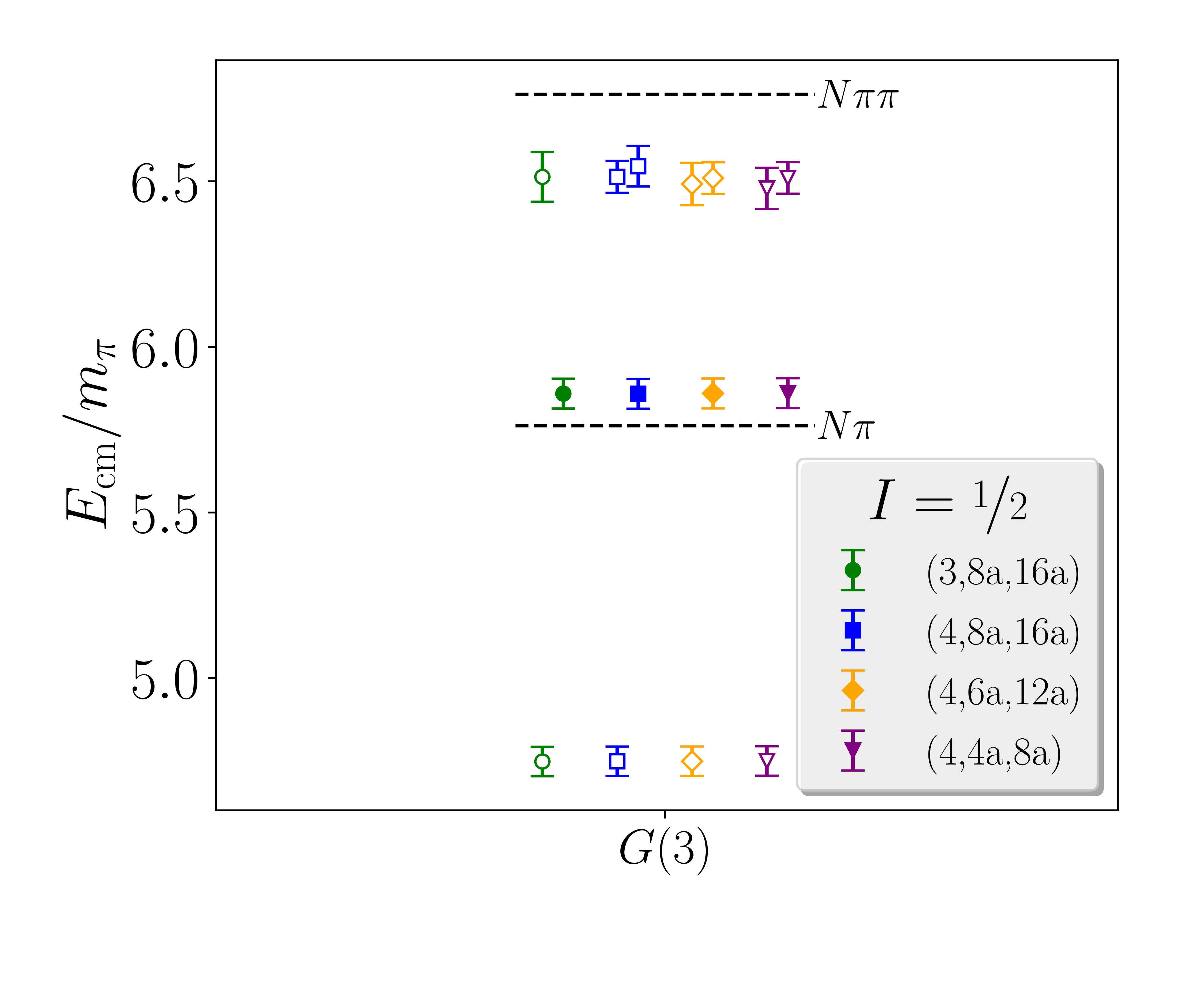
$$C^N_{\text{geom.}}(t) = \frac{Ae^{-E_0 t}}{1 - \sum_{n=1}^{N-1} B_n e^{-\Delta E_n t}}$$



Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

❏ Parity Odd Results - S-wave Nπ

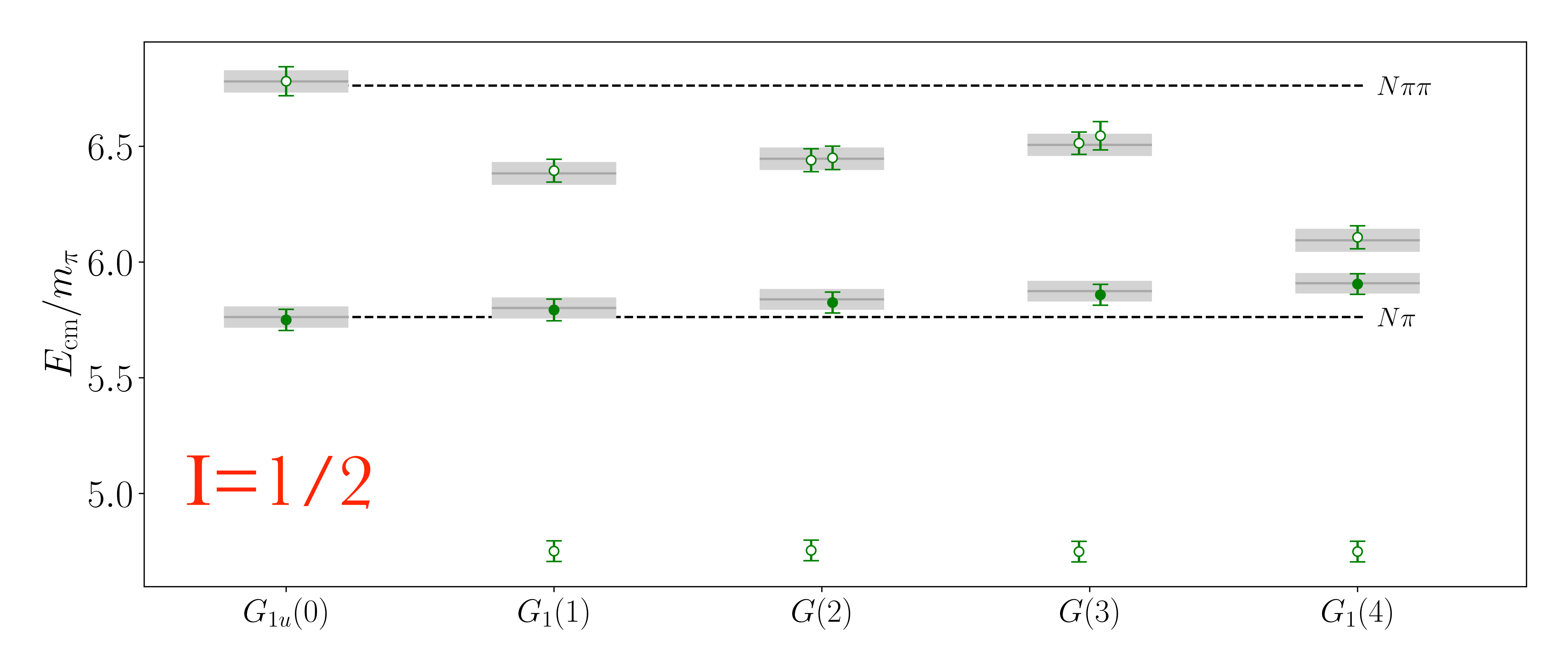


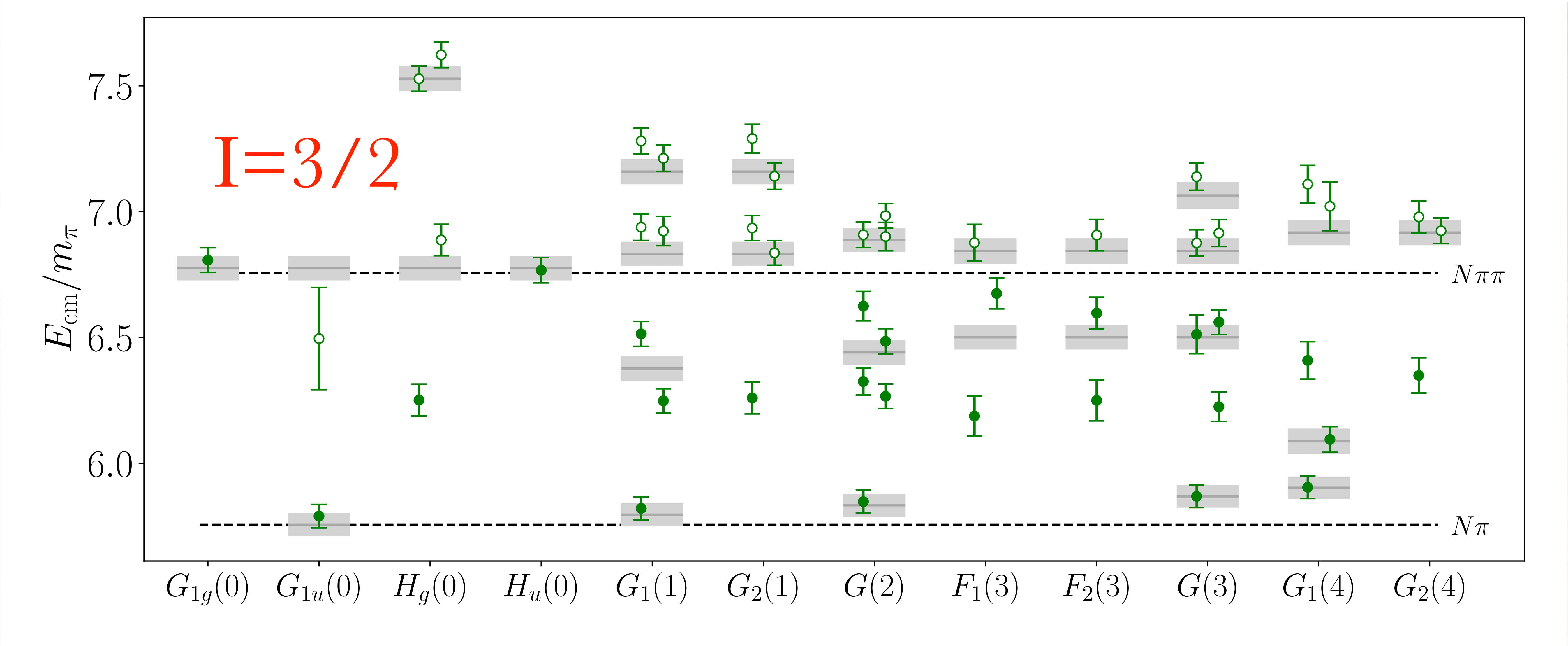


Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

❑ Various irreps used to determine the spectrum

$\boldsymbol{d}$	Λ	dim.	contributing $(2J,\ell)^{n_{\rm occ}}$ for $\ell_{\max}=2$
$(0,0,0)$	$G_{1\rm u}$	2	$(1,0)$
	$G_{1\rm g}$	2	$(1,1)$
	$H_{\rm g}$	4	$(3,1)$, $(5,2)$
	$H_{\rm u}$	4	$(3,2)$, $5,2)$
	$G_{2\rm g}$	2	$(5,2)$
$(0,0,n)$	G_1	2	$(1,0)$, $(1,1)$, $(3,1)$, $(3,2)$, $(5,2)$
	G_2	2	$(3,1)$, $(3,2)$, $(5,2)^2$
$(0,n,n)$	G	2	$(1,0)$, $(1,1)$, $(3,1)^2$, $(3,2)^2$, $(5,2)^3$
(n,n,n)	G	2	$(1,0)$, $(1,1)$, $(3,1)$, $(3,2)$, $(5,2)^2$
	F_1	1	$(3,1)$, $(3,2)$, $(5,2)$
	F_2	1	$(3,1)$, $(3,2)$, $(5,2)$





Note: the gray bands and green energy levels are correlated, which is not reflected visually in the plots

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

- FV Spectrum to Scattering Amplitudes [Lüscher, … many others]

$$\det[\tilde{K}^{-1}(E_{\rm cm}) - B^{\boldsymbol{P}}(E_{\rm cm})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$$

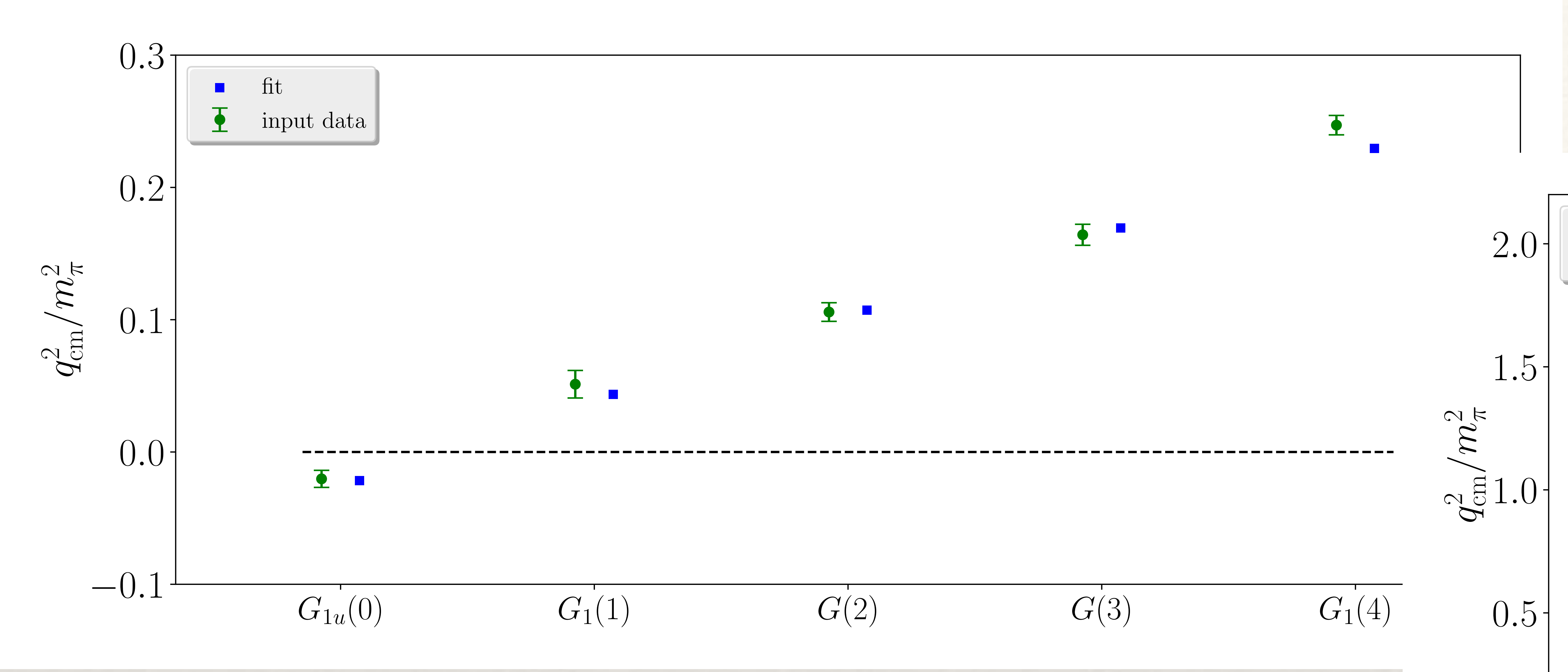
- $\tilde{K}$ proportional to the K-matrix
- $B^{\rm P}(E_{\rm cm})$ is the "Box Matrix" that encodes information about the finite-volume and BCs

- Solving this expression is equivalent to looking for poles in a coupled-channel scattering amplitude
  - for a single channel

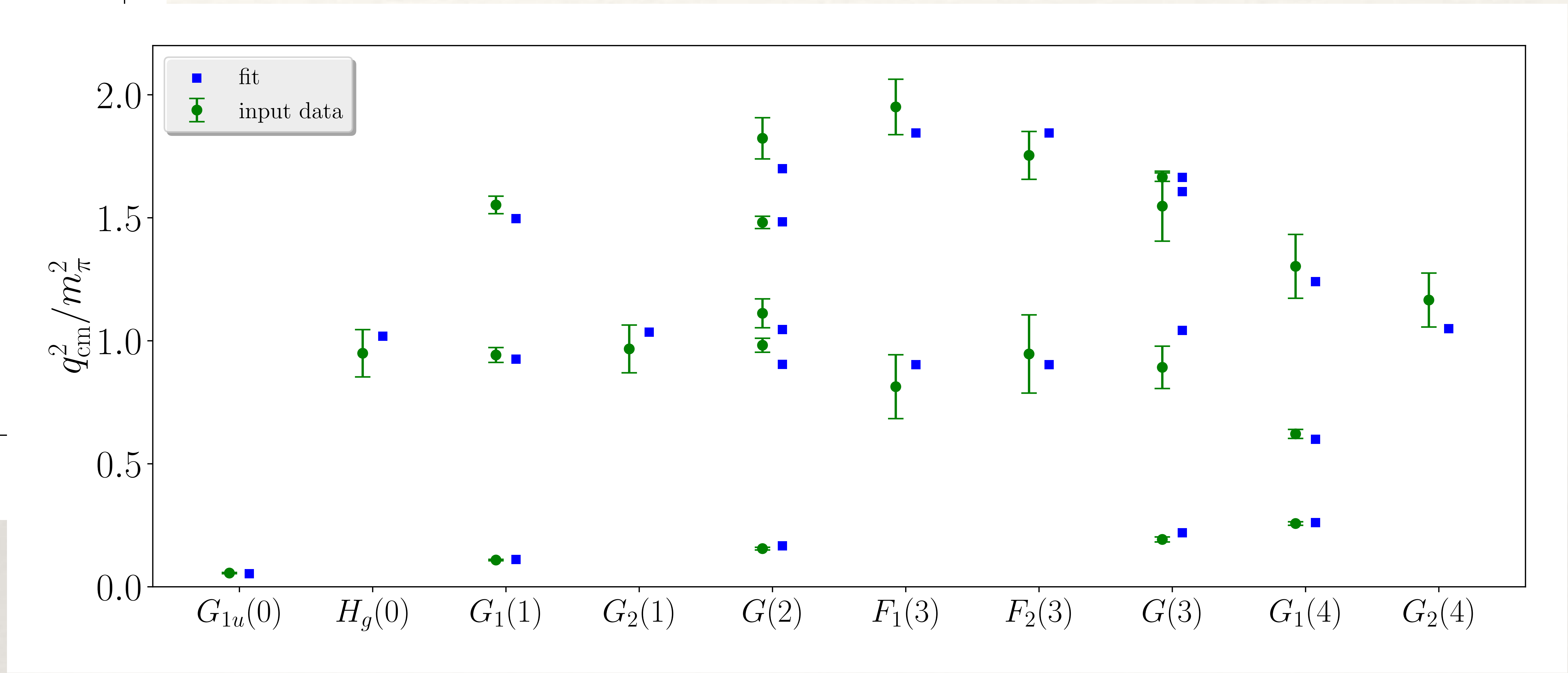
$$p\cot\delta - ip = 0 \longrightarrow p\cot\delta - \frac{1}{\pi L}\lim_{\Lambda\to\infty}\left(\sum_{|\vec{n}|<\Lambda}\frac{1}{|\vec{n}|^2 - \frac{p^2L^2}{4\pi^2}} - 4\pi\Lambda\right) = 0$$

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

FV Spectrum to Scattering Amplitudes - spectrum method comparison



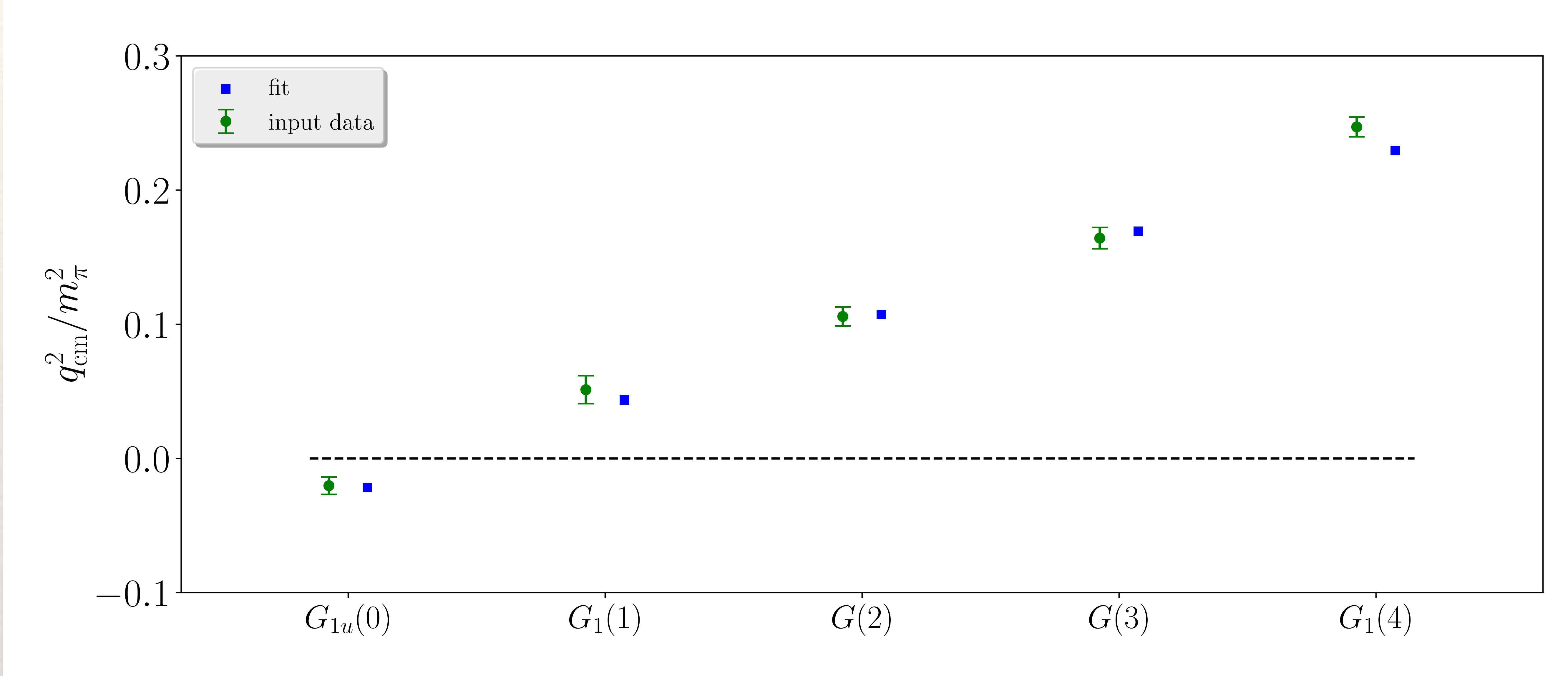
I=1/2 fit using s-wave only approximation

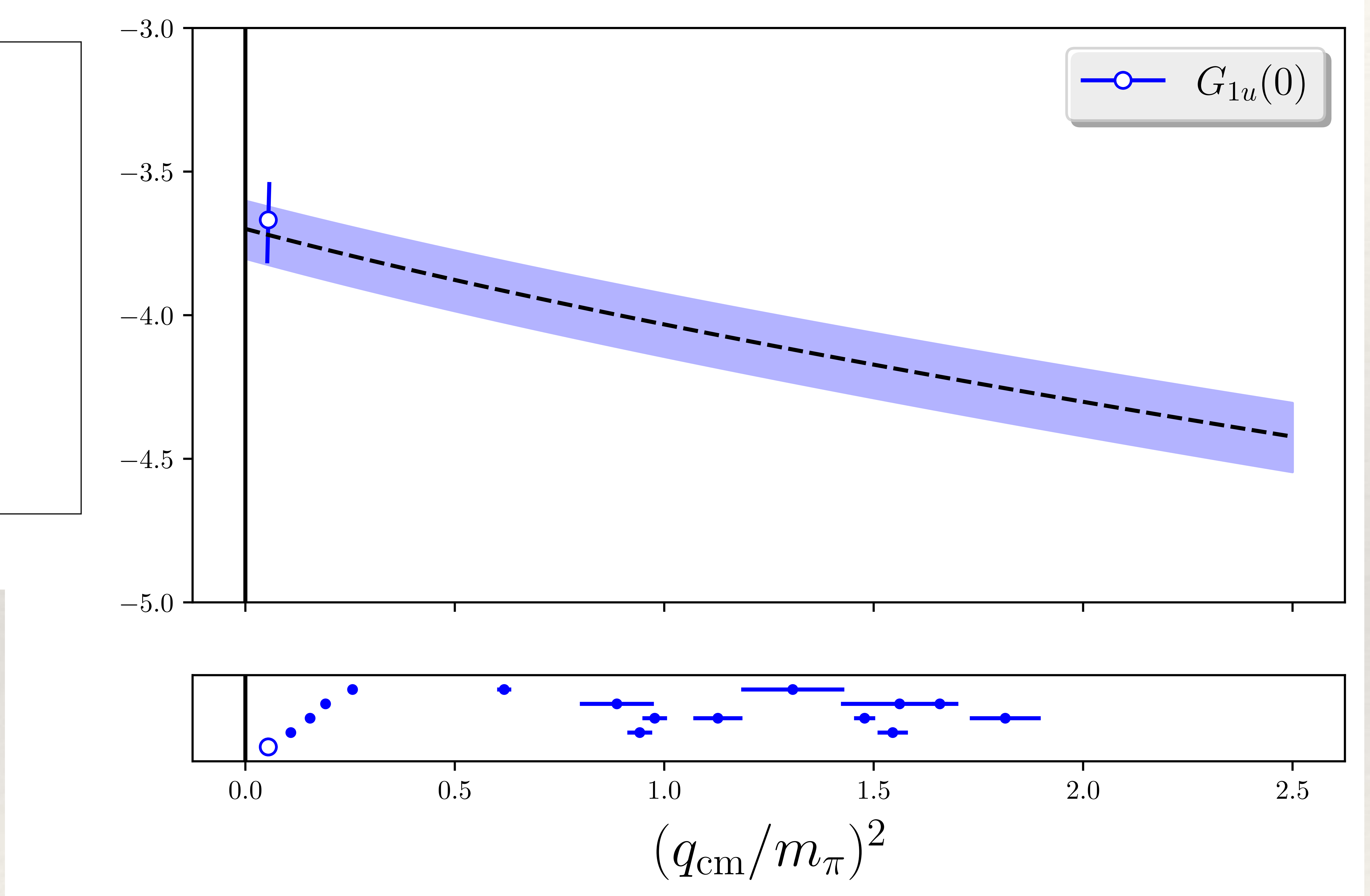


I=3/2 fit using s- and p-wave approximation

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

- FV Spectrum to Scattering Amplitudes - spectrum method comparison - resulting amplitude





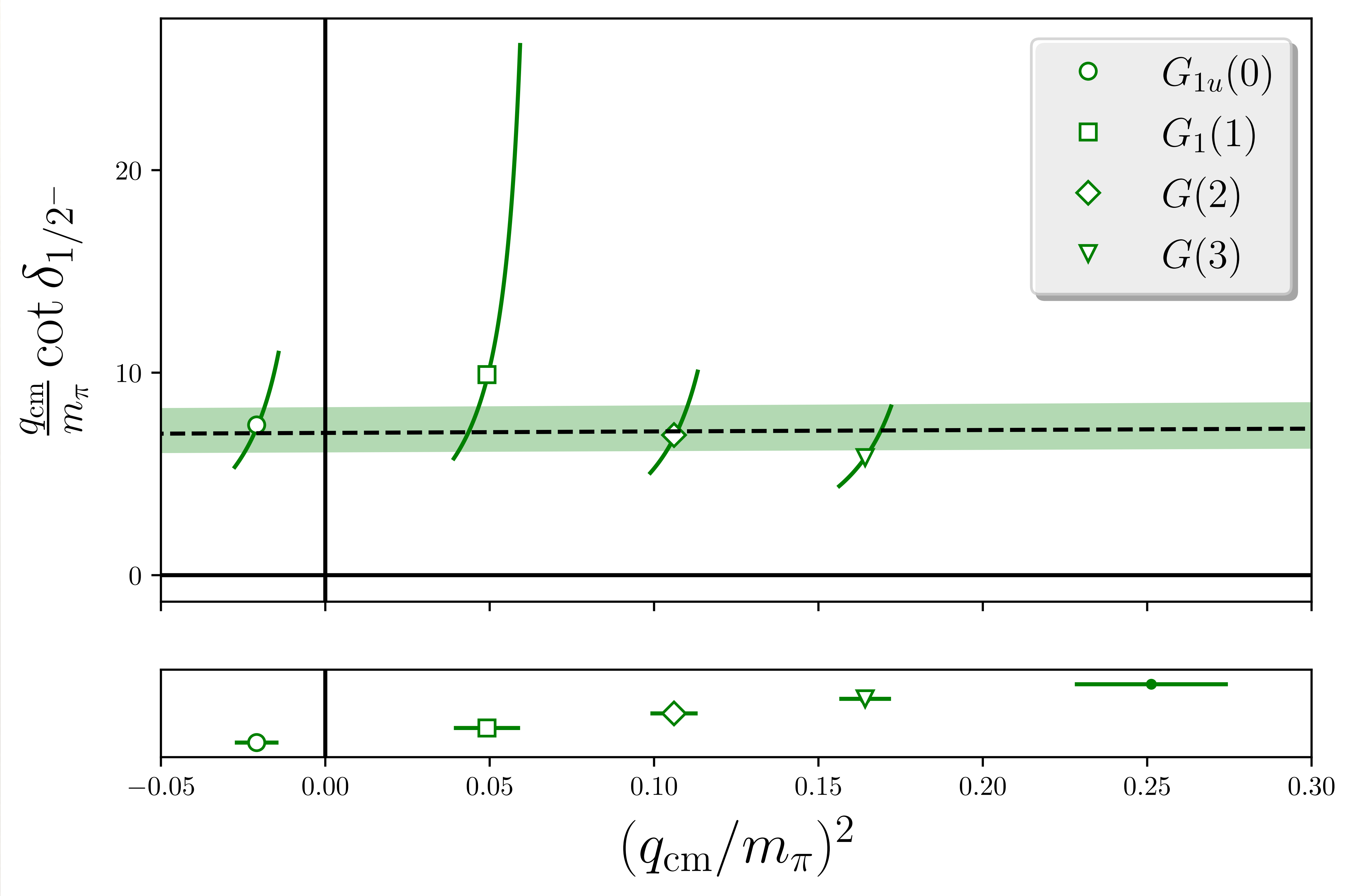
I=1/2 fit using s-wave only approximation

NOTE: global fit - only showing data from 1 irrep

open symbol: contributes to single partial wave
closed symbol: contributes to both partial waves

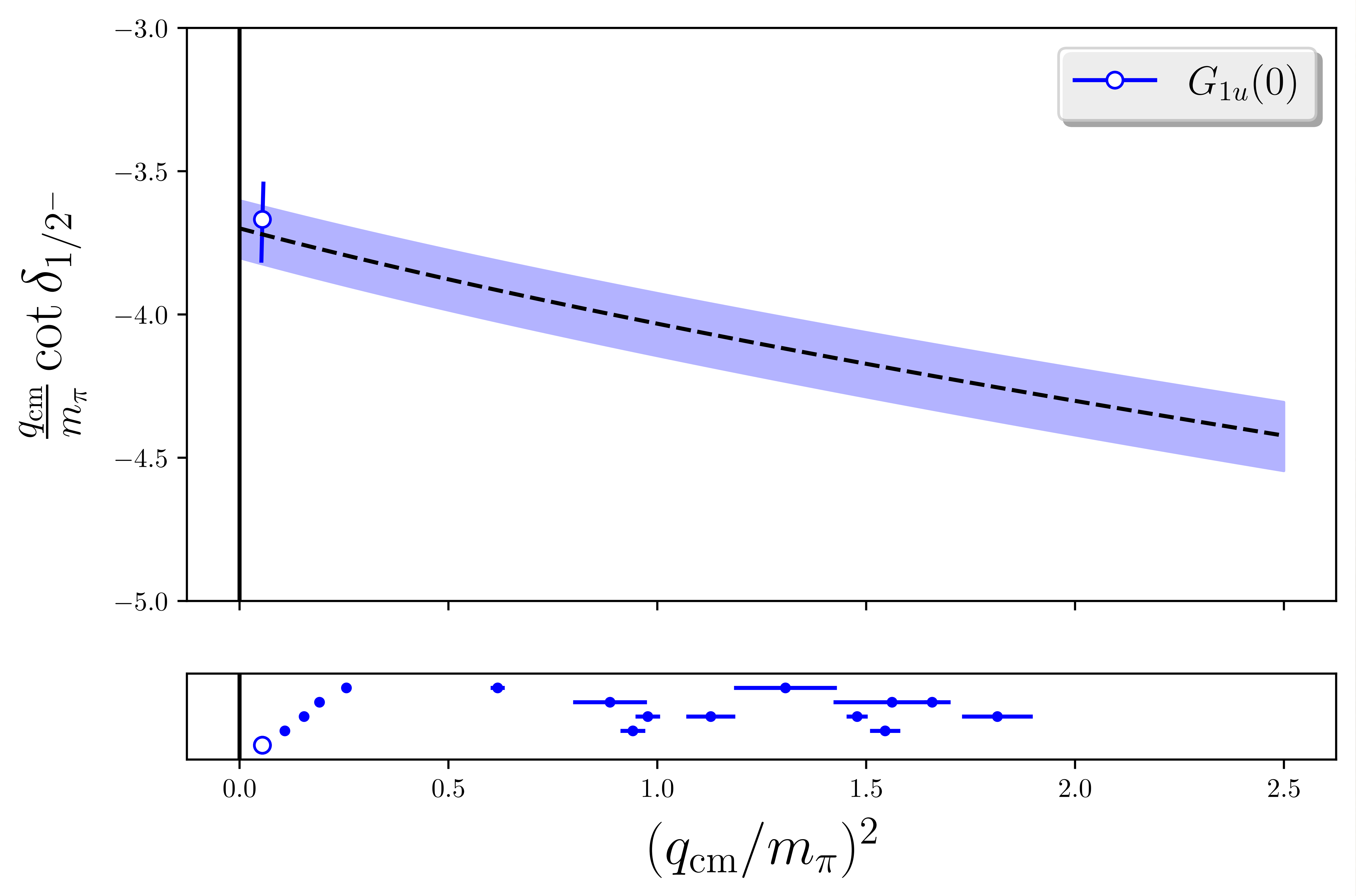
Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

FV Spectrum to Scattering Amplitudes - spectrum method comparison - resulting amplitude



I=1/2 fit using s-wave only approximation

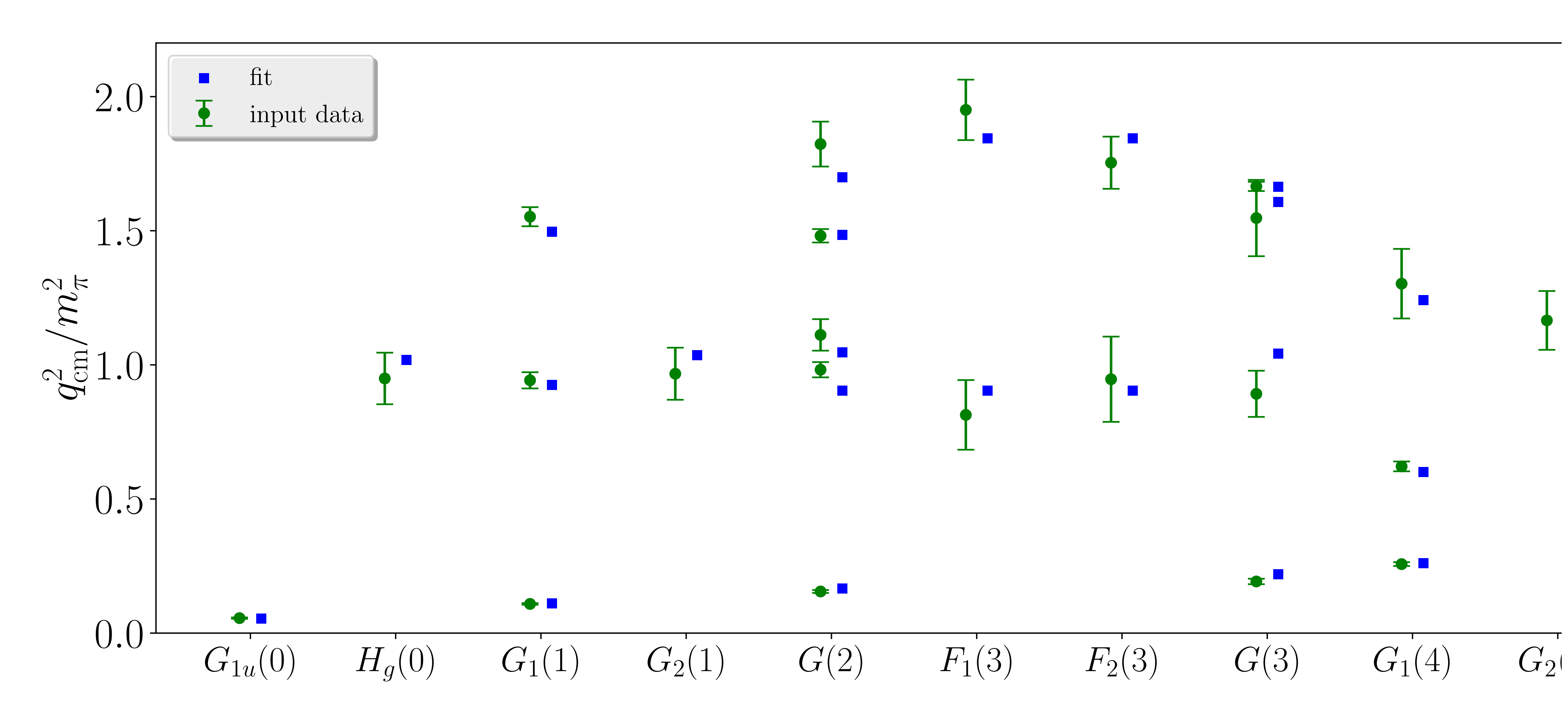
NOTE: global fit - only showing data from 1 irrep

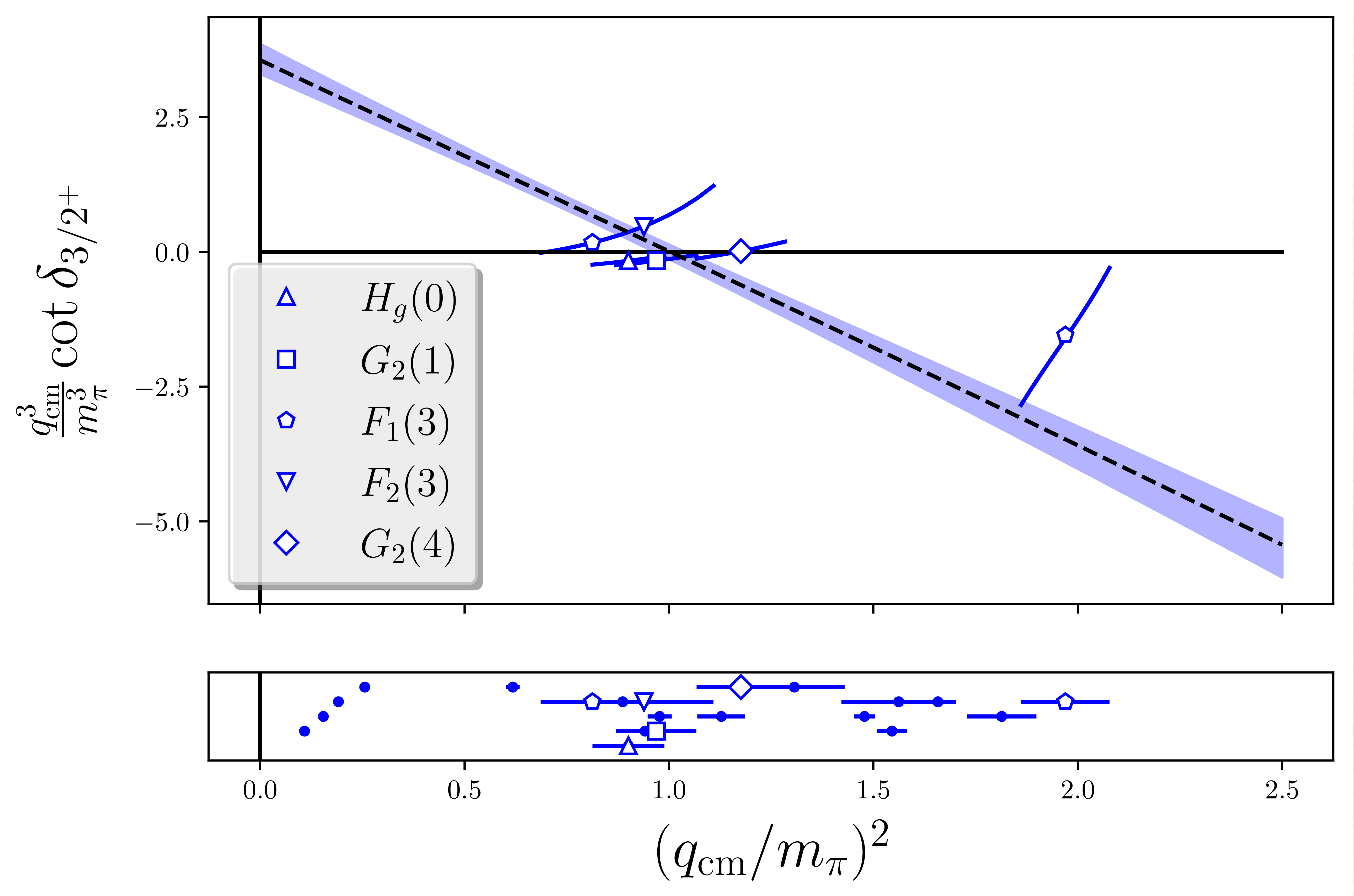


open symbol: contributes to single partial wave
closed symbol: contributes to both partial waves

Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

FV Spectrum to Scattering Amplitudes - spectrum method comparison - resulting amplitude





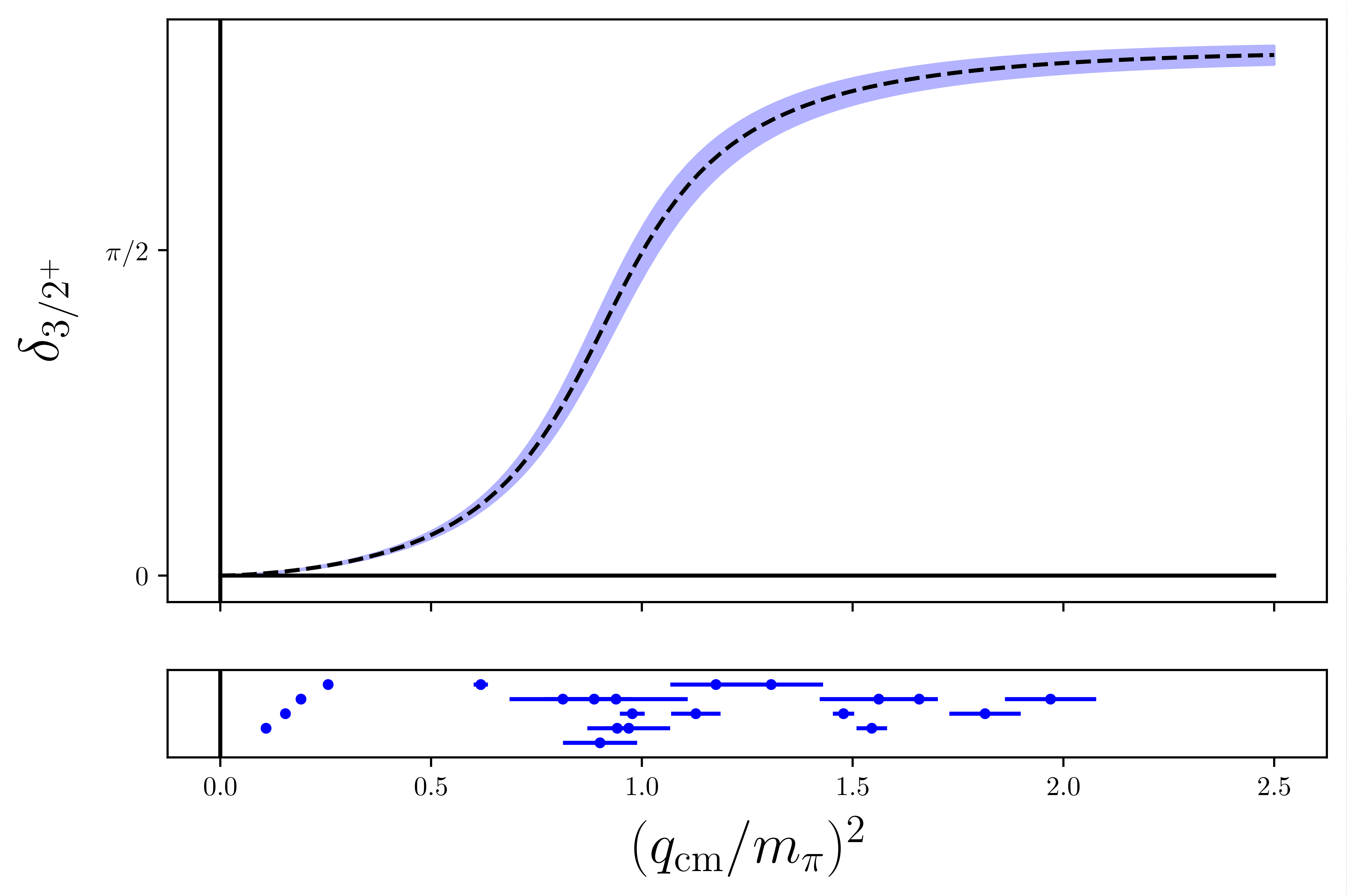
$$\frac{q^3_{\rm cm}}{m^3_\pi}\cot\delta_{3/2^+} = \frac{6\pi\sqrt{s}}{m^3_\pi g^2_{\Delta,\rm BW}}(m^2_\Delta - s)$$

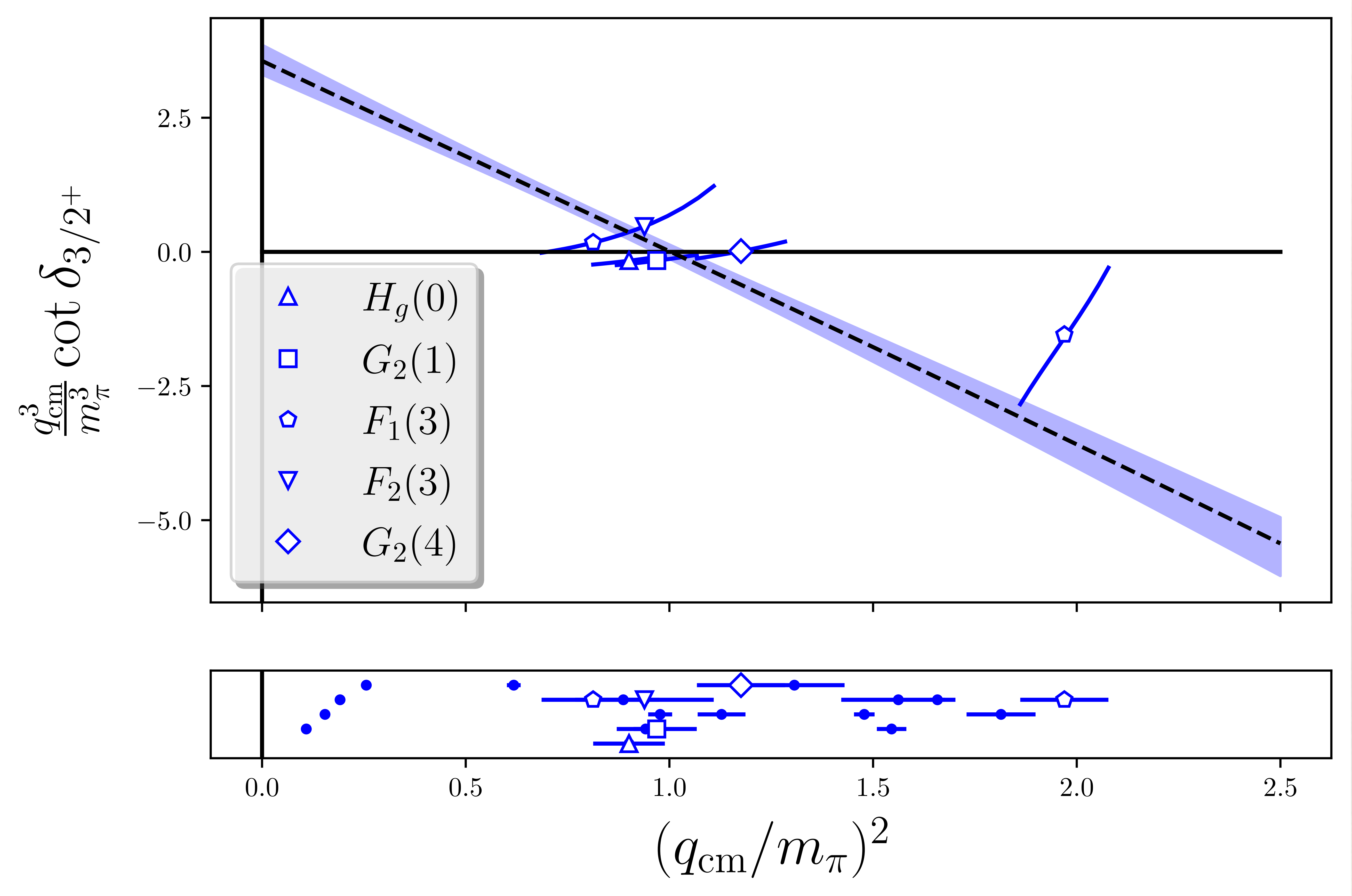
I=3/2 fit using s- and p-wave approximation

open symbol: contributes to single partial wave
closed symbol: contributes to both partial waves

Elastic nucleon-pion scattering at Mπ ≈ 200 MeV from lattice QCD

❑ FV Spectrum to Scattering Amplitudes - spectrum method comparison - resulting amplitude





I=3/2 fit using s- and p-wave approximation

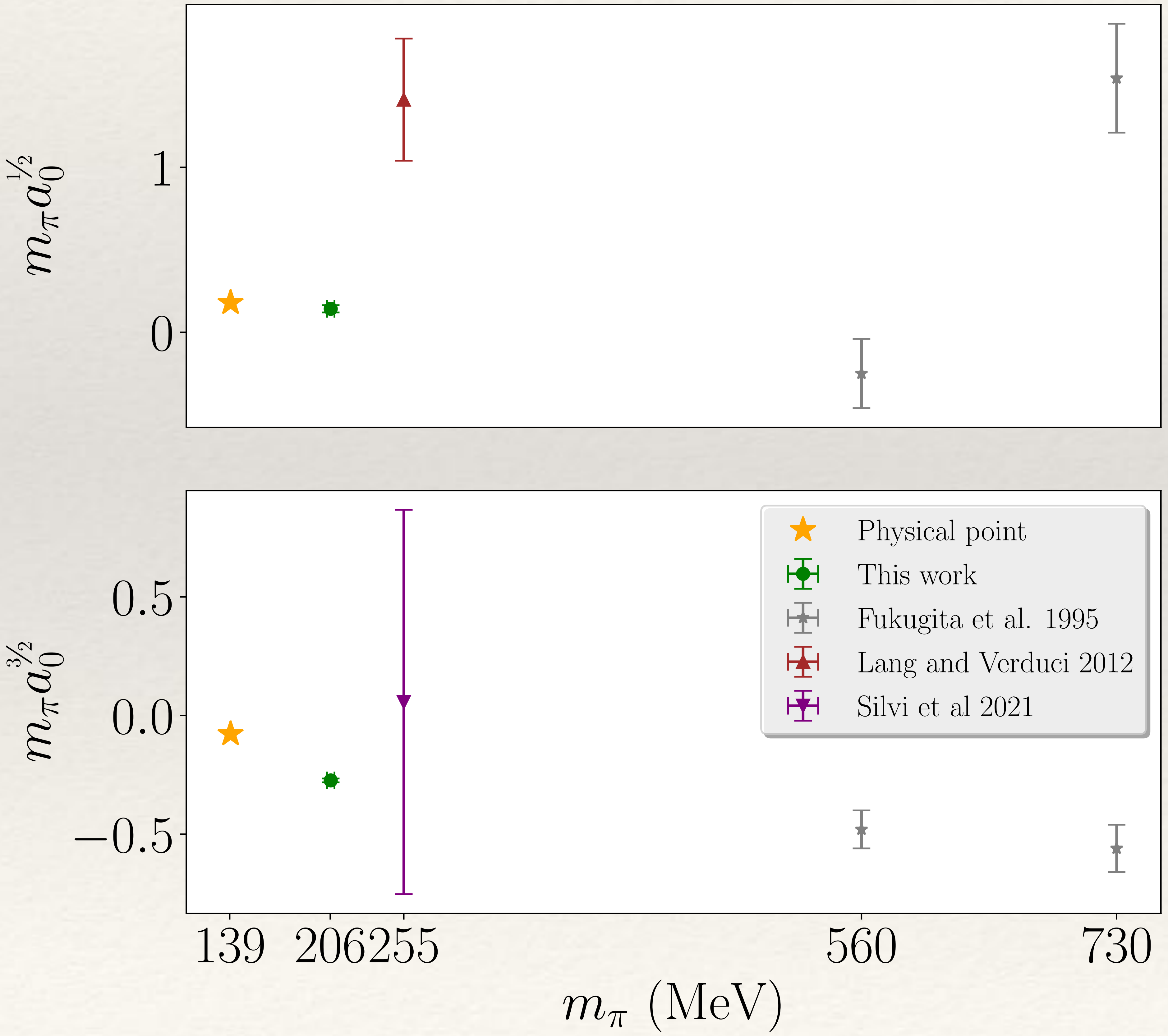
open symbol: contributes to single partial wave
closed symbol: contributes to both partial waves

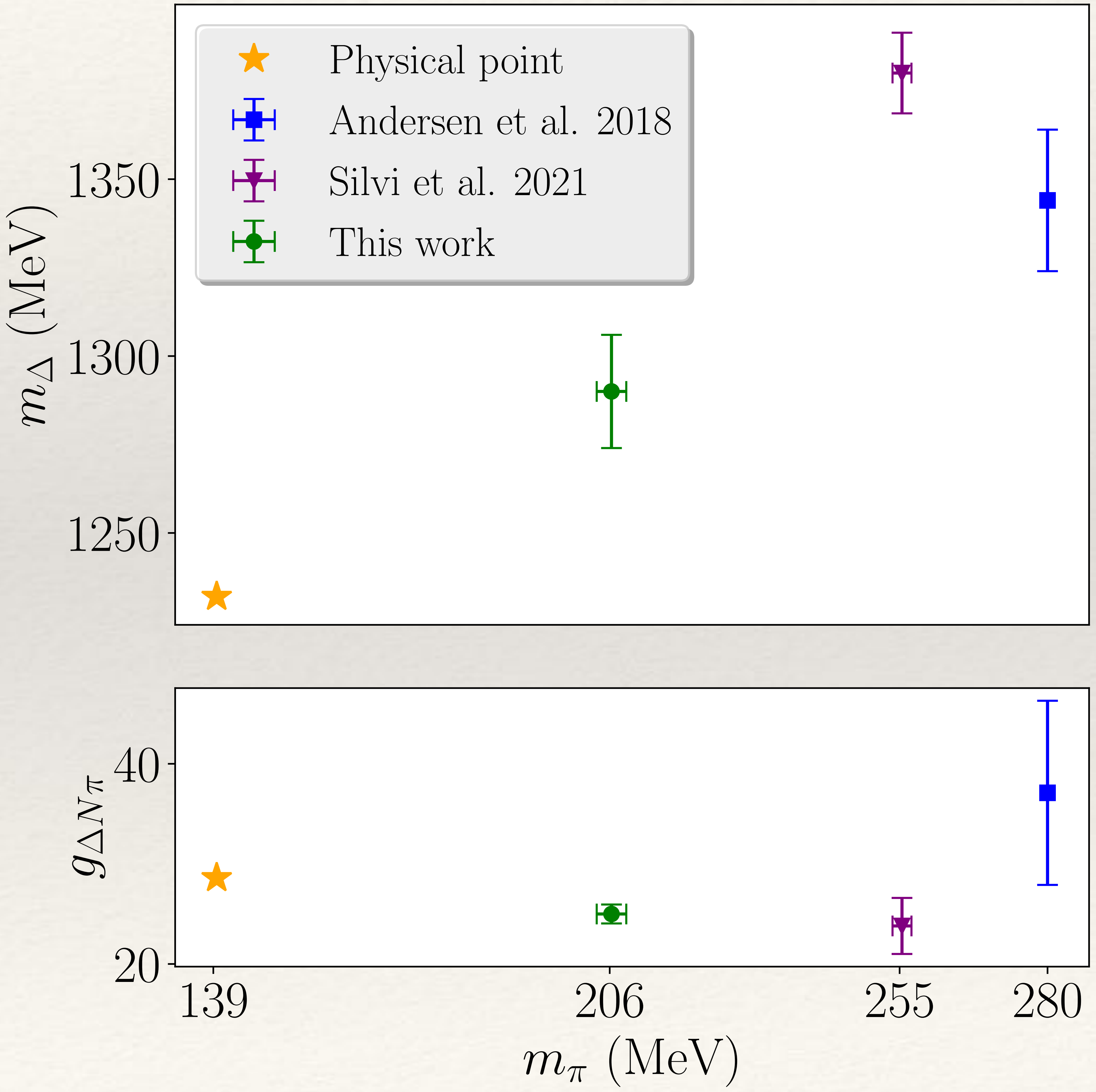
Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

❑ Results for scattering lengths and effective Delta-resonance parameters

$$m_\Delta = 1268(17)\ \text{MeV} \qquad \frac{m_\Delta}{m_\pi} = 6.257(35), \qquad g_{\Delta N\pi} = 14.41(53)$$

$$m_\pi a_0^{3/2} = -0.2735(81)\,, \qquad m_\pi a_0^{1/2} = 0.142(22),$$





Compare with χPT

- The formula for the scattering length are known at 4th order in the chiral expansion (w/o Δ)
  - They are expressed in terms of what is called scalar and vector scattering lengths

$$a_0^{3/2} = a_0^+ - a_0^- \,, \qquad a_0^{1/2} = a_0^+ + 2a_0^-$$

- At NLO, these are given by

- Hoferichter et al, 1510.06039, Hoferichter et al, 1507.07552
- Fettes, Meissner [Steininger] [hep-ph/9803266] hep-ph/0002162

$$m_\pi a_0^{3/2}[\text{NLO}] = -\epsilon_\pi^2 \frac{2\pi}{1+\mu}\left\{1 + \frac{\epsilon_\pi}{2}\frac{\Lambda_\chi}{m_N}(g_A^2 + 8C)\right\},$$

$$m_\pi a_0^{1/2}[\text{NLO}] = \epsilon_\pi^2 \frac{2\pi}{1+\mu}\left\{1 - \frac{\epsilon_\pi}{4}\frac{\Lambda_\chi}{m_N}(g_A^2 + 8C)\right\},$$

```
      COMPARISON of C = mN * (2c1 - c2 - c3)
order   pheno          D200 Fit
nlo       0.300(24)    0.648(62)
n2lo    -0.019(24)            NA
n3lo      0.244(29)           NA
```

$$C = M_N(2c_1 - c_2 - c_3)$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}, \quad \mu = \frac{m_\pi}{M_N}, \quad \Lambda_\chi = 4\pi F_\pi$$

$$\epsilon_\pi^{\text{D200}} = 0.1759(12), \qquad \mu^{\text{D200}} = 0.2102(19),$$

$$\epsilon_\pi^{\text{phys}} = 0.12064(74), \qquad \mu^{\text{phys}} = 0.14875(05)$$

Compare with χPT

- The formula for the scattering length are known at 4th order in the chiral expansion
  - They are expressed in terms of what is called scalar and vector scattering lengths

$$a_0^{3/2} = a_0^+ - a_0^- , \qquad a_0^{1/2} = a_0^+ + 2a_0^-$$

  - At NLO, these are given by

- Hoferichter et al, 1510.06039, Hoferichter et al, 1507.07552
- Fettes, Meissner [Steininger] [hep-ph/9803266] hep-ph/0002162

$$m_\pi a_0^{3/2}[\mathrm{NLO}] = -\epsilon_\pi^2 \frac{2\pi}{1+\mu}\left\{1 + \frac{\epsilon_\pi}{2}\frac{\Lambda_\chi}{m_N}(g_A^2 + 8C)\right\},$$

$$m_\pi a_0^{1/2}[\mathrm{NLO}] = \quad \epsilon_\pi^2 \frac{2\pi}{1+\mu}\left\{1 - \frac{\epsilon_\pi}{4}\frac{\Lambda_\chi}{m_N}(g_A^2 + 8C)\right\},$$

```
    COMPARISON of C = mN * (2c1 - c2 - c3)
order  pheno          D200 Fit
nlo      0.300(24)    0.648(62)
n2lo    -0.019(24)           NA
n3lo     0.244(29)           NA
```

	m_π (MeV)	$m_\pi a_0^{1/2}$	$m_\pi a_0^{3/2}$
This work	200	0.142(22)	−0.2735(81)
LO χPT	200	0.321(04)(57)	−0.161(02)(28)
LO χPT	140	0.159(02)(19)	−0.080(01)(10)
Pheno. (isospin limit)[27]	140	0.1788(38)	−0.0775(35)

Conclusions

- We have performed Nπ scattering — for the first time — at a pion mass of $m_\pi \approx 200$ MeV
- We used the stochastic Laplacian Heaviside (sLapH) method - a stochastic variant of distillation
- We found that our results are in tension with predictions from SU(2) χPT and LECs determined to high-precision Nπ scattering phase shift data
- We are therefore not yet in a position to usefully contribute to the nucleon-pion sigma term "puzzle" — the contrast between the pheno determination and most precise LQCD ones
- Lighter pion masses seem reachable (given our results) and are necessary to understand the apparent tension with SU(2) χPT predictions

Thank You