Quantifying EFT uncertainties with lattice QCD

INT 23-1b: New Physics Searches at the Precision Frontier
May, 2023

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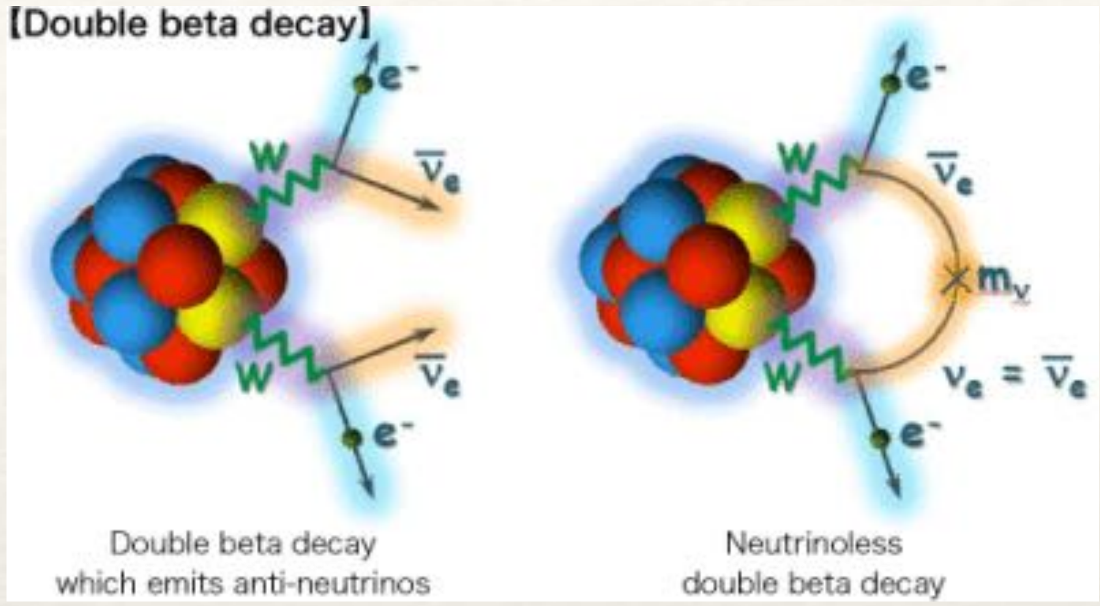
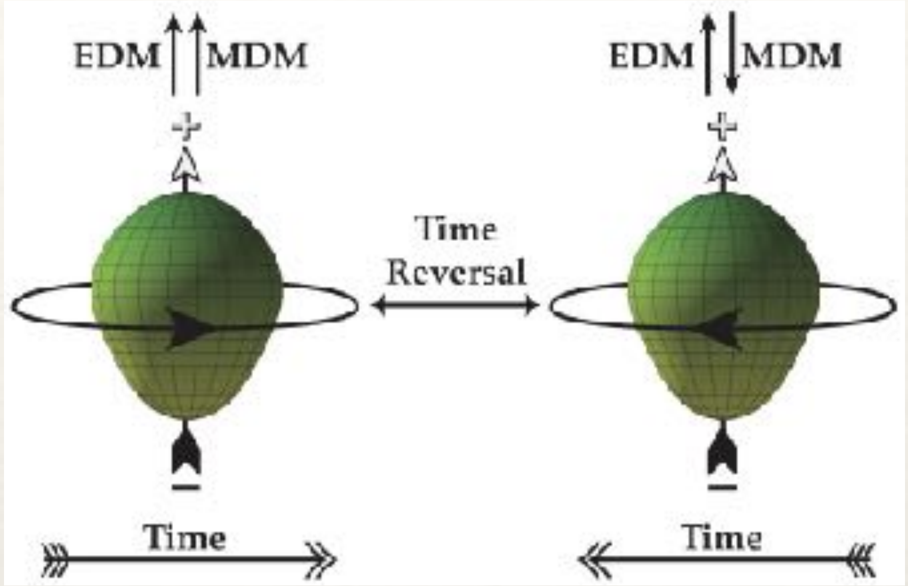
Motivation

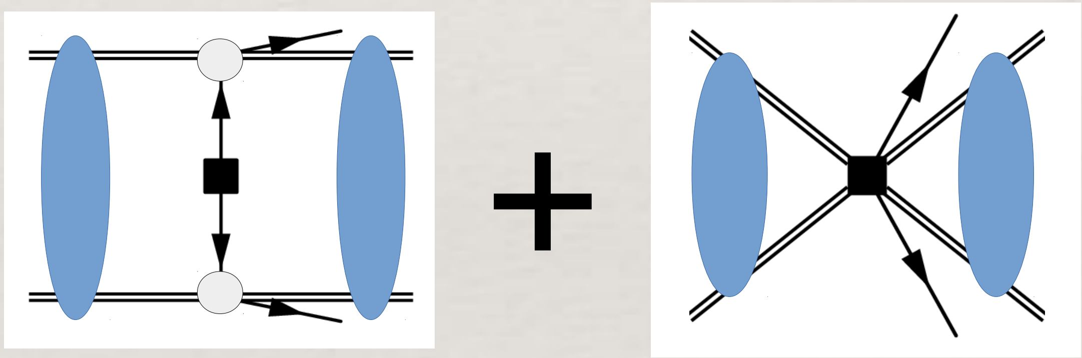
- Various observables — with little or no experimental data — and we'd like to know how well we can predict them (Uncertainty Quantification — UQ)
  - neutrinoless double beta-decay ($0\nu\beta\beta$)
  - nucleon and nuclear EDMs
  - hyperon-nucleon, NNN, YNN interactions
  - …





- $0\nu\beta\beta$ — what is the importance of the short-distance contribution to the nn→pp(ee) amplitude

  Cirigliano et al. PRC 97 (2018) [1710.01729]
  Cirigliano et al. PRL 120 (2018) [1802.10097]
  Cirigliano et al. PRC 100 (2019) [1907.11254]



known (predictive) unknown coupling (LEC)

- Can we predict everything we need using just lattice QCD (LQCD)?

  OR - do we need to rely upon extrapolating the LQCD calculations to the physical pion mass?

- How effective are our Effective (Field) Theories (EFTs)?

Motivation

- Historically — LQCD and EFT (χPT) have a very symbiotic relationship
  - EFT was necessary to extrapolate LQCD results to the physical pion mass
    (and assisted with infinite volume extrapolation and continuum extrapolation)
  - In turn — unknown low-energy-constants (LECs) would be determined through the extrapolation
    LECs are universal — determine them in one quantity, predict another

- < **2013** : EFT was necessary to extrapolate LQCD results to m_π^{phys}
- > **2013** : LQCD calculations carried out @ m_π^{phys} for mesons
- > **2018** : LQCD calculations carried out for simple nucleon quantities @ m_π^{phys}
  (but precision of final result still aided by results at heavier m_π)
- > **202X** : LQCD calculations of two-nucleon systems carried out at m_π^{phys}

for the foreseeable future — it will be necessary to extrapolate NN results to m_π^{phys}
how reliable are those extrapolations?
does the power-counting change as a function of m_π?

Motivation

- **Can we map out the convergence pattern of our EFTs versus m_π?**
  - m_π, m_K, F_π, F_K: MILC Collaboration has demonstrated that SU(3) XPT provides a qualitative, but not a precise quantitative description at $m_s \approx m_s^{\rm phys}$
    C. Bernard, CD2015 [1510.02180]
    - $F_{K^\pm}/F_{\pi^\pm} = 1.1934(19)$ [FLAG 2021] — 0.15% uncertainty
      - roughly speaking: NLO $\approx 20\,\%$ $\rightarrow$ N^{2}LO $\approx 4\,\%$, N^{3}LO $\approx 0.8\,\%$, N^{4}LO $\approx 0.16\,\%$
      - Relying upon SU(3) XPT to achieve this precision is not realistic…
  - M_B: SU(3) heavy baryon XPT (HBXPT) is not a convergent expansion @ $m_s^{\rm phys}$
    LHP Collaboration [0806.4549] — baryon spectrum
    PACS-CS Collaboration [0905.0962] — baryon spectrum
    NPLQCD Collaboration [0912.4243] — meson-baryon scattering lengths

Motivation

- **I believe SU(2) baryon XPT (w/o Δ) is most likely not convergent at** $m_\pi^{\rm phys}$
  - Based on LQCD results we have generated since ~2018
- What can we do with LQCD to conclusively show this is true or not?
- If this is true — what does it mean about NN EFT (with pions) @ $m_\pi^{\rm phys}$?
  - It seems to me, this would essentially invalidate the convergence pattern of NN as well

- It is possible that adding explicit Δ degrees of freedom (dof) will restore convergence
  - Testing this requires more LQCD calculations including
    - πN scattering in the Δ resonance region
    - $N \to \Delta$ transition matrix elements

Motivation

- Before discussing LQCD results

  discuss in high-level detail various extrapolations needed for LQCD

  this will highlight the symbiotic relationship between LQCD and EFT in general

  and hopefully give you a feeling for the complexities of the systematics we aim to control

Chiral, Continuum, Infinite Volume Extrapolations

- LQCD calculations must be extrapolated to the continuum and infinite volume limits and extrapolated/ interpolated to the physical quark-mass limit (in order to compare with experiment)

- To carry out these extrapolations — very useful to define small, dimensionless parameters that parameterize the various effects
  - useful if the parameters are defined in terms of quantities that can be "measured" in the calculation

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi} \qquad \epsilon_a = \frac{a}{2w_0} \qquad \delta_L \approx \frac{e^{-m_\pi L}}{(m_\pi L)^z}, \quad z = \frac{1}{2}, 1, \ldots$$

chiral | continuum | infinite volume

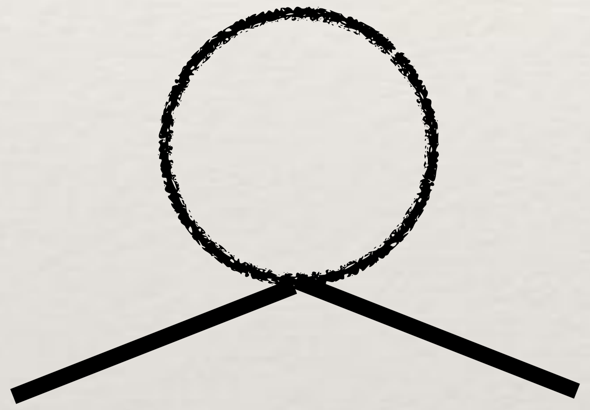
Chiral Extrapolations: $\epsilon_\pi = \dfrac{m_\pi}{4\pi F_\pi}$

- χPT (and its extensions)
  - systematic description of low-energy hadronic/nuclear physics about $m_\pi = 0$ limit
  - theoretical truncation errors scale (in principle) as ϵ_π^{n+1} if one has worked to O(ϵ_π^n)
    - there may be additional small/large scales that invalidate this power-counting, eg. $\Delta \equiv M_\Delta - M_N$
  - all quark mass (pion mass) dependence is explicit

- Nearly all quantities of interest are known to 1-loop order
  (loop order and ϵ_π^n-order are often not synonymous)
- Precision matrix elements: need 2-loop order
  known for most simple quantities
  unknown for some quantities of interest (particularly involving nucleons)

Infinite Volume Extrapolations: $\delta_L \approx \dfrac{e^{-m_\pi L}}{(m_\pi L)^z}$

- Finite Volume (FV) effects are easily incorporated in χPT (and its extensions)
  - inherently IR effects — to large extent, separable from short-distance effects (LECs)
    ie. the leading FV corrections to observables does not depend upon (unknown) LECs
- FV effects are not universal — they depend upon the quantity
- Determined by considering $T \to \infty$ limit at finite L



$$\approx \frac{2B\hat{m}_l}{F^2}\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m_\pi^2} \longrightarrow \frac{2B\hat{m}_l}{F^2}\int \frac{dk_0}{2\pi}\frac{1}{L^3}\sum_{\vec{k}} \frac{i}{(k_0 - \omega_k)(k_0 + \omega_k)}$$

$$= \frac{2B\hat{m}_l}{F^2}\left[I^\infty(m_\pi) + \delta_L I(m_\pi, L)\right]$$

$$= 2B\hat{m}_l\, \epsilon_\pi^2 \left[\ln\left(\frac{m_\pi^2}{\mu^2}\right) + 4\sum_{\vec{n}\neq 0}\frac{K_1(|\vec{n}|m_\pi L)}{|\vec{n}|m_\pi L}\right]$$

$$K_1(|\vec{n}|m_\pi L) = \sqrt{\frac{\pi}{2}}\frac{e^{-|\vec{n}|m_\pi L}}{\sqrt{|\vec{n}|m_\pi L}}\left[1 + \mathrm{O}\left(\frac{1}{|\vec{n}|m_\pi L}\right)\right]$$

- The leading FV corrections from such pion-loop effects provides a good qualitative estimate for $m_\pi L \gtrsim 3.5$
  For the precision of LQCD results for many quantities, 2-loop corrections are needed for accurate determination of FV corrections — Colangelo, Durr, Haefeli, NPB721 (2005) [hep-lat/0503014]

Continuum Extrapolations: $\epsilon_a = \dfrac{a}{2w_0}$

- The continuum extrapolation can be carried out in at least two ways
  1. For fixed quark mass, take the continuum limit of a given quantity
  2. Perform a simultaneous extrapolation in ϵ_π and ϵ_a
- In practice, 1. is challenging to carry out
  - as one varies the lattice spacing, choosing input parameters that hold eg. the pion mass fixed in physical units requires fine-tuning
  - holding the physical volume fixed is nearly impossible — small volume corrections will get mixed in with continuum extrapolation (which will also have small changes in the quark mass mixed in)
- For both options, the first step is what is known as the Symanzik Expansion (an EFT):
  - Expand the discretized lattice action for small lattice spacing, a, about the continuum limit
  - organize the operators in a series expansion in powers of a

$$\mathcal{L}^{LQCD} = a^4 \mathcal{L}^{QCD} + \sum_{n=1}^{\infty} a^{4+n} \underbrace{c_{4+n}}_{\text{Wilson coefficients}} \, O^{4+n}(x)$$

Operators of mass-dimension 4+n

Wilson coefficients

Continuum Extrapolations: $\epsilon_a = \frac{a}{2w_0}$

- Symanzik Expansion: example of Wilson fermions

$$S^{LQCD} = a^4 \sum_n \bar{\psi}(n)\left[\gamma_\mu D_\mu + m_0\right]\psi(n) + a^5 \underbrace{\sum_n \bar{\psi}(n) D_\mu D_\mu \psi(n)}_{\text{Wilson Operator}} + S^G$$

$$D_\mu \psi(n) = \frac{1}{2a}\left[U_\mu(n)\psi(n+\mu) - U_\mu^\dagger(n)\psi(n-\mu)\right]$$

- Things to note:
  - Wilson Operator breaks chiral symmetry
  - UV momentum modes, $p \approx \pi/a$, lead to an additive mass term that scales like $1/a$
- Symanzik Expansion (after EOM to remove redundant operators)

$$S^{LQCD} = \int d^4x\ \bar{\psi}(x)\left[\gamma_\mu D_\mu + m_0 + m_c\right]\psi(x) + a\, c_{SW}\, \bar{\psi}(x)\sigma_{\mu\nu}G_{\mu\nu}\psi(x) + \frac{1}{4g^2}G_{\mu\nu}G_{\mu\nu} + \mathrm{O}(a^2)$$

  - One fine-tunes m_0 such that $m_0 + m_c$ gives a small quark mass
  - One can (usually does) add an operator like c_{SW} to remove the $\mathrm{O}(a)$ effects
  - Lorentz violation begins at $\mathrm{O}(a^2)$: eg. $a^2 \bar{\psi}(x)\gamma_\mu D_\mu D_\mu D_\mu \psi(x)$
    (Lorentz symmetry is an "accidental" symmetry of LQCD)

Continuum Extrapolations: $\epsilon_a = \dfrac{a}{2w_0}$

- Including discretization errors in χPT — Sharpe and Singleton, PRD 48 (1998) [hep-lat/9804028]
  - Perform Symanzik expansion for a given lattice action
  - Map the operators, including higher dimensional ones into a chiral Lagrangian using spurions

$$S^{LQCD} = \int d^4x\, \bar{\psi}(x)\left[\gamma_\mu D_\mu + m_0 + m_c\right]\psi(x) + a\, c_{SW}\, \bar{\psi}(x)\sigma_{\mu\nu}G_{\mu\nu}\psi(x) + \frac{1}{4g^2}G_{\mu\nu}G_{\mu\nu} + \mathrm{O}(a^2)$$

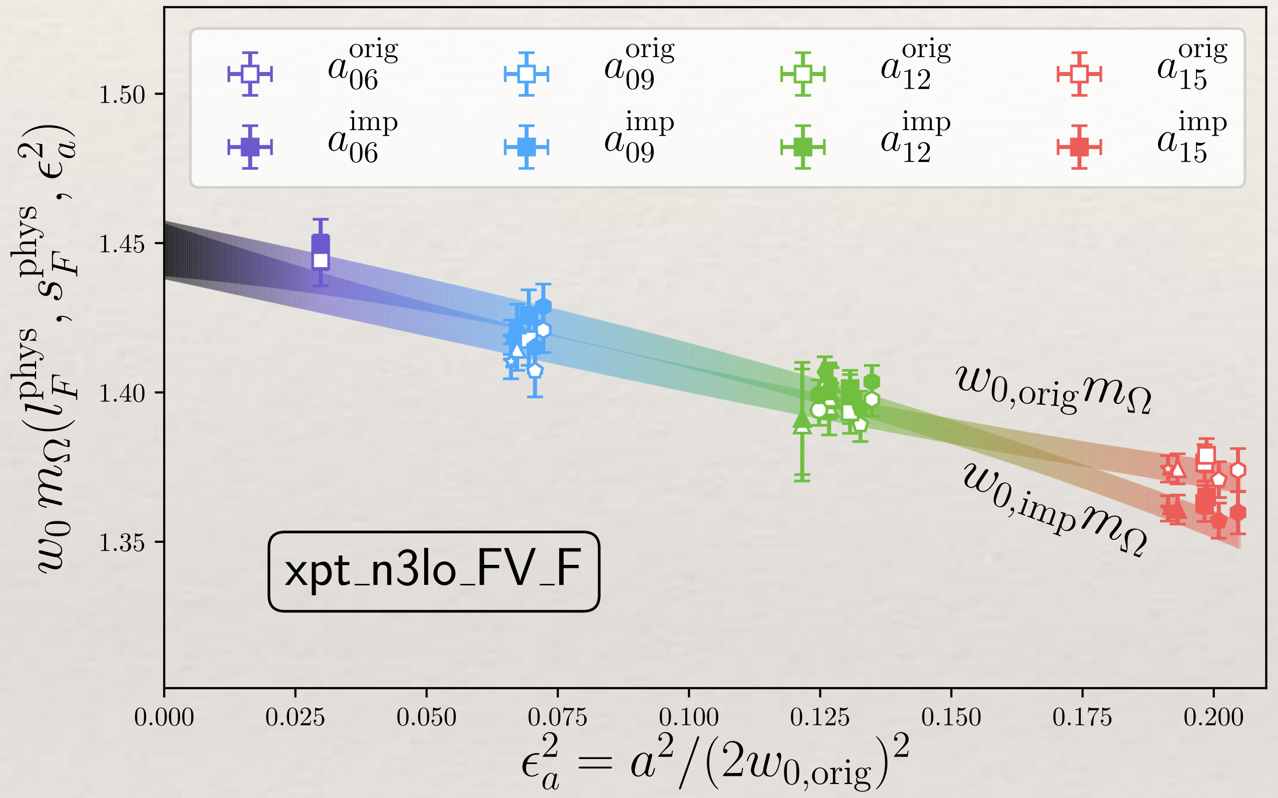
$$\Downarrow$$

$$\mathscr{L}_a^{\chi PT} = \frac{F^2}{4}\mathrm{Tr}\left(D_\mu\Sigma D_\mu\Sigma^\dagger\right) + \frac{F^2}{4}\mathrm{Tr}\left(2B_0M_q\Sigma^\dagger + (2B_0M_q)^\dagger\Sigma\right) + a\mathrm{Tr}\left(2W_0c_{SW}\Sigma^\dagger + (2W_0c_{SW})^\dagger\Sigma\right) + \mathrm{O}(M_q^2, aM_q, a^2)$$

- NOTE:
  - The mixed discretization - quark mass effects can be significant
  - unlike quark mass effects — the LECs parameterizing discretization effects have implicit dependence upon the lattice spacing ($\ln(a)$ effects from radiative gluon corrections)

Scale Setting versus dimensionless ratios

- The optimal way to perform an extrapolation is in terms of a dimensionless quantity, formed from a ratio of two dimensionful ones, if necessary — why?
- Example of recent scale setting I was involved in: Miller et al (CalLat) PRD 103 (2021) [2011.12166]
  - Used Ω-baryon mass, combined with what are known as a Gradient-Flow scales w_0, t_0



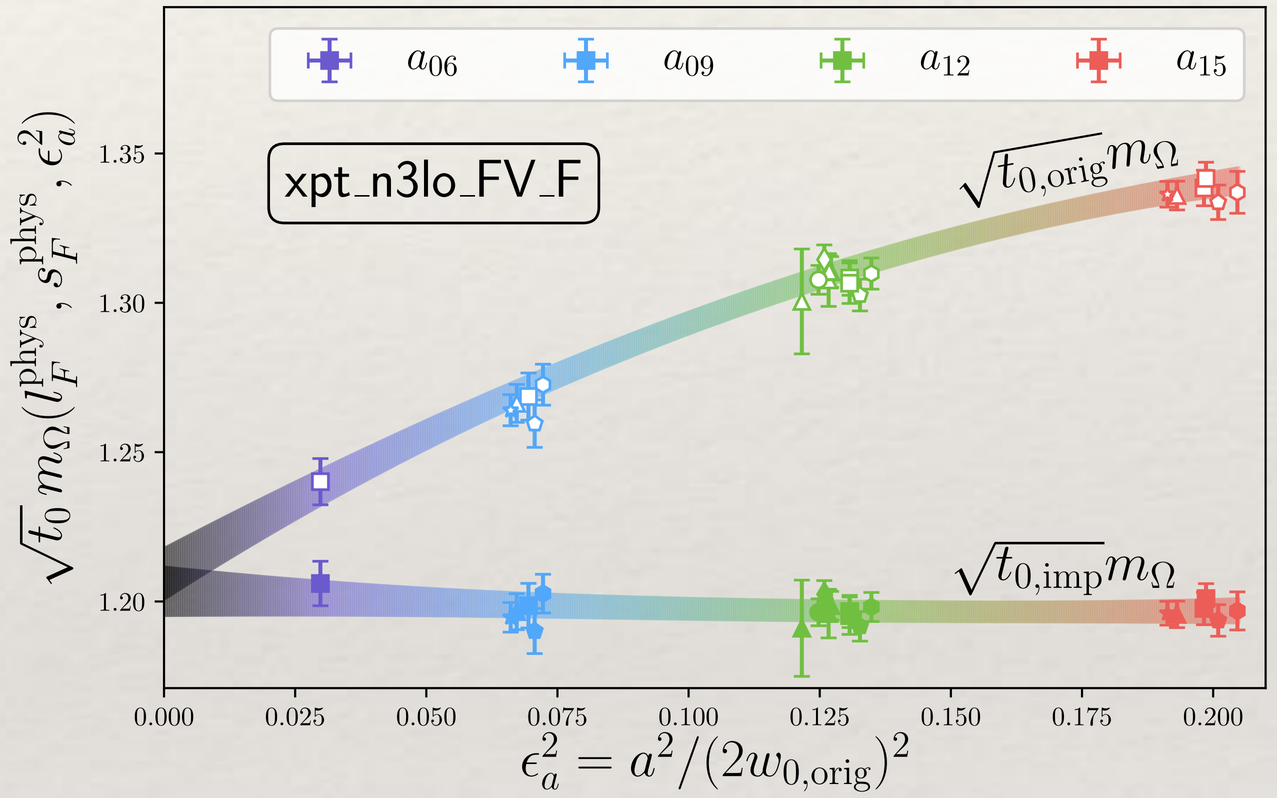
scheme	a_{15}/fm	a_{12}/fm	a_{09}/fm	a_{06}/fm
$t_{0,{\rm orig}}/a^2$	0.1284(10)	0.10788(83)	0.08196(64)	0.05564(44)
$t_{0,{\rm imp}}/a^2$	0.1428(10)	0.11735(87)	0.08632(65)	0.05693(44)
$w_{0,{\rm orig}}/a$	0.1492(10)	0.12126(87)	0.08789(71)	0.05717(51)
$w_{0,{\rm imp}}/a$	0.1505(10)	0.12066(88)	0.08730(70)	0.05691(51)

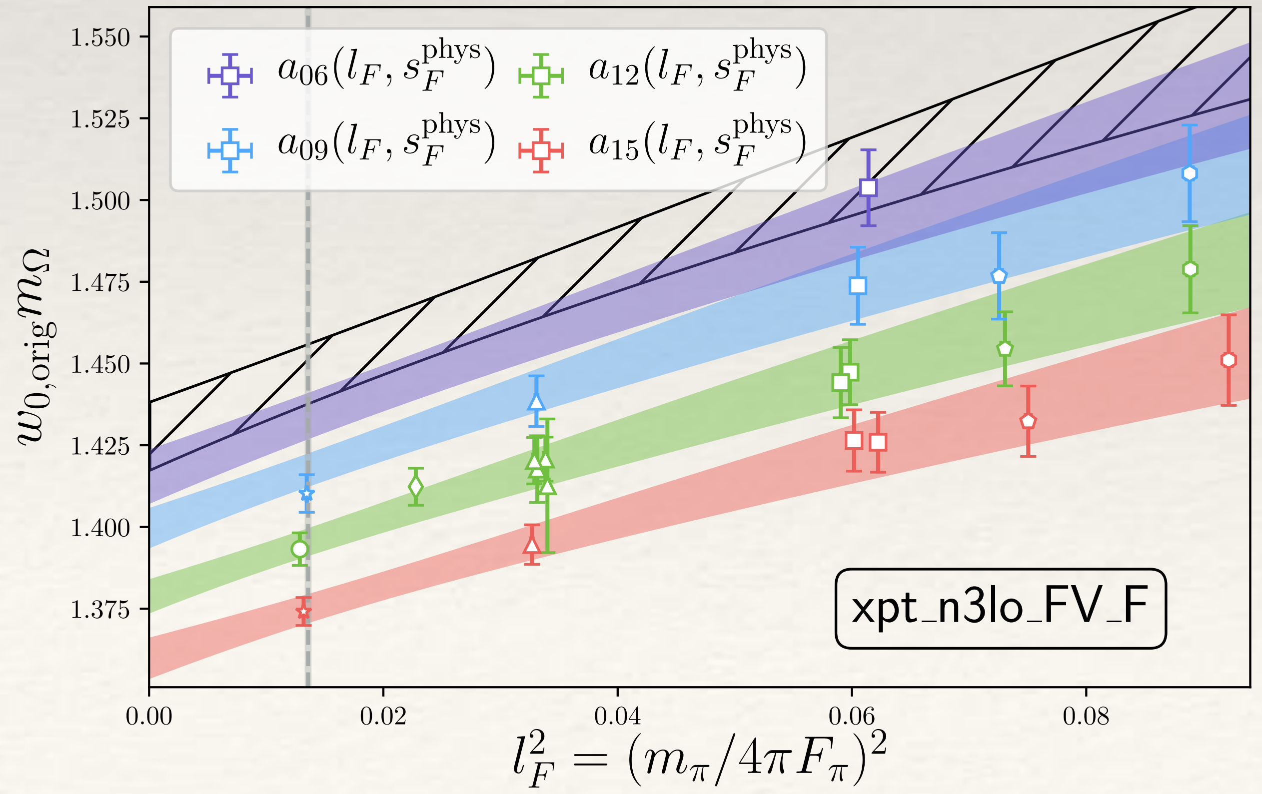
$$w_0 m_\Omega = 1.4483(82)^s(15)^\chi(45)^a(00)^V(26)^{\rm phys}(18)^M = 1.4483(97)$$

$$\frac{w_0}{\rm fm} = 0.1709(10)^s(02)^\chi(05)^a(00)^V(03)^{\rm phys}(02)^M = 0.1709(11)\,,$$

$$\sqrt{t_0} m_\Omega = 1.2051(82)^s(15)^\chi(46)^a(00)^V(21)^{\rm phys}(61)^M = 1.205(12)\,,$$

$$\frac{\sqrt{t_0}}{\rm fm} = 0.1422(09)^s(02)^\chi(05)^a(00)^V(02)^{\rm phys}(07)^M = 0.1422(14)\,,$$





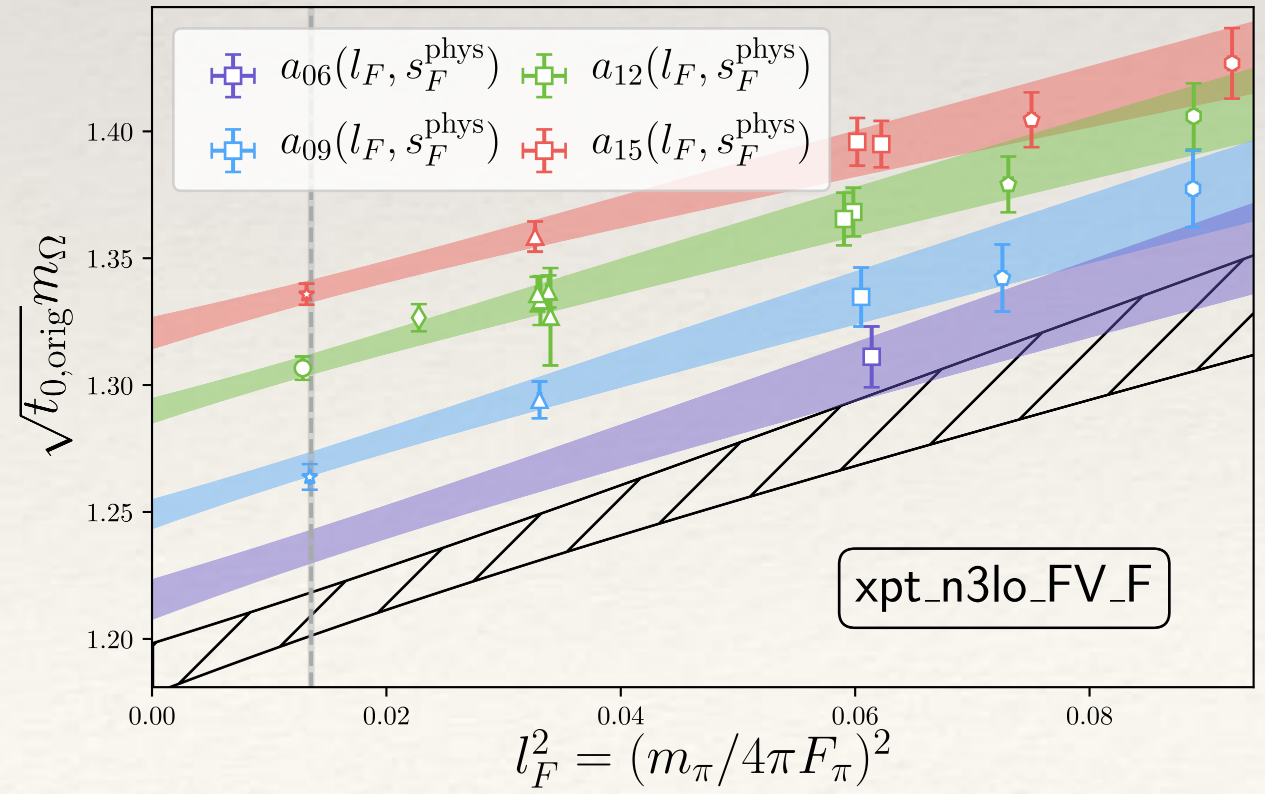
w_0 : 0.64% uncertainty
a : 0.66 — 0.90% uncert.

$\sqrt{t_0}$: 0.98% uncertainty
a : 0.70 — 0.77% uncert.

$$\frac{aM_B}{a} = M_B[\text{MeV}]$$

the most significant uncertainty often comes from a and it introduces a correlation between all ensembles

$\sigma_{aM_B}/aM_B \approx 0.2\,\%$



Scale Setting versus dimensionless ratios

- If Scale Setting introduces dominant uncertainty, what about forming a dimensionless ratio?

$$w_0 M_N, \quad \frac{M_N}{4\pi F_\pi}, \quad \frac{M_N}{M_\Omega}, \quad \cdots$$

- The problem with such options is that each other quantity also depends upon the pion mass
  - LECs are pion-mass independent
    we can not ignore this pion mass dependence as it would pollute our determination of LECs
  - A choice that is possibly the easiest to control the systematics for is a quantity for which we have a good understanding of the chiral corrections — F_π (plus ϵ_a and FV corrections)

$$F_\pi = F\left\{1 + \epsilon_\pi^2\left(\bar{l}_4^r - \ln \epsilon_\pi^2\right) + \epsilon_\pi^4\left(\frac{5}{4}\ln^2 \epsilon_\pi^2 + (\hat{c}_{1F}^r + 2)\ln \epsilon_\pi^2 + \hat{c}_{2F}^r - 2\bar{l}_4^r\right)\right\}$$

Scale Setting versus dimensionless ratios

- Side-bar — for this expression — Ananthanarayan, Bijnens, Ghosh, EPJC 77 (2017) [1703.00141]

$$F_\pi = F\left\{1 + \epsilon_\pi^2\left(\bar{l}_4^r - \ln\epsilon_\pi^2\right) + \epsilon_\pi^4\left(\frac{5}{4}\ln^2\epsilon_\pi^2 + (\hat{c}_{1F}^r + 2)\ln\epsilon_\pi^2 + \hat{c}_{2F}^r - 2\bar{l}_4^r\right)\right\}$$

we have set the dim-reg scale $\mu = 4\pi F_\pi$, which is not a static quantity
Beane, Bedaque, Orginos, Savage, PRD 75 (2007) [hep-lat/0606023]

However, corrections from this choice arise at a higher order — the NLO log induces an NNLO term

We can correct for this, such that the error made does not appear until N^{3}LO
Miller et al., PRD102 (2020) [2005.04795]
First, start with $\mu = 4\pi F$

$$\ln\frac{m_\pi^2}{(4\pi F)^2} = \ln\left(\frac{m_\pi^2}{(4\pi F_\pi)^2}\frac{F_\pi^2}{F^2}\right) = \ln\epsilon_\pi^2 + \ln\left[1 + 2\epsilon_\pi^2\left(\bar{l}_4^r - \ln\epsilon_\pi^2\right) + \mathrm{O}(\epsilon_\pi^4)\right]$$
$$= \ln\epsilon_\pi^2 + 2\epsilon_\pi^2\left(\bar{l}_4^r - \ln\epsilon_\pi^2\right) + \mathrm{O}(\epsilon_\pi^4)$$

then LECs still defined at $\mu = 4\pi F$

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  - Then perform simultaneous extrapolation of $\frac{M_N}{4\pi F_\pi}, \quad F_\pi$

    to determine LECs describing M_N

The extrapolation of a few quantities and tests of convergence

- M_N
  - $M_n - M_p$
- g_A
- πN scattering lengths

M_N vs m_π

- The nucleon mass is known through $O(m_\pi^5)$ in SU(2) HBχPT
  McGovern, Birse PRD74 (2006) [hep-lat/0608002]
- Generically

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi} \qquad \Lambda_\chi = 4\pi F_\pi \qquad \bar{c}_1 = \frac{c_1}{4\pi F}$$

$$M_N = \underbrace{M_0}_{\text{LO}} + \Lambda_\chi \left[\underbrace{-\epsilon_\pi^2 4\bar{c}_1}_{\text{NLO}} \underbrace{-\epsilon_\pi^3 \frac{3\pi g_A^2}{2}}_{\text{N}^2\text{LO}} + \underbrace{\epsilon_\pi^4 \left(\alpha_4 + \beta_4 \ln \epsilon_\pi^2\right)}_{\text{N}^3\text{LO}} + \underbrace{\epsilon_\pi^5 \left(\frac{3\pi g_A^4}{2} \left(1 + 4 \ln \epsilon_\pi^2\right) + \alpha_5 \right)}_{\text{N}^4\text{LO (2-loop)}} + O(\epsilon_\pi^6) \right]$$

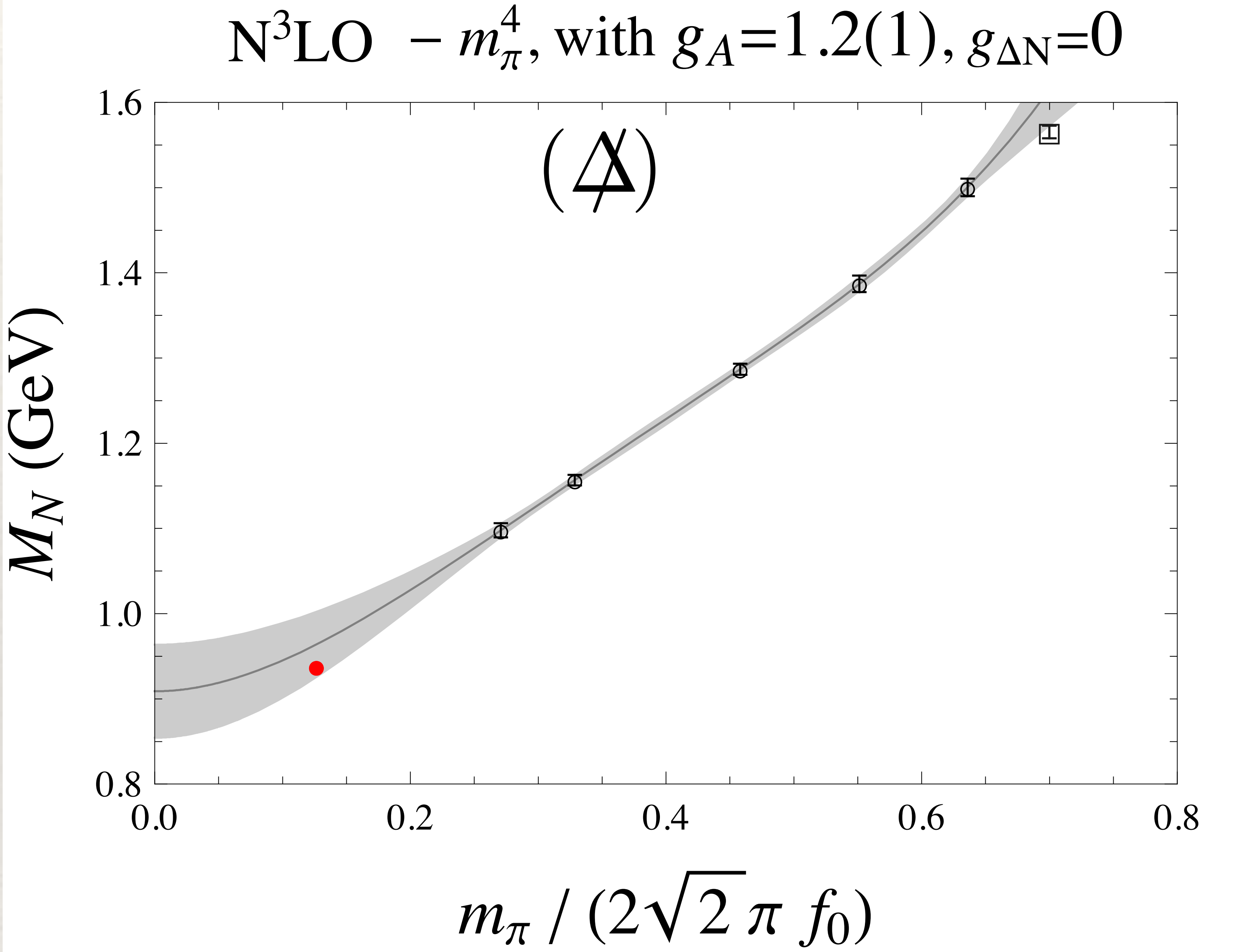
- Note:
  - N^{2}LO term is LEC-free (if we take g_A from other results) and negative and has a large coefficient
  - N^{4}LO term has an even larger coefficient as well $\ln \epsilon_\pi^2$ enhancement (that is negative)
  - If we study M_N/Λ_χ, the known chiral corrections to F_π contribute at N^{3}LO, then N^{5}LO (even powers of ϵ_π only)

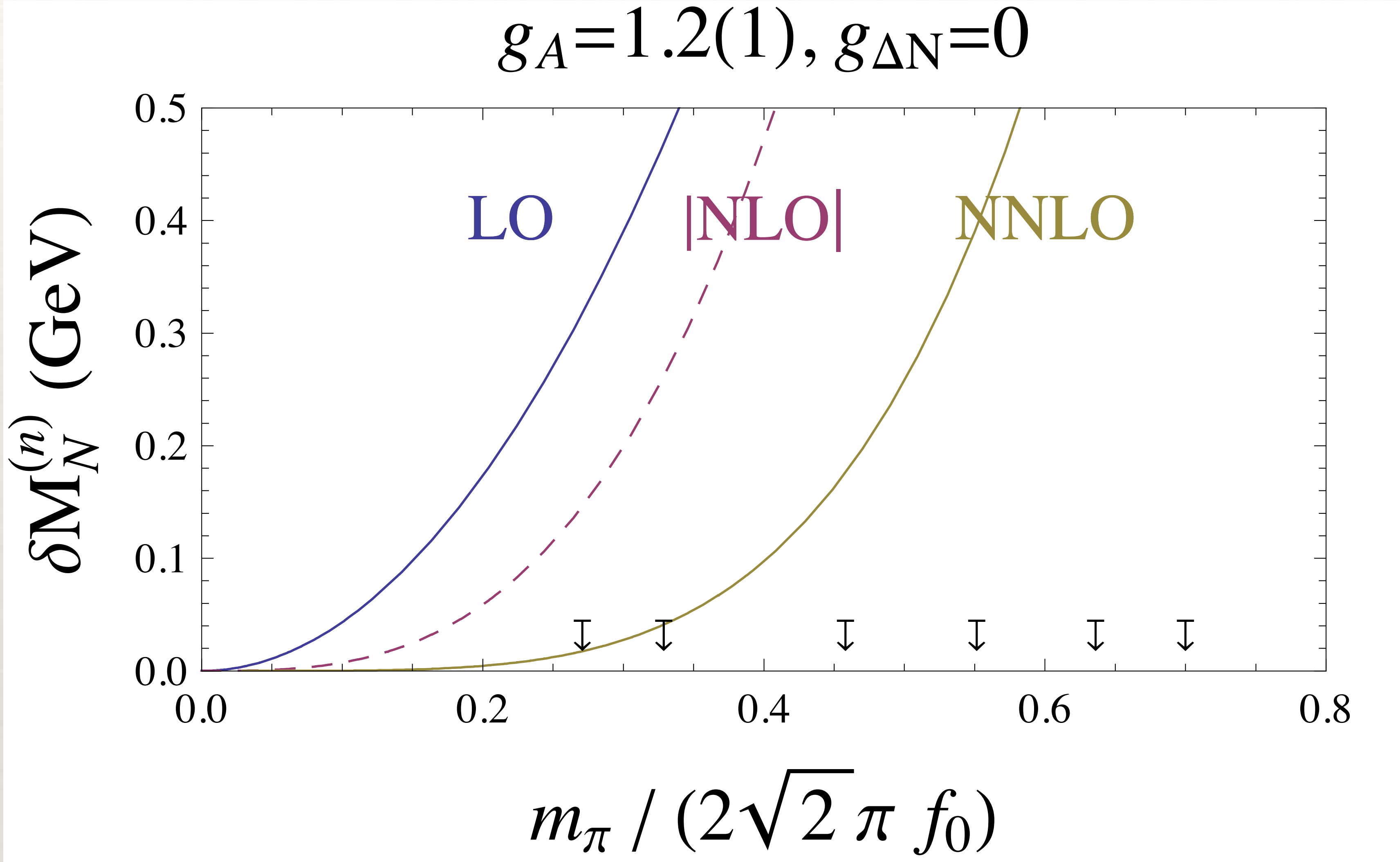
- How does this compare with LQCD results?

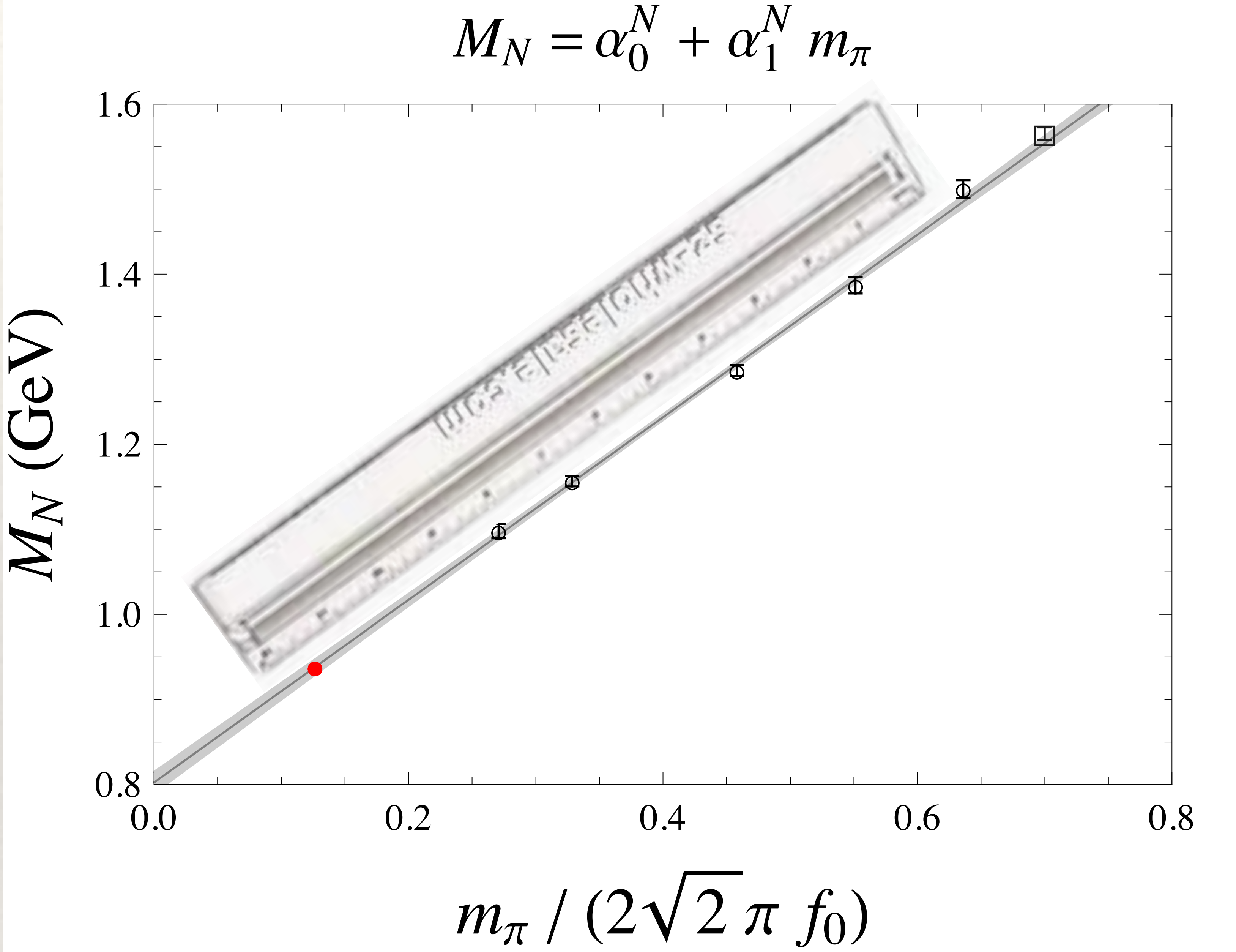
M_N vs m_π

$$M_N = M_0 + \Lambda_\chi \left[-\epsilon_\pi^2\, 4\bar{c}_1 - \epsilon_\pi^3 \frac{3\pi g_A^2}{2} + \epsilon_\pi^4 \left(\alpha_4 + \beta_4 \ln \epsilon_\pi^2\right) + \epsilon_\pi^5 \left(\frac{3\pi g_A^4}{2} \left(1 + 4 \ln \epsilon_\pi^2\right) + \alpha_5 \right) + \mathrm{O}(\epsilon_\pi^6) \right]$$

Walker-Loud et al. (LHP) PRD79 [0806.4549]



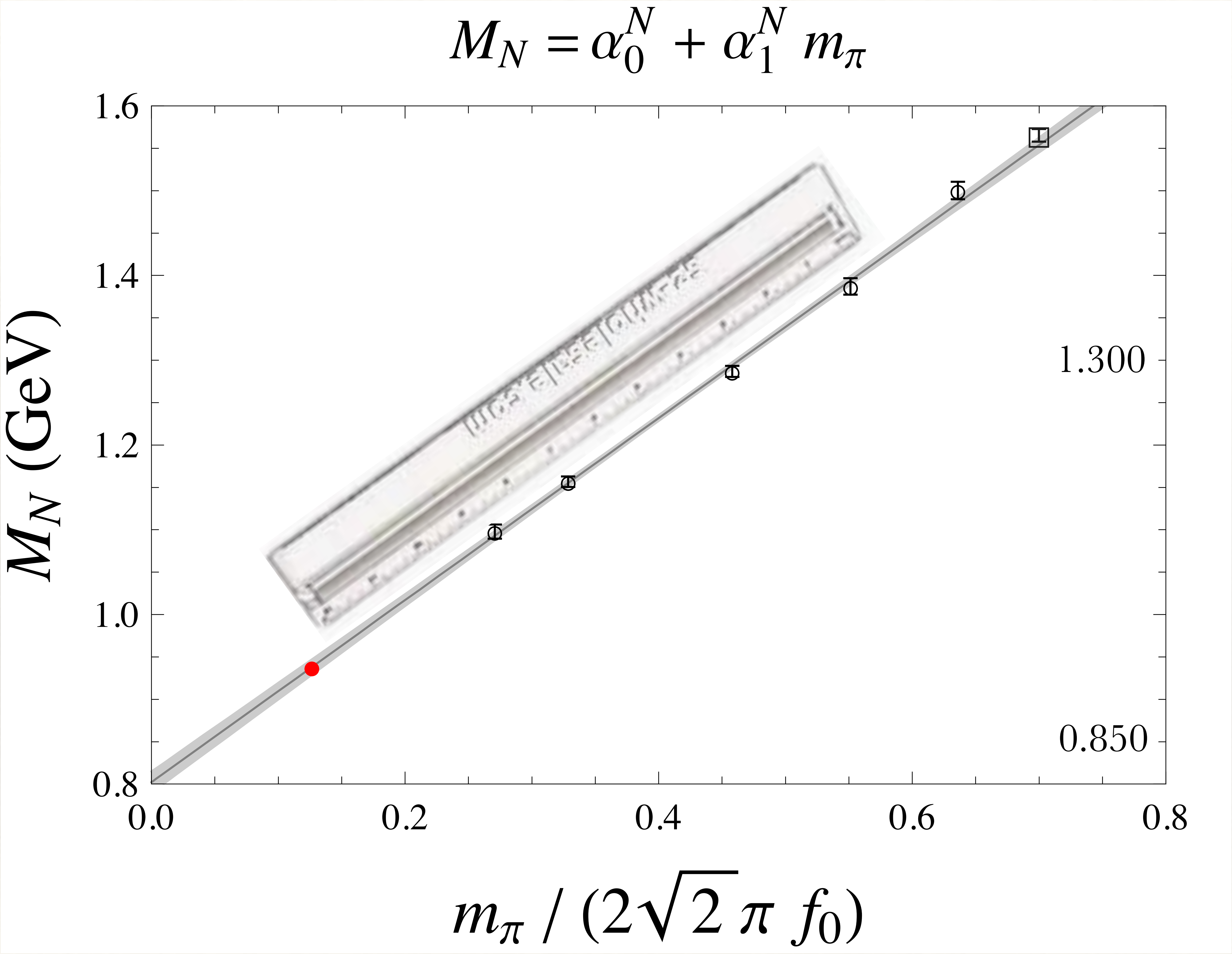




- $m_\pi \gtrsim 300$ MeV, both extrapolations have good χ^2/dof
- Ruler Fit (physical point not included): $M_N \simeq 800 + m_\pi$ $[806(14) + 0.984(49) m_\pi]$

- m_π is clearly too heavy to draw conclusions — how does it compare to more modern results?

M_N vs m_π

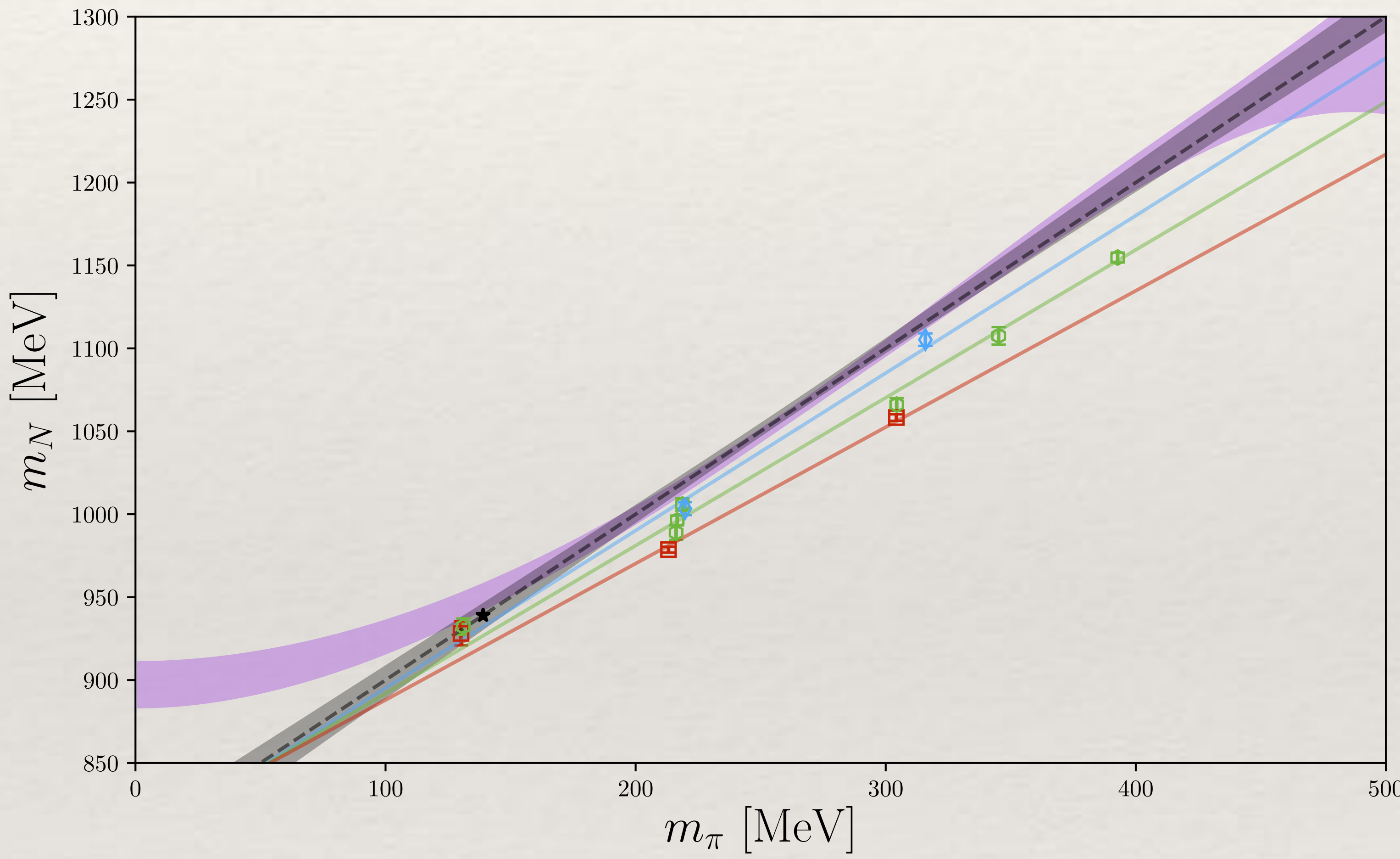


CalLat (under analysis)

= 806(14) + 0.984(49) m_π

– – – – $= 800 + m_\pi$ MeV

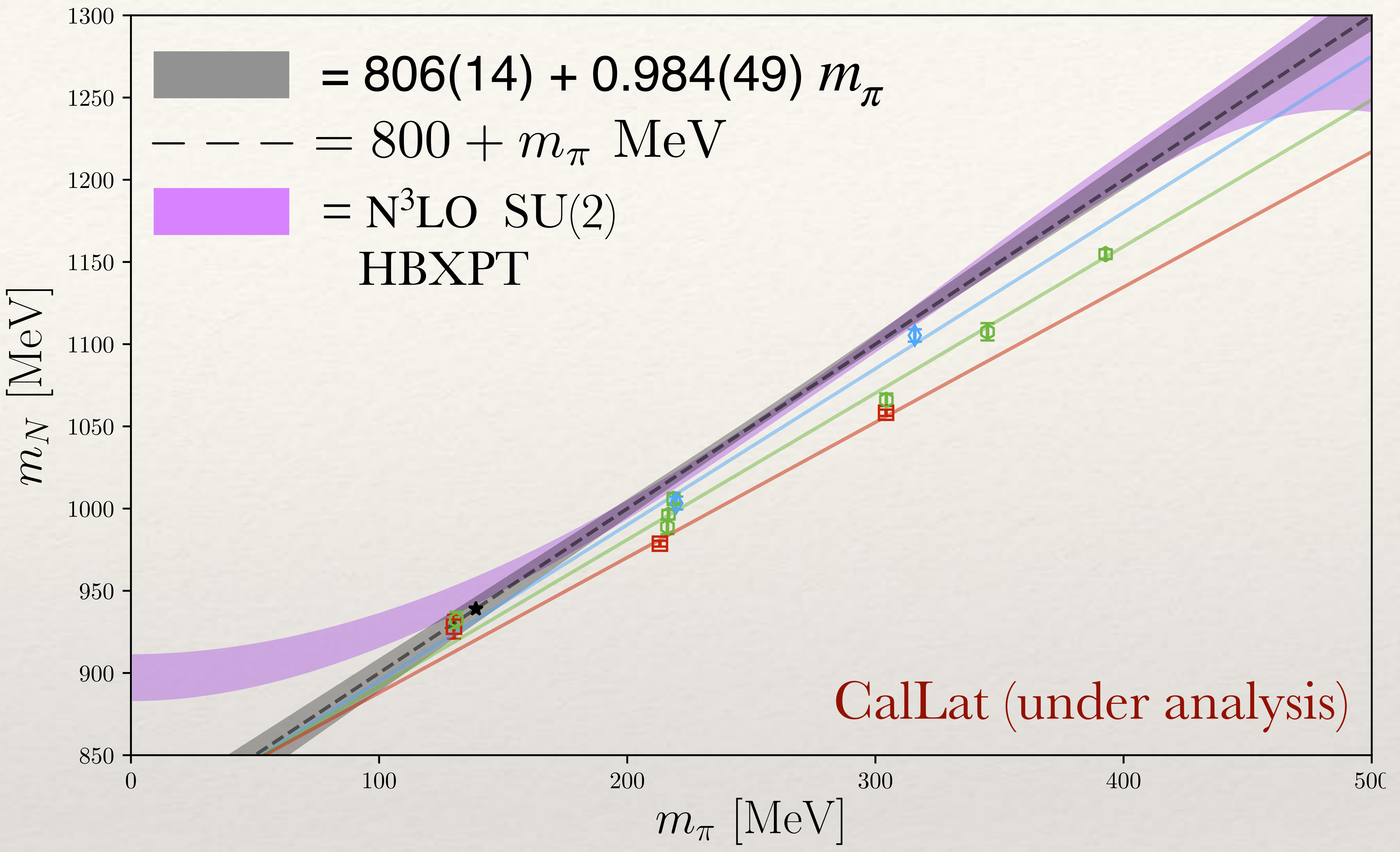
= N^{3}LO SU(2) HBXPT



- ❑ Ruler line is the same (x-axis is not quite the same)
- ❑ Note the large am_π correction $(a^2 m_\pi^2)$ in new results

M_N vs m_π

- **What are the lessons?**
- Nucleon mass goes up while leading non-analytic correction goes down — M_N results want small g_A
- Need simultaneous fit of M_N, g_A to stabilize
- QCD seems to conspire to produce linear in m_π behavior ($\sqrt{\hat{m}_{u,d}}$)
- This requires strong cancellations between different orders — not a sign of a healthy expansion
- At $m_\pi^{\rm phys}$, the series is converging
- Adding explicit Δ makes the convergence worse non-convergent? need more LQCD results



$$M_N = M_0 + \Lambda_\chi \left[-\epsilon_\pi^2\, 4\bar{c}_1 - \epsilon_\pi^3 \frac{3\pi g_A^2}{2} + \epsilon_\pi^4 \left(\alpha_4 + \beta_4 \ln \epsilon_\pi^2\right) + \epsilon_\pi^5 \left(\frac{3\pi g_A^4}{2} \left(1 + 4 \ln \epsilon_\pi^2\right) + \alpha_5 \right) + \mathrm{O}(\epsilon_\pi^6) \right]$$

$M_n - M_p$ vs m_π

- In order to compute strong-isospin breaking quantity, like $M_n - M_p\Big|_{m_d \neq m_u}$
  one can use isospin-symmetric sea-quarks and split the quark mass in the valence sector
  Tiburzi, Walker-Loud, NPA 764 (2006) [hep-lat/0501018]
  Beane, Orginos, Savage, NPB 768 (2007) [hep-lat/0605014]
  Walker-Loud, [0904.2404]

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

$$\begin{aligned} \mathcal{Z}_{u,d} &= \int DU_\mu \ \mathrm{Det}(D + m_l - \delta\tau_3) \ e^{-S[U_\mu]} \\ &= \int DU_\mu \ \mathrm{Det}(D + m_l) \ \det\left(1 - \frac{\delta^2}{(D+m_l)^2}\right) \ e^{-S[U_\mu]} \end{aligned}$$

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Isospin symmetric quantities: error $\mathcal{O}(\delta^2)$
Isospin violating quantities: error $\mathcal{O}(\delta^3)$

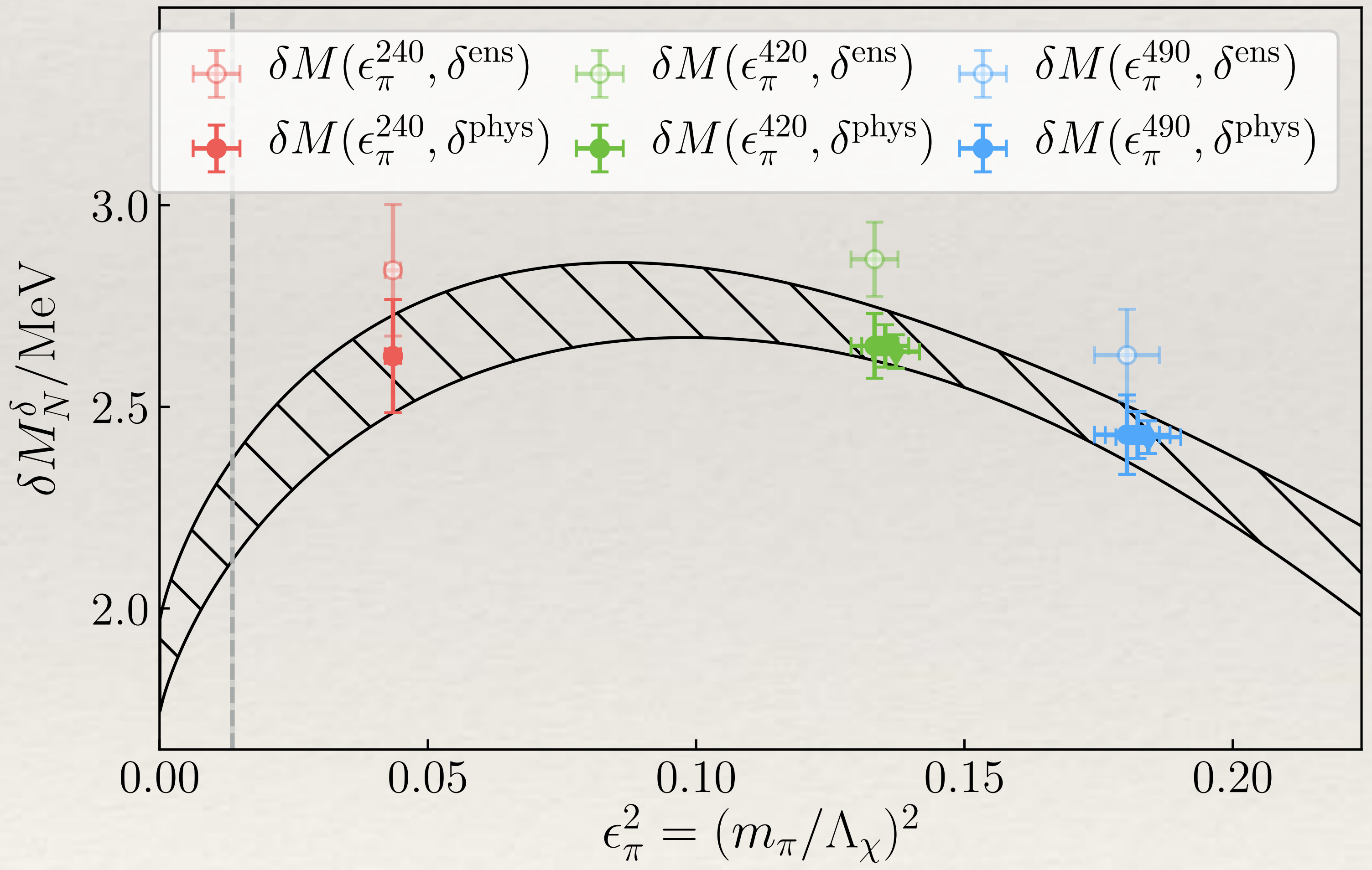
see also
de Divitiis etal JHEP 1204 (2012)
de Divitiis etal Phys. Rev. D87 (2013)

$M_n - M_p$ vs m_π

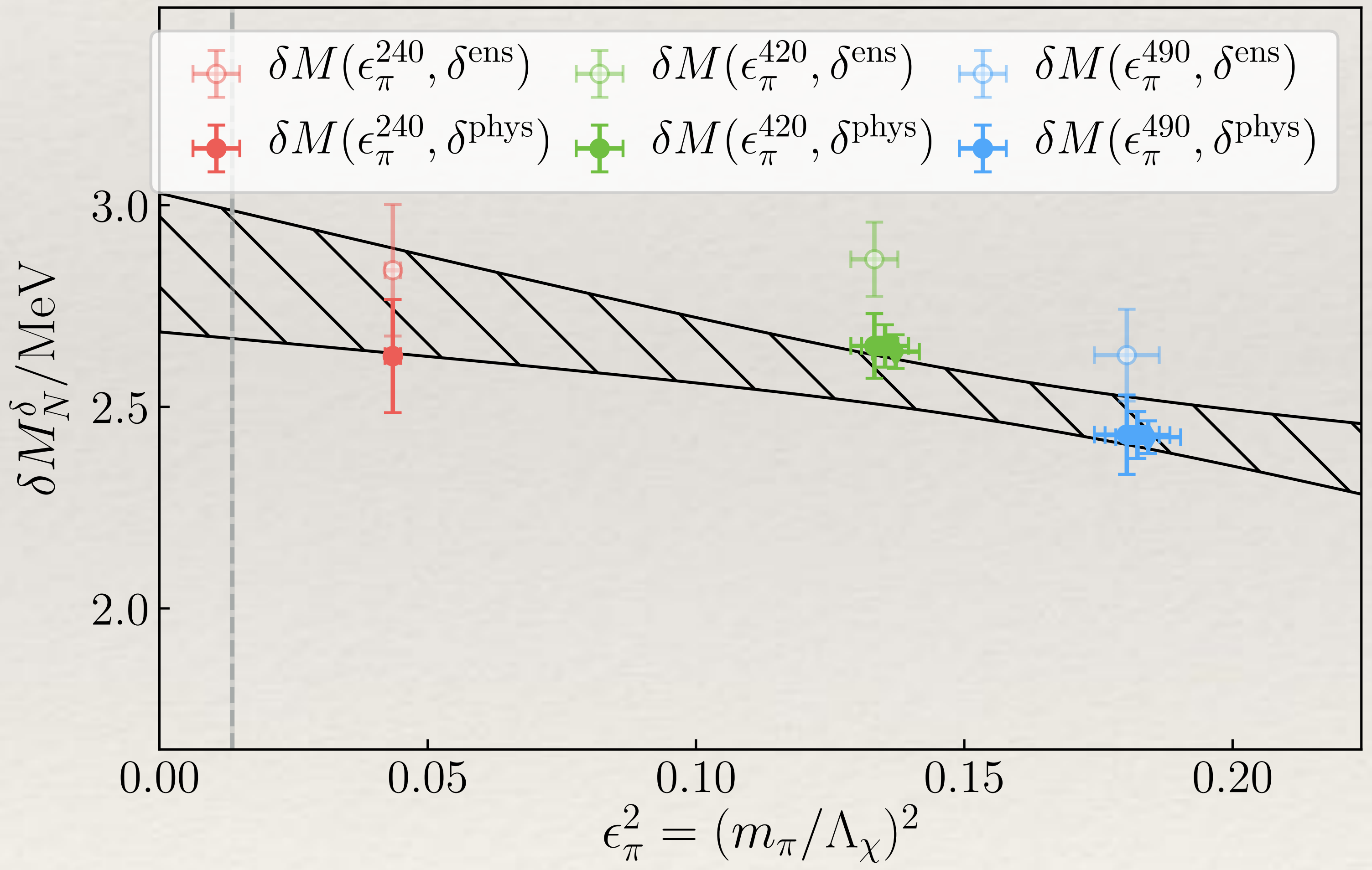
- The iso-vector nucleon mass is known through $O(m_\pi^4)$ in SU(2) HBχPT Walker-Loud [0904.2404]

$$\delta_{M_N}^{m_d - m_u} \equiv M_n - M_p = \delta \left\{ \alpha_N \left[1 - \frac{6g_A^2 + 1}{2} \epsilon_\pi^2 \ln \epsilon_\pi^2 \right] + \beta_N \epsilon_\pi^2 \right\}$$

- Compare with LQCD results, Brantley et al [1612.07733]



HBχPT prediction

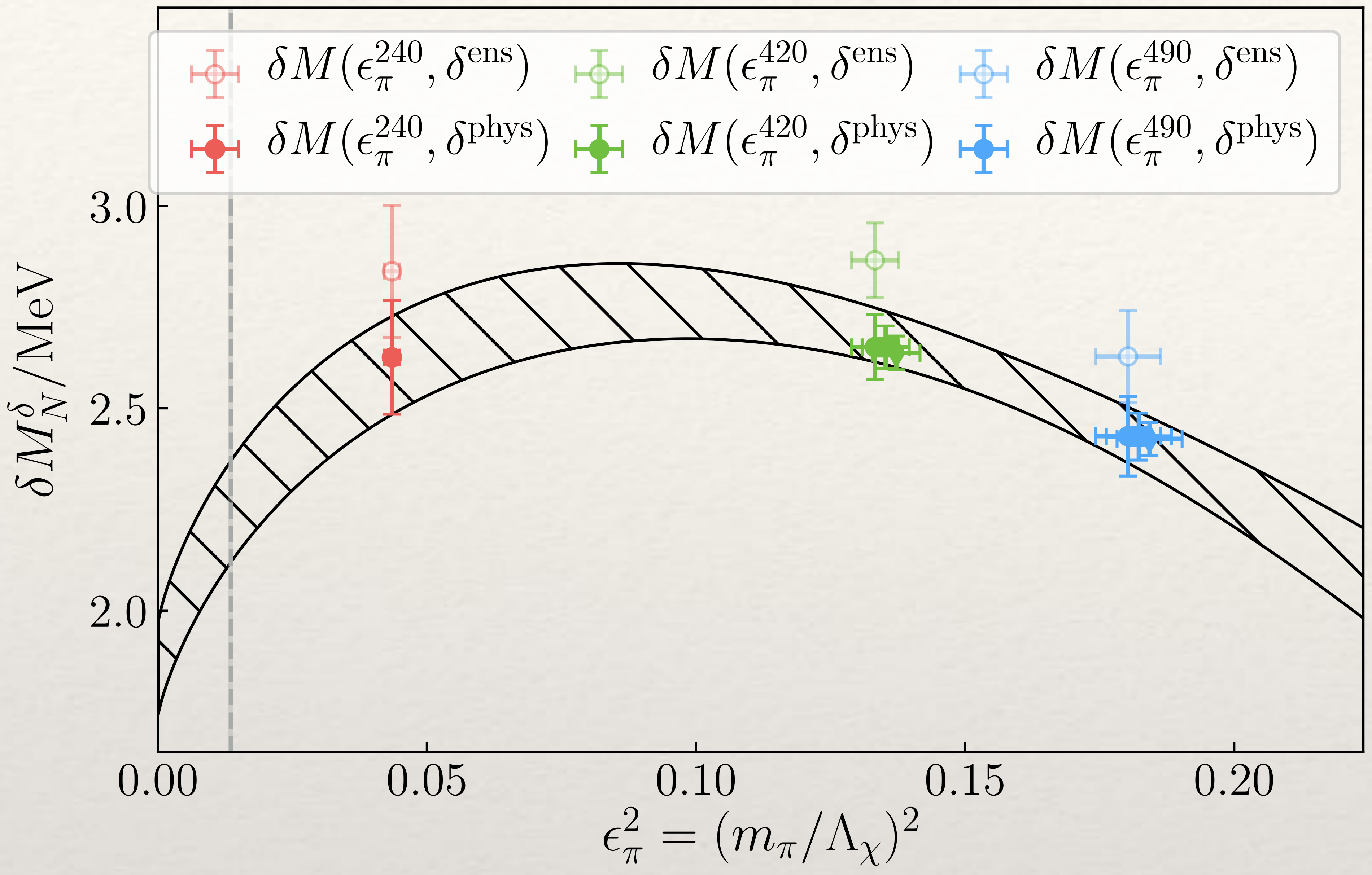


Taylor expansion (polynomial)

- single lattice spacing
- 3 pion masses
- 3 values of $\delta = \dfrac{m_d - m_u}{2}$
- scale setting with m_Ω
- determine $\delta^{\rm phys}$ with kaon mass splitting (after removing estimated QED corrections)
- shift data to $\delta^{\rm phys}$ for plot

$M_n - M_p$ vs m_π

HBχPT prediction



- prior g_A from LQCD result

	posterior	[prior]	logGBF
	$g_A = 1.271(13)$	[1.271(13)]	65.088

- prior g_A "agnostically"

$g_A = 1.15(52)$ [1.3(2.0)] 63.817

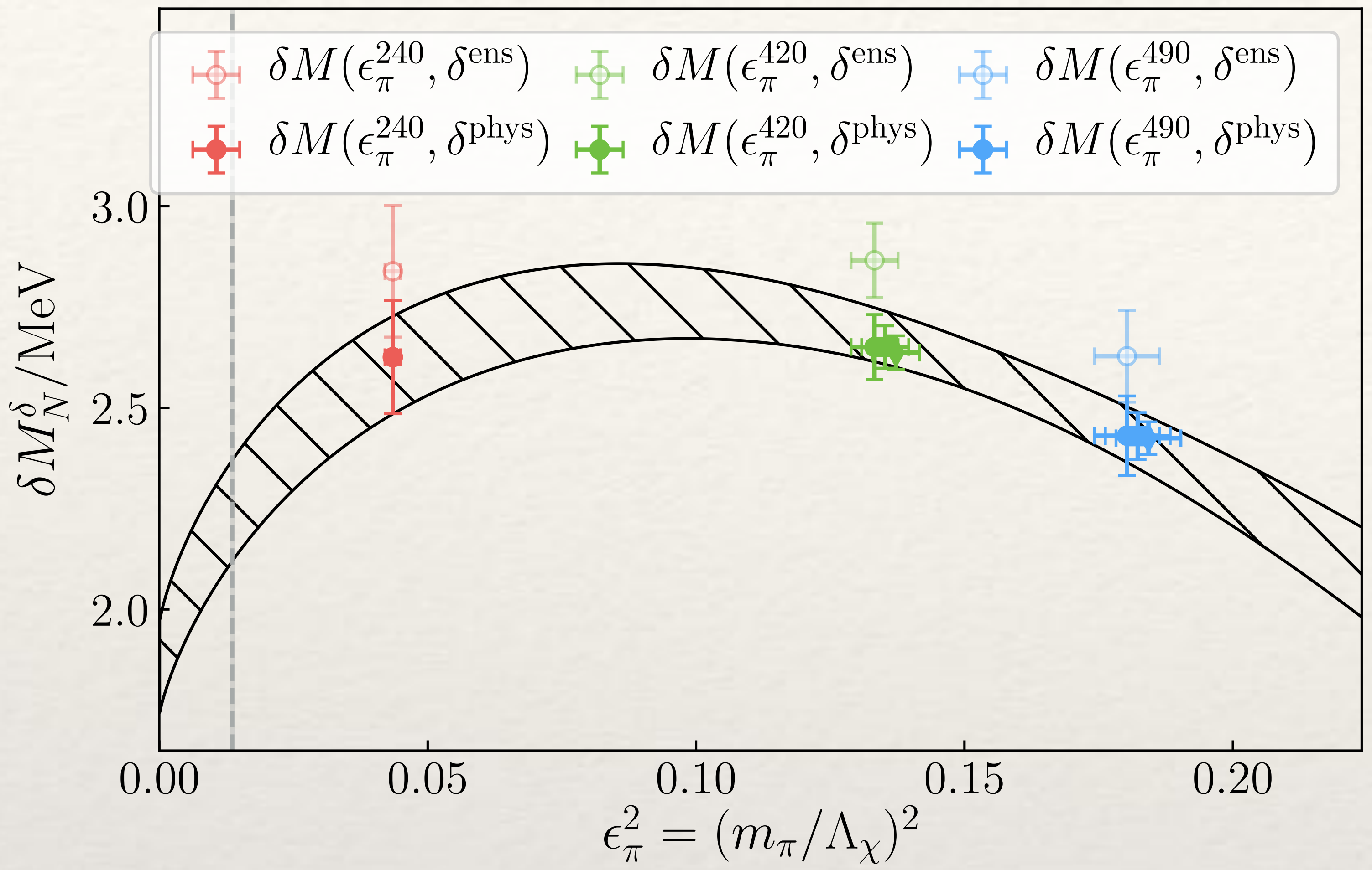
- prior g_A from PDG

$g_A = 1.2754(13)$ [1.2754(13)] 65.084

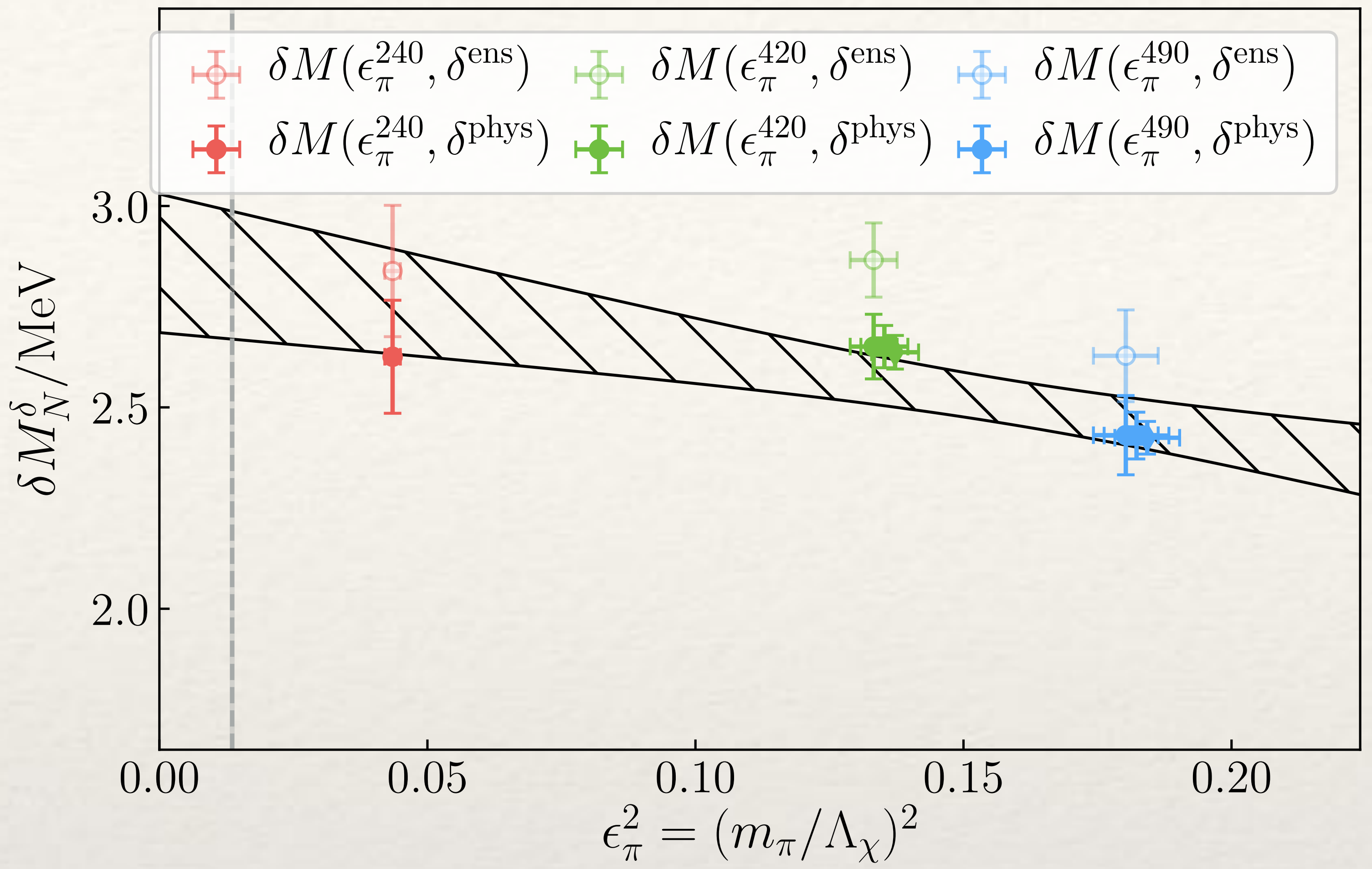
relative weight $w_k = e^{\text{logGBF}_k}$

$M_n - M_p$ vs m_π

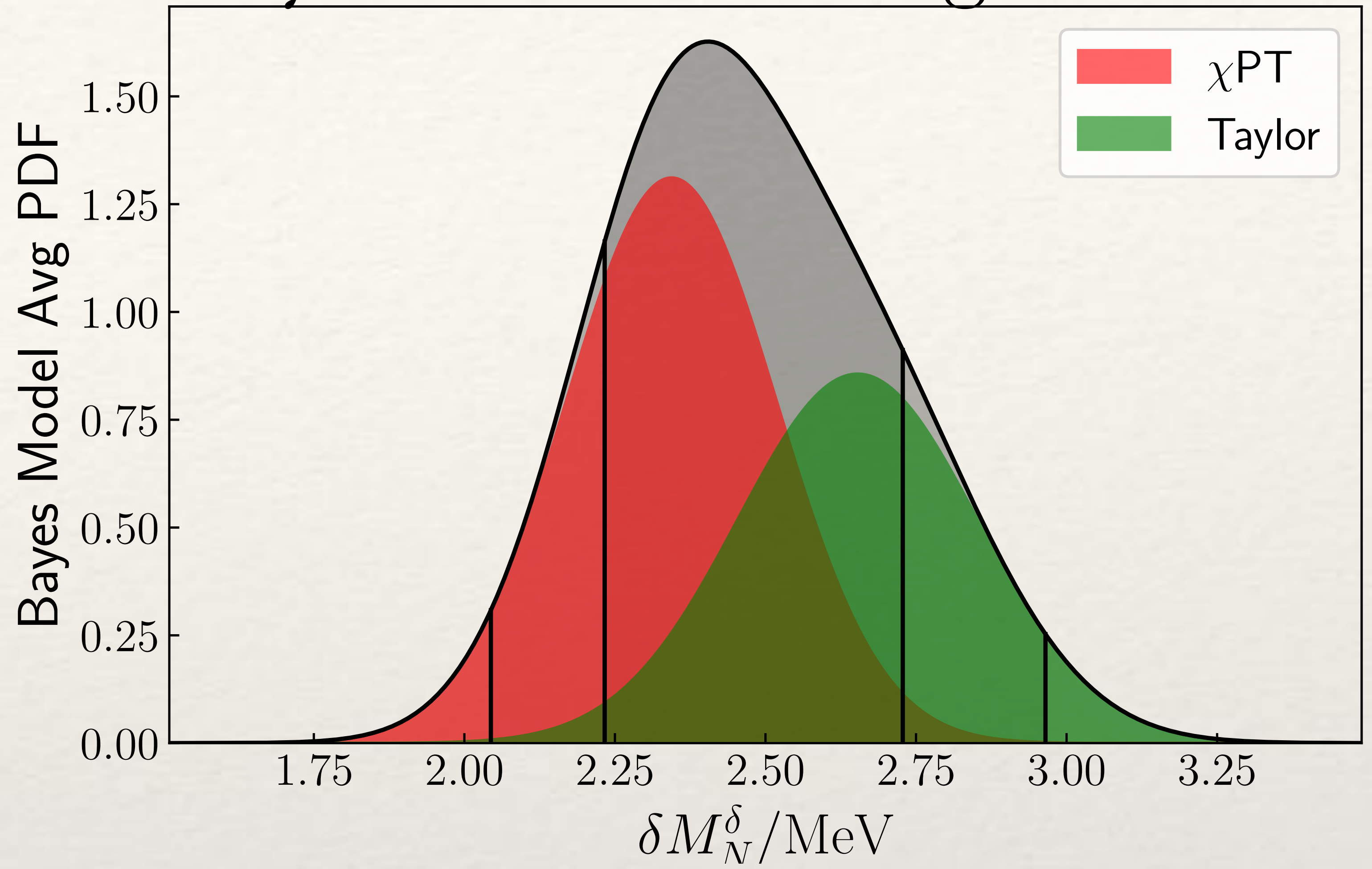
HBχPT prediction

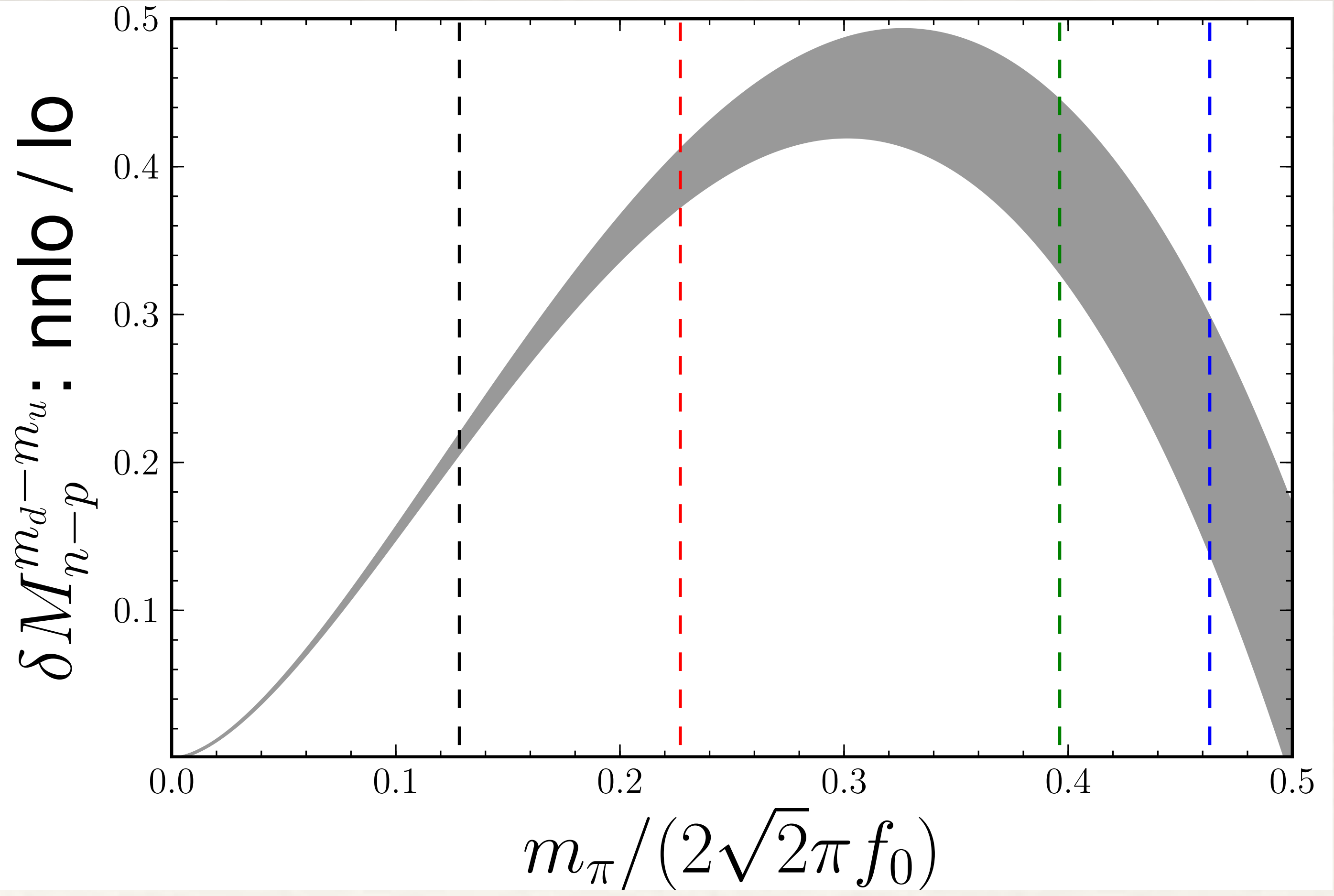


Taylor expansion (polynomial)

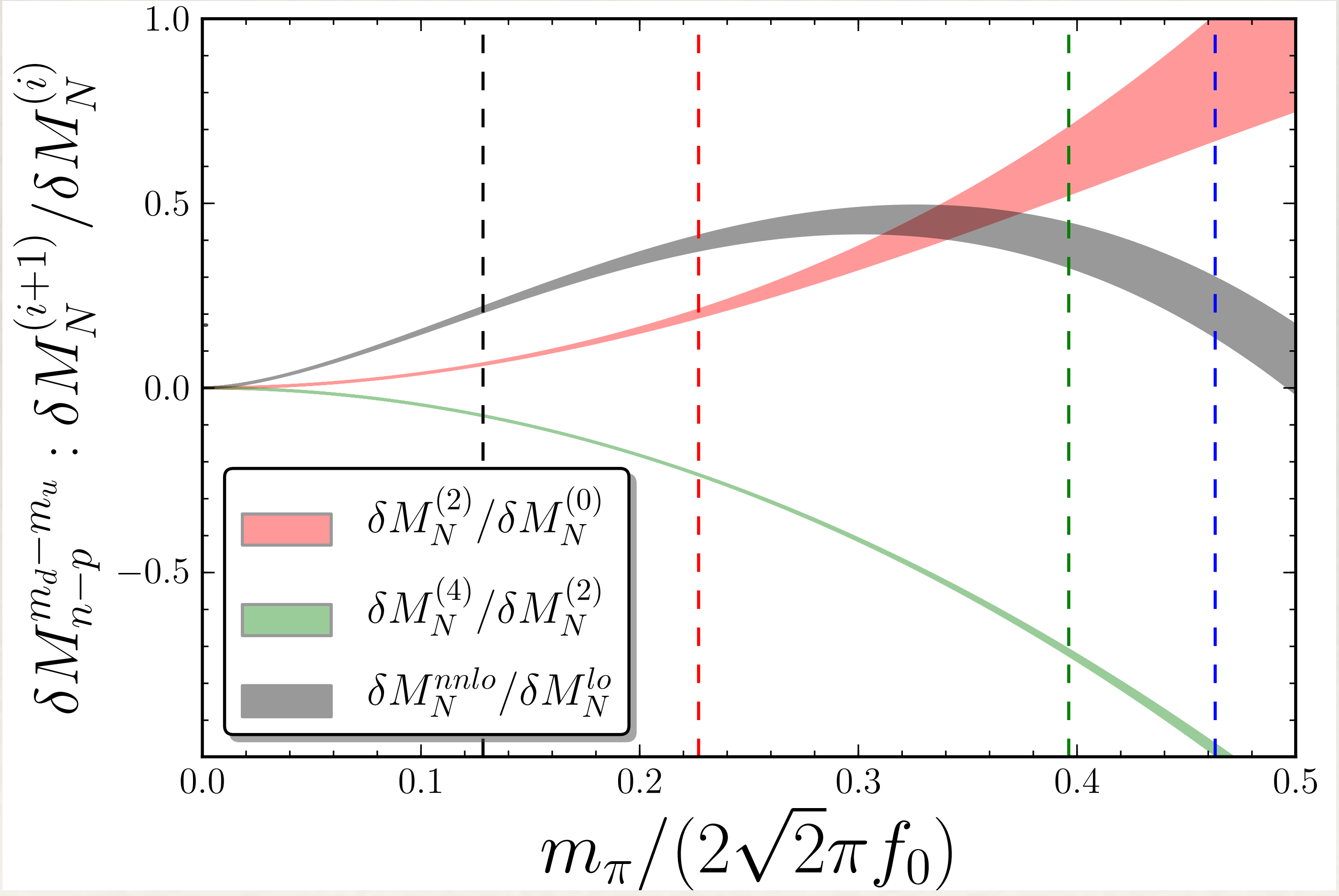


Bayes Model Average





convergence is tolerable



convergence is not very good

$$\delta_{M_N}^{m_d-m_u} = 2.43(13)^s(26)^M(18)^\delta(04)^{\rm scale}$$

Model	χ^2/dof	logGBF	weight
HBχPT	0.347	65.088	0.749
Taylor	0.786	63.993	0.251

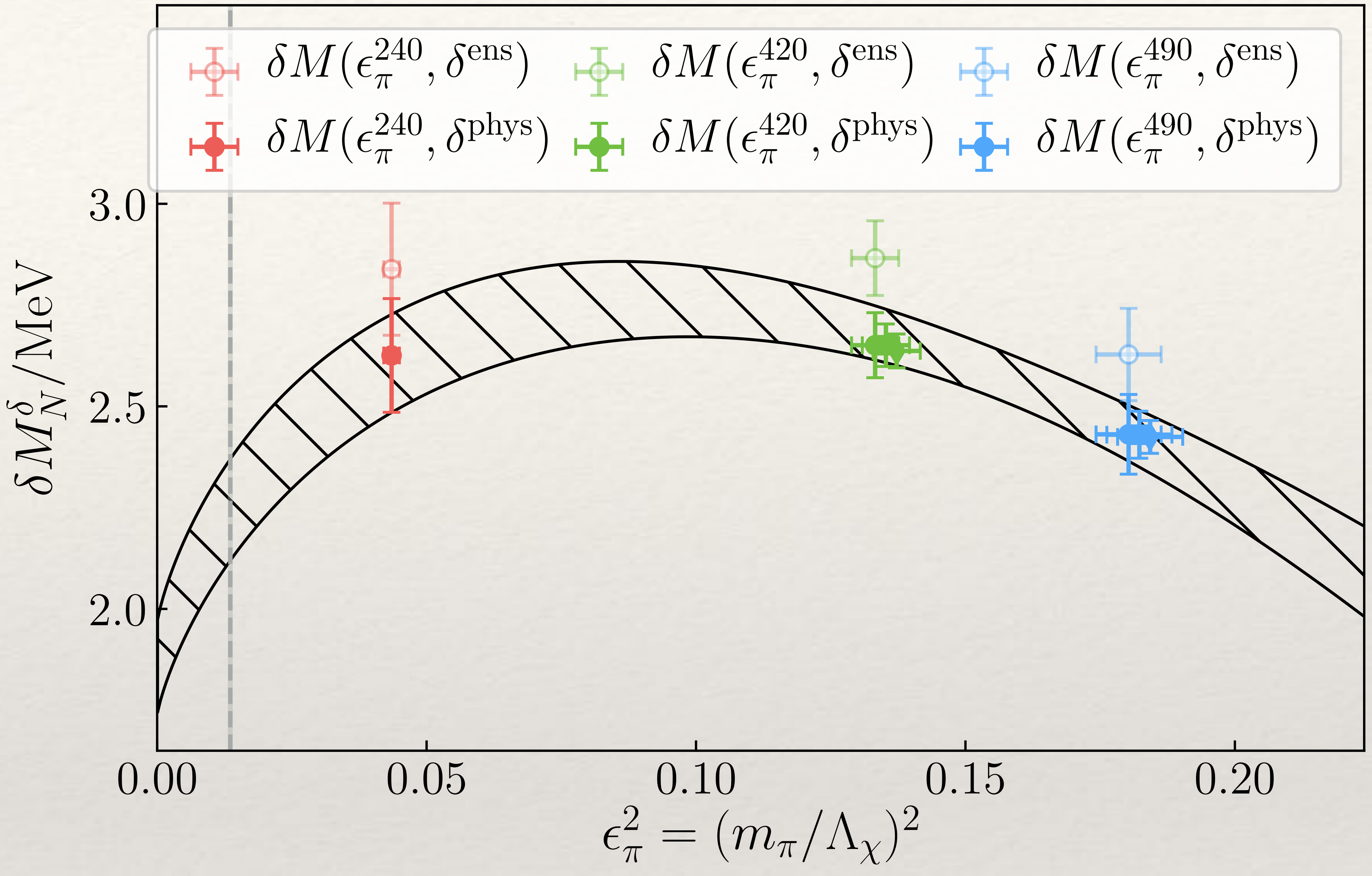
$M_n - M_p$ vs m_π

- **What are the lessons?**

$$\delta^{m_d - m_u}_{M_N} \equiv M_n - M_p = \delta \left\{ \alpha_N \left[1 - \frac{6g_A^2 + 1}{2} \epsilon_\pi^2 \ln \epsilon_\pi^2 \right] + \beta_N \epsilon_\pi^2 \right\}$$

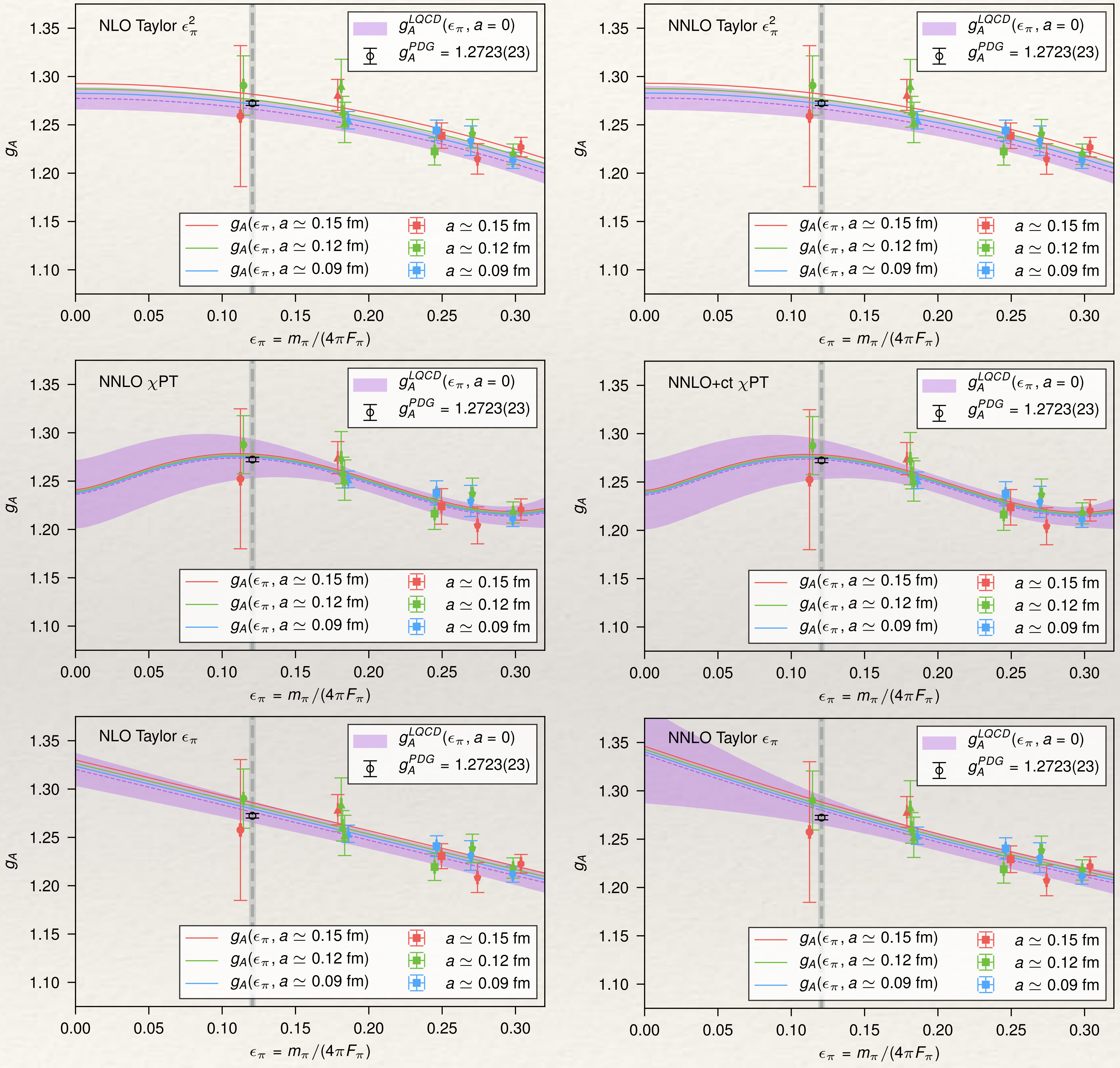
- Leaving g_A unconstrained returns large value of g_A
  The LQCD results prefer a large coefficient
- The LQCD results also favor HBχPT over the Taylor (polynomial) approximation
- Definitive ruling of one model over the other requires results at $m_\pi \lesssim 240$ MeV

- Interesting to note that this iso-vector mass is related to the CP-odd pion-nucleon coupling arising from a QCD θ-term

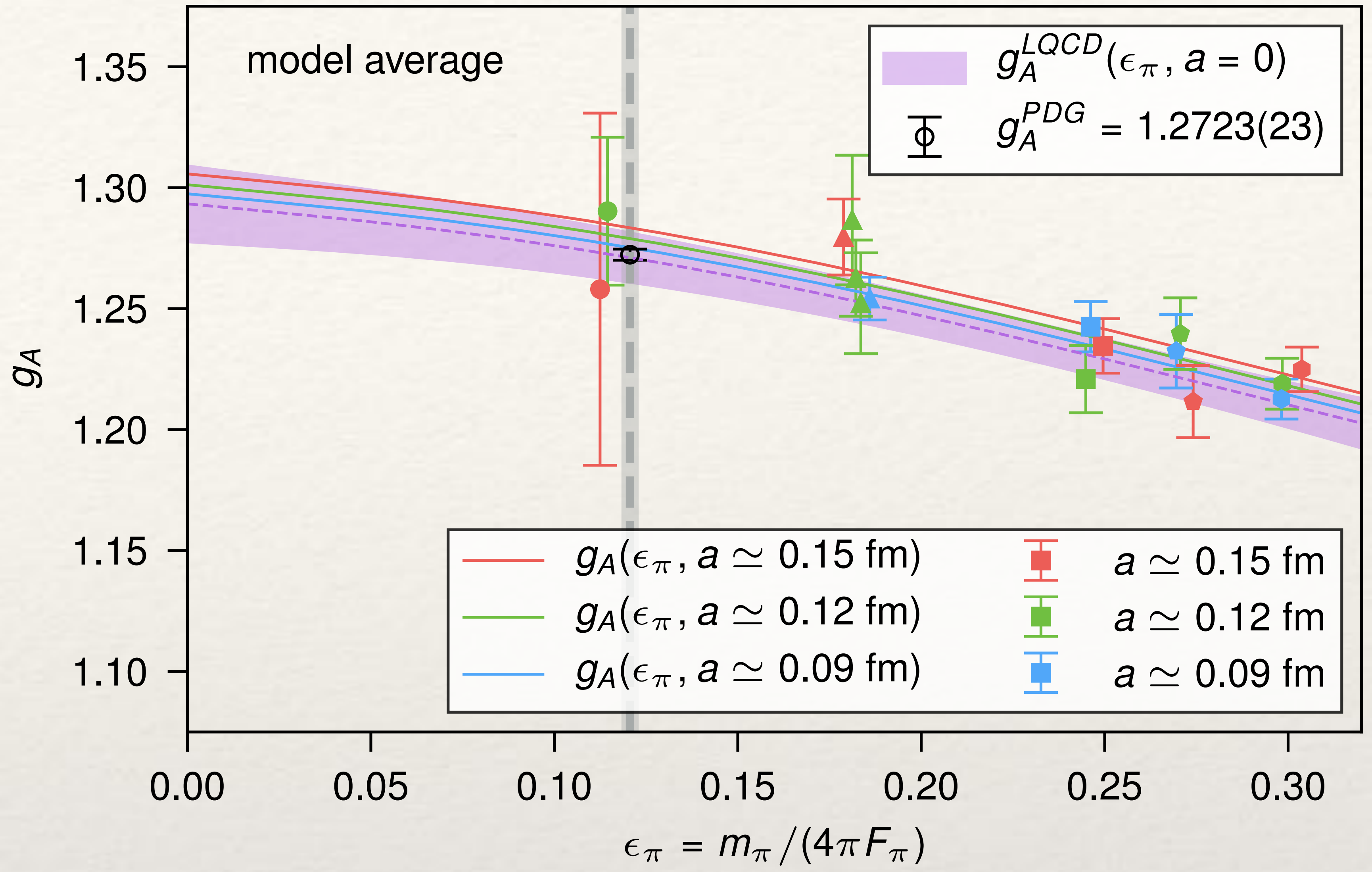


g_A

Use various “models” to extrapolate

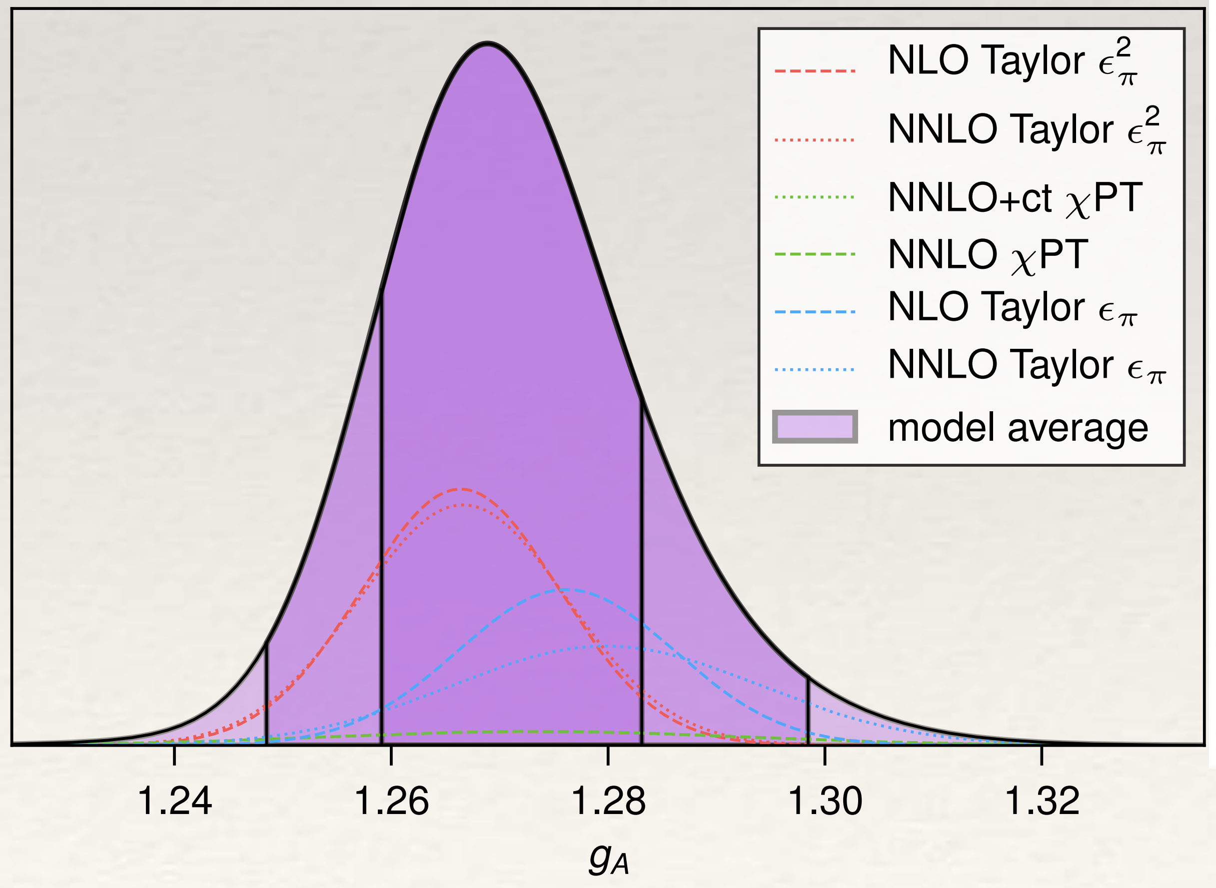


Chang et al (CalLat)
Nature 558 (2018)
[1805.12130]



Final result

$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$



Fit	χ^2/dof	$\mathcal{L}(D\|M_k)$	$P(M_k\|D)$	$P(g_A\|M_k)$
NNLO χPT	0.727	22.734	0.033	1.273(19)
NNLO+ct χPT	0.726	22.729	0.033	1.273(19)
NLO Taylor ϵ_π^2	0.792	24.887	0.287	1.266(09)
NNLO Taylor ϵ_π^2	0.787	24.897	0.284	1.267(10)
NLO Taylor ϵ_π	0.700	24.855	0.191	1.276(10)
NNLO Taylor ϵ_π	0.674	24.848	0.172	1.280(14)
average				1.271(11)(06)

g_A

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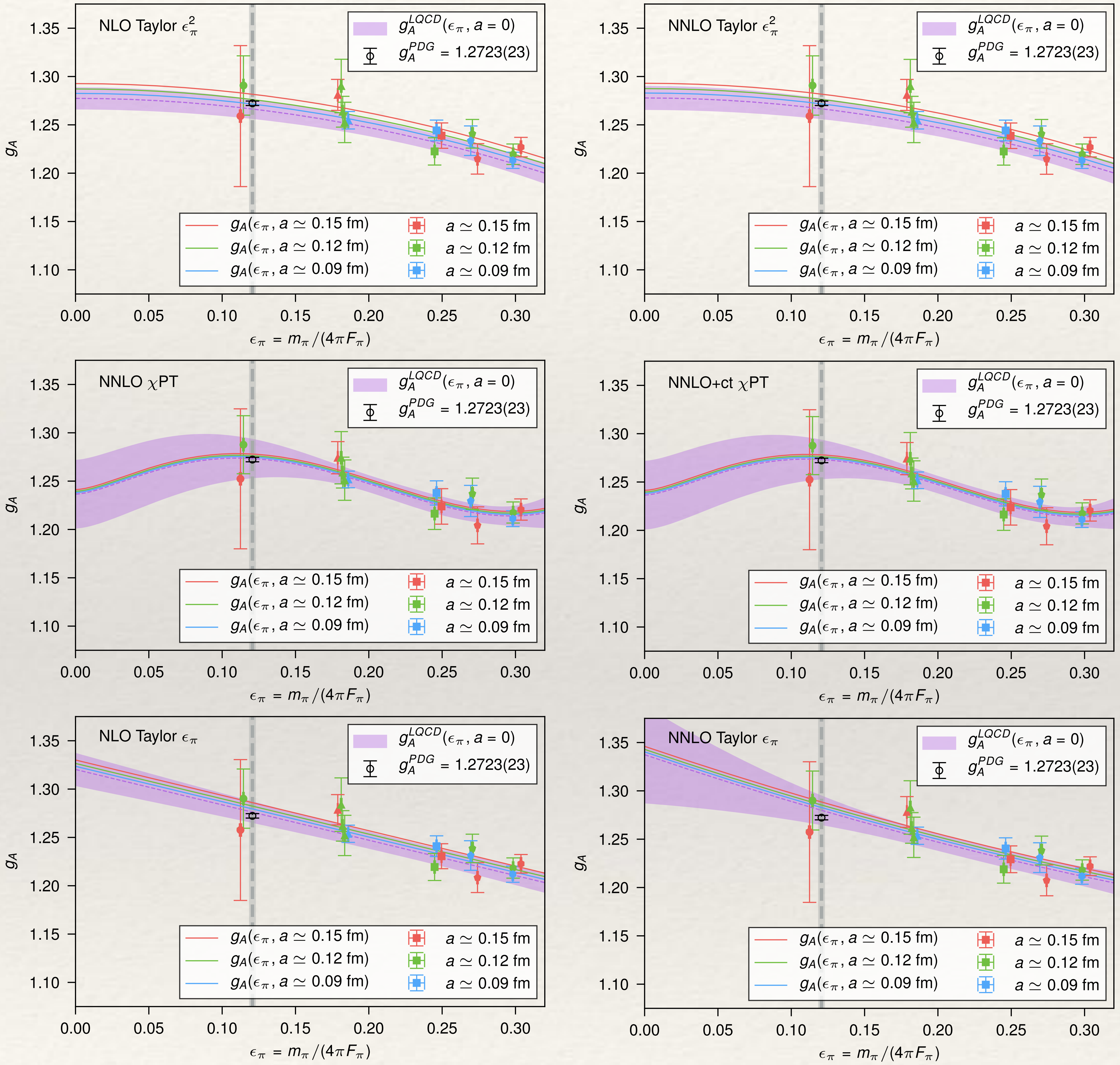
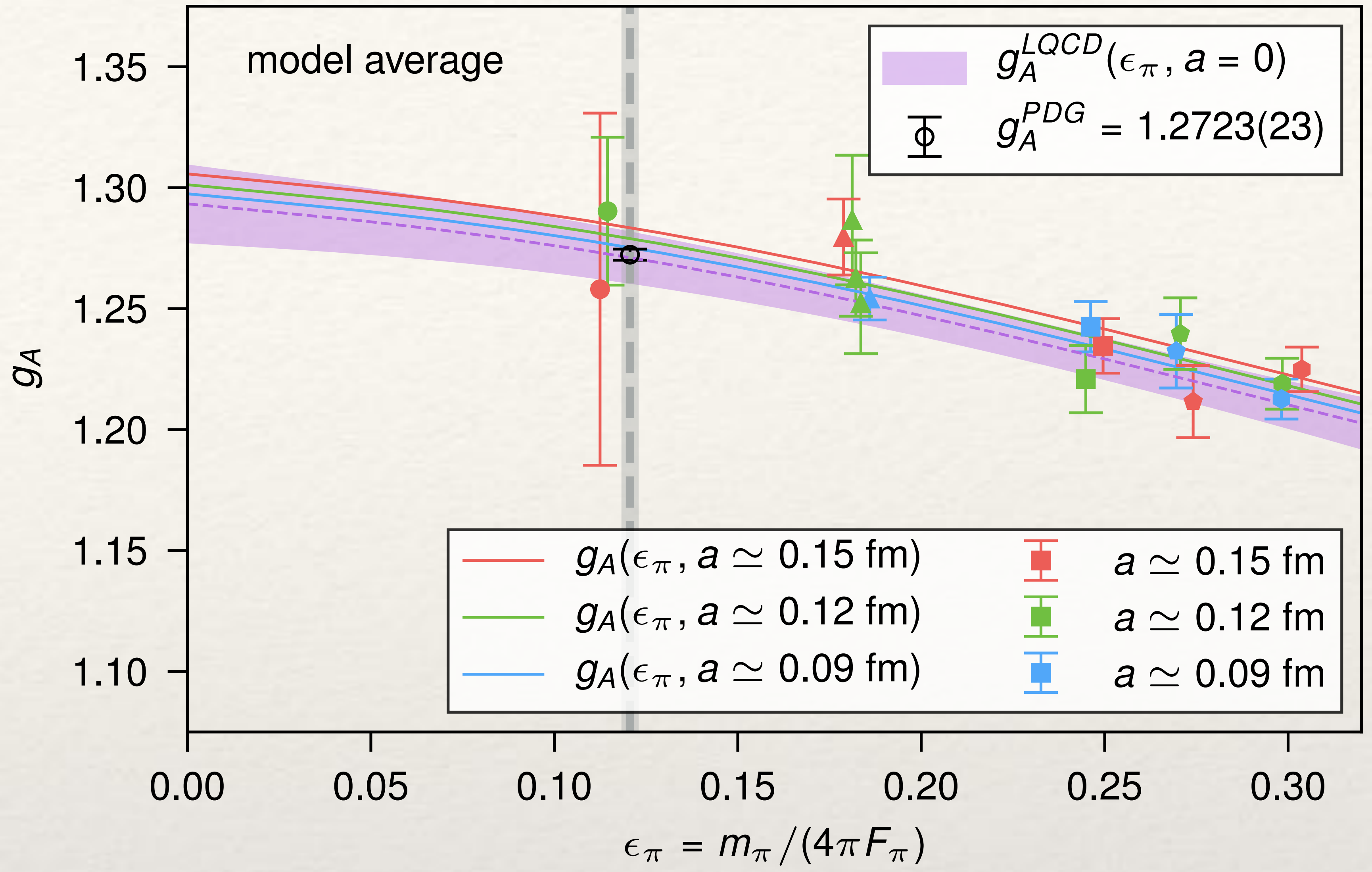


Chang et al (CalLat) Nature 558 (2018) [1805.12130]



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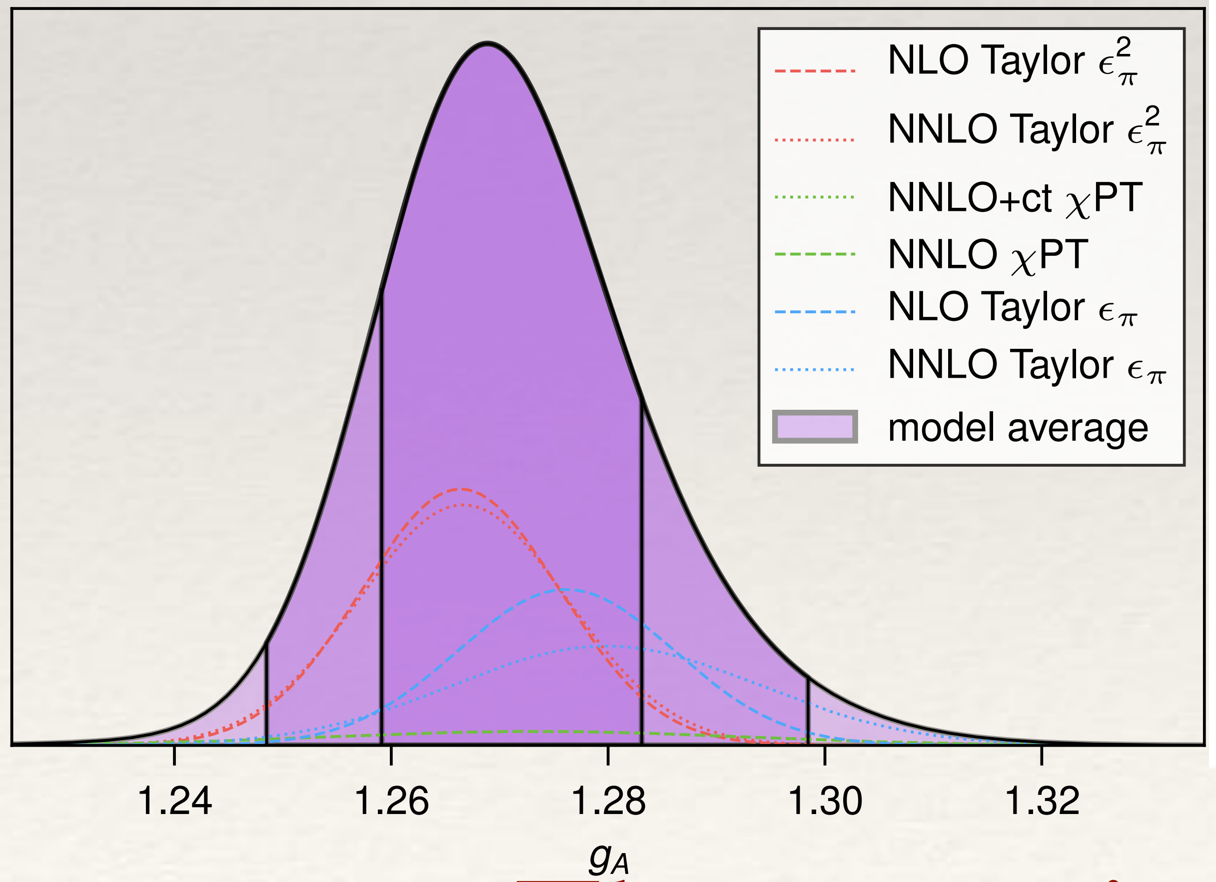

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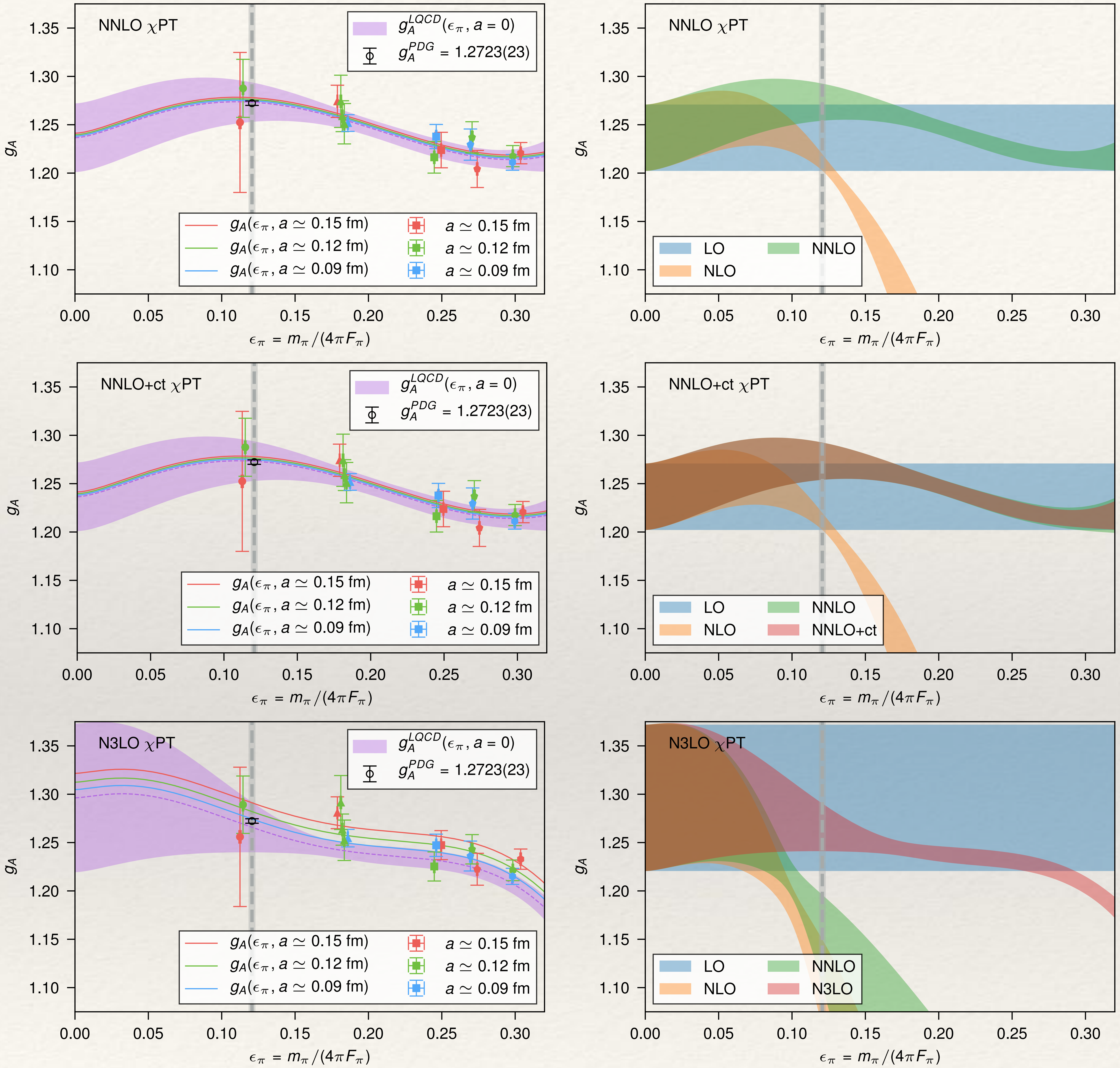
average				1.271(11)(06)

The numerical results "do not like χPT"

convergence of the chiral expansion…



$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + c_4\epsilon_\pi^4$$

❑ Chiral corrections to g_A @ m_π^{phys}

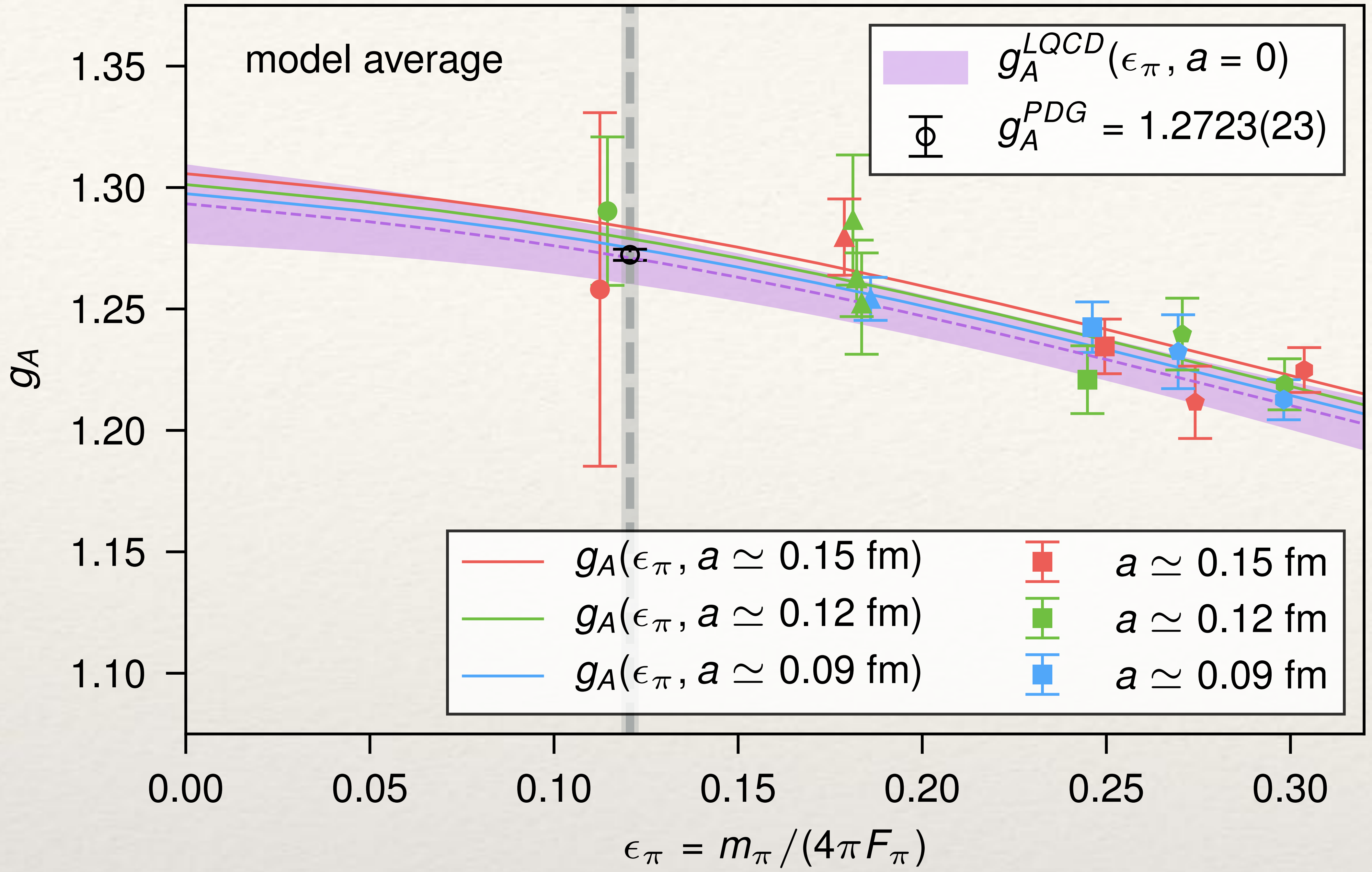
N^nLO	LO	NLO	N^2LO	N^3LO
N^2LO	1.237(34)	-0.026(30)	0.062(14)	–
N^3LO	1.296(76)	-0.19(12)	0.045(63)	0.117(66)

$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + \epsilon_\pi^4\left[c_4 + \tilde{\gamma}_4 \ln(\epsilon_\pi^2) + \left(\frac{2}{3}g_0 + \frac{37}{12}g_0^3 + 4g_0^5\right)\ln^2(\epsilon_\pi^2)\right]$$

Bernard and Meissner (CD06)
Phys.Lett.B639 [hep-lat/0605010]
F —> F_π

Nature 558 (2018) no. 7708, 91-94
Chang et al. [arXiv:1805.12130]

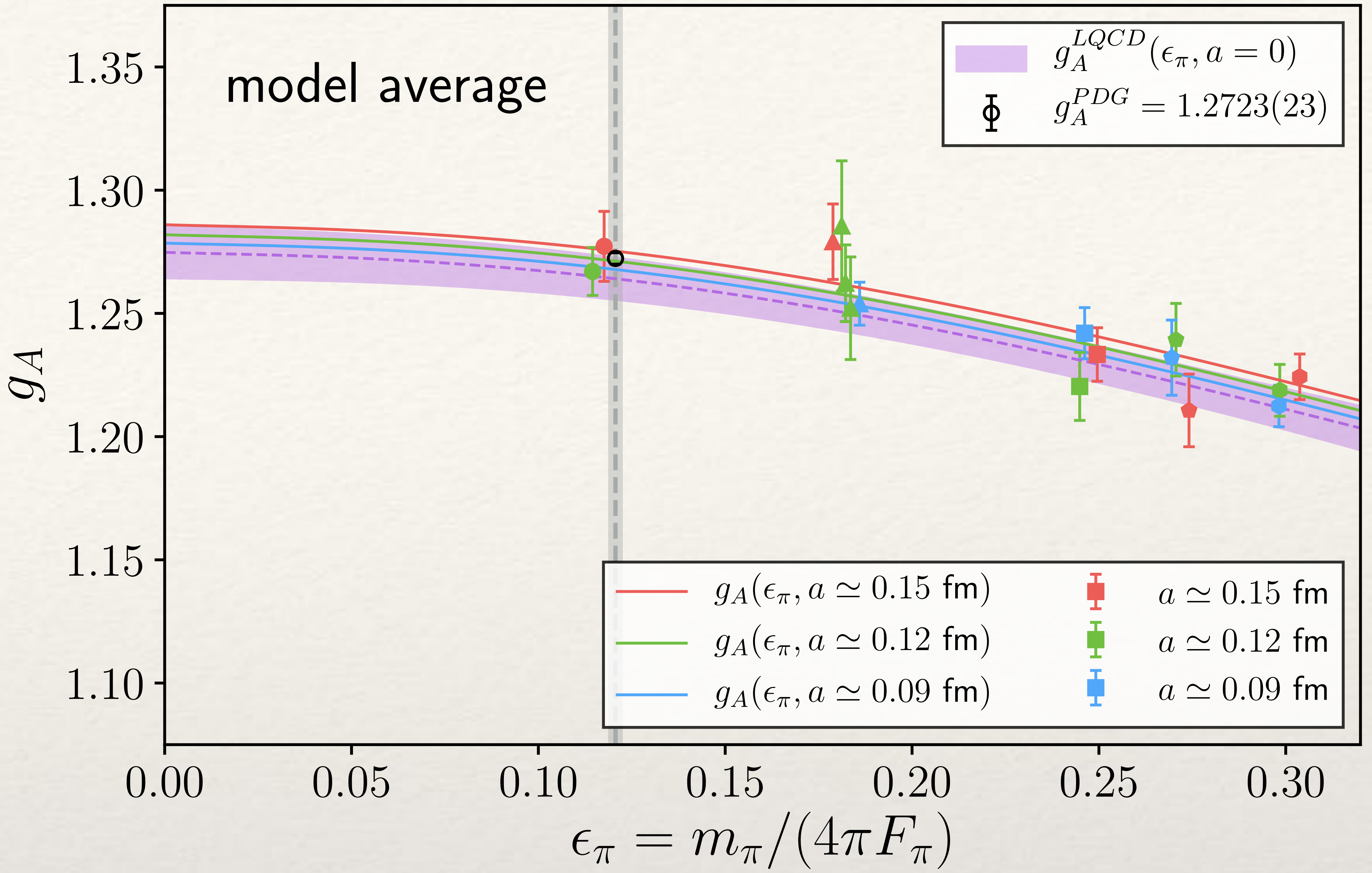
1 year on Titan (ORNL) + 2 years on GPU machines at LLNL



California Lattice Collaboration

Sierra Early Science

PRELIMINARY



- The a12m130 (48^3 x 64 x 20) with 3 sources cost as much as all other ensembles combined
  - 2.5 weekends on Sierra → 16 srcs
  - Now, 32 srcs (un-constrained, 3-state fit)
- We generated a new a15m135XL (48^3 x 64) ensemble (old a15m130 is 32^3 x 48)
  - $M\pi L = 4.93$ (old $M\pi L = 3.2$)
  - $L_5 = 24$, $N_{src} = 16$

Walker-Loud et al (CalLat)
PoS CD2018 [1912.08321]

$$g_A = 1.2711(125) \rightarrow 1.2641(93) \quad [0.74\%]$$

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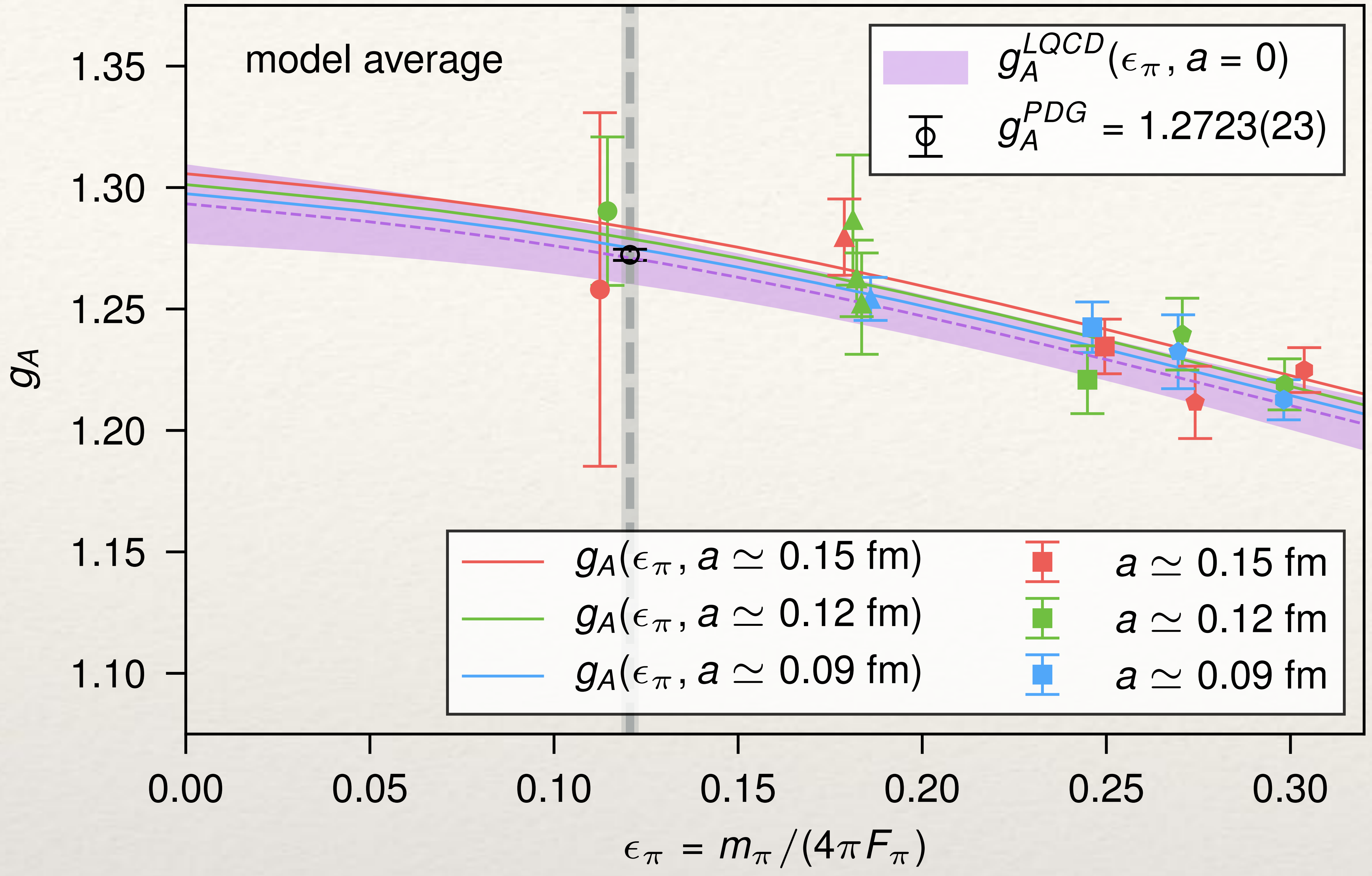

PRELIMINARY

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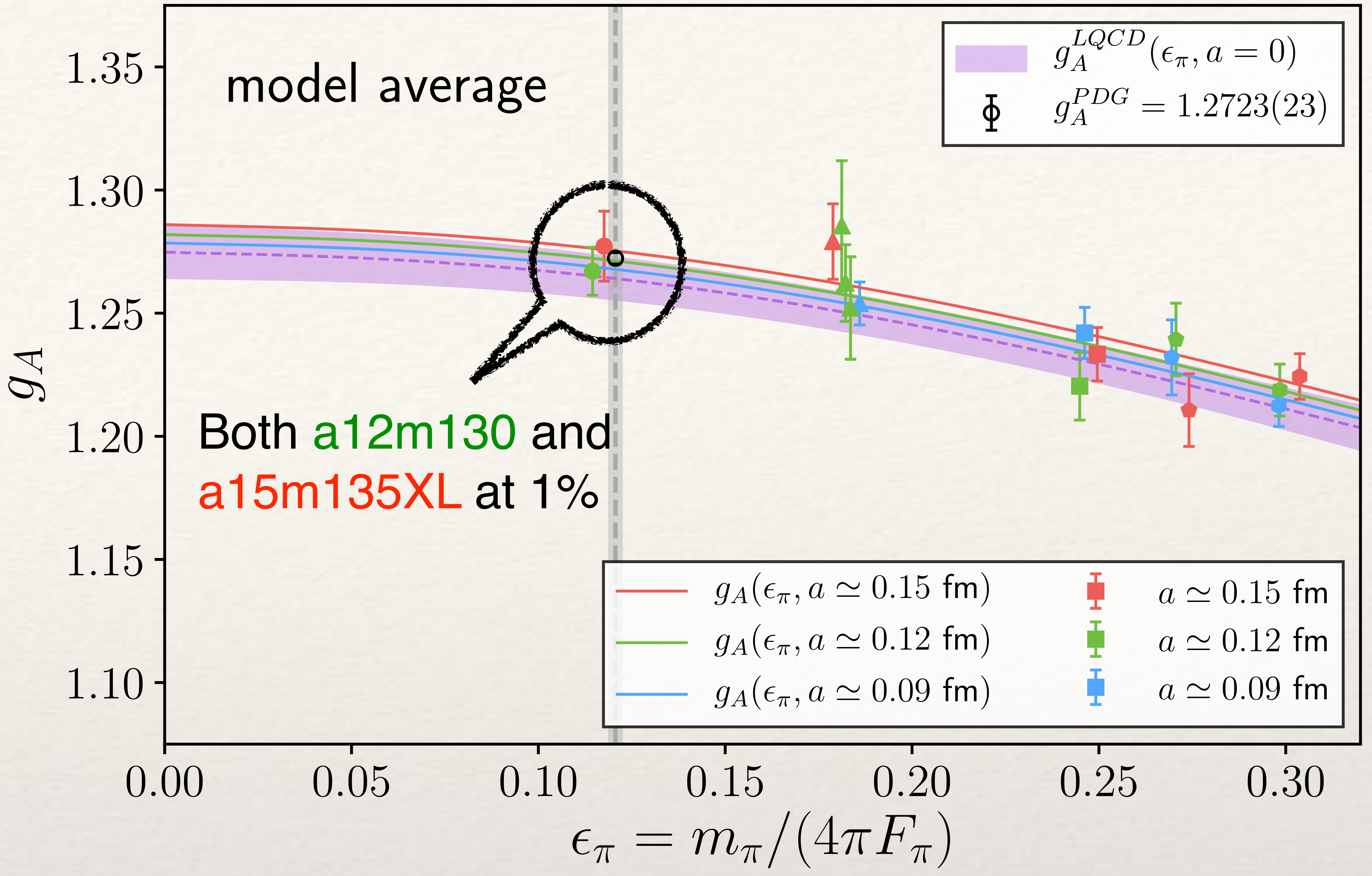

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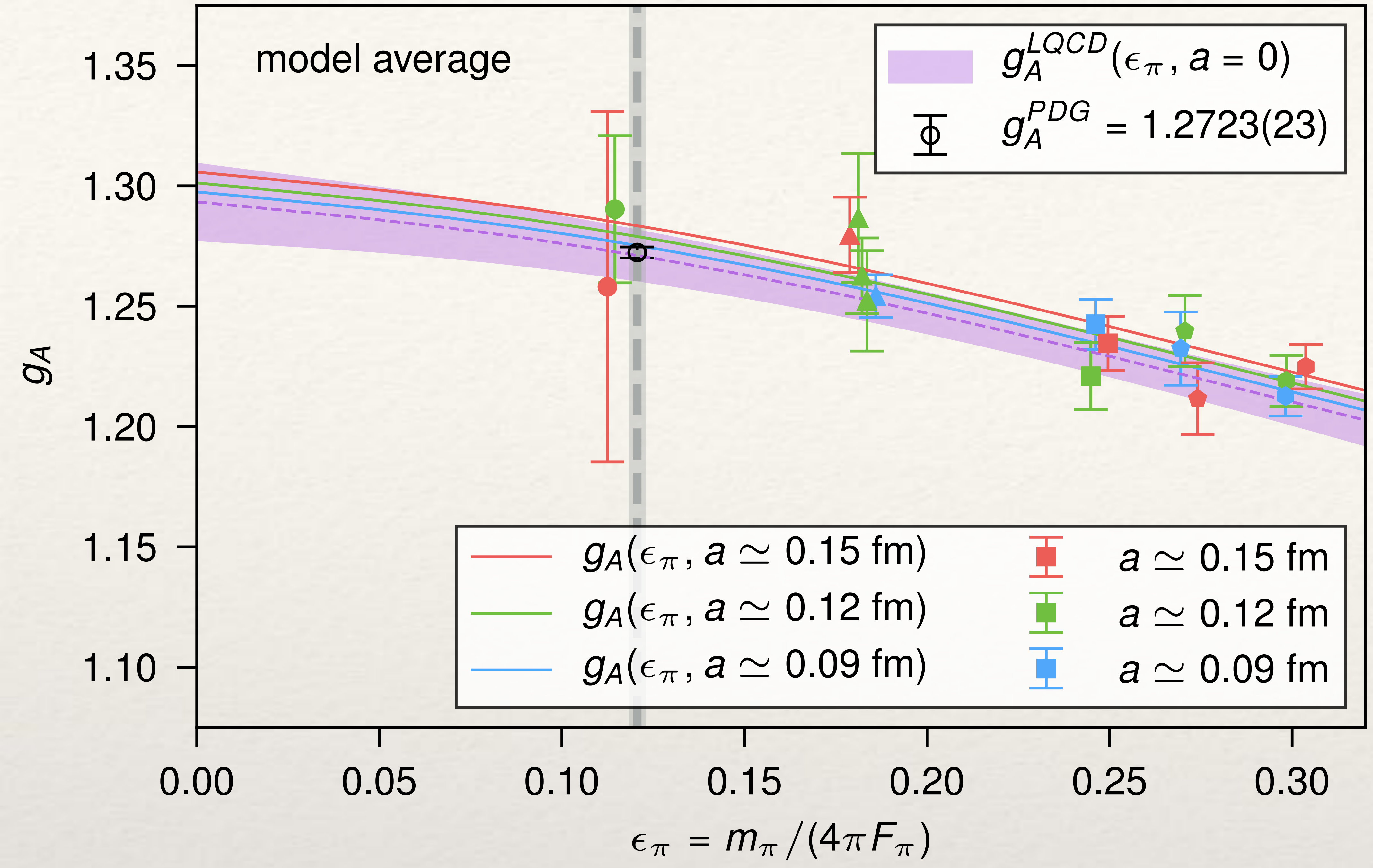

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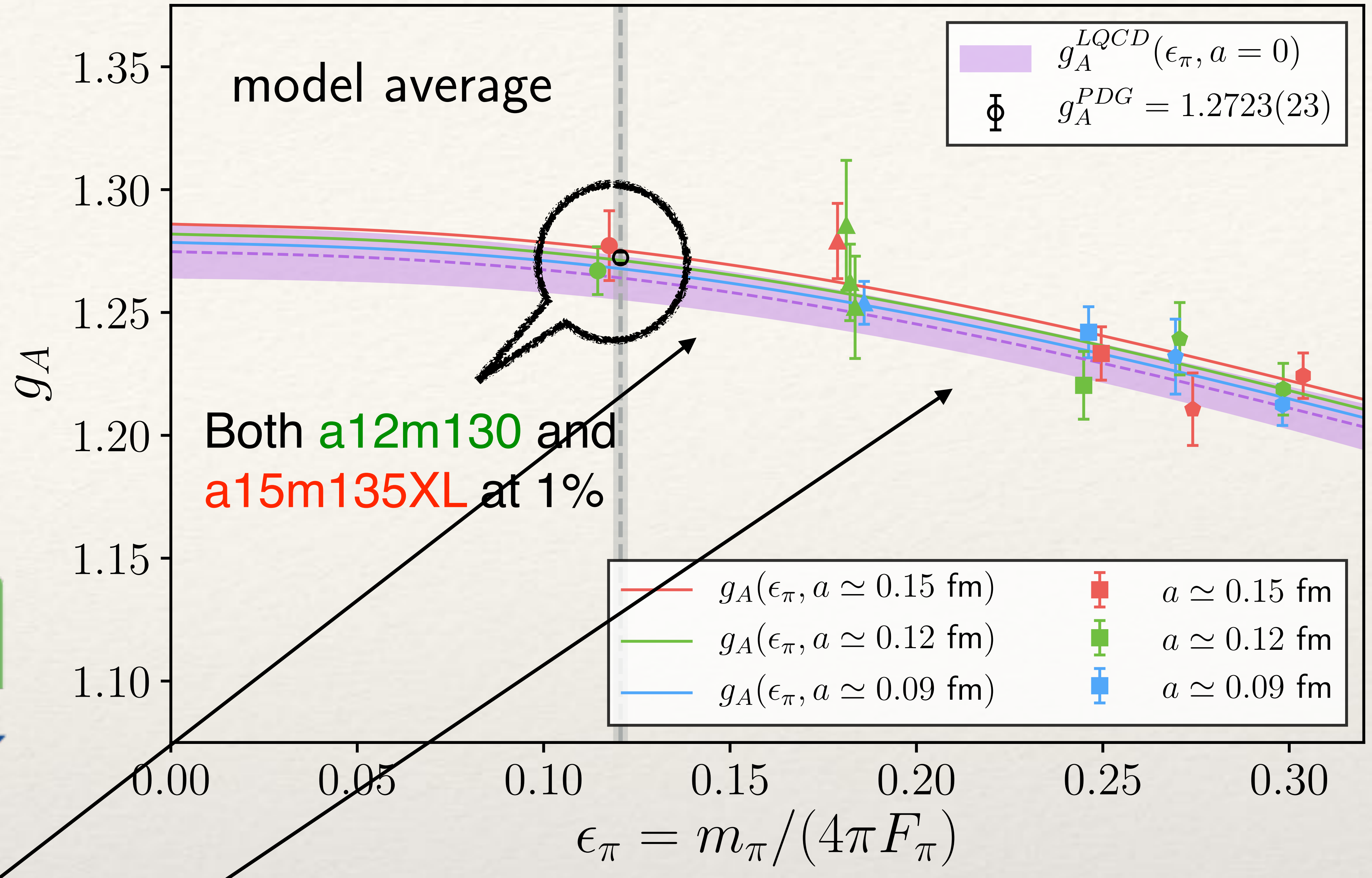
PRELIMINARY



California Lattice Collaboration

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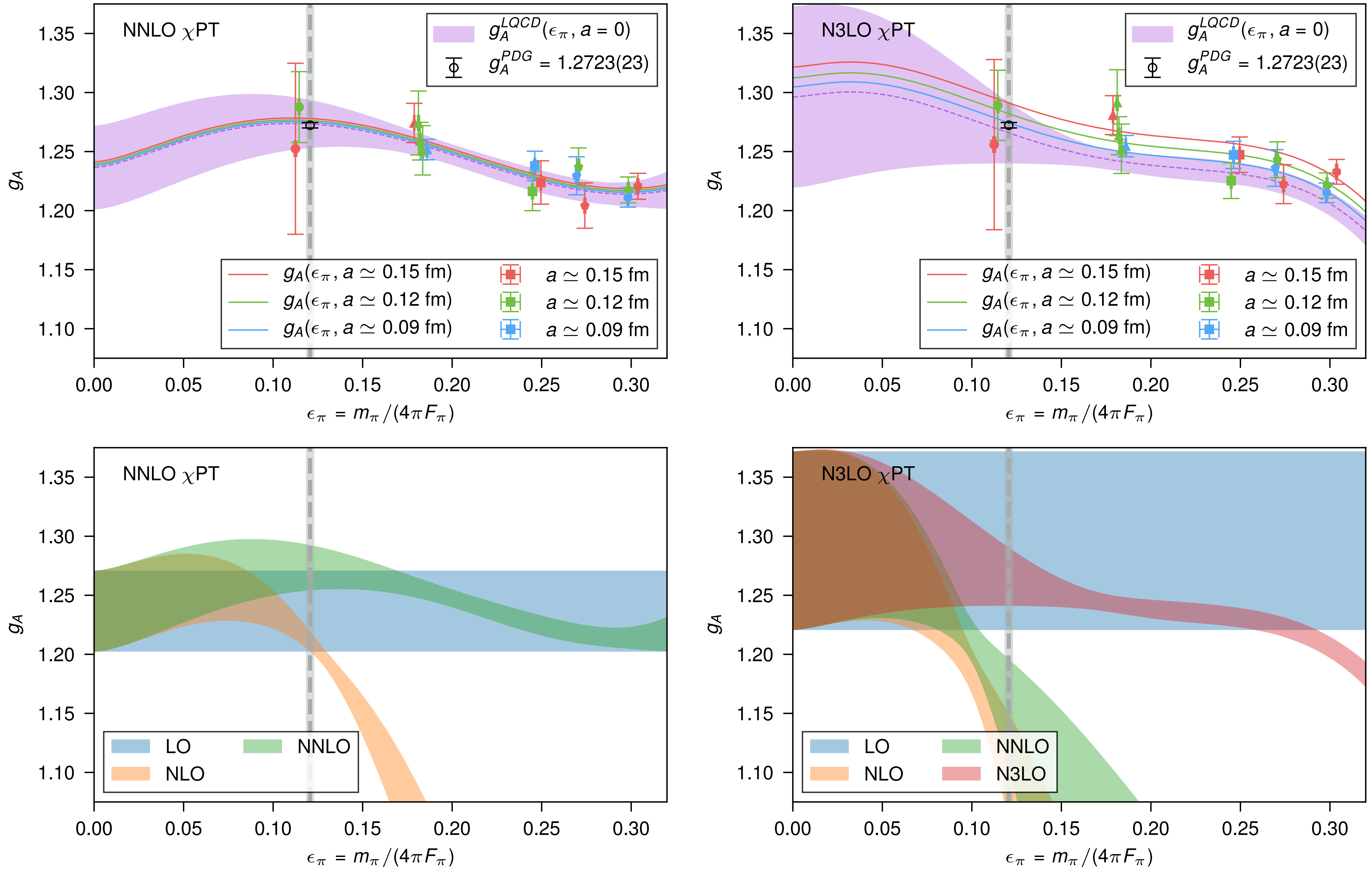
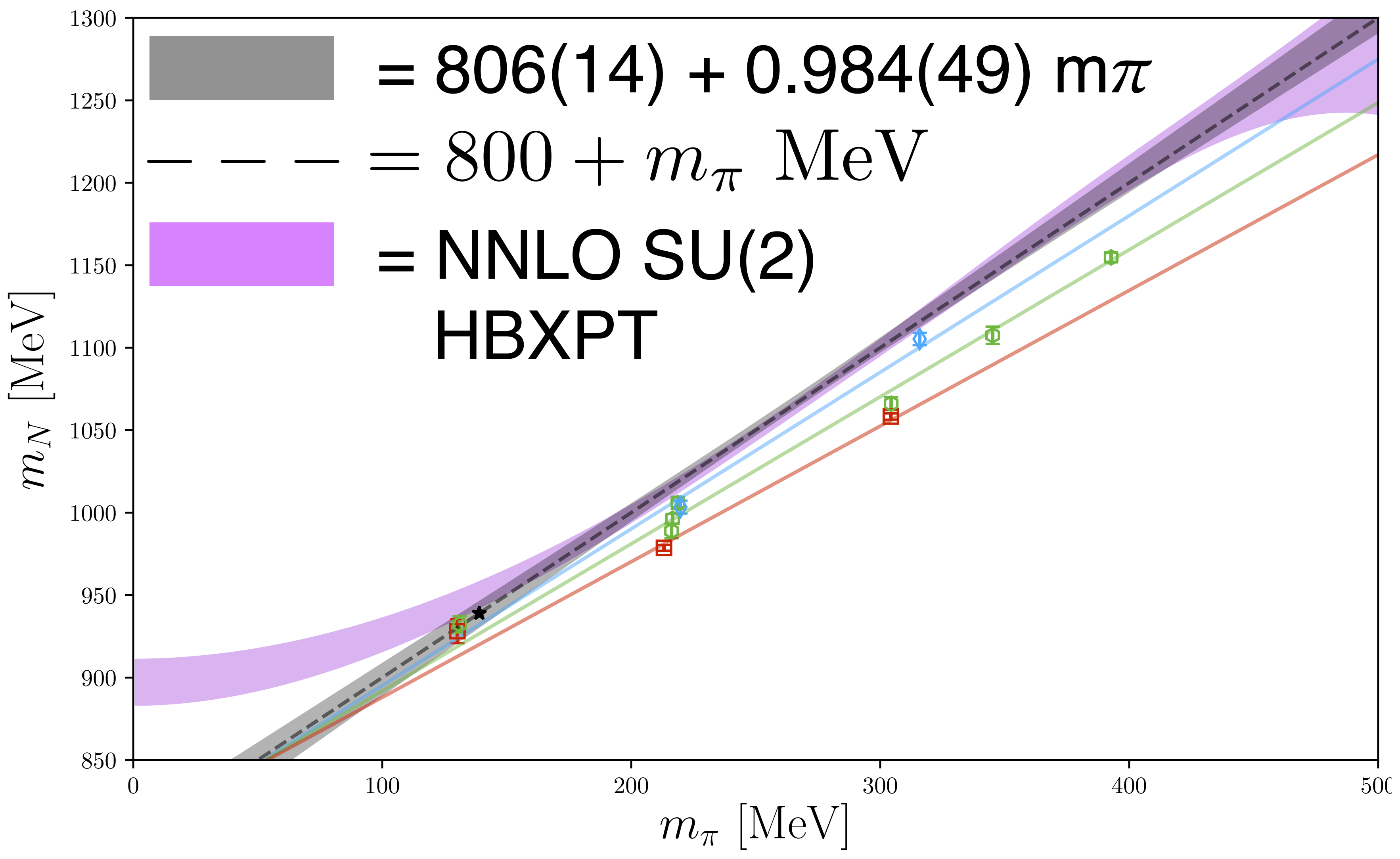
$$g_A = 1.2711(125) \rightarrow 1.2641(93) \quad [0.74\%]$$

- We have 2 additional pion masses (180, 260) and a 4th finer lattice spacing, a≈0.06fm @ $M\pi \approx 220, 310$ MeV
  - We anticipate improving g_A to ~0.5% — we need to address the radiative QED correction to make this useful

convergence of the chiral expansion…



PRELIMINARY 2019



- Chiral corrections to g_A from $SU(2)$ HBχPT($\not\Delta$) at the physical pion mass

N^nLO	LO	NLO	N^2LO	N^3LO
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- Worth noting - if you use $SU(2)$ HBχPT(Δ) and force the delta-axial couplings, the value of the pion-nucleon sigma term is also large
- large Nc gives de-coherent nucleon and delta loop corrections to g_A, but coherent to M_N
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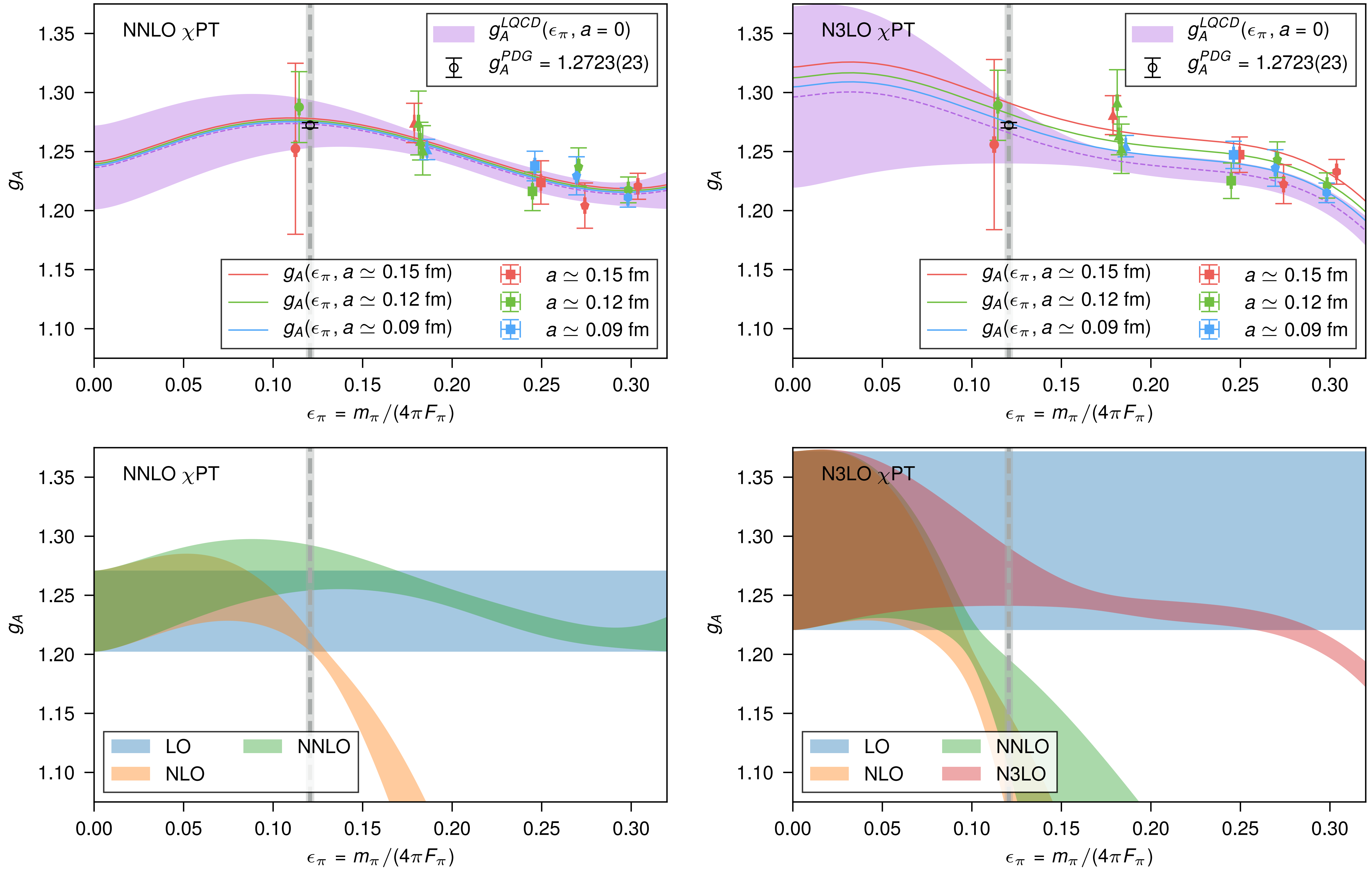

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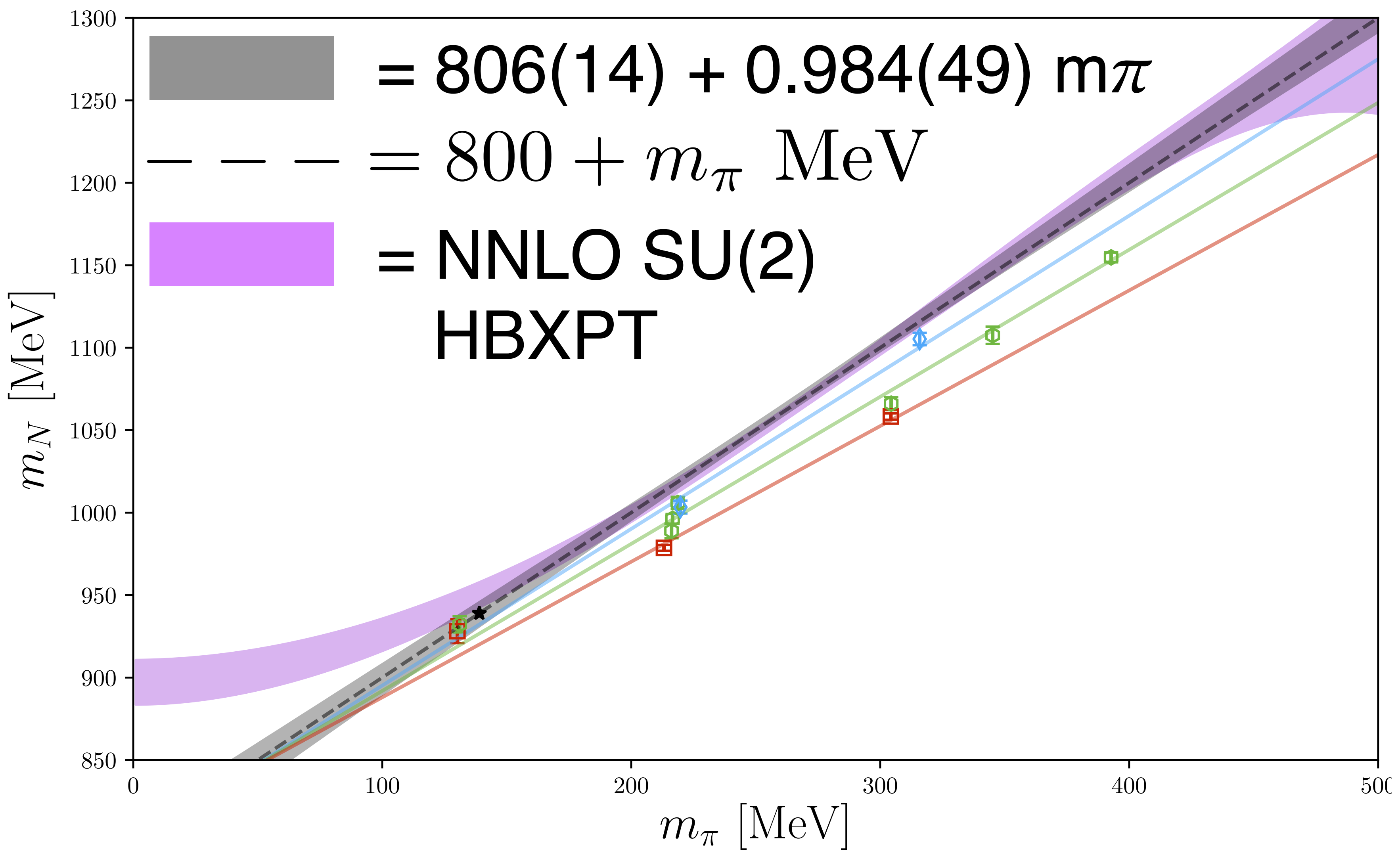

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- We need LQCD results with Δ to study convergence of SU(2) HBχPT (Δ) — πN scattering

- Worth noting - if you use $SU(2)$ HBχPT(Δ) and force the delta-axial couplings, the value of the pion-nucleon sigma term is also large
- large Nc gives de-coherent nucleon and delta loop corrections to g_A, but coherent to M_N
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πN scattering at $m_\pi \approx 200$ MeV

arXiv > hep-lat > arXiv:2208.03867

High Energy Physics – Lattice

[Submitted on 8 Aug 2022 (v1), last revised 7 Feb 2023 (this version, v3)]

Elastic nucleon-pion scattering at $m_\pi = 200$ MeV from lattice QCD

John Bulava, Andrew Hanlon, Ben Hörz, Colin Morningstar, Amy Nicholson, Fernando Romero-López, Sarah Skinner, Pavlos Vranas, André Walker-Loud

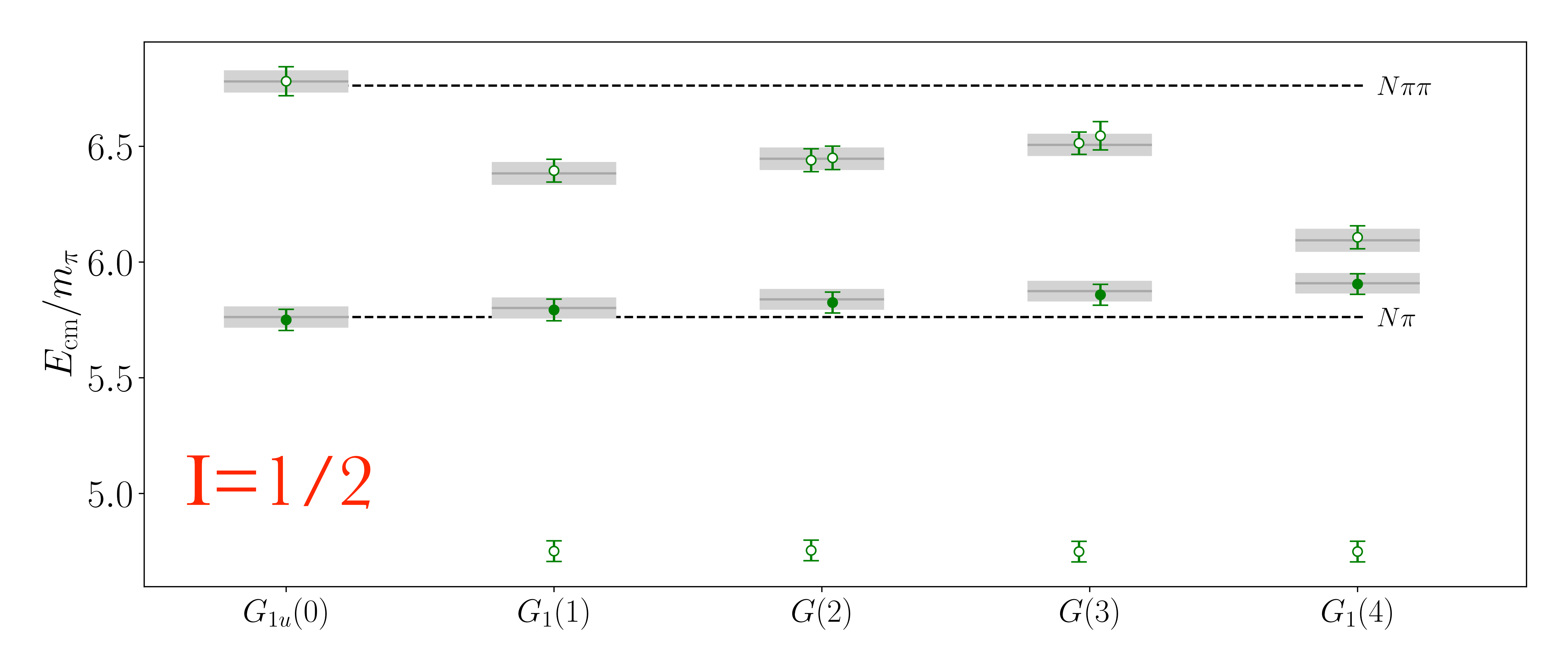
Nucl. Phys. B 987 (2023) 116105

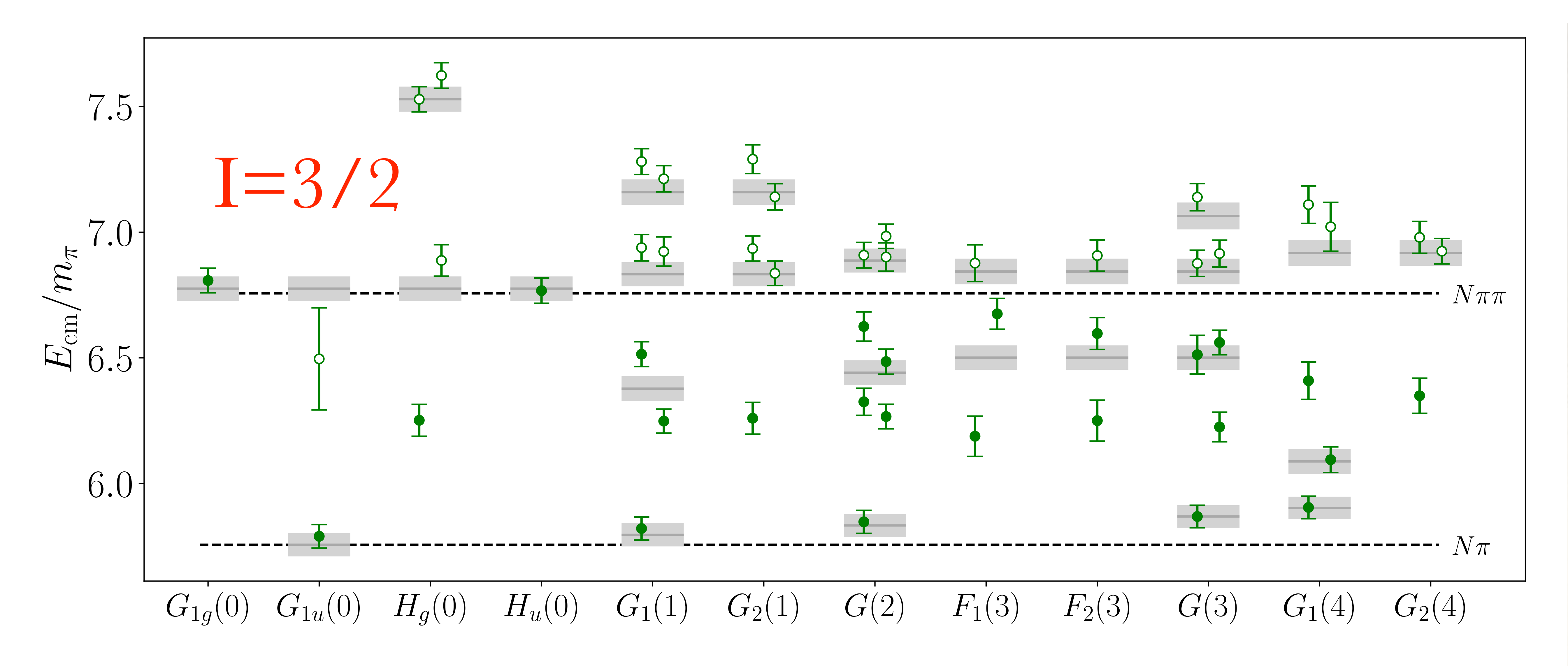
- Exciting in its own right
- Stepping stone towards NN (at this light pion mass)
- m_π is light enough that
  - the Δ is unstable
  - optimistic that EFT could be convergent-ish

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

Various irreps used to determine the spectrum

$\boldsymbol{d}$	Λ	dim.	contributing $(2J, \ell)^{n_{\mathrm{occ}}}$ for $\ell_{\max} = 2$
$(0,0,0)$	$G_{1\mathrm{u}}$	2	$(1,0)$
	$G_{1\mathrm{g}}$	2	$(1,1)$
	H_{g}	4	$(3,1)$, $(5,2)$
	H_{u}	4	$(3,2)$, $5,2)$
	$G_{2\mathrm{g}}$	2	$(5,2)$
$(0,0,n)$	G_1	2	$(1,0)$, $(1,1)$, $(3,1)$, $(3,2)$, $(5,2)$
	G_2	2	$(3,1)$, $(3,2)$, $(5,2)^2$
$(0,n,n)$	G	2	$(1,0)$, $(1,1)$, $(3,1)^2$, $(3,2)^2$, $(5,2)^3$
(n,n,n)	G	2	$(1,0)$, $(1,1)$, $(3,1)$, $(3,2)$, $(5,2)^2$
	F_1	1	$(3,1)$, $(3,2)$, $(5,2)$
	F_2	1	$(3,1)$, $(3,2)$, $(5,2)$





Note: the gray bands and green energy levels are correlated, which is not reflected visually in the plots

Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

- FV Spectrum to Scattering Amplitudes [Lüscher, … many others]

  $$\det[\tilde{K}^{-1}(E_{\rm cm}) - B^{\boldsymbol{P}}(E_{\rm cm})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$$

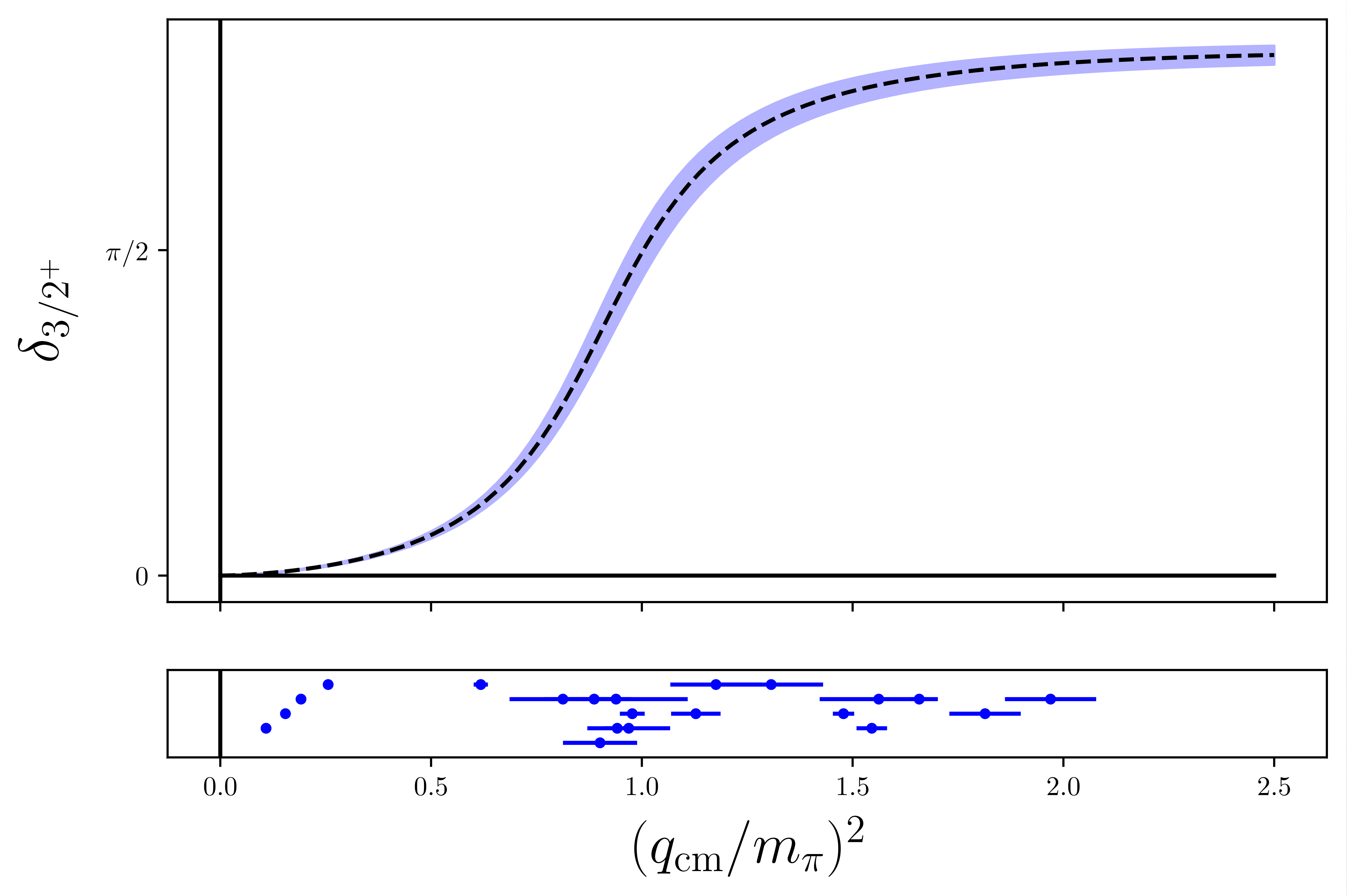
- $\tilde{K}$ proportional to the K-matrix
- $B^{\rm P}(E_{\rm cm})$ is the "Box Matrix" that encodes information about the finite-volume and BCs

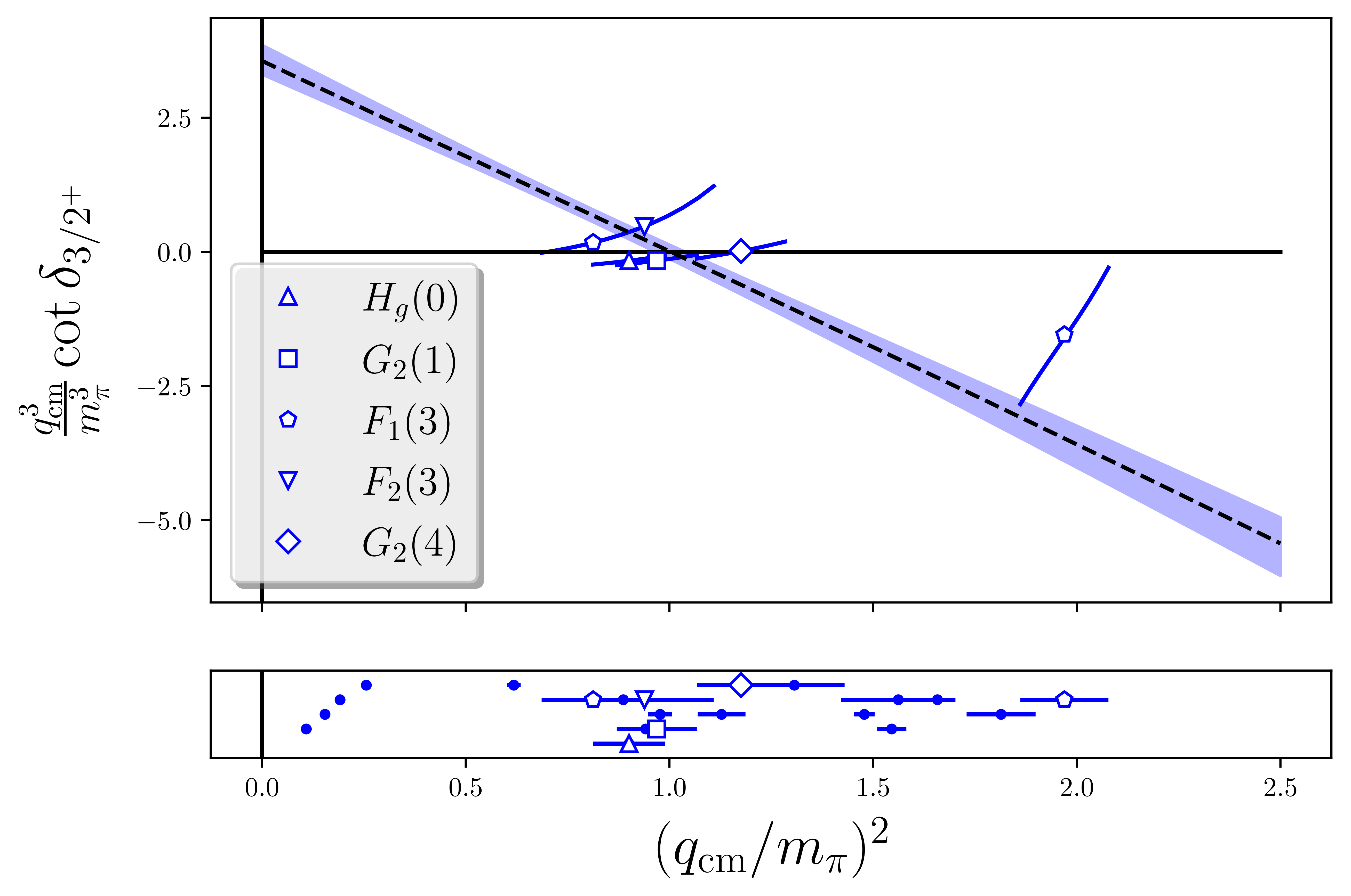
- Solving this expression is equivalent to looking for poles in a coupled-channel scattering amplitude
  - for a single channel

$$p\cot\delta - ip = 0 \longrightarrow p\cot\delta - \frac{1}{\pi L}\lim_{\Lambda\to\infty}\left(\sum_{|\vec{n}|<\Lambda}\frac{1}{|\vec{n}|^2 - \frac{p^2L^2}{4\pi^2}} - 4\pi\Lambda\right) = 0$$

Elastic nucleon-pion scattering at M$\pi \approx$ 200 MeV from lattice QCD [2208.03867]

FV Spectrum to Scattering Amplitudes - spectrum method comparison - resulting amplitude





I=3/2 fit using s- and p-wave approximation

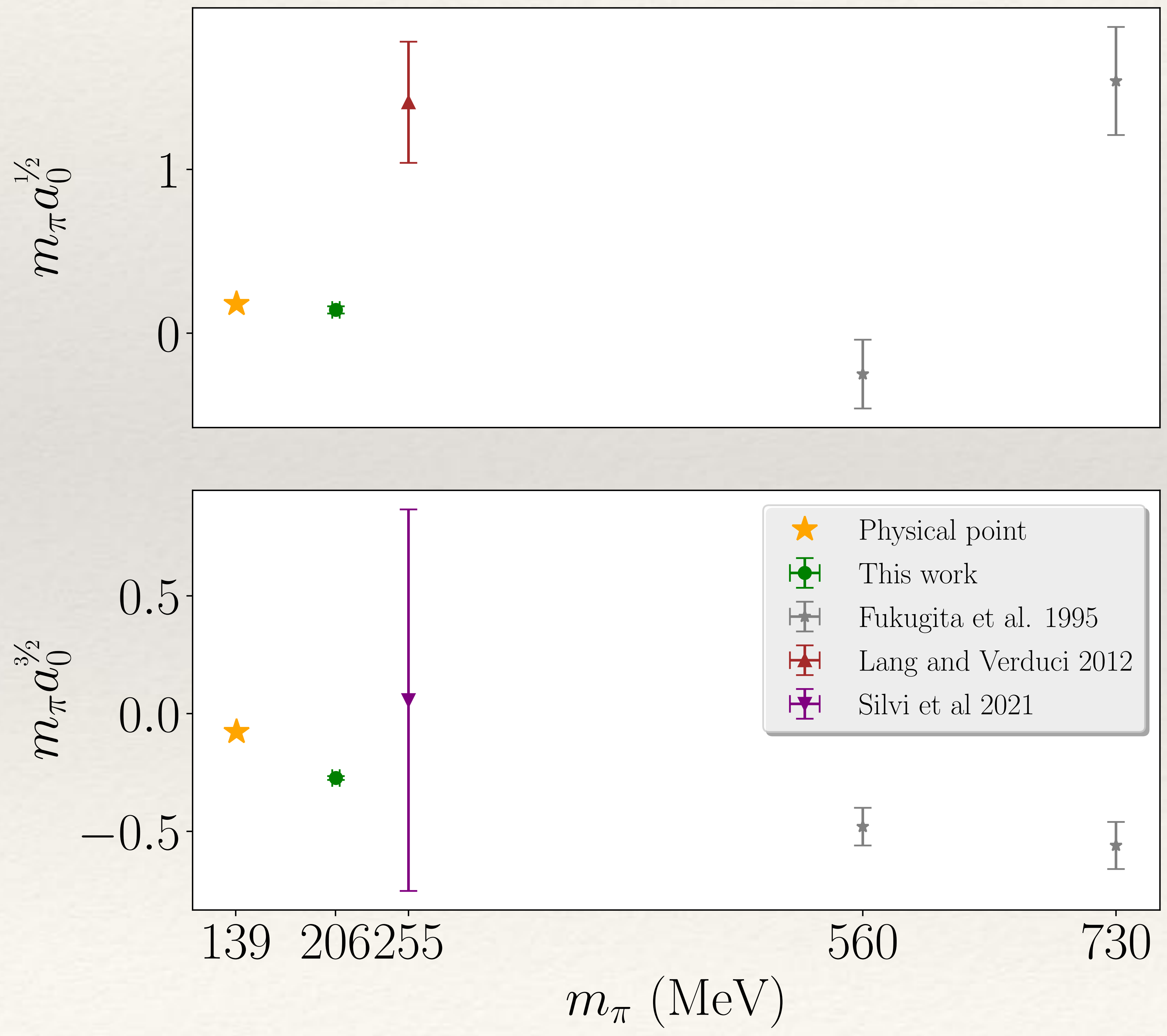
open symbol: contributes to single partial wave
closed symbol: contributes to both partial waves

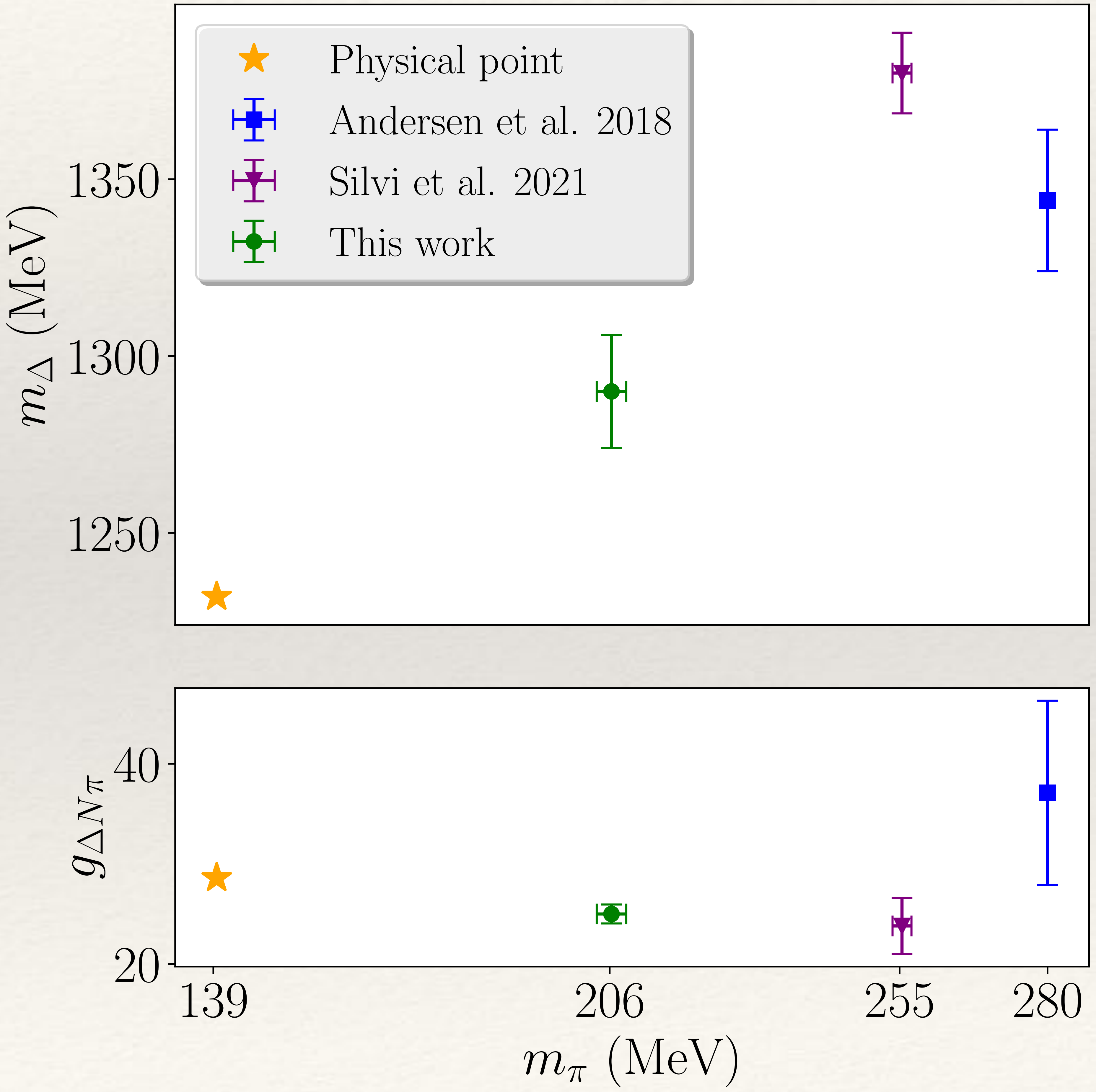
Elastic nucleon-pion scattering at $M\pi \approx 200$ MeV from lattice QCD

Results for scattering lengths and effective Delta-resonance parameters

$$m_\Delta = 1268(17)\ \text{MeV} \qquad \frac{m_\Delta}{m_\pi} = 6.257(35), \qquad g_{\Delta N\pi} = 14.41(53)$$

$$m_\pi a_0^{3/2} = -0.2735(81)\,, \qquad m_\pi a_0^{1/2} = 0.142(22),$$





Compare with χPT

- The formula for the scattering length are known at 4th order in the chiral expansion (w/o Δ)
  - They are expressed in terms of what is called scalar and vector scattering lengths

$$a_0^{3/2} = a_0^+ - a_0^- \,, \qquad a_0^{1/2} = a_0^+ + 2a_0^-$$

  - At NLO, these are given by

- Hoferichter et al, 1510.06039, Hoferichter et al, 1507.07552
- Fettes, Meissner [Steininger] [hep-ph/9803266] hep-ph/0002162

$$m_\pi a_0^{3/2}[\mathrm{NLO}] = -\epsilon_\pi^2 \frac{2\pi}{1+\mu}\left\{1 + \frac{\epsilon_\pi}{2}\frac{\Lambda_\chi}{m_N}(g_A^2 + 8C)\right\},$$

$$m_\pi a_0^{1/2}[\mathrm{NLO}] = \epsilon_\pi^2 \frac{2\pi}{1+\mu}\left\{1 - \frac{\epsilon_\pi}{4}\frac{\Lambda_\chi}{m_N}(g_A^2 + 8C)\right\},$$

```
    COMPARISON of C = mN * (2c1 - c2 - c3)
order   pheno          D200 Fit
nlo      0.300(24)    0.648(62)
n2lo    -0.019(24)           NA
n3lo     0.244(29)           NA
```

$$C = M_N(2c_1 - c_2 - c_3)$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}, \quad \mu = \frac{m_\pi}{M_N}, \quad \Lambda_\chi = 4\pi F_\pi$$

$$\epsilon_\pi^{\mathrm{D200}} = 0.1759(12), \qquad \mu^{\mathrm{D200}} = 0.2102(19),$$

$$\epsilon_\pi^{\mathrm{phys}} = 0.12064(74), \qquad \mu^{\mathrm{phys}} = 0.14875(05)$$

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```

	m_π (MeV)	$m_\pi a_0^{1/2}$	$m_\pi a_0^{3/2}$
This work	200	0.142(22)	−0.2735(81)
LO χPT	200	0.321(04)(57)	−0.161(02)(28)
LO χPT	140	0.159(02)(19)	−0.080(01)(10)
Pheno. (isospin limit)[27]	140	0.1788(38)	−0.0775(35)

Outlook

- There is a growing body of LQCD evidence that SU(2) baryon χPT is not converging @ m_π^{phys}
  - nucleon mass: convergent — adding Δ may make it marginally convergent
  - g_A: not convergent — adding Δ may make it convergent
  - πN scattering lengths: seemingly very different @ $m_\pi \approx 200$ MeV than @ m_π^{phys}

- We are gearing up to perform LQCD calculations with Δ-dof to be able to determine all relevant LECs with LQCD results and not have to rely upon phenol-extractions
  - This will likely take 2-3 years
  - This will enable a QCD determination of the convergence pattern of SU(2) baryon χPT (Δ)

- What additional observables/tests would you like to see to settle this convergence/non-convergence of SU(2) baryon χPT?

- If SU(2) baryon χPT is non-convergent — what does this mean about NN EFT with dynamical pions?
  - It seems to me that this would invalidate a critical foundation of "chiral EFT"

- We (the community) often present EFT as better than models
  - This is true — provided the EFT is converging fast enough (if at all)
  - LQCD is maturing to the point where we can really map out the convergence pattern/radius of nuclear EFTs
  - This scrutiny is essential for us to truly quantify our EFT uncertainties

Thank You

Collaborators

CoSMoN
(Connecting the Standard Model to Nuclei)

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