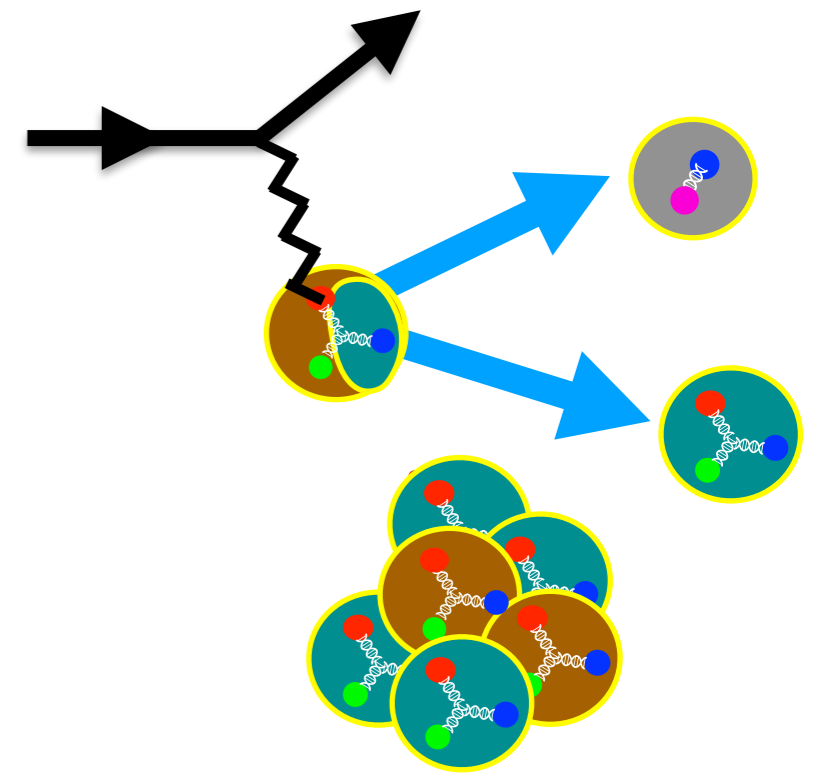
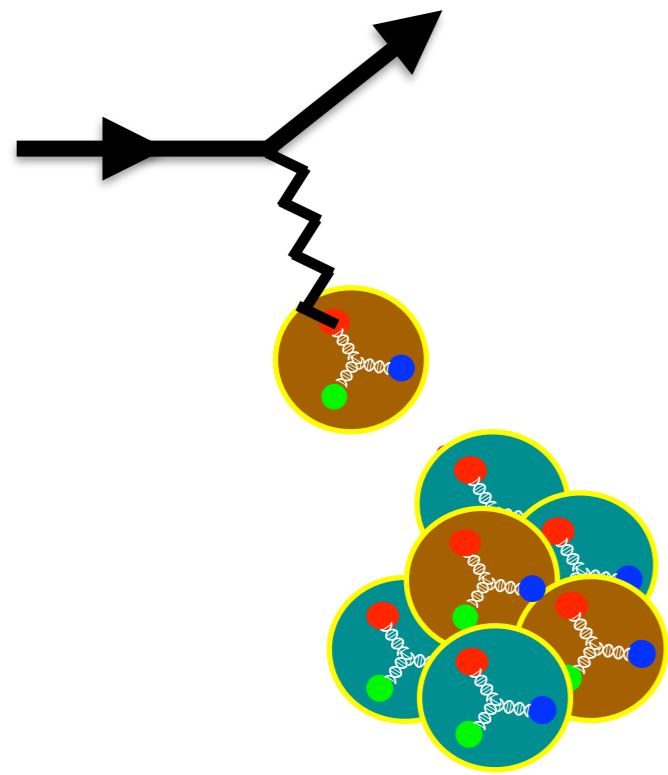


Neutrino-nucleus scattering and lattice QCD



Michael Wagman

INT Program 23-1b

New physics searches at the precision frontier

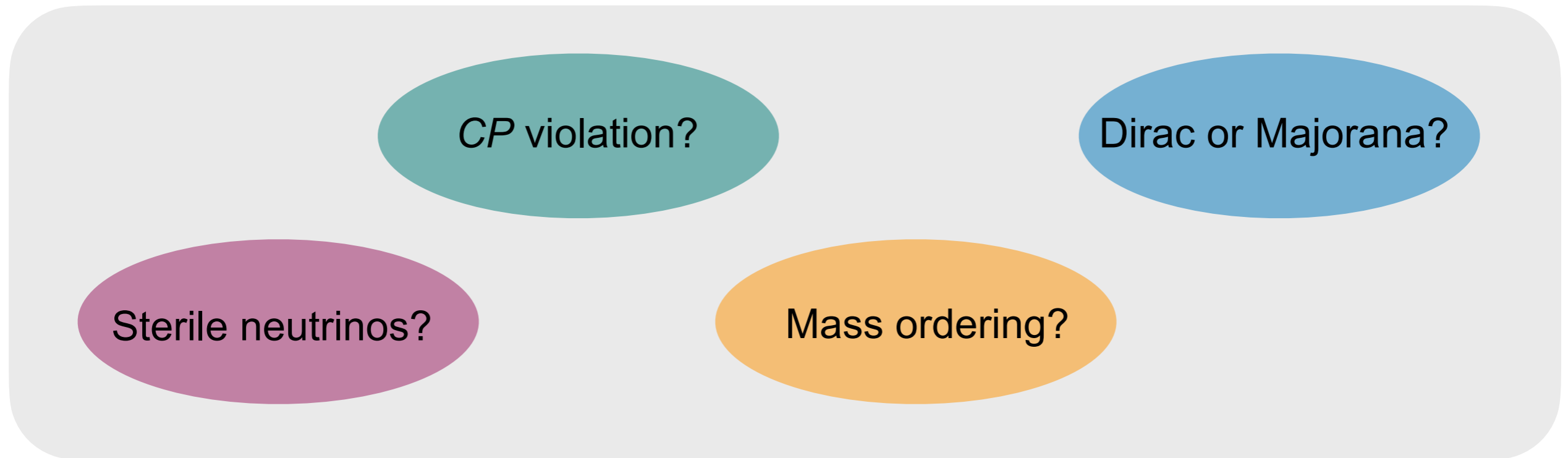


Fermilab

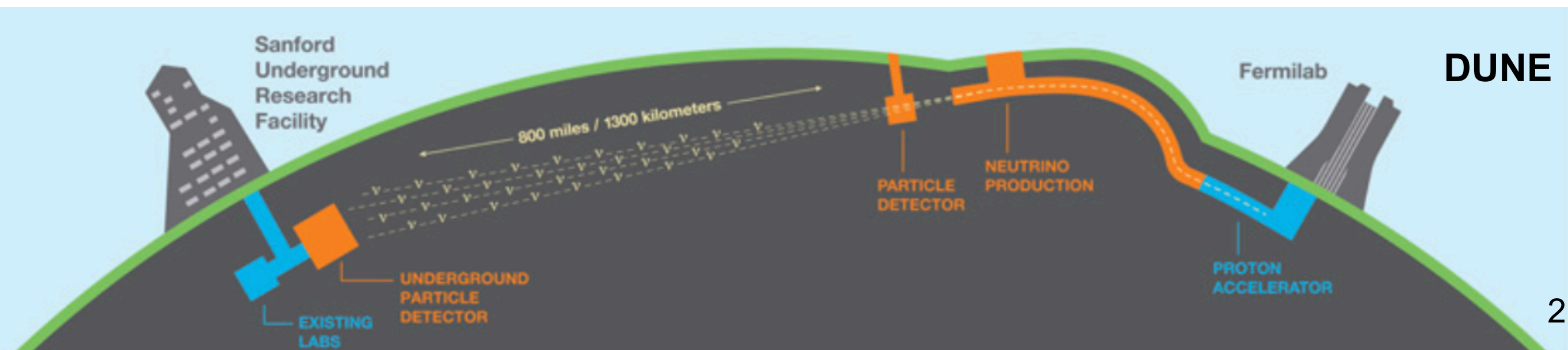
May 9, 2023

Neutrinos and new physics

Neutrino oscillations demonstrate that neutrinos have mass, but the fundamental interactions giving rise to neutrino masses are not well understood

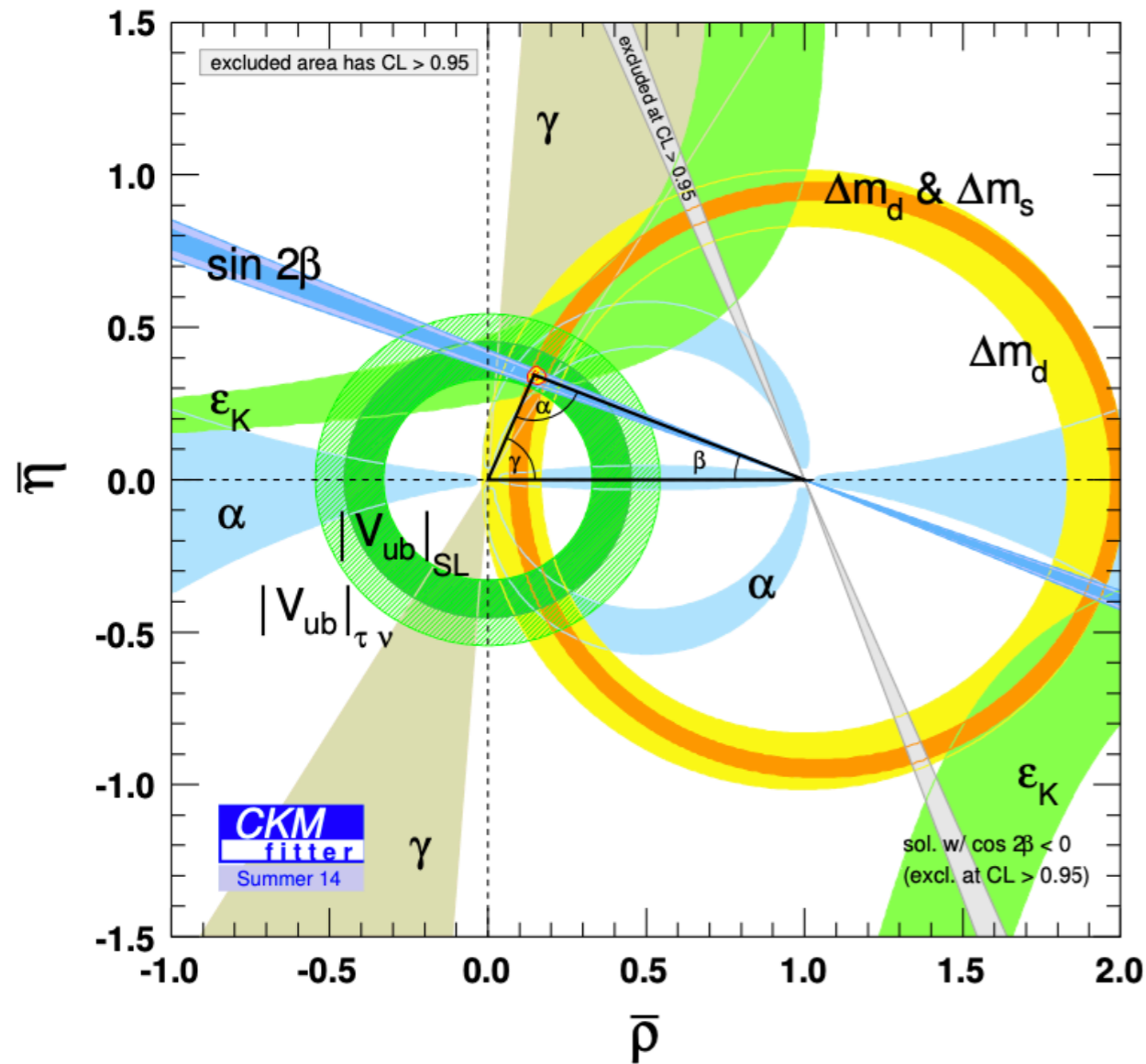


Next-generation neutrino experiments will measure oscillation parameters with unprecedented accuracy to shed light on neutrino masses

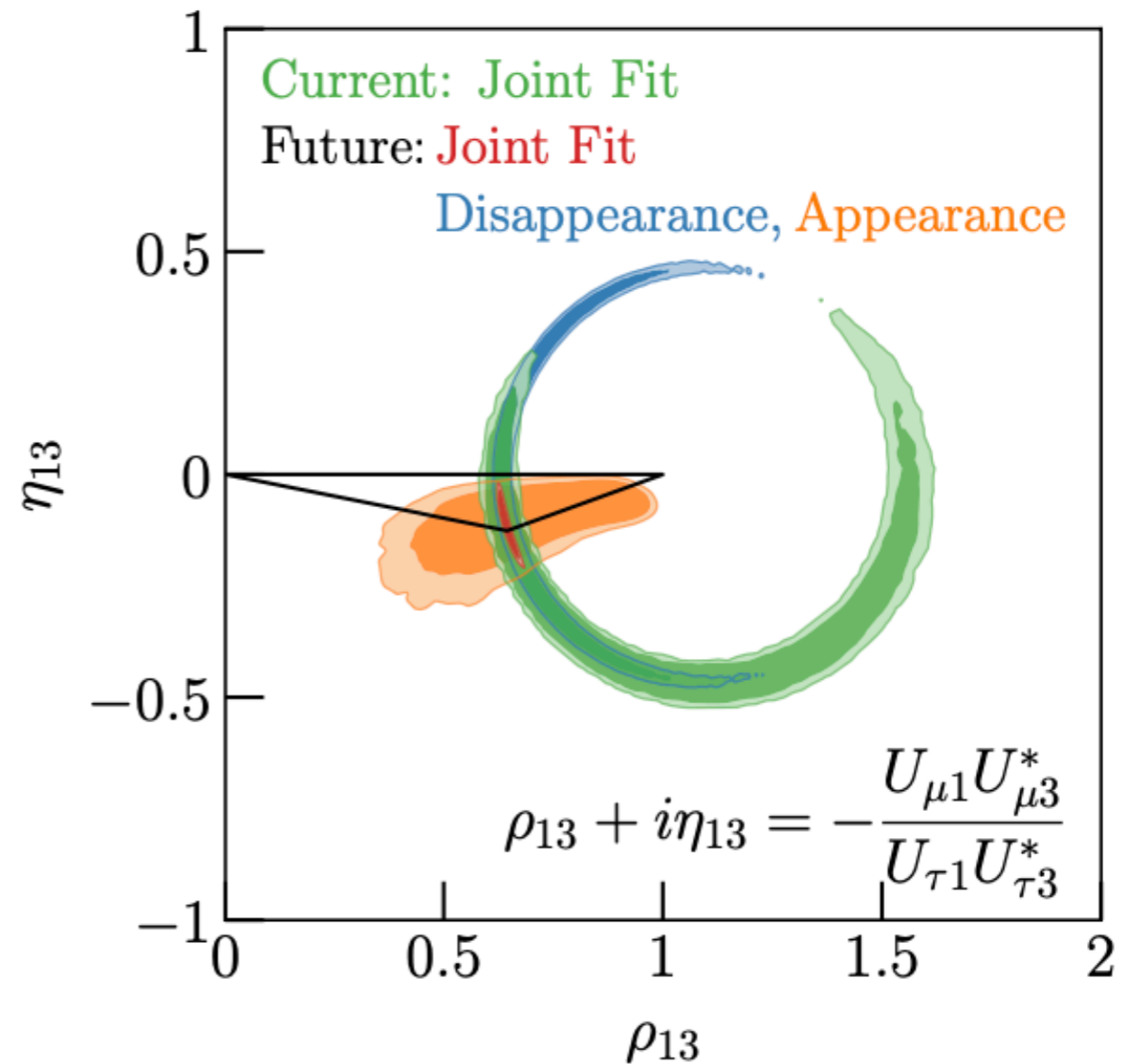


Precision neutrino physics

Next-generation experiments aim to take measurements of neutrino mixing angles from O(10%) to O(1%) precision



Charles et al [CKMfitter], Phys. Rev. D 91 (2015)

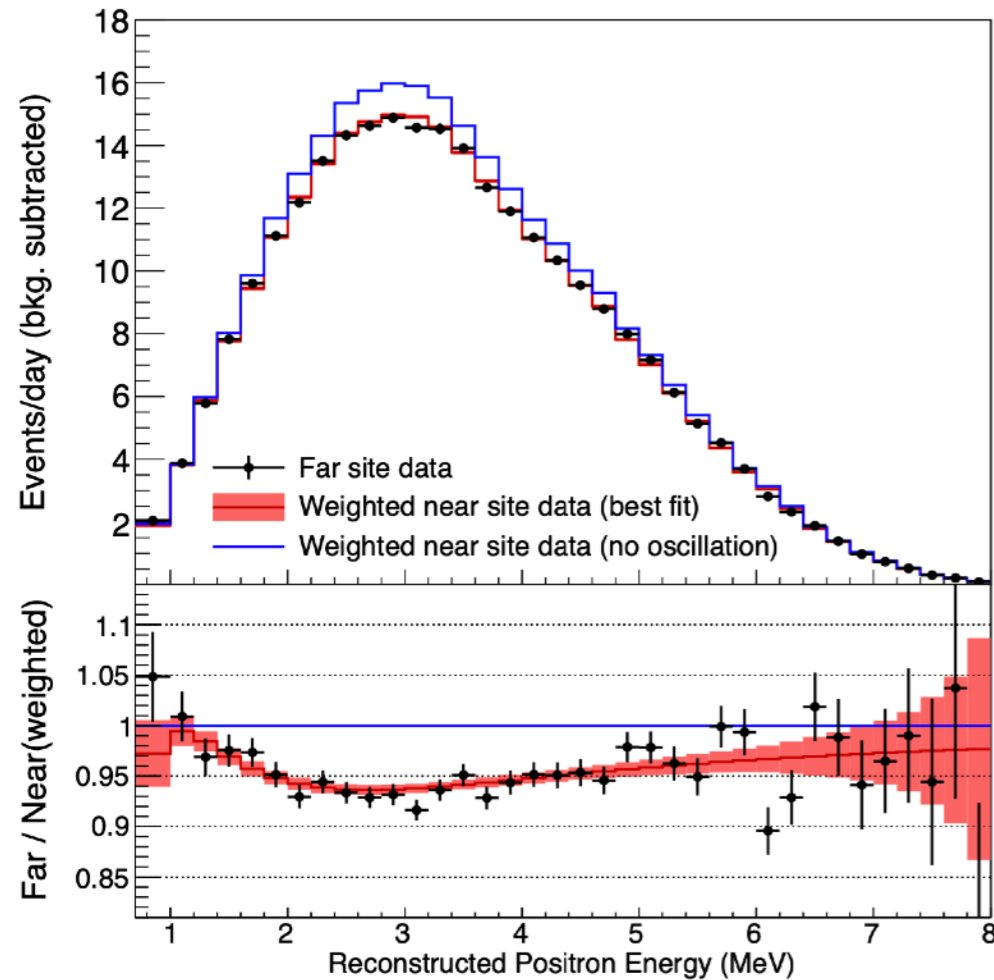


Ellis, Kelly, and Li, Phys. Rev. D 102 (2020)

Are there more surprises hiding in the neutrino sector?

Neutrino oscillations and CP

An et al [Daya Bay], PRL 115 (2015)



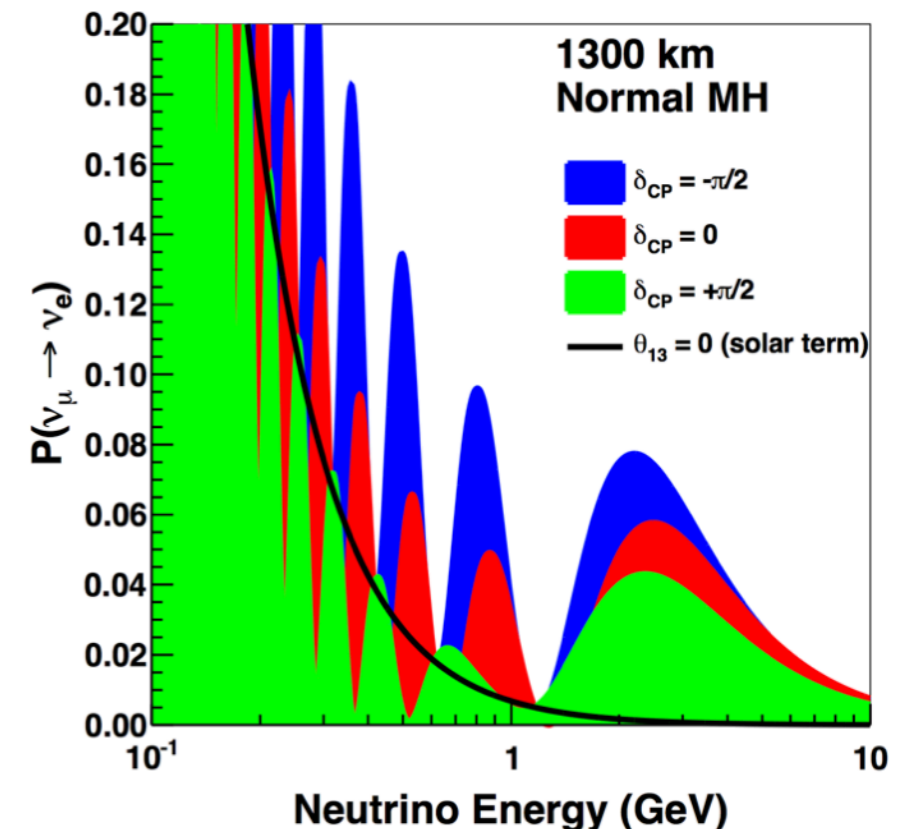
Accelerator / reactor neutrino experiments have found clear evidence of oscillations by comparing near + far detector fluxes

Experiment	Dominant	Important
Solar Experiments	θ_{12}	$\Delta m_{21}^2, \theta_{13}$
Reactor LBL (KamLAND)	Δm_{21}^2	θ_{12}, θ_{13}
Reactor MBL (Daya-Bay, Reno, D-Chooz)	$\theta_{13}, \Delta m_{31,32}^2 $	
Atmospheric Experiments (SK, IC-DC)		$\theta_{23}, \Delta m_{31,32}^2 , \theta_{13}, \delta_{CP}$
Accel LBL $\nu_{\mu}, \bar{\nu}_{\mu}$, Disapp (K2K, MINOS, T2K, NO ν A)	$ \Delta m_{31,32}^2 , \theta_{23}$	
Accel LBL $\nu_e, \bar{\nu}_e$ App (MINOS, T2K, NO ν A)	δ_{CP}	θ_{13}, θ_{23}

Zyla et al [PDG], PTEP 2020 (2020)

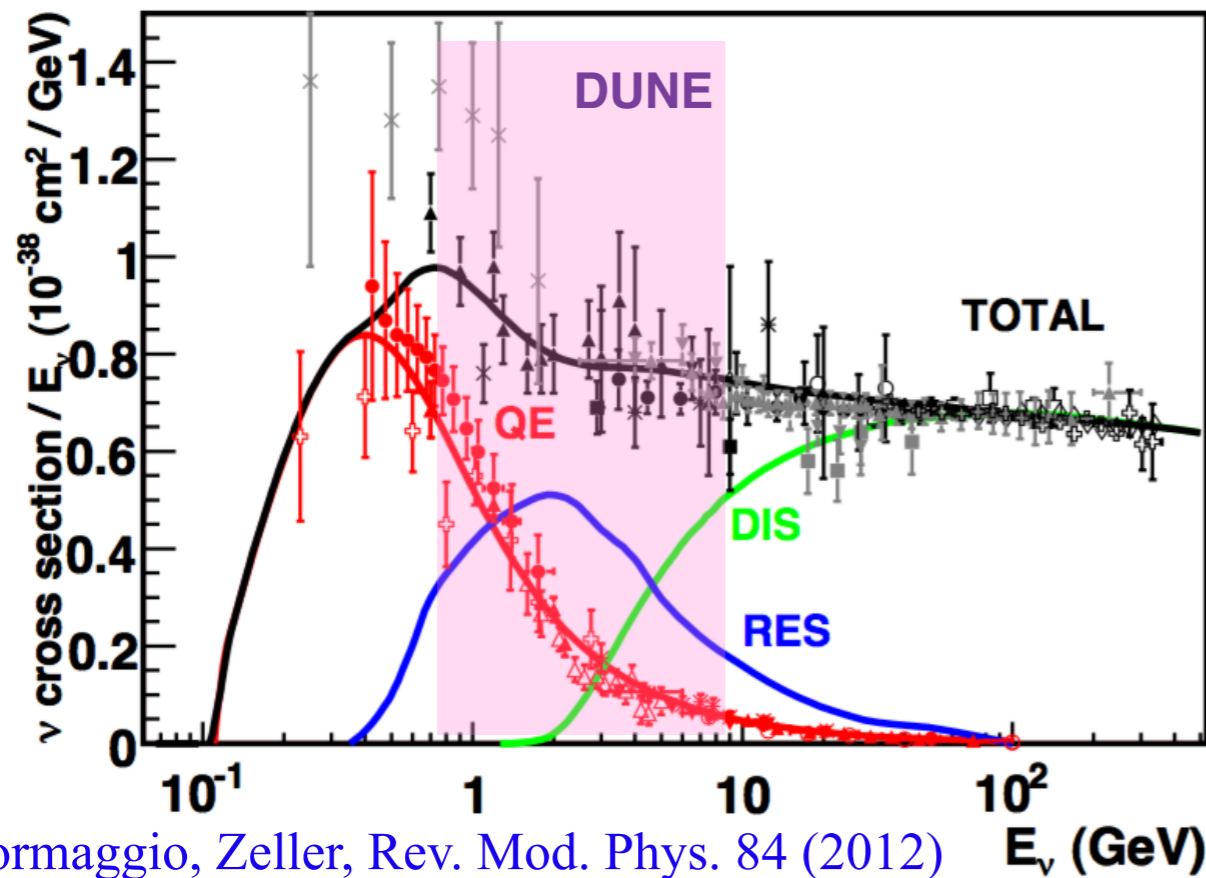
Future DUNE and T2HK experiments aim to precisely constrain subtle features of oscillations sensitive to leptonic CP violation

- Precise and accurate cross section predictions needed to relate near and far detector fluxes will be critical



Acciarri et al [DUNE], arXiv 1512.06148

Neutrino-nucleus scattering



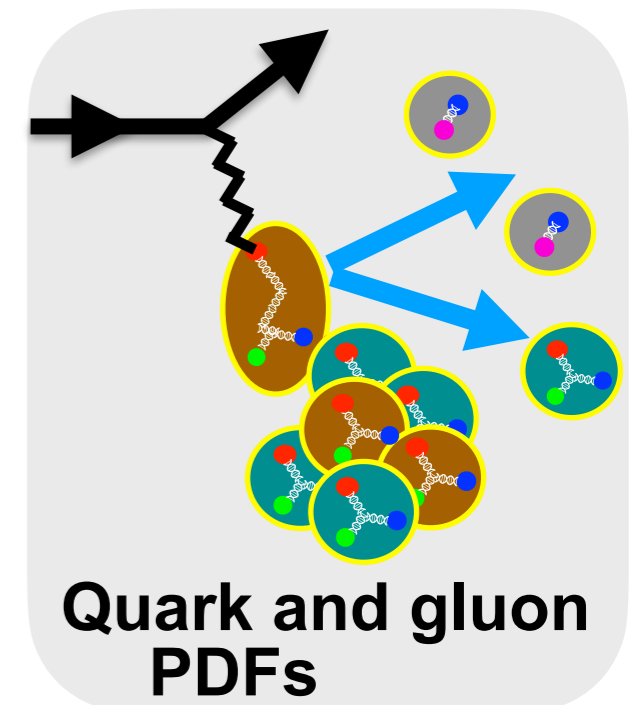
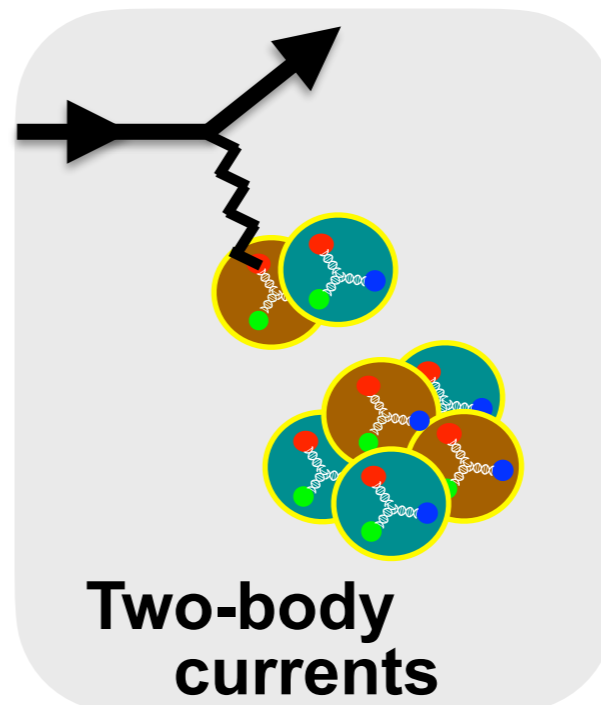
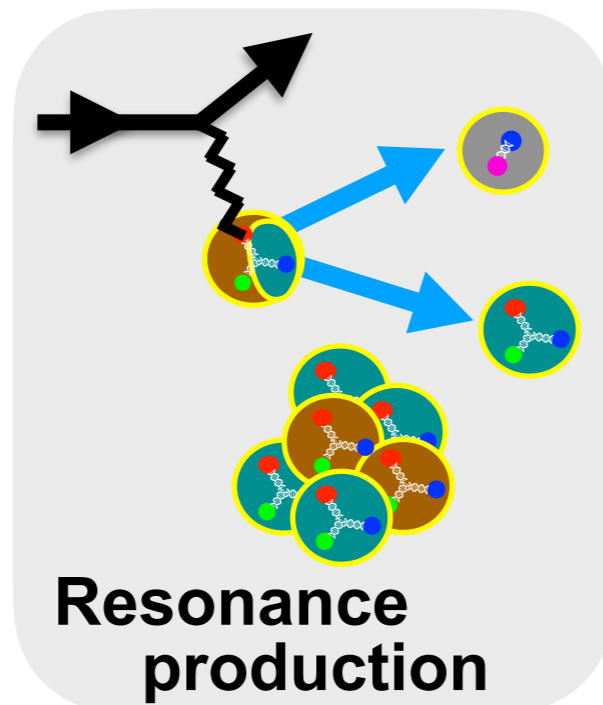
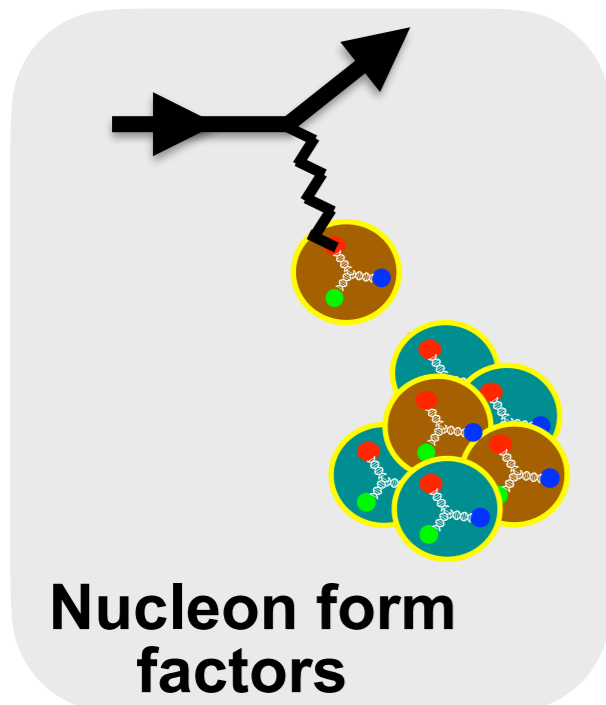
Formaggio, Zeller, Rev. Mod. Phys. 84 (2012)

Accelerator neutrino fluxes cover a wide range of energies where different processes dominate cross-section:

- Quasi-elastic nucleon scattering
- Resonance production
- Deep inelastic scattering

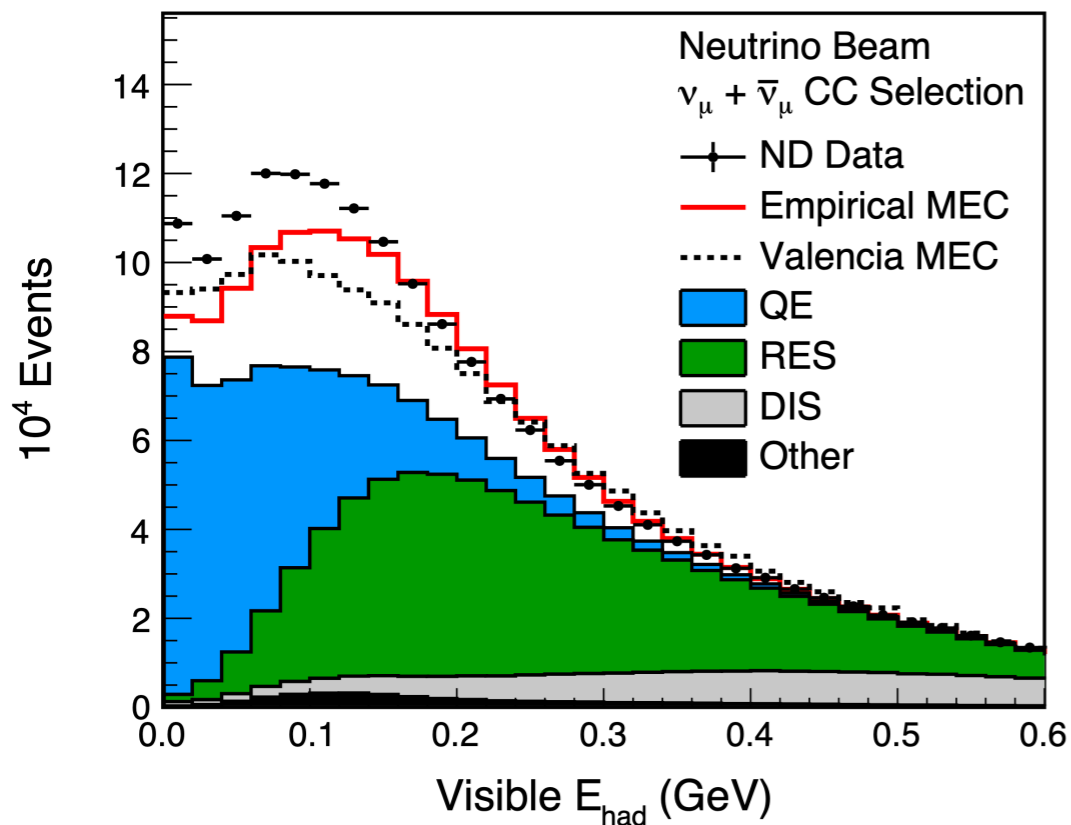
Theory input required to decompose cross section into such processes and therefore predict its energy dependence

Effective theories for different energies require different inputs



Event generators and tuning

Acero et al [NOvA] Eur. Phys. J. C 80 (2020)



Event generators combine models for different reaction mechanisms to make predictions of experimentally observed processes

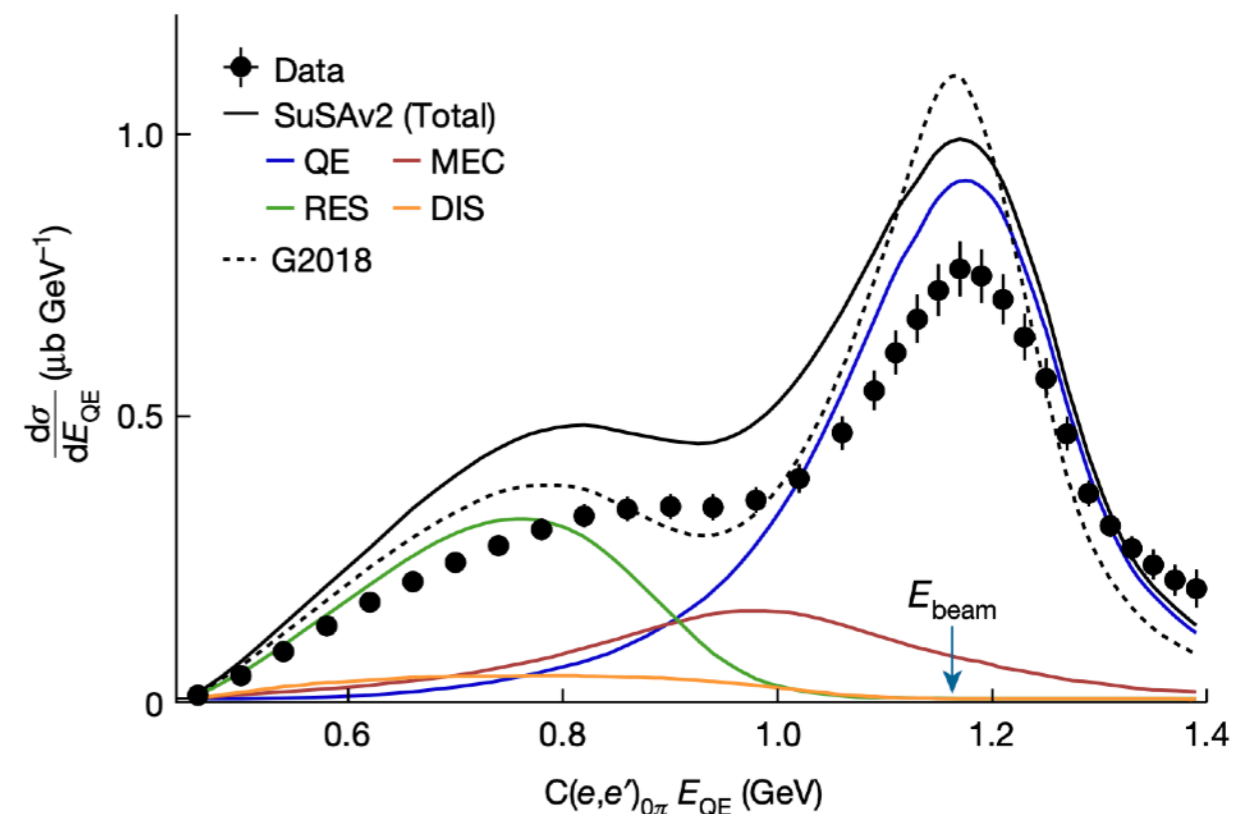
Review: Campell et al, arXiv:2203.11110

Discrepancies between generators and data often corrected by tuning an empirical model of the least well known mechanism: MEC (“meson exchange”/two-body currents)

Neutrino event generators can be validated by comparing predictions for electron scattering with precise data

- Significant discrepancies presently visible
- **Tuning to reproduce one process does not mean other processes/energies will be accurately predicted**
- Contributions from different reaction mechanisms must be isolated

Khachatryan et al [Clas and e4v] Nature 599 (2021)



Tuning can obscure new physics

New physics and νA cross-section both affect shape of far detector flux

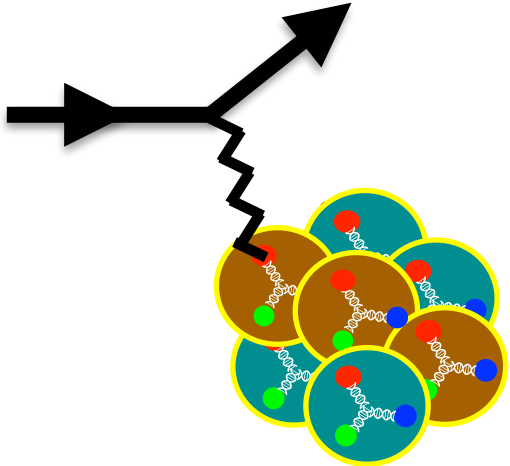
$$\frac{N_{\text{near}}}{N_{\text{far}}} = \frac{\int dE_\nu \Phi_{\text{near}}(E_\nu) \sigma(E_\nu)}{\int dE_\nu \Phi_{\text{far}}(E_\nu) \sigma(E_\nu)}$$

Near-detector neutrino flux and acceptance

Cross-section

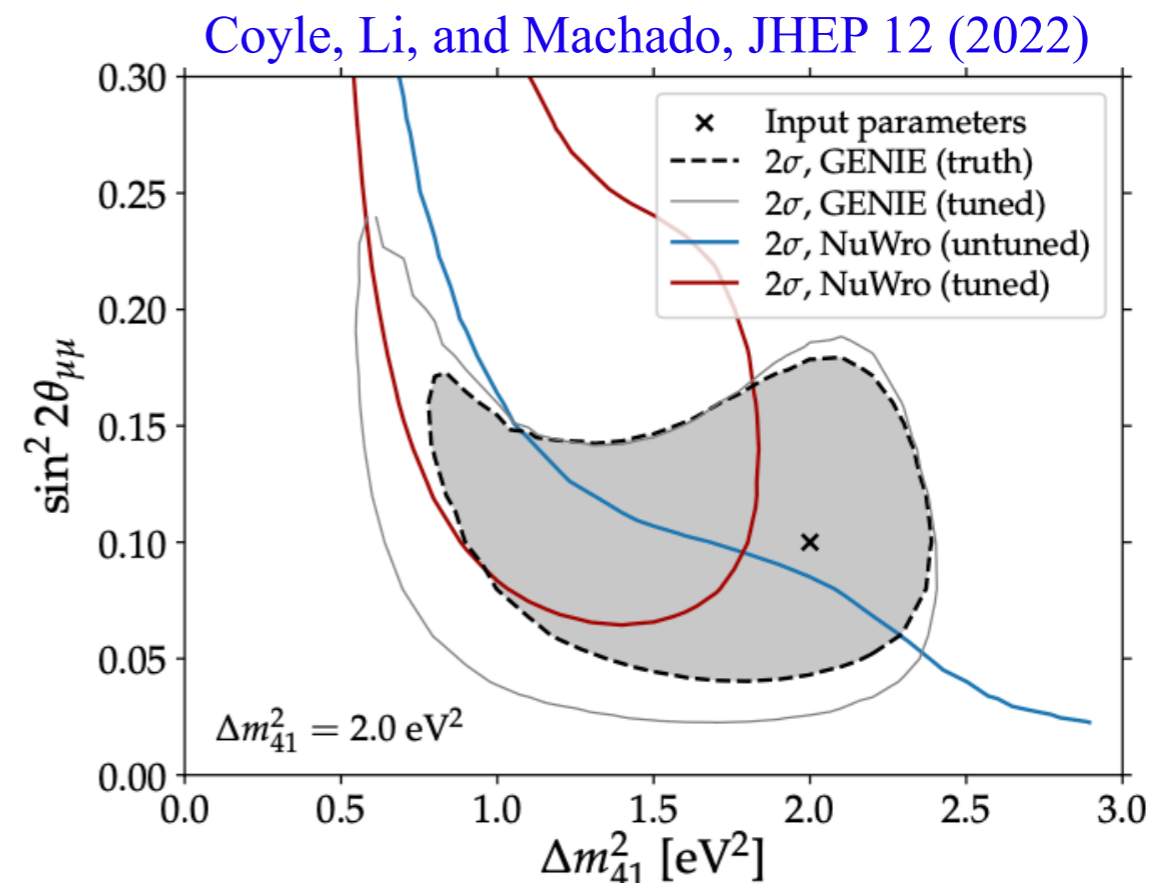
Far-detector flux (depends on oscillation parameters)

Experimentally measured event rates



Inaccuracies in cross-section modeling can distort signatures of new physics such as oscillations into sterile neutrinos

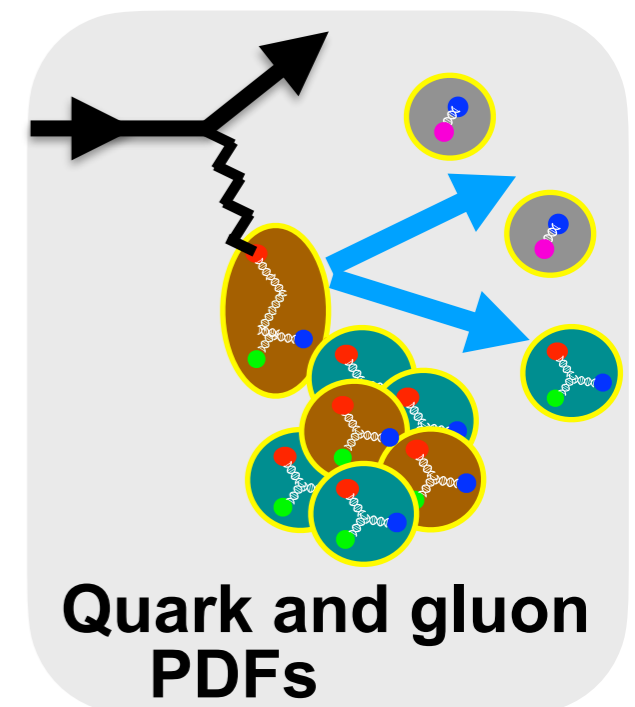
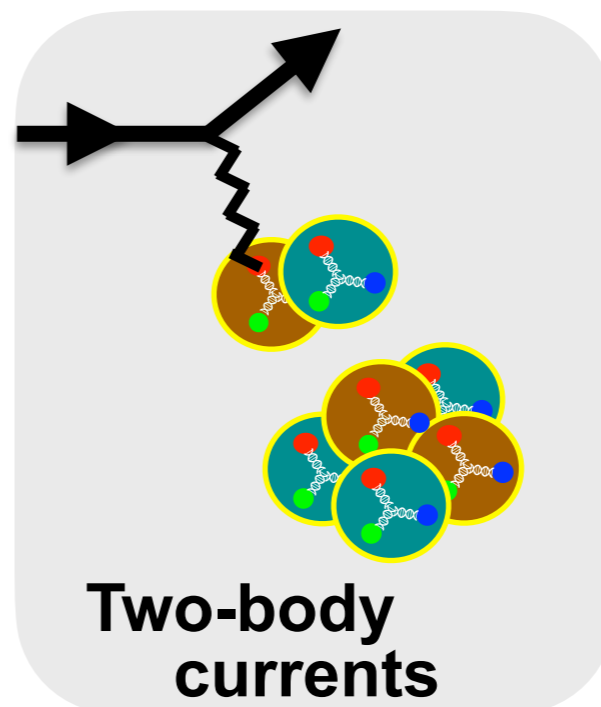
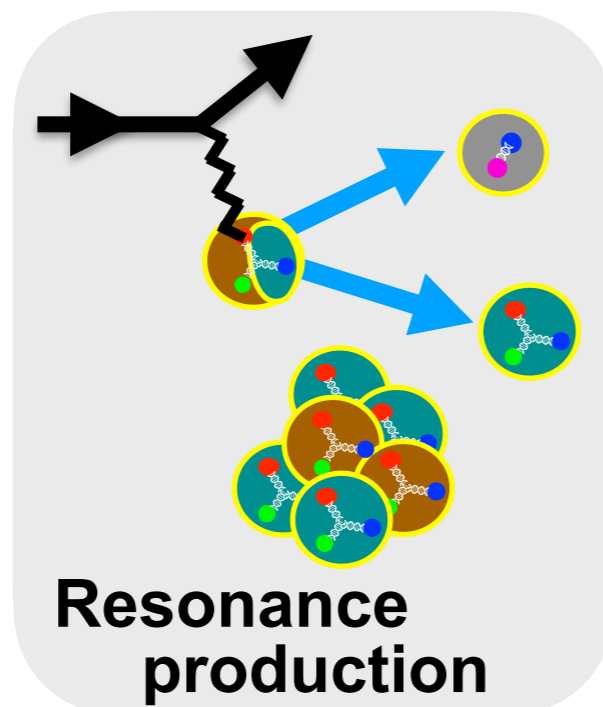
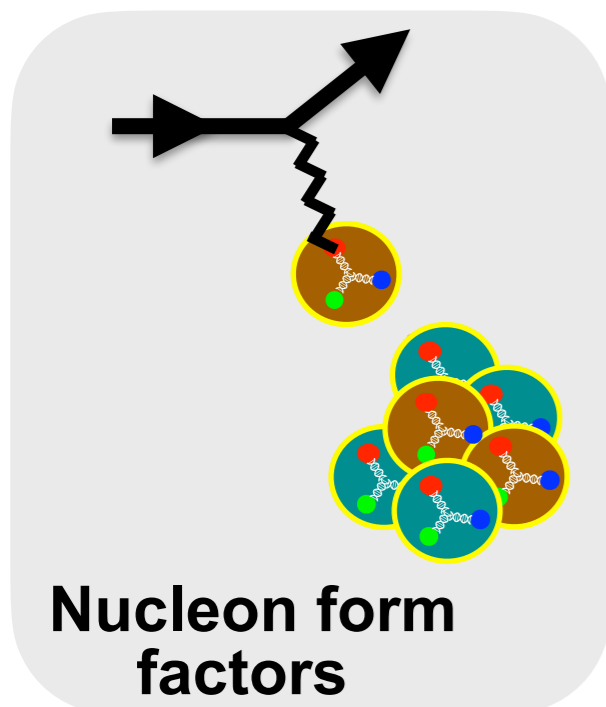
Near-detector tuning is not always sufficient for correctly extracting new physics signals (due to assumptions in cross-section models)



Reaction mechanisms

Process	Neutrino Energy Range	Example Final State
Coherent Elastic Scattering	$\lesssim 50$ MeV	$\nu + A$
Inelastic Scattering	$\lesssim 100$ MeV	$e + {}^A(Z+1)^* (\rightarrow {}^A(Z+1) + n\gamma)$
Quasi-Elastic Scattering	100 MeV–1 GeV	$l + p + X$
Two-Nucleon Emission	1 GeV	$l + 2N + X$
Resonance Production	1–3 GeV	$l + \Delta (\rightarrow N + \pi) + X$
Shallow Inelastic Scattering	3–5 GeV	$l + n\pi + X$
Deep Inelastic Scattering	$\gtrsim 5$ GeV	$l + n\pi + X$

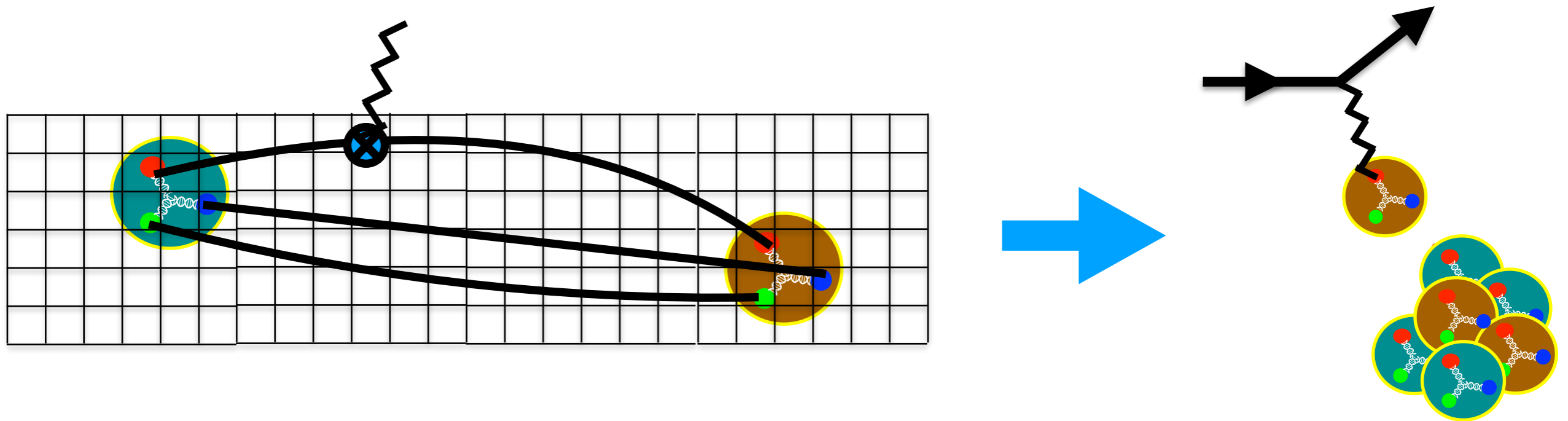
Alvarez-Ruso, MW et al, arXiv:2203.09030



Lattice QCD and neutrino-nucleus

Lattice QCD provides reliable methods for numerically computing properties of QCD including nucleon form factors encoding responses to electroweak currents

Neutrino-nucleon scattering amplitudes can be computed straightforwardly once nucleon electroweak form factors known



Connecting **nucleon** form factors to neutrino-**nucleus** scattering is more complicated

- Lattice QCD can constrain inputs to nuclear EFTs and models
- Constraints from lattice QCD and experiment are often complementary

Quarks and gluons on a lattice

Lattice QCD uses a path integral version of quantum mechanics

- Quark propagators provide explicit solutions to the quark field path integral
- Gluon field path integrals are performed numerically using Monte Carlo: random field values are drawn from a probability distribution similar to the integrand

Compromises:

Lattice spacing

$$a \rightarrow 0$$

Finite-volume

$$L \rightarrow \infty$$

Imaginary time

$$t \rightarrow it$$

Imaginary time turns complex quantum probability amplitudes

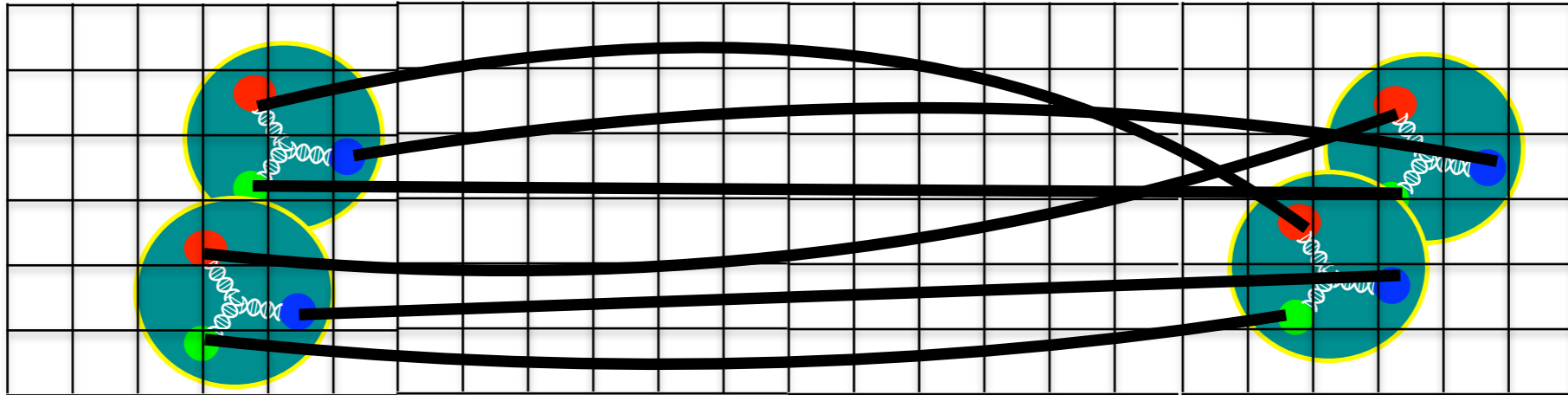
$$\text{probability amplitude} \sim e^{iS}$$

into positive-definite functions that can be interpreted as probabilities for random numbers in a Monte Carlo simulation

$$\text{probability amplitude} \sim e^{-S}$$

Observables in LQCD

LQCD energy spectrum determined from 2-point correlation functions



In imaginary time, correlation functions can be written as sums of exponentials

$$C_A(t) = \langle 0|A(t)A^\dagger(0)|0\rangle = \sum_n \langle 0|A(0)e^{-Ht}|n\rangle \langle n|A^\dagger(0)|0\rangle + \dots$$

$$= \sum_n |Z_n|^2 e^{-E_n t}$$

Imaginary time evolution $e^{-iHt_{\text{real}}} = e^{-H(it_{\text{real}})}$

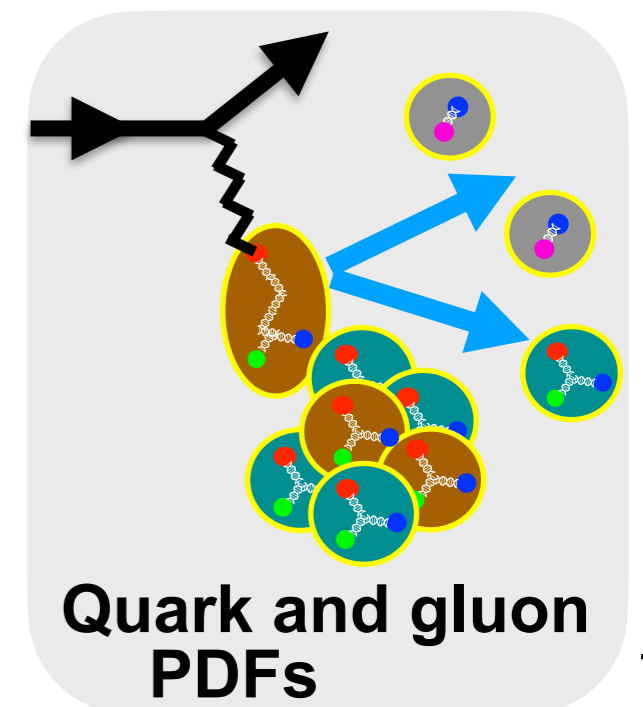
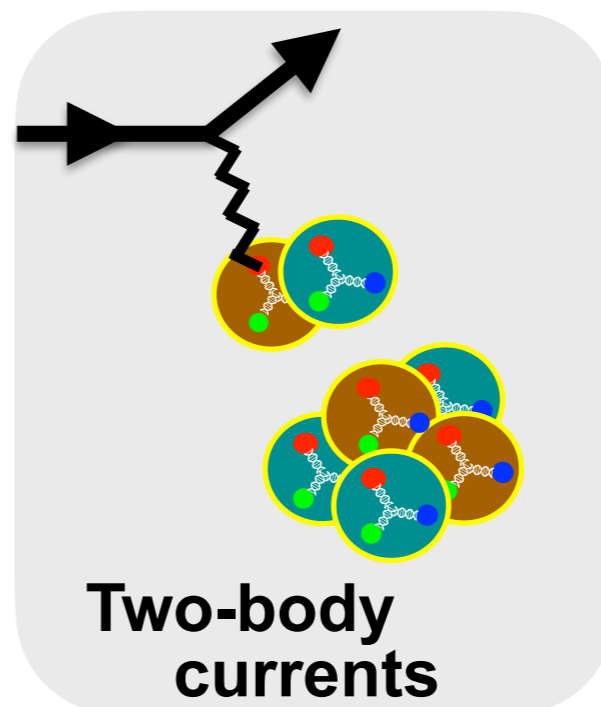
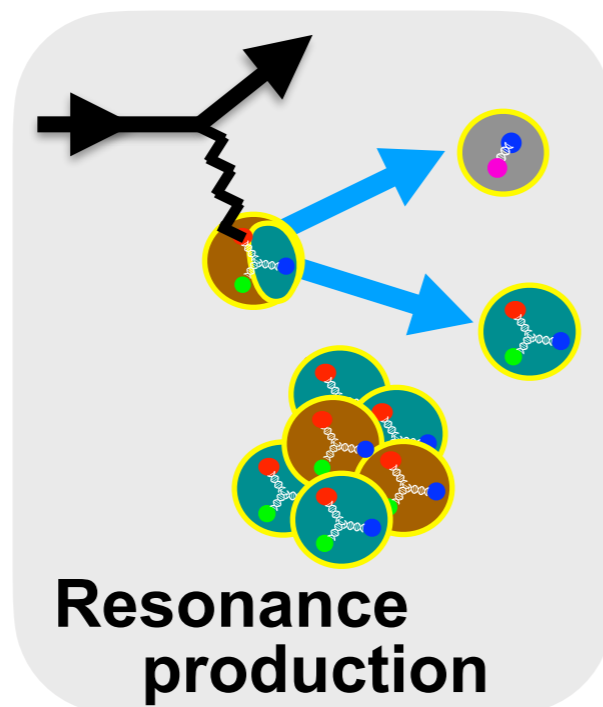
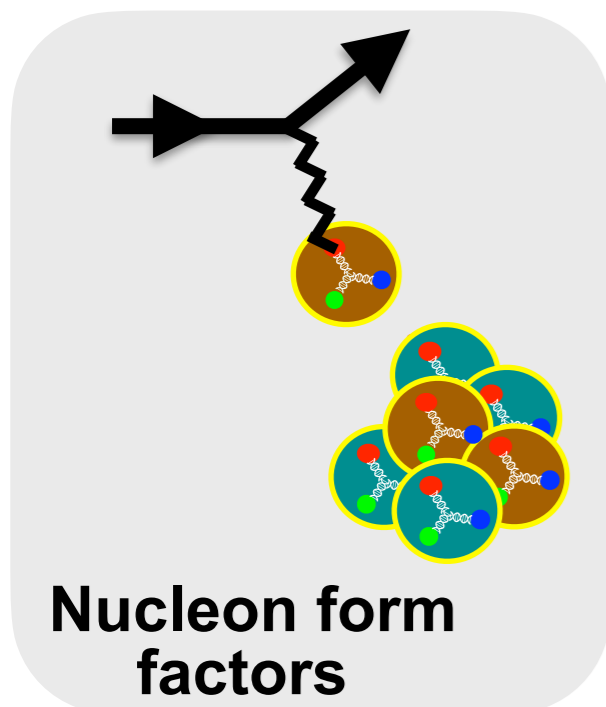
Ground state dominates for large t

$$C_A(t) \propto e^{-E_0 t} + \dots$$

Reaction mechanisms

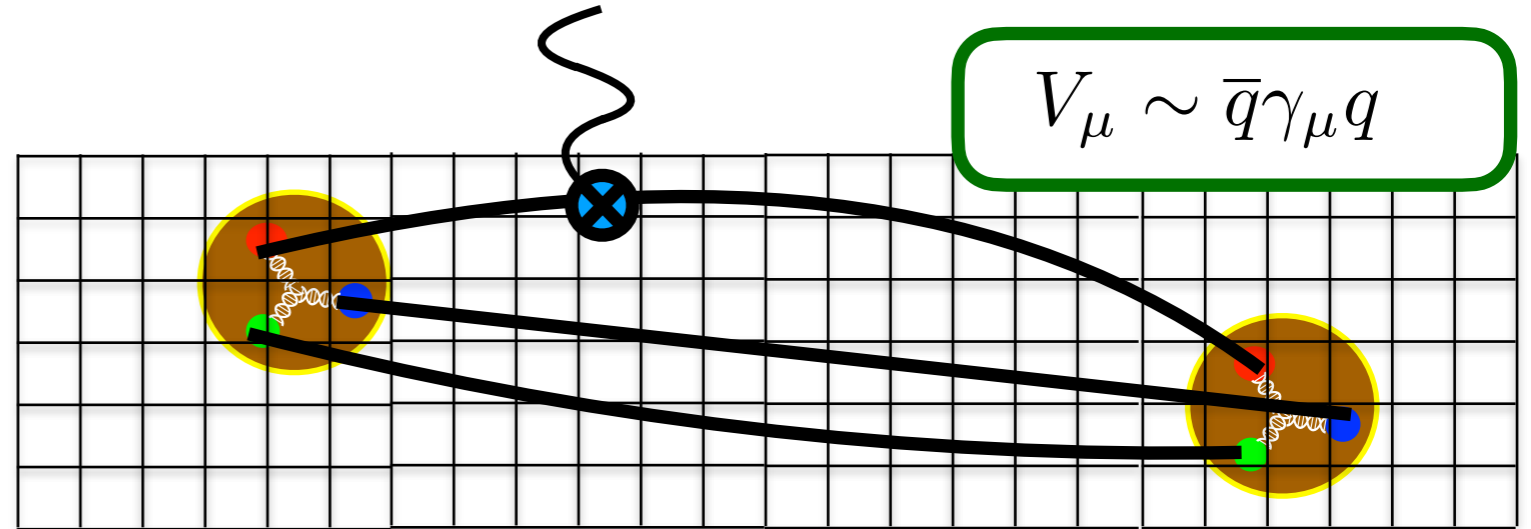
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Alvarez-Ruso, MW et al, arXiv:2203.09030

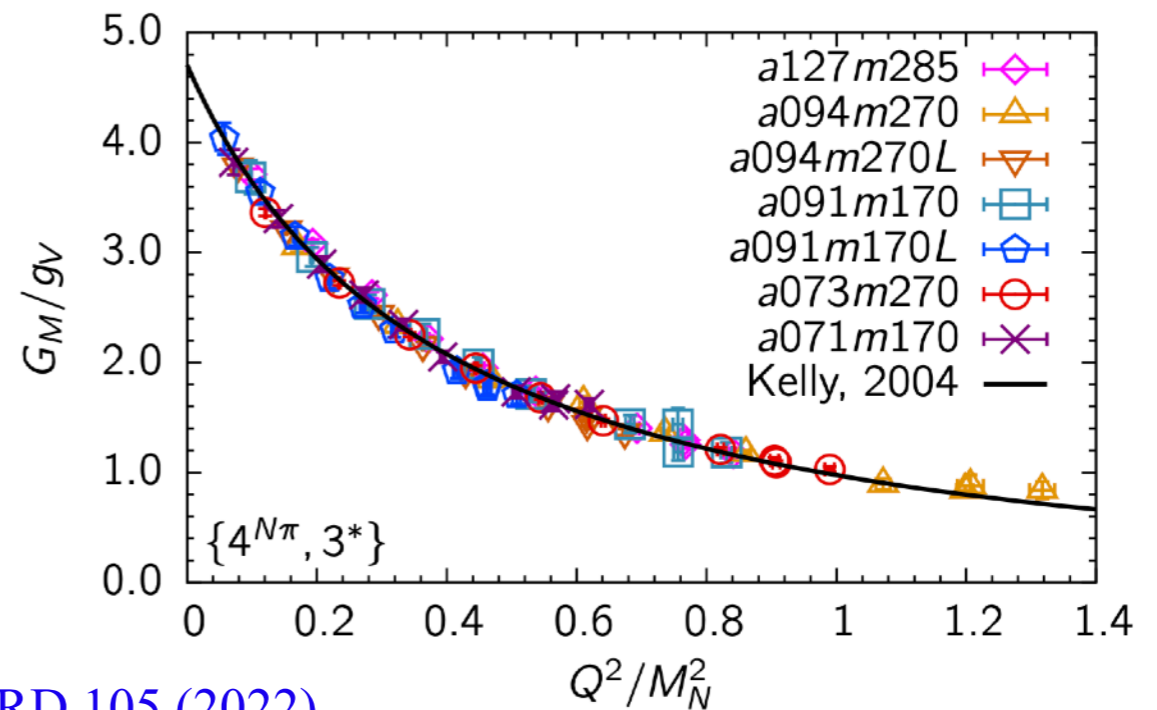
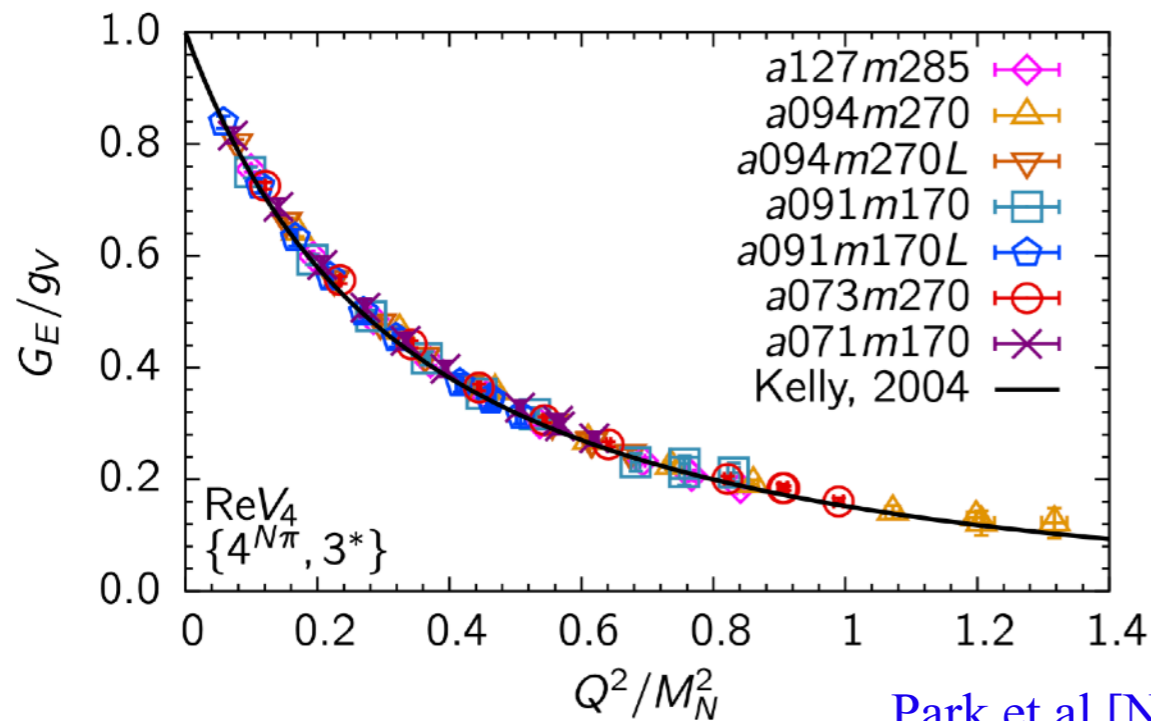


LQCD and nucleon form factors

Nucleon electric and magnetic form factors recently calculated using LQCD with approximately physical quark masses



$$\langle N(\mathbf{p} + \mathbf{q}) | V^\mu | N(\mathbf{p}) \rangle = \bar{u}(\mathbf{p} + \mathbf{q}) \left[F_1(q^2) \gamma^\mu + i \sigma^{\mu\nu} q_\nu \frac{F_2(q^2)}{2M_N} \right] u(\mathbf{p})$$



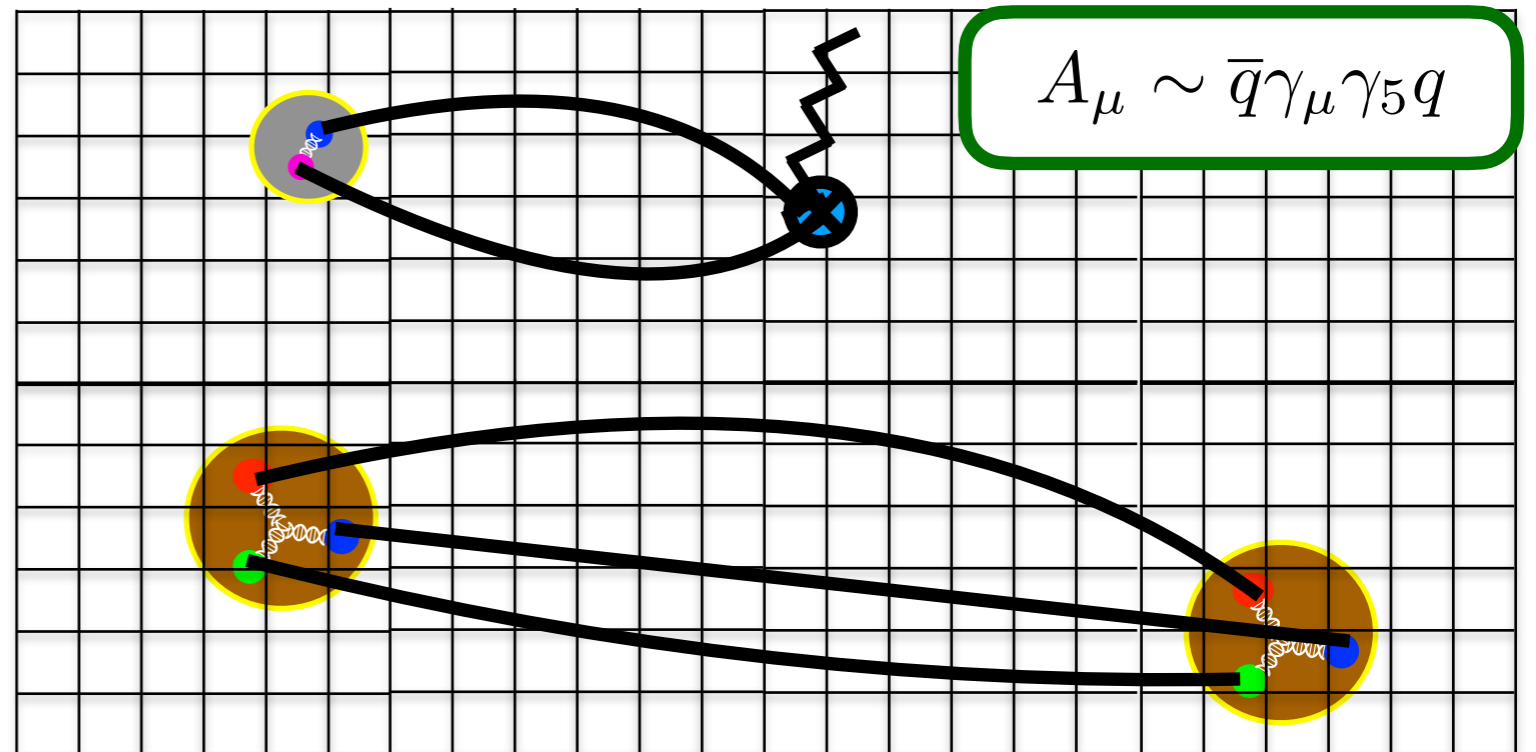
Park et al [NME], PRD 105 (2022)

LQCD results for nucleon electric and magnetic form factors (linear combinations of F_1 and F_2) show good consistency with phenomenological parameterizations

Excited states

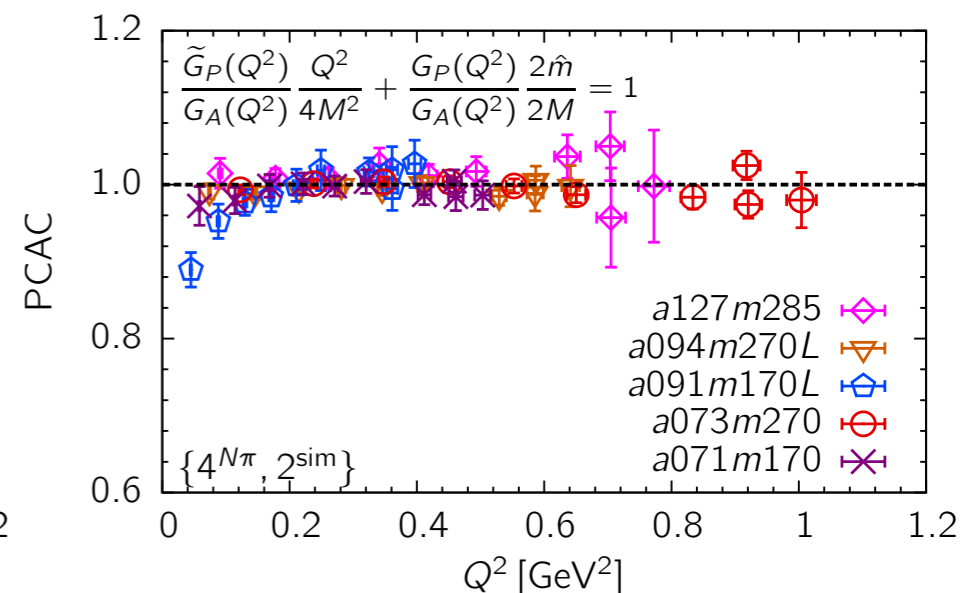
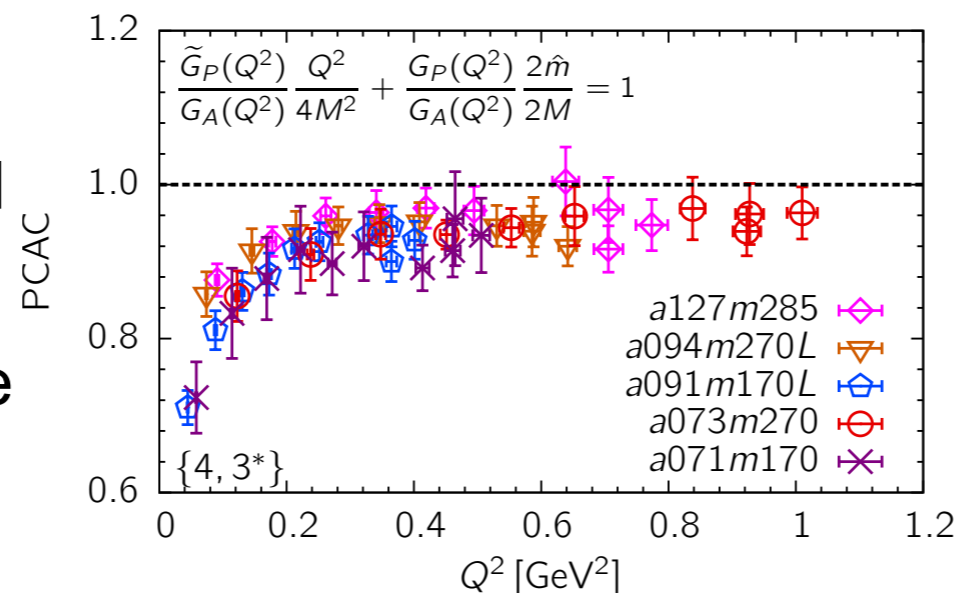
Axial form factor calculations have been performed using analogous methods

Additional excited-state effects arise from the fact that axial currents can act as pion sources



$$\langle N(\mathbf{p} + \mathbf{q}) | A^\mu | N(\mathbf{p}) \rangle = \bar{u}(\mathbf{p} + \mathbf{q}) \left[G_A(q^2) \gamma^\mu \gamma_5 + q^\mu \gamma_5 \frac{\tilde{G}_P(q^2)}{2M_N} \right] u(\mathbf{p})$$

Careful treatment of $N\pi$ excited states required to reproduce known symmetry constraints assuming ground-state dominance of results



Axial form factors

LQCD calculations of nucleon axial form factors with approximately physical quark masses and continuum extrapolations achieved by multiple groups

Bali et al [RQCD], JHEP 05 126 (2020)

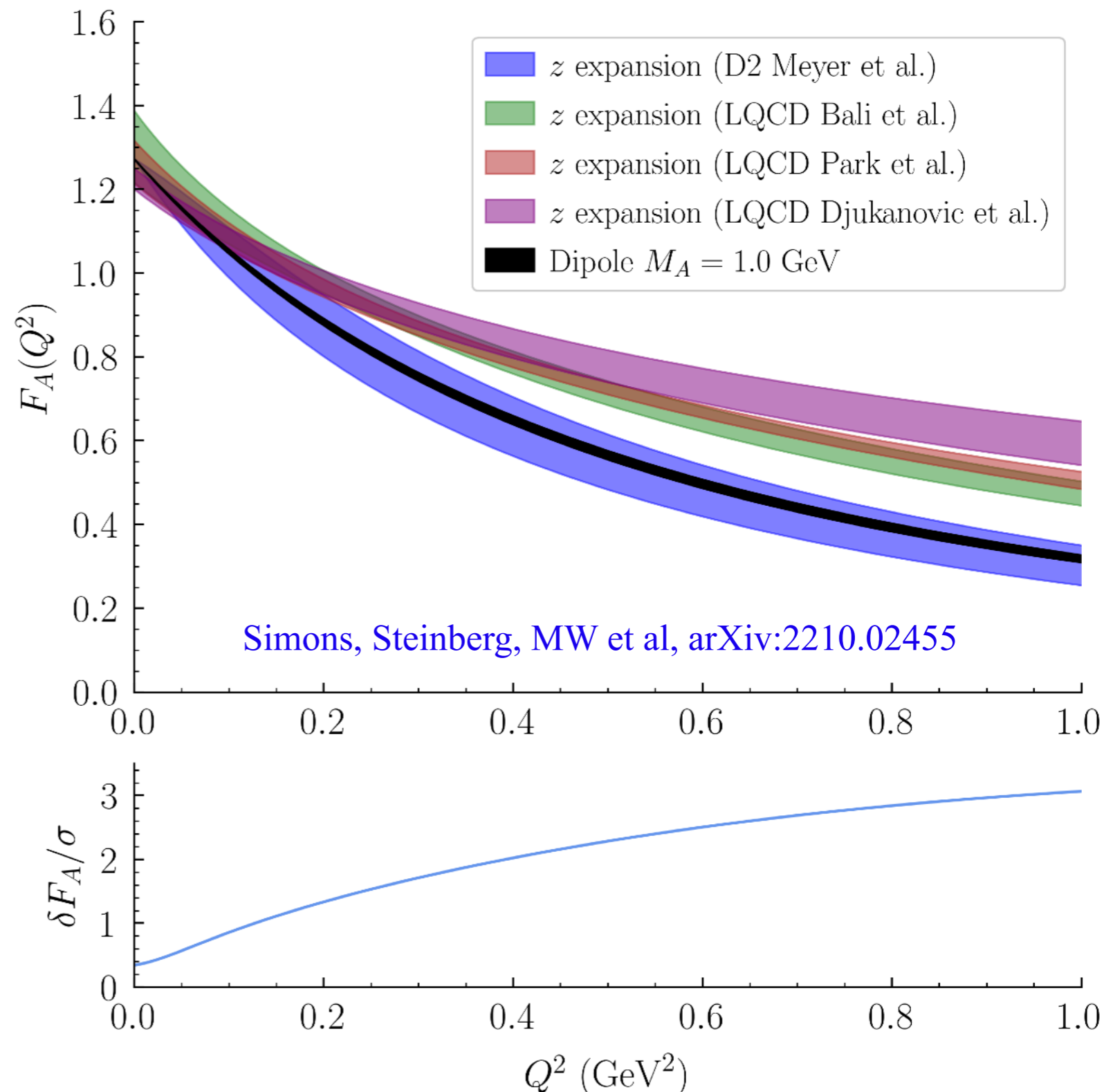
Park et al [NME], PRD 105 (2022)

Djukanovic et al, arXiv:2207.03440

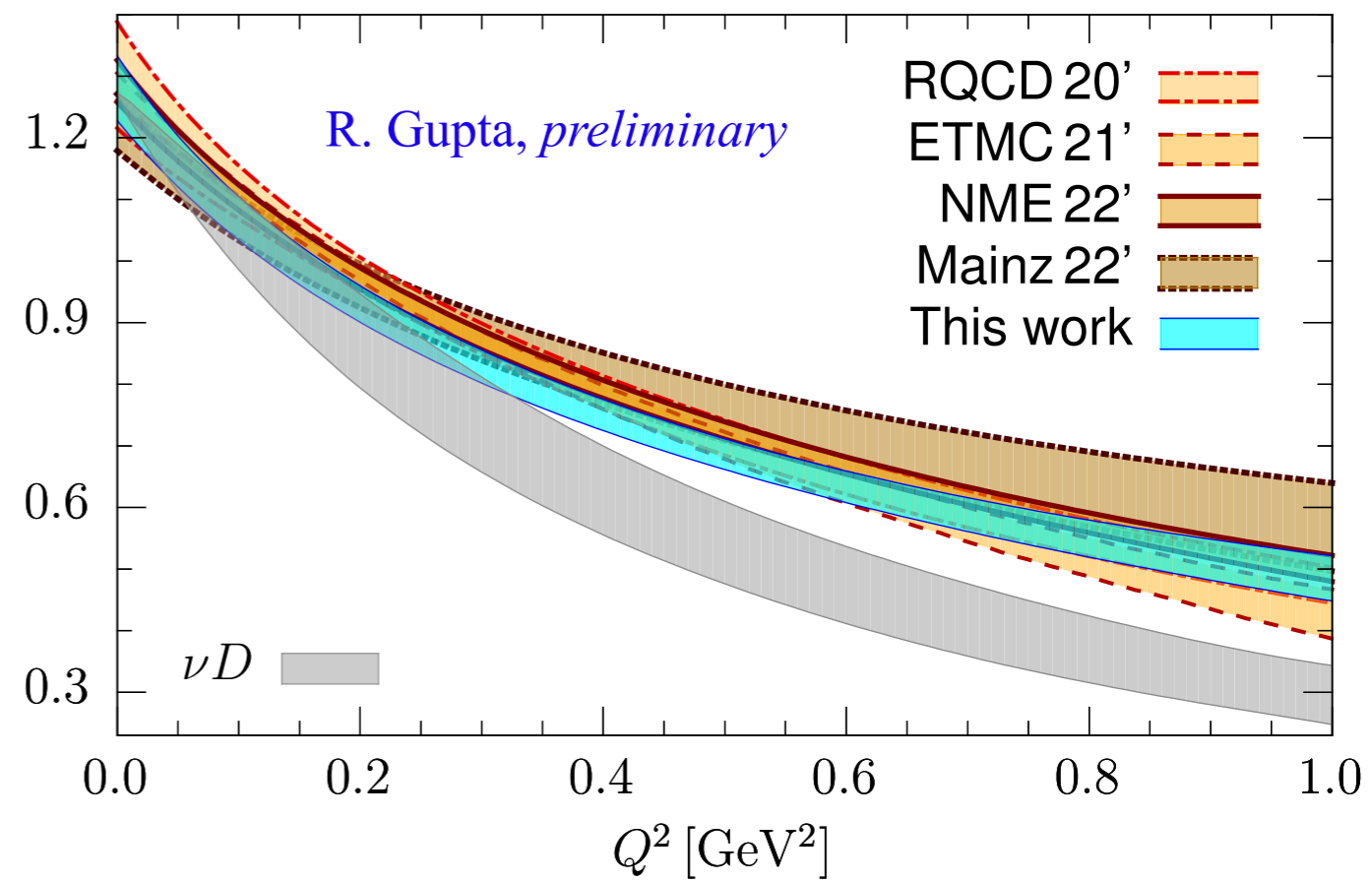
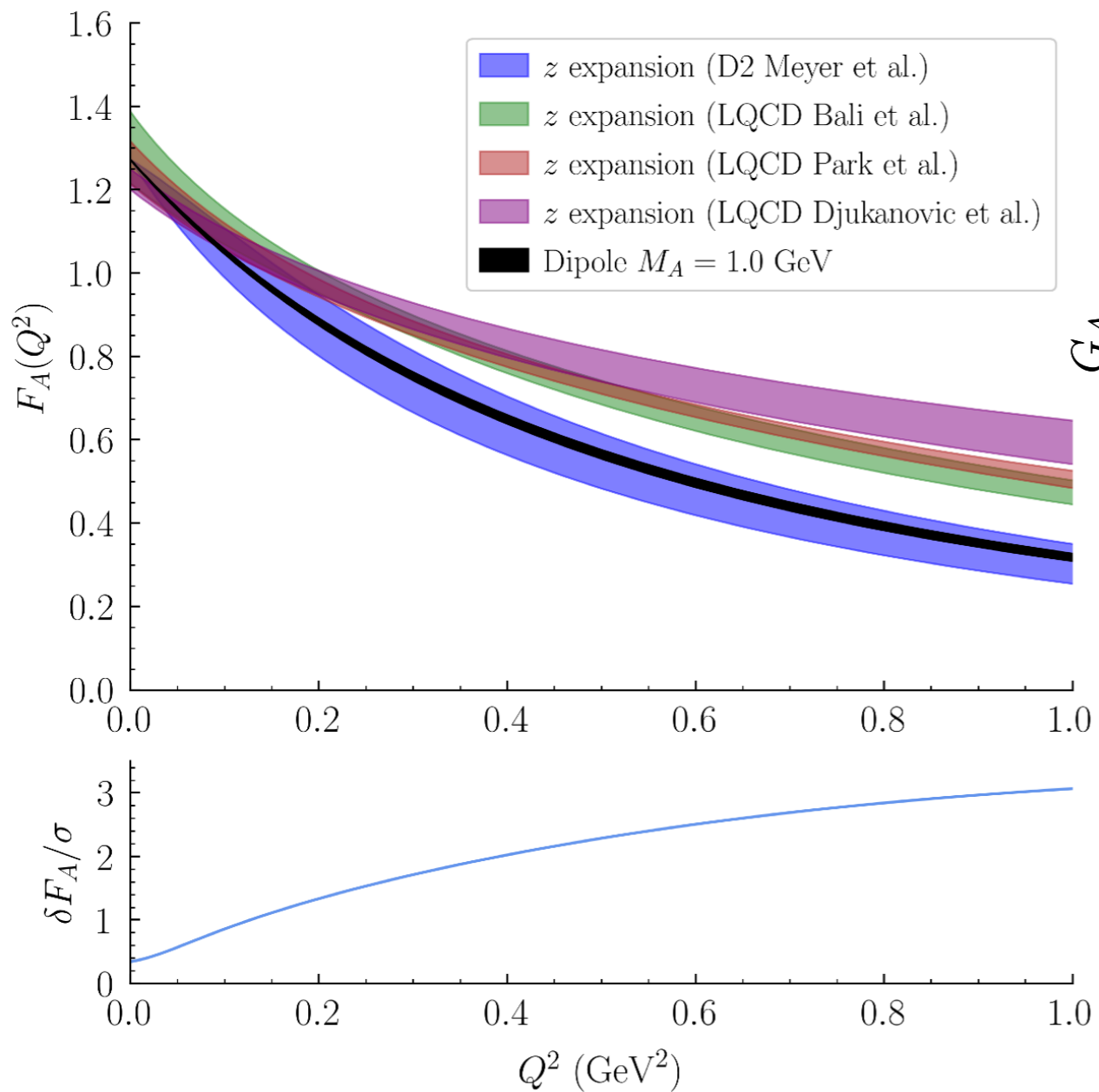
Up to 3 sigma differences between LQCD and experimental axial form factor determinations, could arise from challenging LQCD systematic uncertainties

Differences could also arise from underestimated uncertainties in phenomenological form factor determinations using deuterium bubble chamber data

Meyer, Betancourt, Gran, and Hill, PRD 93 (2016)



Axial form factors

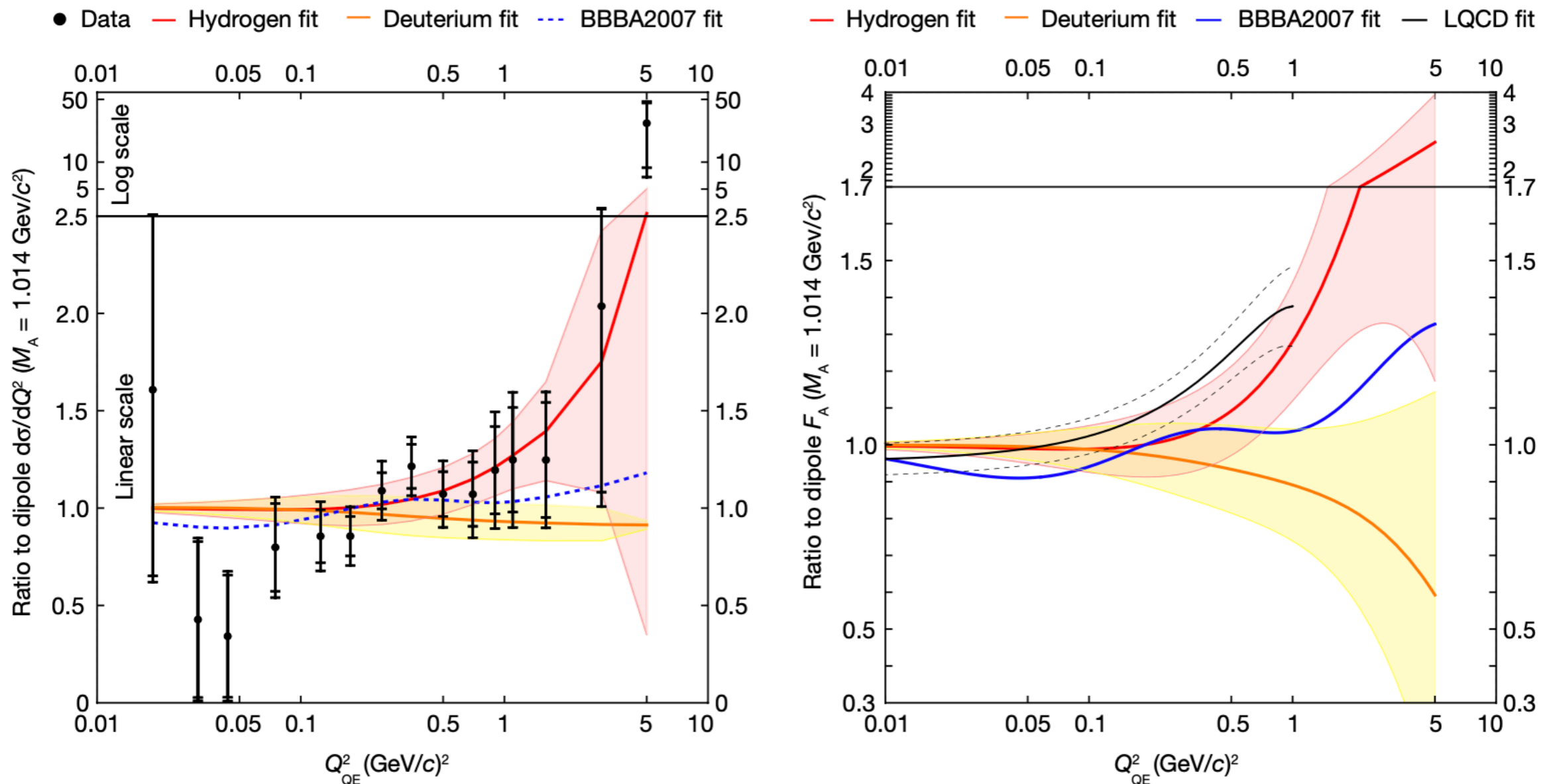


MINERvA results

MINERvA has recently analyzed antineutrino scattering with a hydrocarbon scintillator target (8% H + 89% C + ...) to extract nucleon axial form factor

- Challenging systematics arise in separating nucleon/nuclear events, but broad consistency is seen between MINERvA extraction and LQCD

Cai et al. [MINERvA], Nature 614 (2023)

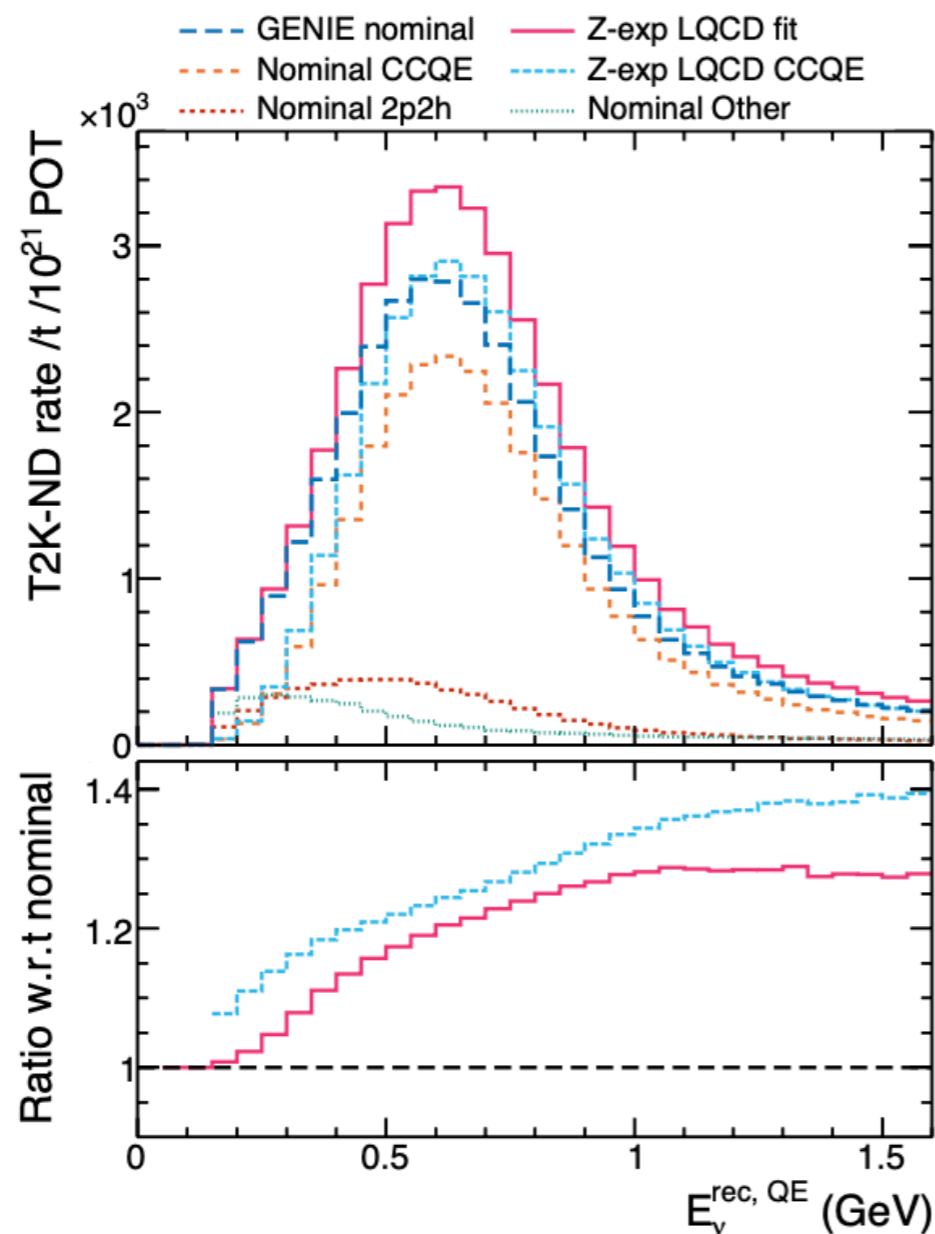


Event generator uncertainties

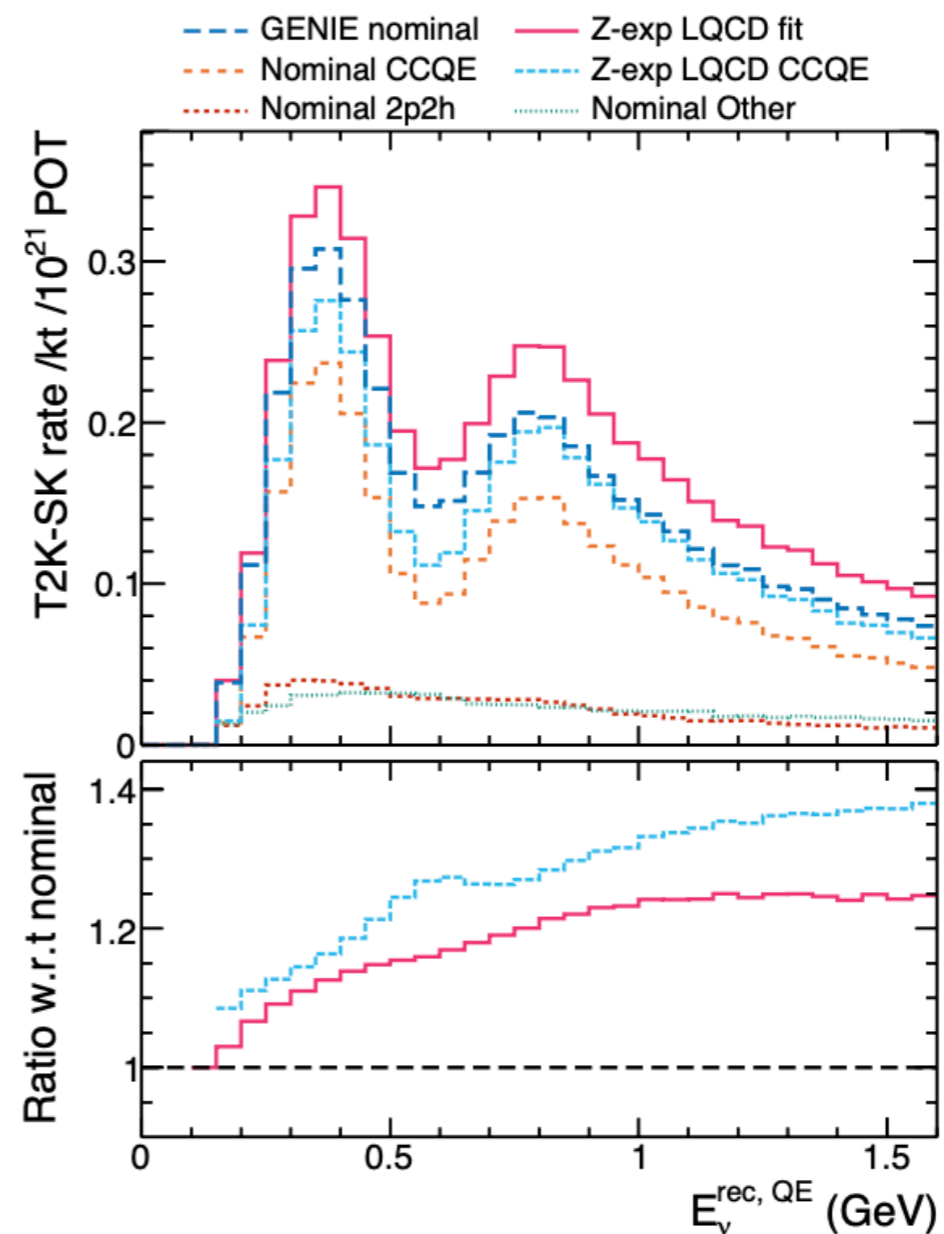
GENIE event generator predictions for T2K event rate using deuterium bubble chamber vs recent LQCD axial form factors differ by $\sim 20\%$

- Effects on near and far detectors differ, understanding discrepancy essential for reliable neutrino oscillation analyses

Meyer, Walker-Loud, Wilkinson, *Ann. Rev. Nucl. Part. Sci.* 72 (2022)



(a) Near detector



(b) Far detector

What precision do we need from LQCD for neutrino physics?

- **Quantitative precision targets** essential for high-performance computing campaigns
- **Cannot assume an overly restrictive form factor shape**, e.g. dipole; assumptions lead to underestimated precision needs
- **Theoretically consistent nuclear models** without tuning to neutrino data needed to disentangle axial form factor from multi-nucleon effects
- **Comparison between multiple nuclear models** needed to study nuclear uncertainties in the role of axial form factors in νA

Simons, Steinberg, Lovato, Meurice, Rocco, MW, arXiv:2210.02455

Predicting νA cross sections

Green's function Monte Carlo (GFMC) methods can accurately solve nuclear many-body problem given a Hamiltonian and electroweak current operators

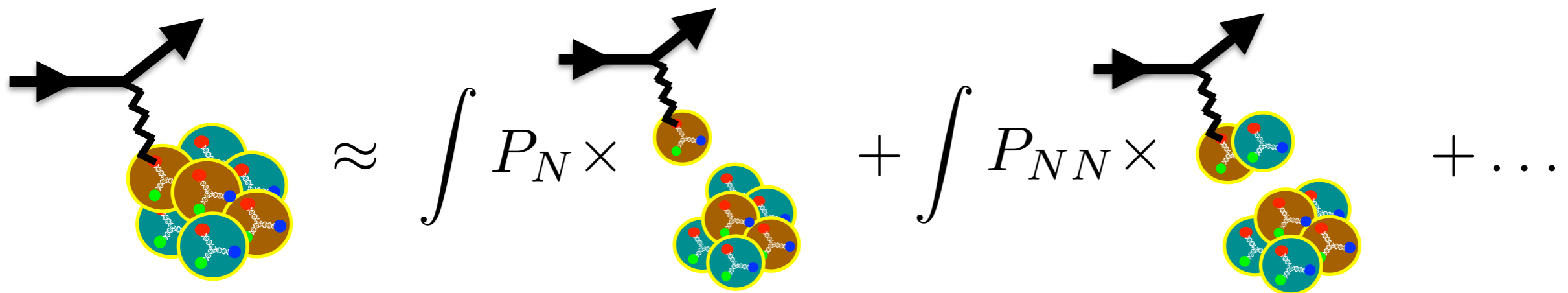
- Cost grows rapidly with nucleon number, computationally limited to $A \lesssim 12$

More computationally tractable: extended factorization scheme using approximate spectral functions — distributions of nucleons (+ NN pairs + ...) in nucleus

Benhar et al, PRD (2005)

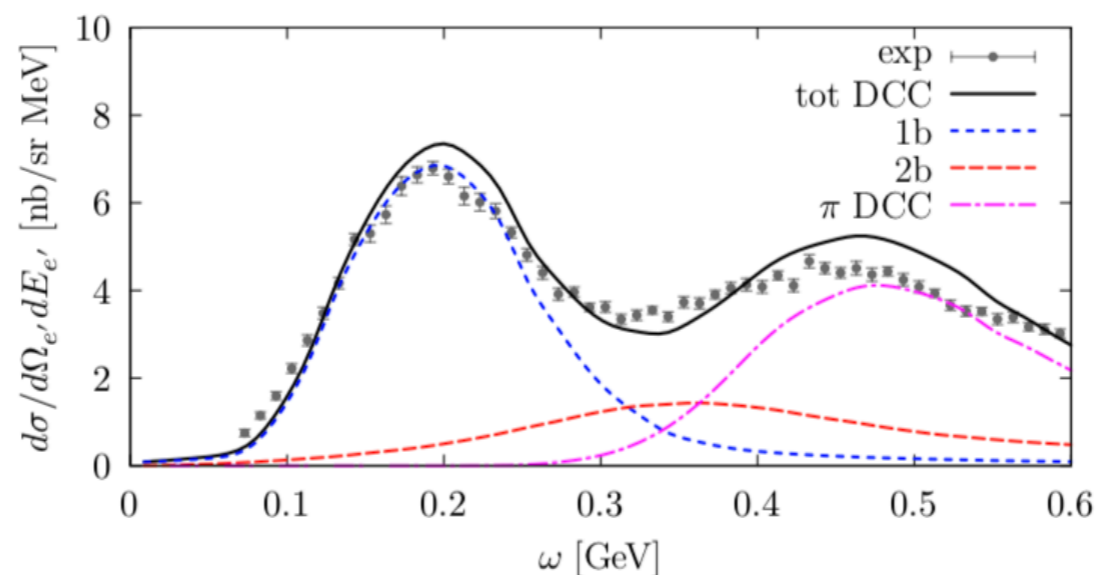
Rocco, Lovato, Benhar PRL 116 (2016)

Rocco et al, PRC 99 (2019) ...



Allows inclusion of 2-body currents and resonance production, computationally feasible for medium-mass nuclei

$E_e=961$ MeV, $\theta_e=37.5^\circ$

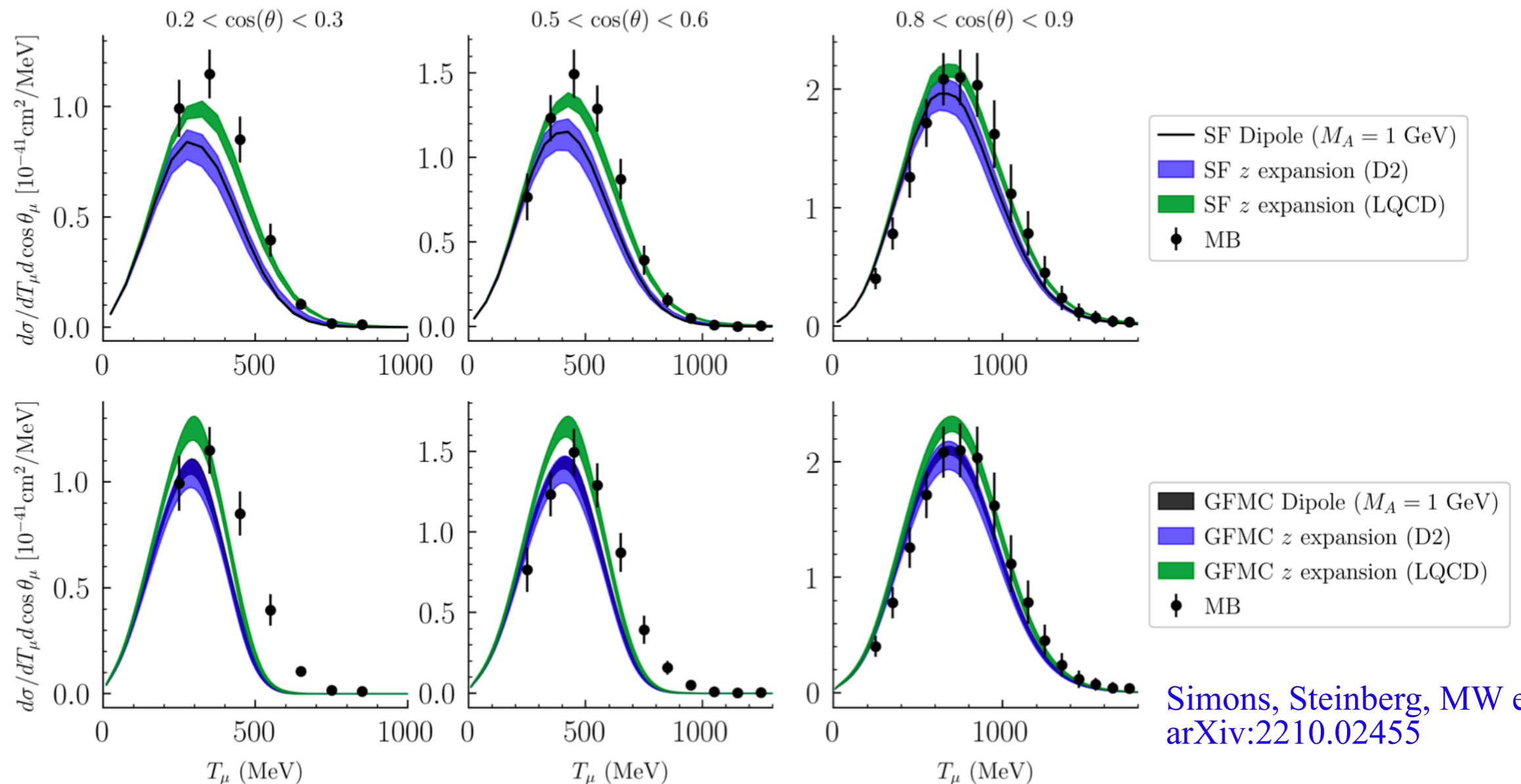


Rocco, Nakamura, Lee, Lovato, PRC 100 (2019)

MiniBooNE results

Comparison of GFMC and spectral function (SF) results for ^{12}C with experimental data and one another provides validation of nuclear many-body methods

- 5-20% differences found between GFMC and SF predictions for MiniBooNE, largest at kinematics where relativistic effects neglected in GFMC are most significant
- 10-20% differences found between LQCD and D2 form factors



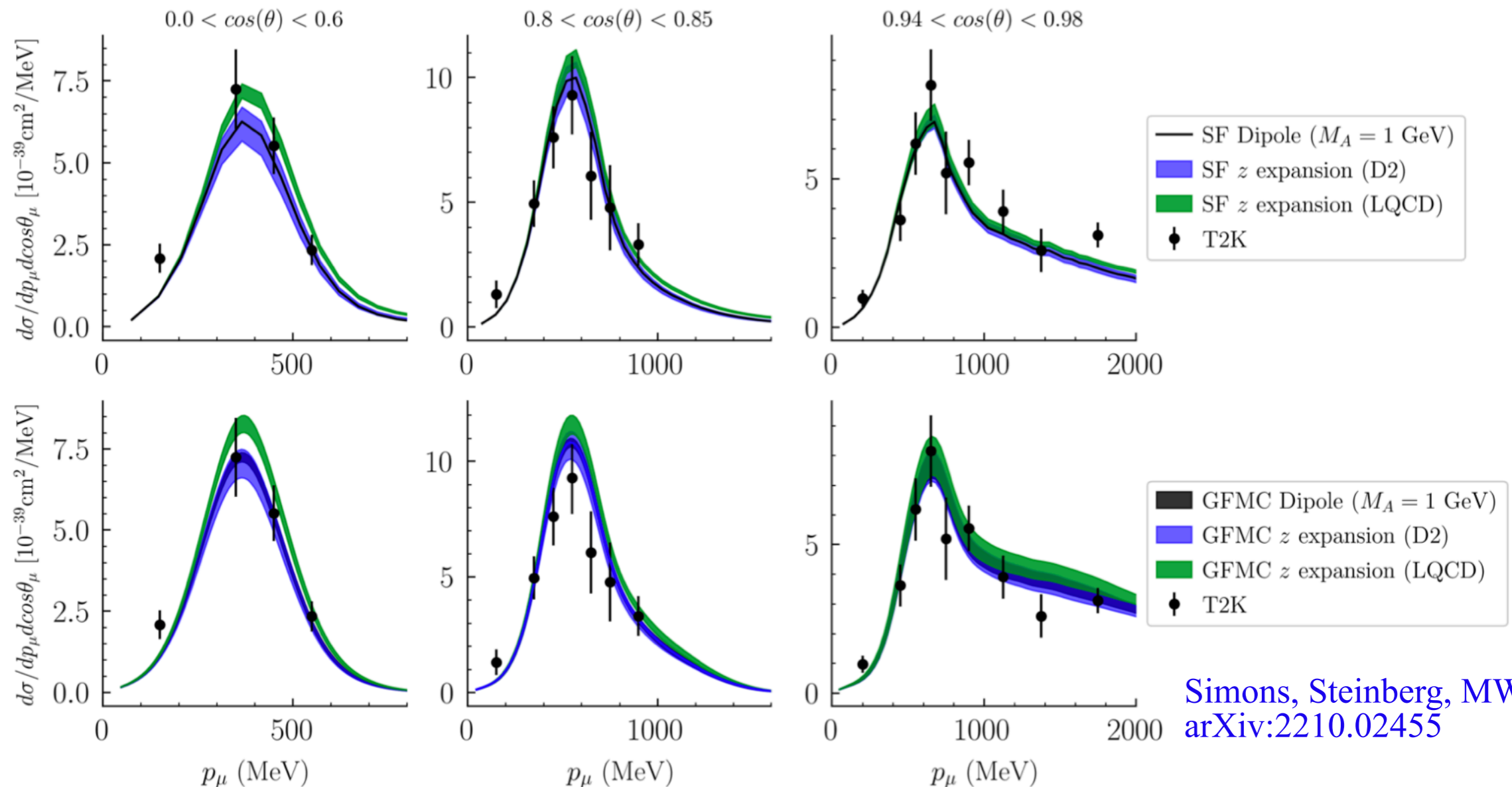
Simons, Steinberg, MW et al,
arXiv:2210.02455

T2K results

Form factor and nuclear model differences both somewhat smaller with T2K kinematics (lower flux of high-energy neutrinos than MiniBooNE)

Changes in nuclear model vs changes in axial form factor are not easy to distinguish

- Consistent treatment of nucleon axial form factor and nuclear many-body effects essential when fitting to neutrino scattering data



Quantifying form factor uncertainties

z expansion — model independent parameterization of axial (and other) form factors that only assumes basic field theory / QCD properties

Hill, eConf C060409, 027 (2006)

Hill and Paz, PRD 82 (2010)

Bhattacharya, Hill, and Paz, PRD 84 (2011)

$$F_A(Q^2) = \sum_{k=0}^{\infty} a_k z(Q^2)^k$$

Free parameters Known function

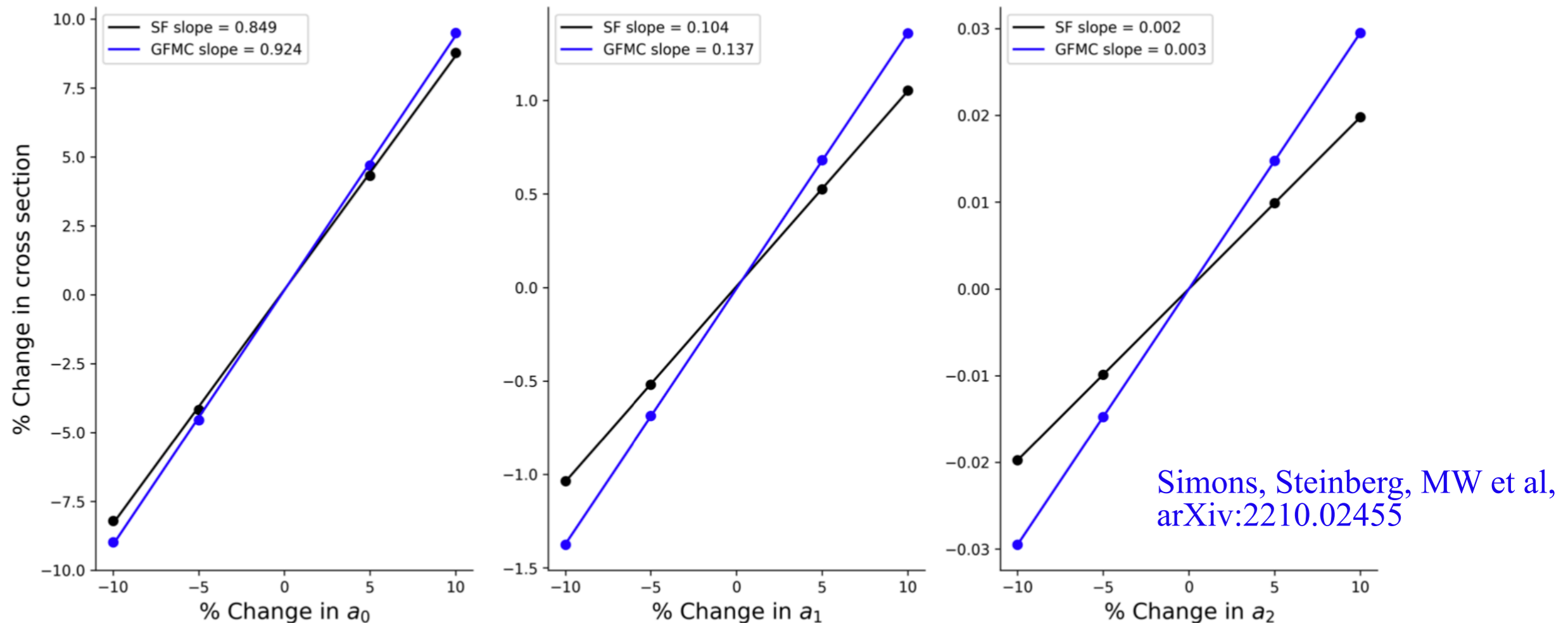
Can be used to quantify relations between nucleon axial form factor uncertainties and neutrino-nucleus cross section uncertainties

$$\delta\sigma = \sum_k \frac{\partial\sigma}{\partial a_k} \delta a_k + \dots$$

Straightforward to determine from calculations with varying a_k

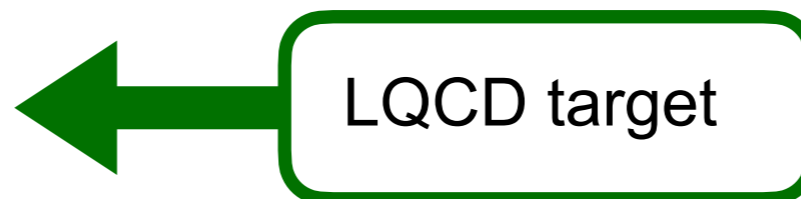
Axial FF uncertainty needs

Uncertainty relations calculated for MiniBooNE cross sections



Achieving 1% cross-section precision for MiniBooNE kinematics requires:

- ~ 1% precision in a_0
- ~ 10% precision in a_1
- Relatively little knowledge of a_2, \dots

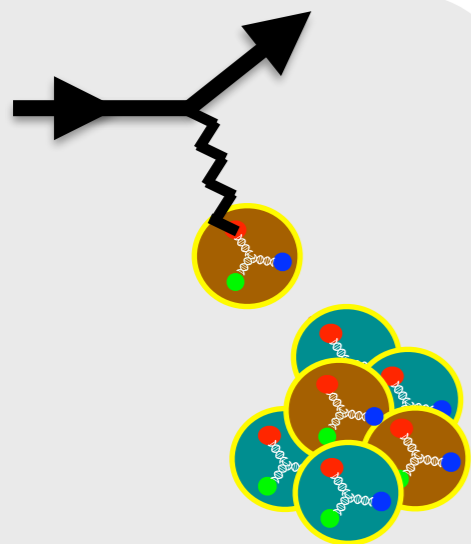


DUNE will be more sensitive to higher coefficients, further dedicated studies needed 24

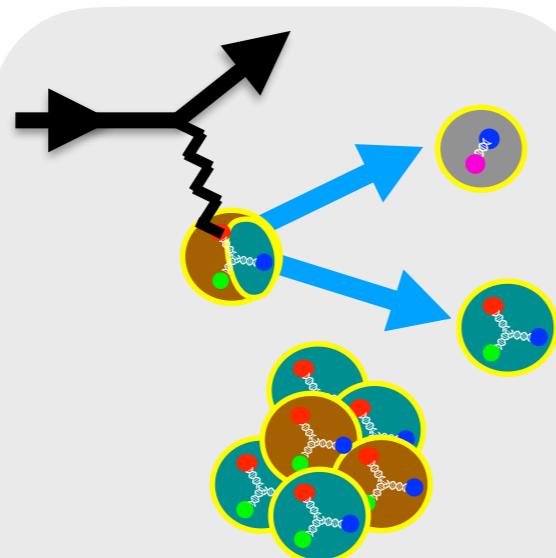
Beyond the axial form factor

Process	Neutrino Energy Range	Example Final State
Coherent Elastic Scattering	$\lesssim 50$ MeV	$\nu + A$
Inelastic Scattering	$\lesssim 100$ MeV	$e + {}^A(Z+1)^*(\rightarrow {}^A(Z+1) + n\gamma)$
Quasi-Elastic Scattering	100 MeV–1 GeV	$l + p + X$
Two Nucleon Emission	1 GeV	$l + 2N + X$
Resonance Production	1–3 GeV	$l + \Delta(\rightarrow N + \pi) + X$
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Deep Inelastic Scattering	$\gtrsim 5$ GeV	$l + n\pi + X$

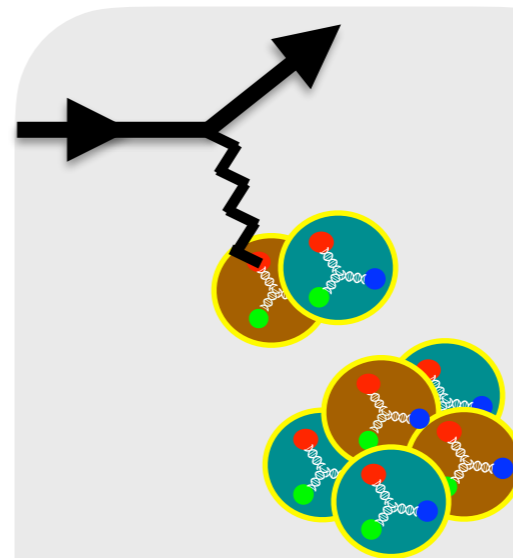
Alvarez-Ruso, MW et al, arXiv:2203.09030



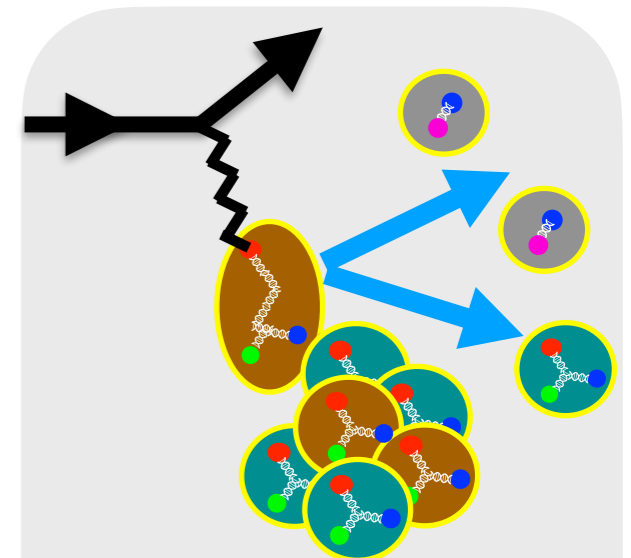
Nucleon form factors



Resonance production



Two-body currents

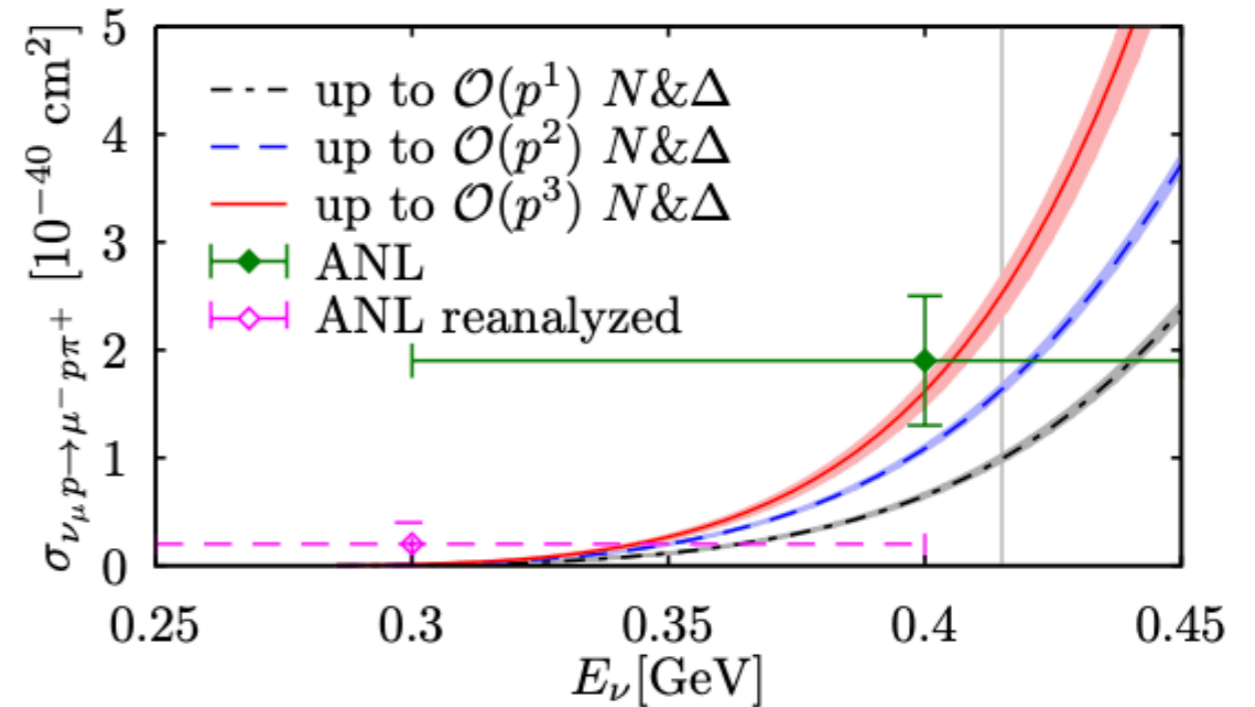


Quark and gluon PDFs

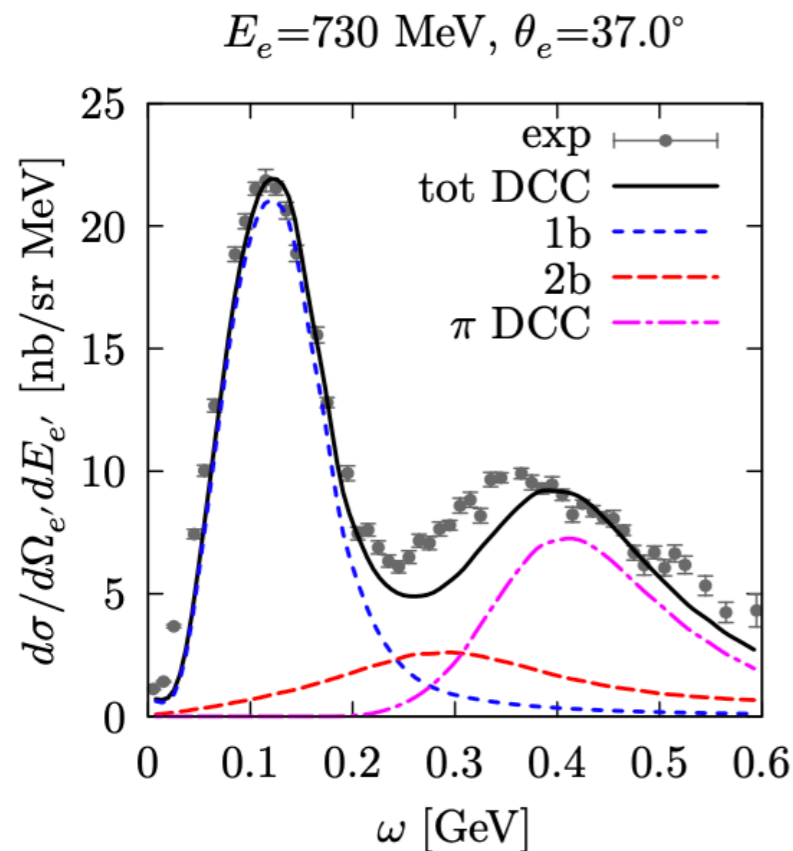
Pion production

Low-energy pion production can be described in relativistic baryon χ PT including $\Delta(1232)$ degrees of freedom

- Δ resonance and nonresonant pion production both significant
- Experimental data on neutrino-induced pion production are scarce



Yao, Alvarez-Ruso, Hiller Blin, and Vicente Vacas, PRD 98 (2018)



Spectral function nuclear model + dynamic coupled-channels (DCC) model of nucleon resonances can reproduce pion electroproduction data

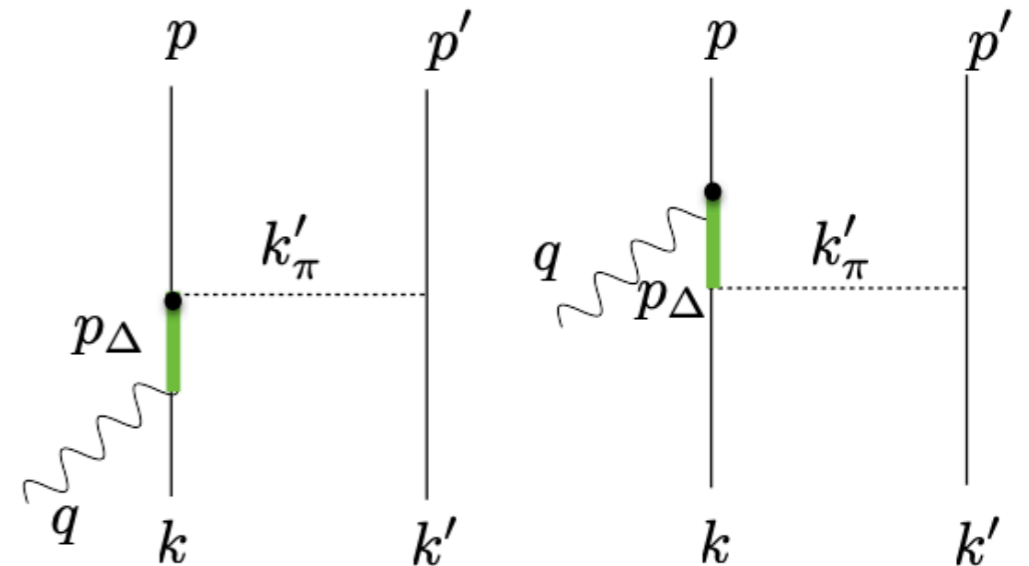
Neutrino predictions include significant uncertainties from nuclear modifications of Δ form factors and axial contributions estimated with tree-level χ PT

Rocco, Nakamura, Lee, and Lovato
PRC 100 (2019)

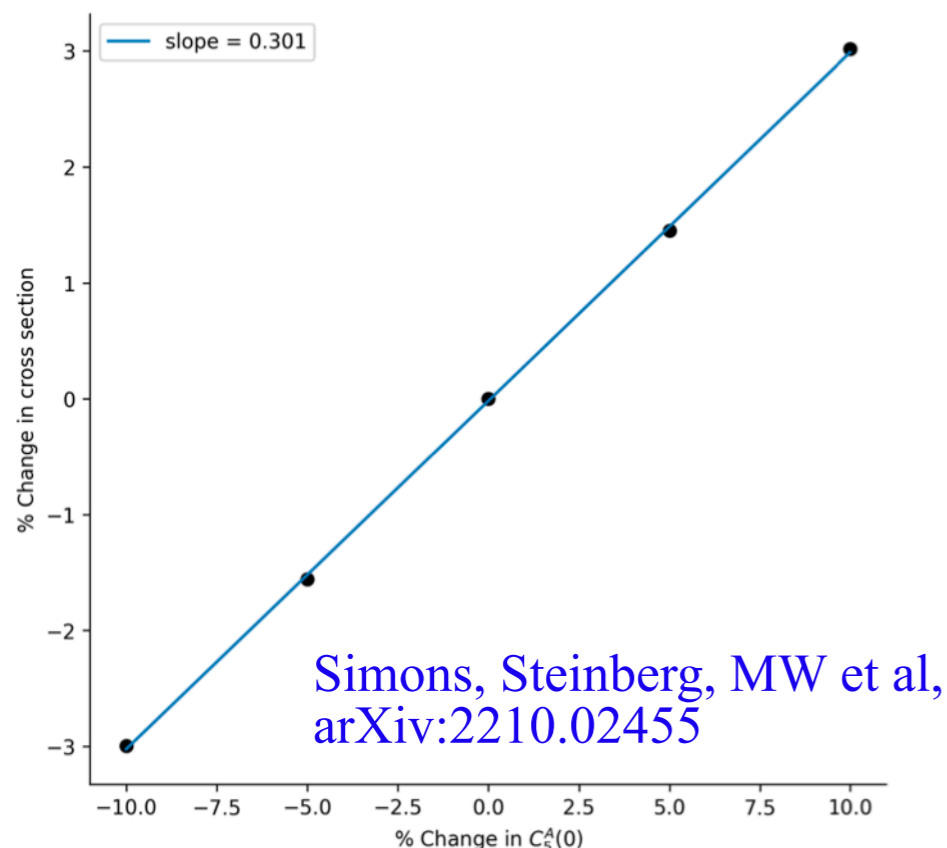
Resonance uncertainty needs

Similar uncertainties quantification can be studied for other cross-section pieces

The largest contributions to two-body currents arise from resonant $N \rightarrow \Delta$ transitions in conjunction with pion exchange



The normalization of the dominant $N \rightarrow \Delta$ transition form factor must be known to 3% precision to achieve 1% cross-section precision for MiniBooNE kinematics



State-of-the-art determinations of this form factor from experimental data on pion electroproduction achieve 10-15% precision (under some assumptions)

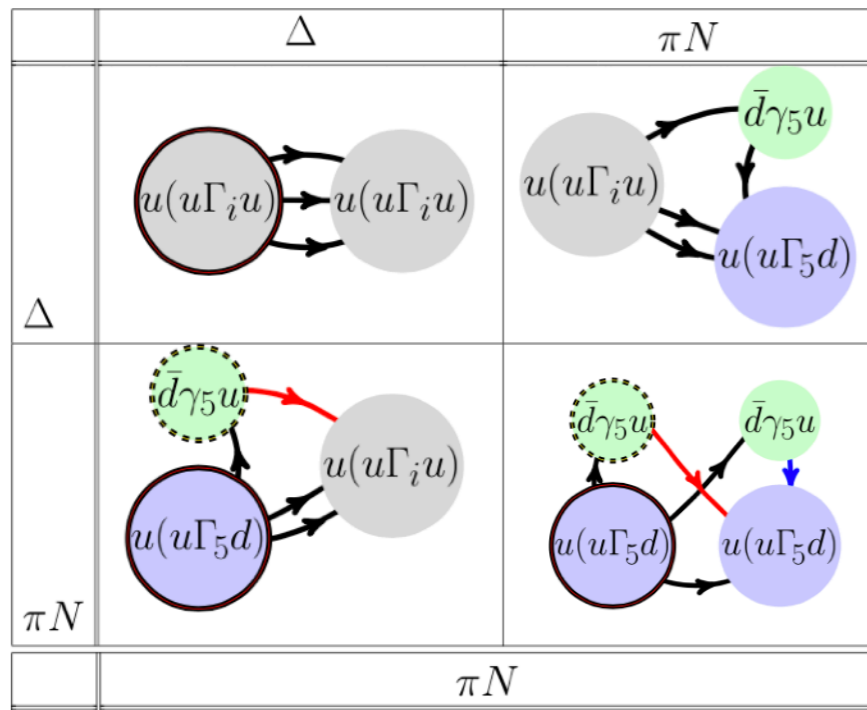
[Hernandez et al, PRD 81 \(2010\)](#)

Further constraints on $N \rightarrow \Delta$ transitions and two-body currents will be necessary to achieve few-percent cross-section precision

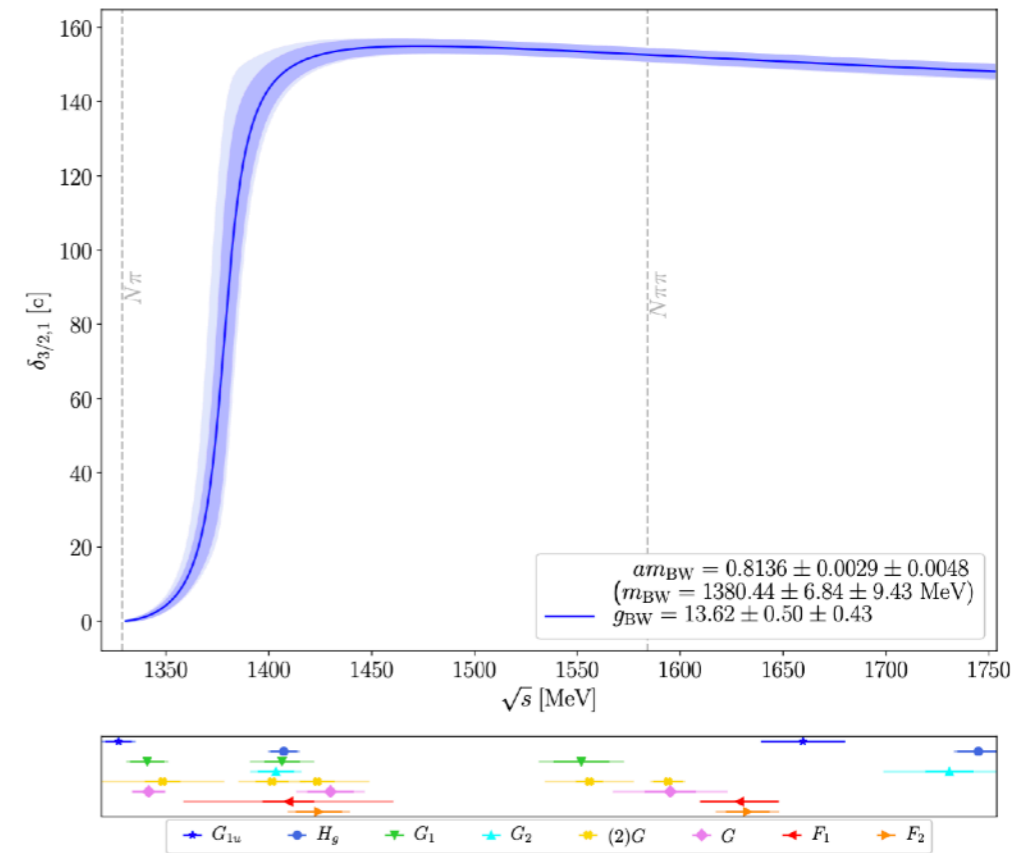
$N\pi$ systems in LQCD

$N\pi$ and Δ systems can be explicitly studied in LQCD

Silvi et al, PRD 23 (2021)



$N\pi$ p-wave phase shift



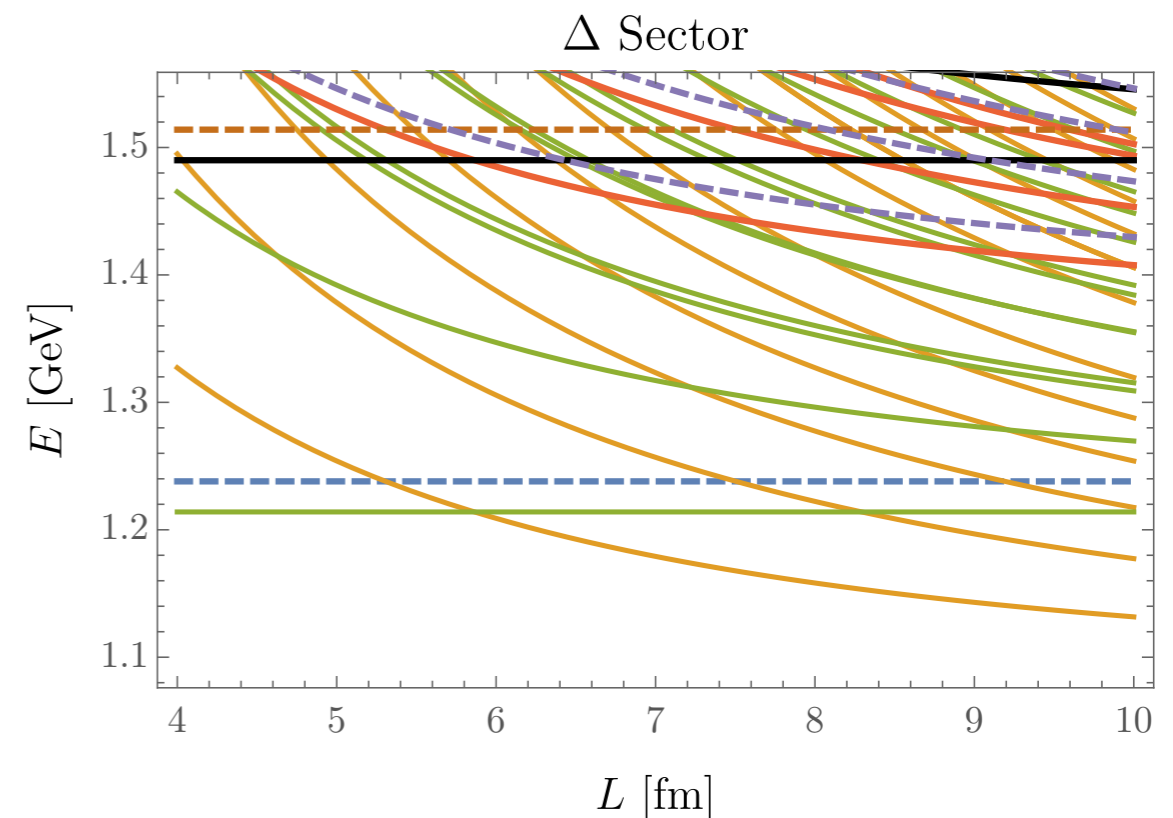
$N \rightarrow \Delta$ transition form factors can also be calculated with variational methods

Barca, Bali, and Collins, PoS LATTICE 2021 (2022)

Fully mapping out spectrum through the Δ resonance region will be challenging

$N\pi\pi$ states relevant

— work in progress with Anthony Grebe

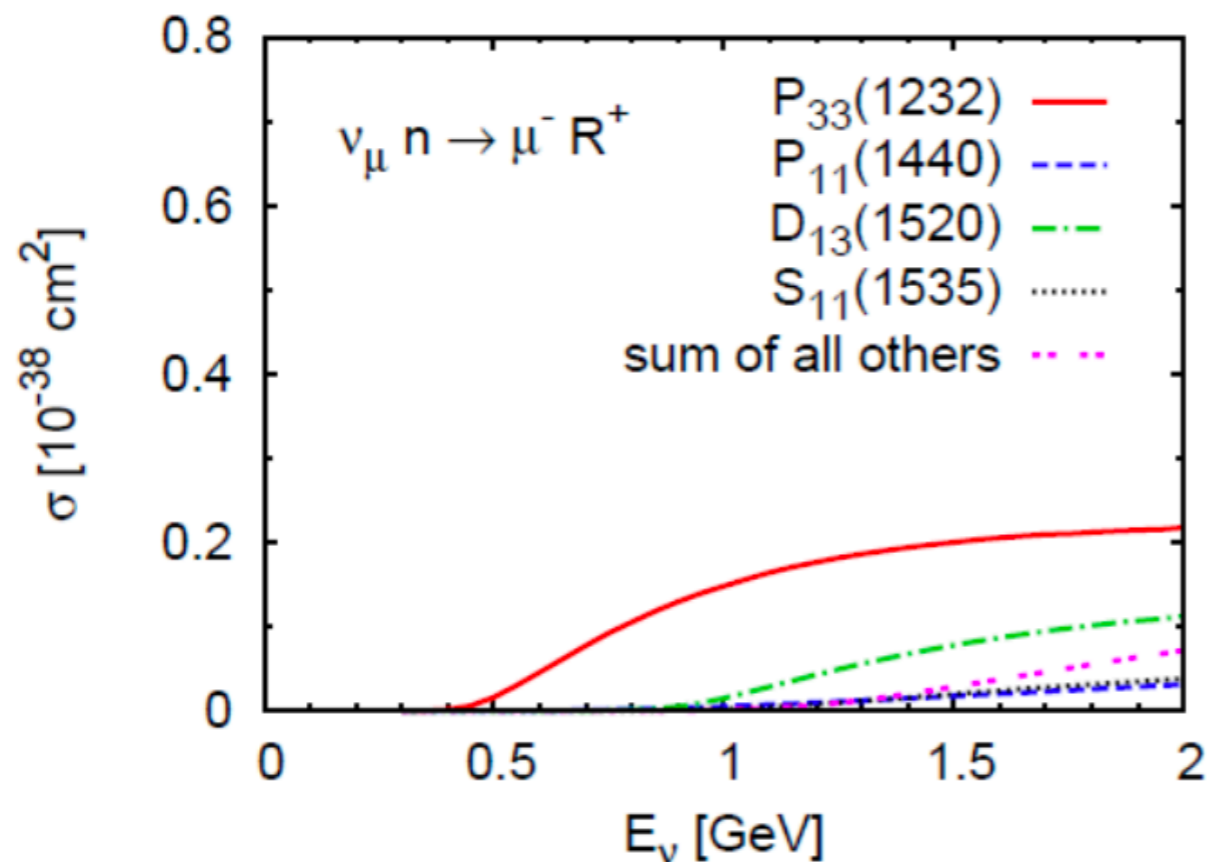


Higher-energy resonances

DCC model from ANL-Osaka describes nucleon resonance production by fitting to experimental data for

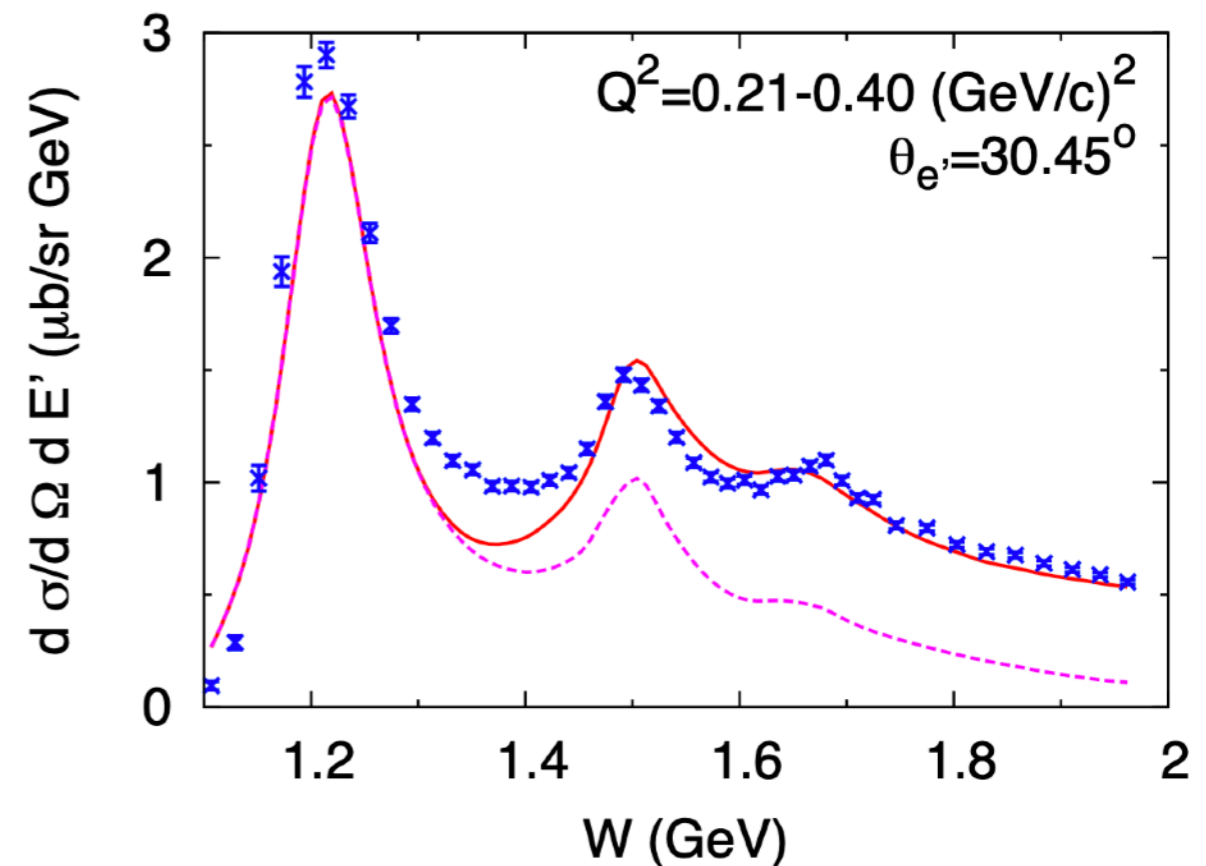
$$\pi N, \gamma N \rightarrow \pi N, \eta N, K \Lambda, K \Sigma$$

Same models can predict neutrino cross-sections, axial contributions constrained by πN through χ PT



Leitner, Buss, Alvarez-Ruso, and Mosel, PRC 79 (2009)

Nakamura, Kamano, and Sato PRD 92 (2015)



GiBUU event generator uses detailed phenomenological resonance model

Vector-current responses taken from MAID analysis, axial-current predicted with χ PT plus neutrino production data for Δ

Improved constraints on N^* resonances (in nuclei!) needed to for precise predictions at DUNE energies

Higher resonances in LQCD

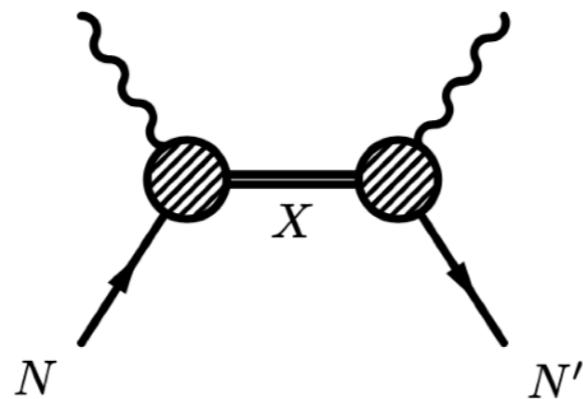
LQCD methods for calculating exclusive cross-sections break down above multi-particle thresholds (≤ 3 hadrons is state-of-the-art)

Reviews: Briceño, Dudek, and Young, *Rev. Mod. Phys.* 90 (2018)

Hansen and Sharpe, *Ann. Rev. Nucl. Part. Sci.* 69 (2019)

How can we use LQCD to constrain higher-energy cross sections, e.g. shallow inelastic scattering region?

The hadron tensor:



Liu and Dong, *PRL* 72 (1994)

Aglietti et al, *Phys. Lett. B* 432 (1998)

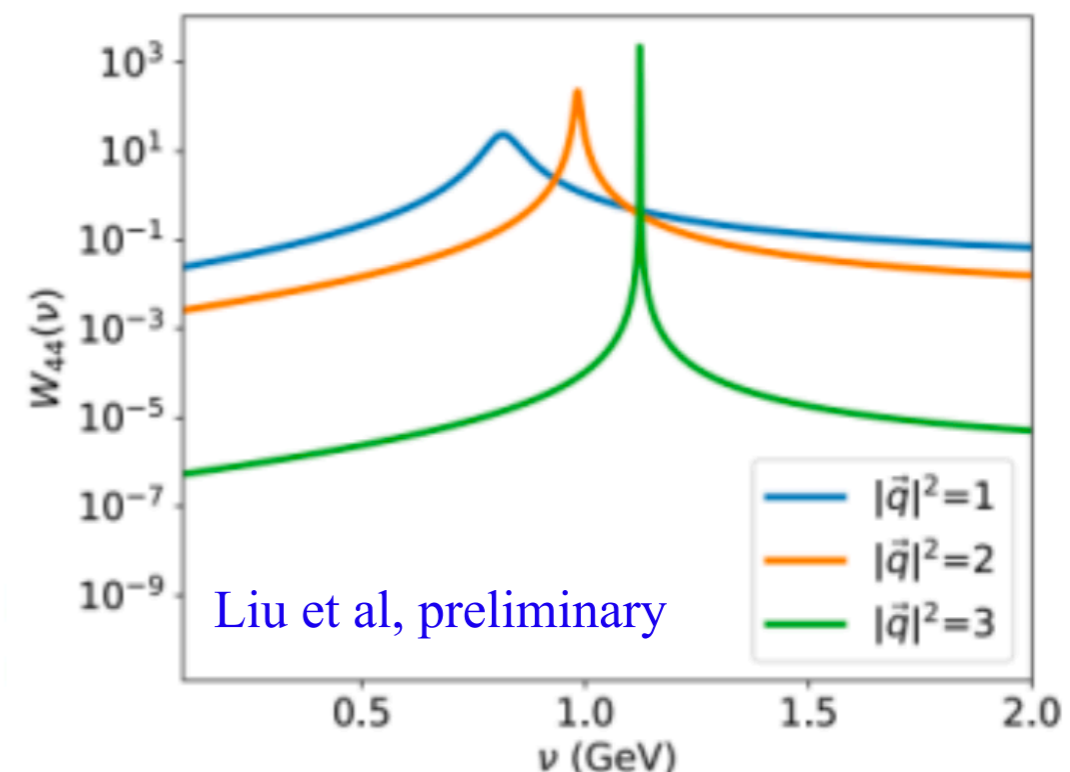
Liu, *PRD* 62 (2000)

Inverse Laplace transform challenging, calculations exploring different methods underway

Liang, Draper, Liu, Rothkopf, and Yang [XQCD] *PRD* 101 (2020)

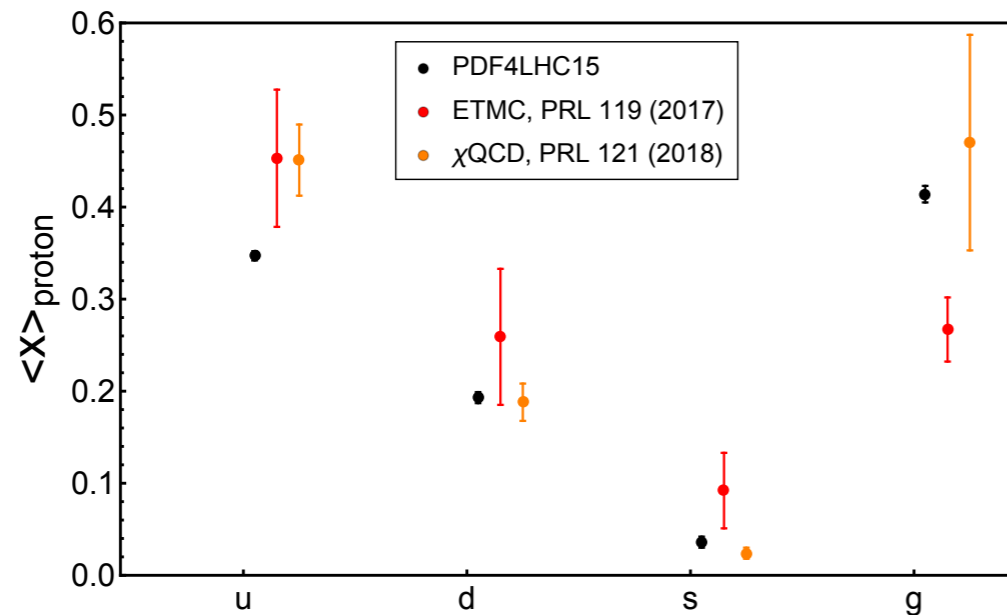
Fukaya, Hashimoto, Kaneko, and Ohki, *PRD* 102 (2020)

After subtracting elastic contributions, resonant behavior consistent with Roper contributions observed in vector-current hadron tensor



DIS and LQCD

LQCD can also compute quantities relevant to neutrino DIS



$\langle x \rangle_{\text{proton}}^q$ and $\langle x \rangle_{\text{proton}}^g$ calculated by several groups

Review: Lin et al, Prog. Part. Nucl. Phys. 100 (2018)

Large momentum effective theory connects Euclidean matrix elements to light-cone PDFs

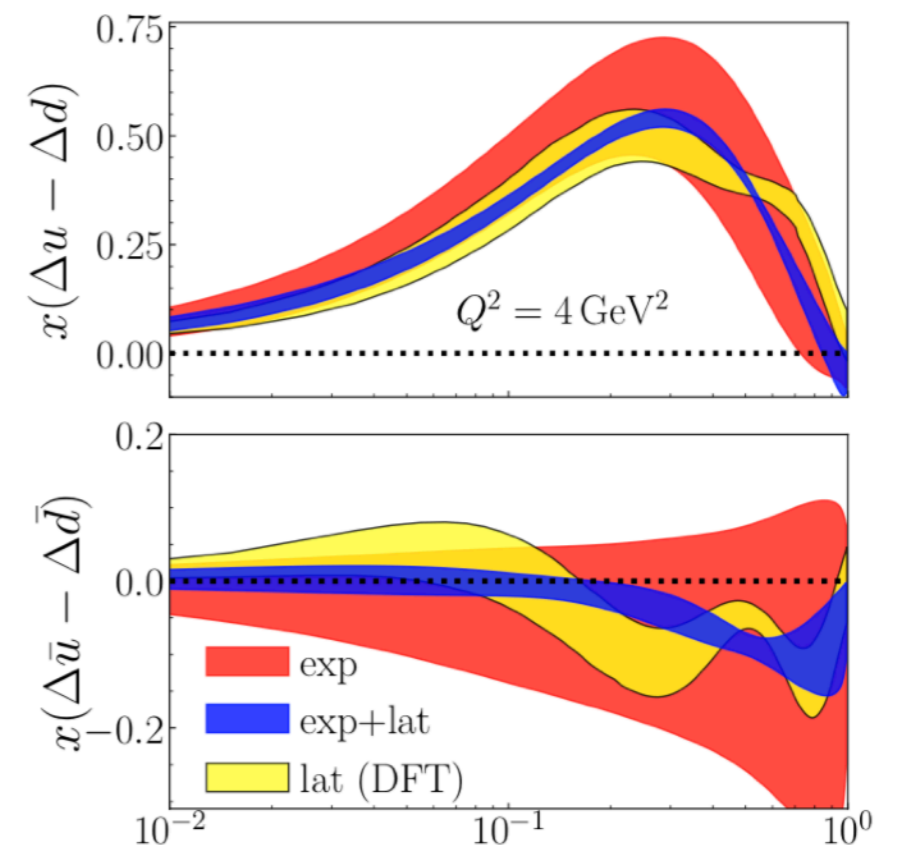
Review: Ji et al, Rev. Mod. Phys. 93 (2021)

Current LQCD results can improve global analyses of isovector polarized PDFs that are relevant for weak interactions in neutrino DIS

Chen, Cohen, Ji, Lin, Zhang, Nucl. Phys. B 911 (2016)

Alexandrou, et al, PRL 121 (2018)

Alexandrou et al, PRL 126 (2021)



Bringewatt et al [JAM], PRD 103 (2021)

Nuclear momentum fractions

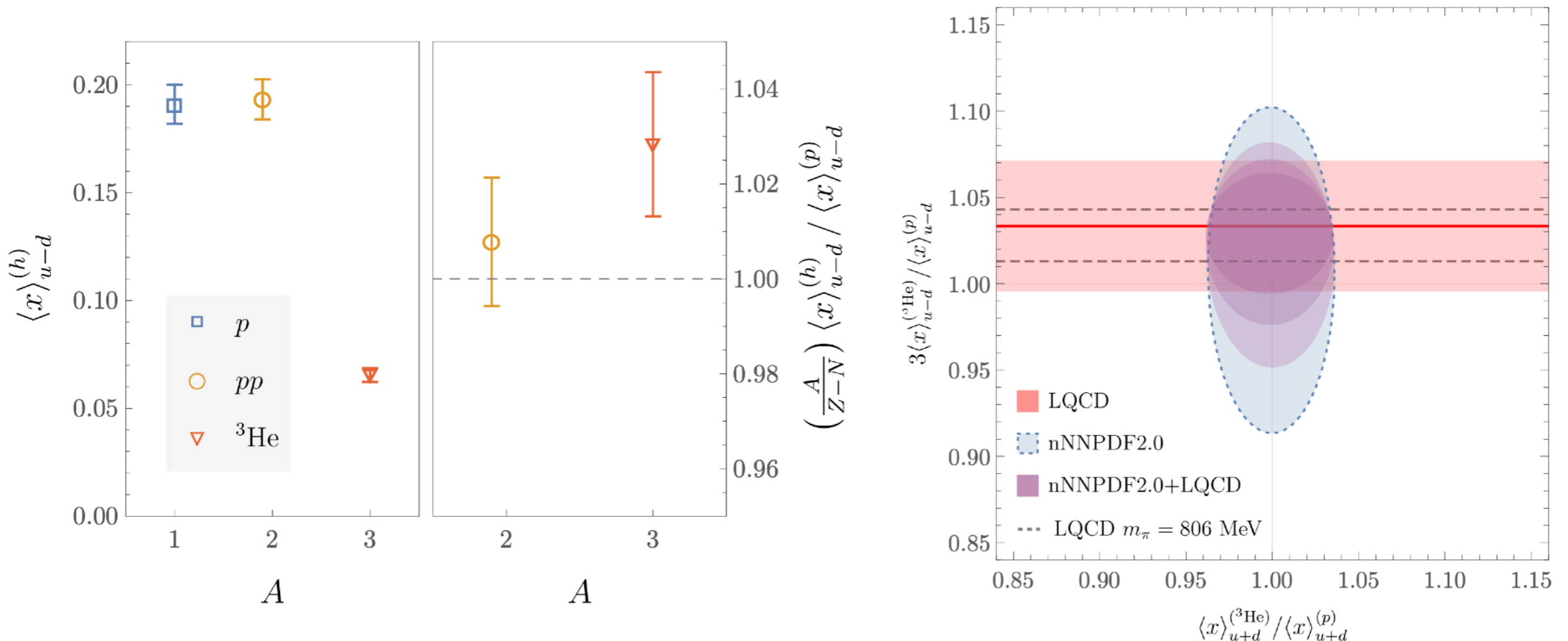
First calculations of gluon and isovector quark momentum fractions of light nuclei

Winter, MW et al [NPLQCD], PRD 96 (2017)

Detmold, MW et al [NPLQCD] PRL 126 (2021)

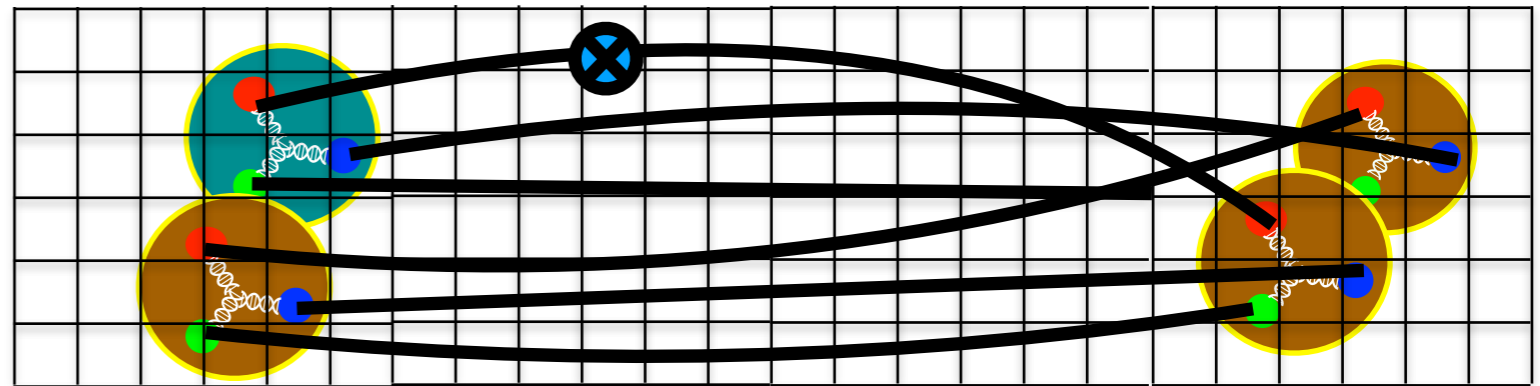
Results matched to pionless EFT to determine two-body current operator relevant to isovector EMC effects

Although systematic uncertainties are not fully controlled (one lattice spacing, volume, quark mass, ...) demonstrates potential for LQCD to usefully constrain nuclear PDFs



Two-body currents in LQCD

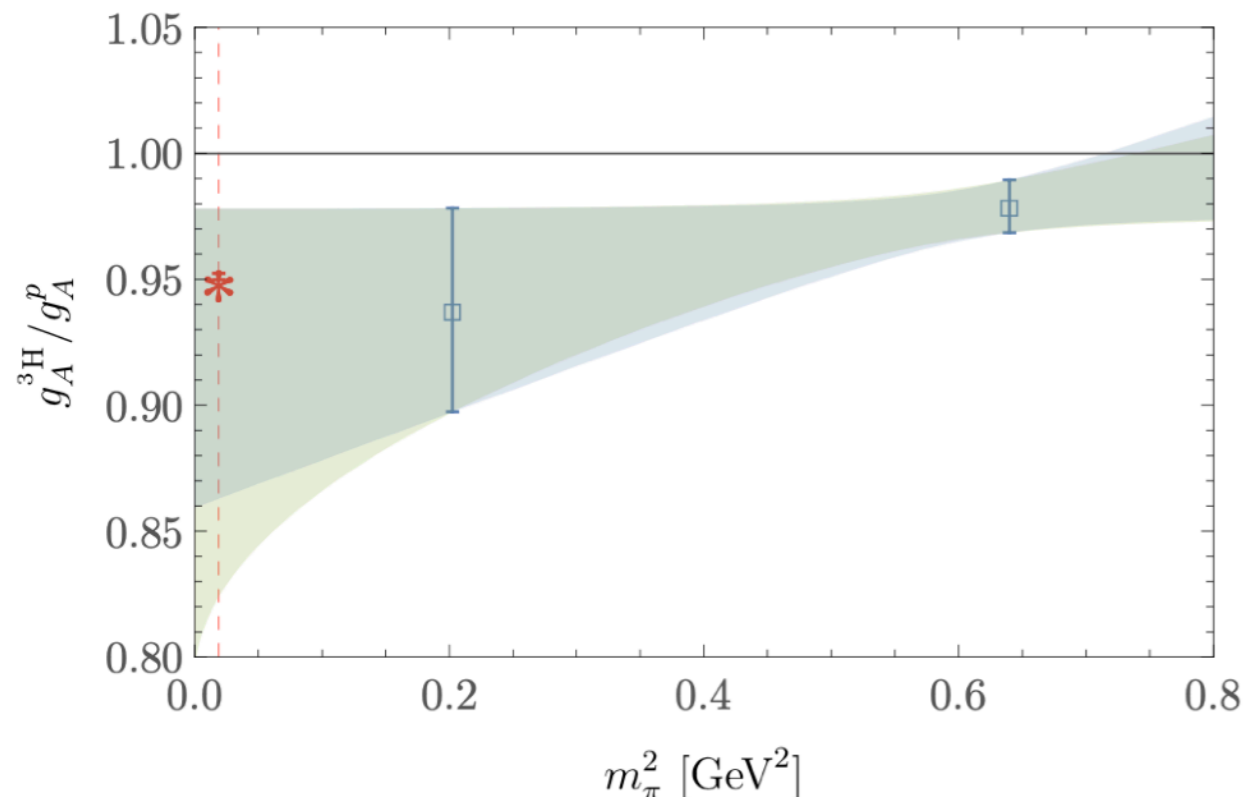
Two-nucleon axial matrix elements relevant for proton-proton fusion computed, used to constrain two-body currents



Savage, MW et al [NPLQCD], PRL 119 (2017)

Flavor decomposition of axial matrix elements of two and three nucleon systems computed with $m_\pi = 806$ MeV

Chang, MW et al [NPLQCD], PRL 120 (2018)



Parreño, MW et al [NPLQCD] PRD 103 (2021)

Axial current matrix element calculations with $m_\pi = 450$ MeV permit preliminary extrapolations to physical masses

Several systematic uncertainties remain, but encouraging agreement with experiment seen

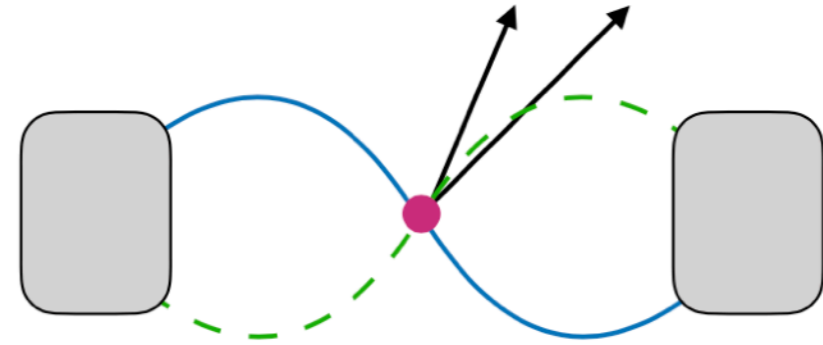
Similar methods used to study two-nucleon matrix elements for double-beta decay:

Shanahan, MW et al [NPLQCD], PRL 119 (2017)

$0\nu\beta\beta$ in LQCD

Matrix elements for $\pi^- \rightarrow \pi^+ e^- e^-$ arising from short-distance new physics mechanisms computed

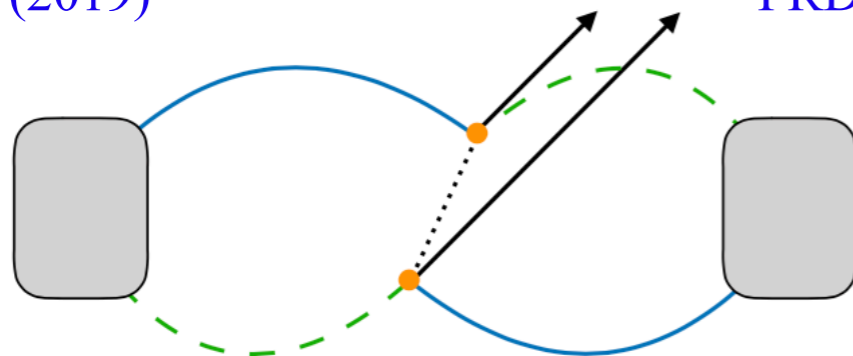
Nicholson et al, PRL 121 (2018)



Cirigliano, Detmold, Nicholson, Shanahan, arXiv:2003.08493

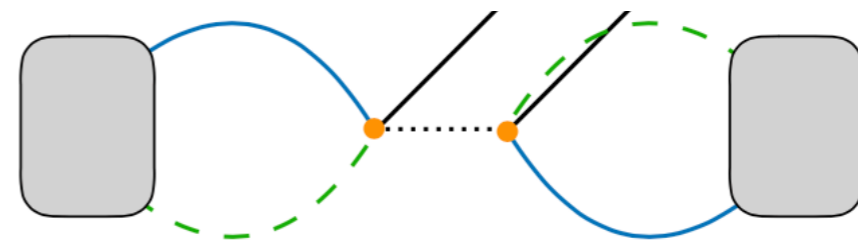
Long-distance Majorana neutrino exchange matrix elements computed for $\pi^- \rightarrow \pi^+ e^- e^-$ and $\pi^- \pi^- \rightarrow e^- e^-$

Feng, Jin, Tuo, and Jia, PRL 122 (2019)



Tuo, Feng, and Jin, PRD 100 (2019)

Detmold and Murphy, arXiv:2004.07404



In nuclear effective theories, short-distance contact operator appears at leading order

Cirigliano et al, PRL 120 (2018)

LQCD calculations of $nn \rightarrow ppe^- e^-$ long-distance matrix elements can constrain two-body contact operator, exploratory calculations underway

Anthony Grebe, Lattice 2023

Systematic uncertainties

Several systematic uncertainties remain to be quantified in detail

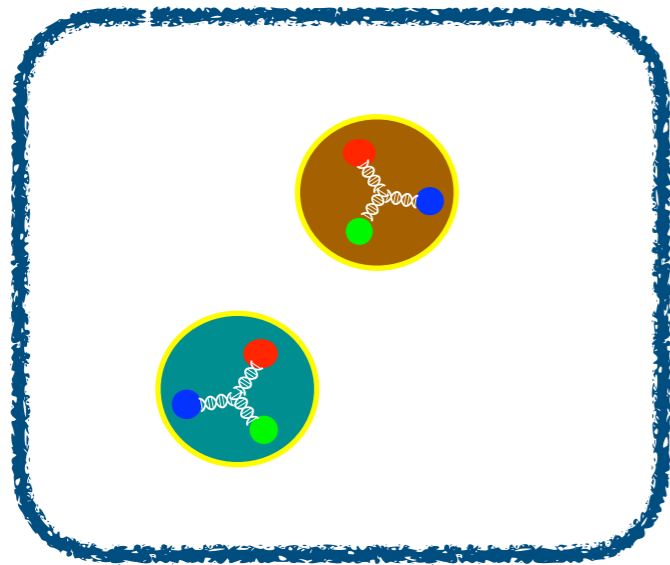
- Heavier than physical quark masses only
- One lattice spacing
- Excited-state effects

Systematic uncertainties

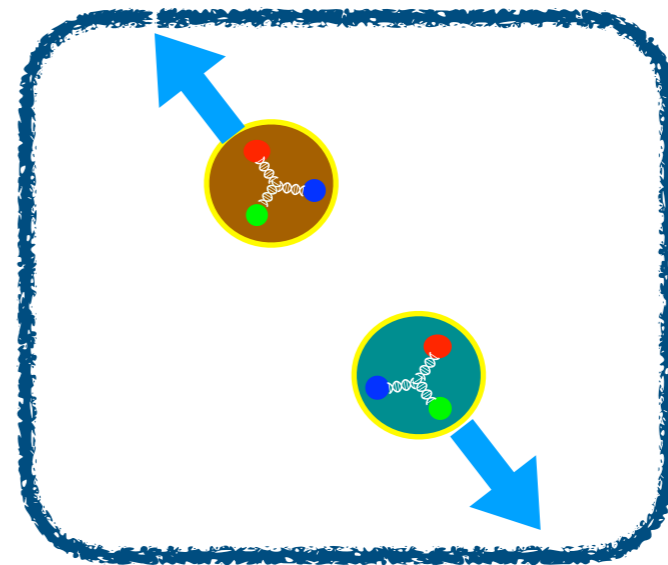
Several systematic uncertainties remain to be quantified in detail

- Heavier than physical quark masses only
- One lattice spacing
- Excited-state effects

Gap between ground and two-nucleon finite-volume “scattering” states becomes small for large volumes



$$E_0 \sim 2M$$



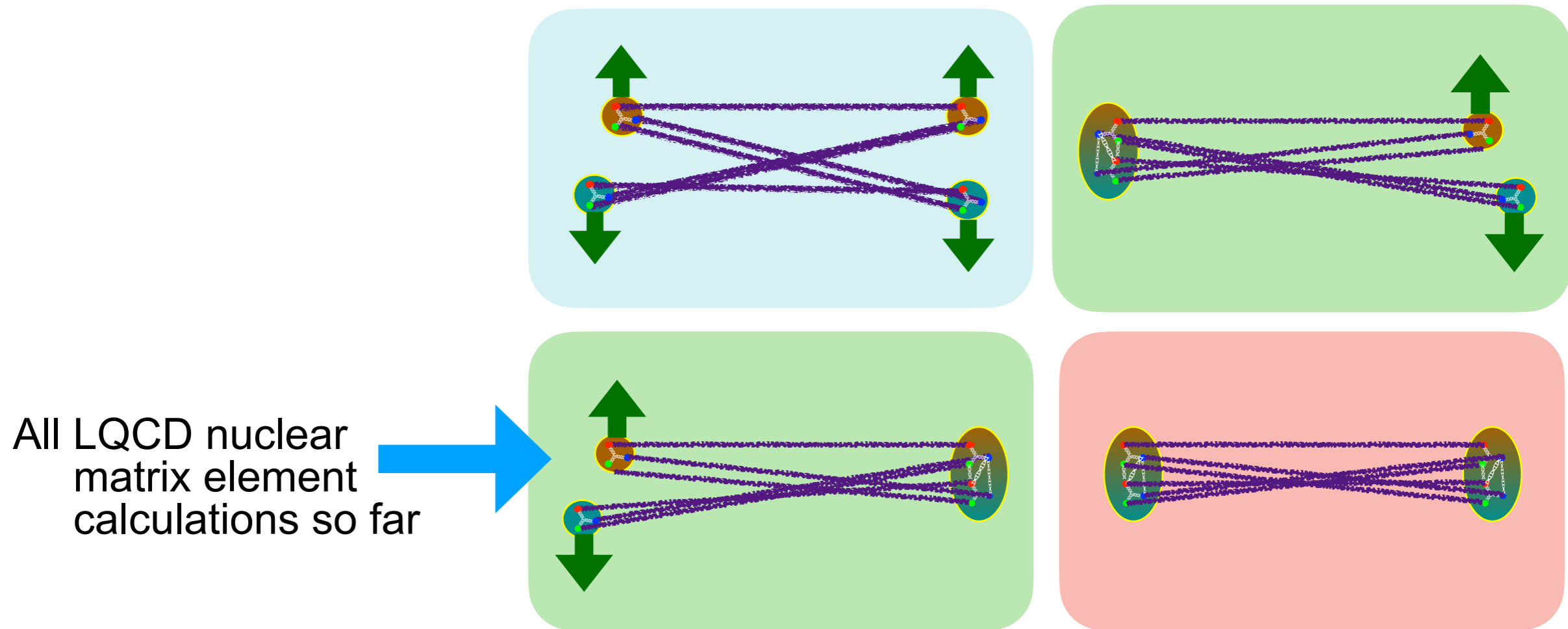
$$E_1 \sim 2M + \delta$$

$$\delta \propto 1/(\text{box length})^2$$

Imaginary-time evolution only effectively suppresses excited states when $e^{-t\delta} \ll 1$, but achieving $t \gg 1/\delta$ is computationally unfeasible for large volumes

Variational methods

Robust upper bounds on energy spectrum can be obtained by diagonalizing symmetric matrices of correlation functions



Although application of variational methods to multi-nucleon systems has long been advocated, it has only recently become computationally feasible

Distillation:

[Peardon et al PRD 80 \(2009\)](#)

[Morningstar et al PRD 83 \(2011\)](#)

Sparsening:

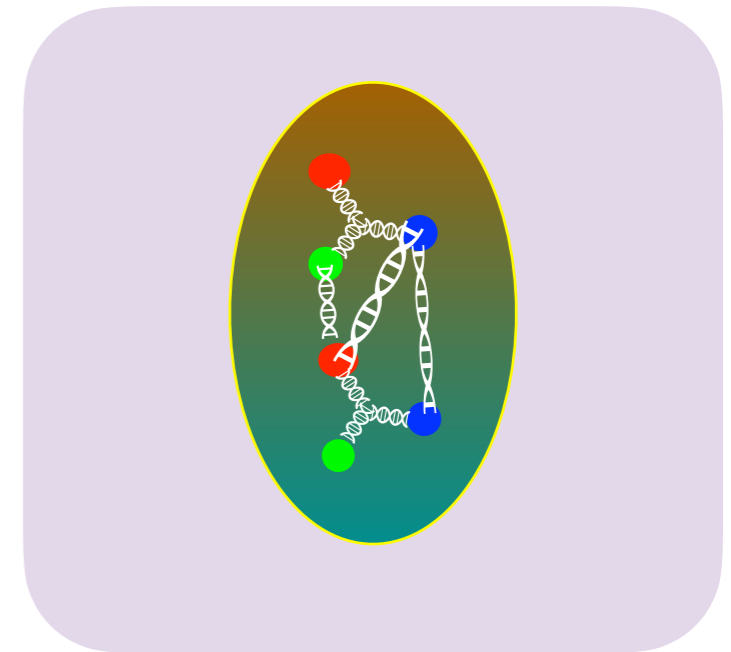
[Detmold, MW et al, PRD 104 \(2021\)](#)

[Li et al, PRD 103 \(2021\)](#)

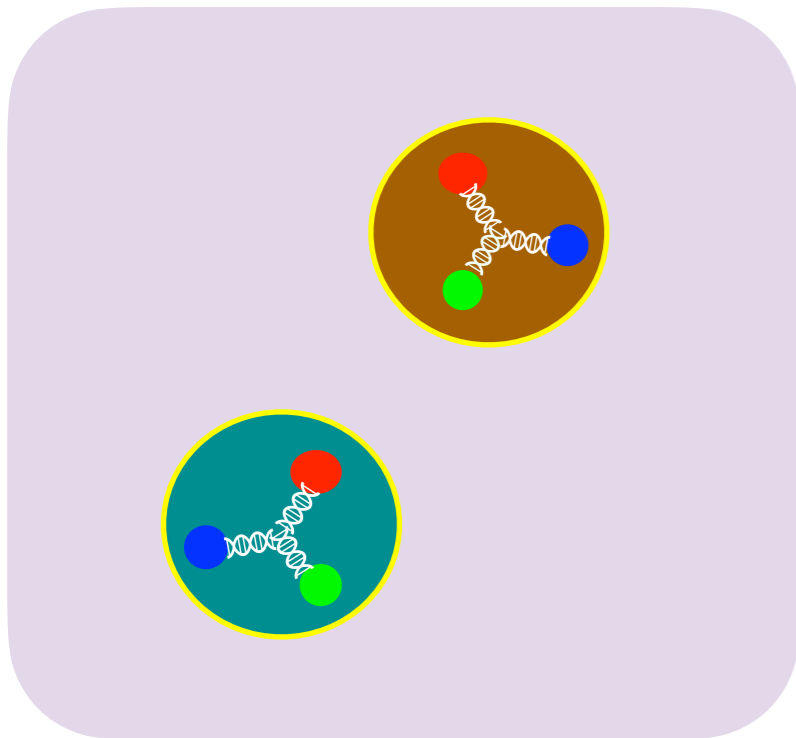
Six-quark operator catalog

Many six-quark operators have the right quantum numbers to describe two-nucleon systems

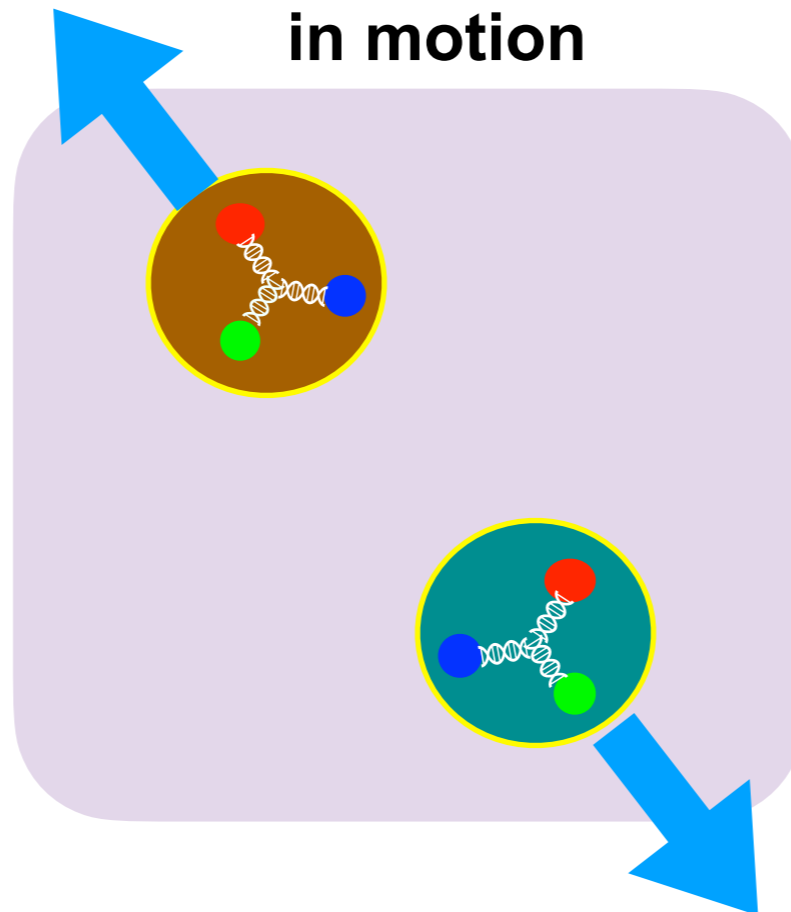
Hexaquark



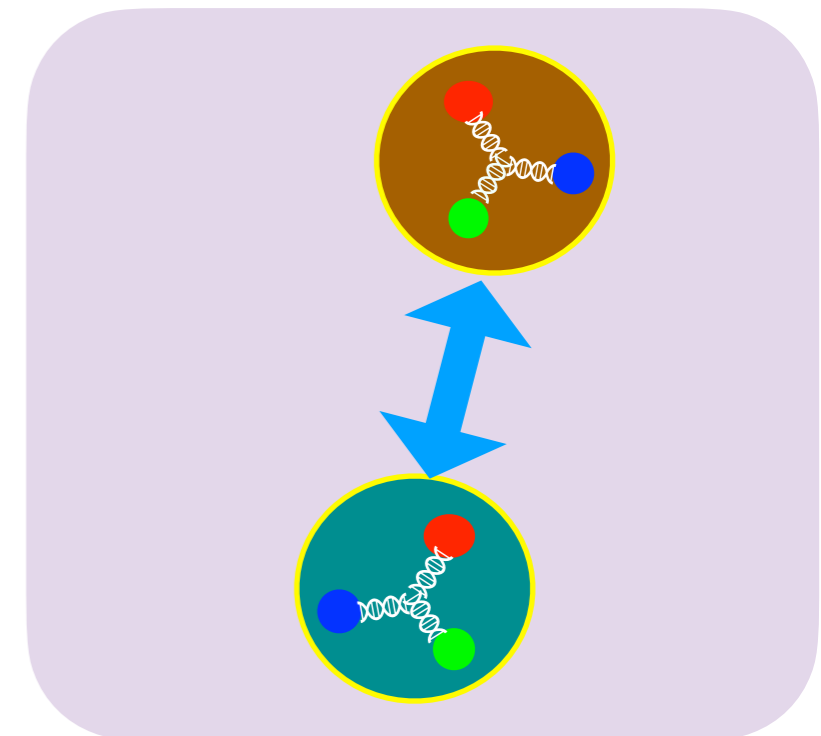
Two nucleons at rest



Two nucleons in motion



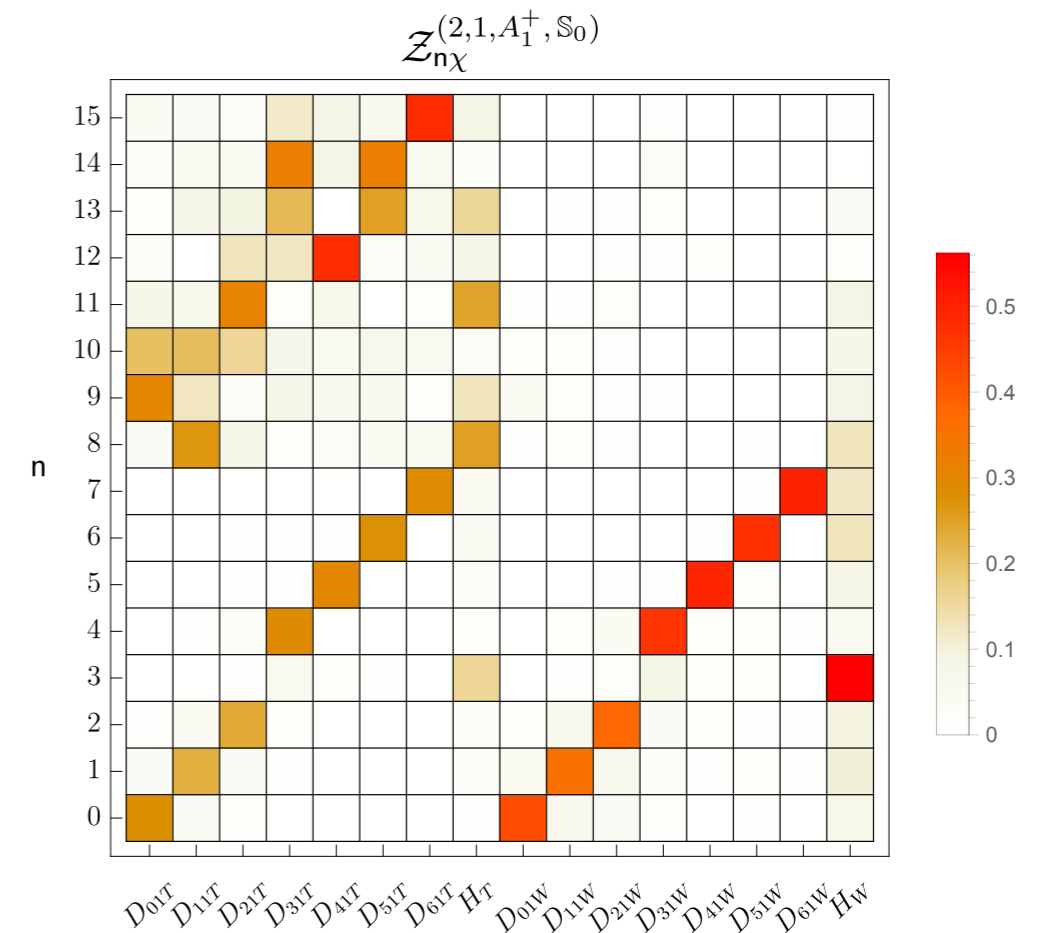
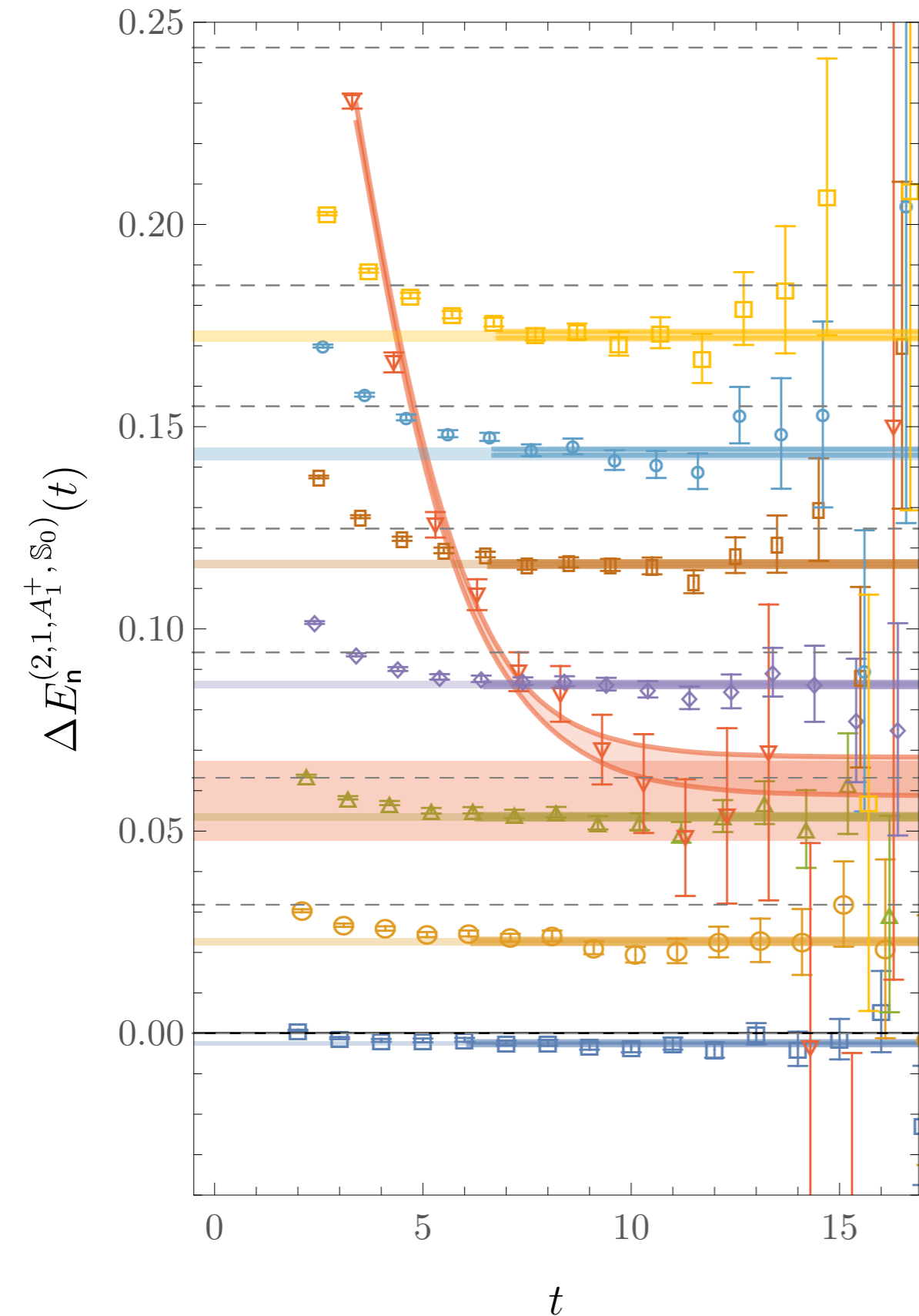
Two exponentially localized nucleons



Two neutrons in a box

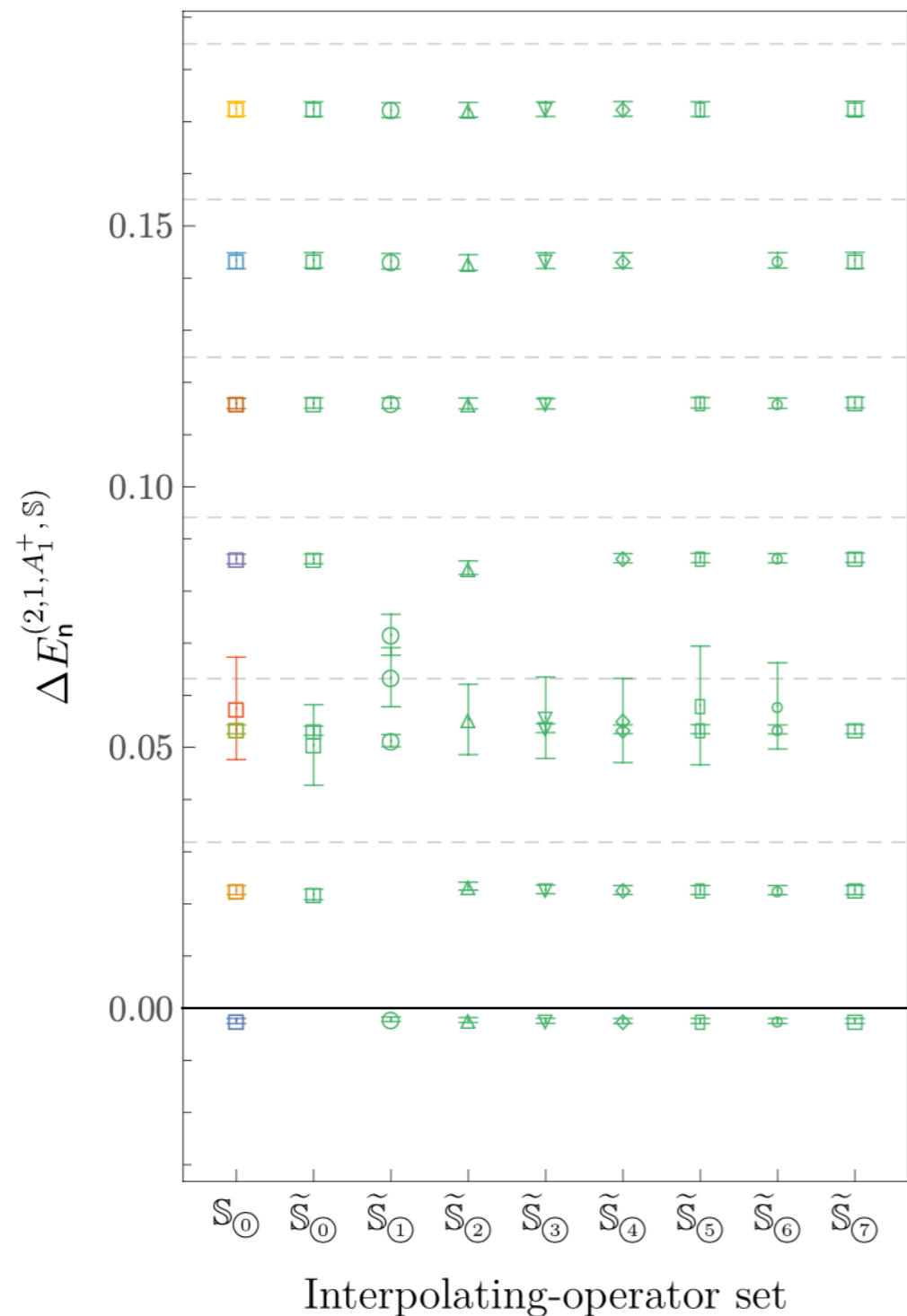
Diagonalization of correlation-function matrices can be used to remove excited-state contamination from states strongly overlapping with other operators

Each energy level dominantly overlaps with one operator structure, sub-dominant operators collectively 30%



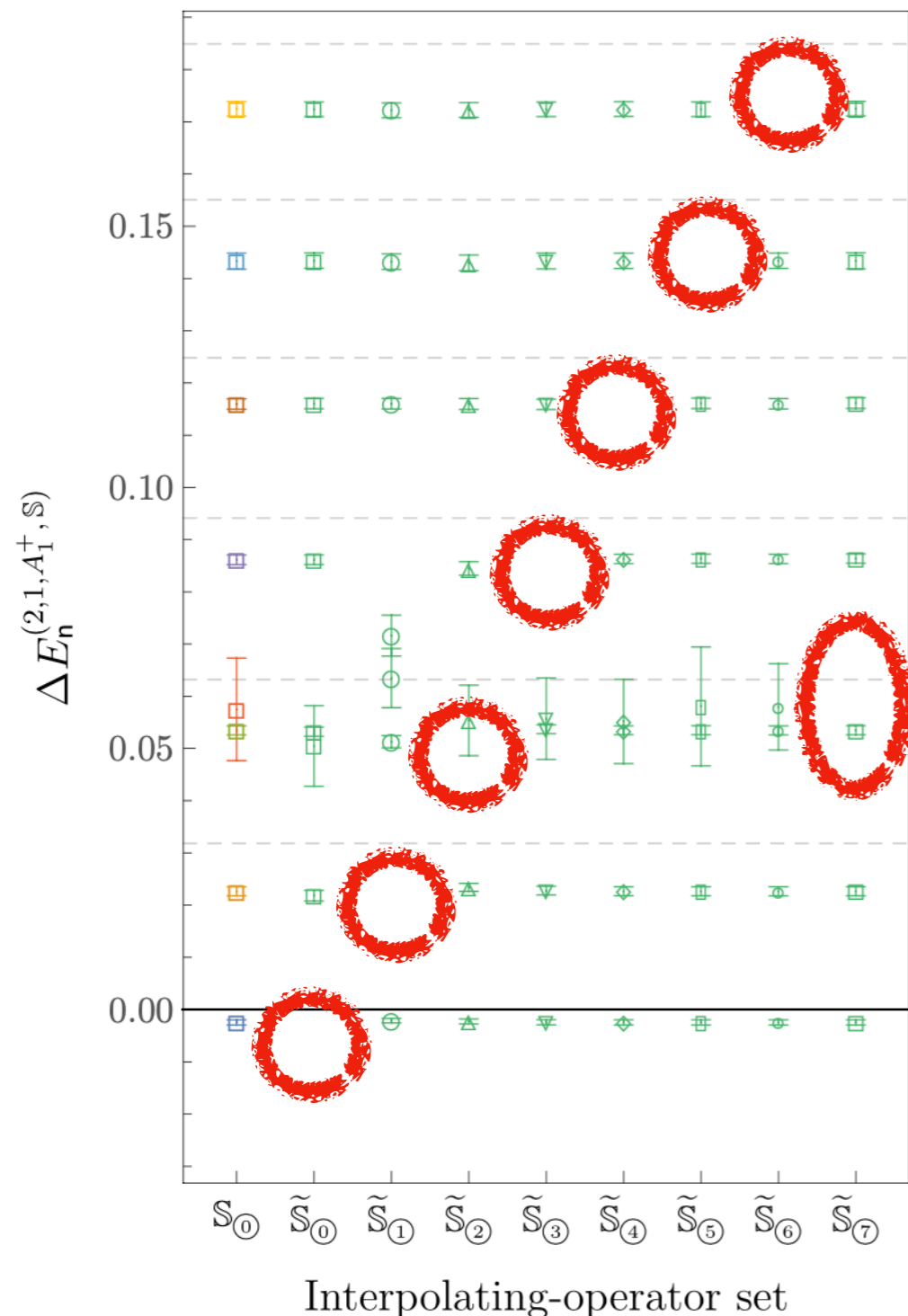
Interpolating operator dependence

Removing interpolating operators leads to “missing energy levels” for states dominantly overlapping with omitted operators

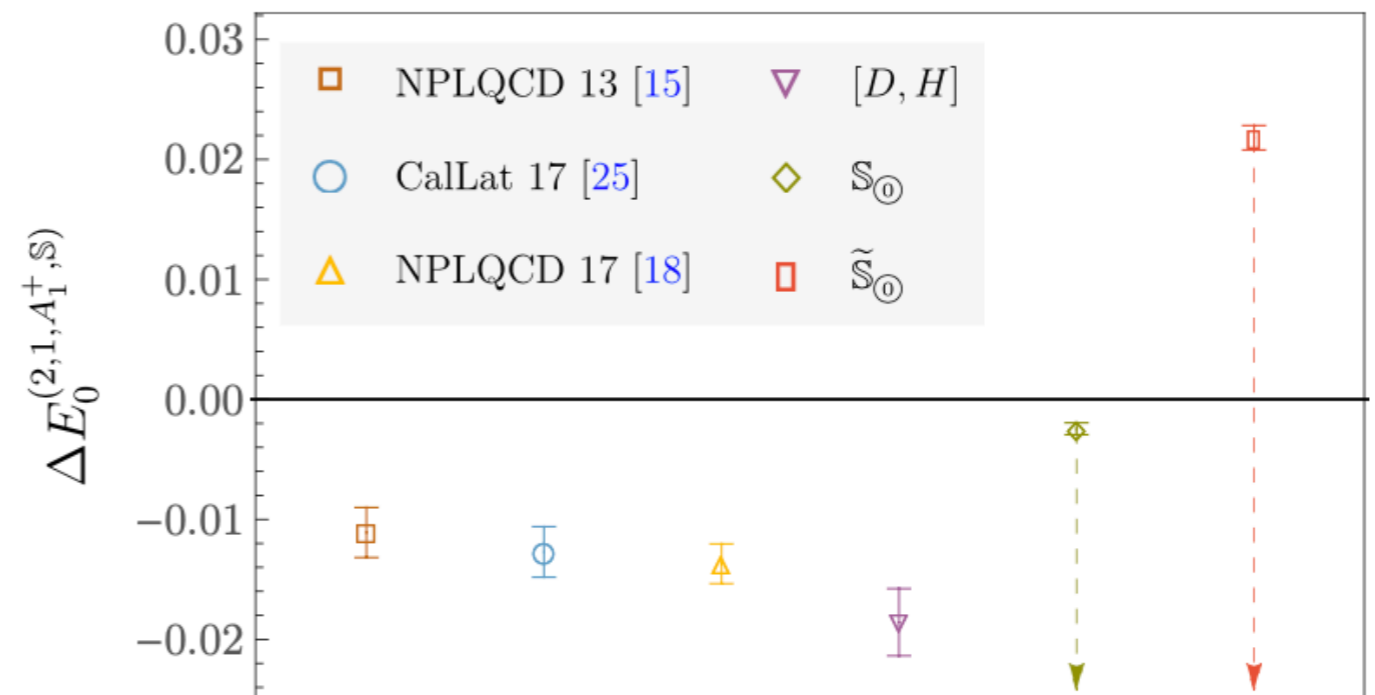


Interpolating operator dependence

Removing interpolating operators leads to “missing energy levels” for states dominantly overlapping with omitted operators



Variational upper bounds obtained using different interpolating operator sets are consistent



Ground-state energy **estimates** using different interpolating-operator sets show large discrepancies

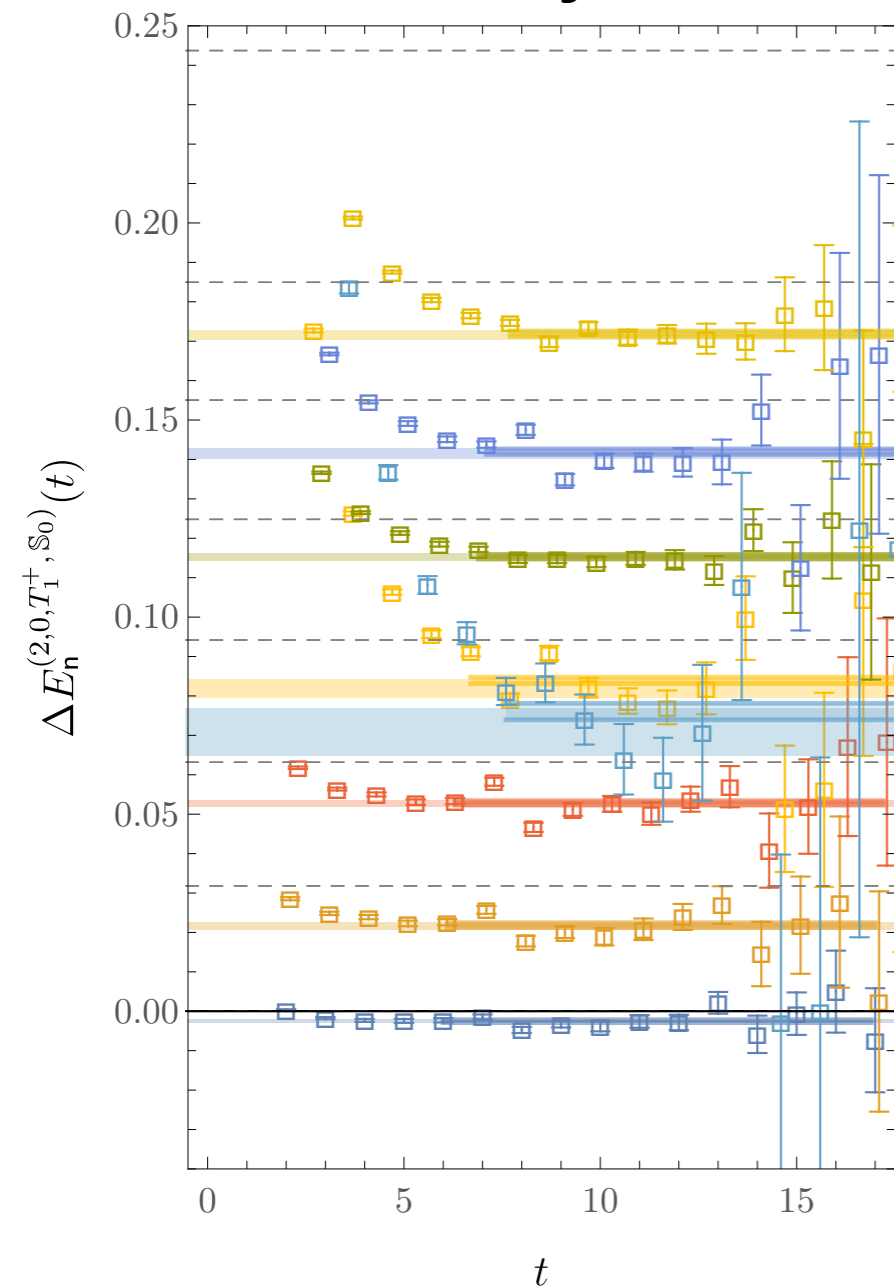


The deuteron channel

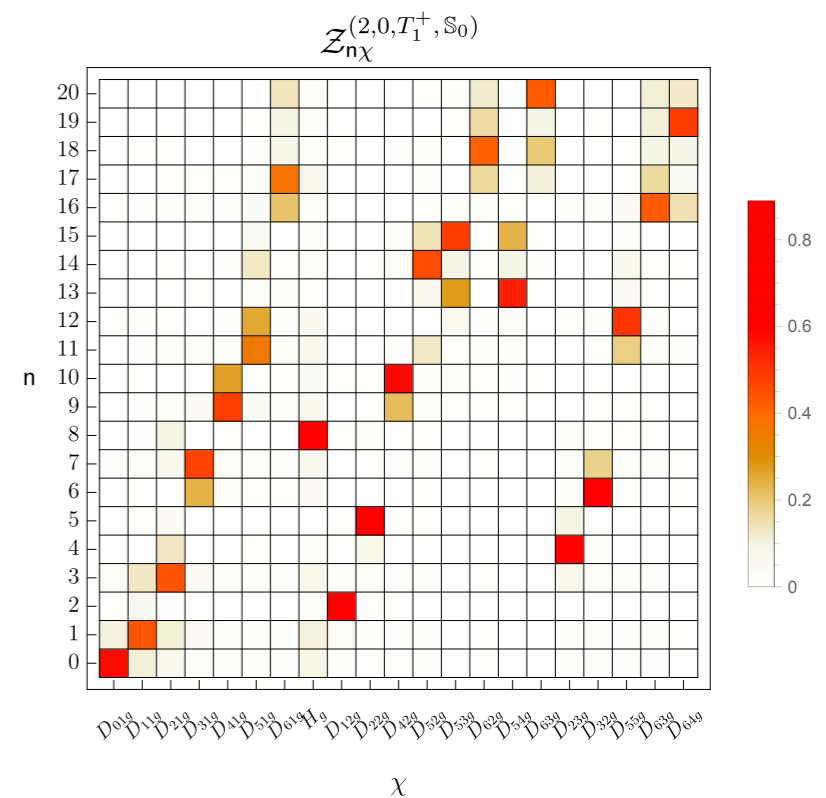
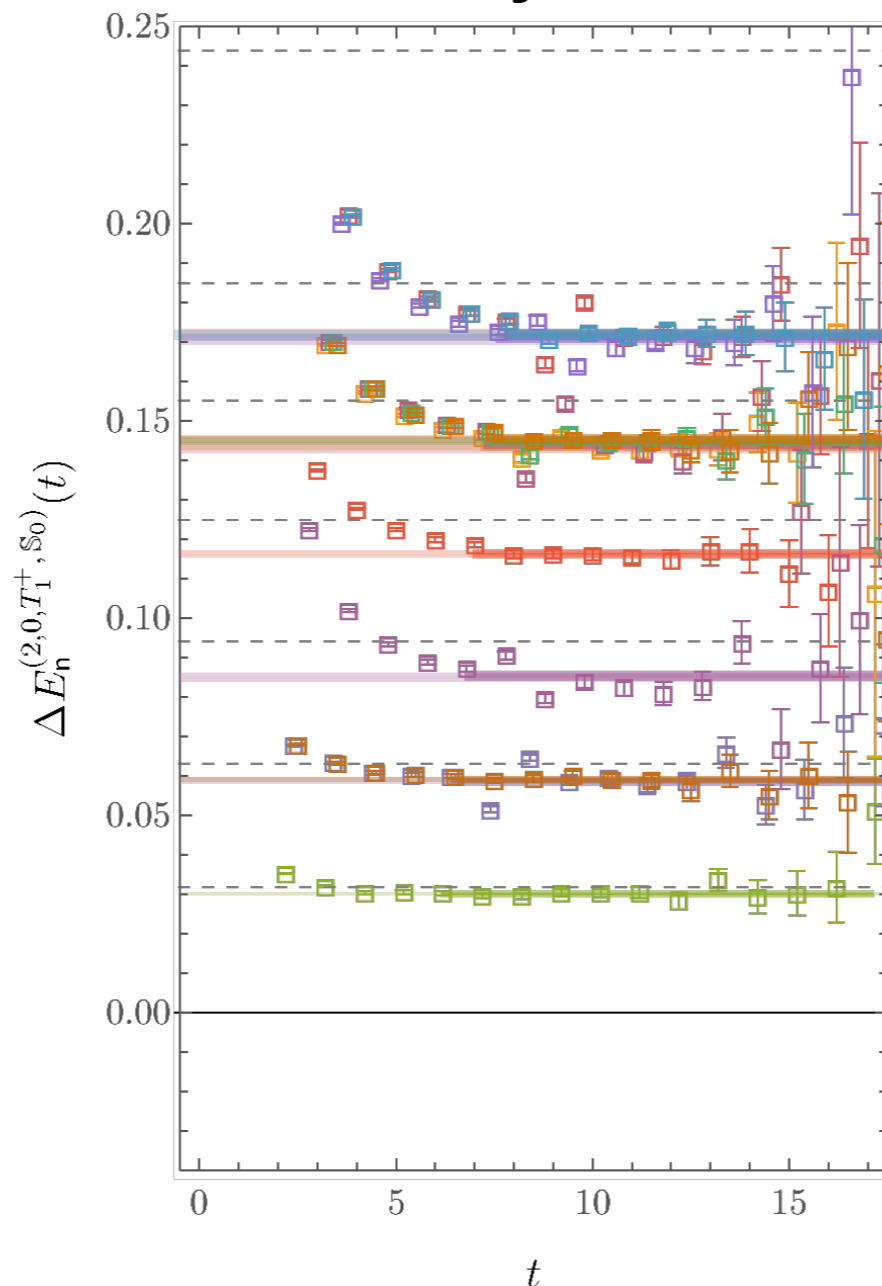
Spin-orbit coupling complicates the deuteron channel

Finite-volume analogs of S -wave and D -wave operators included to provide a complete set of dibaryon operators with sufficiently low relative momentum

Dominantly S-wave



Dominantly D-, G-, or I-wave

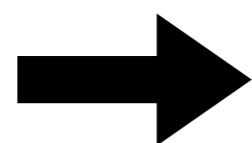
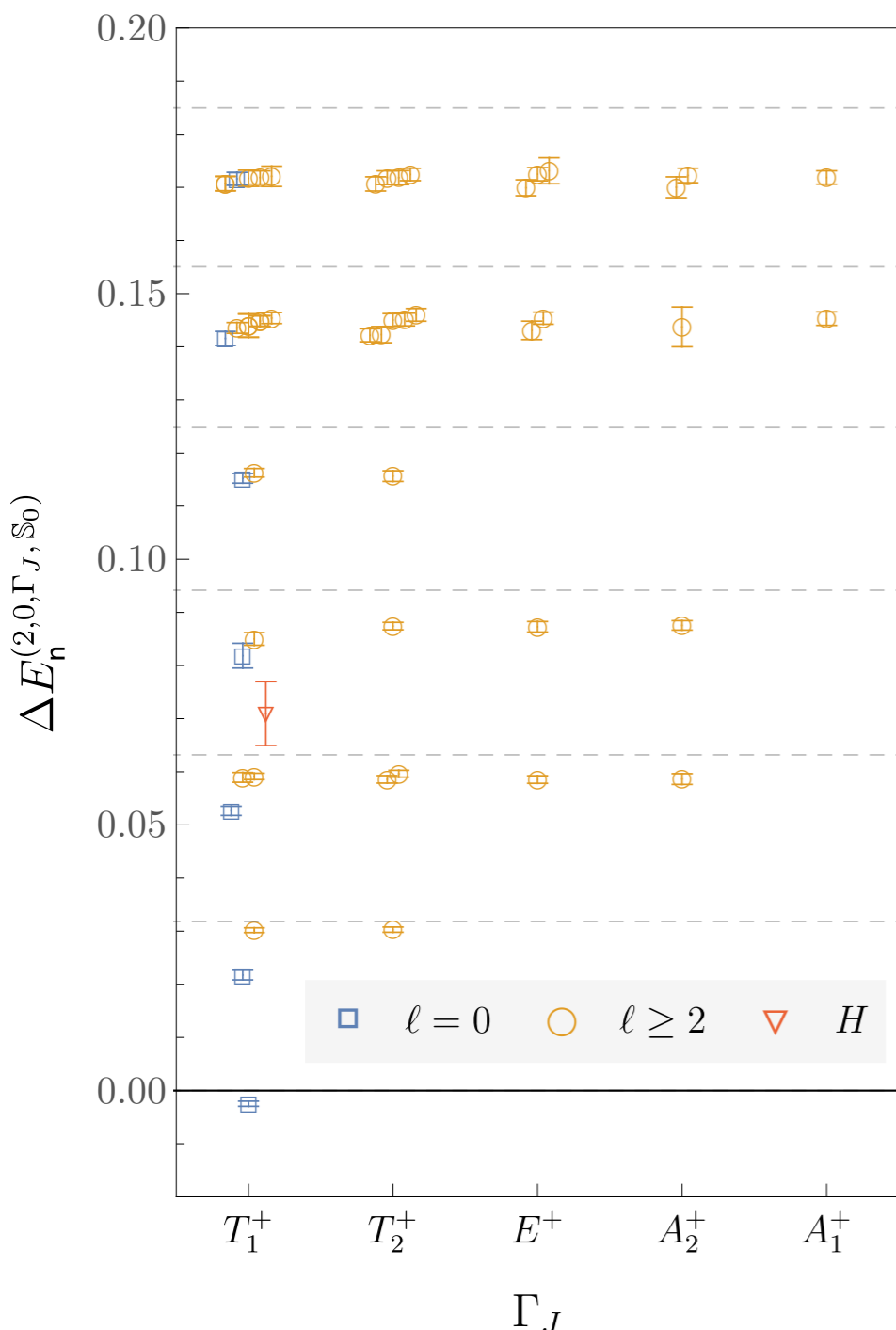


Low-energy states again have majority overlap with 1 operator structure

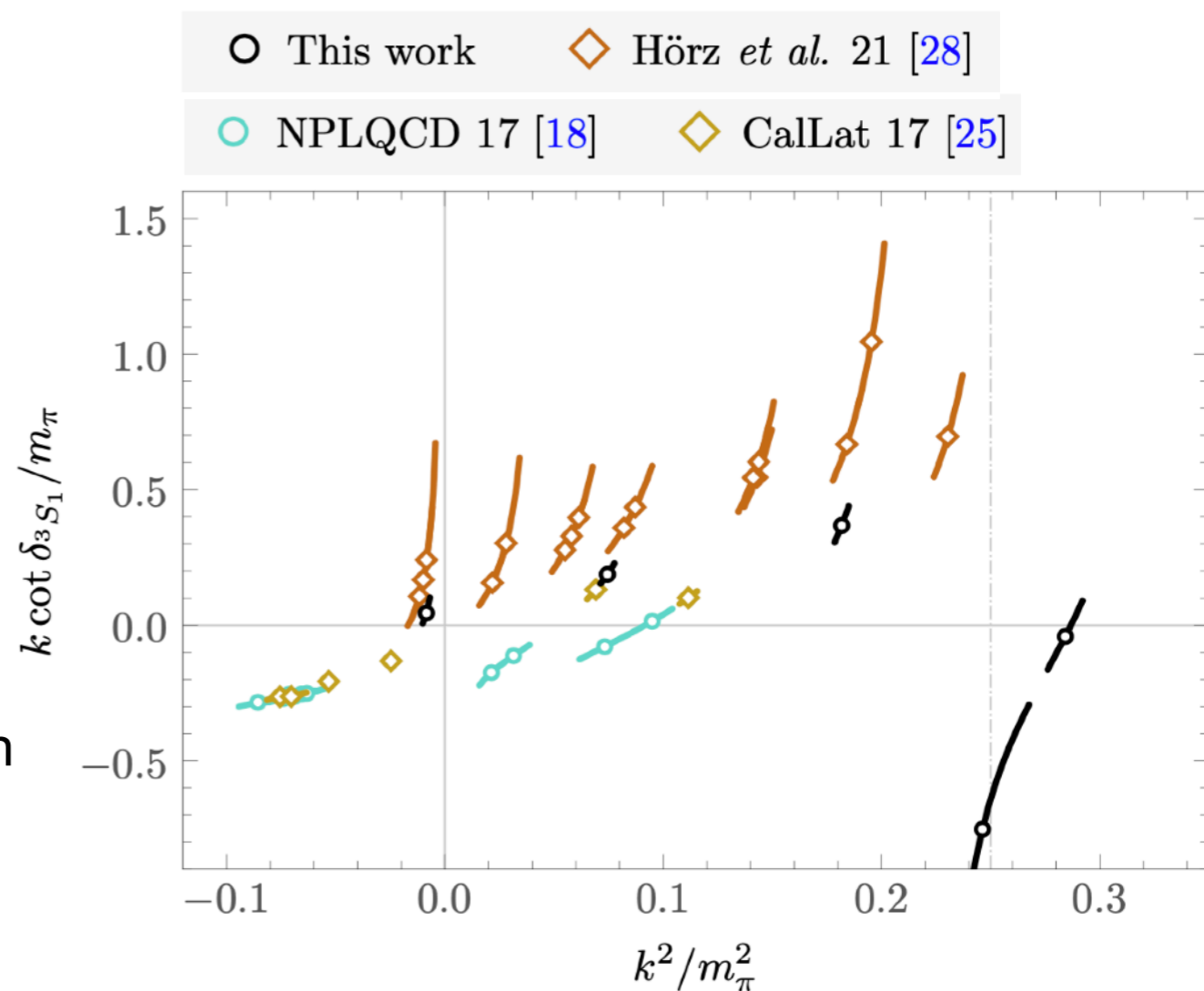
Towards NN scattering from LQCD

Variational calculations including a wide range of two-nucleon operators lead to precise constraints on two-nucleon energy spectra and phase shifts

Deuteron channel GEVP spectrum



Lüscher quantization condition



- Consistency among studies with similar interpolating operators
- Significant discrepancies between calculations with different operators

Further study needed!

Outlook

Discovering lepton CP violation and precisely measuring neutrino oscillations at DUNE and Hyper-Kamiokande will require precise theory predictions of neutrino-nucleus scattering

Uncertainty quantification studies using multiple nuclear models show that more precise one- and two-nucleon inputs are needed for percent-level accuracy

Lattice QCD calculations of axial form factors are becoming mature, stay tuned for precise predictions of pion production amplitudes and two-body currents

