# PDFs and electroweak interactions

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# Electroweak interactions and double logarithms

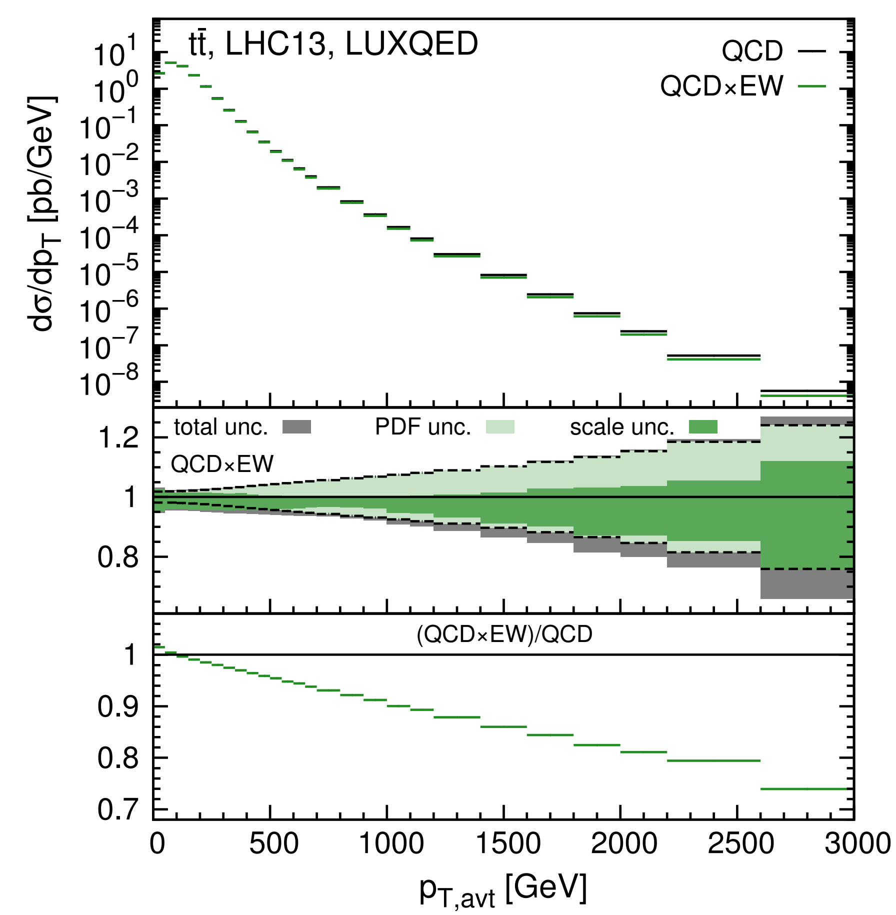
- Size of the couplings:

$$\alpha_s \sim 0.1\,, \quad \alpha \sim 0.01\,, \quad \alpha_2 = \frac{\alpha}{\sin^2\theta_W} \sim 0.03$$

- Expect NNLO QCD ~ NLO EW, but enhanced by double logs:
[Ciafaloni, Comelli; Kuhn, Penin; Fadin et al; Beenakker, Werthenbach; Denner, Pozzorini; Kuhn et al; Denner et al; Chiu et al; …]

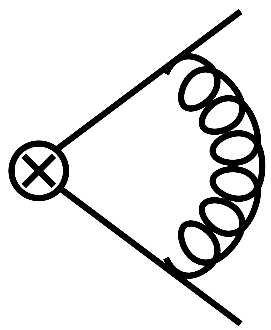
$$\alpha_2 \ln^2 \frac{Q^2}{M_W^2} \sim 0.6 \quad \text{for} \quad Q = 1\ \text{TeV}$$

- Important when searching for new physics in high $p_T$ tails

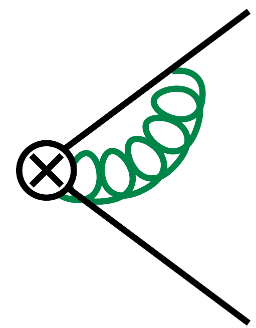



[Czakon et al]

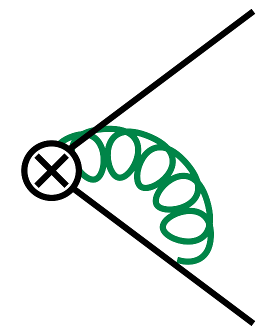
# Example: electroweak Sudakov at one loop



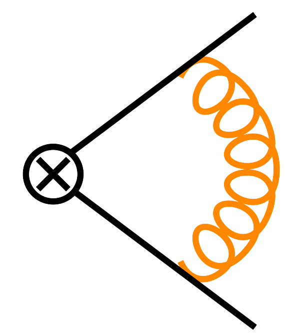
$= C^{(1)}(Q,\mu)+$



$+$



$+$



$+\,\mathcal{O}\left(\frac{M^2}{Q^2}\right)$

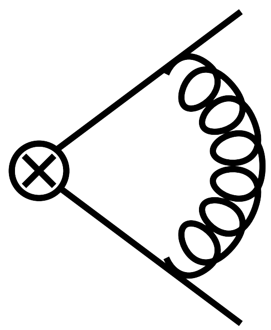
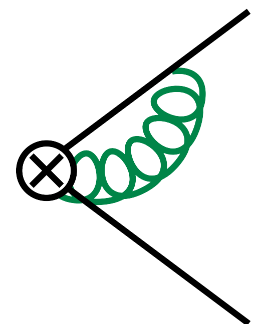
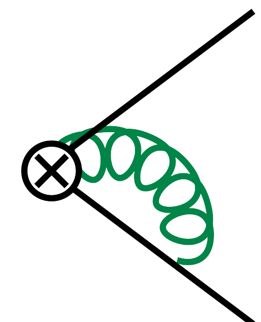
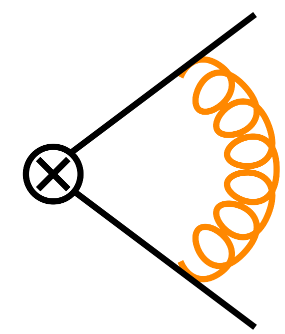
- Sudakov logarithms can be obtained in the **symmetric** phase, expanding in collinear and soft limit:

$$C^{(1)}(Q,\mu) = \frac{\alpha_2 c_F}{4\pi}\left[-\ln^2\frac{Q^2}{\mu^2} + 3\ln\frac{Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

$$\text{coll} + \text{soft} = \frac{\alpha_2 c_F}{4\pi}\left[2\ln\frac{Q^2}{\mu^2}\ln\frac{M^2}{\mu^2} - \ln^2\frac{M^2}{\mu^2} - 3\ln\frac{M^2}{\mu^2} + \frac{9}{2} - \frac{5\pi^2}{6}\right]$$

$$+\quad \frac{\alpha_2 c_F}{4\pi}\left[-\ln^2\frac{Q^2}{M^2} + 3\ln\frac{Q^2}{M^2} - \frac{7}{2} - \frac{2\pi^2}{3}\right]$$

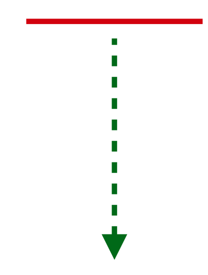
# Example: electroweak Sudakov at one loop

 $= C^{(1)}(Q,\mu) +$  $+$  $+$  $+ \mathcal{O}\Big(\frac{M^2}{Q^2}\Big)$

- Sudakov logarithms can be obtained in the **symmetric** phase, expanding in collinear and soft limit

$$C^{(1)}(Q,\mu) = \frac{\alpha_2 c_F}{4\pi}\left[-\ln^2\frac{Q^2}{\mu^2} + 3\ln\frac{Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

$$\begin{array}{l} \text{coll} + \text{soft} = \dfrac{\alpha_2 c_F}{4\pi}\left[2\ln\dfrac{Q^2}{\mu^2}\ln\dfrac{M^2}{\mu^2} - \ln^2\dfrac{M^2}{\mu^2} - 3\ln\dfrac{M^2}{\mu^2} + \dfrac{9}{2} - \dfrac{5\pi^2}{6}\right] \\ \hline \dfrac{\alpha_2 c_F}{4\pi}\left[-\ln^2\dfrac{Q^2}{M^2} + 3\ln\dfrac{Q^2}{M^2} - \dfrac{7}{2} - \dfrac{2\pi^2}{3}\right] \end{array} +$$




- Factorization of physics at hard scale $Q$ and electroweak scale $M$ → enables resummation of $\alpha_2^n \ln^m(Q^2/M^2)$
- Requires treatment of rapidity logarithms

# Today's talk

- At high energies EW effects should be treated inclusively:
  e.g. W bosons collinear to beam are not resolved ➞ EW PDFs
- We will consider Drell-Yan for definiteness: $pp \to \ell\bar{\ell}X$

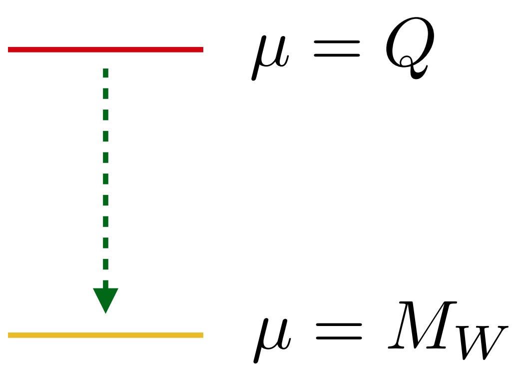
Key features:

- Proton is not EW singlet: $f_u \neq f_d$
  ➞ double logs and soft function in inclusive cross sections
- *SU*(2) is chiral: polarization can't be ignored,
  EW evolution will polarize beams

Based on arXiv:1802.08687, 1803.06347
with Aneesh Manohar and Bartosz Fornal

# Plan: A) factorization, B) evolution, C) matching

- A) Hard scale $Q$: work in symmetric phase, match onto SCET factorize cross section into collinear (and soft!) functions.
- B) Evolve to symmetry-breaking scale $M_W$ including EW effects.
- C) At $M_W$ match onto the broken phase.

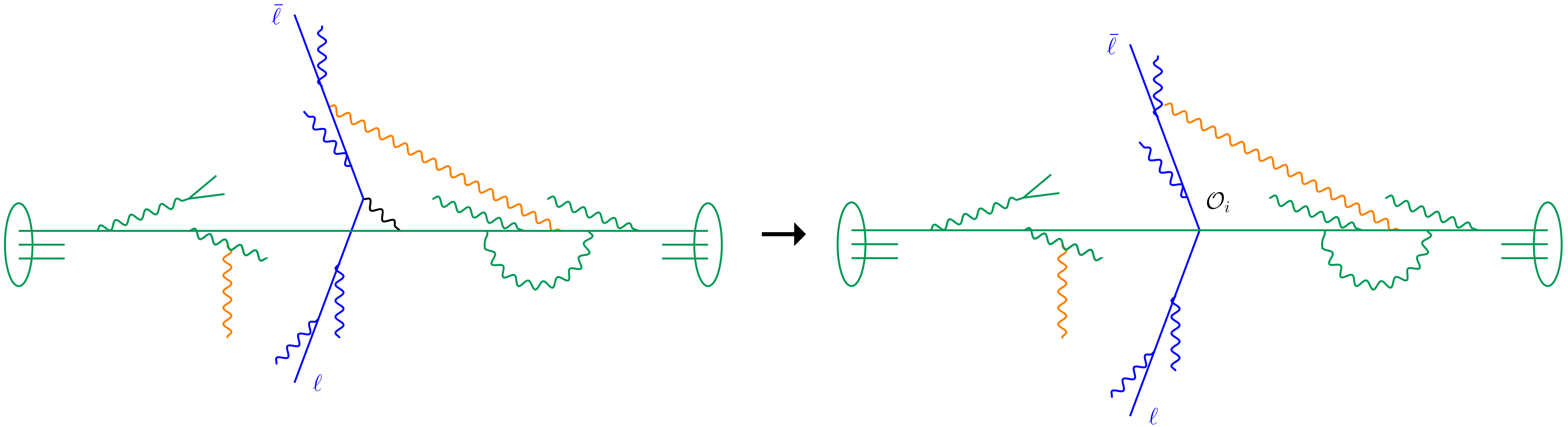



# A) Factorization of hard

- Integrate out hard scattering at $\mu \sim Q$ in **symmetric** phase

$$\mathcal{L}_{\text{hard}} = \sum_i \mathcal{H}_i O_i \qquad \begin{aligned} O_{\ell q}^{(3)} &= (\bar{\ell}_1 \gamma^\mu t^a \ell_2)\,(\bar{q}_3 \gamma_\mu t^a q_4) \\ O_{\ell q} &= (\bar{\ell}_1 \gamma^\mu \ell_2)\,(\bar{q}_3 \gamma_\mu q_4) \\ O_{\ell u} &= (\bar{\ell}_1 \gamma^\mu \ell_2)\,(\bar{u}_3 \gamma_\mu u_4) \\ &\vdots \end{aligned}$$

- Remaining radiation is collinear or soft



# A) Factorization of collinear and soft

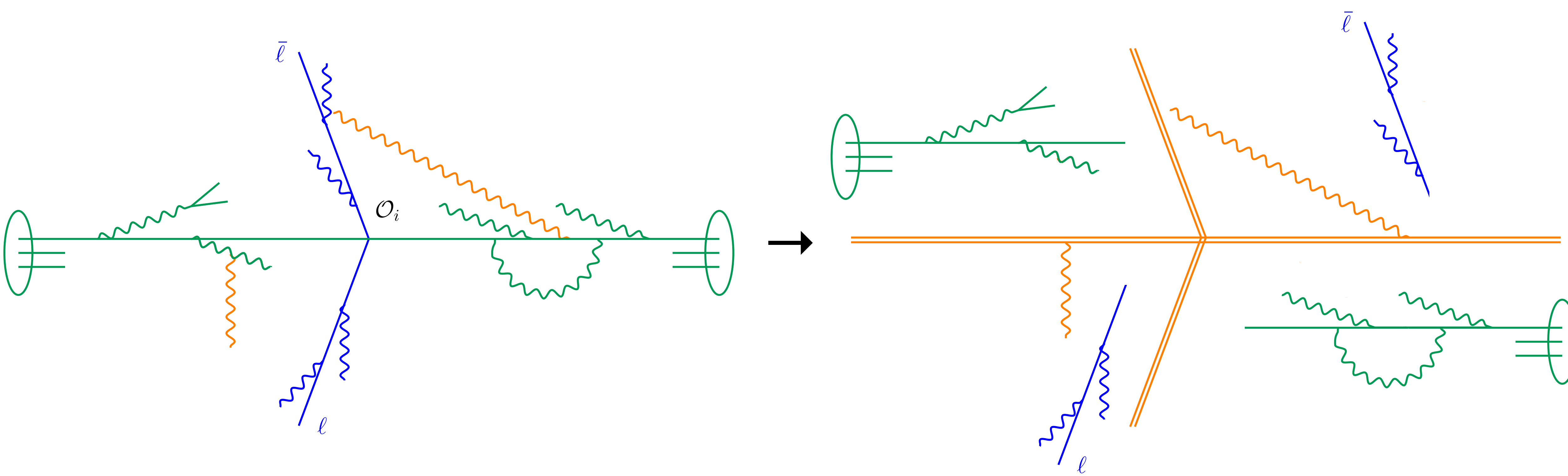
- Soft radiation can be described by emissions from Wilson lines

$$q \to \mathcal{S} q \quad \mathcal{S} = P \exp\left\{ \mathrm{i} \int_{-\infty}^{0} \mathrm{d}s\, n \cdot \left[ \overset{SU(3)}{g_3 A_s(s\, n^\mu)} + \overset{SU(2)}{g_2 W_s(s\, n^\mu)} + \overset{U(1)}{g_1 y_q B_s(s\, n^\mu)} \right] \right\}$$

$$O_{\ell q}^{(3)} \to (\bar{\ell}_1 \mathcal{S}_1^\dagger \gamma^\mu t^a \mathcal{S}_2 \ell_2)\, (\bar{q}_3 \mathcal{S}_3^\dagger \gamma_\mu t^a \mathcal{S}_4 q_4)$$

$$\vdots$$

- There are also collinear Wilson lines (make PDFs/FFs gauge inv.)



# A) Factorization of cross section

- Factorize cross section into PDFs, FFs and a soft function

$$\sigma \sim \sum_X \langle pp|\mathcal{L}_{\rm hard}|\mu^+\mu^- X\rangle\langle\mu^+\mu^- X|\mathcal{L}_{\rm hard}|pp\rangle$$

$$\sim |\mathcal{H}|^2 \underbrace{\langle p|\bar{q}_4 q_4|p\rangle}_{\rm PDF} \underbrace{\langle p|q_3\bar{q}_3|p\rangle}_{\rm PDF} \underbrace{\langle 0|\mathcal{S}_2^\dagger\mathcal{S}_1\mathcal{S}_4^\dagger\mathcal{S}_3\mathcal{S}_1^\dagger\mathcal{S}_2\mathcal{S}_3^\dagger\mathcal{S}_4|0\rangle}_{\rm soft}$$

$$\times \underbrace{\sum_{X_1}\langle 0|\ell_1|\mu^- X_1\rangle\langle\mu^- X_1|\bar{\ell}_1|p\rangle}_{\rm FF} \underbrace{\sum_{X_2}\langle 0|\bar{\ell}_2|\mu^+ X_2\rangle\langle\mu^+ X_2|\ell_2|p\rangle}_{\rm FF} + \ldots$$

# A) Factorization of cross section

- Factorize cross section into PDFs, FFs and a soft function

$$\sigma \sim \sum_X \langle pp|\mathcal{L}_{\rm hard}|\mu^+\mu^- X\rangle\langle\mu^+\mu^- X|\mathcal{L}_{\rm hard}|pp\rangle$$

$$\sim |\mathcal{H}|^2 \underbrace{\langle p|\bar{q}_4 q_4|p\rangle}_{\rm PDF} \underbrace{\langle p|q_3\bar{q}_3|p\rangle}_{\rm PDF} \underbrace{\langle 0|\mathcal{S}_2^\dagger\mathcal{S}_1\mathcal{S}_4^\dagger\mathcal{S}_3\mathcal{S}_1^\dagger\mathcal{S}_2\mathcal{S}_3^\dagger\mathcal{S}_4|0\rangle}_{\rm soft}$$

$$\times \underbrace{\sum_{X_1}\langle 0|\ell_1|\mu^- X_1\rangle\langle\mu^- X_1|\bar{\ell}_1|p\rangle}_{\rm FF} \underbrace{\sum_{X_2}\langle 0|\bar{\ell}_2|\mu^+ X_2\rangle\langle\mu^+ X_2|\ell_2|p\rangle}_{\rm FF} + \ldots$$

- **Nonsinglets** also contribute for EW:

$$\langle p|\bar{q}_4 t^a q_4|p\rangle \quad \langle 0|\mathrm{tr}[\mathcal{S}_1 t^a \mathcal{S}_1^\dagger\,\mathcal{S}_2 t^b \mathcal{S}_2^\dagger]|0\rangle \quad \cdots$$

- Can cancel soft Wilson lines **without** $t^a$ in between, $\mathcal{S}_i^\dagger\mathcal{S}_i = 1$
  This is why no **QCD** soft function for inclusive processes

# C) Matching onto broken phase

- Singlet and triplet fermion PDF are

$$f_q^{(I=0)} \sim \langle p|\bar{q}q|p\rangle \qquad f_q^{(I=1)} \sim \langle p|\bar{q}t^a q|p\rangle$$

- Tree-level matching onto **broken phase** at **electroweak scale**

($h$ = helicity)

$$\begin{aligned}
f_u &= f_{u_R} \\
f_q^{(I=0)} &= f_{u_L} + f_{d_L} \\
f_q^{(I=1,I_3=0)} &= \frac{1}{2} f_{u_L} - \frac{1}{2} f_{d_L} \\
f_{W_h}^{(I=0)} &= f_{W_h^+} + f_{W_h^-} + \cos^2\theta_W f_{Z_h} \\
&\quad + \sin^2\theta_W f_{\gamma_h} + \sin\theta_W \cos\theta_W \left(f_{Z_h\gamma_h} + f_{\gamma_h Z_h}\right) \\
f_{W_h}^{(I=1,I_3=0)} &= f_{W_h^+} - f_{W_h^-} \\
&\vdots
\end{aligned}$$

- Matching for $I_3 \neq 0$ vanishes due to charge conservation

# B) Renormalization group equations

- PDFs (and FFs), where $\bar{n}\cdot r = x E_{\rm cm}$

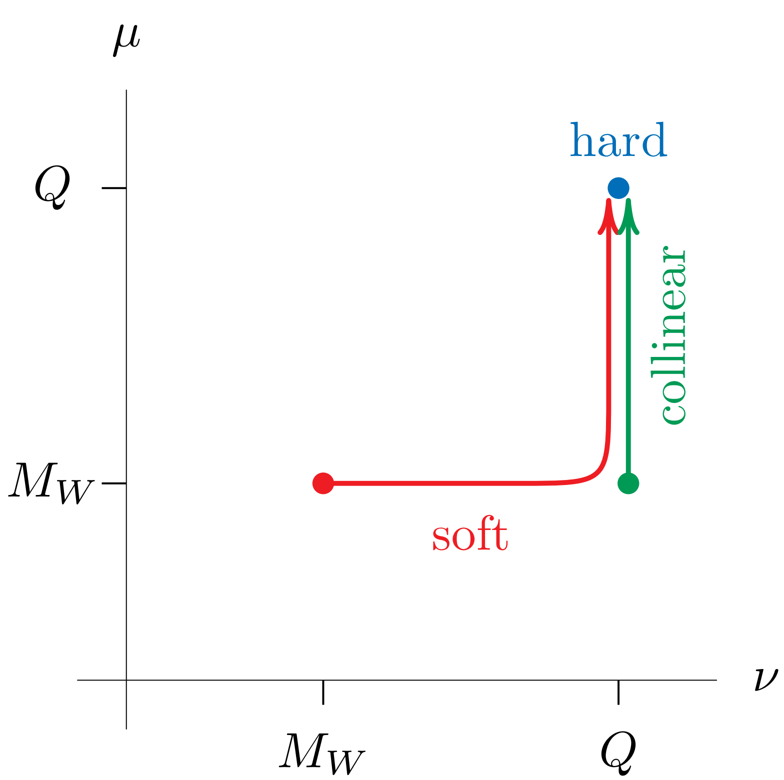
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_i(x,\mu,\nu) = \sum_j \int_0^1 \frac{\mathrm{d}z}{z}\, \frac{\alpha}{\pi}\, \hat{\gamma}_{\mu,ij}\Big(z,\mu,\frac{\nu}{\bar{n}\cdot r}\Big)\, f_j\Big(\frac{x}{z},\mu,\nu\Big)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\nu} f_i(x,\mu,\nu) = \frac{\alpha}{\pi}\, \hat{\gamma}_{\nu,i}(\mu)\, f_i(x,\mu,\nu)$$

- Soft function:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \mathcal{S}(\mu,\nu) = \frac{\alpha}{\pi}\, \hat{\gamma}_{\mu,\mathcal{S}}\Big(\mu,\frac{\nu}{M}\Big)\, \mathcal{S}(\mu,\nu)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\nu} \mathcal{S}(\mu,\nu) = \frac{\alpha}{\pi}\, \hat{\gamma}_{\nu,\mathcal{S}}(\mu)\, \mathcal{S}(\mu,\nu)$$




- We use the rapidity renormalization group

[Chiu et al; Other choices possible: Collins; Becher, Neubert; Becher, Bell; Scimemi et al]

# B) Fermion PDF anomalous dimension

- Singlet and adjoint:

$$f_q^{(I=0)} \sim \langle p|\bar{q}q|p\rangle$$

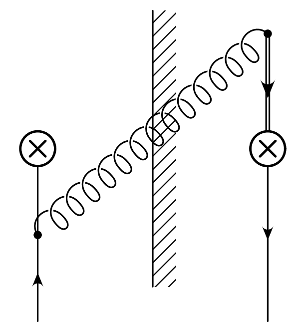
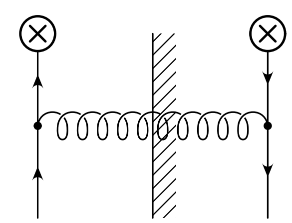
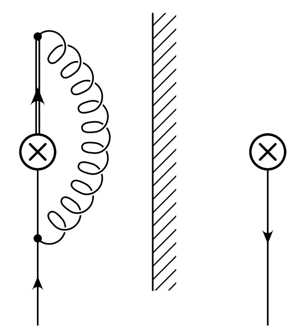
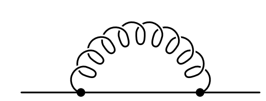
$$f_q^{(I=1)} \sim \langle p|\bar{q}t^a q|p\rangle$$

- Virtual diagrams have $c_F$

- Real diagrams have

$$t^b t^b = c_F$$

$$t^b t^a t^b = (c_F - \tfrac{1}{2}c_A)t^a$$

| Graph | $\hat{\gamma}_\mu$ | $\hat{\gamma}_\nu$ |
|---|---|---|
|  | $\frac{2}{(1-z)_+} - 2 - 2\,\delta(1-z)\ln\frac{\nu}{\bar{n}\cdot r}$ | $-\ln\frac{\mu^2}{M^2}$ |
|  | $1-z$ | $0$ |
| Total$_1$ | $\frac{2}{(1-z)_+} - z - 1 - 2\,\delta(1-z)\ln\frac{\nu}{\bar{n}\cdot r}$ | $-\ln\frac{\mu^2}{M^2}$ |
|  | $2\left(\ln\frac{\nu}{\bar{n}\cdot r}+1\right)\delta(1-z)$ | $\ln\frac{\mu^2}{M^2}$ |
|  | $-\frac{1}{2}\delta(1-z)$ | $0$ |
| Total$_2$ | $\left(2\,\ln\frac{\nu}{\bar{n}\cdot r}+\frac{3}{2}\right)\delta(1-z)$ | $\ln\frac{\mu^2}{M^2}$ |

# B) Fermion PDF anomalous dimension

$$\hat{\gamma}^{(R)}_{\mu,qq} = c^{(R)}_{qq} P_{QQ}(z) + \left[c_F - c^{(R)}_{qq}\right] \left(2 \ln \frac{\nu}{\bar{n}\cdot r} + \frac{3}{2}\right) \delta(1-z)$$

$$\hat{\gamma}^{(R)}_{\nu,q} = \left[c_F - c^{(R)}_{qq}\right] \ln \frac{\mu^2}{M^2}$$

- Color factors: $c^{(I=0)}_{qq} = c_F \,, \quad c^{(I=1)}_{qq} = c_F - \frac{1}{2} c_A$
- *I*=0 has standard $P_{QQ}$ splitting function evolution
- *I*=1 involves **double** logs and **rapidity** logs

# B) Gauge boson PDF anomalous dimension

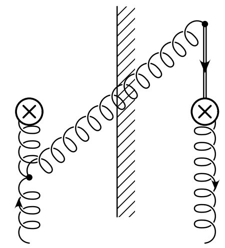
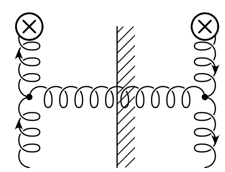
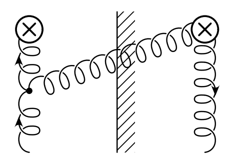
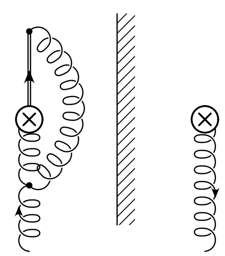
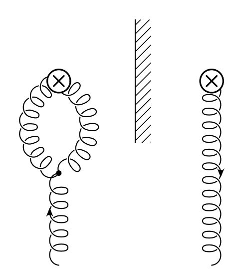
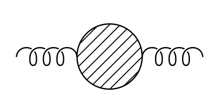
- Virtual diagrams have $c_A$ and $b_0$
- Real diagrams have

$$f_W^{(I=0)} : \quad c_A$$

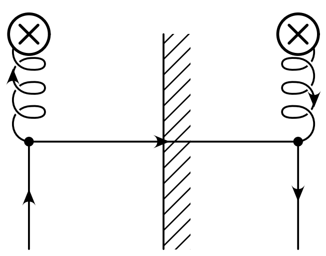
$$f_W^{(I=1)} : \quad \frac{1}{2} c_A$$

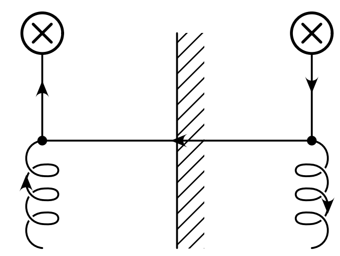
$$f_W^{(I=2)} : \quad -1$$

- Need to keep track of helicity of gauge bosons

| Graph | Helicity stays same | | flipped | |
|---|---|---|---|---|
| | $\hat{\gamma}_\mu$ | $\hat{\gamma}_\nu$ | $\hat{\gamma}_\mu$ | $\hat{\gamma}_\nu$ |
|  | $\frac{2}{(1-z)_+} - 1 - 2\ln\frac{\nu}{\bar{n}\cdot r}\,\delta(1-z)$ | $-\ln\frac{\mu^2}{M^2}$ | $0$ | $0$ |
|  | $\frac{1}{z} + 1 - z^2$ | $0$ | $\frac{(1-z)^3}{z}$ | $0$ |
|  | $-1 - z$ | $0$ | $0$ | $0$ |
| Total$_1$ | $\frac{2}{(1-z)_+} + \frac{1}{z} - 1 - z - z^2 - 2\ln\frac{\nu}{\bar{n}\cdot r}\,\delta(1-z)$ | $-\ln\frac{\mu^2}{M^2}$ | $\frac{(1-z)^3}{z}$ | $0$ |
|  | $c_A\big(2\ln\frac{\nu}{\bar{n}\cdot r} + \frac{5}{2}\big)\delta(1-z)$ | $c_A\ln\frac{\mu^2}{M^2}$ | $0$ | $0$ |
|  | $-\frac{3}{2}c_A\delta(1-z)$ | $0$ | $0$ | $0$ |
|  | $\big(\frac{b_0}{2} - c_A\big)\delta(1-z)$ | $0$ | $0$ | $0$ |
| Total$_2$ | $\big(\frac{b_0}{2} + 2c_A\ln\frac{\nu}{\bar{n}\cdot r}\big)\delta(1-z)$ | $c_A\ln\frac{\mu^2}{M^2}$ | $0$ | $0$ |

# B) Mixing anomalous dimension

| Graph | $P_{G_+Q_+}$ | | $P_{G_+Q_-}$ | |
|---|---|---|---|---|
| | $\hat{\gamma}_\mu$ | $\hat{\gamma}_\nu$ | $\hat{\gamma}_\mu$ | $\hat{\gamma}_\nu$ |
|  | $\frac{1}{z}$ | $0$ | $\frac{(1-z)^2}{z}$ | $0$ |

| Graph | $P_{Q_+G_+}$ | | $P_{Q_+G_-}$ | |
|---|---|---|---|---|
| | $\hat{\gamma}_\mu$ | $\hat{\gamma}_\nu$ | $\hat{\gamma}_\mu$ | $\hat{\gamma}_\nu$ |
|  | $z^2$ | $0$ | $(1-z)^2$ | $0$ |

- Fermions and gauge boson PDFs of **same** SU(2) repr. mix
- $SU(2)_L$ evolution **polarizes** electroweak gauge boson PDFs, and consequently fermion and even gluon PDFs

# B) PDF evolution in Standard Model at one loop

Includes:

- $SU(3)$x$SU(2)$x$U(1)$
- Spin dependence
- Yukawa for top
- Higgs
- Longitudinal $W$, $Z$ (Goldstone equiv.)
- $\gamma Z$ interference

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=1)}_{q,r,s} &= \frac{\alpha_3}{\pi}\frac{4}{3}\widetilde{P}_{Q_-Q_-}\otimes f^{(I=1)}_{q,r,s}\\
&+\frac{\alpha_2}{\pi}\left[-\frac{1}{4}\widetilde{P}_{Q_-Q_-}\otimes f^{(I=1)}_{q,r,s}+\Gamma_1 f^{(I=1)}_{q,r,s}(z)+\frac{1}{4}N_c\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f^{(I=1)}_{W_+}+\frac{1}{4}N_c\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f^{(I=1)}_{W_-}\right]\\
&+\frac{\alpha_1}{\pi}\mathrm{y}_q^2\widetilde{P}_{Q_-Q_-}\otimes f^{(I=1)}_{q,r,s}+\frac{g_1g_2}{4\pi^2}\mathrm{y}_qN_c\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes\left(f^{(I=1)}_{W_+B_+}+f^{(I=1)}_{B_+W_+}\right)\\
&+\frac{g_1g_2}{4\pi^2}\mathrm{y}_qN_c\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes\left(f^{(I=1)}_{W_-B_-}+f^{(I=1)}_{B_-W_-}\right)\\
&+\frac{Y_t^2}{4\pi^2}\left[-\frac{1}{8}\delta_{r3}f^{(I=1)}_{q,3,s}(z)-\frac{1}{8}\delta_{s3}f^{(I=1)}_{q,r,3}(z)+\frac{N_c}{2}\delta_{r3}\delta_{s3}\,1\otimes f^{(I=1)}_{\bar{H}}\right],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=1)}_{\ell,r,s} &= \frac{\alpha_2}{\pi}\left[-\frac{1}{4}\widetilde{P}_{Q_-Q_-}\otimes f^{(I=1)}_{\ell,r,s}+\Gamma_1 f^{(I=1)}_{\ell,r,s}(z)+\frac{1}{4}\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f^{(I=1)}_{W_+}+\frac{1}{4}\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f^{(I=1)}_{W_-}\right]\\
&+\frac{\alpha_1}{\pi}\mathrm{y}_\ell^2\widetilde{P}_{Q_-Q_-}\otimes f^{(I=1)}_{\ell,r,s}+\frac{g_1g_2}{4\pi^2}\mathrm{y}_\ell\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes\left(f^{(I=1)}_{W_+B_+}+f^{(I=1)}_{B_+W_+}\right)\\
&+\frac{g_1g_2}{4\pi^2}\mathrm{y}_\ell\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes\left(f^{(I=1)}_{W_-B_-}+f^{(I=1)}_{B_-W_-}\right),
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=1)}_{W_\pm} &= \frac{\alpha_2}{\pi}\Bigg[\widetilde{P}_{G_\pm G_+}\otimes f^{(I=1)}_{W_+}+\widetilde{P}_{G_\pm G_-}\otimes f^{(I=1)}_{W_-}+\Gamma_2 f^{(I=1)}_{W_\pm}(z)+P_{G_\pm Q_+}\otimes\sum_{\substack{i=\bar{q},\bar{\ell}\\ r=1,\dots,n_g}}f^{(I=1)}_{i,r,r}\\
&+P_{G_\pm Q_-}\otimes\sum_{\substack{i=q,\ell,\\ r=1,\dots,n_g}}f^{(I=1)}_{i,r,r}+P_{G_\pm H}(z)\otimes\sum_{i=H,\bar{H}}f^{(I=1)}_{i}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=1)}_{W_\pm B_\pm} &= \left[\frac{\alpha_2}{\pi}\Gamma_3+\frac{\alpha_1}{\pi}\frac{1}{4}b_{0,1}\right]f^{(I=1)}_{W_\pm B_\pm}(z)+\frac{g_1g_2}{4\pi^2}\widetilde{P}_{G_\pm Q_-}\otimes\sum_{i=q,\ell,r=1,\dots,n_g}\mathrm{y}_i f^{(I=1)}_{i,r,r}\\
&-\frac{g_1g_2}{4\pi^2}\widetilde{P}_{G_\pm Q_+}\otimes\sum_{i=\bar{q},\bar{\ell},r=1,\dots,n_g}f^{(I=1)}_{i,r,r},
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=1)}_{H} &= \frac{\alpha_2}{\pi}\left[-\frac{1}{4}\widetilde{P}_{HH}\otimes f^{(I=1)}_{H}+\Gamma_4 f^{(I=1)}_{H}(z)+\frac{1}{4}\widetilde{P}_{HG_+}\otimes f^{(I=1)}_{W_+}+\frac{1}{4}\widetilde{P}_{HG_-}\otimes f^{(I=1)}_{W_-}\right]\\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_H^2\widetilde{P}_{HH}\otimes f^{(I=1)}_{H}\right]+\frac{Y_t^2}{8\pi^2}\left[z\otimes f^{(I=1)}_{\bar{q},3,3}-N_cf^{(I=1)}_{H}(z)\right],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=1)}_{\widetilde{H}H} &= \frac{\alpha_2}{\pi}\left[-\frac{1}{4}\widetilde{P}_{HH}\otimes f^{(I=1)}_{\widetilde{H}H}+\Gamma_4 f^{(I=1)}_{\widetilde{H}H}(z)\right]\\
&+\frac{\alpha_1}{\pi}\left[-\mathrm{y}_H^2\widetilde{P}_{HH}\otimes f^{(I=1)}_{\widetilde{H}H}+2\mathrm{y}_H^2\Gamma_4 f^{(I=1)}_{\widetilde{H}H}(z)\right]-\frac{Y_t^2}{8\pi^2}N_cf^{(I=1)}_{\widetilde{H}H}(z).
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=0)}_{B_\pm} &= \frac{\alpha_1}{\pi}\Bigg[\frac{1}{2}b_{0,1}f^{(I=0)}_{B_\pm}(z)+\widetilde{P}_{G_\pm Q_+}\otimes\sum_{\substack{i=\bar{q},u,d,\bar{\ell},e\\ r=1,\dots,n_g}}\mathrm{y}_i^2 f^{(I=0)}_{i,r,r}\\
&+\widetilde{P}_{G_\pm Q_-}\otimes\sum_{\substack{i=q,\bar{u},\bar{d},\ell,\bar{e}\\ r=1,\dots,n_g}}\mathrm{y}_i^2 f^{(I=0)}_{i,r,r}+\mathrm{y}_H^2\widetilde{P}_{G_\pm H}\otimes\sum_{i=H,\bar{H}}f^{(I=0)}_{i}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=0)}_{H} &= \frac{\alpha_2}{\pi}\left[\frac{3}{4}\widetilde{P}_{HH}(z)\otimes f^{(I=0)}_{H}+\frac{1}{2}\widetilde{P}_{HG_+}\otimes f^{(I=0)}_{W_+}+\frac{1}{2}\widetilde{P}_{HG_-}\otimes f^{(I=0)}_{W_-}\right]\\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_H^2\widetilde{P}_{HH}(z)\otimes f^{(I=0)}_{H}+\mathrm{y}_H^2\widetilde{P}_{HG_+}\otimes f^{(I=0)}_{B_+}+\mathrm{y}_H^2\widetilde{P}_{HG_-}\otimes f^{(I=0)}_{B_-}\right]\\
&+\frac{Y_t^2}{8\pi^2}\left[z\otimes\left(f^{(I=0)}_{\bar{q},3,3}+2f^{(I=0)}_{\bar{u},3,3}\right)-N_cf^{(I=0)}_{H}(z)\right],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=0)}_{q,r,s}(z) &= \frac{\alpha_3}{\pi}\left[\frac{4}{3}\widetilde{P}_{Q_-Q_-}\otimes f^{(I=0)}_{q,r,s}+\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f^{(I=0)}_{g_+}+\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f^{(I=0)}_{g_-}\right]\\
&+\frac{\alpha_2}{\pi}\left[\frac{3}{4}\widetilde{P}_{Q_-Q_-}\otimes f^{(I=0)}_{q,r,s}+\frac{N_c}{2}\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f^{(I=0)}_{W_+}+\frac{N_c}{2}\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f^{(I=0)}_{W_-}\right]\\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_q^2\widetilde{P}_{Q_-Q_-}\otimes f^{(I=0)}_{q,r,s}+2N_c\mathrm{y}_q^2\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f^{(I=0)}_{B_+}+2N_c\mathrm{y}_q^2\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f^{(I=0)}_{B_-}\right]\\
&+\frac{Y_t^2}{4\pi^2}\Bigg[\delta_{r3}\delta_{s3}(1-z)\otimes f^{(I=0)}_{u,3,3}-\frac{1}{8}\delta_{r3}f^{(I=0)}_{q,3,s}(z)-\frac{1}{8}\delta_{s3}f^{(I=0)}_{q,r,3}(z)\\
&+\frac{N_c}{2}\delta_{r3}\delta_{s3}\,1\otimes f^{(I=0)}_{\bar{H}}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=0)}_{u,r,s} &= \frac{\alpha_3}{\pi}\left[\frac{4}{3}\widetilde{P}_{Q_+Q_+}\otimes f^{(I=0)}_{u,r,s}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_+G_+}\otimes f^{(I=0)}_{g_+}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_+G_-}\otimes f^{(I=0)}_{g_-}\right]\\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_u^2\widetilde{P}_{Q_+Q_+}\otimes f^{(I=0)}_{u,r,s}+N_c\mathrm{y}_u^2\delta_{rs}\widetilde{P}_{Q_+G_+}\otimes f^{(I=0)}_{B_+}+N_c\mathrm{y}_u^2\delta_{rs}\widetilde{P}_{Q_+G_-}\otimes f^{(I=0)}_{B_-}\right]\\
&+\frac{Y_t^2}{4\pi^2}\Bigg[\frac{1}{2}(1-z)\delta_{r3}\delta_{s3}\otimes f^{(I=0)}_{q,3,3}-\frac{1}{4}\delta_{r3}f^{(I=0)}_{u,3,s}(z)-\frac{1}{4}\delta_{s3}f^{(I=0)}_{u,r,3}(z)\\
&+\frac{N_c}{2}\delta_{r3}\delta_{s3}\,1\otimes f^{(I=0)}_{H}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=0)}_{d,r,s} &= \frac{\alpha_3}{\pi}\left[\frac{4}{3}\widetilde{P}_{Q_+Q_+}\otimes f^{(I=0)}_{d,r,s}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_+G_+}\otimes f^{(I=0)}_{g_+}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_+G_-}\otimes f^{(I=0)}_{g_-}\right]\\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_d^2\widetilde{P}_{Q_+Q_+}\otimes f^{(I=0)}_{d,r,s}+N_c\mathrm{y}_d^2\delta_{rs}\widetilde{P}_{Q_+G_+}\otimes f^{(I=0)}_{B_+}+N_c\mathrm{y}_d^2\delta_{rs}\widetilde{P}_{Q_+G_-}\otimes f^{(I=0)}_{B_-}\right],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=0)}_{\ell,r,s} &= \frac{\alpha_2}{\pi}\left[\frac{3}{4}\widetilde{P}_{Q_-Q_-}\otimes f^{(I=0)}_{\ell,r,s}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f^{(I=0)}_{W_+}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f^{(I=0)}_{W_-}\right]\\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_\ell^2\widetilde{P}_{Q_-Q_-}\otimes f^{(I=0)}_{\ell,r,s}+\mathrm{y}_\ell^2\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f^{(I=0)}_{B_+}+\mathrm{y}_\ell^2\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f^{(I=0)}_{B_-}\right],
\end{aligned}
$$

$$
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=0)}_{e,r,s} = \frac{\alpha_1}{\pi}\left[\mathrm{y}_e^2\widetilde{P}_{Q_+Q_+}\otimes f^{(I=0)}_{e,r,s}+\mathrm{y}_e^2\delta_{rs}\widetilde{P}_{Q_+G_+}\otimes f^{(I=0)}_{B_+}+\mathrm{y}_e^2\delta_{rs}\widetilde{P}_{Q_+G_-}\otimes f^{(I=0)}_{B_-}\right],
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=0)}_{g_\pm} &= \frac{\alpha_3}{\pi}\Bigg[3\widetilde{P}_{G_\pm G_+}\otimes f^{(I=0)}_{g_+}+3\widetilde{P}_{G_\pm G_-}\otimes f^{(I=0)}_{g_-}+\frac{1}{2}b_{0,3}f^{(I=0)}_{g_\pm}(z)\\
&+\frac{4}{3}\widetilde{P}_{G_\pm Q_+}\otimes\sum_{\substack{i=\bar{q},u,d,\\ r=1,\dots,n_g}}f^{(I=0)}_{i,r,r}+\frac{4}{3}\widetilde{P}_{G_\pm Q_-}\otimes\sum_{\substack{i=q,\bar{u},\bar{d}\\ r=1,\dots,n_g}}f^{(I=0)}_{i,r,r}\Bigg],
\end{aligned}
$$

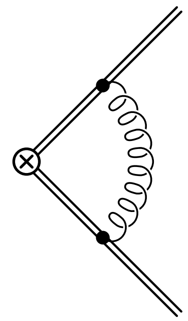
$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=0)}_{W_\pm} &= \frac{\alpha_2}{\pi}\Bigg[2\widetilde{P}_{G_\pm G_+}(z)\otimes f^{(I=0)}_{W_+}+2\widetilde{P}_{G_\pm G_-}(z)\otimes f^{(I=0)}_{W_-}+\frac{1}{2}b_{0,2}f^{(I=0)}_{W_\pm}(z)\\
&+\frac{3}{4}\widetilde{P}_{G_\pm Q_+}\otimes\sum_{\substack{i=\bar{q},\bar{\ell}\\ r=1,\dots,n_g}}f^{(I=0)}_{i,r,r}+\frac{3}{4}\widetilde{P}_{G_\pm Q_-}\otimes\sum_{\substack{i=q,\ell,\\ r=1,\dots,n_g}}f^{(I=0)}_{i,r,r}+\frac{3}{4}\widetilde{P}_{G_\pm H}\otimes\sum_{i=H,\bar{H}}f^{(I=0)}_{i}\Bigg],
\end{aligned}
$$

$$
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f^{(I=2)}_{W_\pm} = \frac{\alpha_2}{\pi}\left[-\widetilde{P}_{G_\pm G_+}(z)\otimes f^{(I=2)}_{W_+}-\widetilde{P}_{G_\pm G_-}(z)\otimes f^{(I=2)}_{W_-}+\left(\frac{b_{0,2}}{2}+6\ln\frac{\nu}{\bar{n}\cdot r}\right)f^{(I=2)}_{W_\pm}(z)\right]
$$

$$
\begin{aligned}
\nu\frac{\mathrm{d}}{\mathrm{d}\nu}f^{(I=0)}_{i} &= 0,\\
\nu\frac{\mathrm{d}}{\mathrm{d}\nu}f^{(I=1,I_3=0)}_{i} &= \frac{\alpha_2}{\pi}\ln\frac{\mu^2}{M_W^2}f^{(I=1,I_3=0)}_{i},\\
\nu\frac{\mathrm{d}}{\mathrm{d}\nu}f^{(I=2,I_3=0)}_{i} &= \frac{3\alpha_2}{\pi}\ln\frac{\mu^2}{M_W^2}f^{(I=2,I_3=0)}_{i},
\end{aligned}
$$

$$
\begin{aligned}
\nu\frac{\mathrm{d}}{\mathrm{d}\nu}f^{(I=1,I_3=1)}_{\widetilde{H}H} &= \left[\frac{\alpha_2}{2\pi}\ln\frac{\mu^2}{M_W^2}+\frac{(\alpha_2+4\mathrm{y}_H^2\alpha_1)}{2\pi}\ln\frac{\mu^2}{M_Z^2}\right]f^{(I=1,I_3=1)}_{\widetilde{H}H}\\
&= \left[\frac{\alpha_2}{2\pi}\ln\frac{\mu^2}{M_W^2}+\frac{\alpha_{\mathrm{em}}}{2\pi\sin^2\theta_W\cos^2\theta_W}\ln\frac{\mu^2}{M_Z^2}\right]f^{(I=1,I_3=1)}_{\widetilde{H}H}.
\end{aligned}
$$

# B) Soft function anomalous dimension

| Graph | $\hat{\gamma}_\mu$ | $\hat{\gamma}_\nu$ |
|---|---|---|
|  | $\ln \frac{(-n_i\cdot n_j - \mathrm{i}0)\nu^2}{2\mu^2}$ | $\ln \frac{\mu^2}{M^2}$ |

- Direction of Wilson line $\mathcal{S}_i$ denoted by $n_i = \pm(1, \hat{n}_i)$
  In- vs. outgoing ($\pm$) turns out not to matter

- $\nu$-evolution cancels against that of PDFs/FFs:

$$\hat{\gamma}_\nu = -(n^{(I=1)} + 3n^{(I=2)}) \ln \frac{\mu^2}{M^2}$$

number of PDFs/FFs in this representation

- $\mu$-evolution depends on angles. For two Wilson lines:

$$\langle 0|\mathcal{S}_{12}^{ab}|0\rangle \equiv \langle 0|\mathrm{tr}(\mathcal{S}_1 t^a \mathcal{S}_1^\dagger \mathcal{S}_2 t^b \mathcal{S}_2^\dagger)|0\rangle : \quad \hat{\gamma}_\mu = c_A\left[\ln\frac{\mu^2}{\nu^2} - \ln\left|\frac{n_1\cdot n_2}{2}\right|\right]$$

# B) Electroweak resummation

- $\nu$-evolution vanishes for $\mu = M_W$ (at NLL)

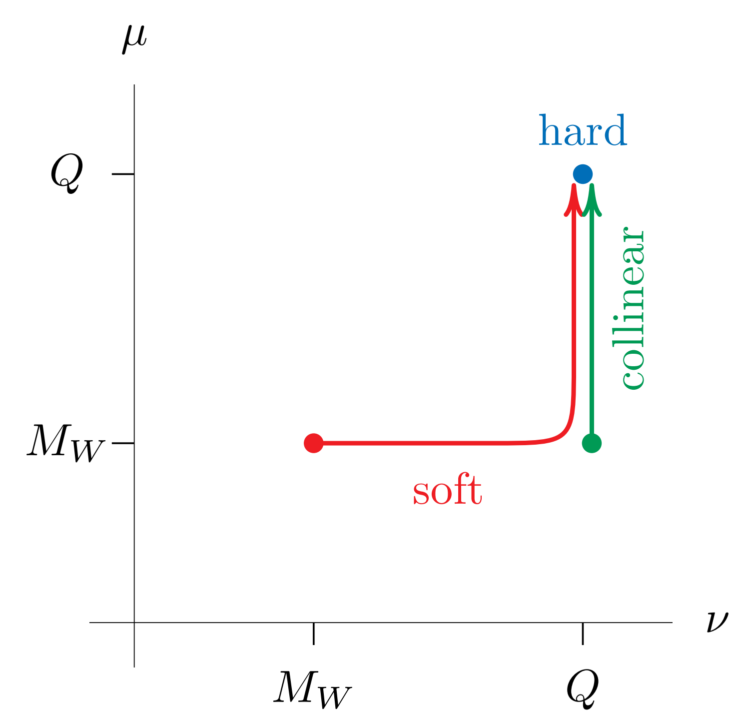
$$U_\nu = \exp\left[\int_{M_W}^{Q} \frac{\mathrm{d}\nu}{\nu}\,\gamma_{\nu,\mathcal{S}}\right] = \exp\left[-\left(n^{(I=1)} + 3n^{(I=2)}\right)\frac{\alpha_2(\mu)}{\pi}\ln\frac{Q}{M_W}\ln\frac{\mu^2}{M_W^2}\right]$$

- $\mu$-evolution gives rise to double logarithms

$$U_\mu^{\mathrm{DL}} = \exp\left[-\left(n^{(I=1)} + 3n^{(I=2)}\right)\frac{\alpha_2}{\pi}\ln^2\frac{Q}{M_W}\right]$$

leading to Sudakov suppression of nonsinglets [See Ciafaloni et al]

- Single logarithms for nonsinglets:
  - Different coefficient splitting function
  - Angular dependence through soft function



# Comparison with Bauer, Ferland, Webber

- They cut off soft singularity in PDF evolution and fix remainder using momentum sum rules [Following Ciafaloni, Ciafaloni, Comelli]

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_q^{(I=1)}(x,\mu) = \frac{\alpha_2}{\pi}\Big\{\int_0^{1-M/\mu} \mathrm{d}z \Big[-\frac{1}{4}P_{QQ}(z)\, f_q^{(I=1)}\Big(\frac{x}{z},\mu\Big) + \frac{1}{4}N_c P_{QG}(z)\, f_W^{(I=1)}\Big(\frac{x}{z},\mu\Big) + \ldots\Big] + \Big(\frac{3}{2}\ln\frac{M}{\mu}+\frac{9}{8}\Big) f_q^{(I=1)}(x,\mu)\Big\}$$

- Agrees with our result at LL, not NLL, though they initially forgot to account for the polarization of gauge bosons

# Summary

- Electroweak corrections are enhanced by logs in high-energy tails → important to include for BSM searches
- Electroweak corrections for inclusive processes:
  - involve double logs because initial/final particles are not singlet
  - evolution polarizes the PDFs, because EW is chiral
  - depends on the angle due to the soft function

# Summary

- Electroweak corrections are enhanced by logs in high-energy tails → important to include for BSM searches
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  - involve double logs because initial/final particles are not singlet
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Thank you!