Proton number cumulants in heavy-ion collisions and search for the QCD critical point

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INT Workshop INT-20R-1C "Chirality and Criticality: Novel Phenomena in Heavy-Ion Collisions"

August 23, 2023

INSTITUTE for NUCLEAR THEORY

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QCD phase structure

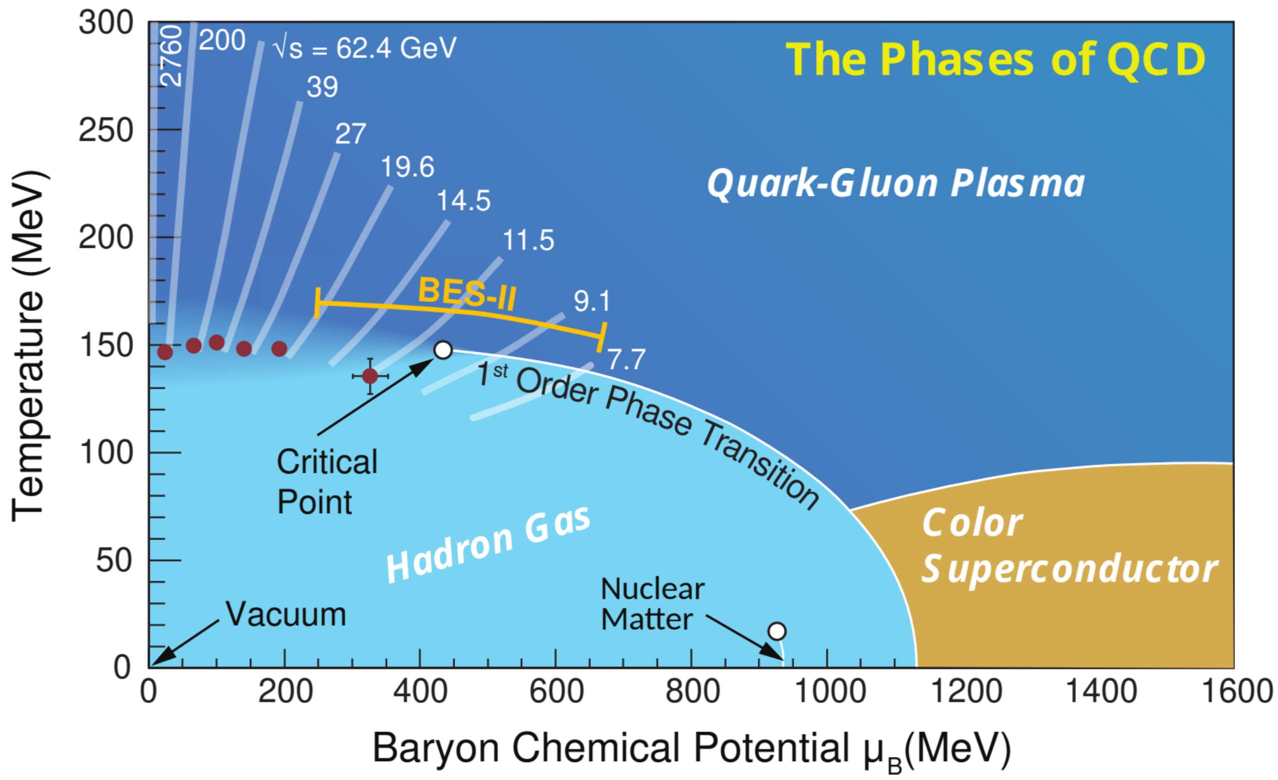

Figure from Bzdak et al., Phys. Rept. '20

- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

Is there a critical point and how to find it?

Where is the critical point? Ask AI

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ChatGPT struggles

Is there a QCD critical point and where is it?

As of my knowledge cutoff in September 2021, the existence and precise location of a critical point in the phase diagram of Quantum Chromodynamics (QCD), the theory of the strong interaction between quarks and gluons, was not definitively established.

...

However, finding the QCD critical point using lattice QCD is a particularly challenging problem due to the so-called "sign problem." The sign problem arises when trying to extend lattice QCD calculations from zero to non-zero baryon chemical potential, which is necessary to locate the critical point on the phase diagram. At non-zero chemical potential, the action of the QCD path integral becomes complex, which makes the standard Monte Carlo techniques used in lattice QCD simulations ineffective.

...

Experimental searches for the QCD critical point in heavy-ion collisions generally rely on identifying fluctuations and correlations in the properties of the particles produced in the collisions, which are expected to exhibit unique features near the critical point.

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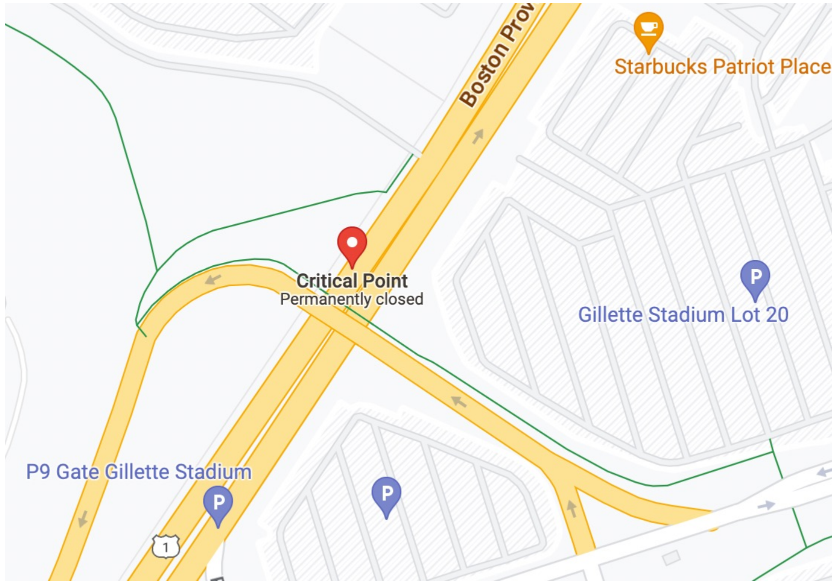
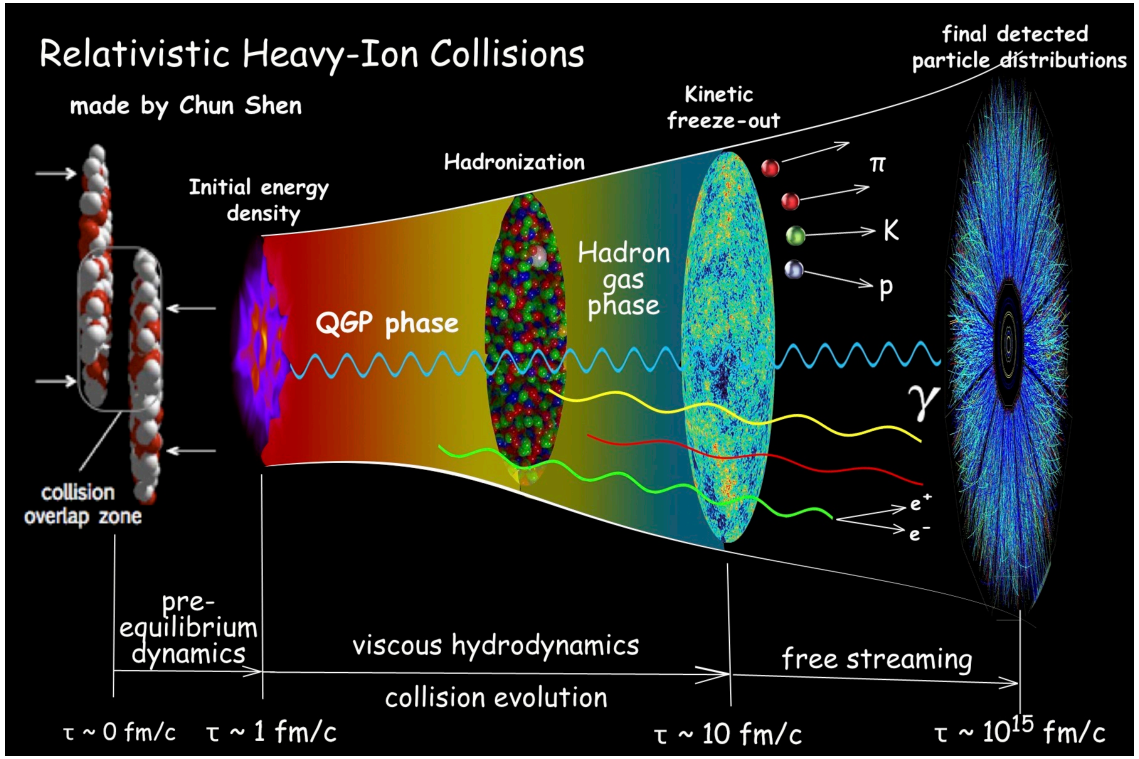
No issues for Google



Relativistic heavy-ion collisions



Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
- Size of the collision region

Measurements

- Final hadron abundances and momentum distributions **event-by-event**

Event-by-event fluctuations and statistical mechanics

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Cumulant generating function

$$K_N(t) = \ln \langle e^{tN} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

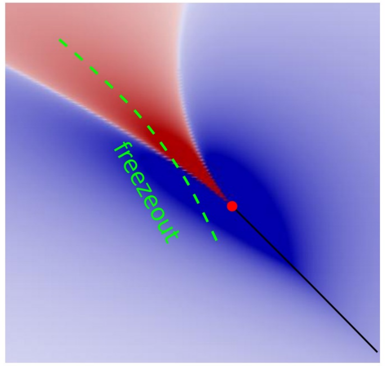
Grand partition function

$$\ln Z^{\text{gce}}(T,V,\mu) = \ln \left[\sum_N e^{\mu N/T} Z^{\text{ce}}(T,V,N) \right]$$

$$\kappa_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point** – large correlation length, critical fluctuations of baryon number



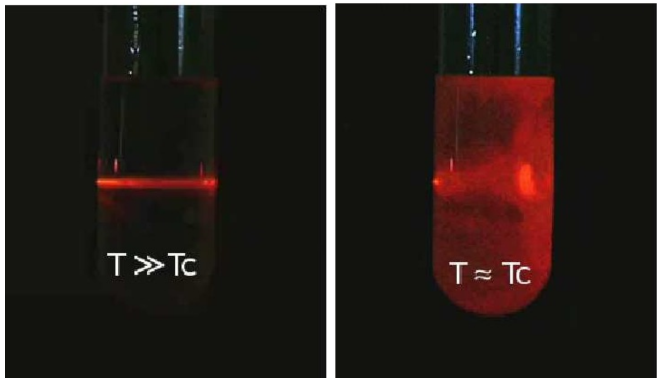
M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

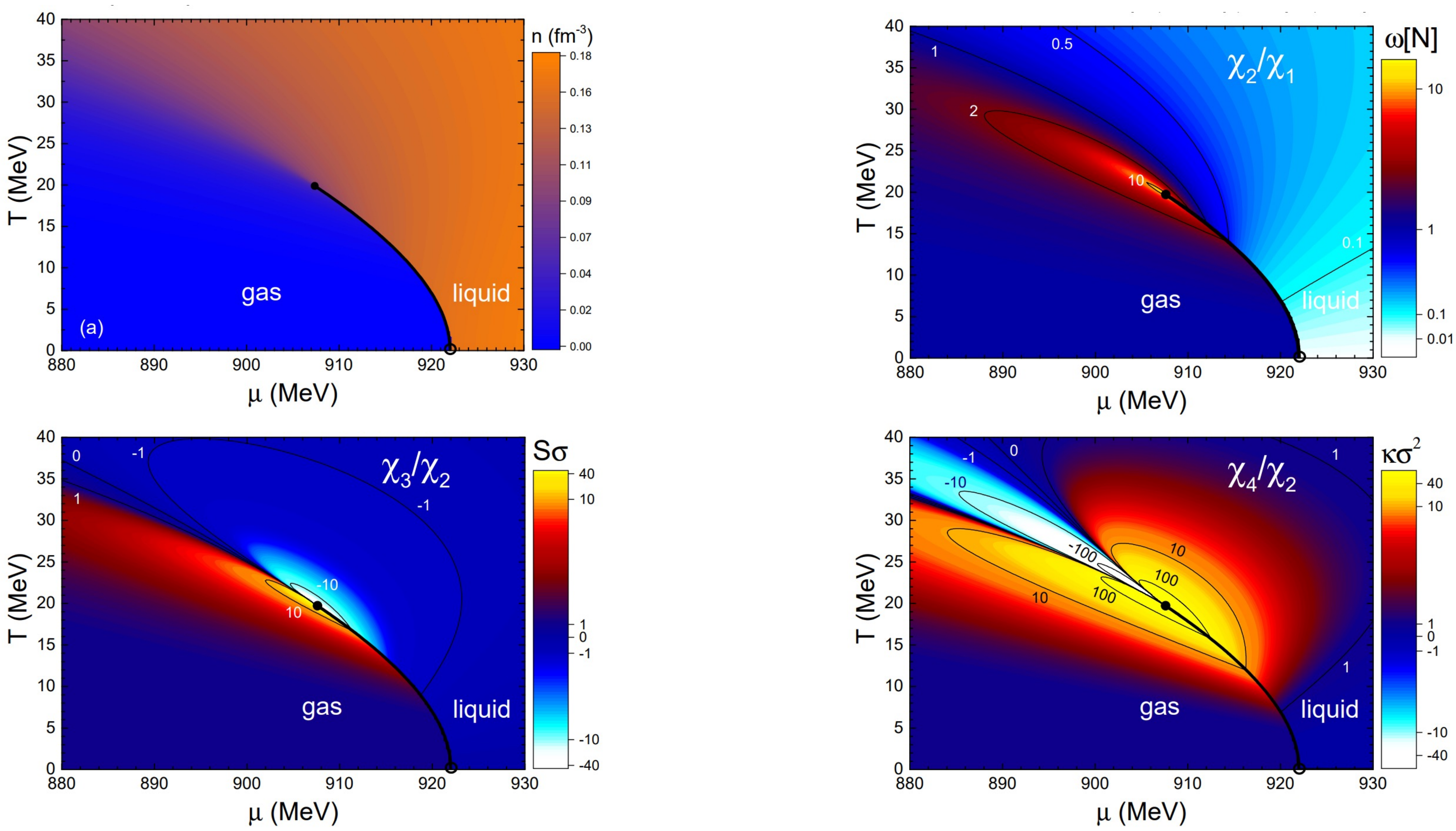
$$\xi \to \infty$$

Looking for enhanced fluctuations
and non-monotonicities

Critical opalescence



Example: Nuclear liquid-gas transition



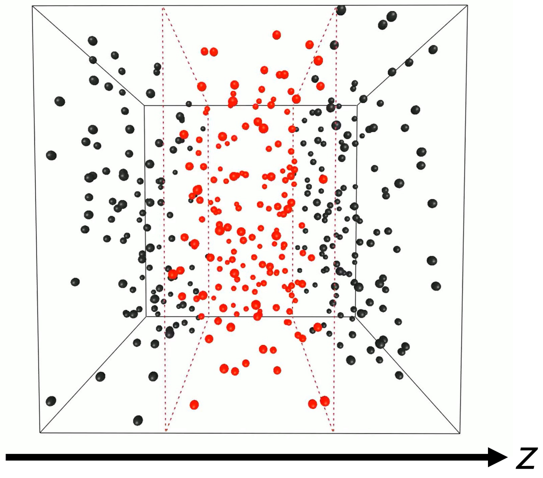
VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Example: Lennard-Jones fluid

Kuznietsov, Savchuk, Gorenstein, Koch, VV, Phys. Rev. C 105, 044903 (2022)

Classical molecular dynamics simulations* of a **Lennard-Jones fluid** along the (super)critical isotherm ($T \approx 1.06 T_c$) of the liquid-gas transition

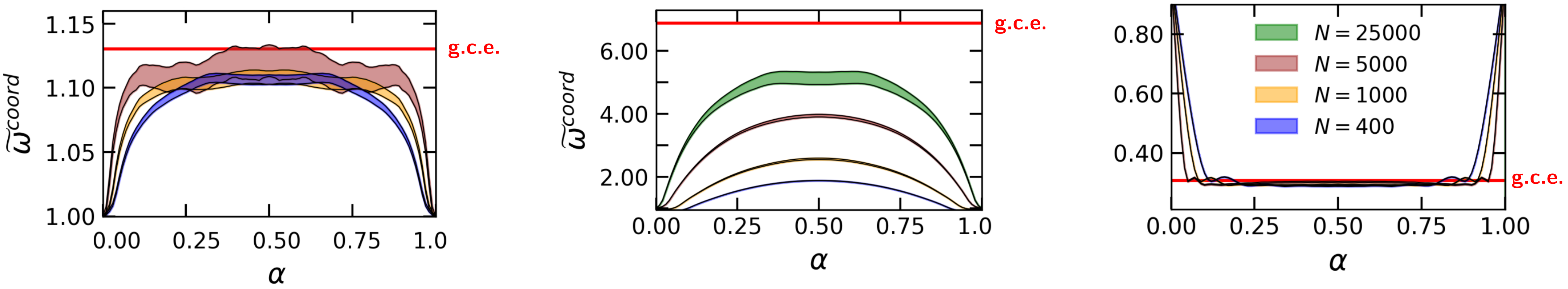
Microcanonical (const. *EVN*) ensemble with periodic boundary conditions



Variance of conserved particle number distribution inside coordinate space subvolume $|z| < z^{max}$ as time average

$$\tilde{\omega}^{\text{coord}} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$

$n \approx 0.15\, n_c$ $\qquad$ $n \approx n_c$ $\qquad$ $n \approx 2 n_c$

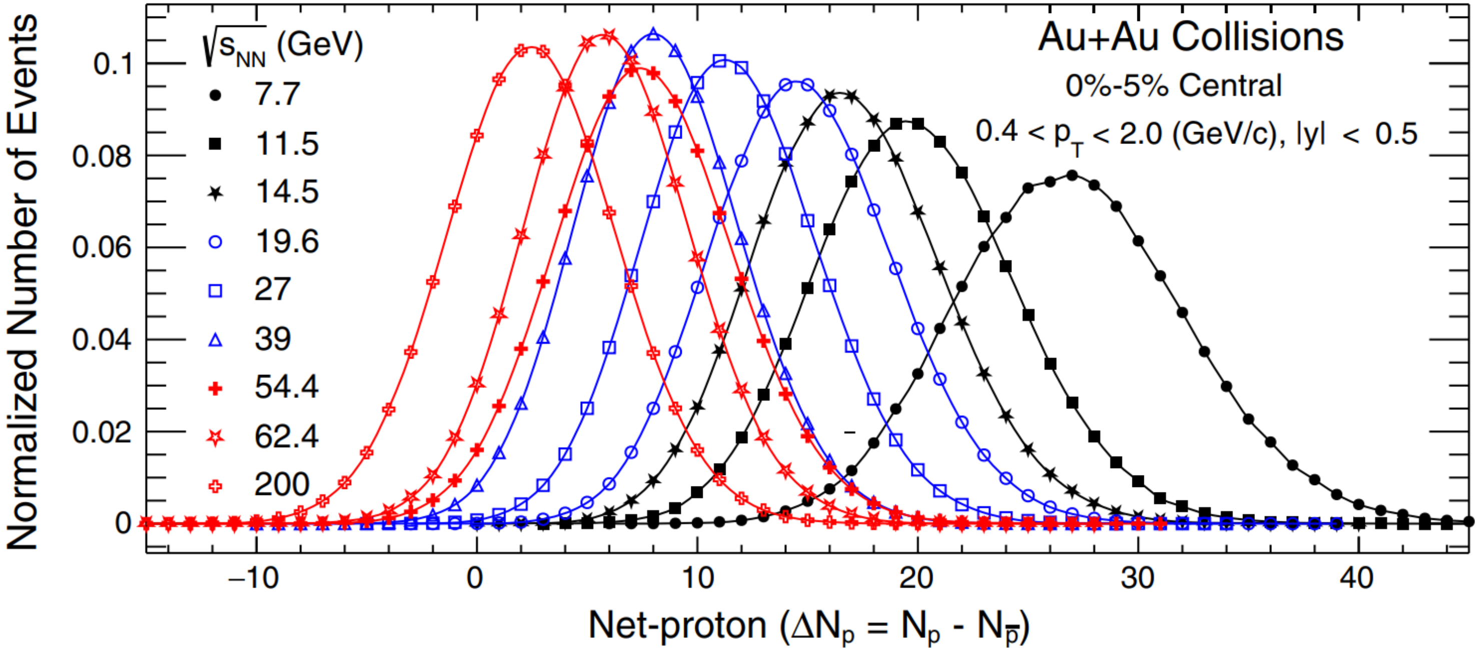


*Molecular dynamics code from https://github.com/vlvovch/lennard-jones-cuda

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons $P(\Delta N_p) \sim \frac{N_{events}(\Delta N_p)}{N_{events}^{total}}$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

Experimental measurements

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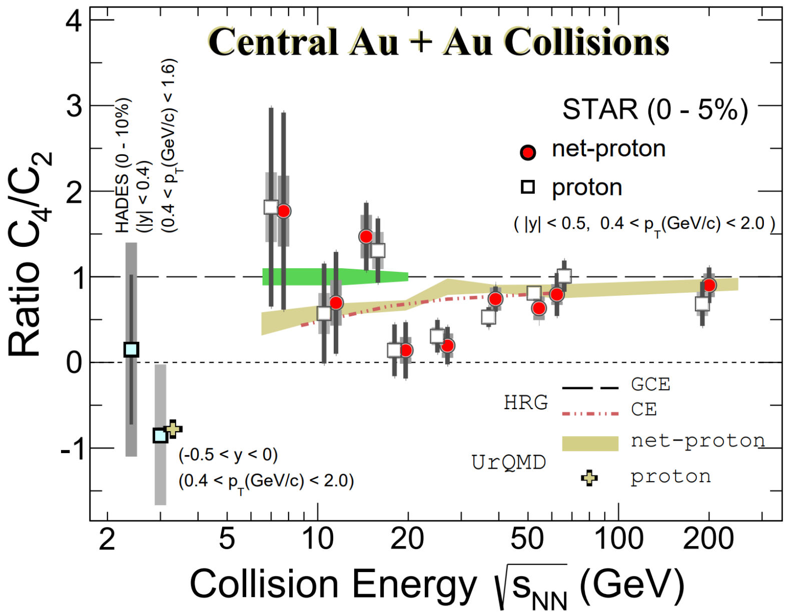
Beam energy scan in search for the critical point (STAR Coll.)



STAR Coll., PRL 126, 092301 (2021), PRL 128, 202303 (2022)

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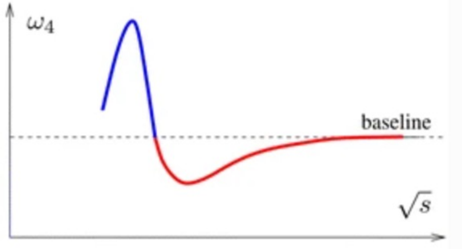
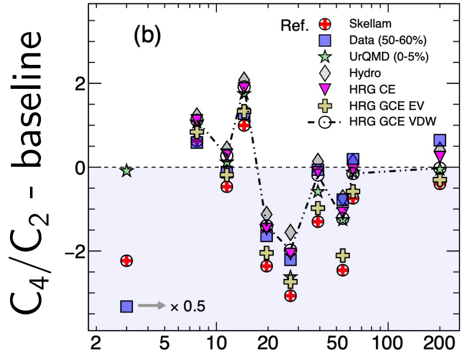
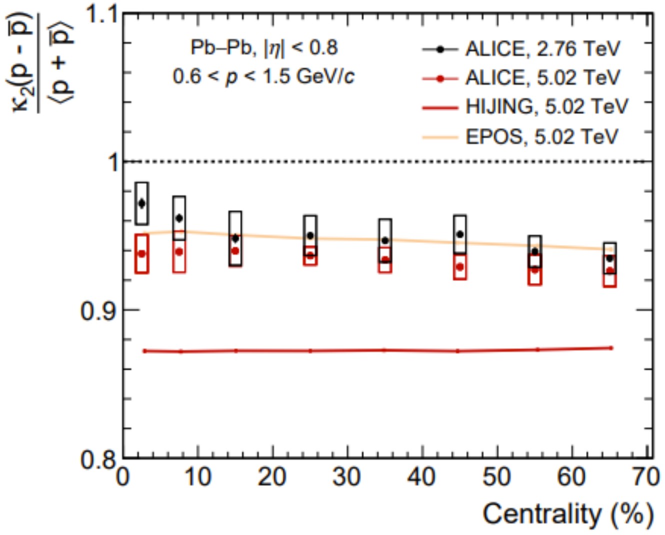


M. Stephanov, Phys. Rev. Lett. (2011)



A. Pandav's talk



ALICE Coll., arXiv:2206.03343

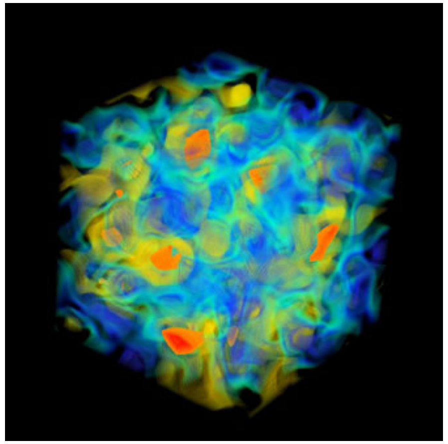
Reduced errors (better statistics), more energies, to come soon from RHIC-BES-II program, STAR-FXT etc.

Can we learn more from the more accurate data available for κ_2 and κ_3?

Other measurements: LHC-ALICE, GSI-HADES & CERN-NA61/SHINE Collaborations

Theory vs experiment: Challenges for fluctuations

Theory



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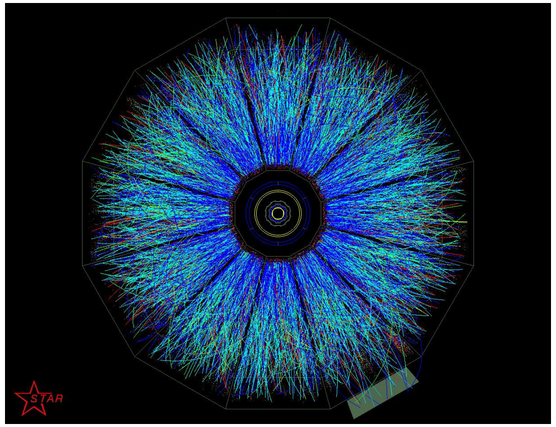
- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

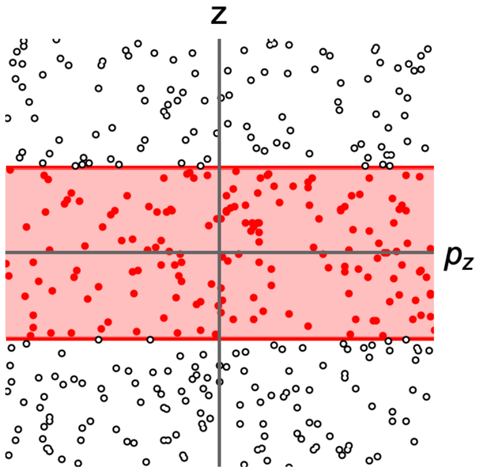
- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

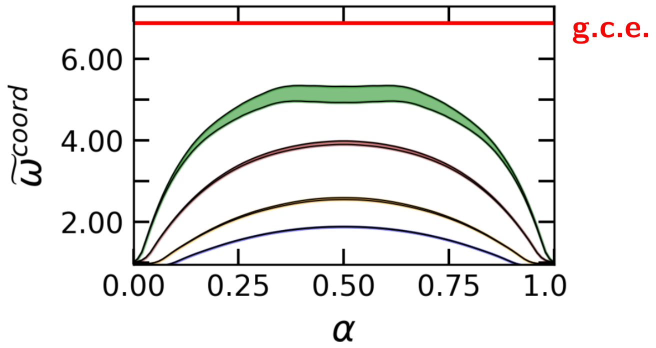
Need dynamical description

Coordinate vs Momentum space

Box setup: Coordinates and momenta are uncorrelated

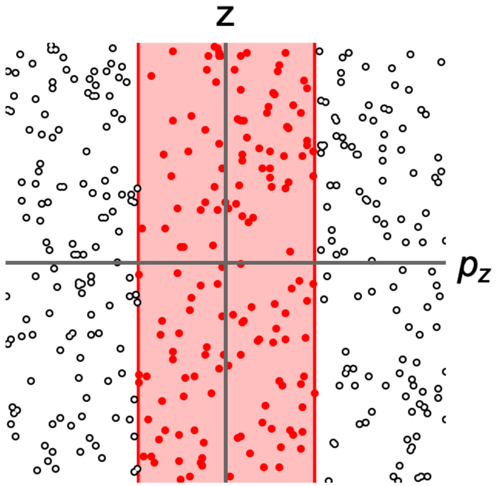
Coordinate space cut

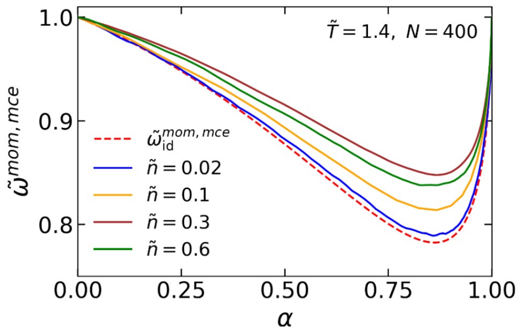




Large correlations

Momentum space cut





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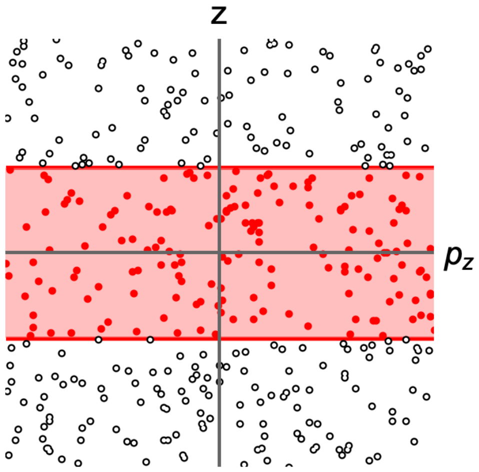
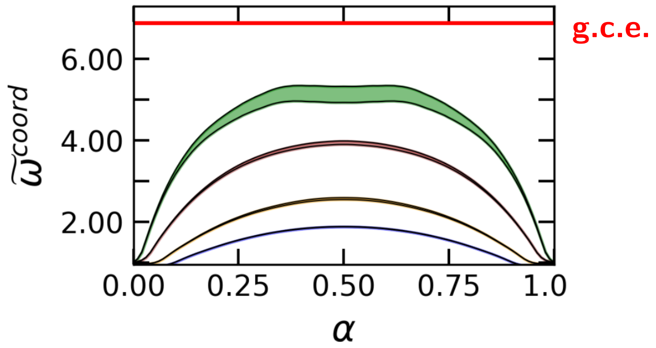
Coordinate vs Momentum space

Box setup: Coordinates and momenta are uncorrelated

Coordinate space cut





Large correlations

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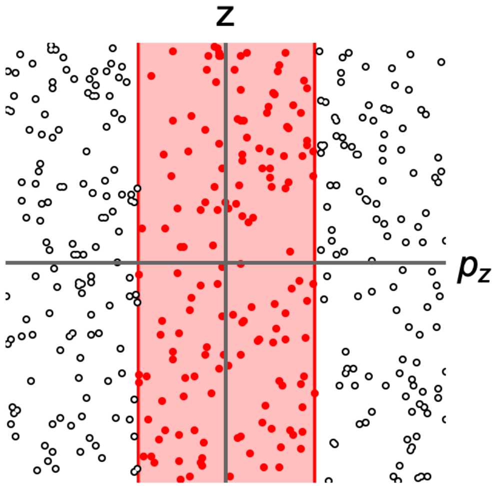

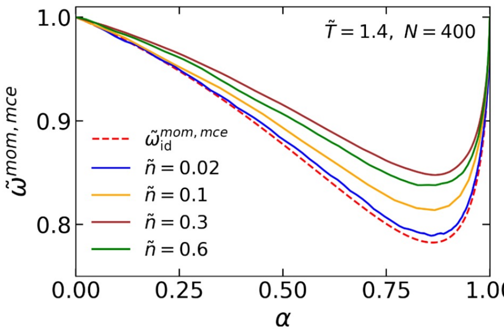

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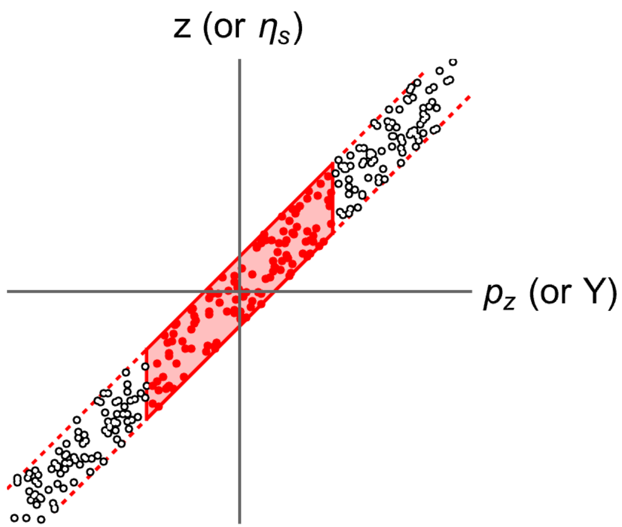
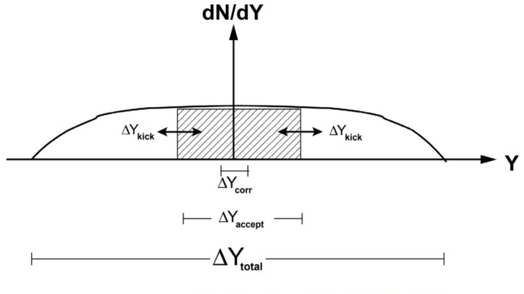
HICs: Flow (e.g. Bjorken)





V. Koch, arXiv:0810.2520

momentum cut $\sim$ coordinate cut $+$ smearing

Dynamical approaches to the QCD critical point search

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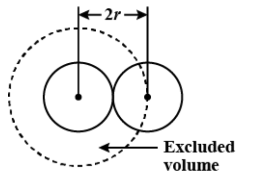
1. Dynamical model calculations of critical fluctuations BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]
   - Fluctuating hydrodynamics (hydro+) or molecular dynamics
   - Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
   - Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

Alternatives at high μ_B: hadronic transport/molecular dynamics with a critical point
[A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznietsov et al., PRC 105, 044903 (2022)]



2. **Deviations from precision calculations of non-critical fluctuations**
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

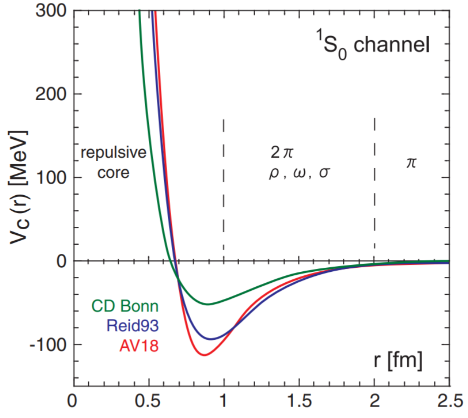


Figure from Ishii et al., PRL '07

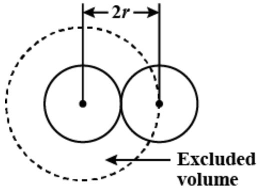
Excluded volume effect

Incorporate repulsive baryon (nucleon) hard core via excluded volume

VV, M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

Amounts to a van der Waals correction for baryons in the HRG model

$$V \to V - bN$$



$$p^{\mathrm{ev}}_{B(\bar{B})} = p^{\mathrm{id}}_{B(\bar{B})}\, e^{-b p^{\mathrm{ev}}_{B(\bar{B})}/T}$$

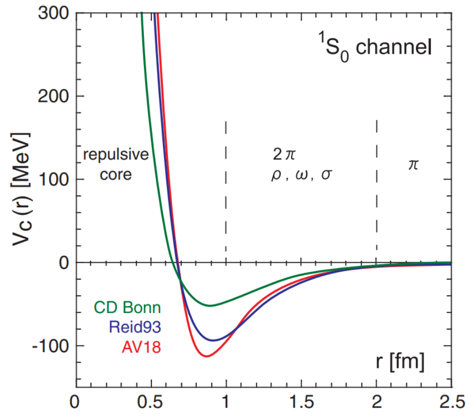
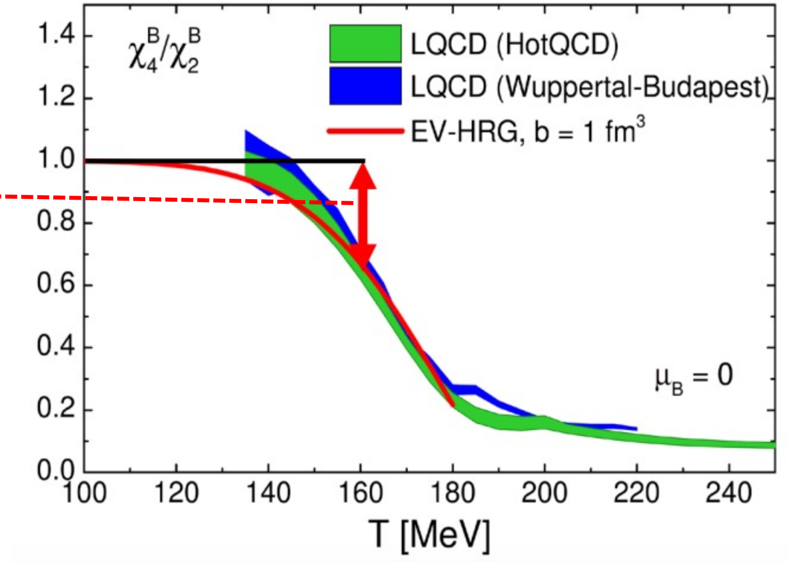


Figure from Ishii et al., PRL '07

- Net baryon kurtosis suppressed as in lattice QCD*

$$\frac{\chi_4^B}{\chi_2^B} \simeq 1 - 12 b \phi_B(T) + O(b^2)$$



- Reproduces virial coefficients of baryon interaction from lattice QCD

VV, A. Pasztor, S. Katz, Z. Fodor, H. Stoecker, Phys. Lett. B 755, 71 (2017)

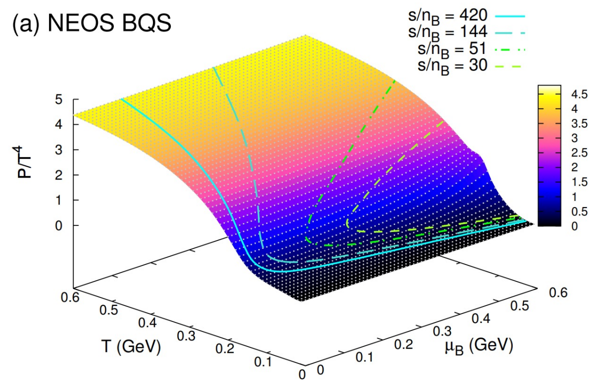
Excluded volume from lattice QCD: **b ≈ 1 fm^3**

*J.M. Karthein, V. Koch, C. Ratti, VV, PRD 104, 094009 (2021)

Hydrodynamic description within non-critical physics

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

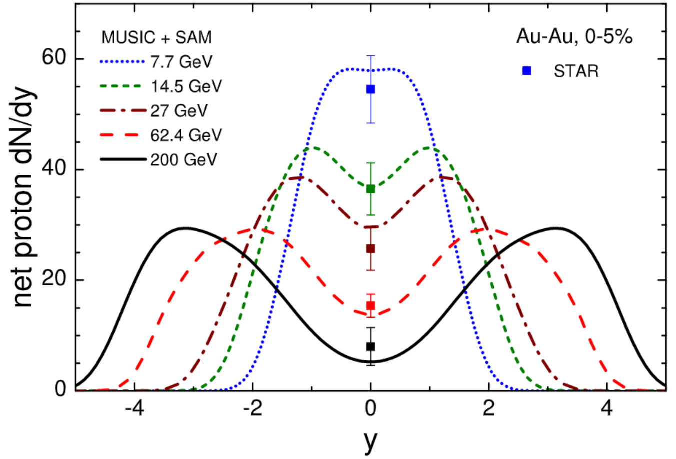
- Collision geometry based 3D initial state
  - Constrained to net proton distributions [Shen, Alzhrani, Phys. Rev. C 102, 014909 (2020)]
- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]



- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu\, \frac{d_j\, \lambda_j^{\rm ev}(x)}{(2\pi)^3}\, \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

- Particlization respects QCD-based baryon number distribution
  - Incorporated via baryon excluded volume b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]



- Incorporates exact global baryon conservation (and other charges) [VV, Phys. Rev. C 105, 014903 (2022)]

- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

Calculating cumulants from hydrodynamics

- **Analytic approach** VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)
  - Calculate proton cumulants in the experimental acceptance in the grand-canonical limit using the Cooper-Frye formula to model acceptance effect
  - Apply correction for the exact global baryon number conservation (SAM-2.0)
    VV, Phys. Rev. C 105, 014903 (2022)

  Pros: Calculate high-order cumulants (up to 8^{th} order) without the need for large statistics

  Cons: The method is approximate and not easily extendable to other observables

- **Monte Carlo approach (FIST sampler)** VV, Phys. Rev. C 106, 064906 (2022) https://github.com/vlvovch/fist-sampler
  - Event generator (Cooper-Frye particlization)
  - Conservation laws (baryon number, also charge and strangeness) via rejection sampling
  - Excluded volume effect by rejecting coordinate space overlap of baryons

  Pros: Flexibility of an event generator, more accurate

  Cons: Need large statistics for high-order cumulants

RHIC-BES: Net proton cumulant ratios (MUSIC)

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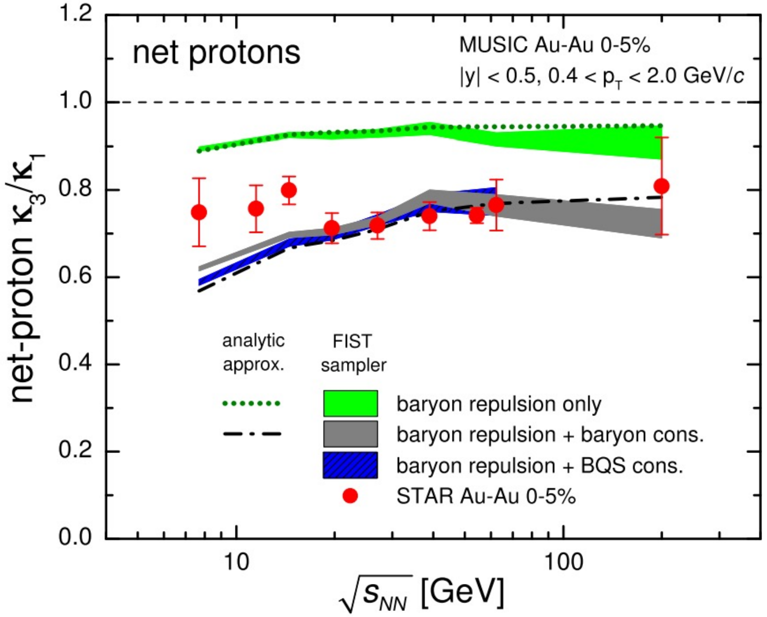
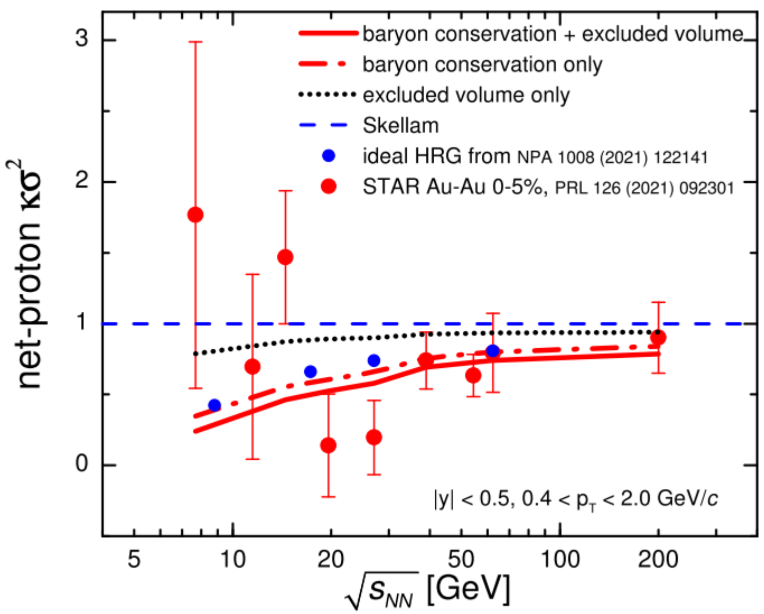
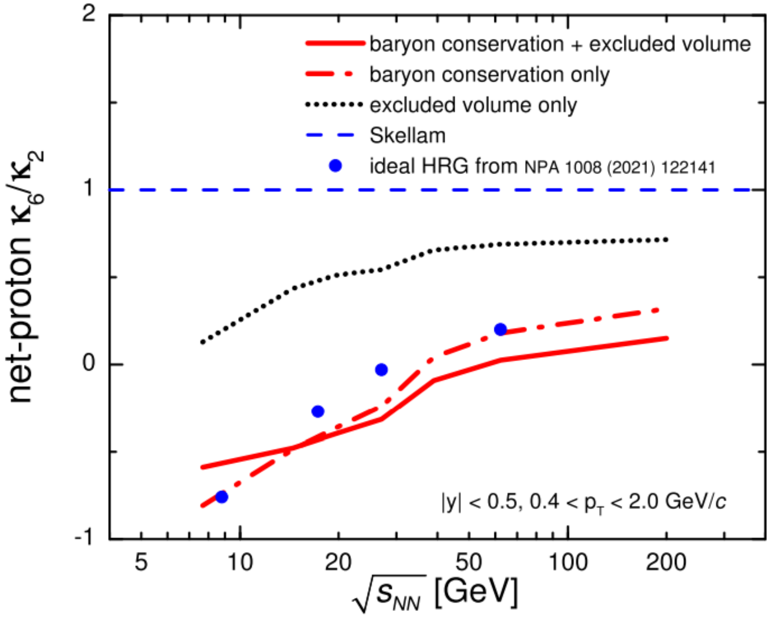
κ_3/κ_1



κ_4/κ_2



κ_6/κ_2



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Canonical ideal HRG limit is consistent with the data-driven study of [Braun-Munzinger et al., 2007.02463]
- κ_6/κ_2 turns negative at $\sqrt{s_{NN}}\sim 50$ GeV

Removing the "net" part: Proton variance

Net-proton $\kappa_2/\kappa_1 \sim \frac{<p+\bar{p}>}{<p-\bar{p}>} \sim \coth(\frac{\mu_B}{T})$ in free gas

Proton $\kappa_2/\kappa_1 \sim \frac{<p>}{<p>} = 1$ in free gas

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022); VV, Phys. Rev. C 106, 064906 (2022)

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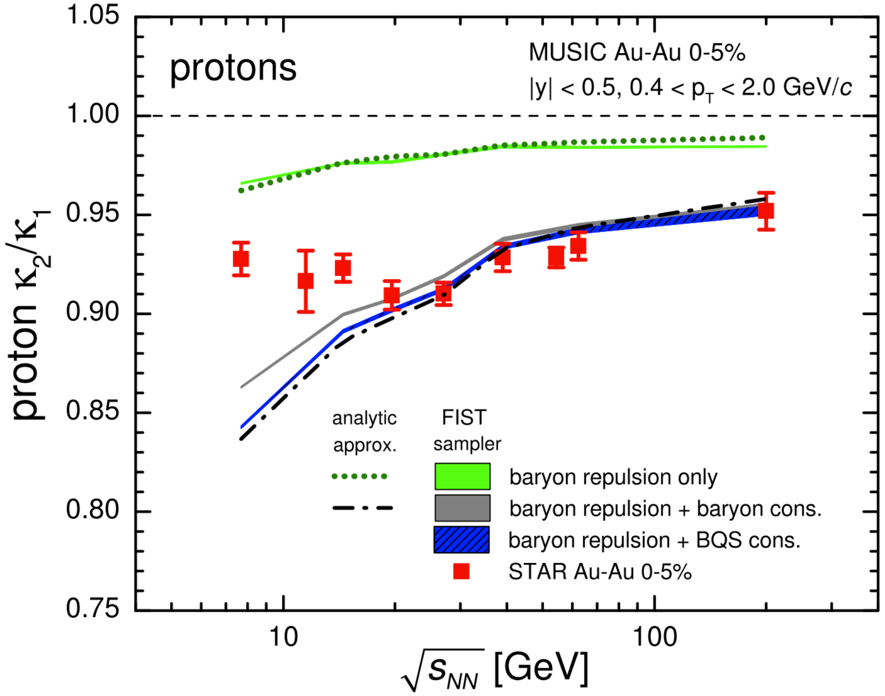
Proton κ_2/κ_1



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Excess of fluctuations in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

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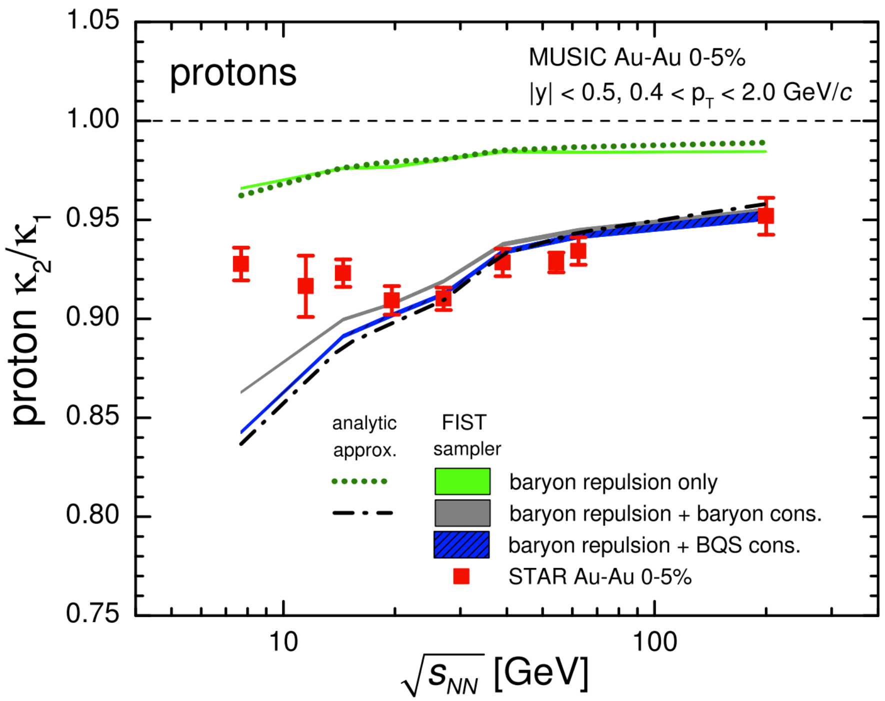
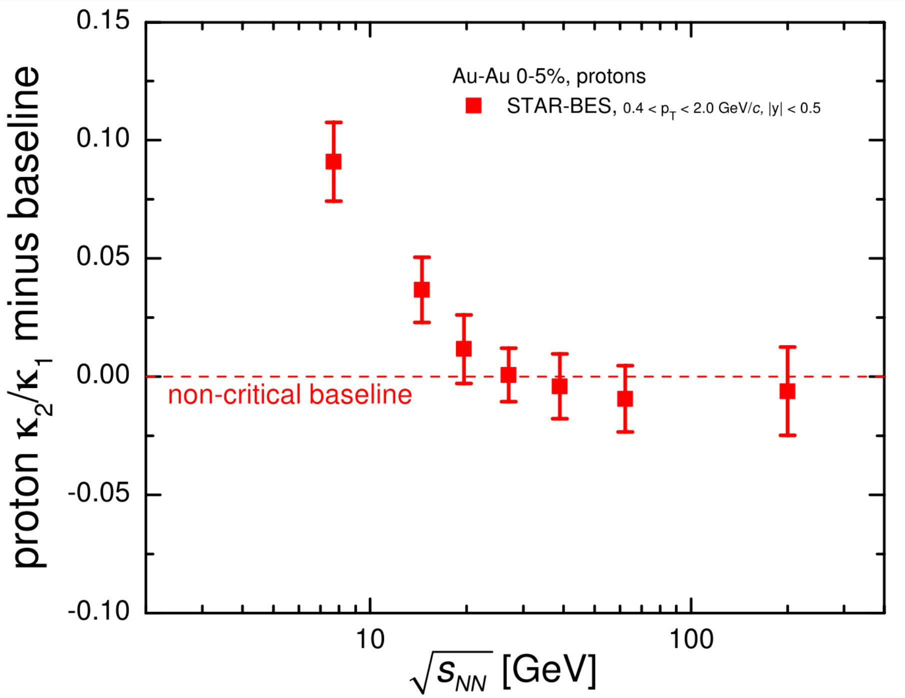
Proton κ_2/κ_1



Proton κ_2/κ_1 excess over baseline



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VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022); VV, Phys. Rev. C 106, 064906 (2022)

Correlation Functions (factorial cumulants)

- Analyze genuine multi-particle correlations via **factorial cumulants** $\hat{C}_n$
  [Bzdak, Koch, Strodthoff, Phys. Rev. C '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

- Three- and four-particle correlations are small without a CP

$$\hat{C}_n^{\text{cons}} \propto \alpha^n, \qquad \hat{C}_n^{\text{EV}} \propto b^n$$

[Bzdak, Koch, Skokov, EPJC '17] [VV et al, PLB '17]

  - Multi-particle correlations expected near the critical point [Ling, Stephanov, PRC '15]

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  [Bzdak, Koch, Strodthoff, Phys. Rev. C '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

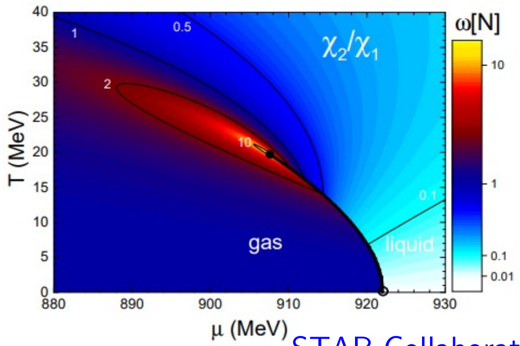
- Three- and four-particle correlations are small without a CP

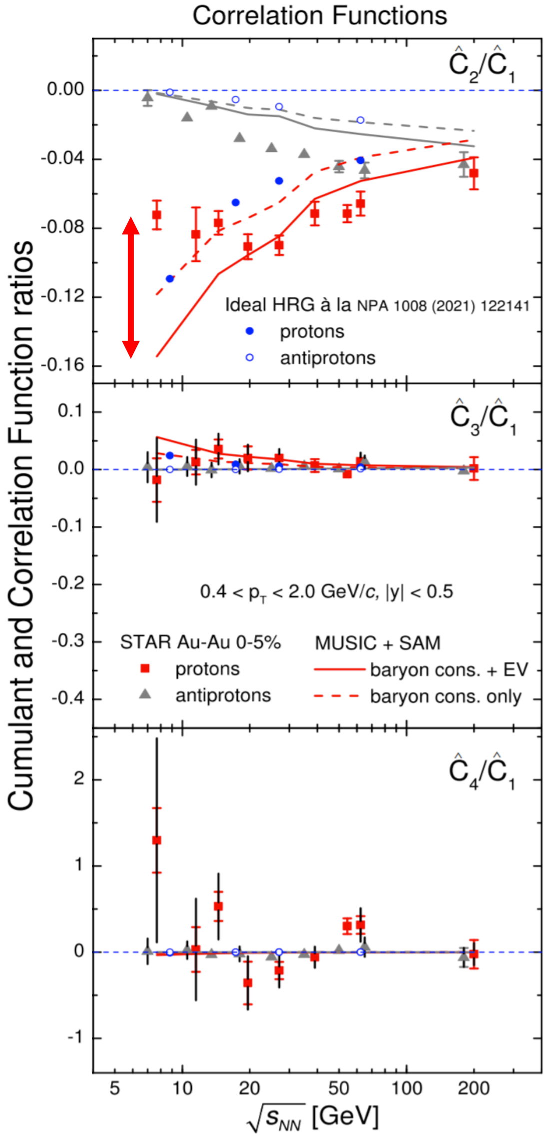
$$\hat{C}_n^{\text{cons}} \propto \alpha^n, \qquad \hat{C}_n^{\text{EV}} \propto b^n$$

[Bzdak, Koch, Skokov, EPJC '17] [VV et al, PLB '17]

  - Multi-particle correlations expected near the critical point [Ling, Stephanov, PRC '15]

- Signals from the data at $\sqrt{s_{NN}} \leq 20$ GeV
  - Excess of two-proton correlations
  - Possibility of significant 4-proton correlations
  - Centrality selection?
  - ***Critical point?***

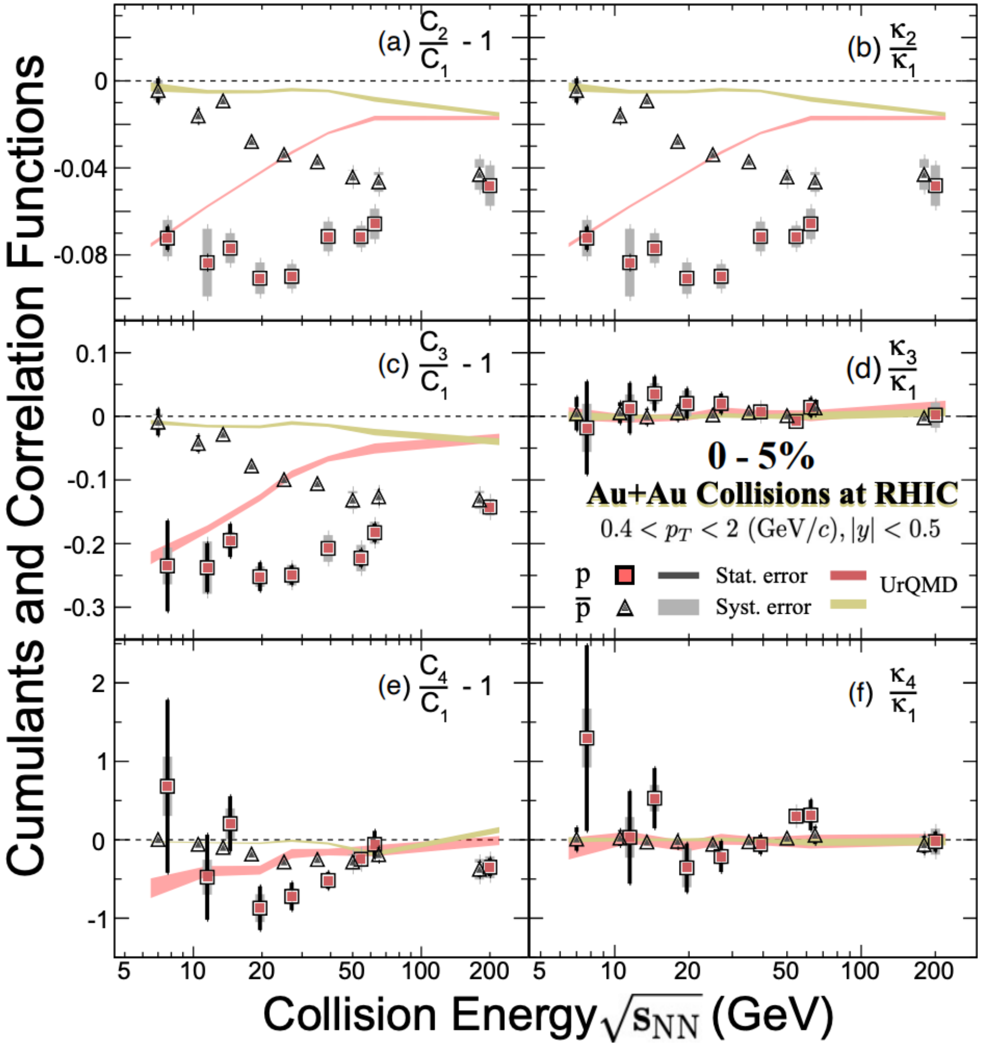




STAR Collaboration, Phys. Rev. C 104 (2021) 024902

Factorial cumulants in UrQMD

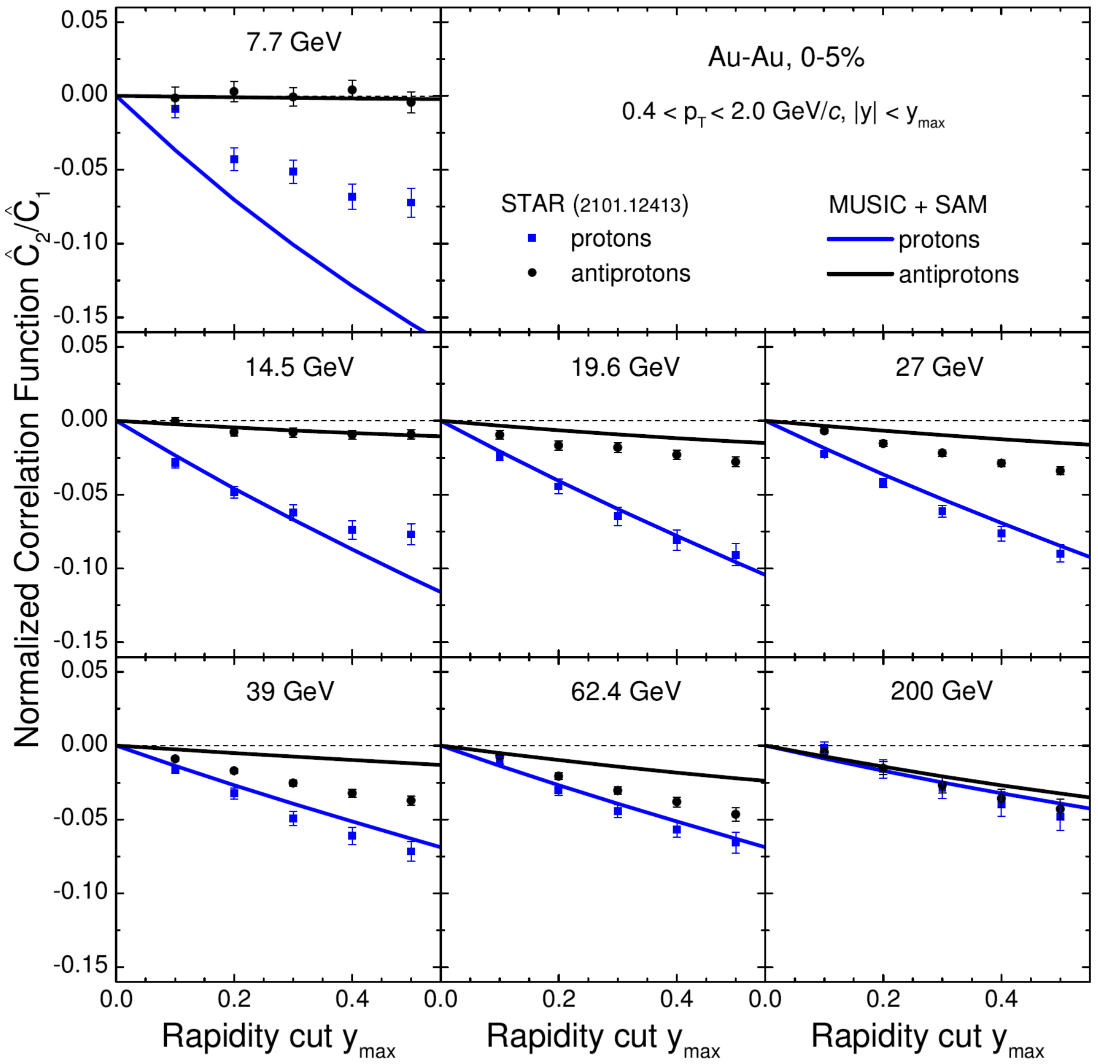
ordinary cumulants factorial cumulants



STAR Collaboration, Phys. Rev. C 104 (2021) 024902

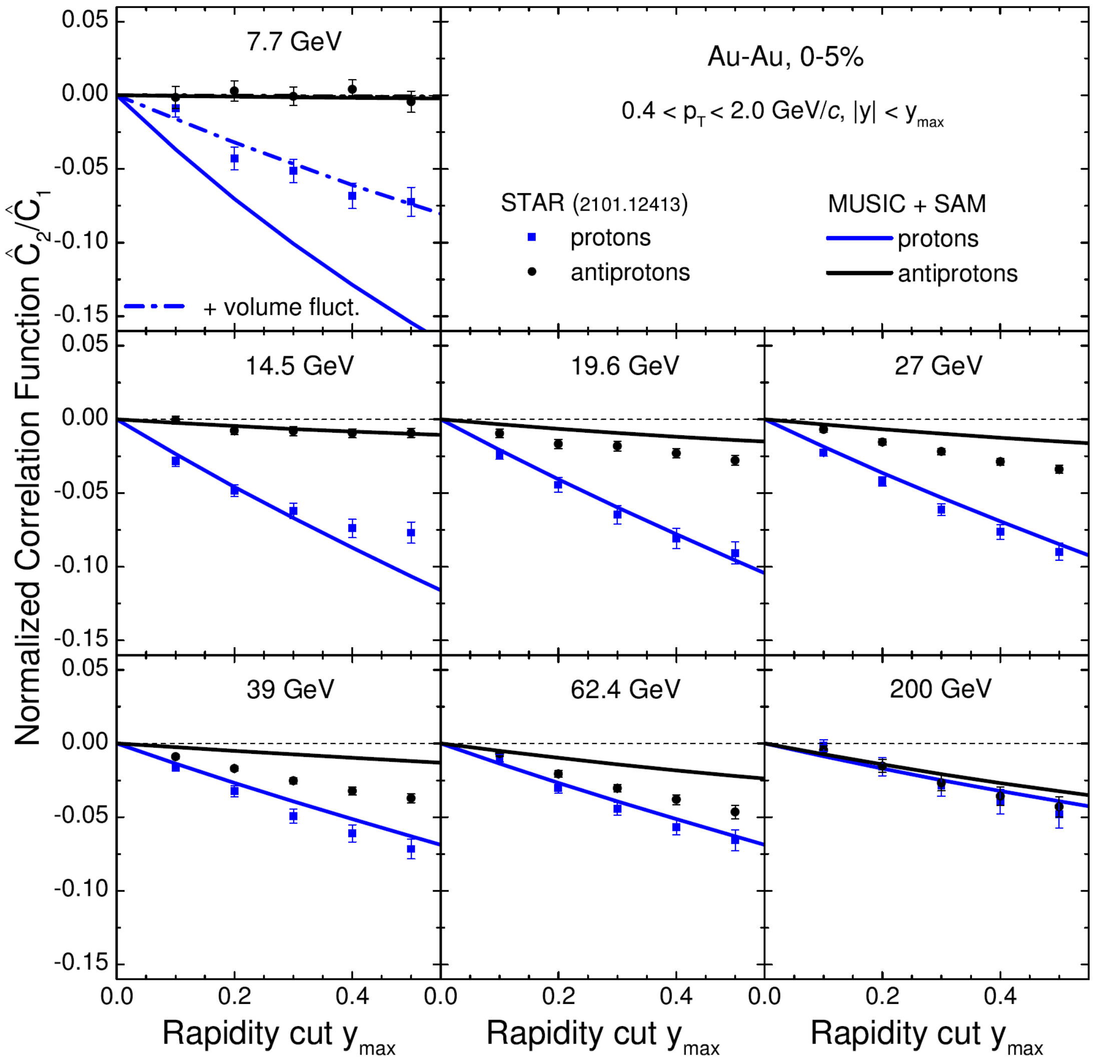
Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?



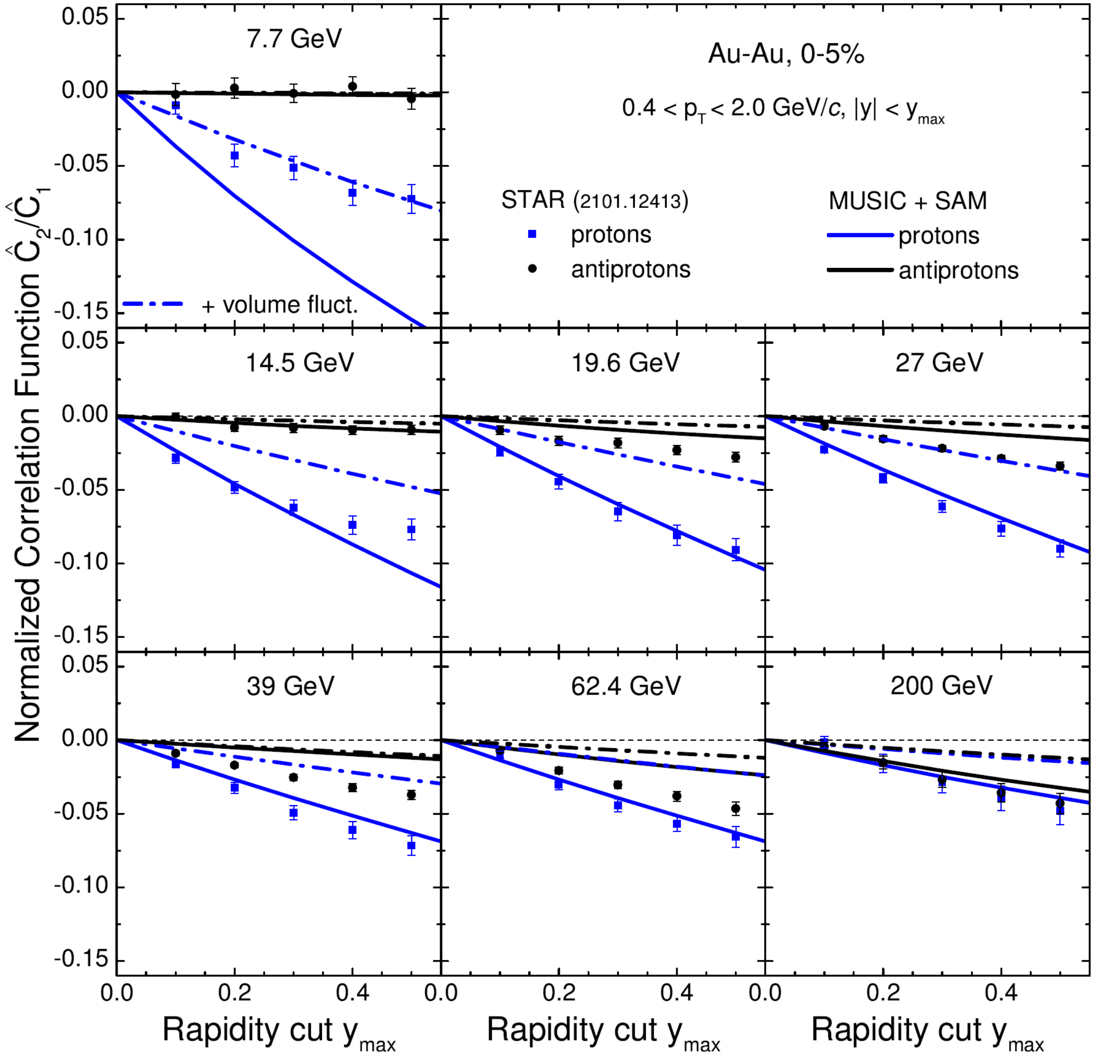
Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * \Delta v^2$



Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 += C_1 * \Delta v^2$
  - Can improve low energies but spoil high energies?



Acceptance dependence of two-particle correlations

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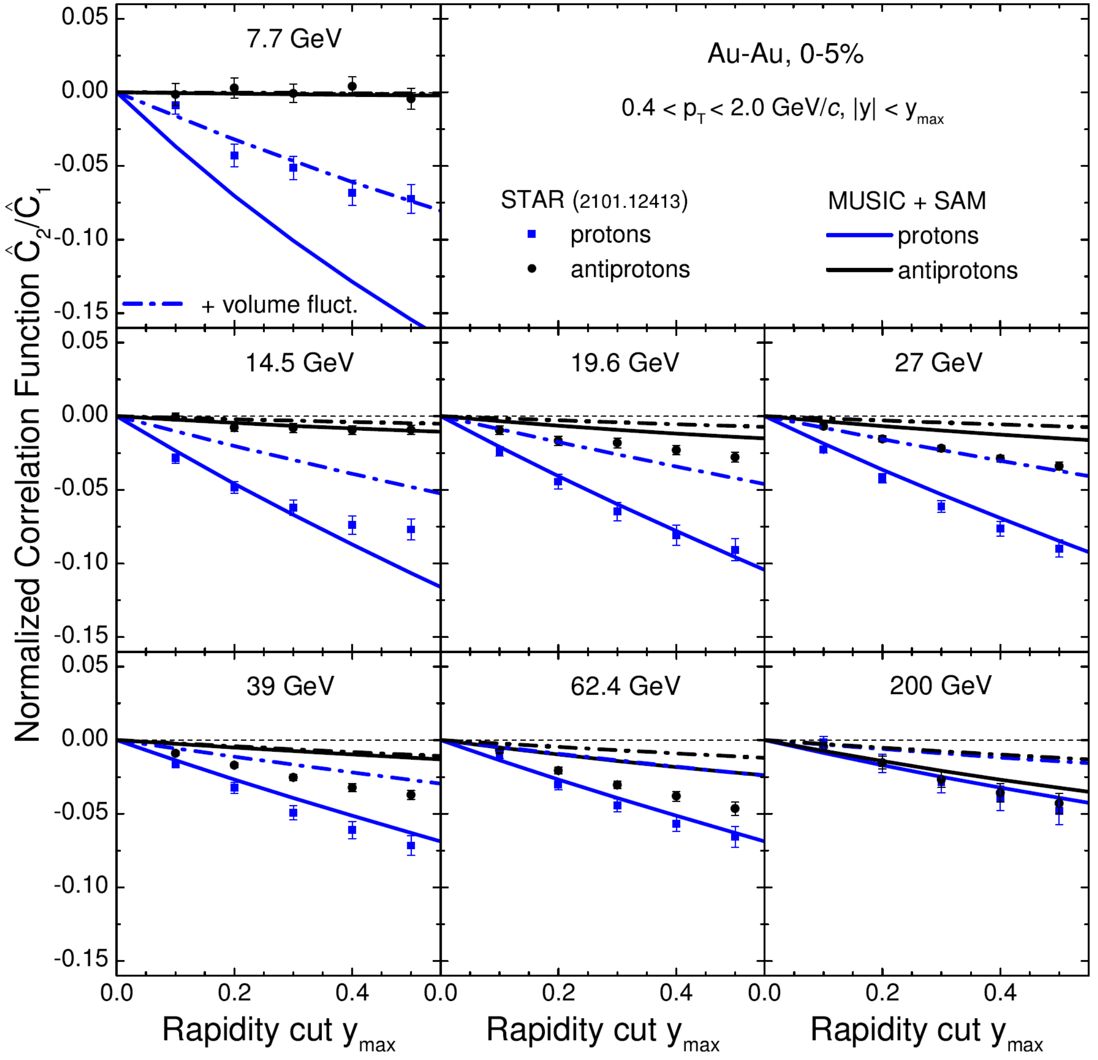
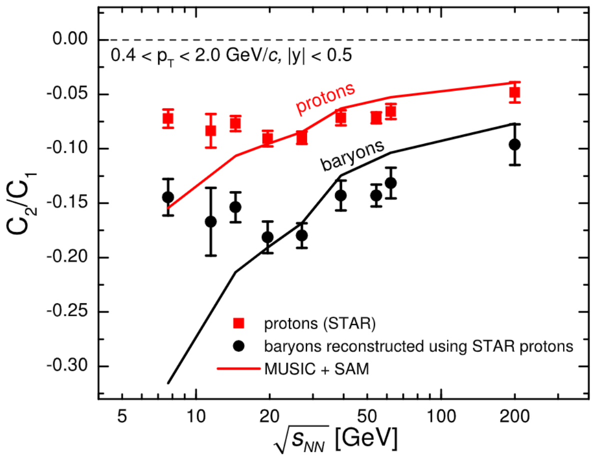
- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * \Delta v^2$
  - Can improve low energies but spoil high energies?
- **Attractive interactions?**
  - Could work if baryon repulsion turns into attraction in the high-μ_B regime
  - **Critical point?**



Baryon cumulants from protons

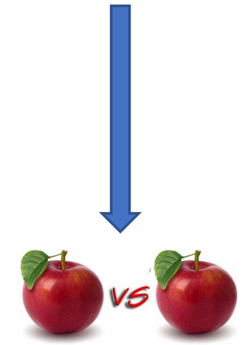
- **net baryon** $\neq$ **net proton** (protons are ***subset*** of all baryons)

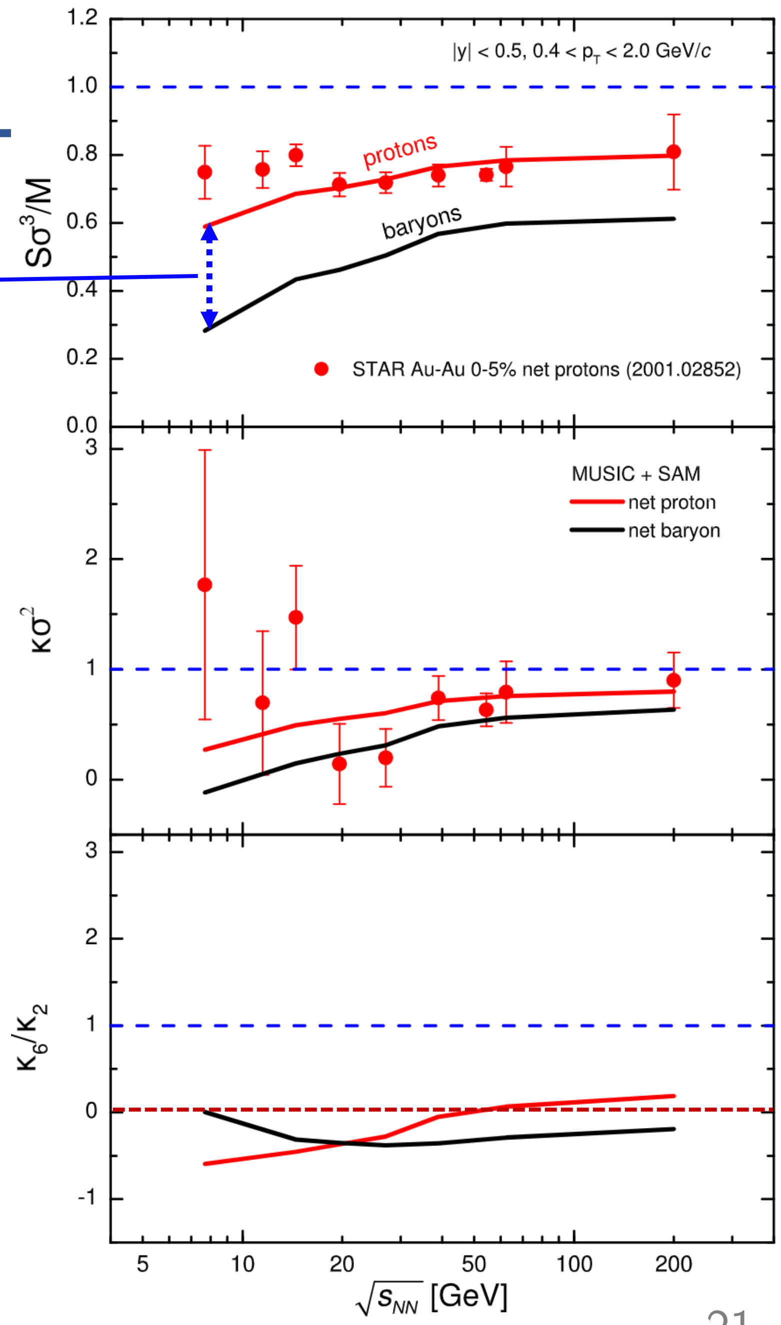
- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Amounts to an additional "efficiency correction" and requires the use of joint factorial moments, only experiment can do it model-independently



$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$

unfolding





Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

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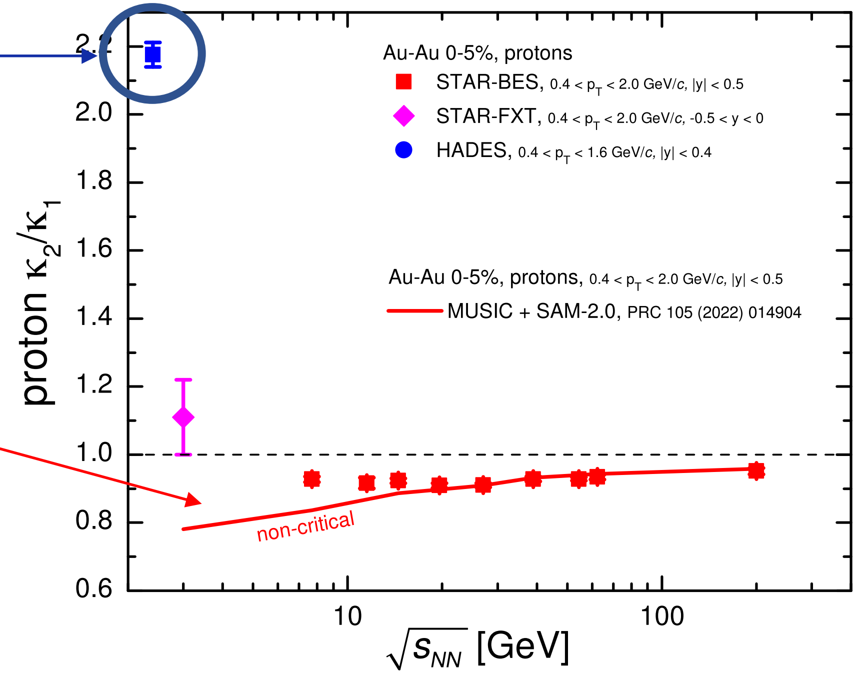
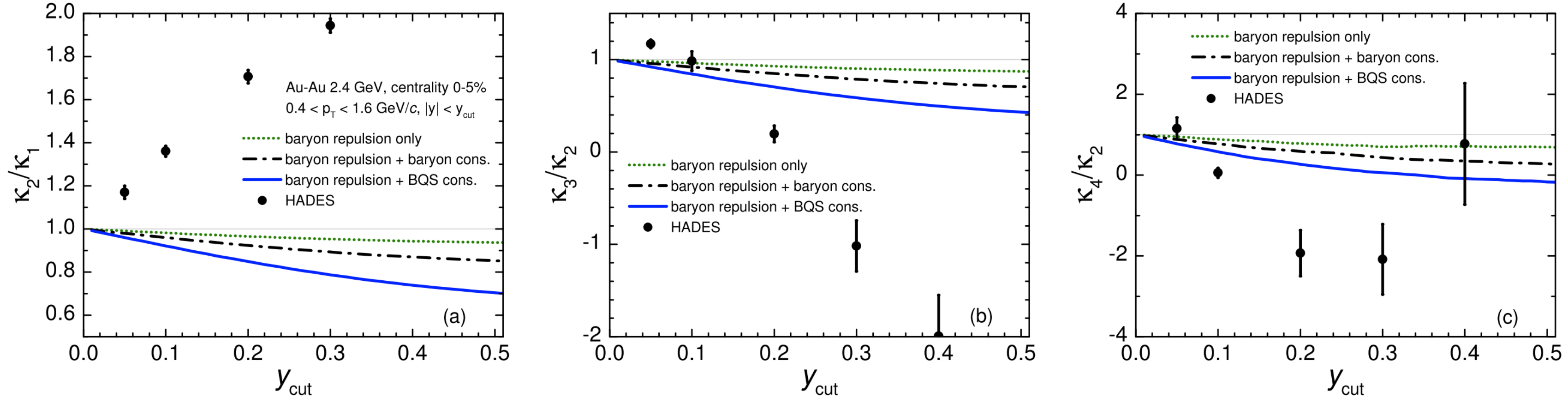
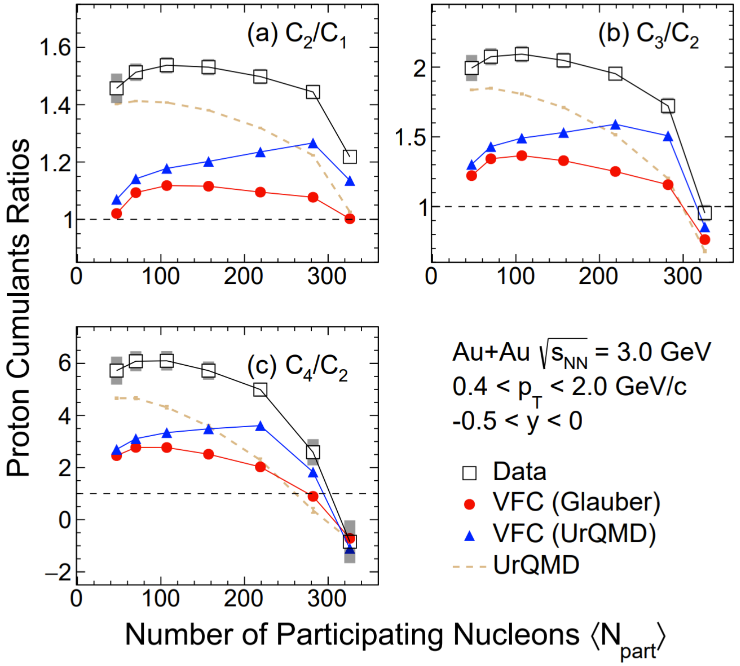
- Intriguing hints from HADES@ 2.4 GeV and STAR-FXT@3GeV: huge excess of two-proton correlations! [HADES Collaboration, Phys. Rev. C 102, 024914 (2020)] [STAR Collaboration, Phys. Rev. Lett. 128, 202303 (2022)]
- No change of trend in the non-critical reference
- Additional mechanisms:
  - Nuclear liquid-gas transition
  - Light nuclei formation/fragmentation





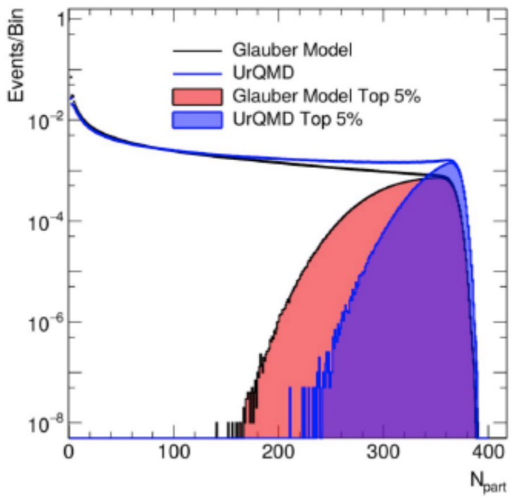
VV, Phys. Rev. C 106, 064906 (2022)

Closer look at data at lower energies



STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303

STAR-FXT



HADES

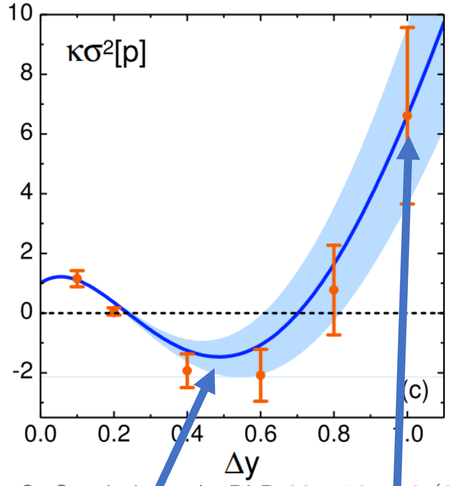


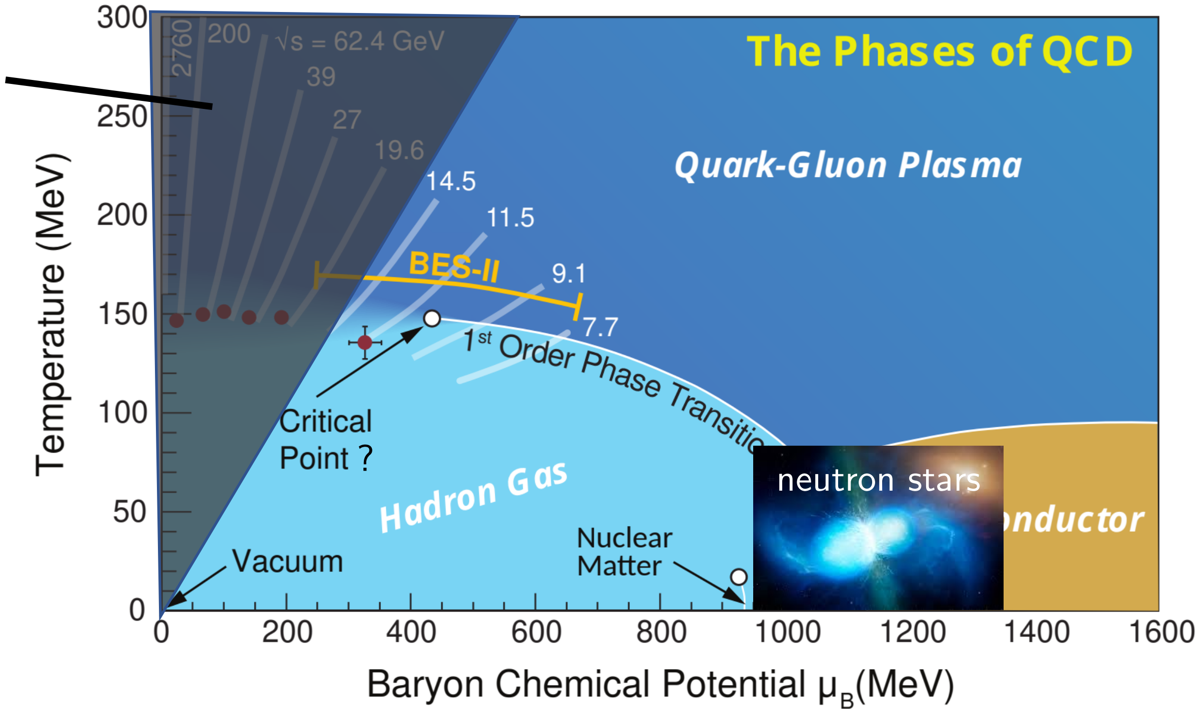
Figure from O. Savchuk et al., PLB 835, 137540 (2022)

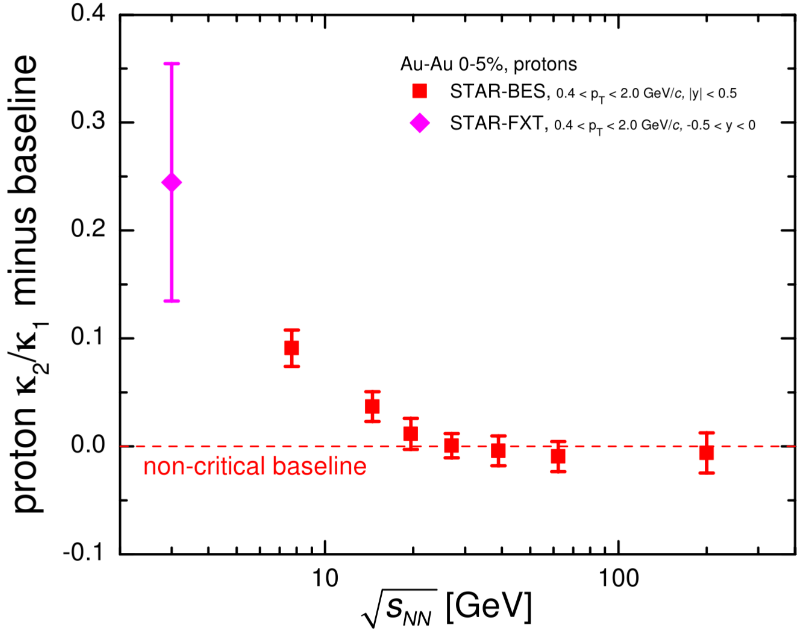
- Volume fluctuations/centrality selection appear to play an important role
  - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (purely hadronic?), note -0.5<y<0 instead of |y|<0.5

Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

Summary: What we learned so far from fluctuations

Critical point disfavored





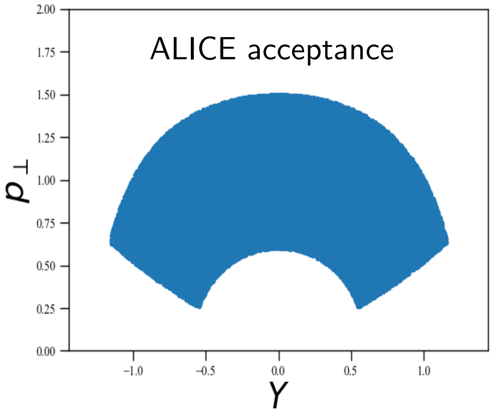
- Data at high energies ($\sqrt{s_{NN}} \geq 20$ GeV) consistent with "non-critical" physics
  - Disfavors QCD critical point at μ_B/T<2-3, consistent with what we know from lattice QCD
- Interesting indications for (multi)-proton correlations at $\sqrt{s_{NN}} \leq 7.7$ GeV, better modeling is required

Thanks for your attention!

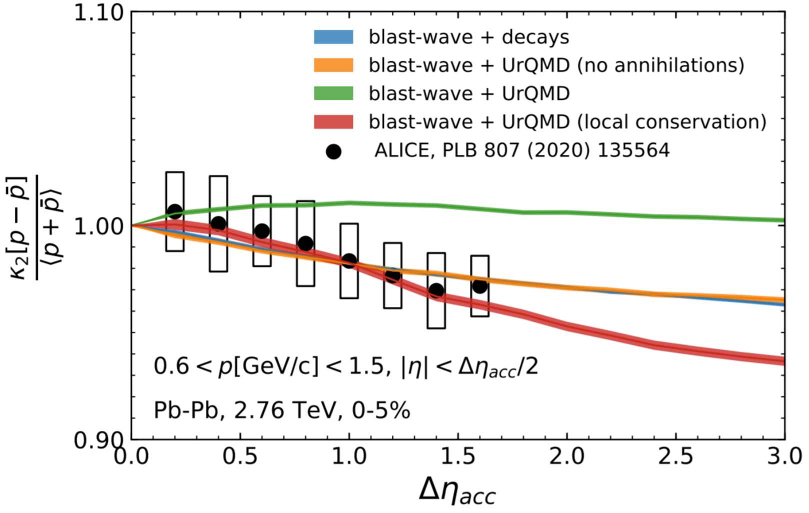
Backup slides

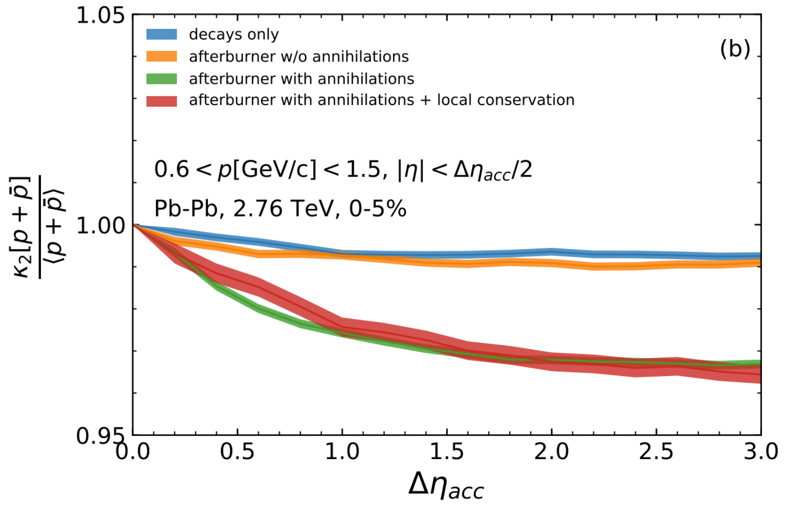
Net-particle fluctuations at the LHC (blast-wave model)

- Net protons described within errors and consistent with either   VV, Koch, Phys. Rev. C 103, 044903 (2021)
  - global baryon conservation without $B\bar{B}$ annihilations in the hadronic phase
    see e.g. ALICE Coll. arXiv:2206.03343
  - or local baryon conservation with $B\bar{B}$ annihilations in the hadronic phase
    O. Savchuk et al., Phys. Lett. B 827, 136983 (2022)



$0.6 < p < 1.5$ GeV/c, $\Delta\eta_{acc} = 1.6$





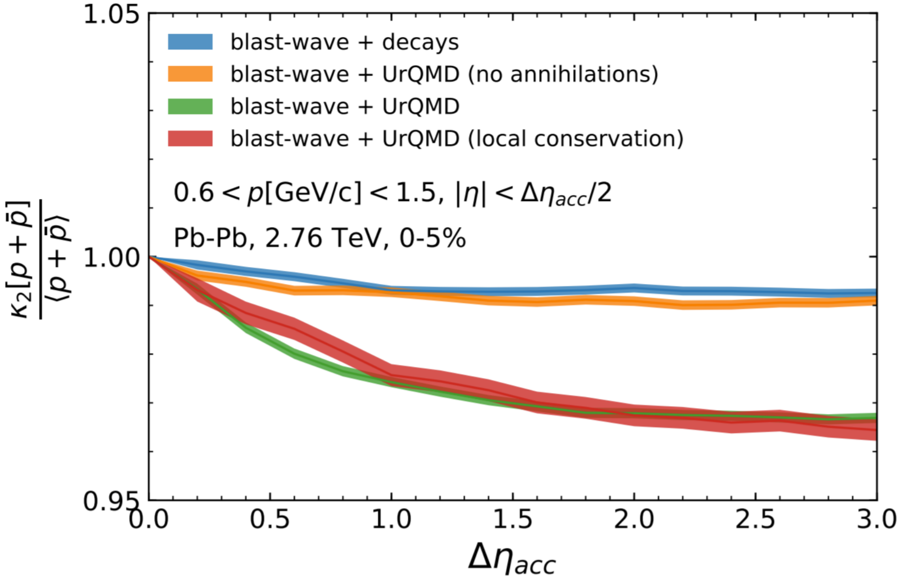
Data on (net-)proton fluctuations can constrain the effect of annihilations in the hadronic phase

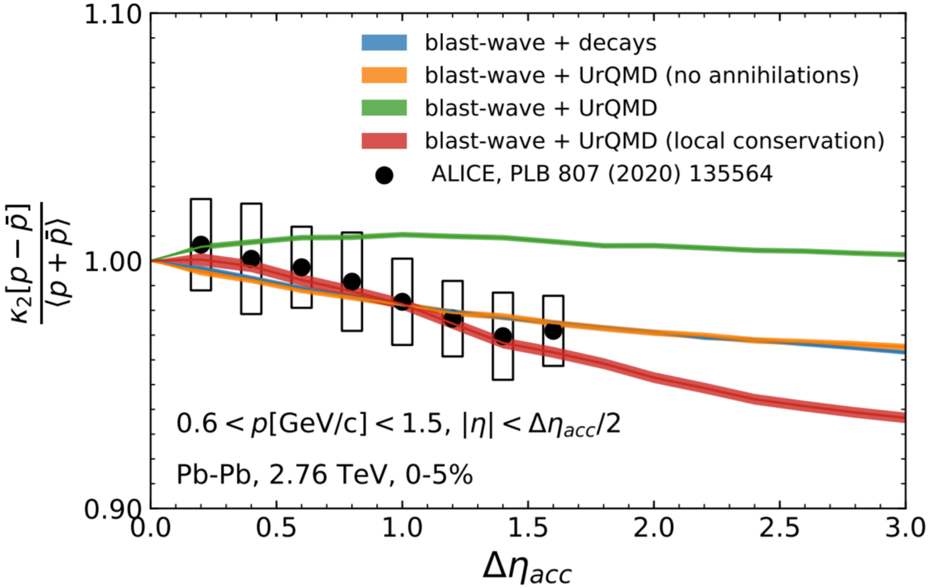
Effects of baryon annihilation and local conservation

O. Savchuk, V.V., V. Koch, J. Steinheimer, H. Stoecker, arXiv:2106.08239

Baryon annihilation $B\bar{B} \to n\pi$ in afterburners (UrQMD, SMASH) suppresses baryon yields

$\langle p + \bar{p} \rangle \searrow$ → $\frac{\kappa_2[p-\bar{p}]}{\langle p+\bar{p} \rangle} \nearrow$





- ALICE data requires local baryon conservation across $\Delta y \sim \pm 1.5$ with UrQMD annihilations (no regenerations) or global conservation ($\Delta y \sim \Delta y_{tot}$) without annihilations
- Local conservation and $B\bar{B}$ annihilation can be constrained from data through the combined analysis of $\kappa_2[p-\bar{p}]$ and $\kappa_2[p+\bar{p}]$

Thermodynamic analysis of HADES data

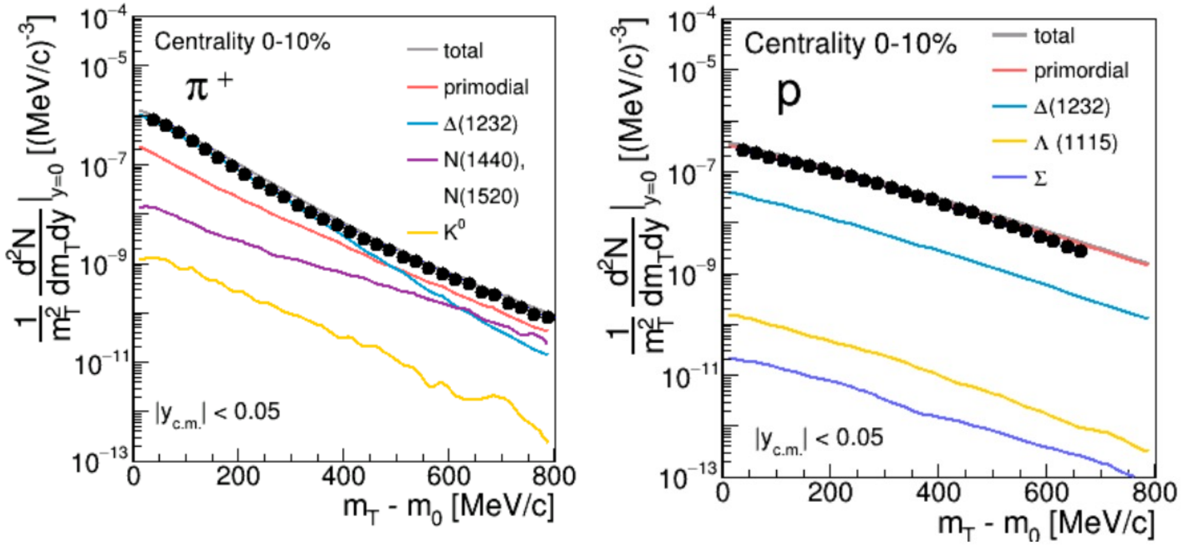
VV, Koch, Phys. Lett. B 833, 137368 (2022)

- Single freeze-out scenario: Emission from Siemens-Rasmussen hypersurface with Hubble-like flow

  → Pion and proton spectra o.k.

  [S. Harabasz et al., PRC 102, 054903 (2020)]

- Uniform $T \approx 70$ MeV, $\mu_B \approx 875$ MeV across the fireball [A. Motornenko et al., PLB 822, 136703 (2021)]



- Fluctuations:
  - Same as before but incorporate additional binomial filtering to account for protons bound in light nuclei
  - Uniform fireball → Final proton cumulants are linear combinations of baryon susceptibilities χ_n^B at freeze-out

$$\kappa_n^p = \sum_{m=1}^{n} \alpha_{n,m}\, \chi_m^B$$

Extract χ_n^B directly from experimental data

A closer look at the HADES data

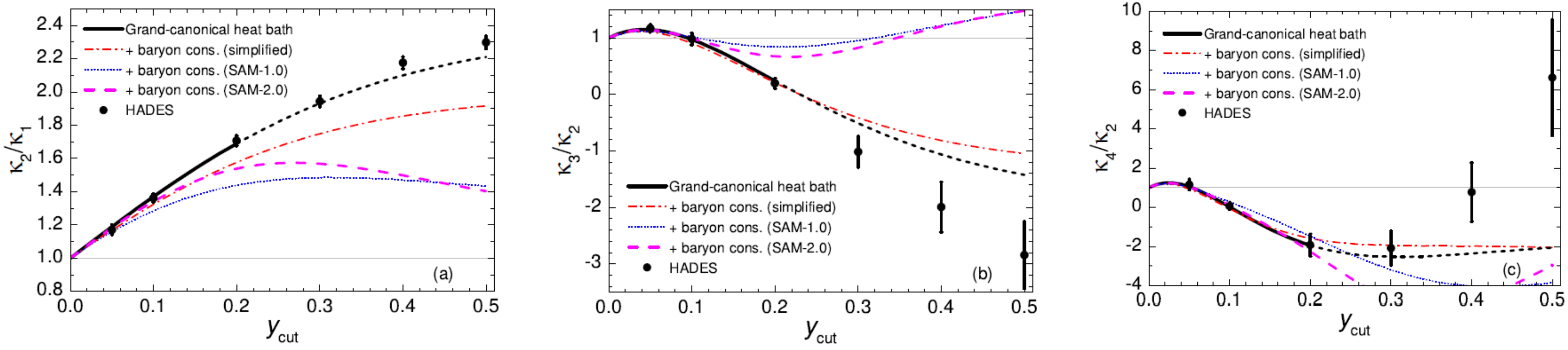
VV, Koch, Phys. Lett. B 833, 137368 (2022)

- Fit baryon susceptibilities to data within a fireball model (Siemens-Rasmussen*)

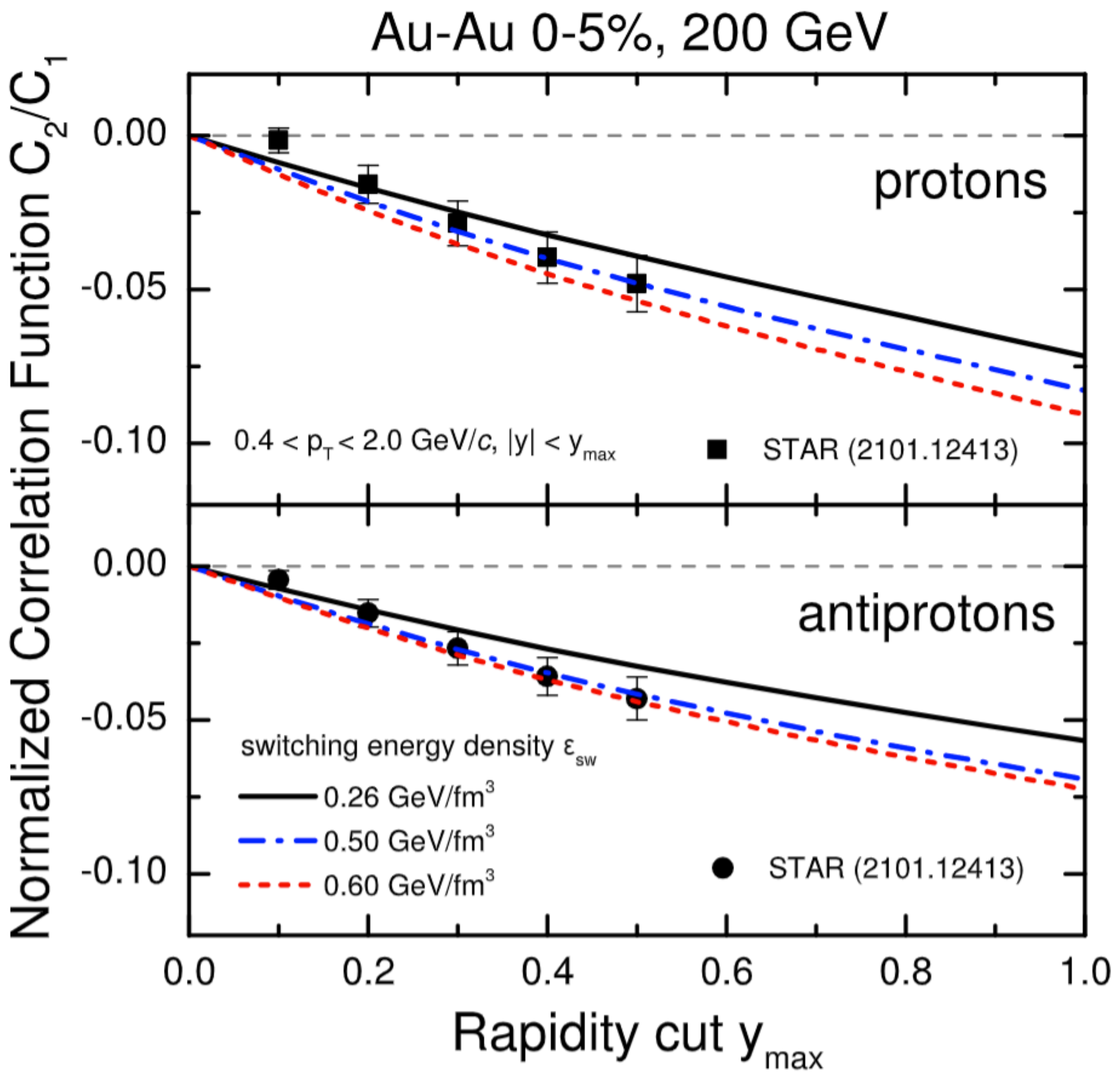
- In the grand-canonical limit (no baryon conservation, small y_{cut}) the data are described well with

$$\frac{\chi_2^B}{\chi_1^B} \sim 9.17 \pm 0.21, \qquad \frac{\chi_3^B}{\chi_2^B} \sim -33.1 \pm 0.8, \qquad \frac{\chi_4^B}{\chi_2^B} \sim 691 \pm 50, \qquad \text{i.e.} \qquad \chi_4^B \gg -\chi_3^B \gg \chi_2^B \gg \chi_1^B$$

- Could be indicative of a *critical point* near the HADES freeze-out at T~70 MeV, μ_B~875 MeV
- However, the results for $y_{cut} > 0.2$ are challenging to describe with baryon conservation included

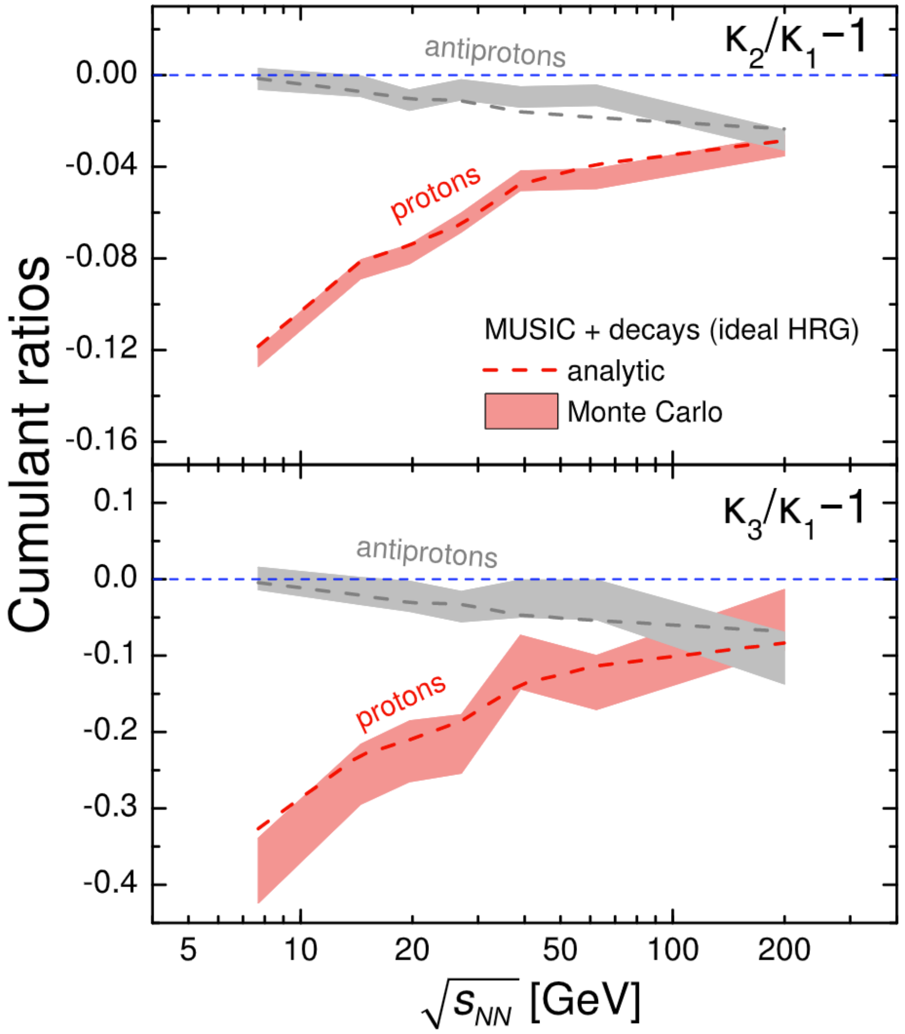


Dependence on the switching energy density



Cross-checking the cumulants with Monte Carlo

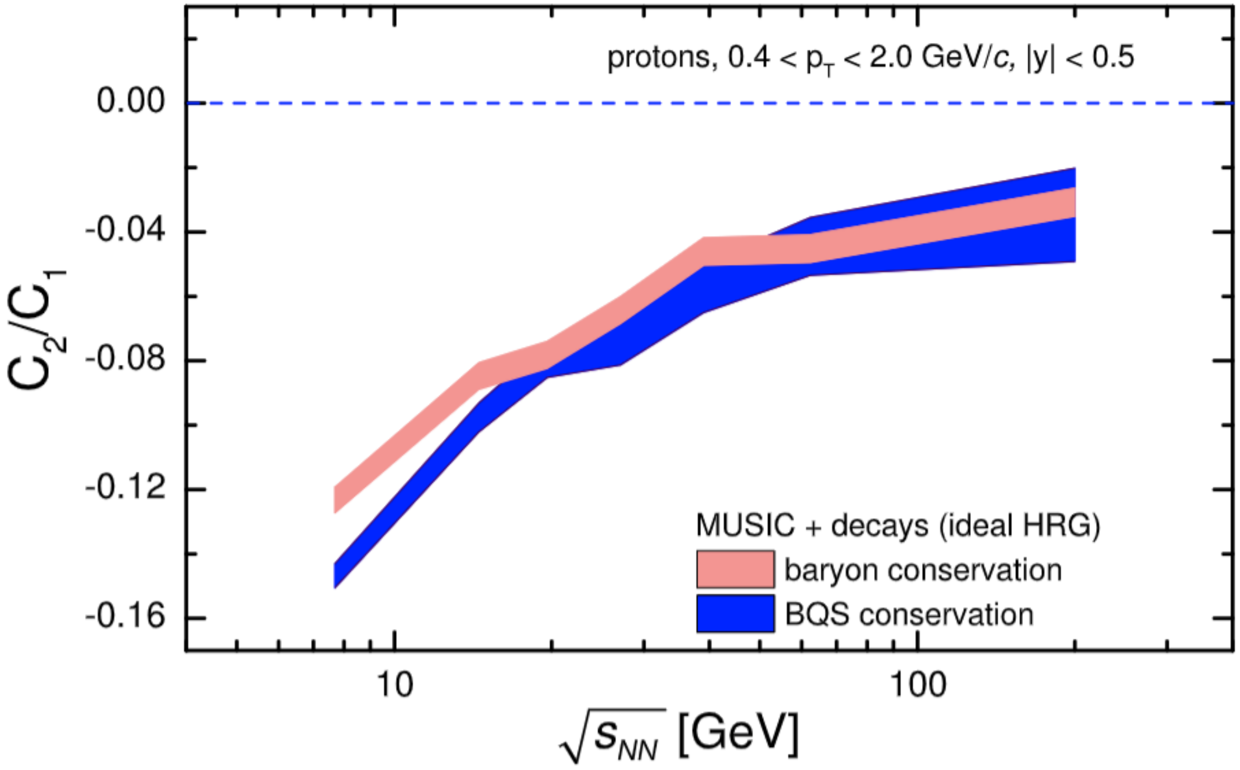
- Sample canonical ideal HRG model at particlization with Thermal-FIST
- Analytic results agree with Monte Carlo within errors



Exact conservation of electric charge

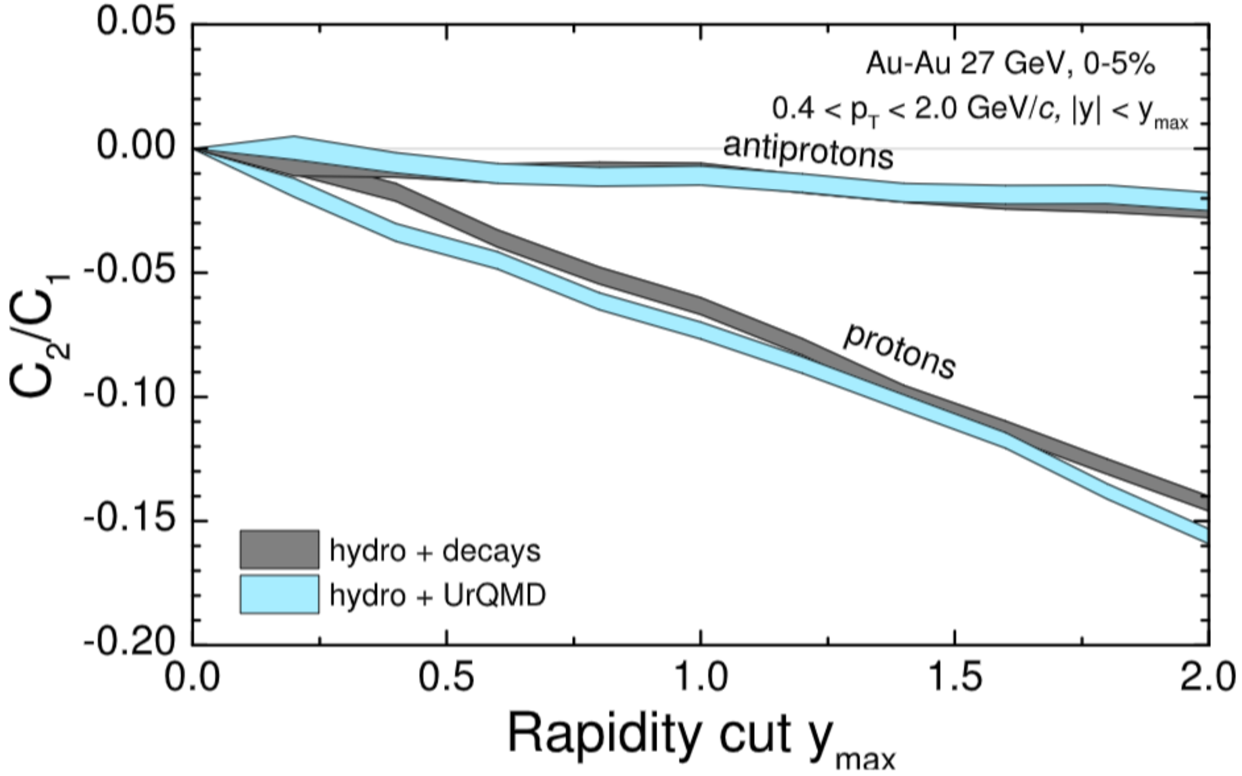
- Sample ideal HRG model at particlization with exact conservation of baryon number, electric charge, and strangeness using Thermal-FIST

- Protons are affected by electric charge conservation at $\sqrt{s_{NN}} \leq 14.5$ GeV



Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

e.g. $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=160MeV}^{\mathrm{GCE}} \simeq 0.67 \neq \left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{acc}=1}^{\mathrm{HIC}} \simeq 0.56$

"lattice QCD" *experiment*

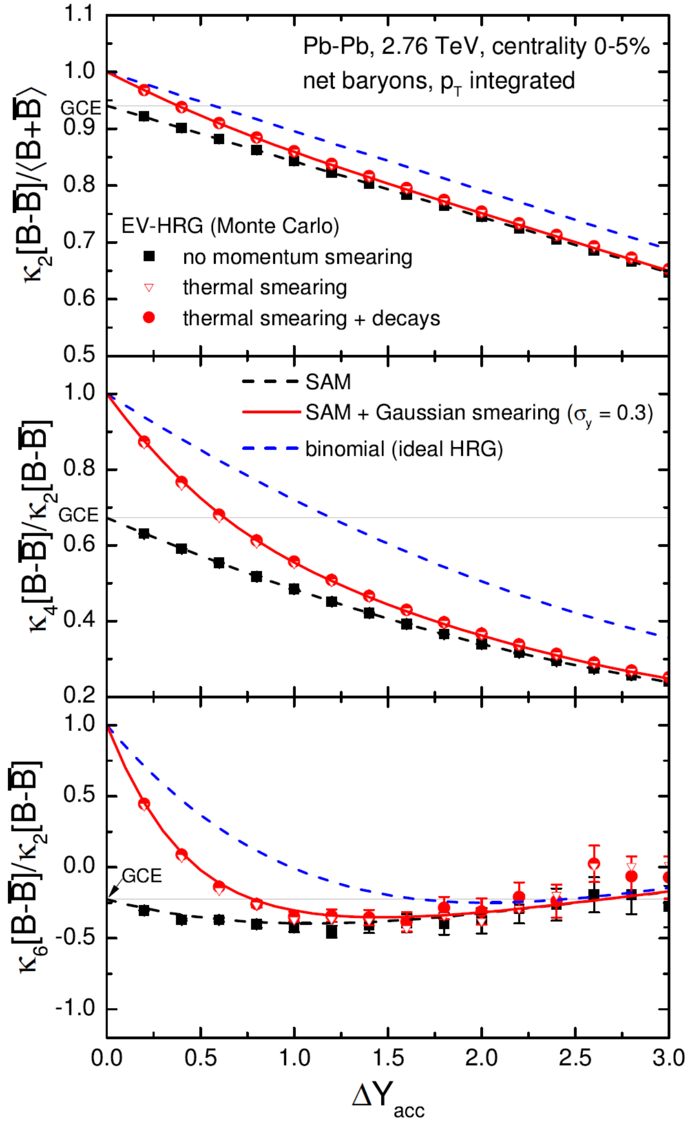
- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

$$\frac{\kappa_2}{\langle B+\bar{B}\rangle} = (1-\alpha)\frac{\kappa_2^{\mathrm{gce}}}{\langle B+\bar{B}\rangle}, \qquad \alpha = \frac{\Delta Y_{\mathrm{acc}}}{9.6}, \quad \beta \equiv 1-\alpha$$

$$\frac{\kappa_4}{\kappa_2} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$

- Effect of resonance decays is negligible



VV, Koch, arXiv:2012.09954

Net baryon vs net proton 

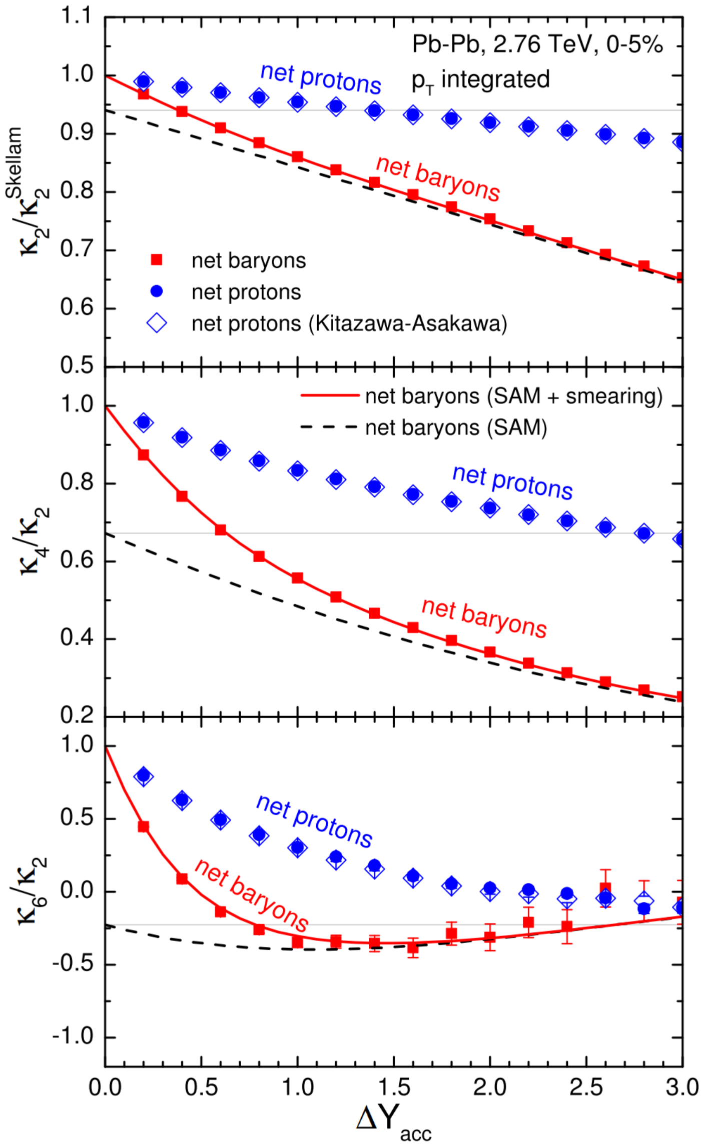
- Thermal smearing distorts the signal at $\Delta Y_{accept} \leq 1$. Net baryons converge to model-independent SAM result at larger ΔY_{accept}

- net baryon ≠ net proton, e.g.

$$\left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{\text{acc}}=1}^{\text{HIC}} \simeq 0.56 \quad \neq \quad \left.\frac{\chi_4^p}{\chi_2^p}\right|_{\Delta Y_{\text{acc}}=1}^{\text{HIC}} \simeq 0.83$$

- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Requires the use of joint factorial moments, only experiment can do it model-independently

unfolding →



VV, Koch, arXiv:2012.09954