Probing the EoS of dense matter with fluctuation observables in heavy-ion collisions

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INT Workshop INT-22-84W "Dense Nuclear Matter Equation of State from Heavy-Ion Collisions"

September 7, 2022

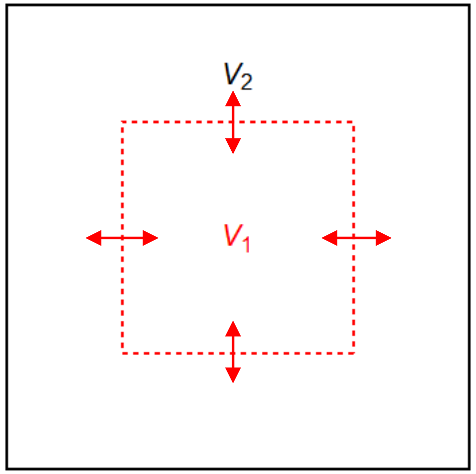
Outline

- Cumulants of event-by-event fluctuations and statistical mechanics
    - Degrees of freedom
    - QCD critical point
    - Speed of sound
    - Density-dependent mean field

- Experimental results

- Towards fluctuations in transport/molecular dynamics

Event-by-event fluctuations and statistical mechanics



Cumulant generating function

$$K_N(t) = \ln\langle e^{tN} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

Grand partition function

$$\ln Z^{\mathrm{gce}}(T, V, \mu) = \ln \left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T, V, N) \right]$$

$$\kappa_n \propto \frac{\partial^n (\ln Z^{\mathrm{gce}})}{\partial \mu^n}$$

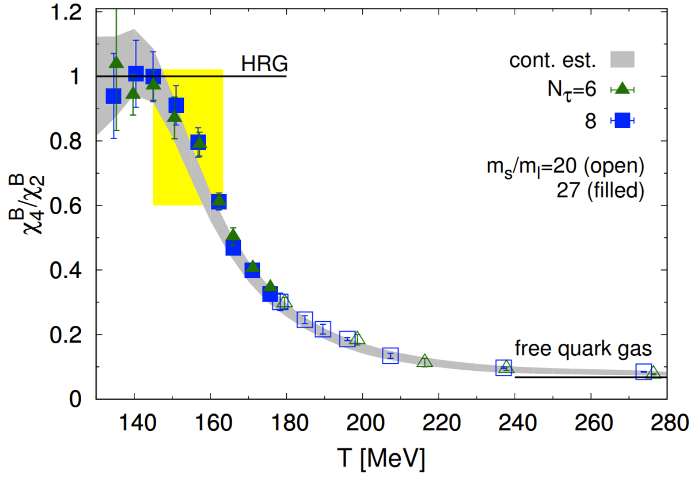
Cumulants measure chemical potential derivatives of the (QCD) equation of state

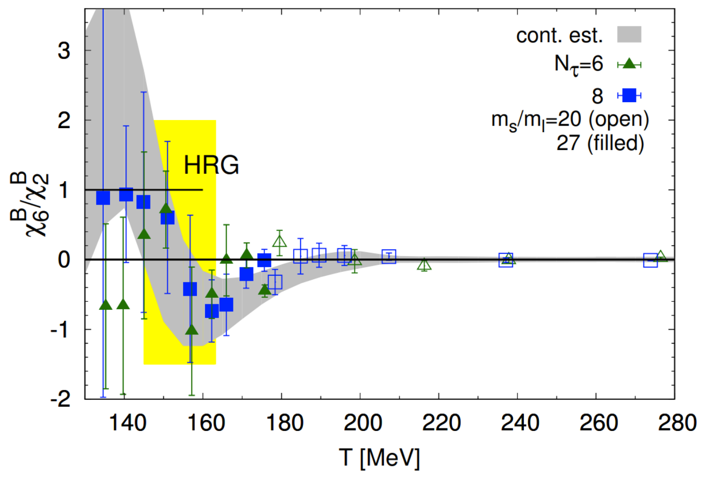
Cumulants and degrees of freedom

- Sensitive to QCD transition and change of **DoFs**
- Suppression in QGP due to fractional charge carriers

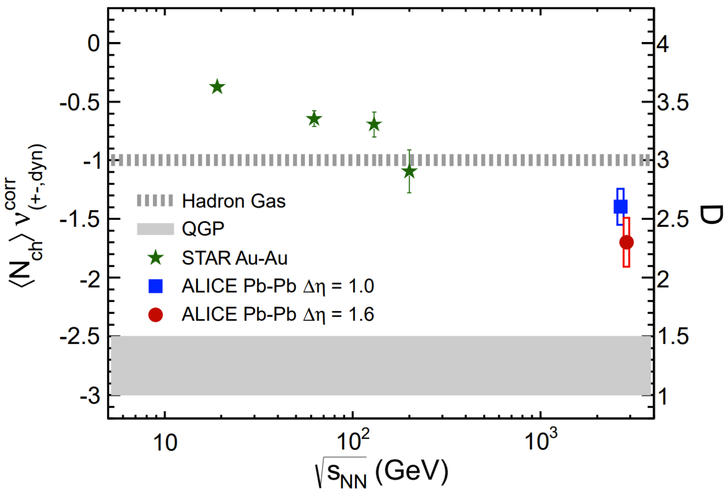
$\langle \delta B^2 \rangle \sim \langle B_i^2 \rangle$ $\langle \delta B^4 \rangle \sim \langle B_i^4 \rangle$

Jeon, Koch, PRL 85 (2000) 2076;
Asakawa, Heinz, Muller, PRL 85 (2000) 2072



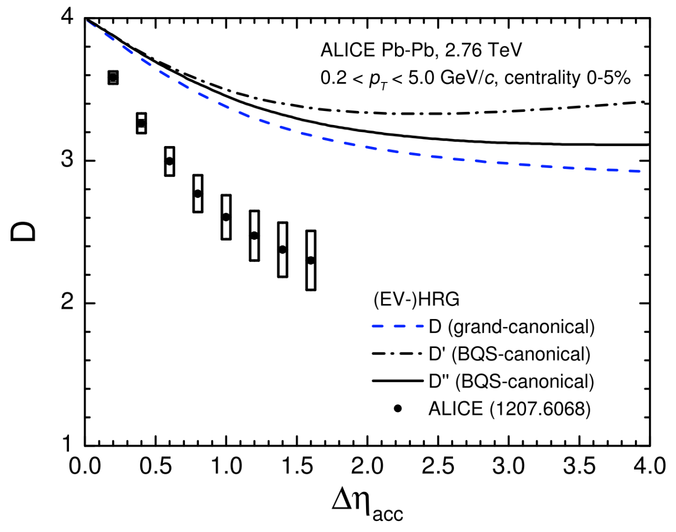


HotQCD Coll., Phys. Rev. D 95 (2017) 054504



ALICE Coll., PRL 110 (2013) 152301

$$D = 4\frac{\langle \delta Q^2 \rangle}{N_{ch}}$$

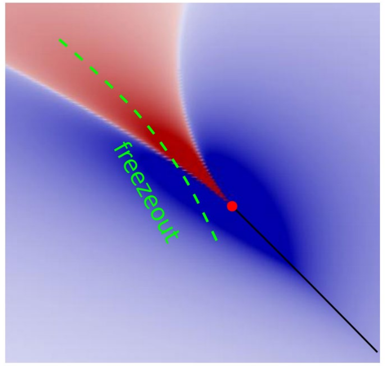


VV, Koch, Phys. Rev. C 95 (2017) 054504

Cumulants probe the degrees of freedom of QCD matter

Cumulants and QCD critical point

- **(QCD) critical point** – large correlation length, critical fluctuations of baryon number



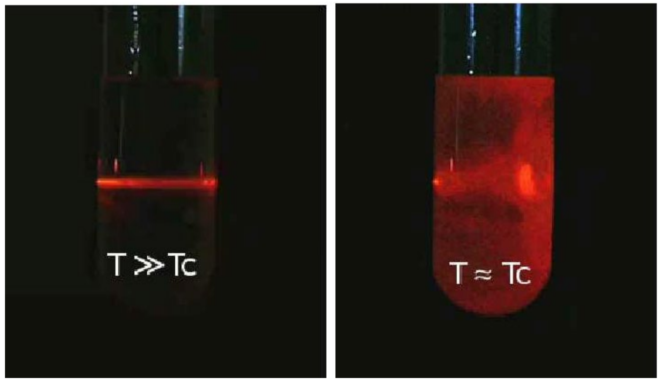
M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

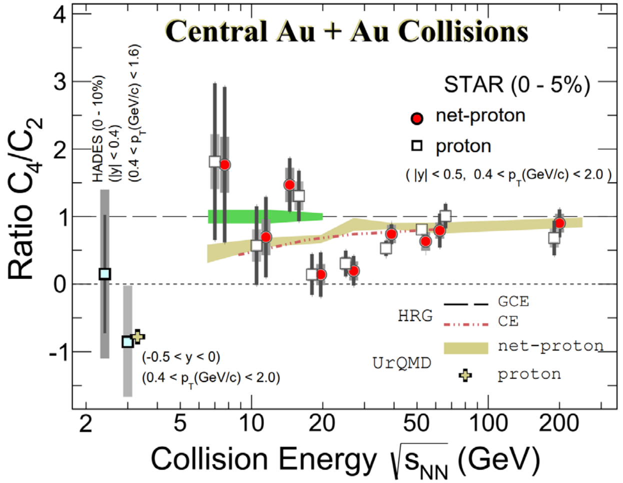
$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

$$\xi \to \infty$$

Looking for enhanced fluctuations
and non-monotonicities

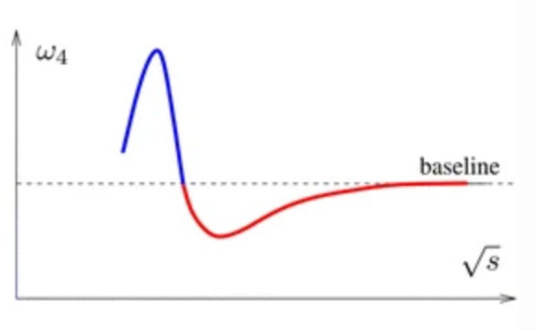
Critical opalescence





STAR Coll., PRL 126, 092301 (2021); PRL 128, 202303 (2022)

?



M. Stephanov, Phys. Rev. Lett. (2011)

Cumulants probe the QCD critical point

Cumulants and the speed of sound

- Isothermal **speed of sound**, $c_T^2 = \left(\frac{dP}{d\varepsilon}\right)_T$

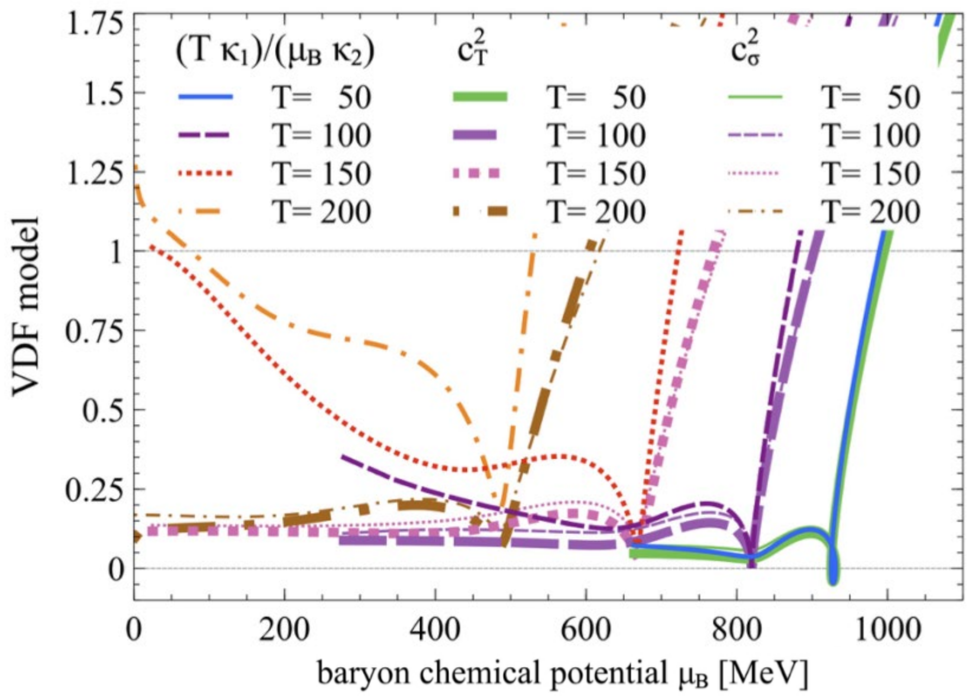
$$c_T^2 = \left(\frac{dP}{d\varepsilon}\right)_T = \frac{\left(\frac{dP}{d\mu_B}\right)_T}{\left(\frac{d\varepsilon}{d\mu_B}\right)_T} = \frac{\left(\frac{dP}{d\mu_B}\right)_T}{\mu_B\left(\frac{d^2P}{d\mu_B^2}\right)_T + T\left(\frac{ds}{dn_B}\right)_T}$$

$$= \frac{T}{\mu_B}\frac{\kappa_1^B}{\kappa_2^B}\frac{1}{1+\frac{T}{\mu_B}\left(\frac{ds}{dn_B}\right)_T}$$

Dense matter, $\frac{\mu_B}{T} \gg 1$:

$$c_T^2 \approx \frac{T}{\mu_B}\frac{\kappa_1^B}{\kappa_2^B} \approx c_\sigma^2$$

Derivatives wrt to n_B through high-order cumulants



Sorensen, Oliinychenko, Koch, McLerran, PRL 127, 042303 (2021)

Cumulants can measure the speed of sound in dense matter

Cumulants and the density-dependent mean field

- Density-dependent **mean field** $\mu_B \to \mu_B - U(n_B)$

$$U(n_B) = \frac{d\,[n_B V(n_B)]}{dn_B}$$

For a Maxwell-Boltzmann gas

$$\frac{\kappa_2^B}{\kappa_1^B} = \frac{1}{1 + \frac{n_B}{T} U'(n_B)}$$

Higher-order cumulants probe higher-order derivatives of $U(n_B)$

Momentum-dependent interactions(?) -> can be particularly sensitive due to momentum cuts in experiments

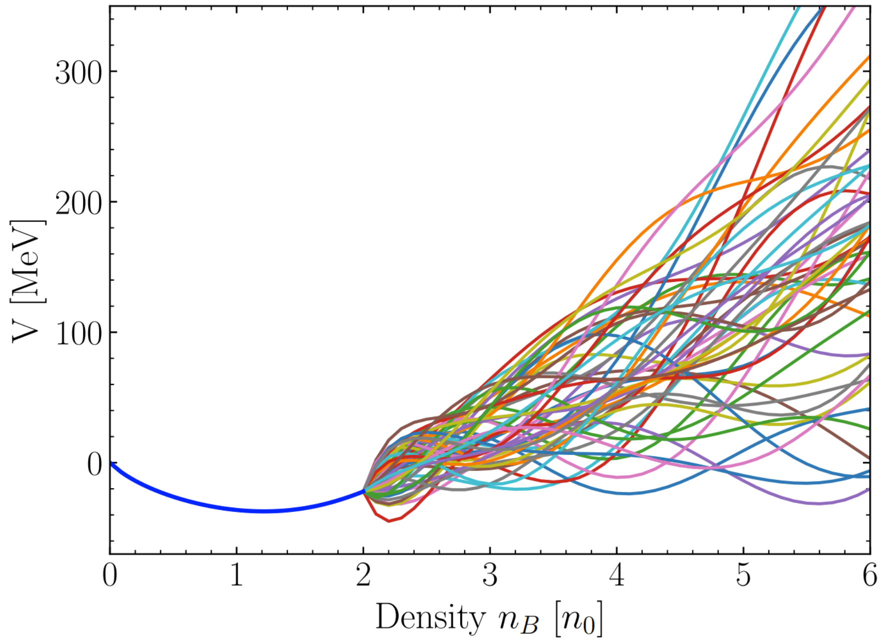
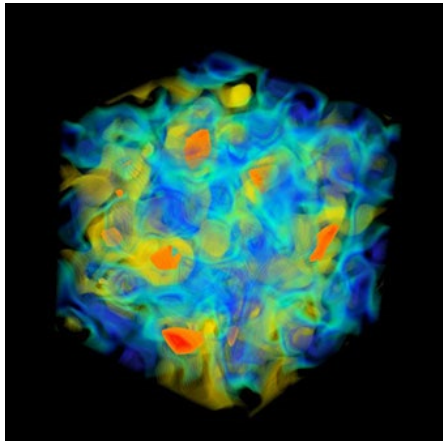


Figure from M. Kuttan, J. Steinheimer, K. Zhou, H. Stoecker, 2211.11670

Cumulants probe the density-dependence and momentum-dependence of single particle potential

Theory vs experiment: Challenges for fluctuations

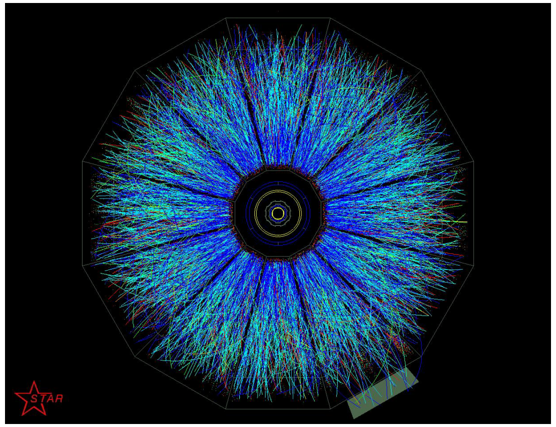
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

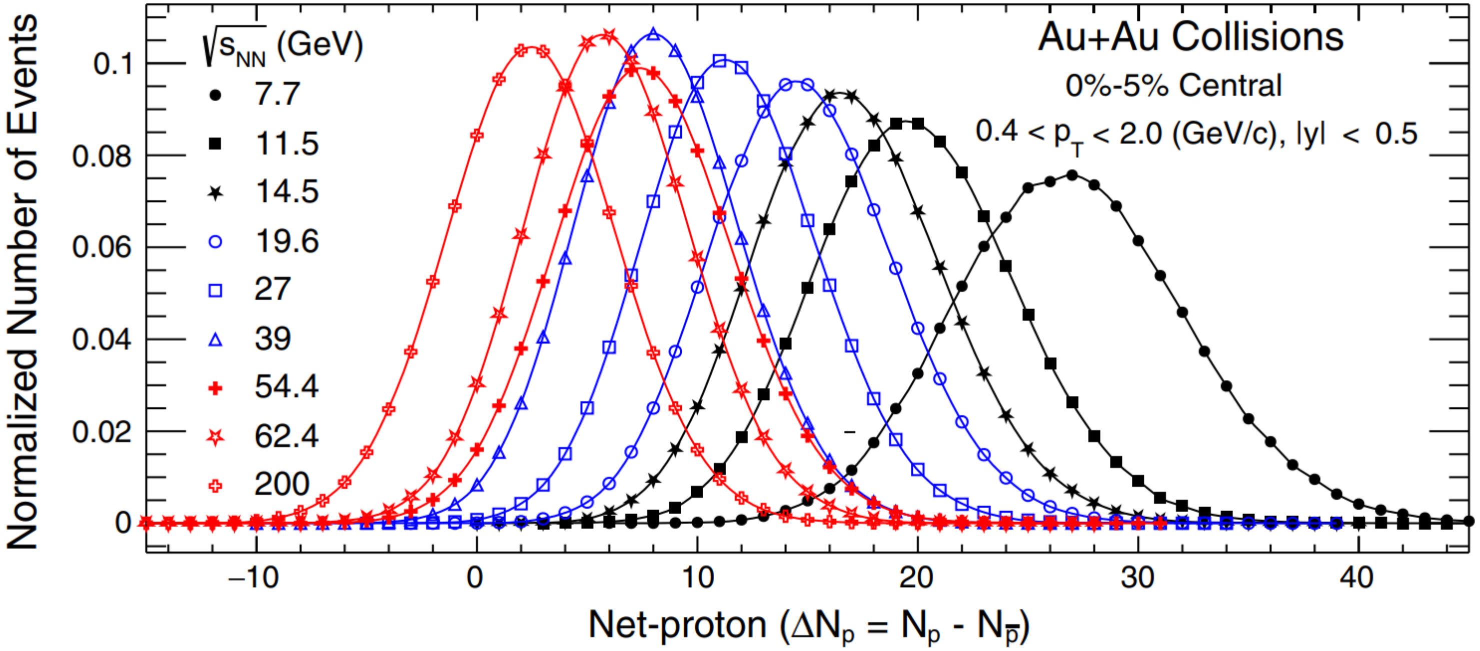
- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume/centrality selection

Need to implement fluctuations in a dynamical setup

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons $P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{events}^{total}}$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)

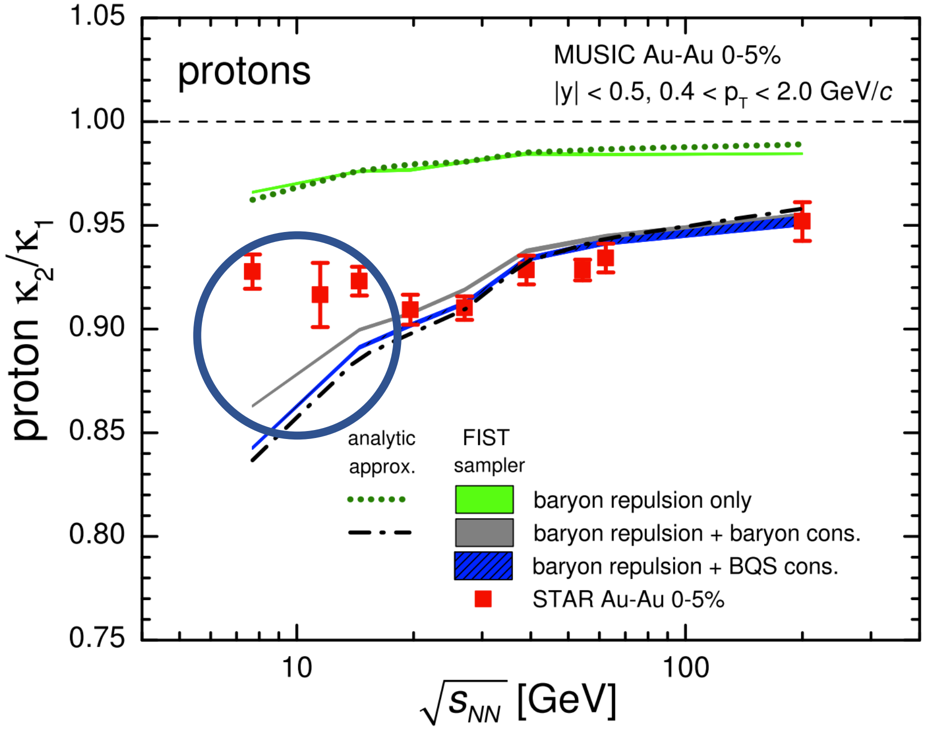


Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

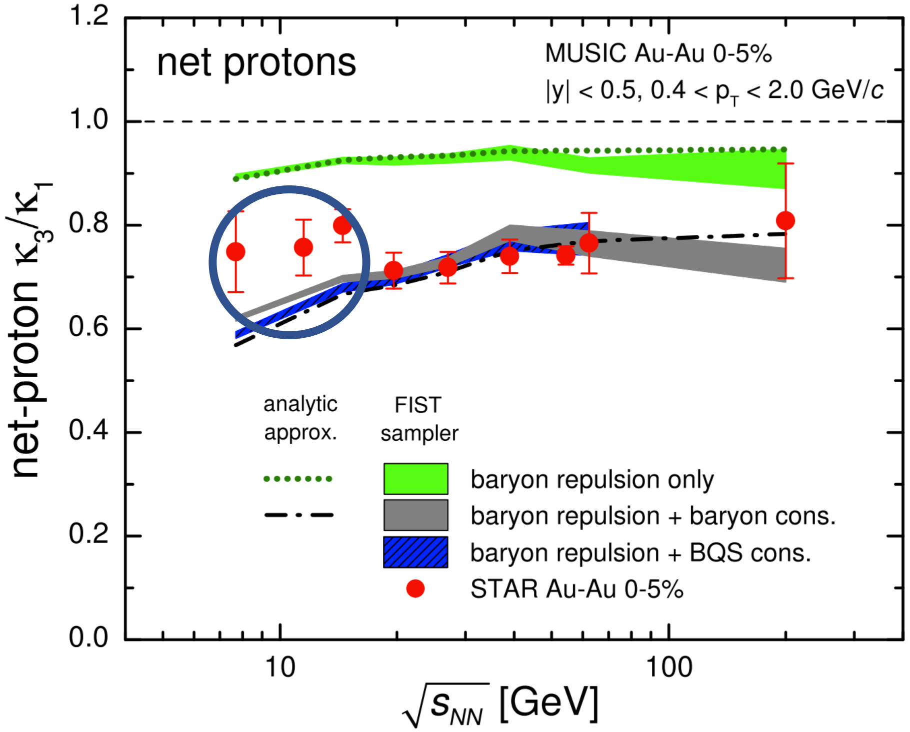
$$\frac{\kappa_2}{\langle N \rangle}, \qquad \frac{\kappa_3}{\kappa_2}, \qquad \frac{\kappa_4}{\kappa_2}$$

RHIC-BES: Net proton cumulant ratios (MUSIC)

Proton κ_2/κ_1



Net-proton κ_3/κ_1



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of fluctuations in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022); VV, arXiv:2208.13693

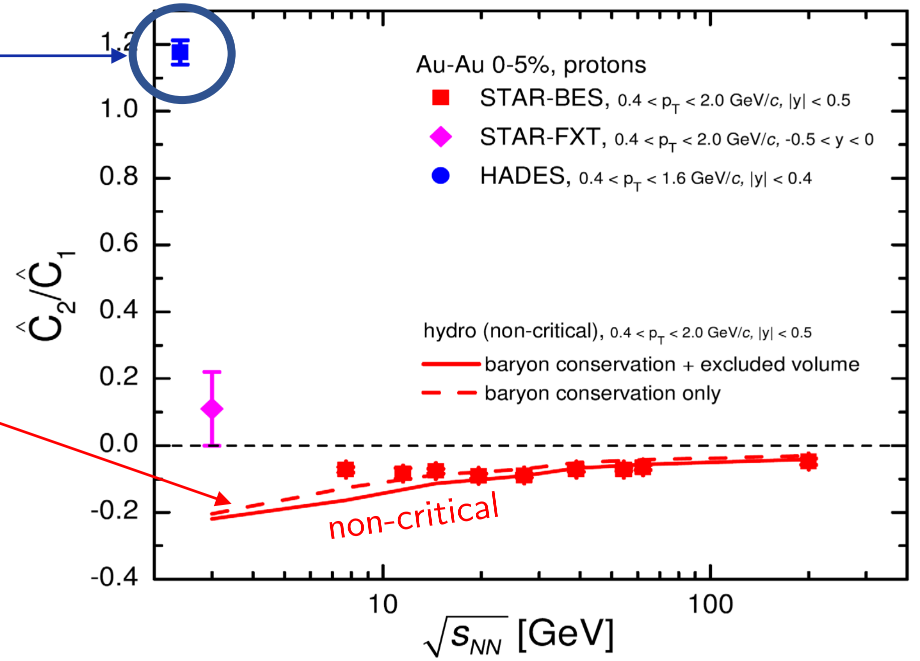
Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

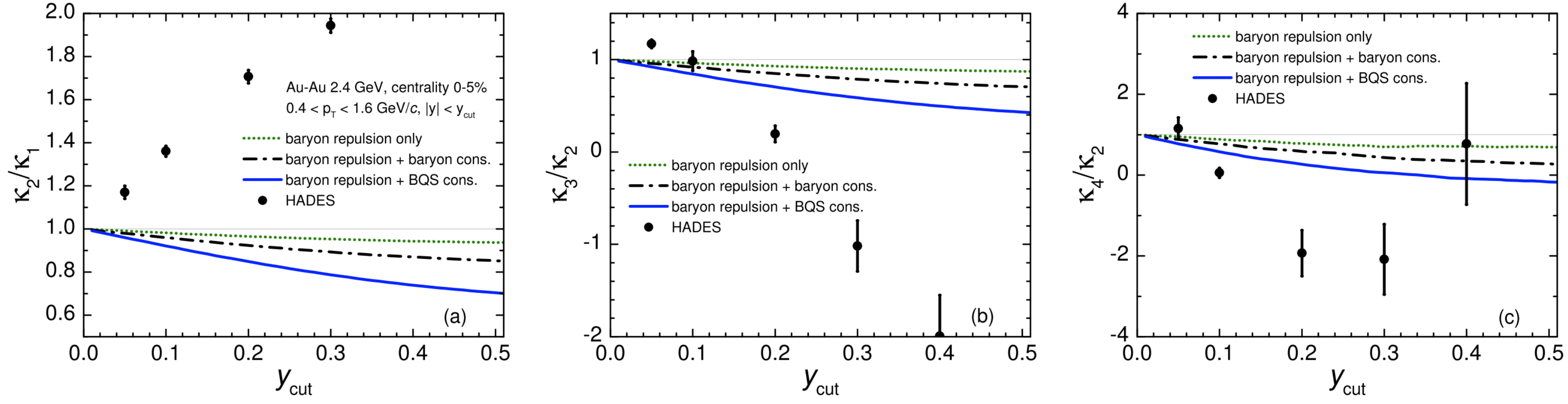
- Intriguing hint from HADES @ $\sqrt{s_{NN}} = 2.4$ GeV: huge excess of two-proton correlations!

  [HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]

- No change of trend in the non-critical reference

- Additional mechanisms:
  - Nuclear liquid-gas transition
  - Light nuclei formation

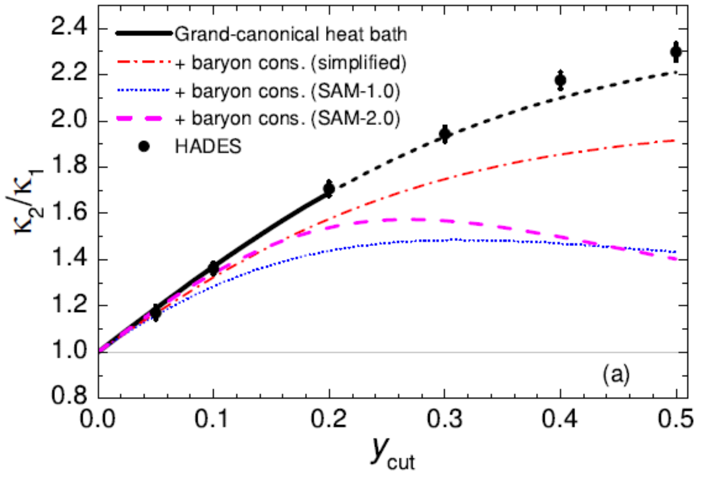




VV, arXiv:2208.13693

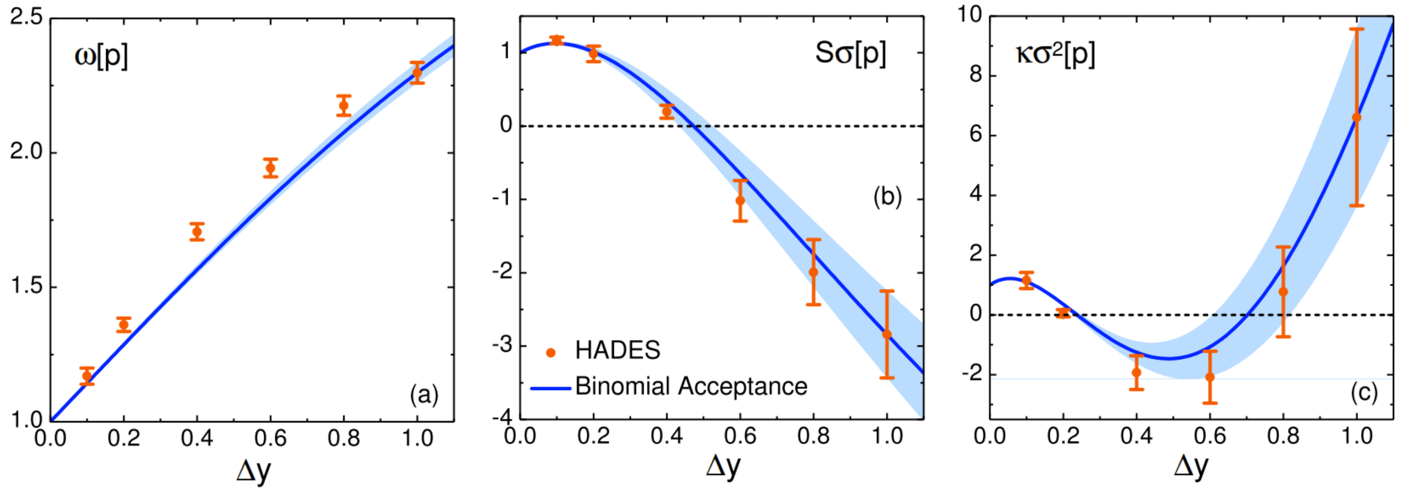
A closer look at the HADES data

Fireball model (Siemens-Rasmussen)



VV, Koch, Phys. Lett. B 833, 137368 (2022)

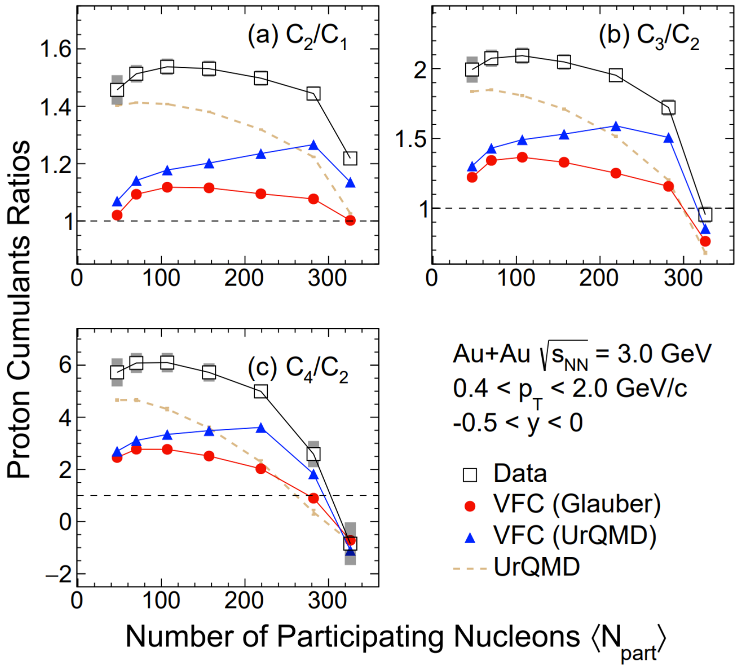
Global proton number fluct. + binomial acceptance



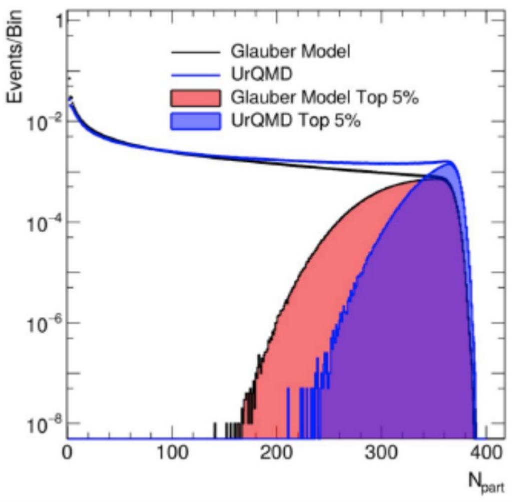
Savchuk, Poberezhnyuk, Goresntein, Phys. Lett. B 835, 137540 (2022)

- Large non-Gaussian proton number fluctuations in acceptance $|y|<0.5$
  - Challenging to understand in terms of baryon conservation (already on κ_2 level)
- Acceptance dependence can nicely be fitted by folding with binomial
  - Likely weak correlations in momentum space

STAR-FXT 3 GeV



STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303



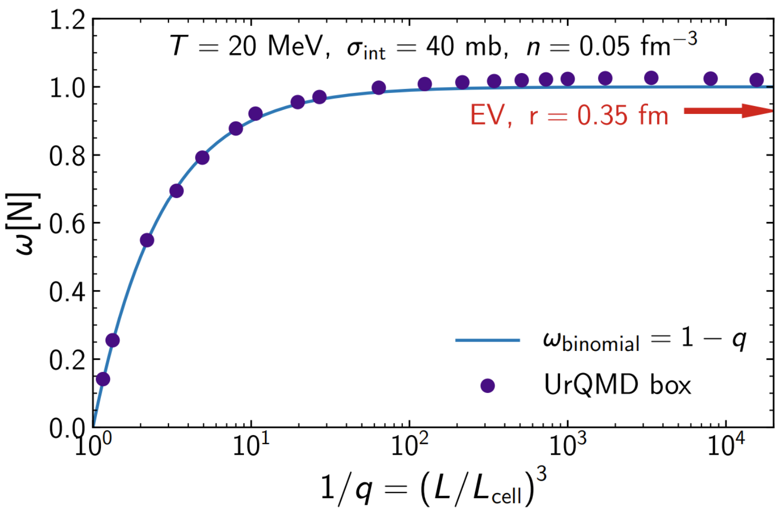
- Volume fluctuations/centrality selection appear to play an important role
  - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (baryon conservation), note -0.5<y<0 instead of |y|<0.5

Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

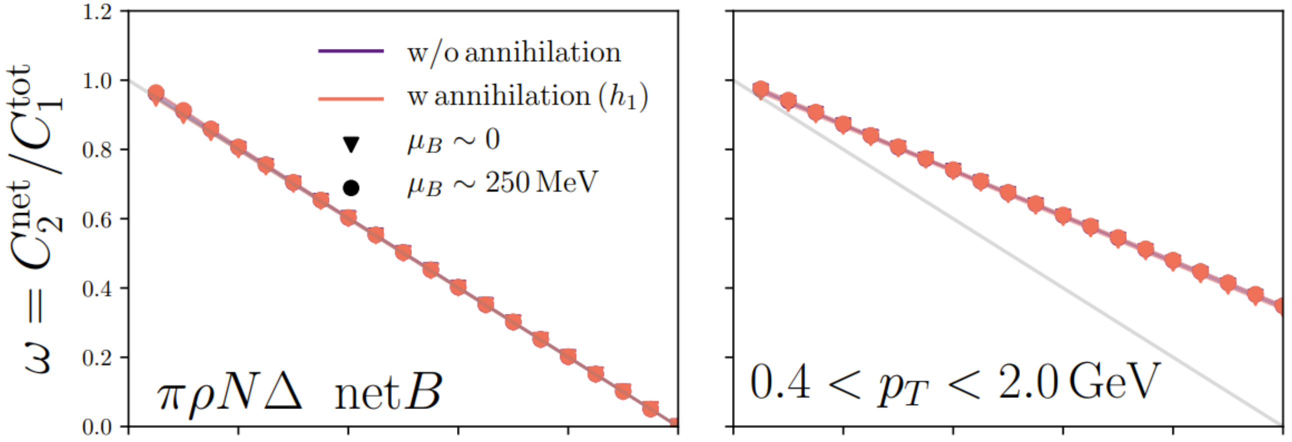
Fluctuations in hadronic transport/molecular dynamics

Existing hadronic cascades

- Conserve baryon number/charge/strangeness
- Allow for centrality selection/volume fluctuations/acceptance cuts like in experiment
- Provide a baseline but do not probe the nuclear EoS



A. Motornenko et al., J.Phys.G 45 (2018) 035101



J. Hammelmann, H. Elfner, 2202.11417

Can we use transport to model non-trivial fluctuations due to the EoS?

Example: Lennard-Jones fluid

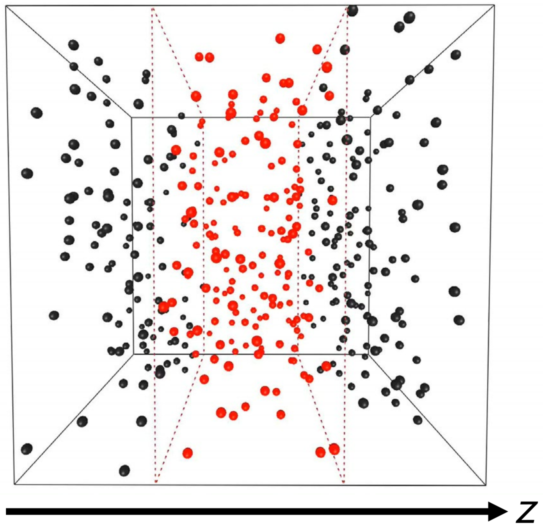
Kuznietsov, Savchuk, Gorenstein, Koch, VV, Phys. Rev. C 105, 044903 (2022)

Classical molecular dynamics simulations* of a **Lennard-Jones fluid** along the (super)critical isotherm of the liquid-gas transition

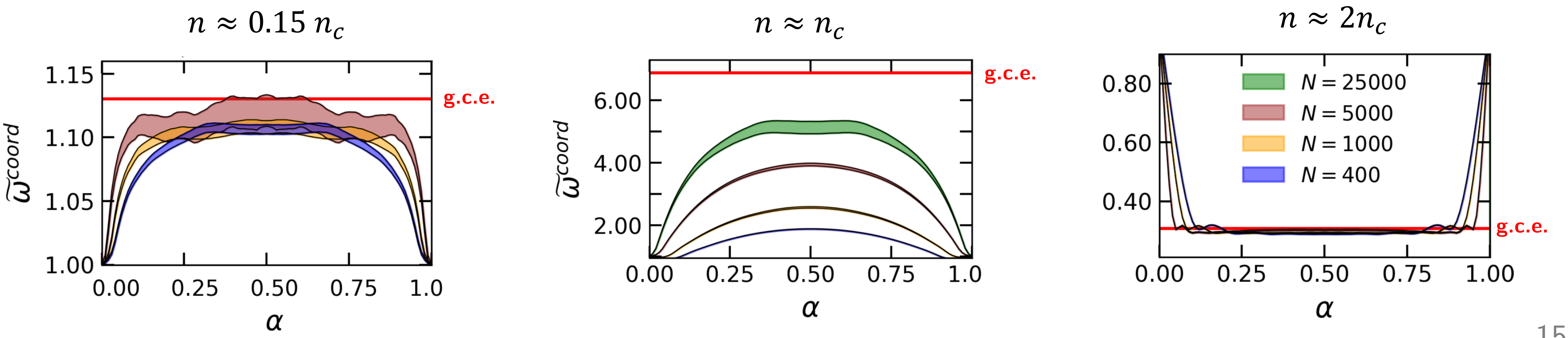
Microcanonical (const. *EVN*) ensemble with periodic boundary conditions

Scaled variance of conserved particle number inside coordinate space subvolume $|z| < z^{max}$ as time average

$$\tilde{\omega}^{\text{coord}} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$



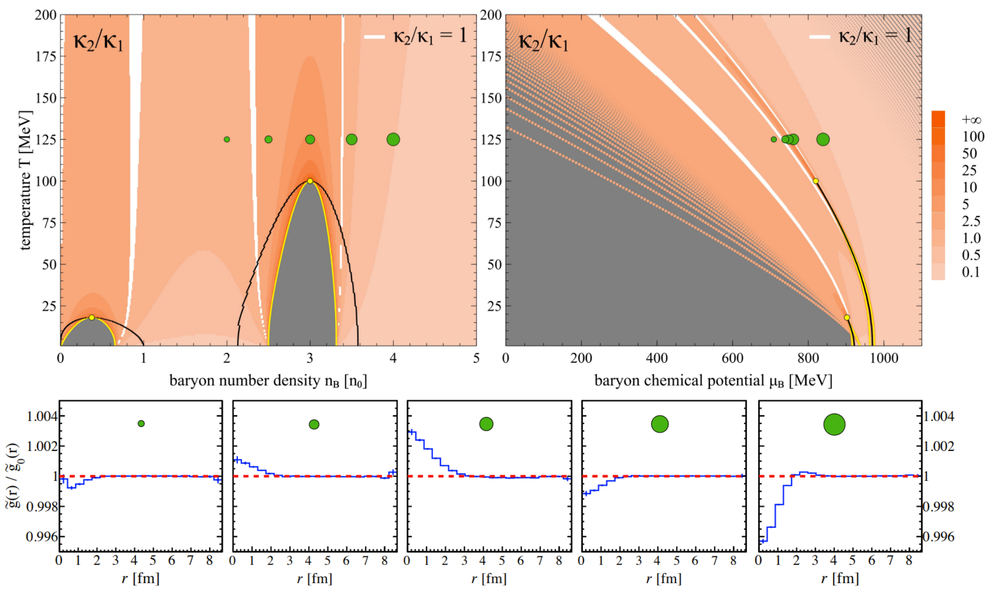
Stat. mech. expectation: $\tilde{\omega}^{\text{coord}} \overset{\langle N \rangle \to \infty}{\longrightarrow} \omega^{\text{gce}}$



*Molecular dynamics code from https://github.com/vlvovch/lennard-jones-cuda

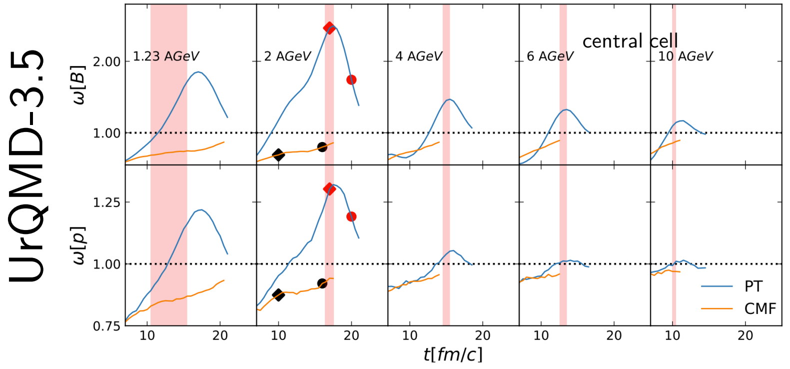
SMASH/UrQMD with density-dependent EoS

SMASH-VDF



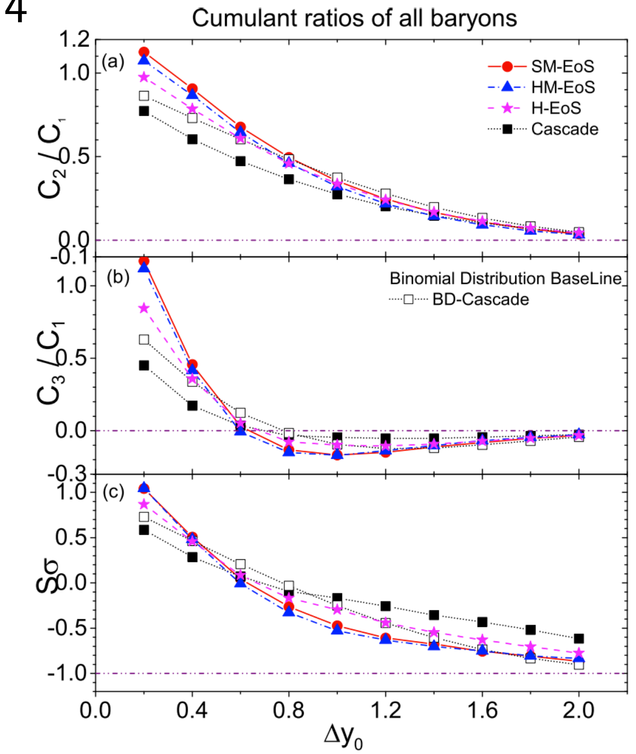
A. Sorensen, V. Koch, Phys. Rev. C 104 (2021) 034904

UrQMD-3.5



O. Savchuk et al., 2211.13200

UrQMD-3.4



Y. Ye, et al., Phys. Rev. C 98 (2018) 054620

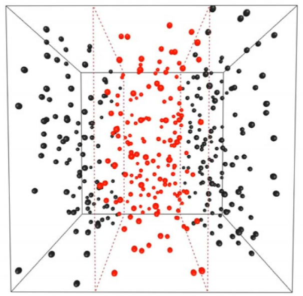
Qualitative behaviors seem consistent with expectations, but do these implementations meet a quantitative test?

$$\frac{1}{1-\alpha}\,\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle} \overset{?}{=} \frac{1}{1+\frac{n_B}{T}U'(n_B)}$$

Probing the EoS of dense matter with (proton) cumulants

Possible road map

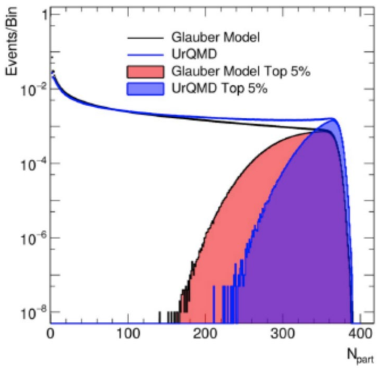
1. Implement the given EoS/interactions into transport/molecular dynamics (SMASH-VDF, UrQMD-3.5, etc.)



2. Box simulations to reproduce the EoS and GCE flucts.

$$\frac{1}{1-\alpha}\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle} \overset{?}{=} \frac{1}{1+\frac{n_B}{T}U'(n_B)}$$

3. Event-by-event simulations of HICs
   - Momentum cuts
   - Same centrality selection as in experiment (e.g. doing b<3 fm is *not enough*)
   - Light nuclei/fragmentation
   - If needed, mixed phase dynamics



4. Ultimately, Bayesian analysis(?)

Connection to workshop questions

- What other observables could enable the extraction of the EOS?
  - Cumulants of proton number are sensitive to density and momentum dependence of nuclear interactions, probe the speed of sound
- What improvements on the constraints on the EOS can we expect from future heavy-ion experiments?
  - Cumulants are being measured by HADES, RHIC BES-II & FXT, and, in the future by the CBM experiment
  - With improved theoretical modeling these data can further constrain the EoS of symmetric matter
- What development is necessary for transport codes to address the above questions?
  - Proper treatment of fluctuations (different implementations of mean field, test particles, momentum dependence)
  - Box simulation tests of statistical mechanics expectations

Summary

- Cumulants measure chemical potential derivatives of the EoS
  - High energies: degrees of freedom, QCD critical point
  - Dense matter: speed of sound, density dependence of the mean field *very little explored*
- Experimental data
  - RHIC-BES: disfavor QCD CP at $\sqrt{s_{NN}} > 20$ GeV
  - HADES and FXT: indications for an enhanced κ_2, difficult to explain especially HADES
  - Should be understood before moving on to higher orders
- Fluctuations and transport theory/MD in dense matter
  - Arguably more applicable than hydro
  - If done properly can alleviate many of the issues, such as centrality selection, charge conservation, acceptance, etc.

Thanks for your attention!