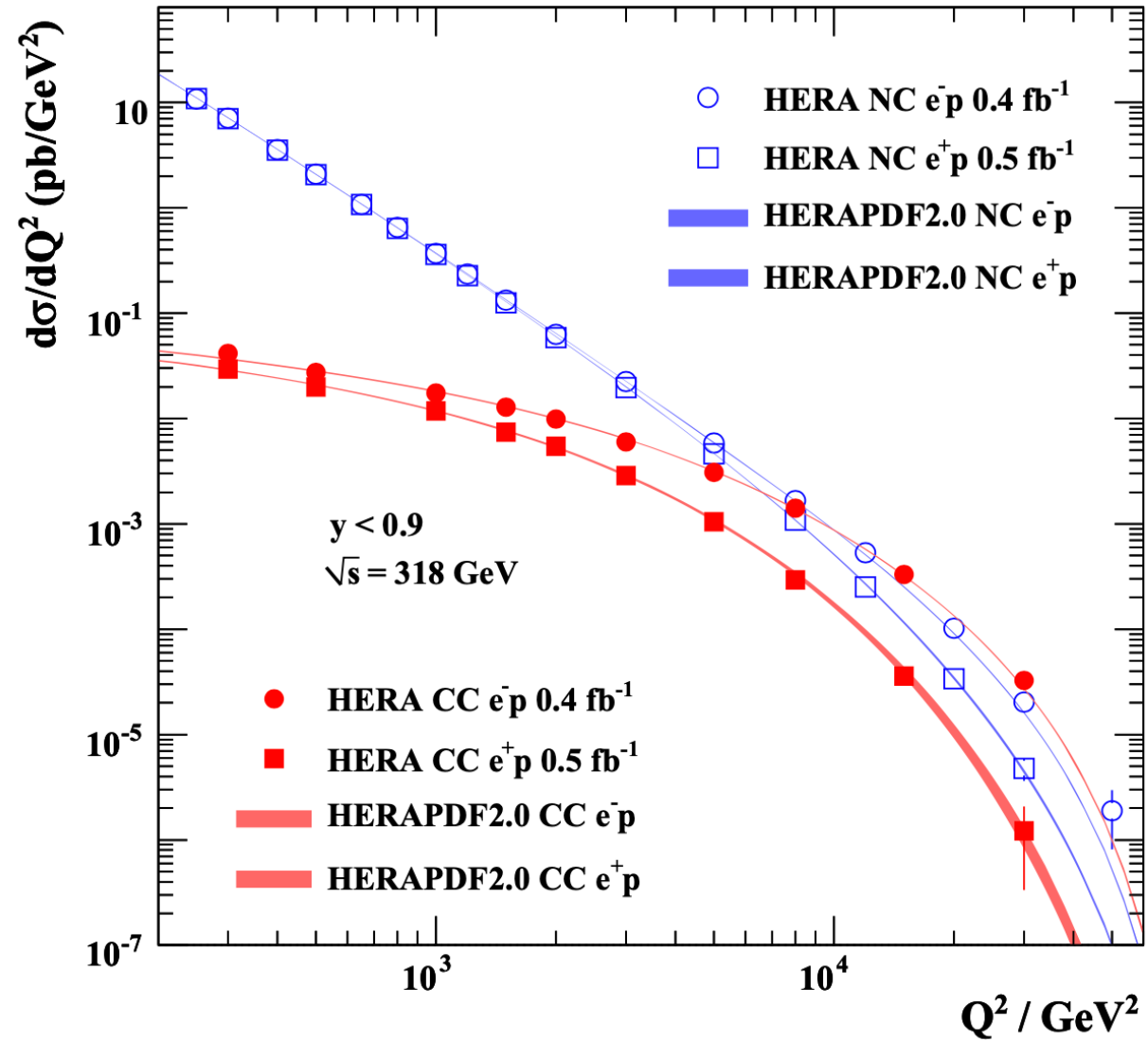


Charged-current observables at the EIC and the proton's polarized PDFs

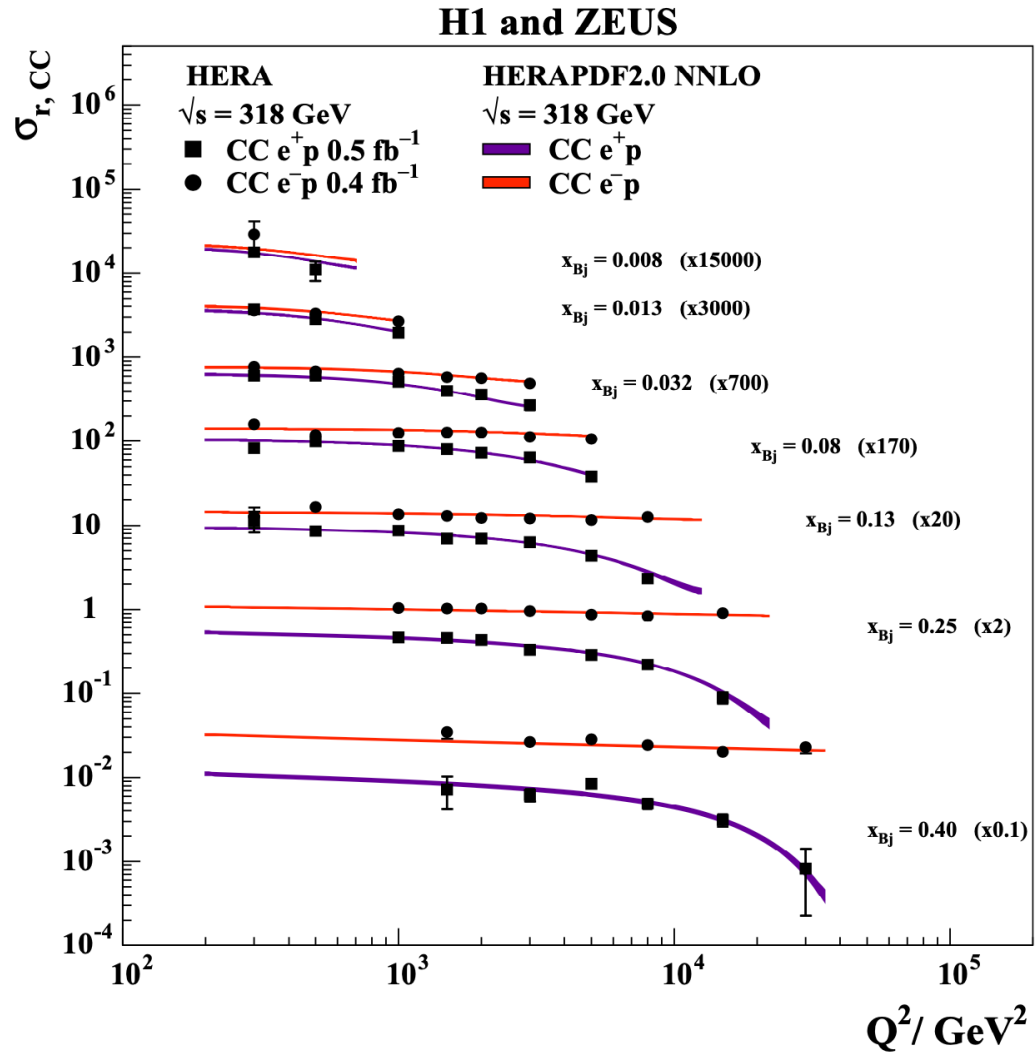
Werner Vogelsang
Univ. of Tübingen

INT, 02/16/2024

H1 and ZEUS



H1, ZEUS 2015



$$\sigma_{r, \text{CC}}^{\pm} = \frac{2\pi x}{G_F^2} \left[\frac{M_W^2 + Q^2}{M_W^2} \right]^2 \frac{d^2 \sigma_{\text{CC}}^{e^{\pm}p}}{dx dQ^2}$$

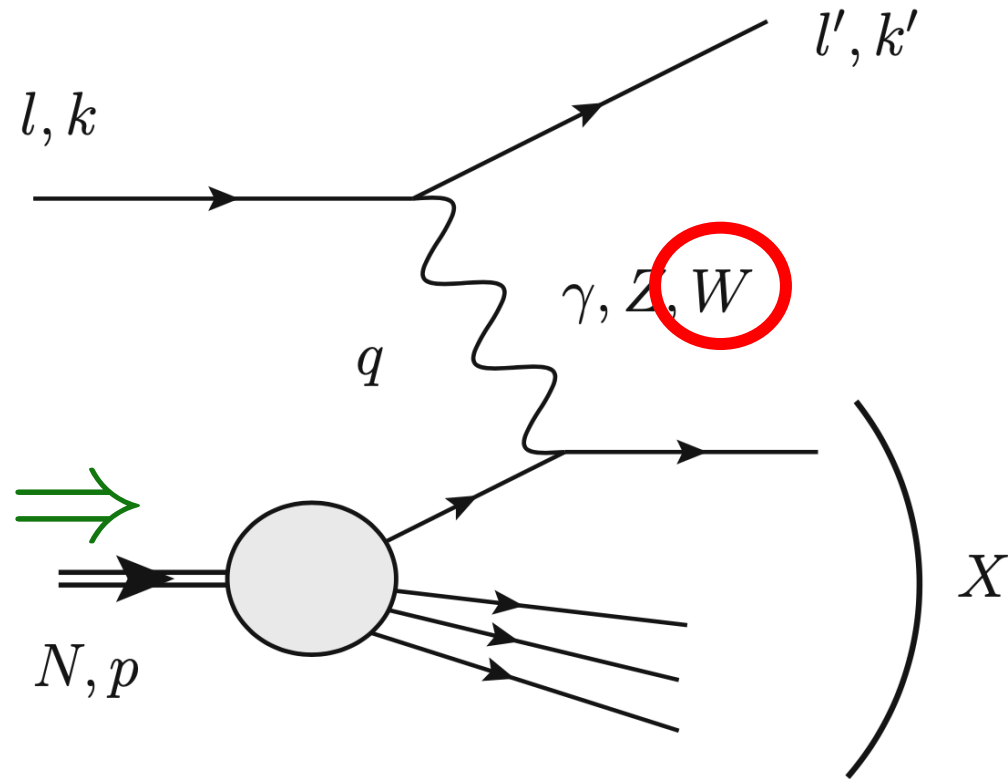
$$= \frac{1 + (1 - y)^2}{2} F_2^{\pm} \mp \frac{1 - (1 - y)^2}{2} x F_3^{\pm} - \frac{y^2}{2} F_L^{\pm}$$

$$\sigma_{r, \text{CC}}^+ \approx (x\bar{U} + (1 - y)^2 x D)$$

$$\sigma_{r, \text{CC}}^- \approx (xU + (1 - y)^2 x \bar{D})$$

H1, ZEUS 2015

Polarized charged-current DIS



→ new insights into helicity PDFs

Wray; Derman; Ahmed, Ross; Joshipura, Roy
Lampe; Anselmino, Gambino, Kalinowski
Blümlein, Kochelev; Ravindran, van Neerven
Stratmann, WV, Weber

Forte, Mangano, Ridolfi; de Florian, Sassot
Deshpande, Kumar, Ringer, Riordan, Taneja, WV
Aschenauer, Burton, Martini, Spiesberger, Stratmann

Borsa, de Florian, Pedron

de Florian, Rothstein Habarnau

Arratia, Furltova, Hobbs, Olness, Sekula

Arratia, Kang, Paul, Prokudin, Ringer, Zhao

Outline:

- Helicity PDFs: status
- Recent theory advances
- Charged-current studies at the EIC: inclusive
- Less inclusive observables
- Conclusions

Helicity PDFs: status

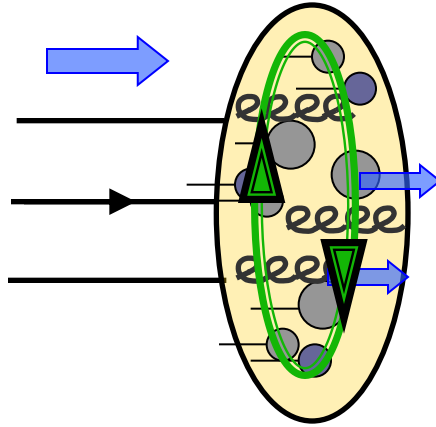
$$\Delta q(x) = \text{[Diagram: Red circle with white dot and right-pointing yellow arrow]} - \text{[Diagram: Red circle with white dot and left-pointing yellow arrow]}$$

$$\Delta g(x) = \text{[Diagram: Red circle with 'eee' and right-pointing yellow arrow]} - \text{[Diagram: Red circle with 'eee' and left-pointing yellow arrow]}$$

- in QCD: operator definition
 → dependence on “resolution” scale μ

proton spin:

Jaffe, Manohar; Chen et al;
Wakamatsu; Hatta; ...



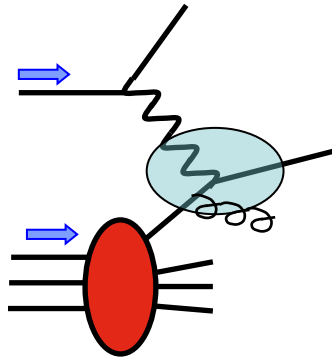
$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L_q + L_g$$

$$\Delta\Sigma = \int_0^1 dx [\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}](x) \ll 1$$

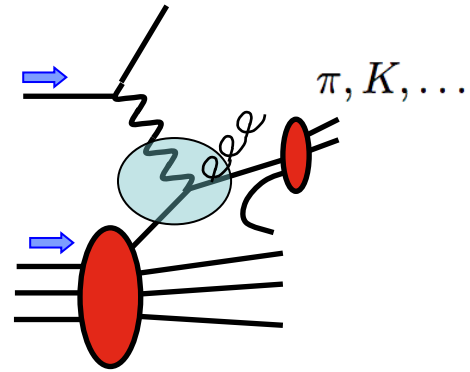
$$\Delta G = \int_0^1 dx \Delta g(x)$$

Helicity PDFs accessible in polarized high-energy scattering:

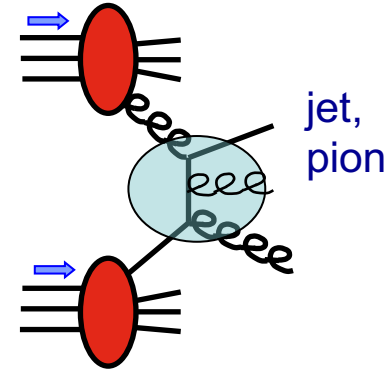
EMC, SMC,
COMPASS,
E142, E143, E154,
E155, HERMES,
CLAS, HALL-A



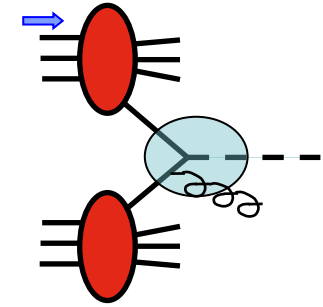
DIS



SIDIS



high- p_T



W bosons

RHIC:
PHENIX,
STAR

e.g.
$$\Delta\sigma_{pp}^{\text{jet}} = \sum_{a,b=q,\bar{q},g} \Delta f_a(x_a, \mu) \otimes \Delta f_b(x_b, \mu) \otimes \Delta\hat{\sigma}_{ab}$$

$$\Delta\hat{\sigma}_{ab} = \Delta\hat{\sigma}_{ab}^{\text{LO}} + \frac{\alpha_s}{\pi} \Delta\hat{\sigma}_{ab}^{\text{NLO}} + \dots$$

DSSV

Borsa,
de Florian, Sassot,
Stratmann, WV

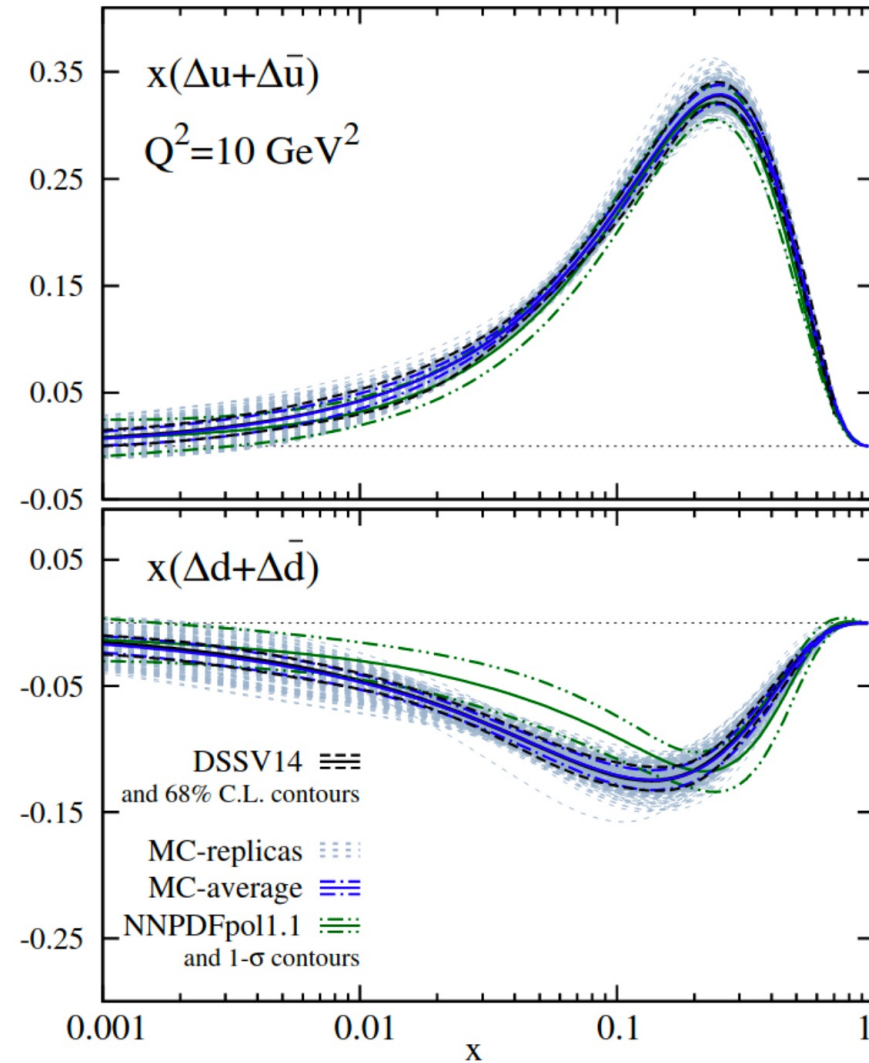
NNPDF

Nocera, Ball, Forte,
Ridolfi, Rojo, ...

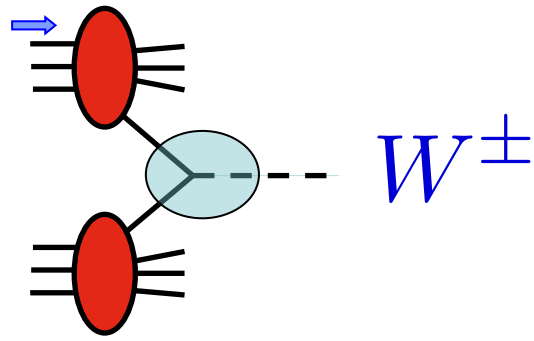


Sato, Cocuzza,
Ethier,
Melnitchouk, ...

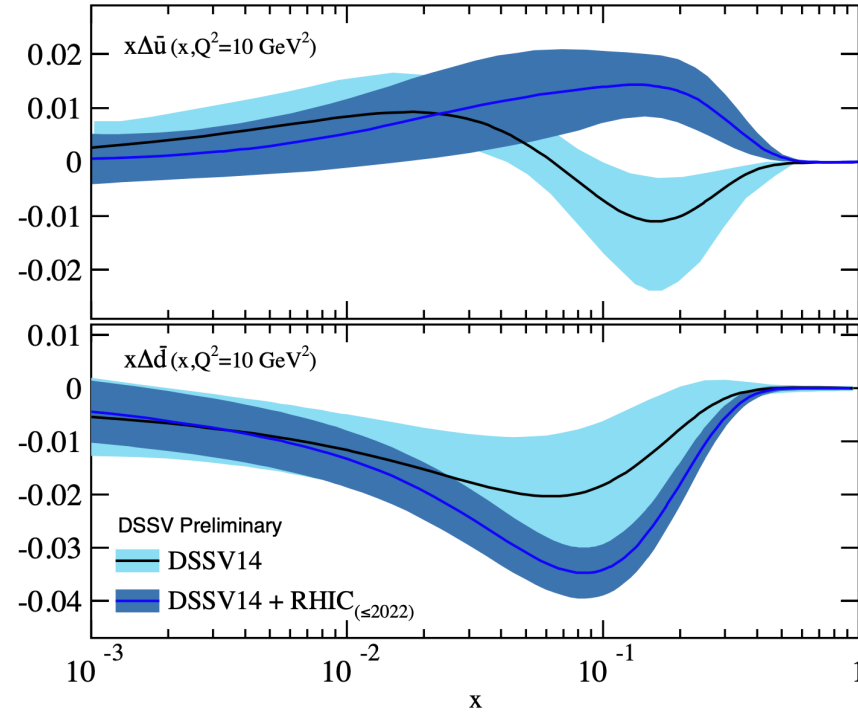
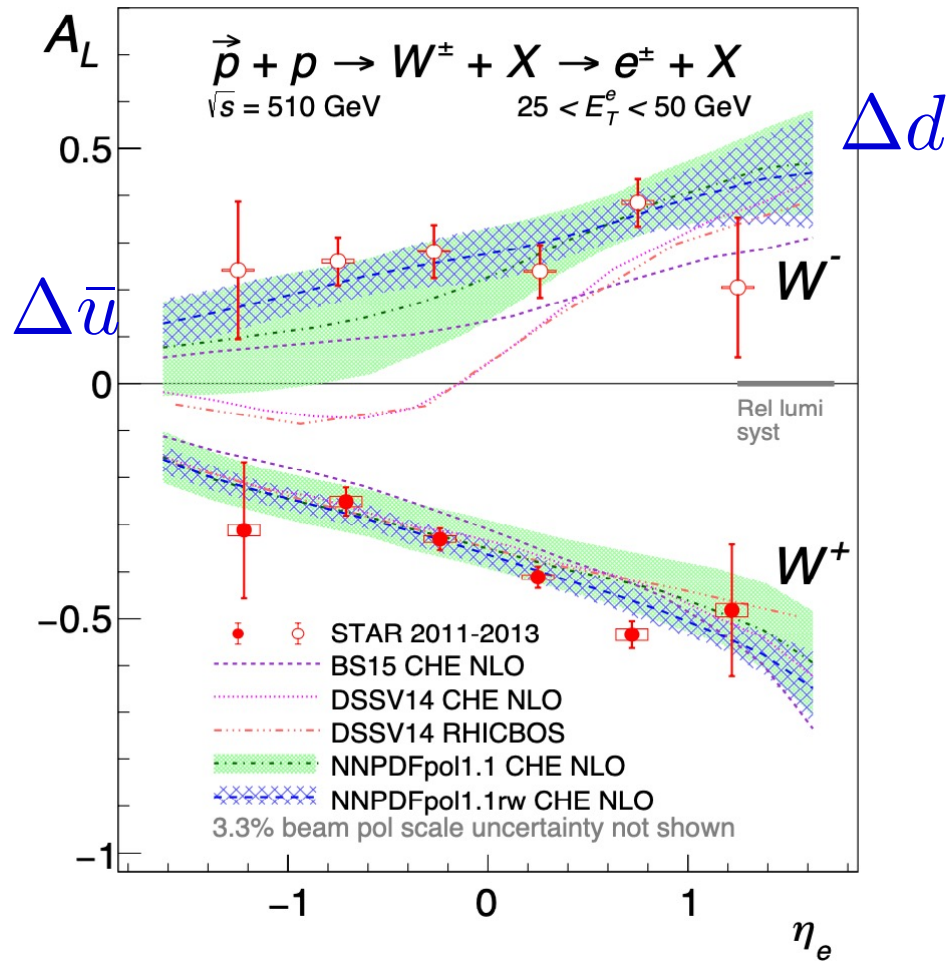
Total up and down polarizations:



Light sea quarks:



(from 2302.00605)

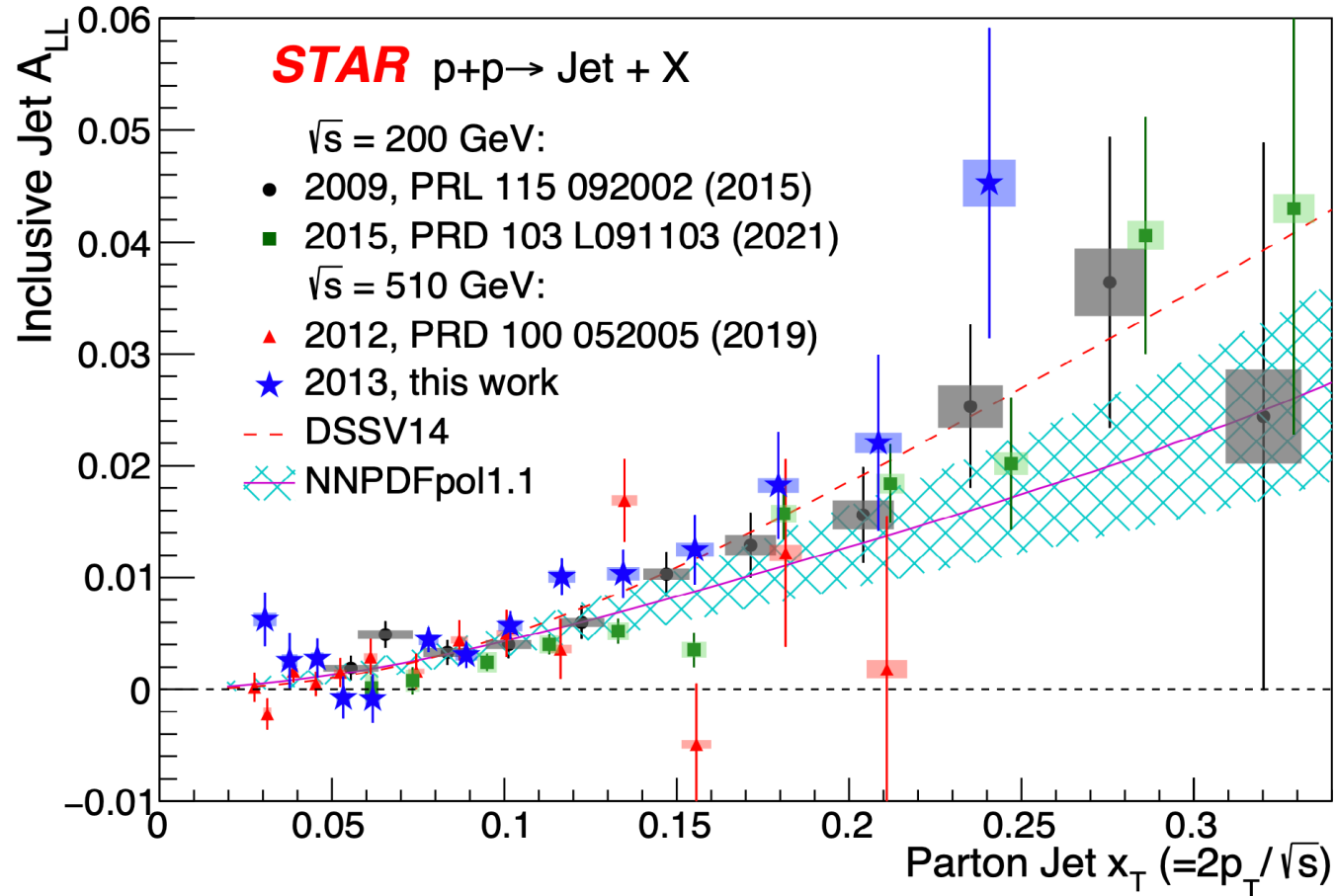


(consistent w/ SIDIS)

also NNPDF, JAM
 (Cocuzza, Melnitchouk, Metz, Sato)

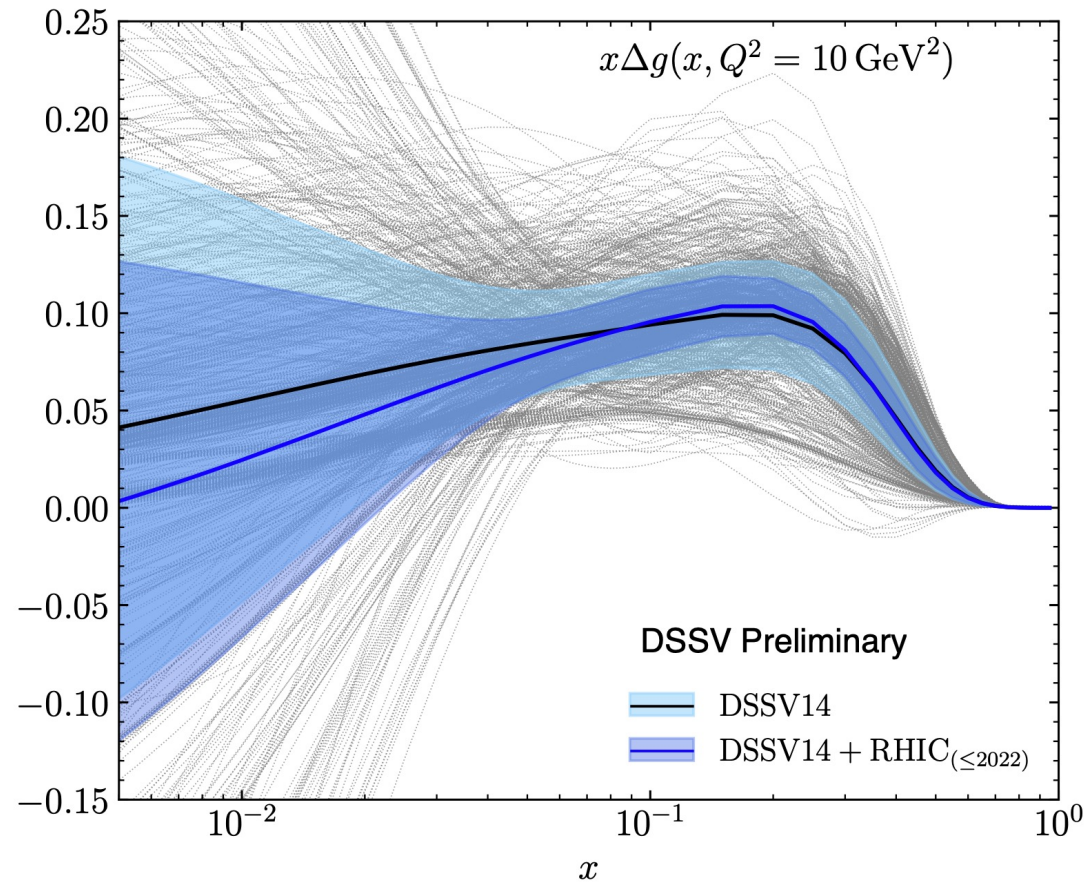
Gluon polarization: 2014 RHIC-discovery that $\Delta g > 0$

→ improvement and consolidation !



+ STAR dijets

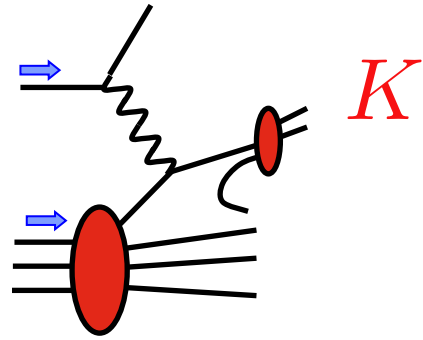
+ Phenix $\pi^{0,\pm}, \gamma$



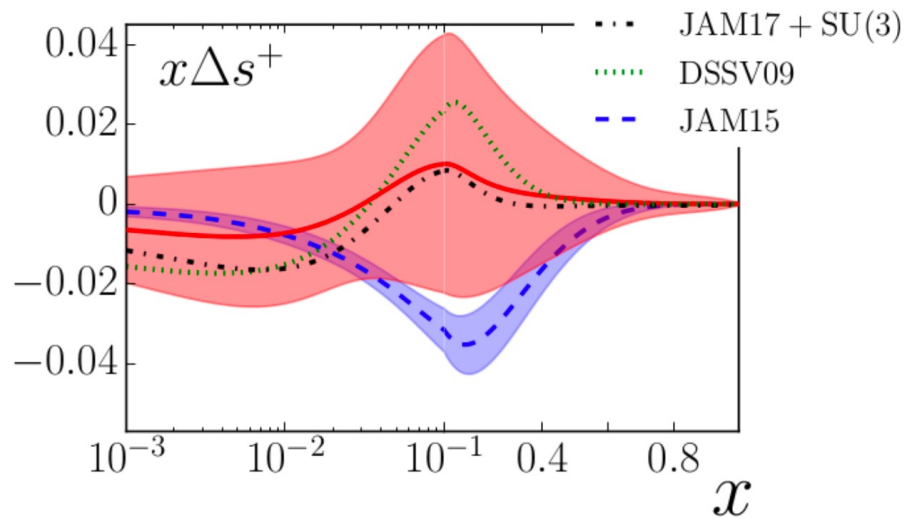
$$\int_{0.01}^1 dx \Delta g(x, Q^2) = 0.3 \pm 0.1$$

(from 2302.00605)

strangeness $\Delta s + \Delta \bar{s}$:



→ little trace of strangeness



JAM'17:

$$\int_0^1 dx (\Delta s + \Delta \bar{s}) = -0.03 \pm 0.10$$

SU(3): sizable negative

$$\int (\Delta s + \Delta \bar{s}) \sim -0.1$$

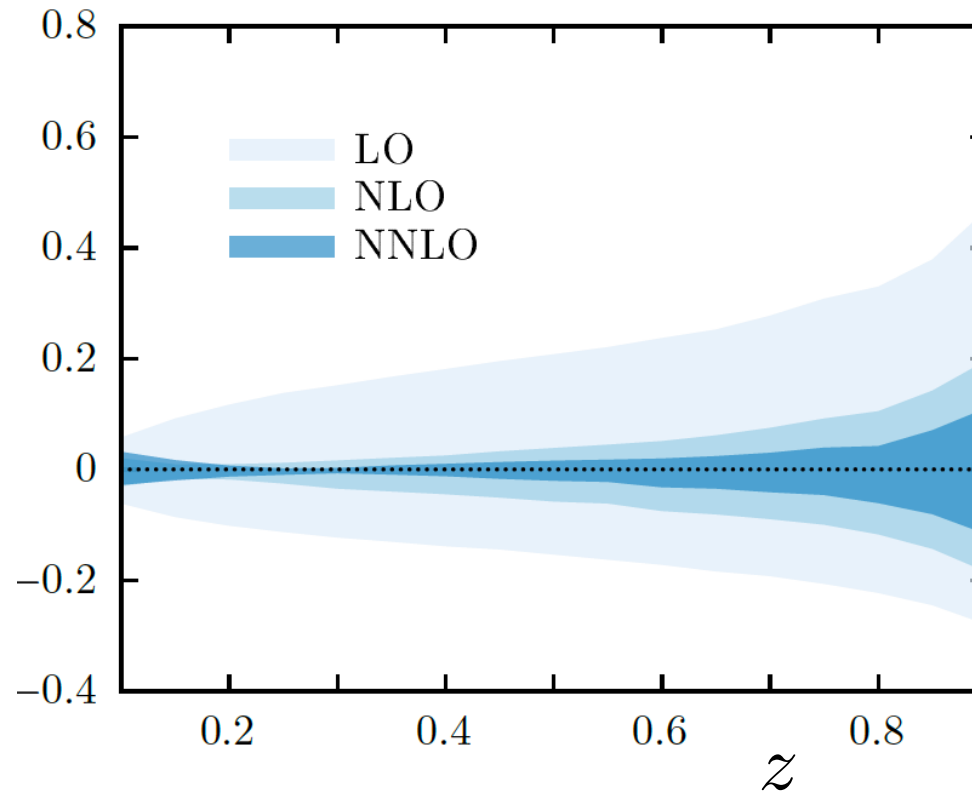
Recent theory advances:
Nucleon helicity structure at NNLO

Why NNLO ?

- need per cent accuracy for EIC and JLab (cf. LHC experience)
- reduce theory uncertainty

$$\frac{\sigma(\mu) - \sigma(Q)}{\sigma(Q)}$$

$$Q/2 \leq \mu_{R,F} \leq 2Q$$



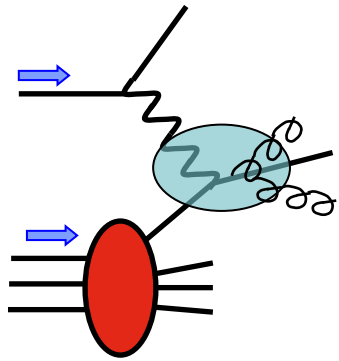
$$Q^2 > 5 \text{ GeV}^2$$

SIDIS@EIC

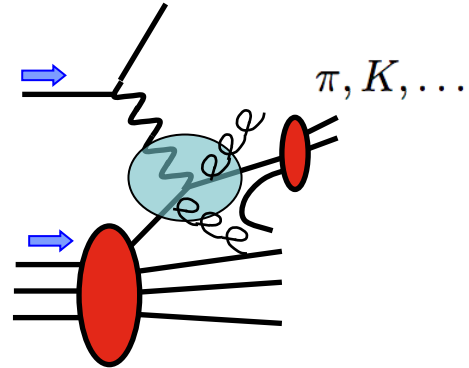
Abele, De Florian, WV

- progress in lattice computations

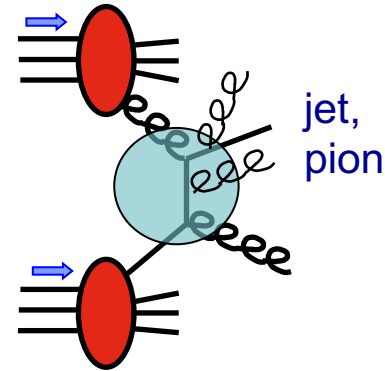
How to get to NNLO ?



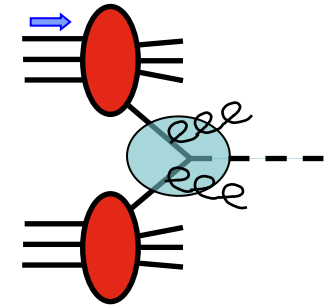
DIS



SIDIS



pp high- p_T



W bosons

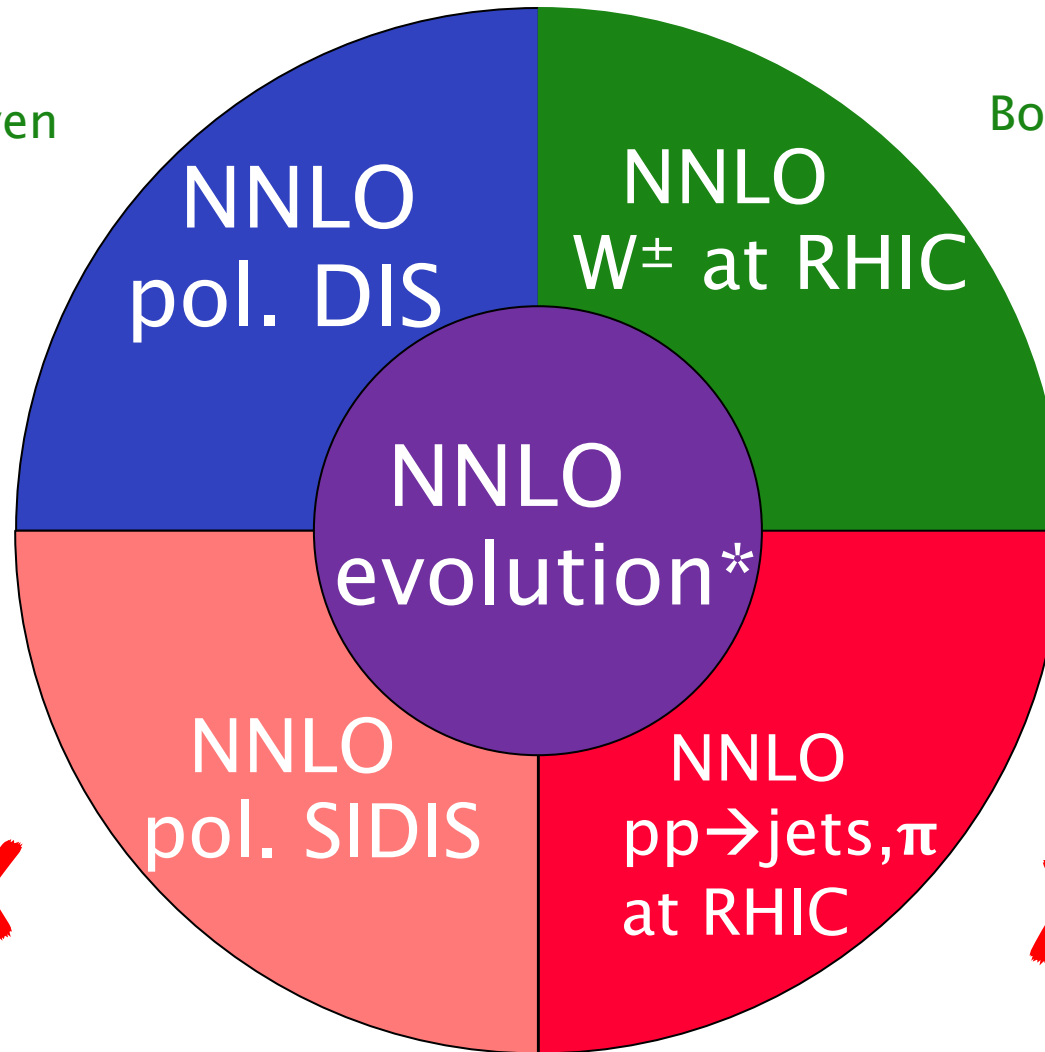
Ingredients for NNLO:

- Partonic hard scattering:
$$\Delta \hat{\sigma}_{ab} = \Delta \hat{\sigma}_{ab}^{\text{LO}} + \frac{\alpha_s}{\pi} \Delta \hat{\sigma}_{ab}^{\text{NLO}} + \left(\frac{\alpha_s}{\pi} \right)^2 \Delta \hat{\sigma}_{ab}^{\text{NNLO}} + \dots$$

- PDF evolution:
$$\Delta \mathcal{P}_{ij} = \frac{\alpha_s}{2\pi} \Delta P_{ij}^{\text{LO}} + \left(\frac{\alpha_s}{2\pi} \right)^2 \Delta P_{ij}^{\text{NLO}} + \left(\frac{\alpha_s}{2\pi} \right)^3 \Delta P_{ij}^{\text{NNLO}} + \dots$$

Zijlstra, van Neerven
1994

Boughezal, Li, Petriello 2021



* Moch, Vogt, Vermaseren
Blümlein, Marquard,
Schneider, Schönwald
QCD Pegasus: A. Vogt

unpolarized:

Boninio, Gehrmann, Stagnitto
Goyal, Moch, Pathak,
Rana, Ravindra (2024)

→ use QCD resummation techniques to obtain approximate NNLO
for SIDIS and pp

Abele, de Florian, WV

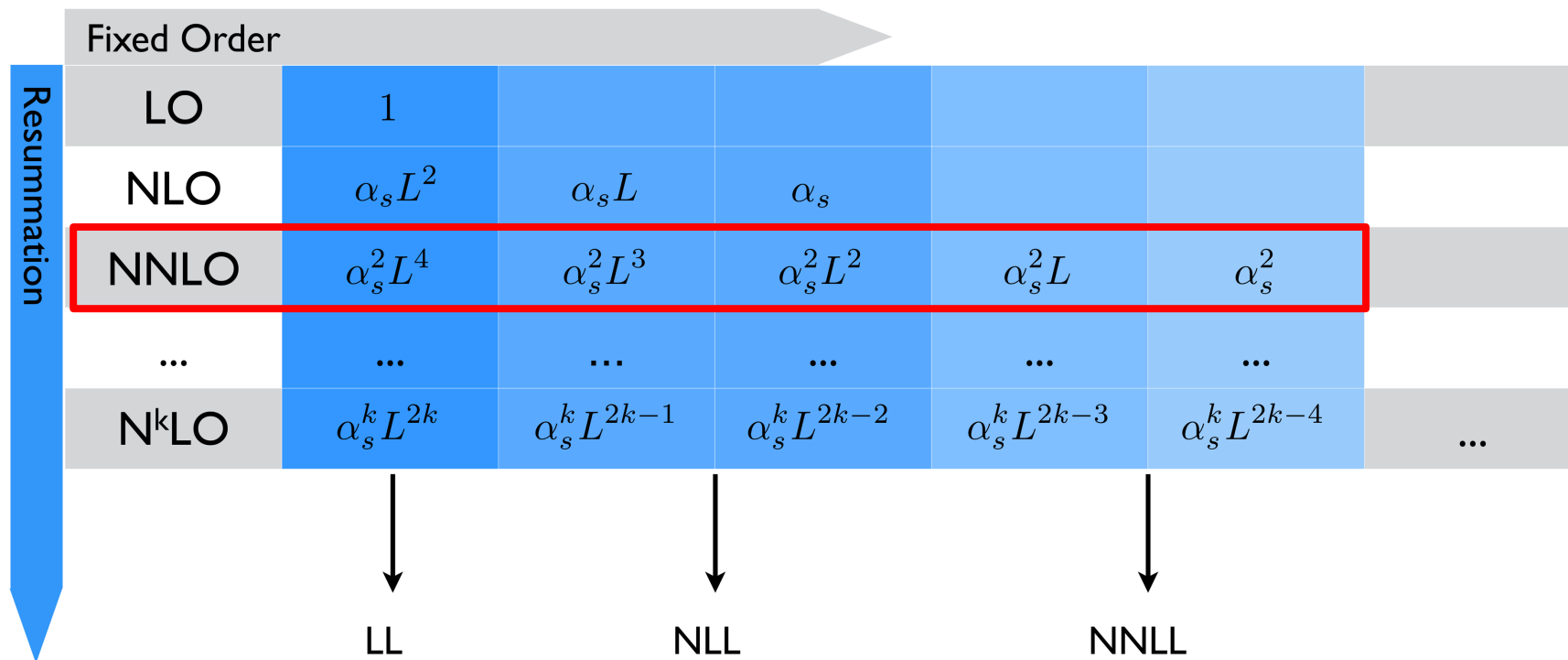
$$\Delta\hat{\sigma}_{qq}^{\text{N}^k\text{LO}}(\hat{x}, \hat{z}) \sim \alpha_s^k \left[\delta(1-\hat{x}) \left(\frac{\ln^{2k-1}(1-\hat{z})}{1-\hat{z}} \right)_+ + \delta(1-\hat{z}) \left(\frac{\ln^{2k-1}(1-\hat{x})}{1-\hat{x}} \right)_+ \right. \\ \left. + \frac{1}{(1-\hat{x})_+} \left(\frac{\ln^{2k-2}(1-\hat{z})}{1-\hat{z}} \right)_+ + \frac{1}{(1-\hat{z})_+} \left(\frac{\ln^{2k-2}(1-\hat{x})}{1-\hat{x}} \right)_+ + \dots \right]$$

$$\hat{x} = \frac{Q^2}{2p_a \cdot q}$$

$$\hat{z} = \frac{p_a \cdot p_b}{p_a \cdot q}$$

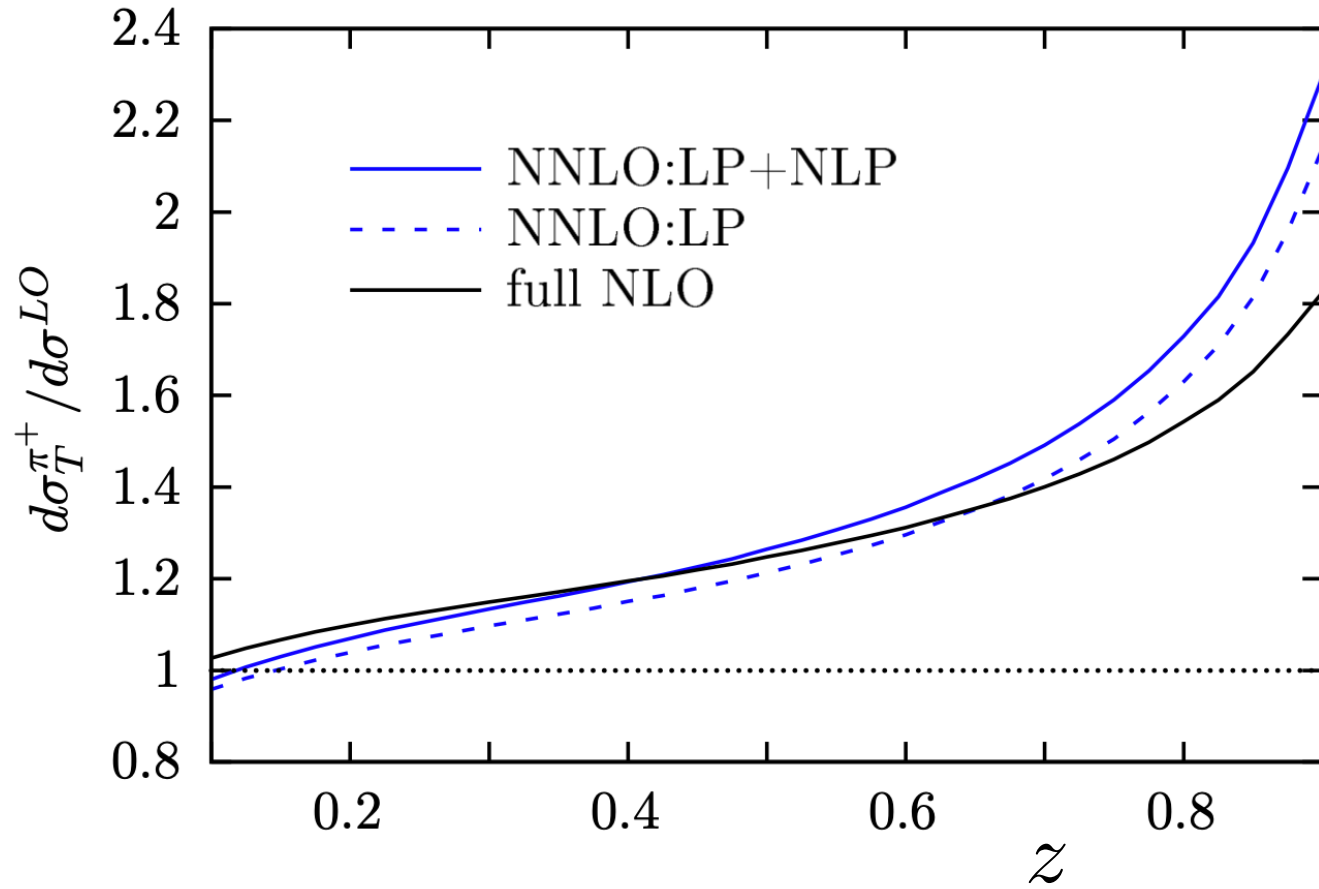
- logs can be resummed to all orders: **threshold resummation**

Anderle, Ringer, WV
Abele, de Florian, WV

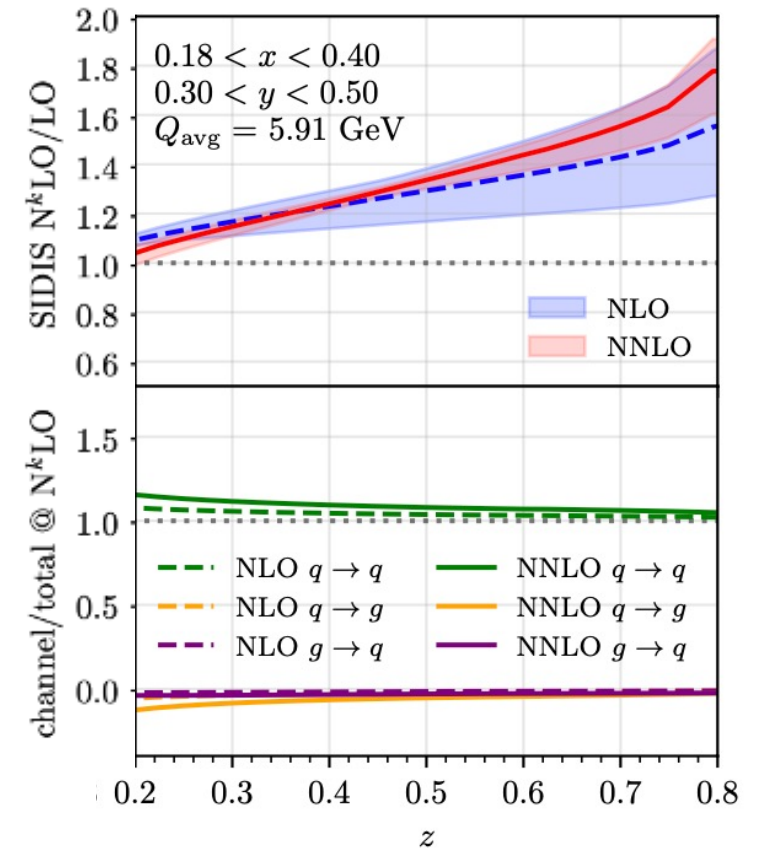


$\mu p \rightarrow \mu \pi^+ X$

COMPASS

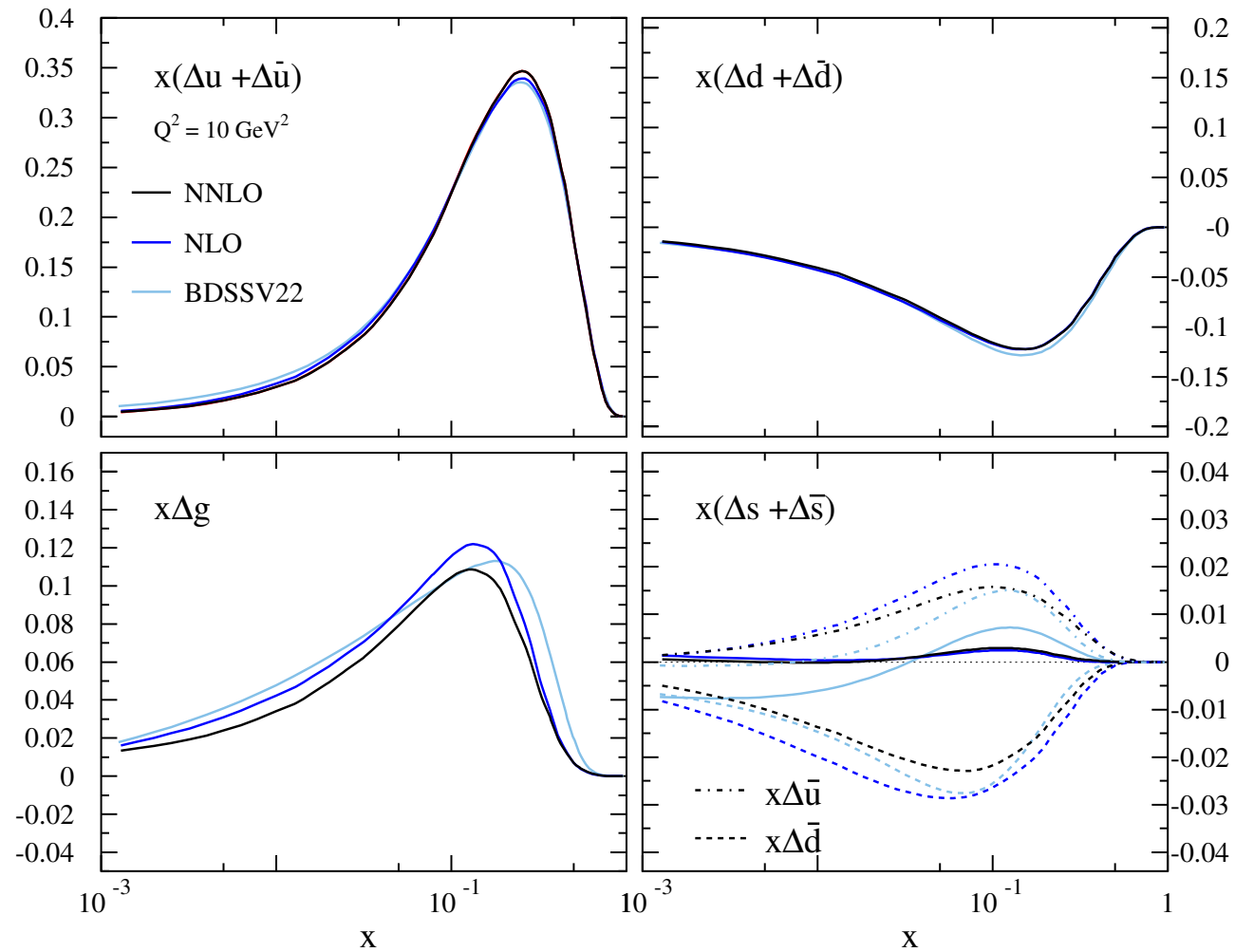
**Full NNLO unpolarized:**

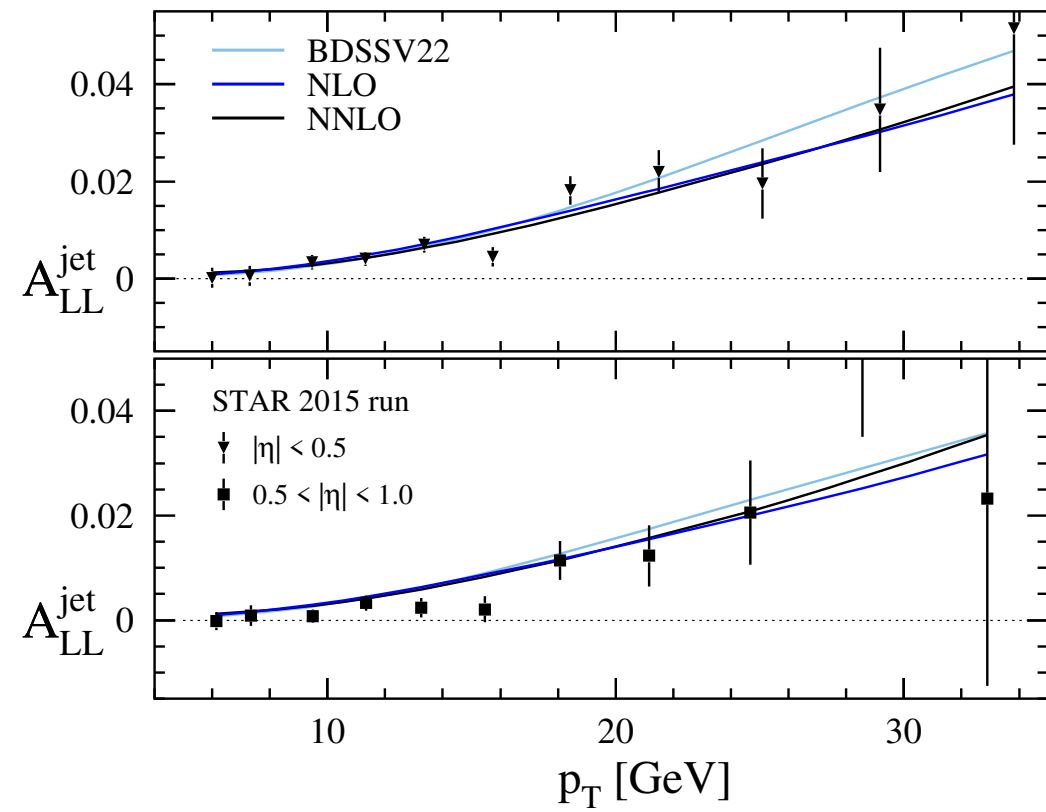
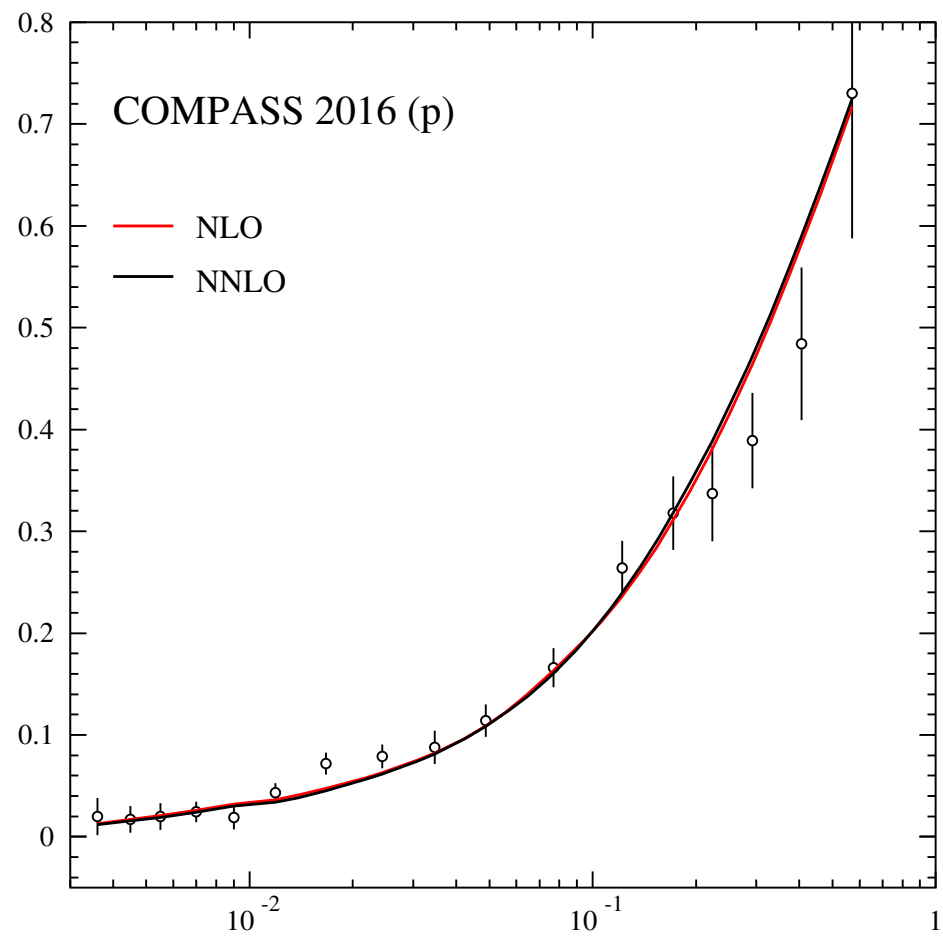
Boninio, Gehrmann, Stagnitto
Goyal, Moch, Pathak, Rana, Ravindran
(2024)



First global NNLO analysis of helicity PDFs:

Borsa, de Florian, Sassot, Stratmann, WV **BDSSV**





Charged-current studies at EIC: inclusive

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} F_2(x, Q^2) + i \epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha P^\beta}{2P \cdot q} F_3(x, Q^2) \\ - i \epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha S^\beta}{P \cdot q} g_1(x, Q^2) + \frac{S \cdot q}{P \cdot q} \left[\frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} g_4(x, Q^2) + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) g_5(x, Q^2) \right]$$

$$\hat{P}_\mu = P_\mu - \frac{P \cdot q}{q^2} q_\mu$$

CC:

$$\frac{d^2 \sigma^{\lambda_e}}{dx dy} = \frac{G_F^2 s}{\pi(1 + Q^2/M_W^2)^2} \left[-\lambda_e y(2-y)x F_3 + 2(1-y) F_2 + 2xy^2 F_1 \right]$$

$$\frac{d^2 \Delta \sigma^{\lambda_e}}{dx dy} = \frac{G_F^2 s}{\pi(1 + Q^2/M_W^2)^2} \left[-\lambda_e y(2-y)x g_1 - (1-y) g_4 - xy^2 g_5 \right]$$

$\lambda_e = -1$ electron

$\lambda_e = +1$ positron

LO, 4 active flavors:

$$g_4 = 2xg_5 \quad (\text{Dicus})$$

$$\begin{aligned} & \left. \begin{aligned} g_1^{W^-}(x, Q^2) &= [\Delta u + \Delta \bar{d} + \Delta c + \Delta \bar{s}](x, Q^2) \\ g_5^{W^-}(x, Q^2) &= [-\Delta u + \Delta \bar{d} - \Delta c + \Delta \bar{s}](x, Q^2) \\ g_1^{W^+}(x, Q^2) &= [\Delta \bar{u} + \Delta d + \Delta \bar{c} + \Delta s](x, Q^2) \\ g_5^{W^+}(x, Q^2) &= [\Delta \bar{u} - \Delta d + \Delta \bar{c} - \Delta s](x, Q^2) \end{aligned} \right\} + \end{aligned}$$

$$g_1^{W^+} + g_1^{W^-} = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} + \Delta c + \Delta \bar{c}$$

$$g_5^{W^+} + g_5^{W^-} \sim -\Delta u_V - \Delta d_V \quad g_1^{W^+} - g_1^{W^-} \sim -\Delta u_V + \Delta d_V$$

Define HERA-like "reduced" cross sections:

$$\Delta\sigma_r^{e^\pm} = \frac{\pi x}{2G_F^2} \left[\frac{M_W^2 + Q^2}{M_W^2} \right]^2 \frac{d^2 \Delta\sigma^{e^\pm p}}{dx dQ^2}$$

$$\approx \begin{cases} x \left(\Delta u + \Delta c - (1-y)^2 (\Delta \bar{d} + \Delta \bar{s}) \right) \\ x \left(-\Delta \bar{u} - \Delta \bar{c} + (1-y)^2 (\Delta d + \Delta s) \right) \end{cases}$$

electrons

positrons

If neutrons available, e.g.

$$g_{5,p}^{W^-}(x, Q^2) = \left[-\Delta u + \Delta \bar{d} - \Delta c + \Delta \bar{s} \right] (x, Q^2)$$

$$g_{5,n}^{W^-}(x, Q^2) = \left[-\Delta d + \Delta \bar{u} - \Delta c + \Delta \bar{s} \right] (x, Q^2)$$

$$g_{5,p}^{W^-} - g_{5,n}^{W^-} = -(\Delta u + \Delta \bar{u}) + \Delta d + \Delta \bar{d} = -6 \left(g_{1,p}^{\text{e.m.}} - g_{1,n}^{\text{e.m.}} \right)$$

especially,

$$\int_0^1 dx \left(g_{5,p}^{W^-} - g_{5,n}^{W^-} \right) = -g_A \left(1 - \frac{2\alpha_s}{3\pi} \right)$$

Stratmann, WV, Weber

$$\int_0^1 dx \left(g_{1,p}^{\text{e.m.}} - g_{1,n}^{\text{e.m.}} \right) = \frac{g_A}{6} \left(1 - \frac{\alpha_s}{\pi} \right)$$

Bjorken sum rule

$$\int_0^1 dx \left(g_{5,p}^{W^-} - g_{5,n}^{W^-} \right) = -g_A \left(1 - \frac{2\alpha_s}{3\pi} + \mathcal{O}(\alpha_s^2) \right)$$

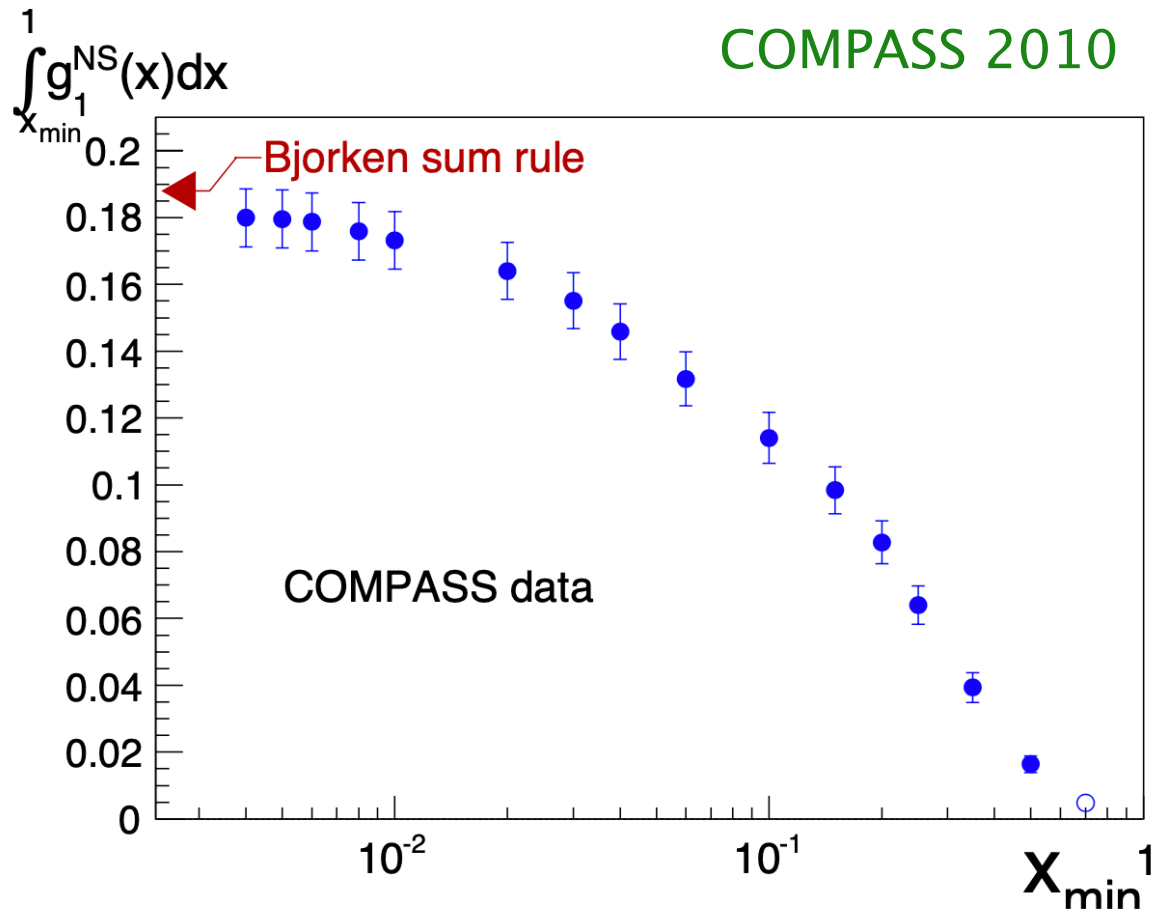
cf. unpolarized Bjorken sum rule:

$$\int_0^1 dx \left(F_{1,p}^{W^-} - F_{1,n}^{W^-} \right) = \left(1 - \frac{2\alpha_s}{3\pi} + \mathcal{O}(\alpha_s^2) \right) \int_0^1 dx (u_V - d_V) = 1 \left(1 - \frac{2\alpha_s}{3\pi} + \mathcal{O}(\alpha_s^2) \right)$$

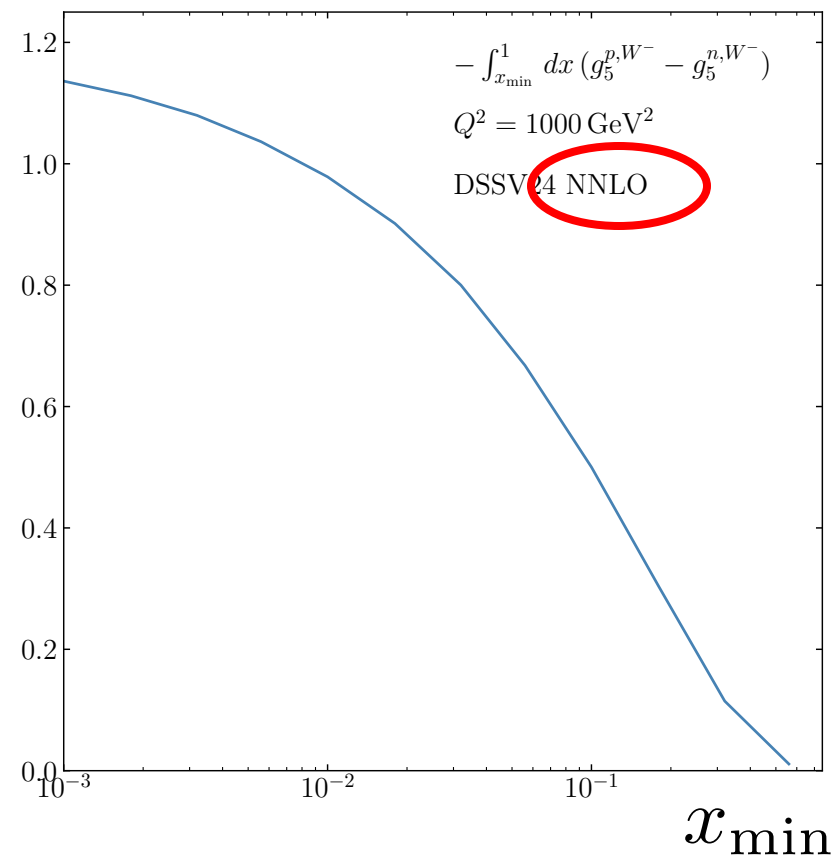
Gross Llewellyn-Smith sum rule:

$$\frac{1}{2} \int_0^1 dx \left(F_{3,p}^{W^-} + F_{3,p}^{W^+} \right) = \left(1 - \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right) \int_0^1 dx (u_V + d_V) = 3 \left(1 - \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right)$$

COMPASS 2010

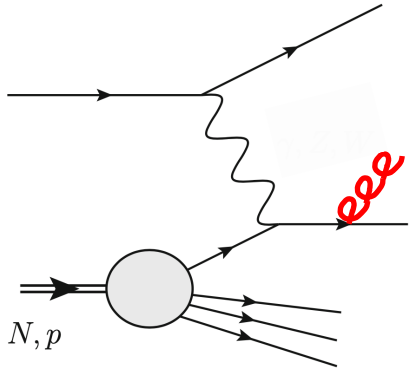


$$-\int_{x_{\min}}^1 dx \left(g_{5,p}^{W^-} - g_{5,n}^{W^-} \right)$$



courtesy I. Borsa

at NLO:



$$g_1^{W^\pm; \text{NLO}}(x, Q^2) = \Delta C_{q,1} \otimes g_1^{W^\pm; \text{LO}} + N_f \Delta C_{g,1} \otimes \Delta g$$

$$g_4^{W^\pm; \text{NLO}}(x, Q^2) = \Delta C_{q,4} \otimes g_4^{W^\pm; \text{LO}}$$

$$g_5^{W^\pm; \text{NLO}}(x, Q^2) = \Delta C_{q,5} \otimes g_5^{W^\pm; \text{LO}}$$

$$\Delta C_{q,i} = \delta(1-x) + \frac{\alpha_s}{2\pi} \Delta C_{q,i}^{(1)} + \dots$$

$$\Delta C_{g,1} = \frac{\alpha_s}{2\pi} \Delta C_g^{(1)} + \dots$$

de Florian, Sassot;
Stratmann, Weber, WV;
Forte, Mangano, Ridolfi

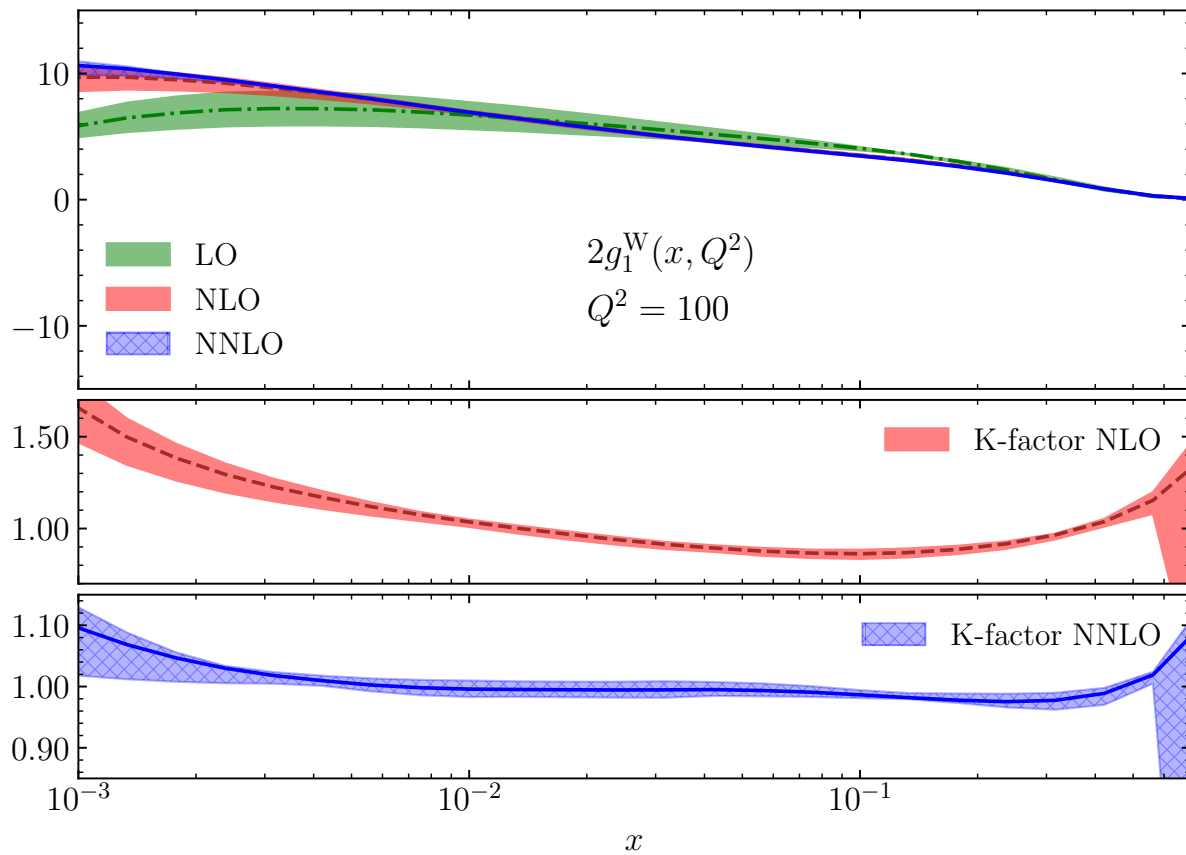
NNLO corrections:
(exploit similarities

van Neerven, Zijlstra 1991

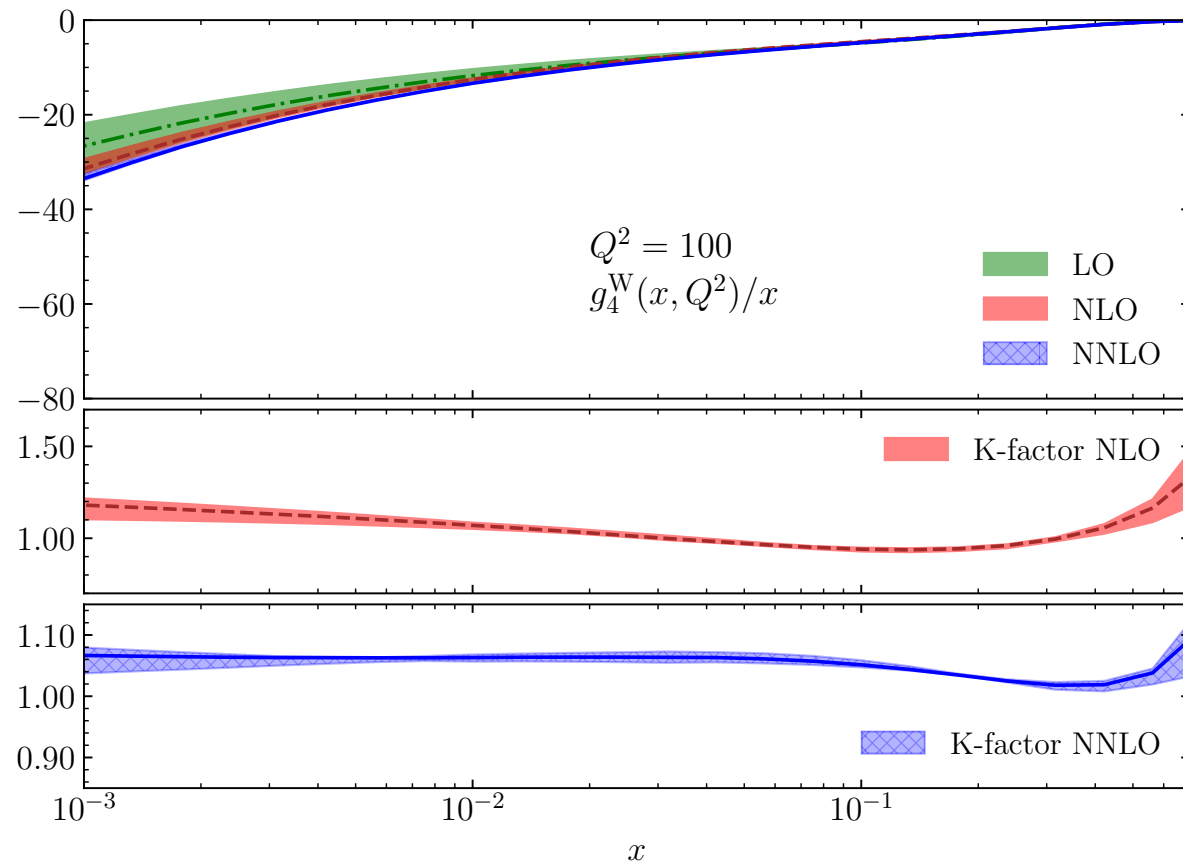
Borsa, de Florian, Pedron, 2022

$$g_1 \leftrightarrow F_3 \quad g_4, g_5 \leftrightarrow F_2, F_1)$$

$$2g_1^{W^-}$$



$$g_4^{W^-} / x$$

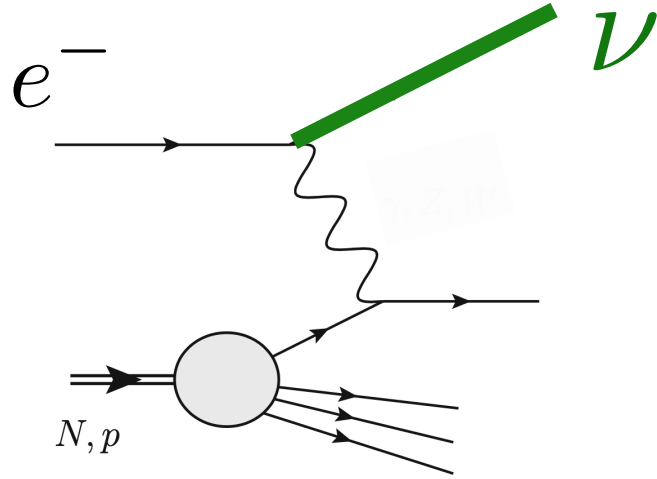


(with NNLO PDFs!)

courtesy I. Borsa

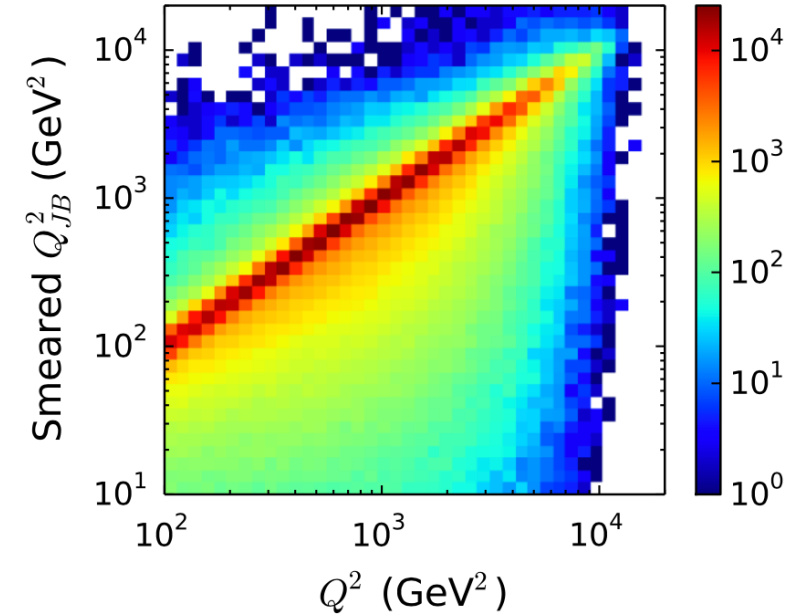
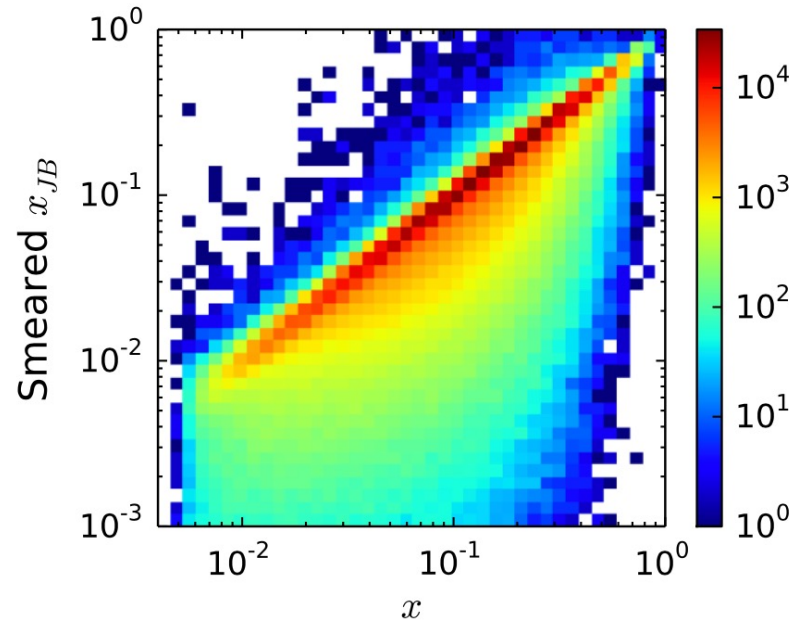
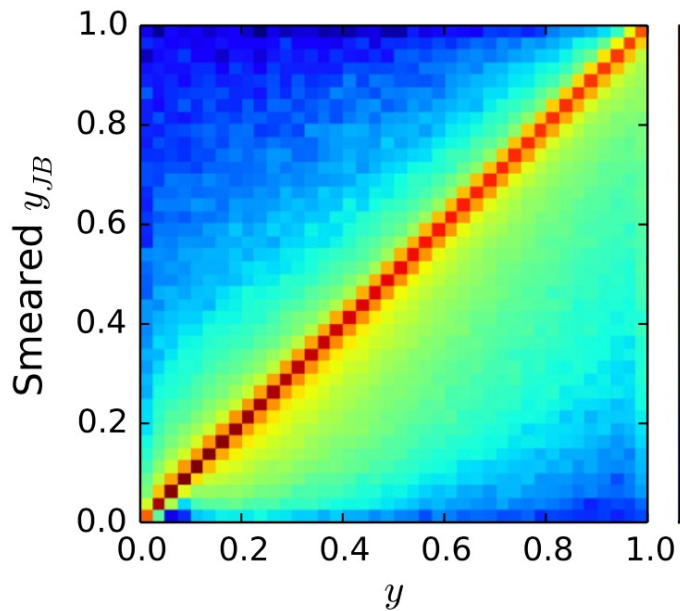
Meeting reality of CC at EIC:

Aschenauer, Burton, Martini,
Spiesberger, Stratmann 2013

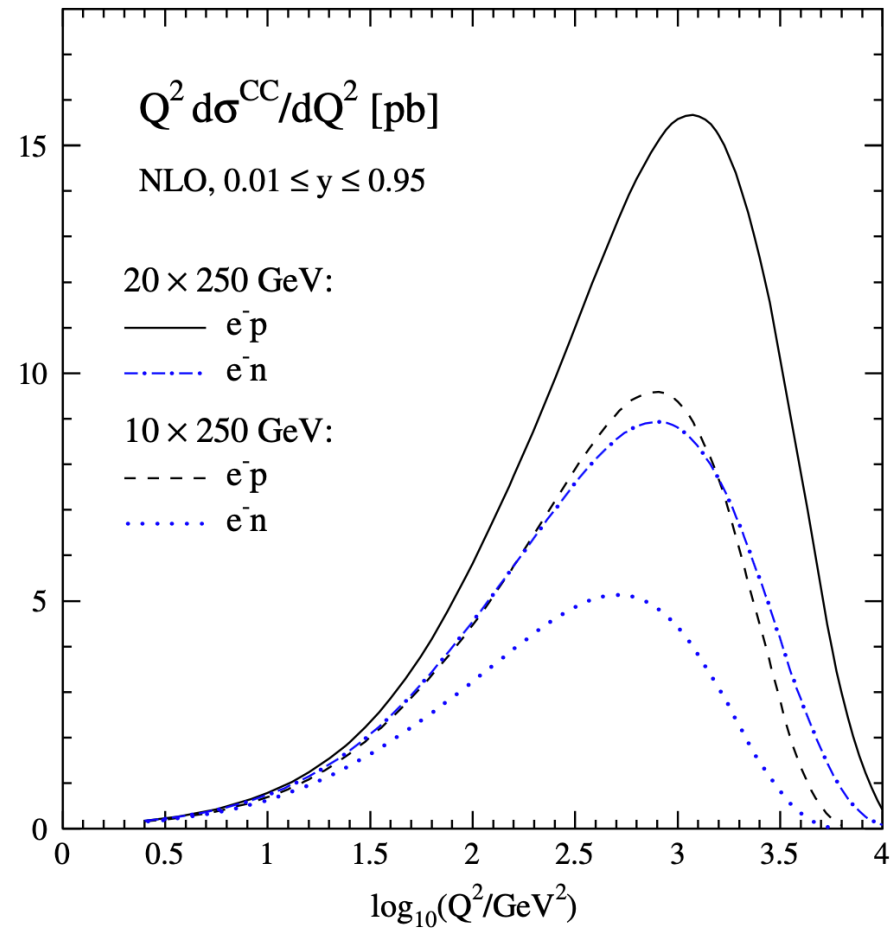
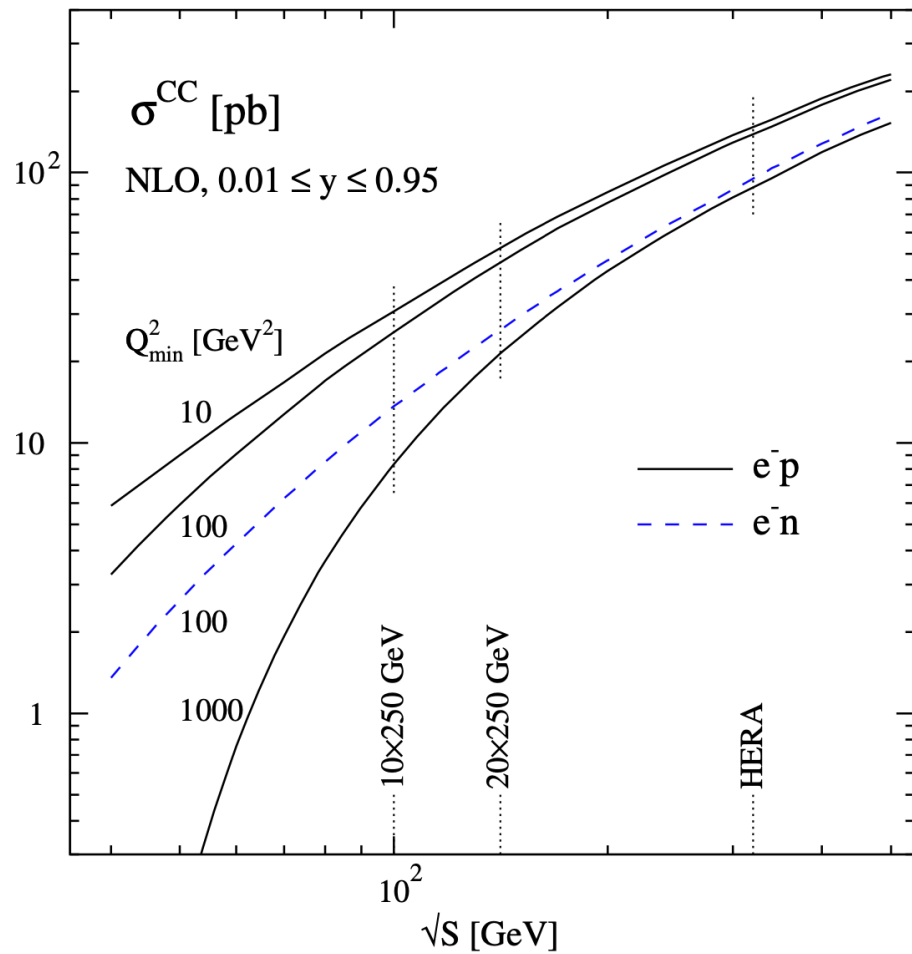


Jacquet-Blondel method:

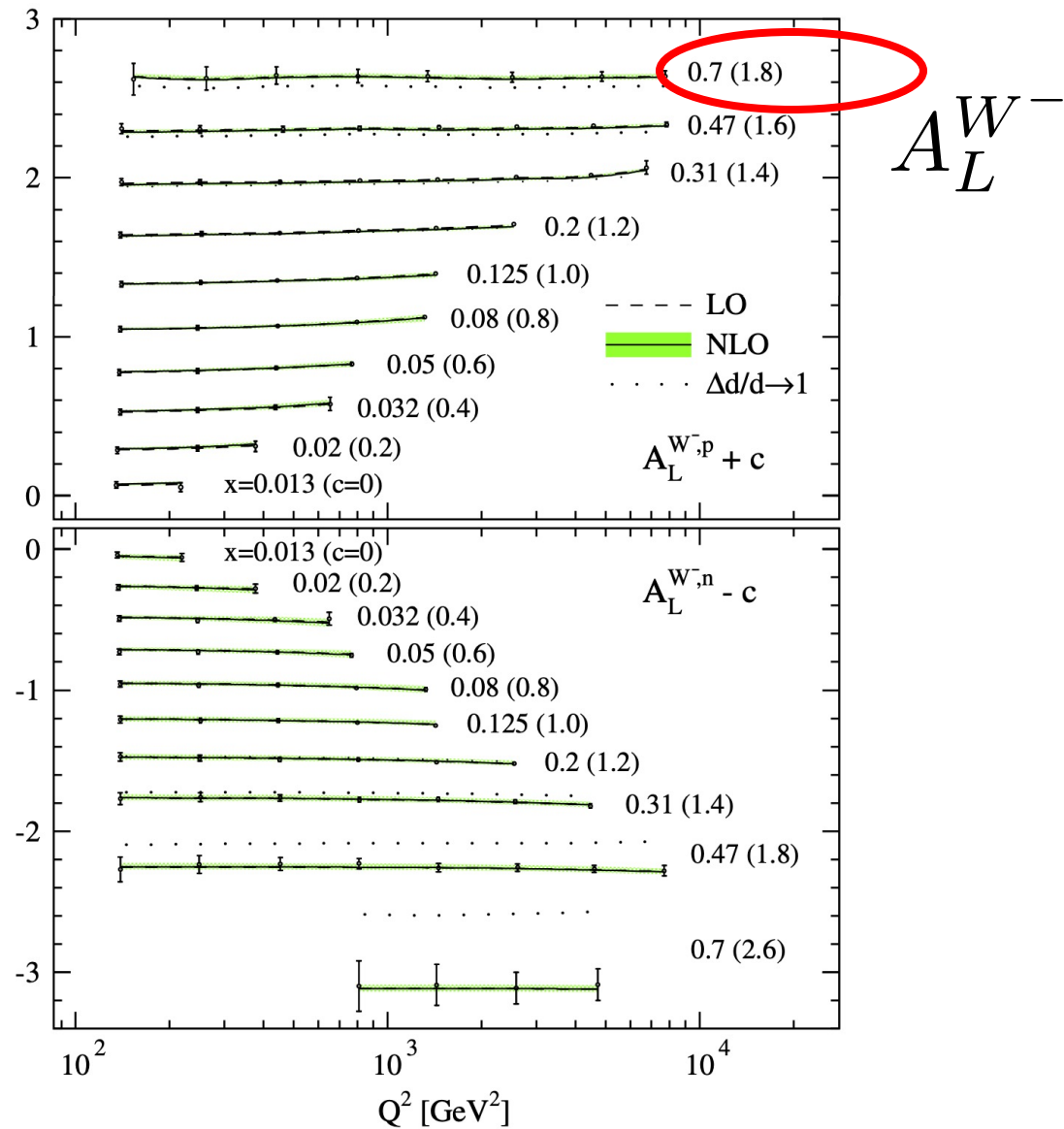
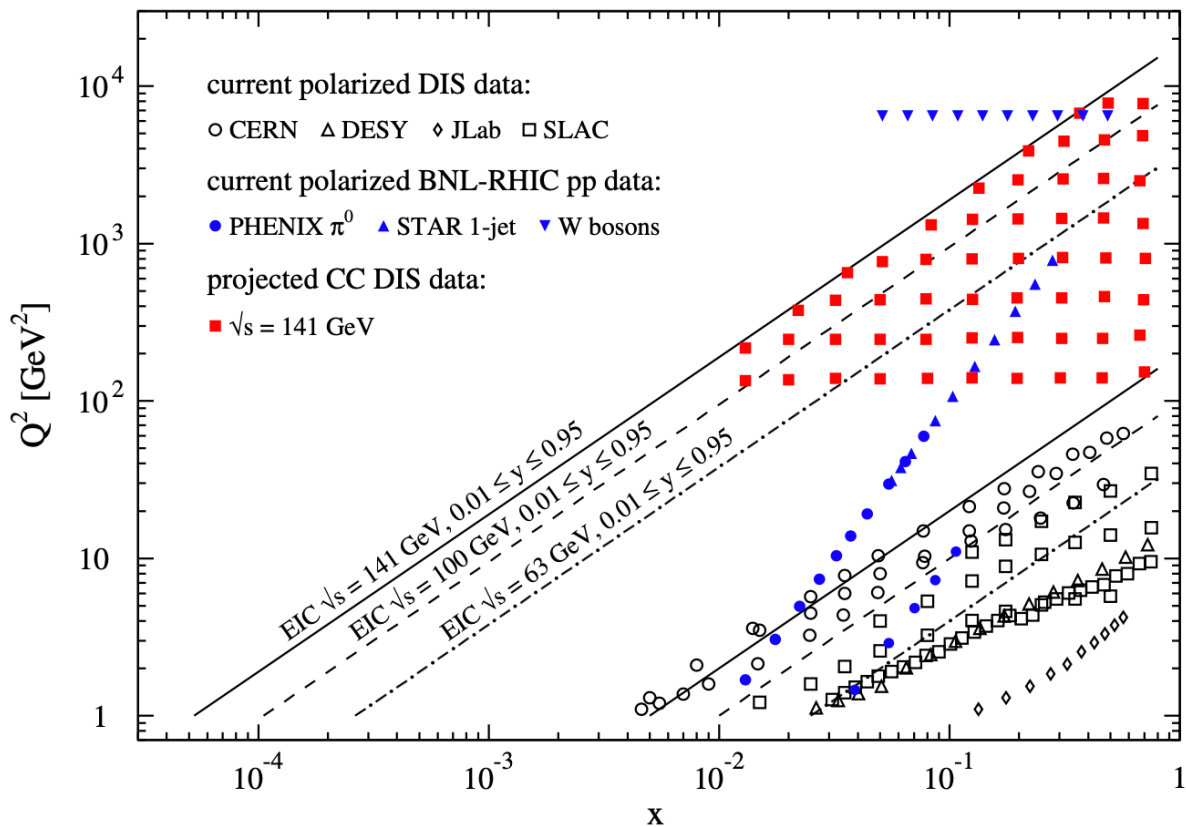
$$y_{\text{JB}} = \frac{\sum_i (E_i - p_{z,i})}{2E_e}, \quad Q_{\text{JB}}^2 = \frac{p_{T,h}^2}{1 - y_{\text{JB}}}, \quad x_{\text{JB}} = \frac{Q_{\text{JB}}^2}{y_{\text{JB}} S}$$



includes detector resolution (GEANT4), QED radiative effects (DJANGO)



$$\langle Q^2 \rangle \sim 1000 \text{ GeV}^2 \quad \langle x \rangle \sim 0.2$$

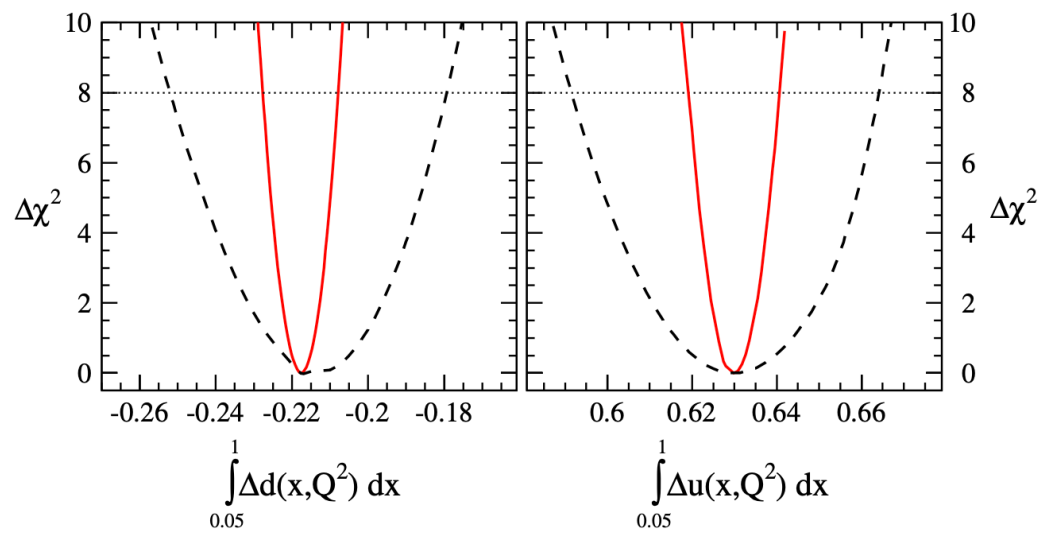
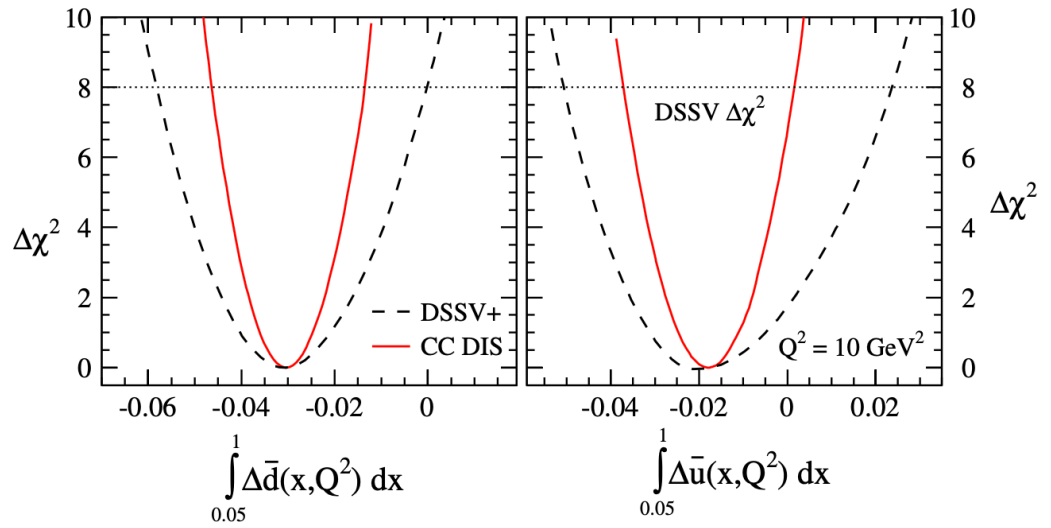


$$A_L^{W^-} = \frac{2b g_1^{W^-} - a g_5^{W^-}}{a F_1^{W^-} + b F_3^{W^-}}$$

$$a = 2(y^2 - 2y + 2)$$

$$b = y(2 - y)$$

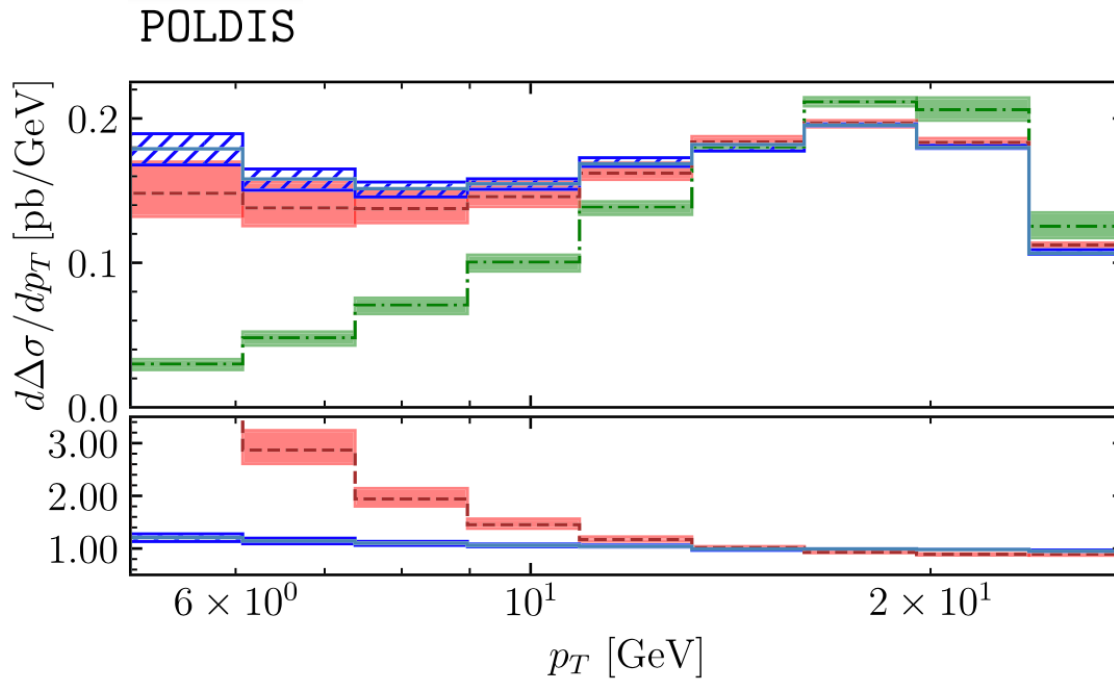
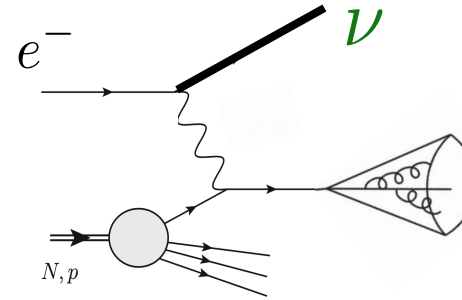
Aschenauer, Burton, Martini,
Spiesberger, Stratmann 2013



Less inclusive observables

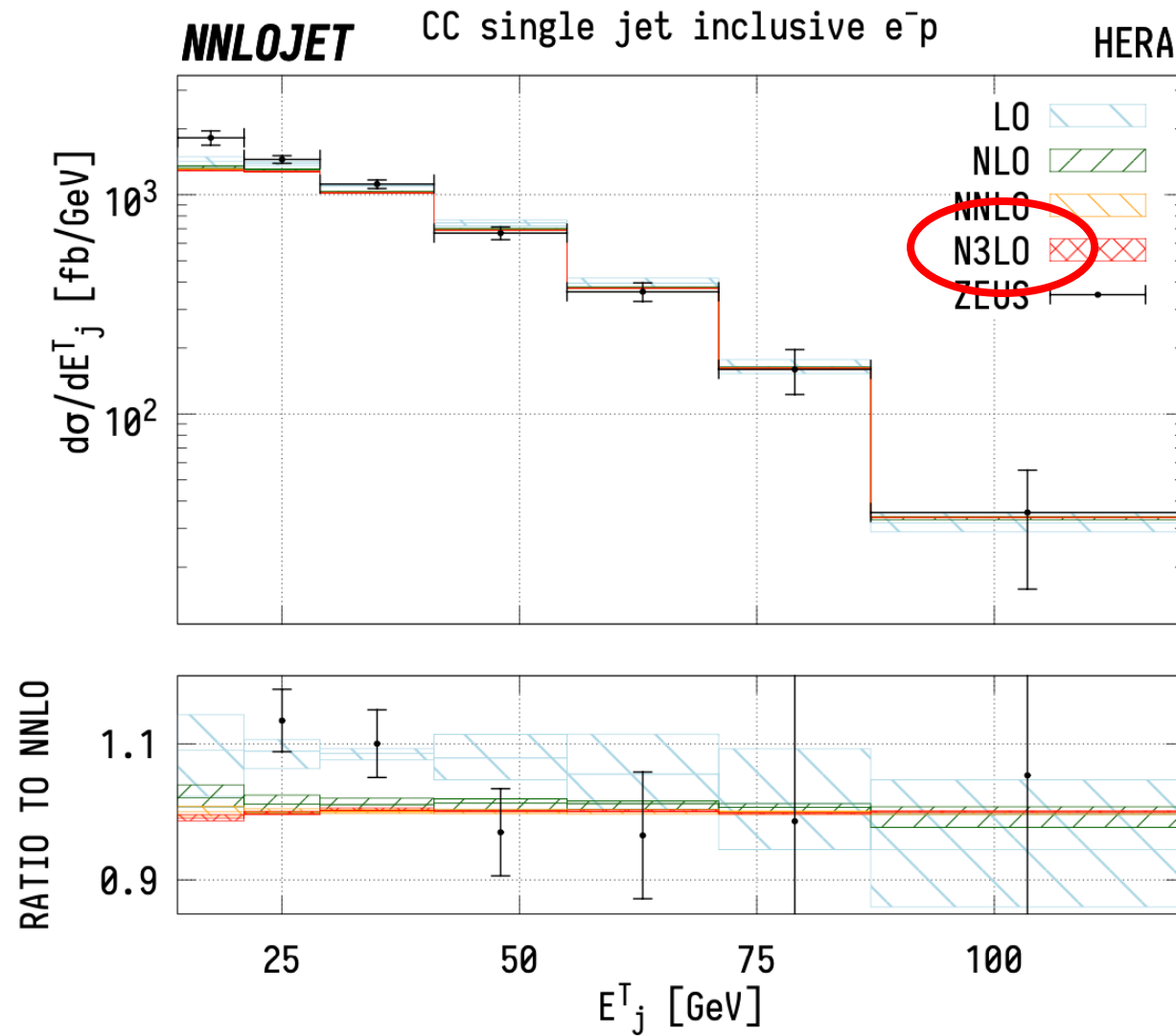
inclusive-jet production in CC DIS:

Borsa, de Florian, Pedron 2023

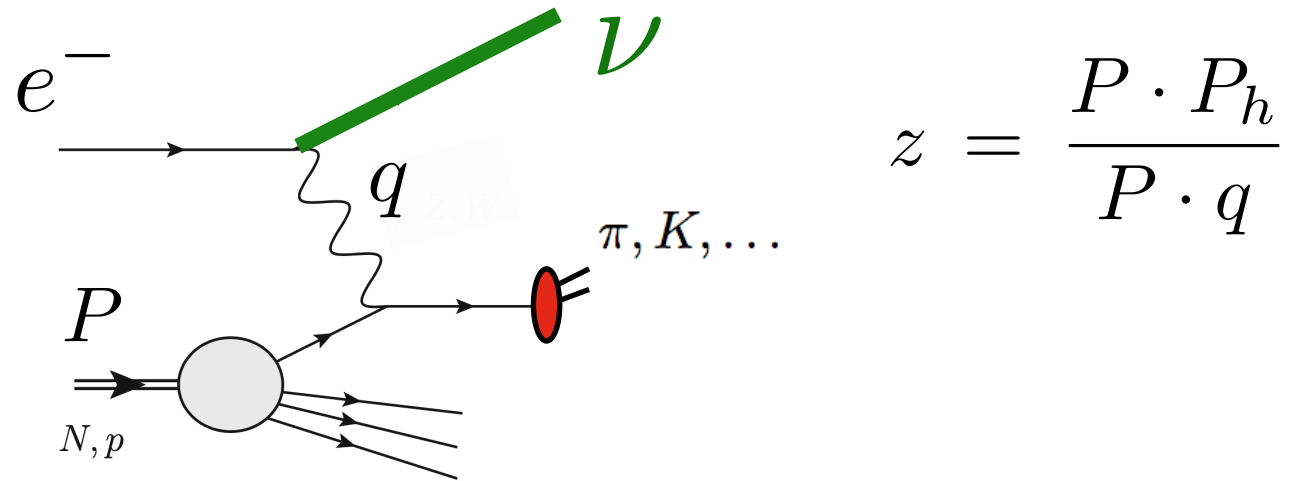


CC $ep \rightarrow \text{jet} + e + X$
 $\sqrt{s} = 140 \text{ GeV}$

K-factor NLO (red) K-factor NNLO (blue hatched)



CC semi-inclusive DIS:



$$g_1^{W^-,h}(x, z, Q^2) = \left[\Delta u(x) D_u^h(z) + \Delta \bar{d}(x) D_{\bar{u}}^h(z) + \Delta c(x) D_s^h(z) + \Delta \bar{s}(x) D_{\bar{c}}^h(z) \right]_{Q^2}$$

$$g_5^{W^-,h}(x, z, Q^2) = \left[-\Delta u(x) D_u^h(z) + \Delta \bar{d}(x) D_{\bar{u}}^h(z) - \Delta c(x) D_s^h(z) + \Delta \bar{s}(x) D_{\bar{c}}^h(z) \right]_{Q^2}$$

NLO corrections:

de Florian, Rothstein Habarnau 2013

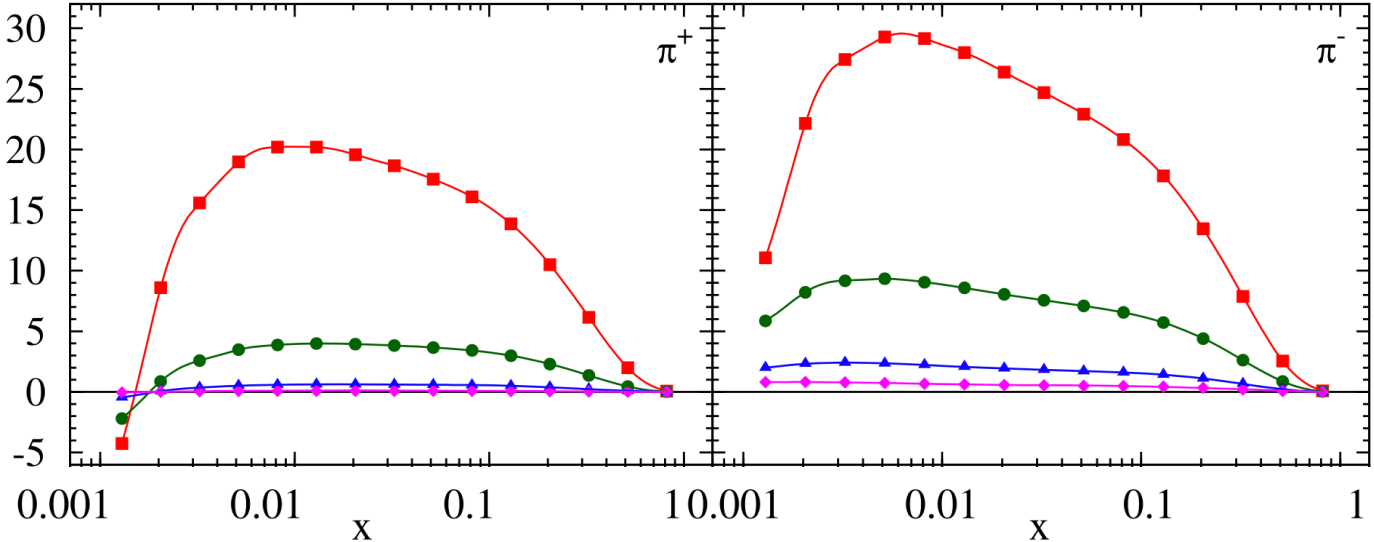
Further improvements for flavor separation:

Kang, Liu, Mantry, Shao 2020

“jet charge”

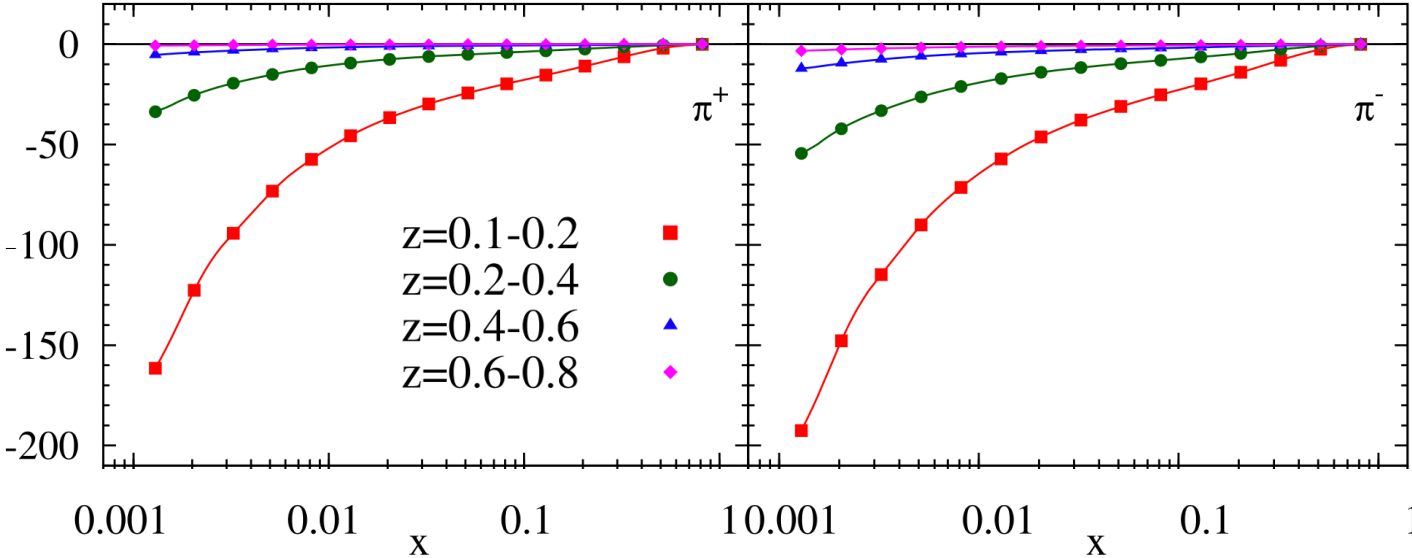
$$Q_\kappa \equiv \sum_{h \in \text{jet}} z_h^\kappa Q_h$$

$g_1^{W^-, \pi^+}$



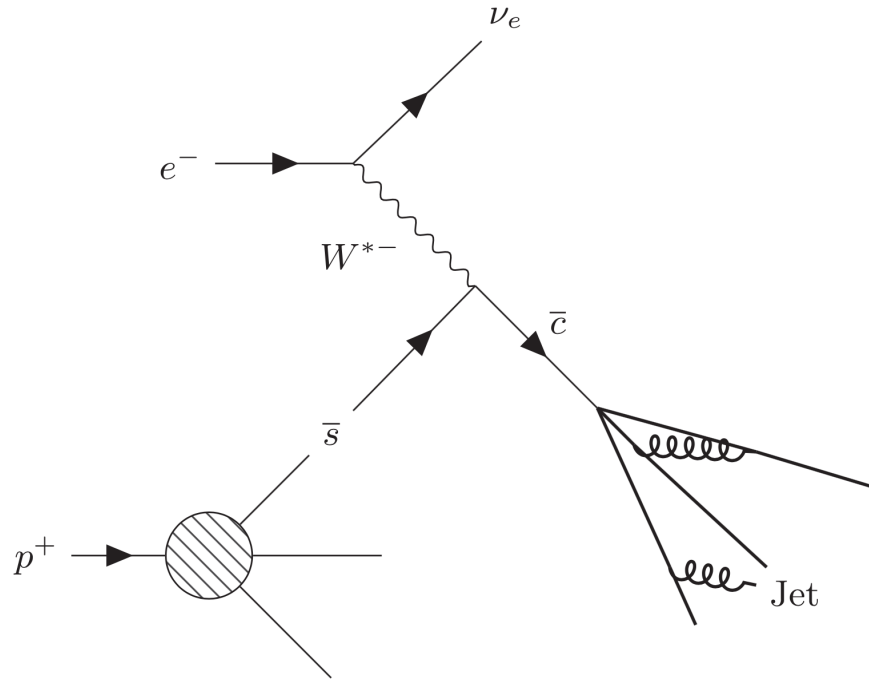
$g_1^{W^-, \pi^-}$

$g_5^{W^-, \pi^+}$



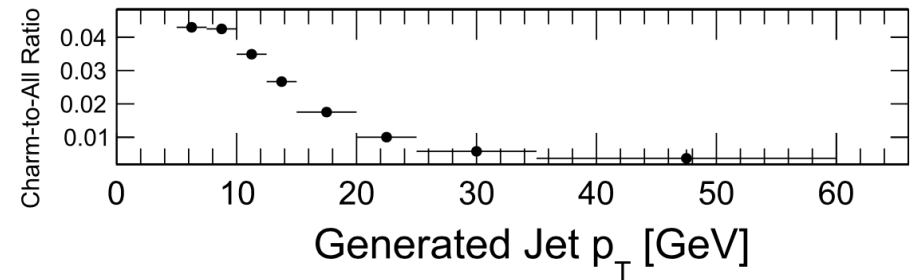
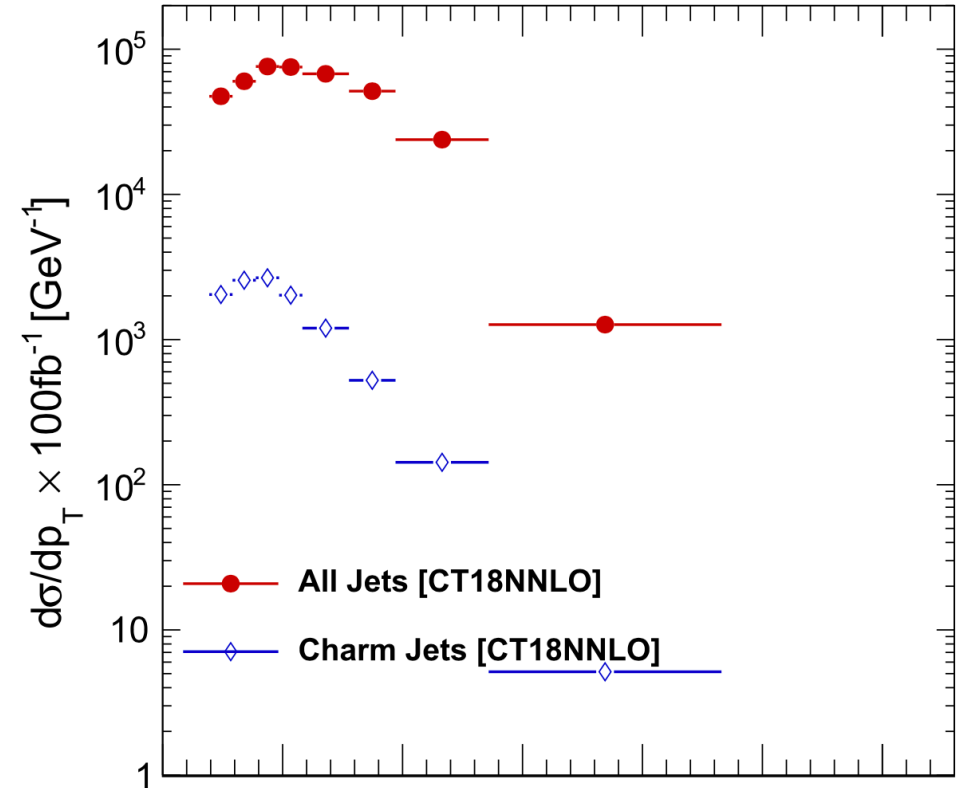
$g_5^{W^-, \pi^-}$

CC charm production:



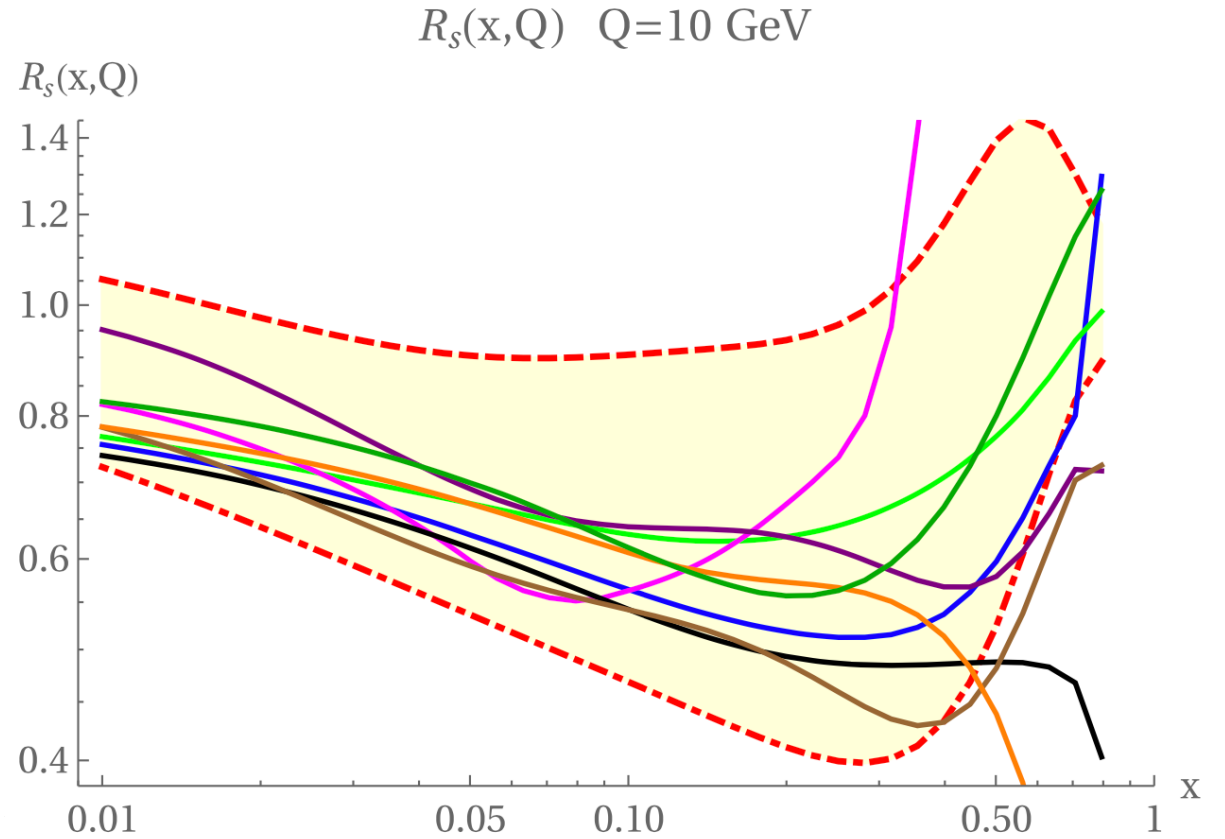
charm jet tagging

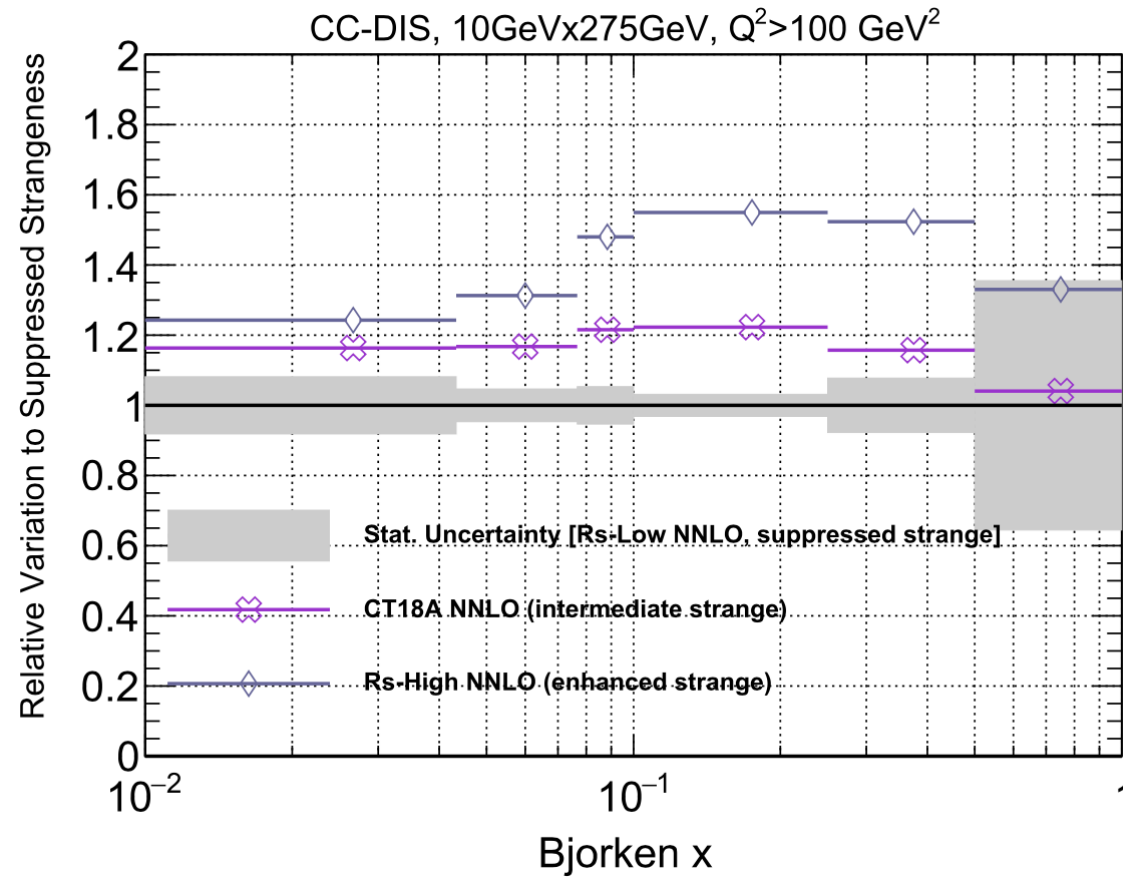
CC-DIS, 10GeVx275GeV, $Q^2 > 100 \text{ GeV}^2$



unpolarized strange PDF ratio

$$R_s(x, Q) = \frac{s(x, Q) + \bar{s}(x, Q)}{\bar{u}(x, Q) + \bar{d}(x, Q)}$$





(unpolarized ep)

Full NLO treatment with charm mass:

Aivazis, Olness, Tung
Kretzer, Stratmann

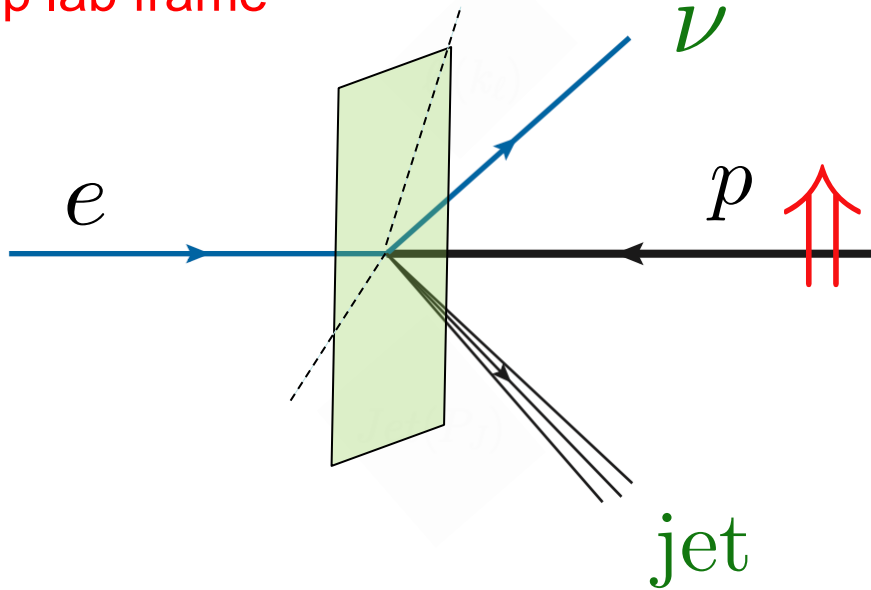
Apply to polarized case?

TMD physics in ν +jet:

Arratia, Kang, Paul, Prokudin, Ringer, Zhao 2023

(Liu, Ringer, WV, Yuan 2019)

ep lab frame

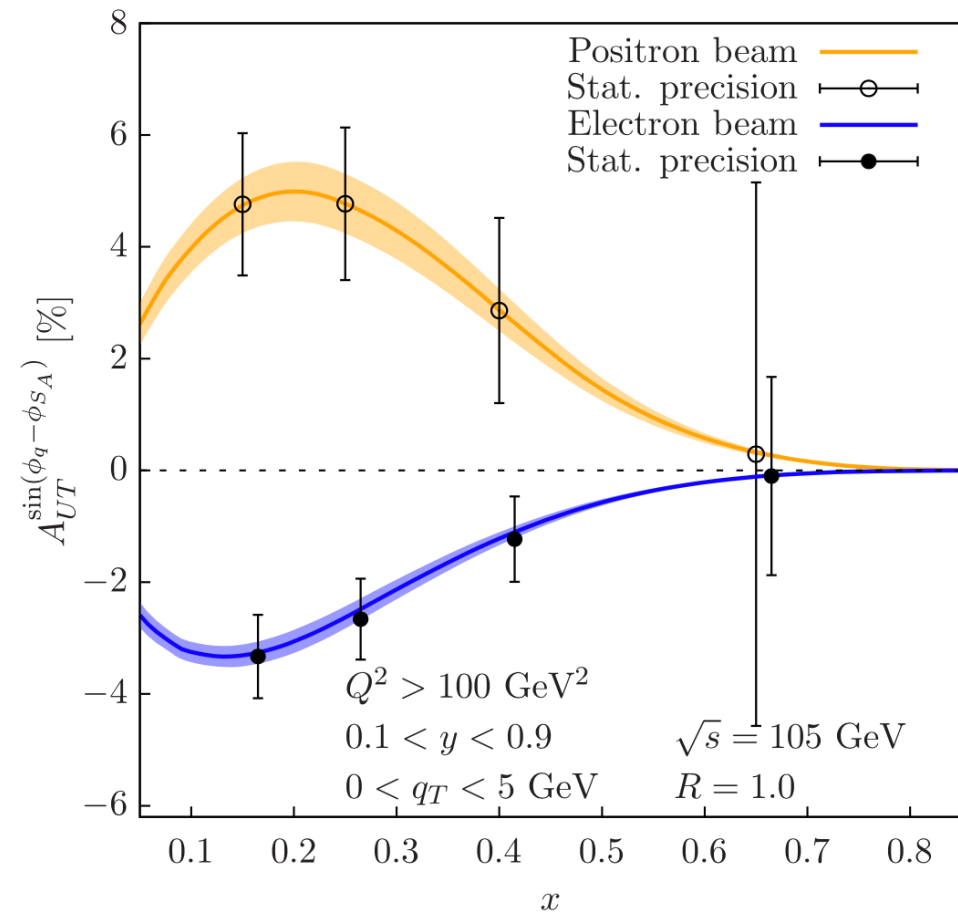
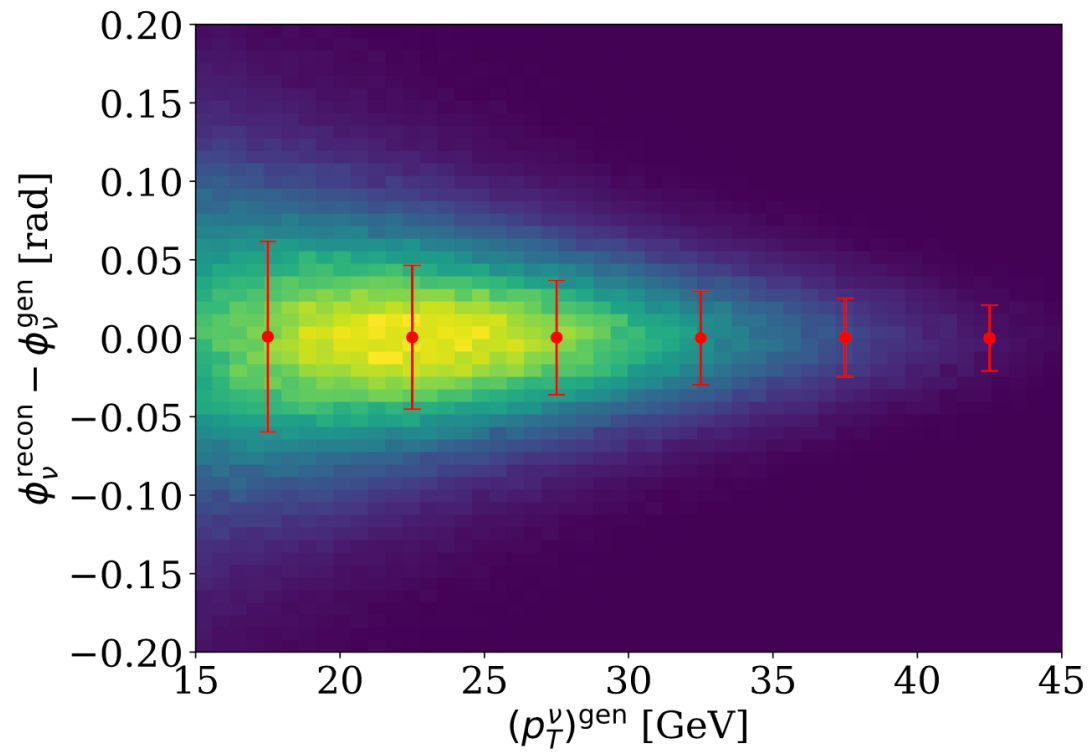


$$\frac{d\sigma^{ep \rightarrow \nu \text{jet} X}}{dx d^2 \vec{p}_T^\nu d^2 q_T}$$

imbalance of $\vec{q}_T = \vec{p}_T^{\text{jet}} + \vec{p}_T^\nu$

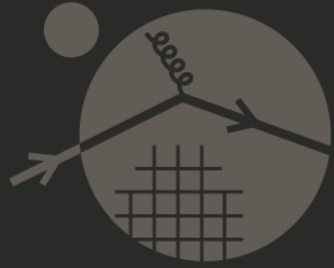
$$= F_{UU} + \lambda_p F_{UL} + |S_T| \left[\sin(\phi_q - \phi_{S_A}) F_{UT}^{\sin(\phi_q - \phi_{S_A})} + \cos(\phi_q - \phi_{S_A}) F_{UT}^{\cos(\phi_q - \phi_{S_A})} \right] + \lambda_e \left[F_{LU} + \lambda_p F_{LL} + |S_T| \sin(\phi_q - \phi_{S_A}) F_{LT}^{\sin(\phi_q - \phi_{S_A})} + |S_T| \cos(\phi_q - \phi_{S_A}) F_{LT}^{\cos(\phi_q - \phi_{S_A})} \right]$$

- TMD factorization for $q_T \ll p_T^{\text{jet}}, p_T^\nu$



Concluding remarks:

- qualitative step forward to NNLO: pQCD analysis “in good shape”
- CC observables at EIC will allow new insights into proton helicity structure
- enormous theoretical progress
- need for updated studies (now that ePIC takes shape)



INSTITUTE for NUCLEAR THEORY

PROGRAM

APRIL 28 - JUNE 20, 2025

Precision QCD with the Electron Ion Collider (INT)

R. Fatemi, H. W. Lin, W. Vogelsang

Joint with the Bridging Theory Workshop

WORKSHOP

JUNE 2 - JUNE 6, 2025

Bridging Theory and Experiment at the Electron- Ion Collider (INT)

A. Bacchetta, W. Cosyn, F. Ringer, A. Stasto

Joint with the Precision QCD Program

+ Miguel Arratia, Daniel de Florian, Thomas Gehrmann, Zhongbo Kang, Matt Sievert