Higher-order QCD corrections and extractions of FFs and PDFs

Werner Vogelsang
Univ. of Tübingen

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Outline:

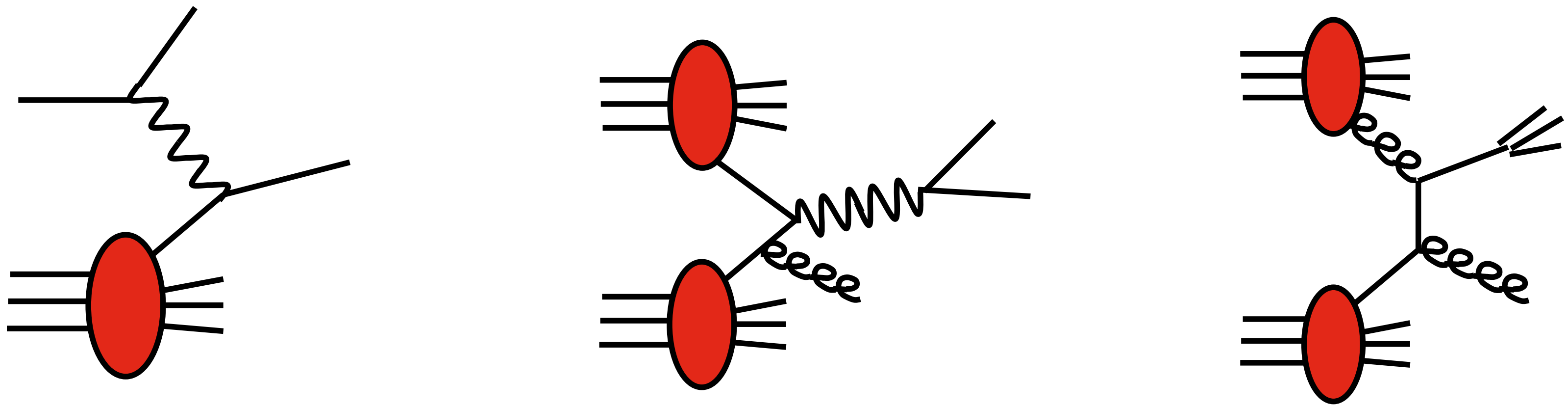
- Basic pQCD framework
- SIDIS   ep → e+π+X
- Phenomenology & fragmentation functions
- pp → π+X
- Conclusions

Maurizio Abele, Daniel de Florian, WV   PRD 2021 & 2022

Ignacio Borsa, Daniel de Florian, Rodolfo Sassot, Marco Stratmann, WV
PRL 2022

Patriz Hinderer, Felix Ringer, George Sterman, WV   PRD 2019

Basic pQCD framework



$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a, \mu) \, f_b(x_b, \mu) \, \omega_{ab} \left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu} \right) + \ldots$$

- $f_{a,b}$ parton distributions: non-pert., but universal
- ω_{ab} partonic cross sections: process-dep., but pQCD
- $\mu \sim Q$ factorization / renormalization scale
- corrections power-suppressed in 1/Q

$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a,\mu) \, f_b(x_b,\mu) \, \omega_{ab} \left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu} \right) + \ldots$$

$$\frac{d}{d\log\mu^2} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} \mathcal{P}_{ij}(z, \alpha_s(\mu)) \, f_j\left(\frac{x}{z}, \mu\right)$$

$$\omega_{ab} = \omega_{ab}^{(0)} + \frac{\alpha_s}{\pi} \omega_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega_{ab}^{(2)} + \ldots$$

LO NLO NNLO

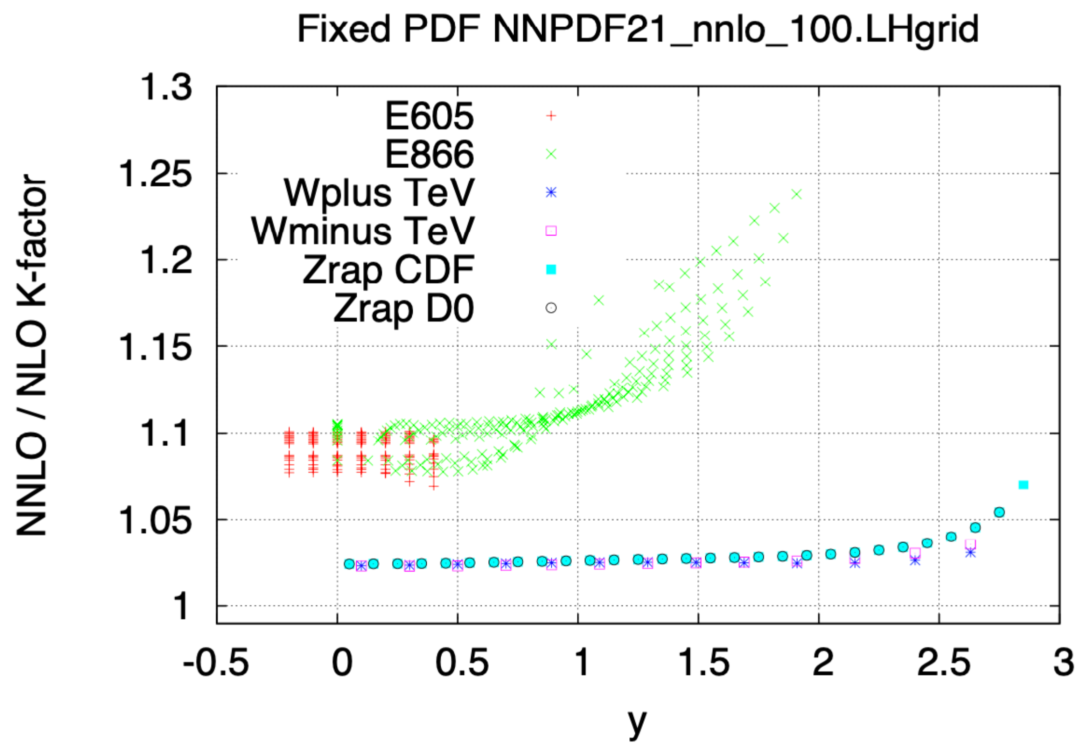
$$\mathcal{P}_{ij} = \frac{\alpha_s}{2\pi} P_{ij}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^3 P_{ij}^{(2)} + \ldots$$

- framework applies to unpolarized, spin, (nuclear) PDFs

MSHT20, 2012.04684

Data set	NLO	NNLO
ATLAS W^+, W^-, Z [119]	34.7/30	29.9/30
CMS W asym. $p_T > 35$ GeV [155]	11.8/11	7.8/11
CMS asym. $p_T > 25, 30$ GeV [156]	11.8/24	7.4/24
LHCb $Z \to e^+e^-$ [157]	14.1/9	22.7/9
LHCb W asym. $p_T > 20$ GeV [158]	10.5/10	12.5/10
CMS $Z \to e^+e^-$ [159]	18.9/35	17.9/35
ATLAS High-mass Drell-Yan [160]	20.7/13	18.9/13
CMS double diff. Drell-Yan [72]	222.2/132	144.5/132
Tevatron, ATLAS, CMS $\sigma_{t\bar{t}}$ [93]- [94]	22.8/17	14.5/17
LHCb 2015 W, Z [95, 96]	114.4/67	99.4/67
LHCb 8 TeV $Z \to ee$ [97]	39.0/17	26.2/17
CMS 8 TeV W [98]	23.2/22	12.7/22
ATLAS 7 TeV jets [18]	226.2/140	221.6/140
CMS 7 TeV $W + c$ [99]	8.2/10	8.6/10
ATLAS 7 TeV high precision W, Z [20]	304.7/61	116.6/61
CMS 7 TeV jets [100]	200.6/158	175.8/158
CMS 8 TeV jets [101]	285.7/174	261.3/174
CMS 2.76 TeV jet [107]	124.2/81	102.9/81
ATLAS 8 TeV Z p_T [75]	235.0/104	188.5/104
ATLAS 8 TeV single diff $t\bar{t}$ [102]	39.1/25	25.6/25
ATLAS 8 TeV single diff $t\bar{t}$ dilepton [103]	4.7/5	3.4/5
CMS 8 TeV double differential $t\bar{t}$ [105]	32.8/15	22.5/15
CMS 8 TeV single differential $t\bar{t}$ [108]	12.9/9	13.2/9
ATLAS 8 TeV High-mass Drell-Yan [73]	85.8/48	56.7/48
ATLAS 8 TeV W [106]	84.6/22	57.4/22
ATLAS 8 TeV $W +$ jets [104]	33.[illegible]/30	1[illegible].1/30
ATLAS 8 TeV double differential Z [74]	157.4/59	85.6/59
Total	5822.0/4363	5121.9/4363

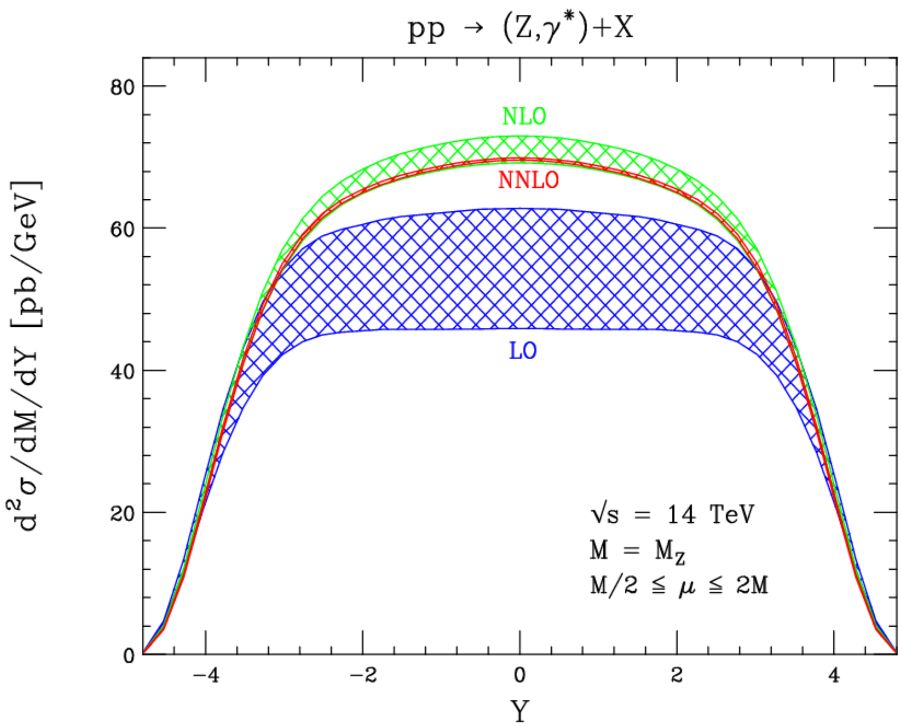
NNPDF, 1107.2652

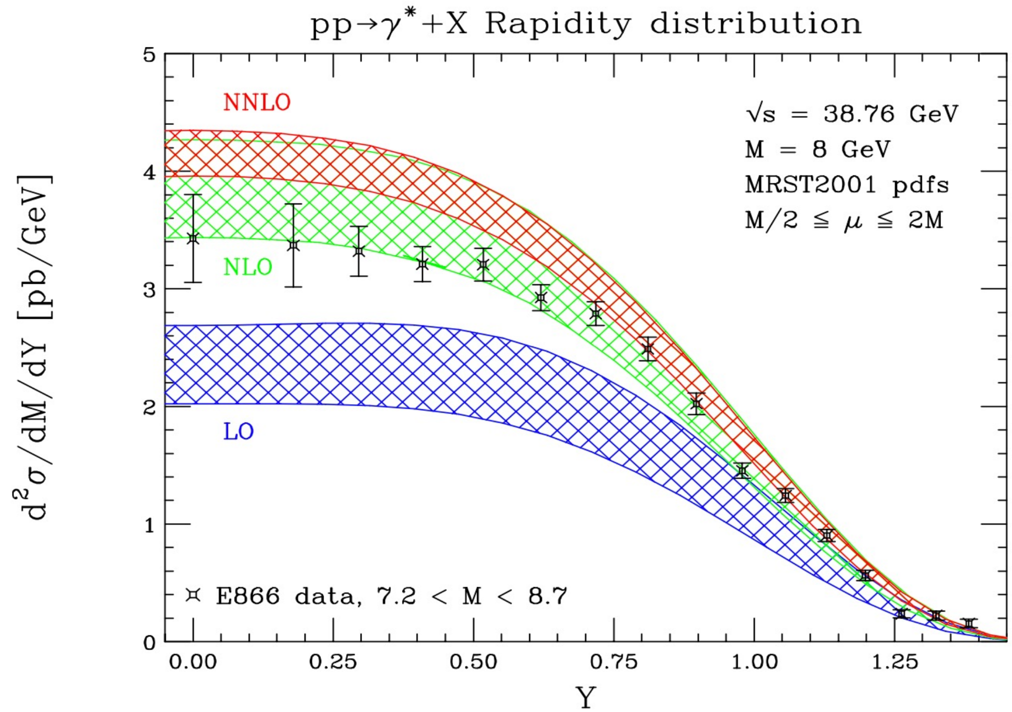


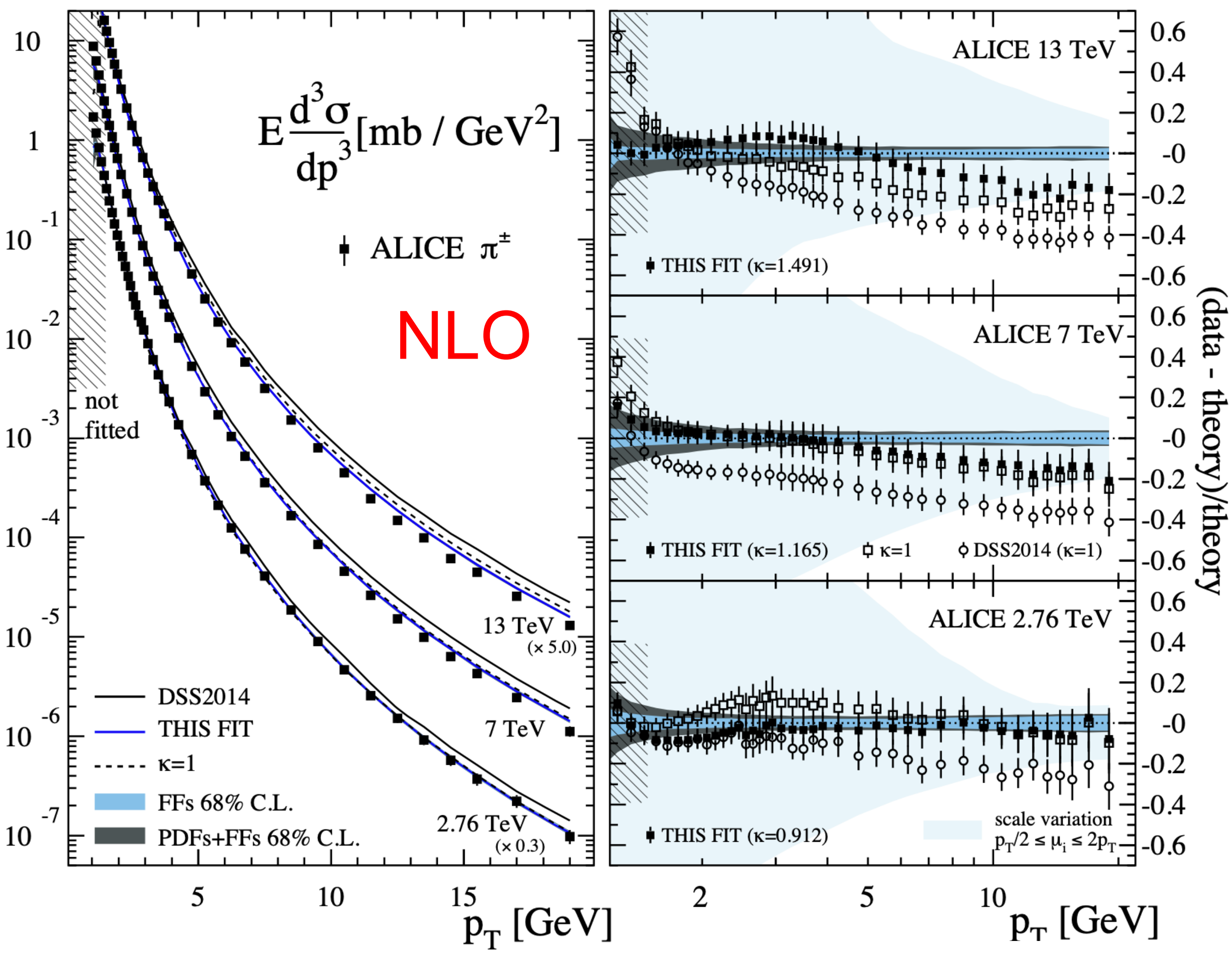
- threshold-resummed PDFs

NNPDF, JAM (P. Barry et al.)

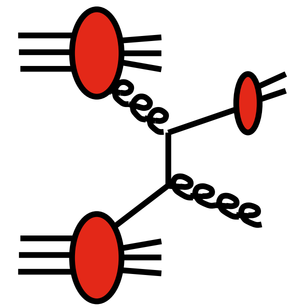
Anastasiou, Dixon, Melnikov, Petriello





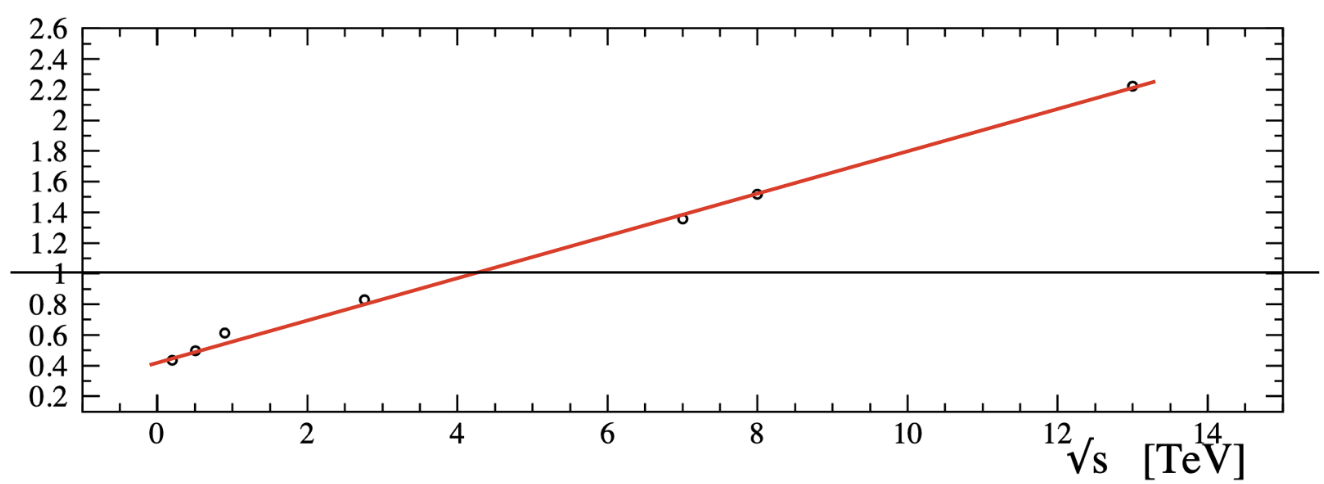


Borsa, de Florian, Sassot, Stratmann



$$\mu = \kappa p_T$$

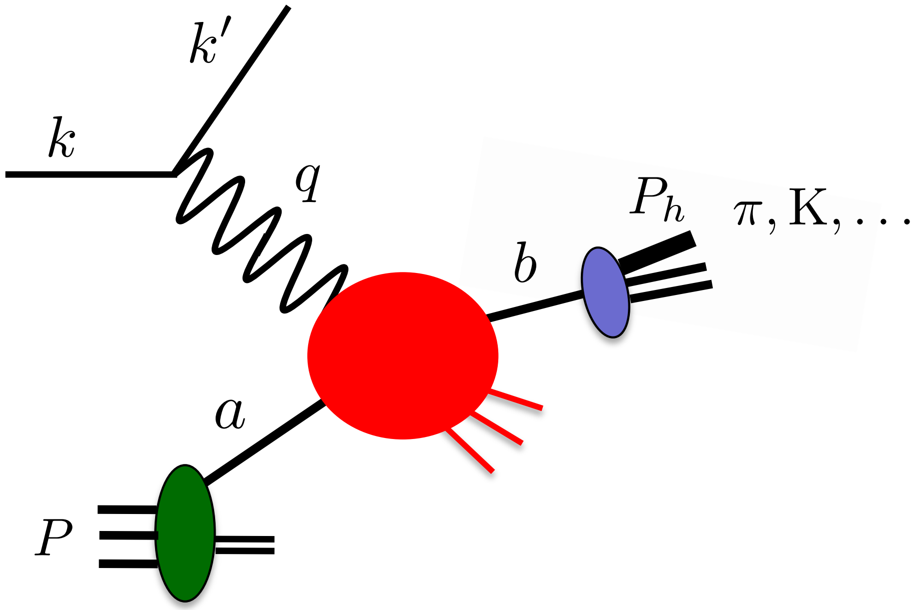
$$\kappa^2 = \frac{\mu^2}{p_T^2}$$



- incorporation of higher-order corrections crucial
- NNLO required
- relevant also for EIC
- in some cases of interest knowledge currently stops at NLO
- … especially in polarized case

Obtain approximate NNLO results? And beyond?

Semi-inclusive DIS



$$x = \frac{Q^2}{2P \cdot q}$$

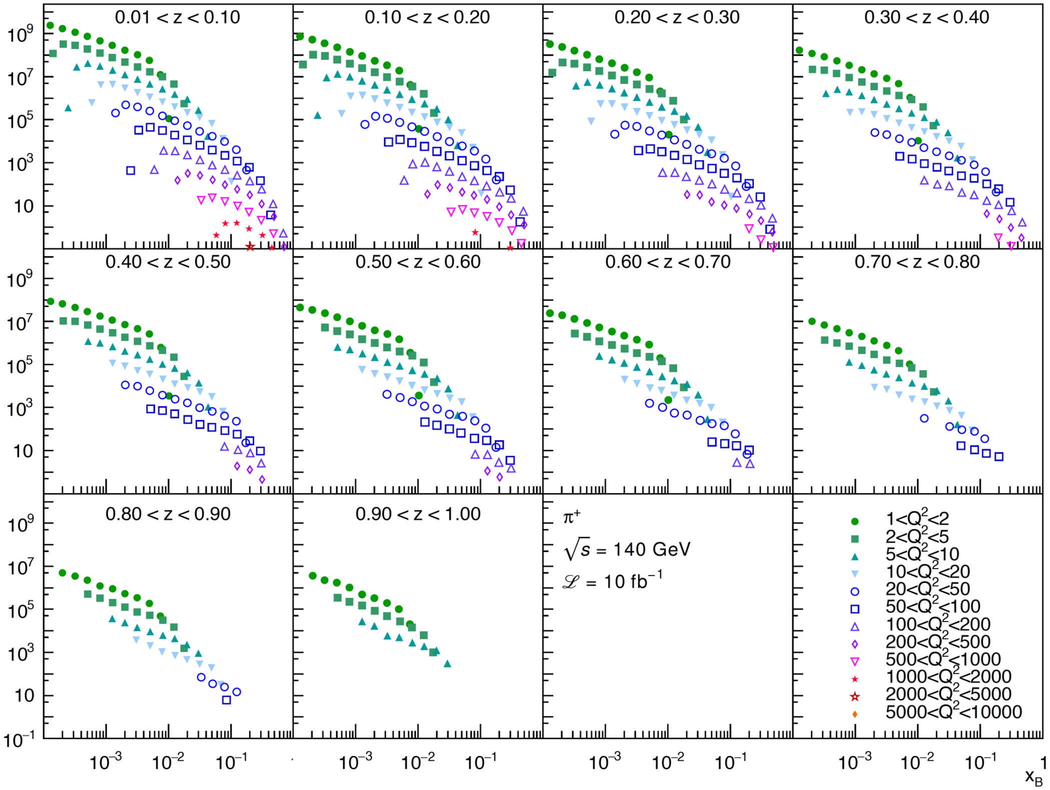
$$z = \frac{P \cdot P_h}{P \cdot q}$$

$$\sigma \propto \sum_q e_q^2 \, f_q\left(x, Q^2\right) D_q^h\left(z, Q^2\right)$$

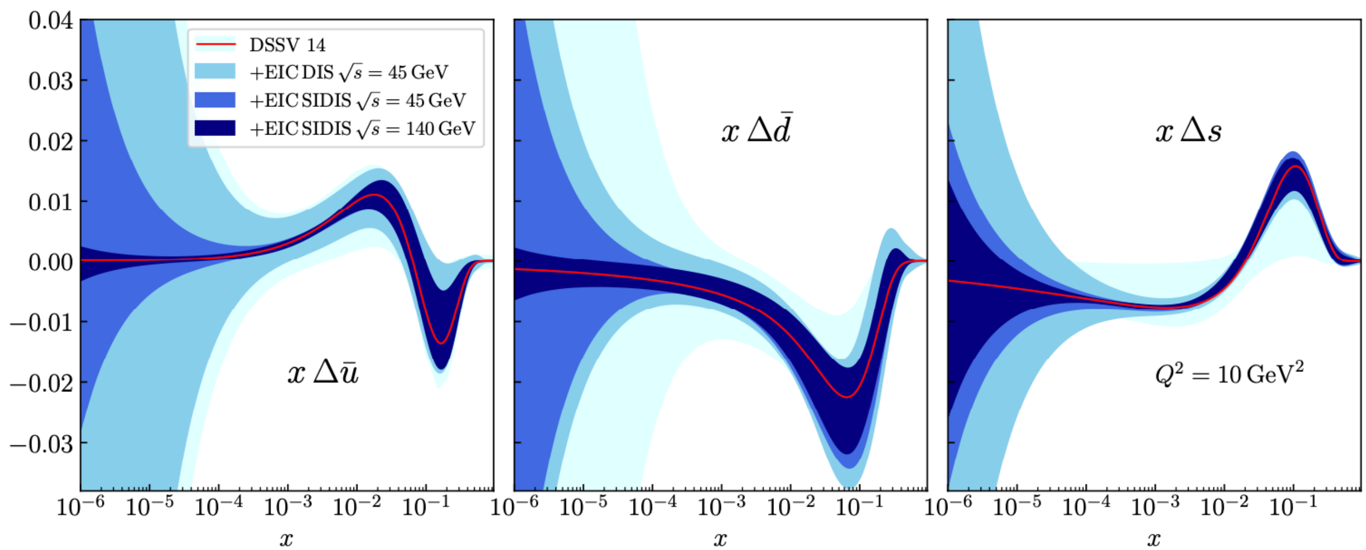
- PDFs (polarized, flavor dependence)
- fragmentation functions
- plus, transverse-momentum dep. (TMDs)   (not today)

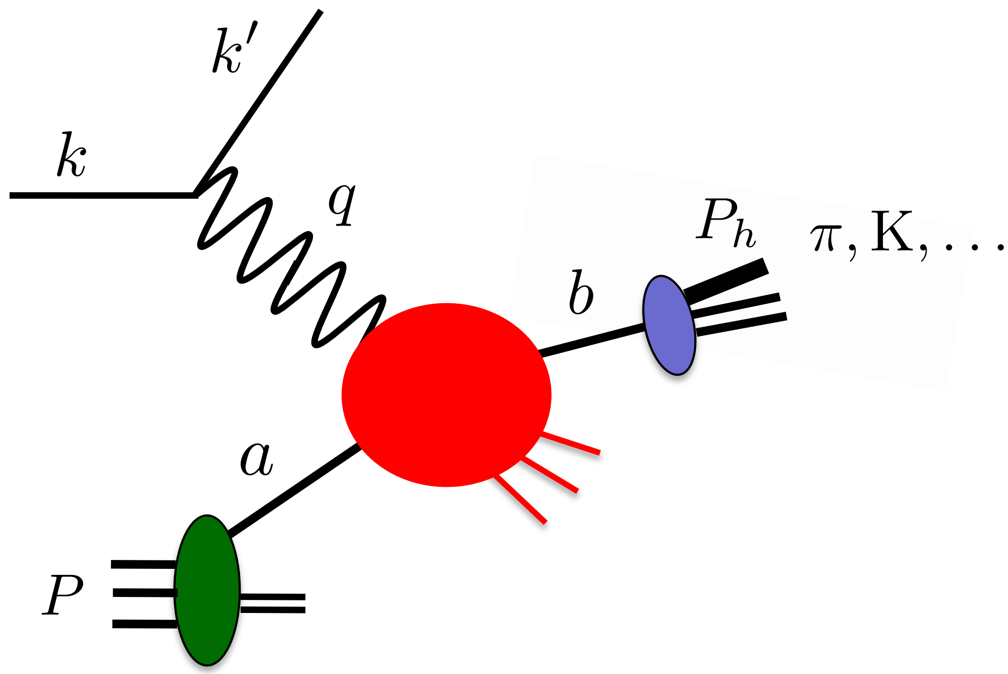
$$\frac{d\sigma}{dx\,dQ^2\,dz}$$

[pb/GeV^2]



Aschenauer, Borsa, Lucero, Nunes, Sassot (2019)





$$x = \frac{Q^2}{2P\cdot q}$$

$$z = \frac{P\cdot P_h}{P\cdot q}$$

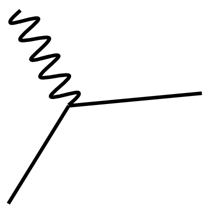
$$\frac{d^3\sigma^h}{dx\,dy\,dz} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}F_T^h(x,z,Q^2) + \frac{1-y}{y}F_L^h(x,z,Q^2)\right]$$

$$F_T^h(x,z,Q^2) = \sum_{a,b}\int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\, f_a\left(\frac{x}{\hat{x}},Q^2\right)\,\omega_{ab}\left(\hat{x},\hat{z},\alpha_s\right)\,D_b^h\left(\frac{z}{\hat{z}},Q^2\right) + \text{P.C.}$$

perturbative

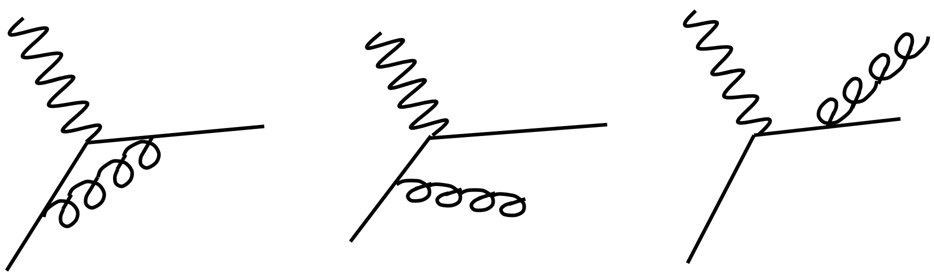
$$\hat{x} = \frac{Q^2}{2p_a\cdot q} \qquad \hat{z} = \frac{p_a\cdot p_b}{p_a\cdot q}$$

$$\omega_{ab} = \omega_{ab}^{(0)} + \frac{\alpha_s}{2\pi}\omega_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \omega_{ab}^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \omega_{ab}^{(3)} + \mathcal{O}(\alpha_s^4)$$

LO: 

$$\omega_{qq}^{(0)}(\hat{x},\hat{z}) = e_q^2\,\delta(1-\hat{x})\,\delta(1-\hat{z})$$

$$\Rightarrow F_T^h(x,z,Q^2) = \sum_q e_q^2\, f_q\left(x,Q^2\right) D_q^h\left(z,Q^2\right)$$

NLO:  $+ \ldots$

Altarelli et al.;
de Florian,
Stratmann, WV

NNLO: so far unknown

(Daleo, Garcia Canal, Sassot;
Anderle, de Florian, Habarnau)

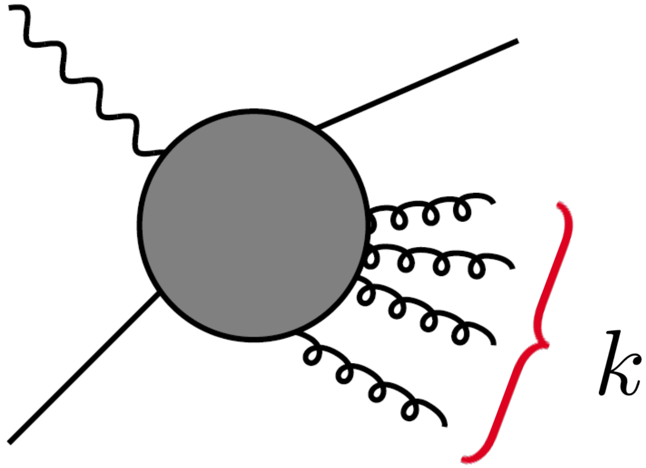
LO: $$\omega_{qq}^{(0)}(\hat{x},\hat{z}) = e_q^2\,\delta(1-\hat{x})\,\delta(1-\hat{z})$$

NLO, as $\hat{x},\hat{z} \longrightarrow 1$:

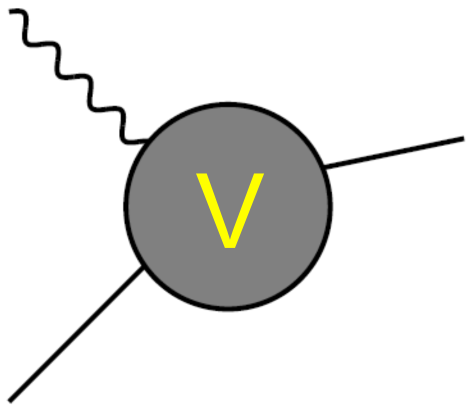
$$\omega_{qq}^{(1)}(\hat{x},\hat{z}) = e_q^2 C_F \left[2\delta(1-\hat{x}) \left(\frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+ + 2\delta(1-\hat{z}) \left(\frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ \right.$$
$$\left. + \frac{2}{(1-\hat{x})_+(1-\hat{z})_+} - 8\delta(1-\hat{x})\,\delta(1-\hat{z}) + \ldots \right]$$

k[th] order of perturbation theory:

$$\alpha_s^k\,\omega_{qq}^{(k)}(\hat{x},\hat{z}) \sim \alpha_s^k \left[\delta(1-\hat{x}) \left(\frac{\ln^{2k-1}(1-\hat{z})}{1-\hat{z}} \right)_+ + \delta(1-\hat{z}) \left(\frac{\ln^{2k-1}(1-\hat{x})}{1-\hat{x}} \right)_+ \right.$$
$$\left. + \frac{1}{(1-\hat{x})_+} \left(\frac{\ln^{2k-2}(1-\hat{z})}{1-\hat{z}} \right)_+ + \frac{1}{(1-\hat{z})_+} \left(\frac{\ln^{2k-2}(1-\hat{x})}{1-\hat{x}} \right)_+ + \ldots \right]$$



$$(1-\hat{x}) + (1-\hat{z}) \approx \frac{2k_0}{Q}$$



$$\hat{x} = \hat{z} = 1$$

- real and virtual contributions “imbalanced”
- logs can be resummed to all orders: threshold resummation

Sterman; Catani,Trentadue; …

- use to determine dominant parts of NNLO, N^{3}LO corrections

Mellin moments:

$$\tilde{F}_T^h(N, M, Q^2) \equiv \int_0^1 dx\, x^{N-1} \int_0^1 dz\, z^{M-1}\, F_T^h(x, z, Q^2)$$

$$= \sum_{a,b} \tilde{f}_a(N, Q^2)\, \tilde{\omega}_{ab}(N, M, \alpha_s)\, \tilde{D}_b^h(M, Q^2)$$

where

$$\tilde{\omega}_{ab}(N, M, \alpha_s) \equiv \int_0^1 d\hat{x}\, \hat{x}^{N-1} \int_0^1 d\hat{z}\, \hat{z}^{M-1}\, \omega_{ab}(\hat{x}, \hat{z}, \alpha_s)$$

recall, NLO:

$$\omega_{qq}^{(1)}(\hat{x},\hat{z}) \;=\; e_q^2 C_F \Bigg[2\delta(1-\hat{x}) \left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ + 2\delta(1-\hat{z}) \left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ + \frac{2}{(1-\hat{x})_+(1-\hat{z})_+} - 8\delta(1-\hat{x})\,\delta(1-\hat{z}) \;+\; \ldots \Bigg]$$

large $\hat{x},\hat{z}$ $\leftrightarrow$ large N, M:

$$\omega_{qq}^{(1)}(N,M) \;=\; e_q^2 C_F \left[\left(\ln \bar{N} + \ln \bar{M}\right)^2 - 8 + \frac{\pi^2}{3} \;+ \ldots \right]$$

$$\bar{N} \equiv N \mathrm{e}^{\gamma_E},\; \bar{M} \equiv M \mathrm{e}^{\gamma_E}$$

k^th order: corrections

$$\alpha_s^k \left(\ln \bar{N} + \ln \bar{M}\right)^{2k} \;+ \ldots$$

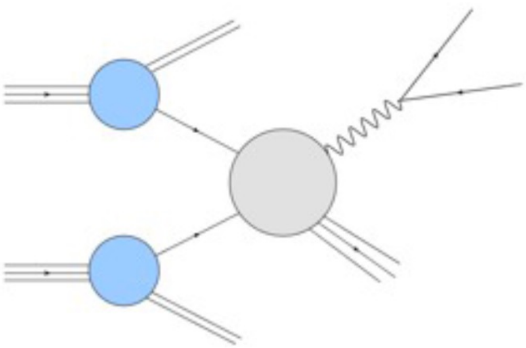
Sterman, WV; Anderle, Ringer, WV

$$\tilde{\omega}_{qq}^{\rm res}(N,M,\alpha_s)$$

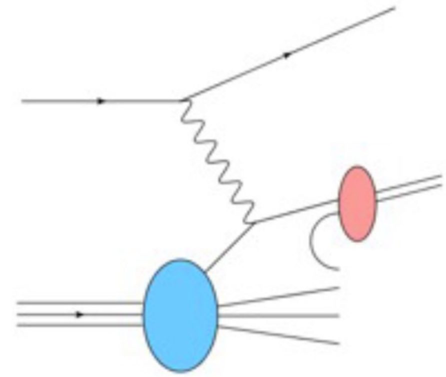
$$\propto \exp\left[\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}A_q(\alpha_s(k_\perp^2))\left\{\int_{\frac{k_\perp^2}{Q}}^{Q}\frac{dk^+}{k^+}\left[{\rm e}^{-(Nk^++Mk^-)/Q}-1\right]+\ln\bar{N}+\ln\bar{M}\right\}\right]$$

cf. inclusive Drell-Yan

SIDIS



$$N\to\sqrt{NM}$$



$$\int_{\frac{k_\perp^2}{Q}}^{Q}\frac{dk^+}{k^+}\left[{\rm e}^{-N(k^++k^-)/Q}-1\right]+2\ln\bar{N}$$

$$\approx\ 2\left[K_0\left(N\frac{2k_\perp}{Q}\right)+\ln\left(\frac{k_\perp}{Q}\bar{N}\right)\right]$$

$$\int_{\frac{k_\perp^2}{Q}}^{Q}\frac{dk^+}{k^+}\left[{\rm e}^{-(Nk^++Mk^-)/Q}-1\right]+\ln\bar{N}+\ln\bar{M}$$

$$\approx\ 2\left[K_0\left(\sqrt{NM}\,\frac{2k_\perp}{Q}\right)+\ln\left(\frac{k_\perp}{Q}\sqrt{\bar{N}\bar{M}}\right)\right]$$

- similar for Drell-Yan w/ rapidity

Full resummed formula becomes (to *all* log order!):

$$\tilde{\omega}_{qq}^{\rm res}(N,M,\alpha_s) \;=\; e_q^2\, C_{qq}(\alpha_s)$$

$$\times\, \exp\left\{\int_{Q^2/(\bar{N}\bar{M})}^{Q^2} \frac{d\mu^2}{\mu^2}\left[A_q(\alpha_s(\mu))\,\ln\left(\frac{\mu^2\bar{N}\bar{M}}{Q^2}\right) - \frac{1}{2}\widehat{D}_q(\alpha_s(\mu))\right]\right\}$$

$$C_{qq}(\alpha_s) \;=\; 1 + \frac{\alpha_s}{\pi}\, C_{qq}^{(1)}(\alpha_s) + \left(\frac{\alpha_s}{\pi}\right)^2 C_{qq}^{(2)}(\alpha_s) + \ldots$$

hard virtual corr.,
N,M-independent

$$A_q(\alpha_s) \;=\; \frac{\alpha_s}{\pi}\, A_q^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_q^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 A_q^{(3)} + \ldots$$

$$\widehat{D}_q(\alpha_s) \;=\; \left(\frac{\alpha_s}{\pi}\right)^2 \widehat{D}_q^{(2)} + \ldots$$

Kodaira, Trentadue; …
Moch, Vermaseren, Vogt;
Catani, de Florian, Grazzini

Hard factor may be determined from spacelike quark form factor:

Catani, Cieri, de Florian, Ferrera, Grazzini

quark form factor

2 loops:

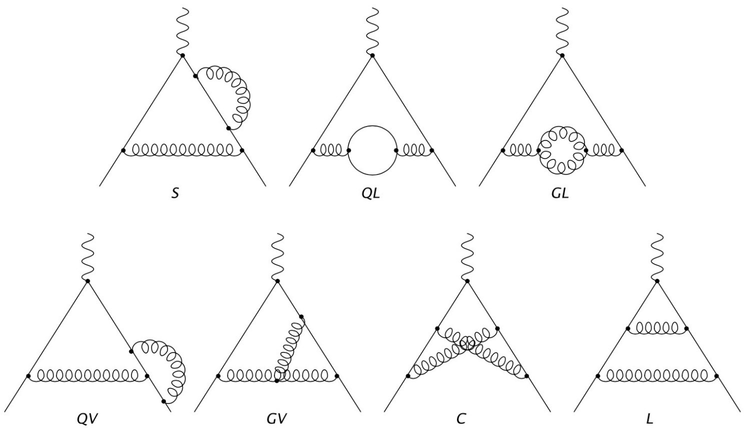
Gehrmann, Huber, Maitre
Moch, Vermaseren, Vogt

3 loops:

Gehrmann, Glover, Huber,
Ikizlerli, Studerus

(4 loops:

Lee , von Manteuffel,
Schabinger, Smirnov,
Smirnov, Steinhauser)



$$C_{qq}^{(1)} = C_F \left(-4 + \frac{\pi^2}{6} \right)$$

$$C_{qq}^{(2)} = C_F^2 \left(-\frac{15\zeta(3)}{4} - \frac{\pi^2}{16} + \frac{511}{64} - \frac{\pi^4}{60} \right) + C_F C_A \left(\frac{151\zeta(3)}{36} + \frac{7\pi^4}{720} - \frac{1535}{192} - \frac{5\pi^2}{16} \right) + C_F N_f \left(\frac{\zeta(3)}{18} + \frac{\pi^2}{24} + \frac{127}{96} \right)$$

Expansion to NNLL or N^{3}LL: $\lambda = b_0\,\alpha_s\,(\ln\bar{N} + \ln\bar{M})$

LL NLL

$$\tilde{\omega}^{\rm res}_{qq}(N,M,\alpha_s) = e_q^2\,C_{qq}(\alpha_s)\,\exp\left\{\frac{\lambda}{b_0\alpha_s}\,h_q^{(1)}(\lambda) + h_q^{(2)}(\lambda) + \alpha_s\,h_q^{(3)}(\lambda) + \alpha_s^2\,h_q^{(4)}(\lambda) + \ldots\right\}$$

NNLL N^{3}LL

$$h_q^{(1)}(\lambda) = \frac{A_q^{(1)}}{\pi b_0 \lambda}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right]$$

$$h_q^{(2)}(\lambda) = \ldots$$

$$h_q^{(3)}(\lambda) = \ldots$$

$$L \equiv \ln \bar{N} + \ln \bar{M}$$

Fixed Order

Resummation

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

LL NLL NNLL

Finally, further expansion to NNLO (and N^{3}LO):

$$\tilde{\omega}_{qq}(N,M,\alpha_s) = 1 + \frac{\alpha_s}{\pi}\,\tilde{\omega}_{qq}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \tilde{\omega}_{qq}^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 \tilde{\omega}_{qq}^{(3)} + \mathcal{O}(\alpha_s^4)$$

$$\mathcal{L} \equiv \frac{1}{2}\left(\ln(\bar{N}) + \ln(\bar{M})\right)$$

dominant subleading terms

$$\tilde{\omega}_{qq}^{(1)}(N,M) = e_q^2 C_F\left[2\mathcal{L}^2 + \frac{\pi^2}{6} - 4\right] + e_q^2 C_F \mathcal{L}\left(\frac{1}{N} + \frac{1}{M}\right)$$

$$\frac{1}{e_q^2}\tilde{\omega}_{qq}^{(2)}(N,M) = 2C_F^2\mathcal{L}^4 + \frac{4\pi b_0 C_F}{3}\mathcal{L}^3 + C_F\mathcal{L}^2\left[C_F\left(-8+\frac{\pi^2}{3}\right) + \left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{5}{9}N_f\right]$$

$$+ C_F\mathcal{L}\left[\left(\frac{101}{27} - \frac{7}{2}\zeta(3)\right)C_A - \frac{14}{27}N_f\right] + C_F^2\left[\frac{511}{64} - \frac{\pi^2}{16} - \frac{\pi^4}{60} - \frac{15}{4}\zeta(3)\right]$$

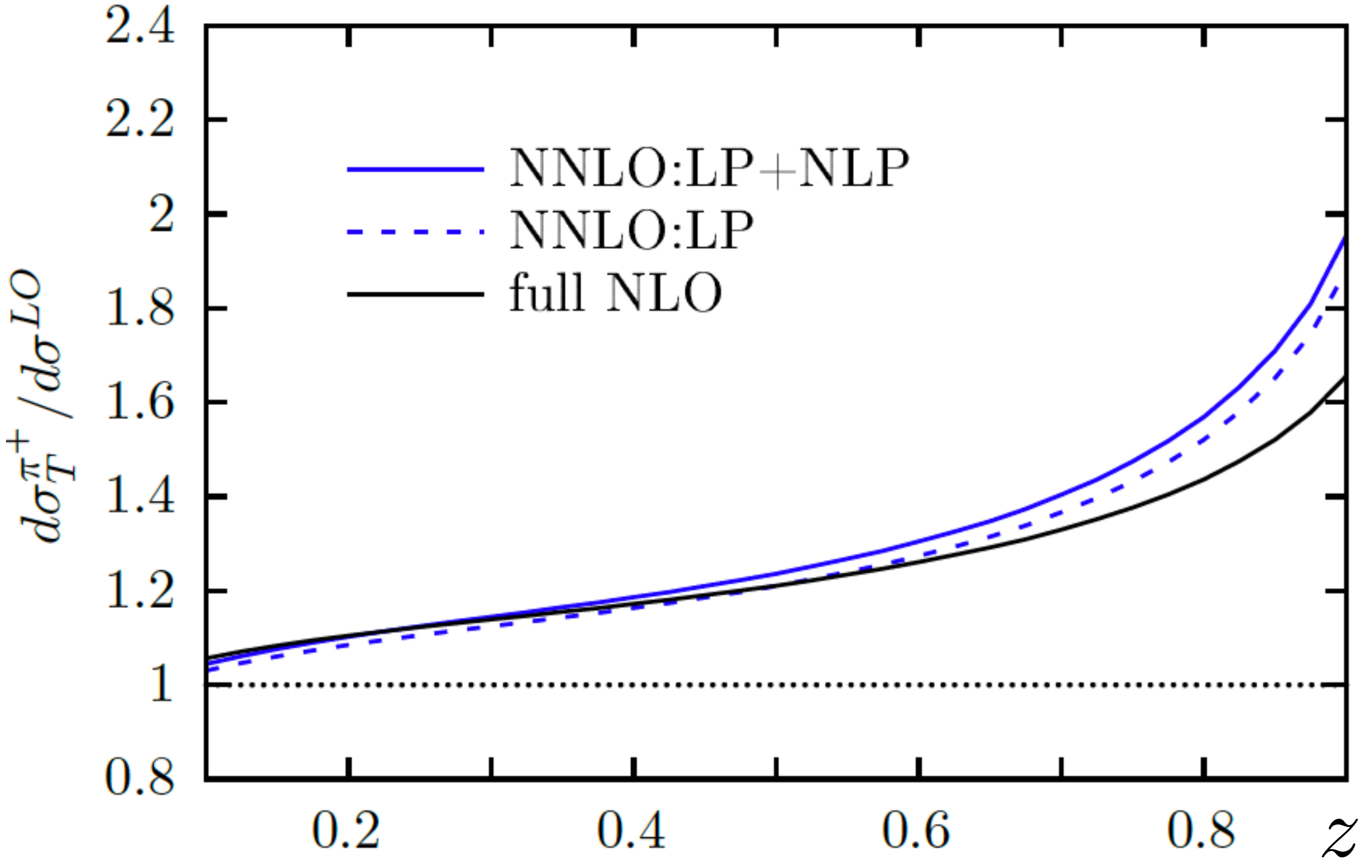
$$+ C_F C_A\left[-\frac{1535}{192} - \frac{5\pi^2}{16} + \frac{7\pi^4}{720} + \frac{151}{36}\zeta(3)\right] + C_F N_f\left[\frac{127}{96} + \frac{\pi^2}{24} + \frac{\zeta(3)}{18}\right]$$

$$+ 2\,C_F^2\,\mathcal{L}^3\left(\frac{1}{N} + \frac{1}{M}\right)$$

same for **pol.** SIDIS !

Phenomenology & FFs

EIC $e^- p \to e^- \pi^+ X$



$\sqrt{s} = 100$ GeV

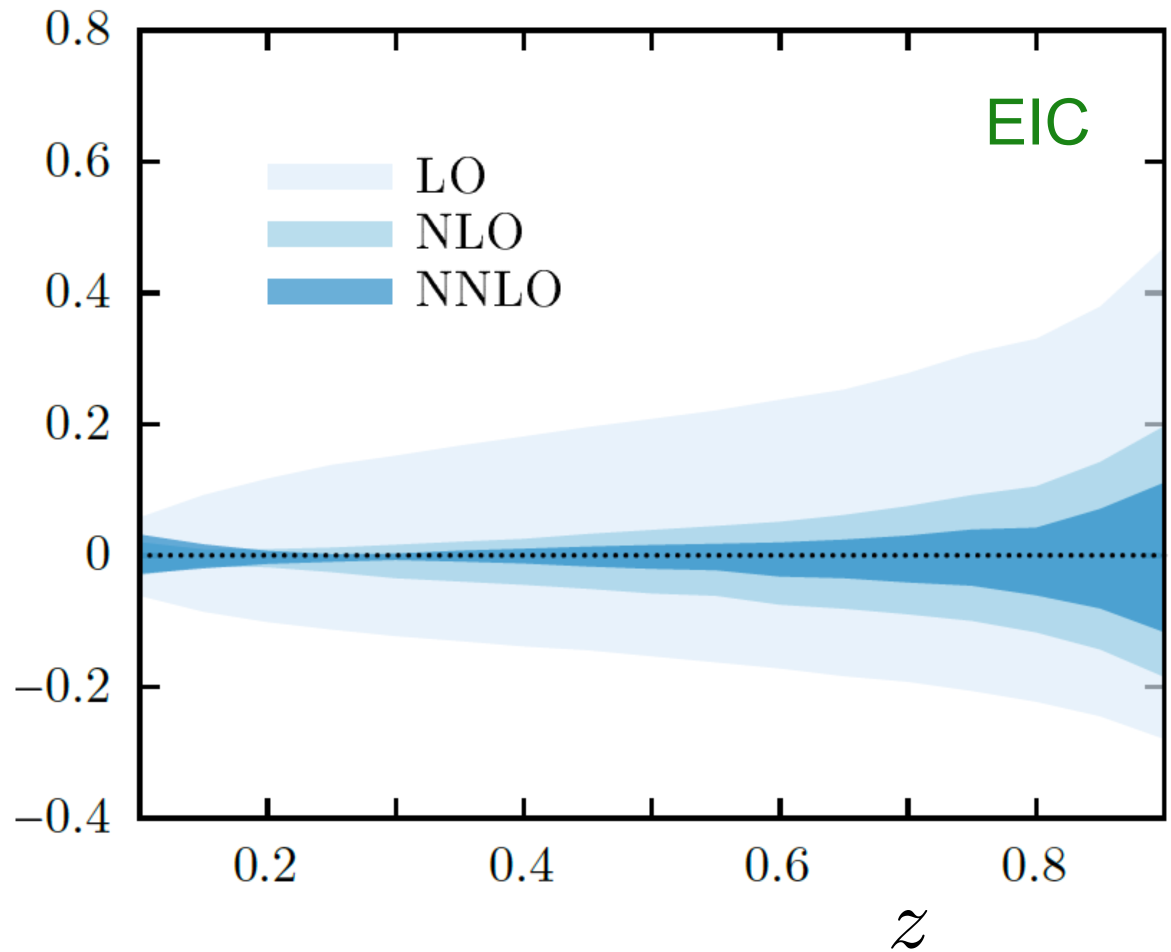
$x \in [0.1, 0.8]$

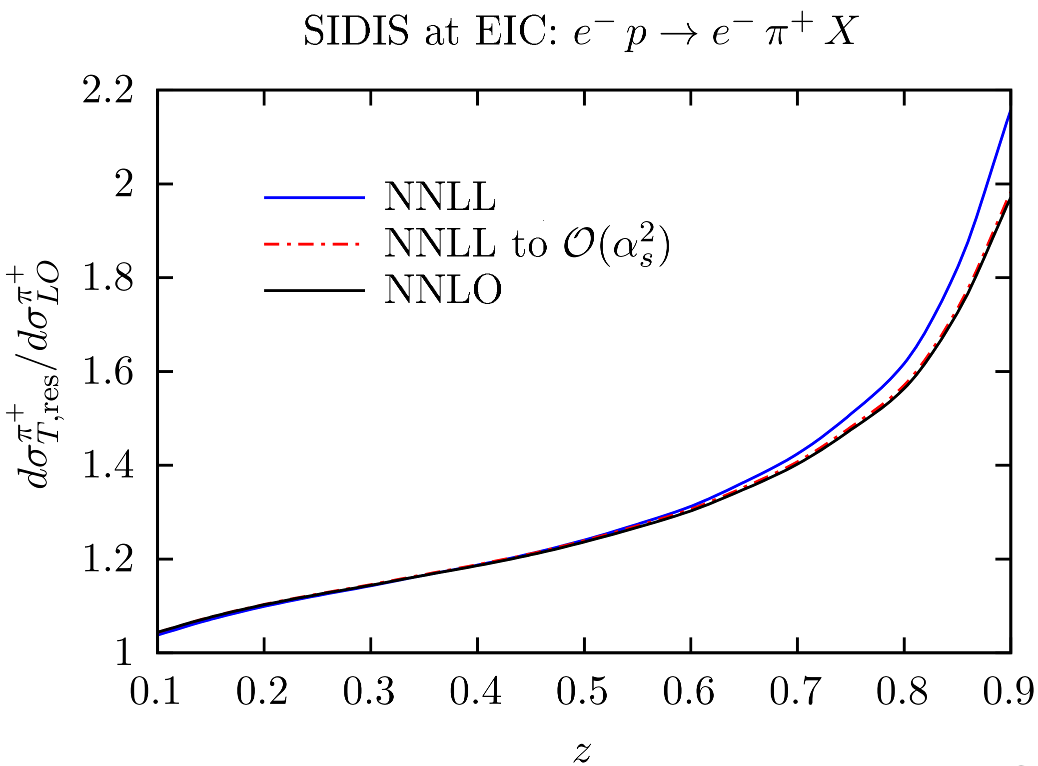
$y \in [0.1, 0.9]$

(normalized to LO cross section)

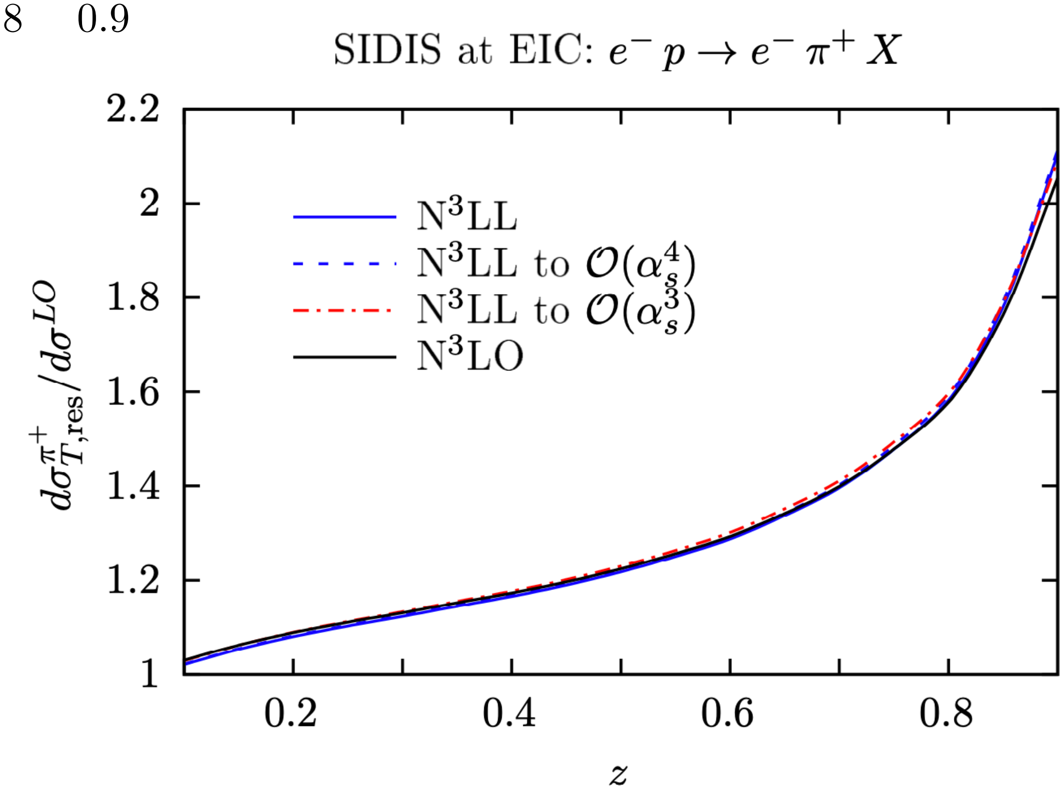
PDFs: CT18, FFs: Anderle, Kaufmann, Ringer, Stratmann scale dependence: $Q/2 \leq \mu_{R,F} \leq 2Q$

$$\frac{\sigma(\mu) - \sigma(Q)}{\sigma(Q)}$$

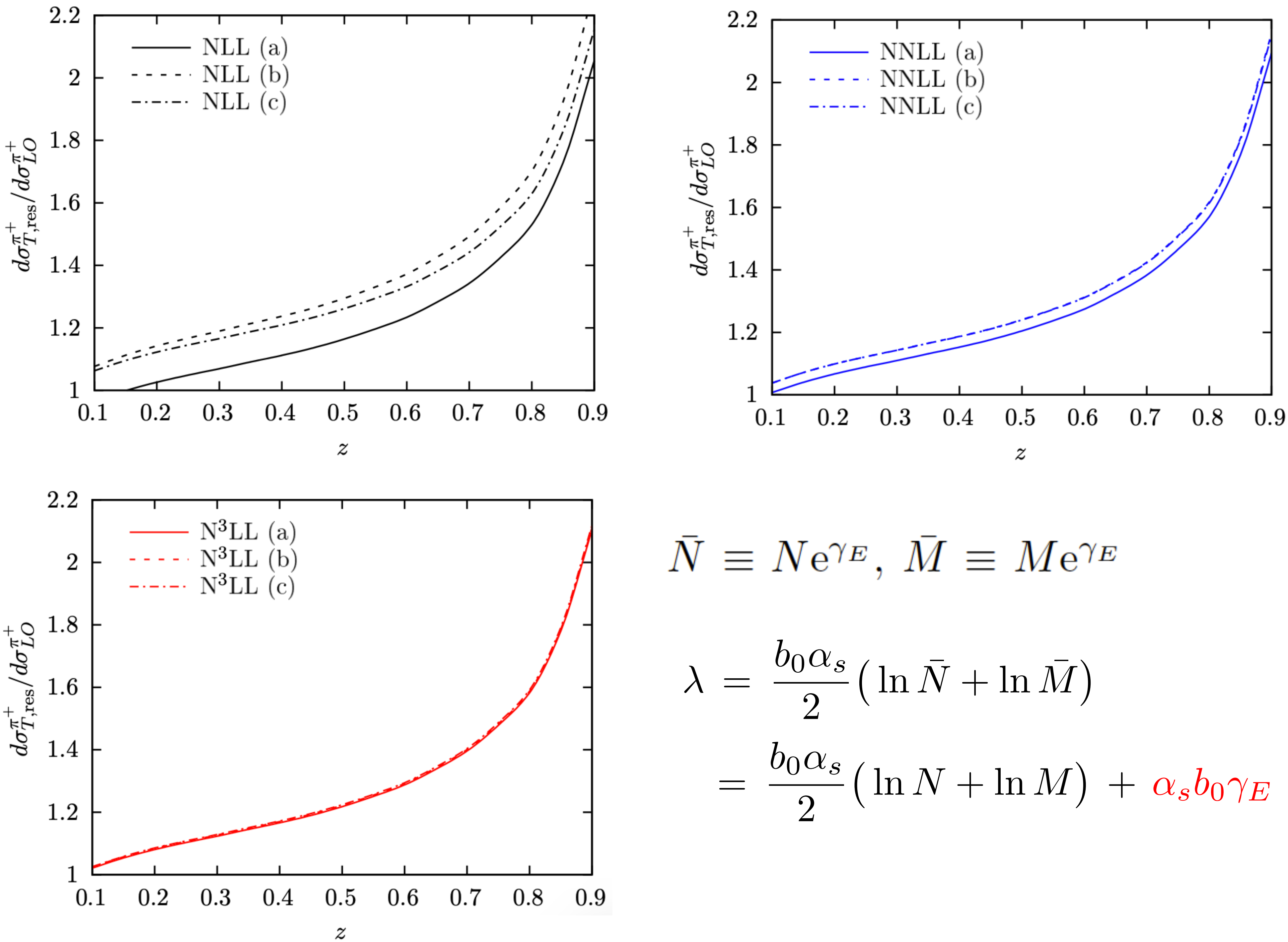




SIDIS at EIC: $e^- p \to e^- \pi^+ X$
NNLL
NNLL to $\mathcal{O}(\alpha_s^2)$
NNLO
$d\sigma^{\pi^+}_{T,\mathrm{res}}/d\sigma^{\pi^+}_{LO}$
z



SIDIS at EIC: $e^- p \to e^- \pi^+ X$
N^3LL
N^3LL to $\mathcal{O}(\alpha_s^4)$
N^3LL to $\mathcal{O}(\alpha_s^3)$
N^3LO
$d\sigma^{\pi^+}_{T,\mathrm{res}}/d\sigma^{LO}$
z



$$\bar{N} \equiv N e^{\gamma_E}, \;\; \bar{M} \equiv M e^{\gamma_E}$$

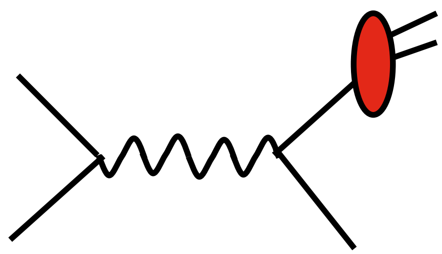
$$\lambda = \frac{b_0 \alpha_s}{2}\left(\ln \bar{N} + \ln \bar{M}\right)$$

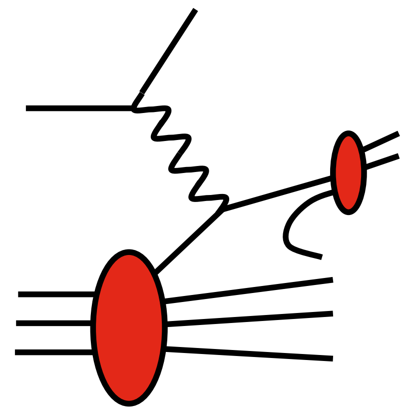
$$= \frac{b_0 \alpha_s}{2}\left(\ln N + \ln M\right) + \alpha_s b_0 \gamma_E$$

Borsa, De Florian, Sassot, Stratmann, WV

- global analysis of fragmentation functions at “nearly NNLO”

$$e^+e^- \to hX, \quad ep \to hX$$



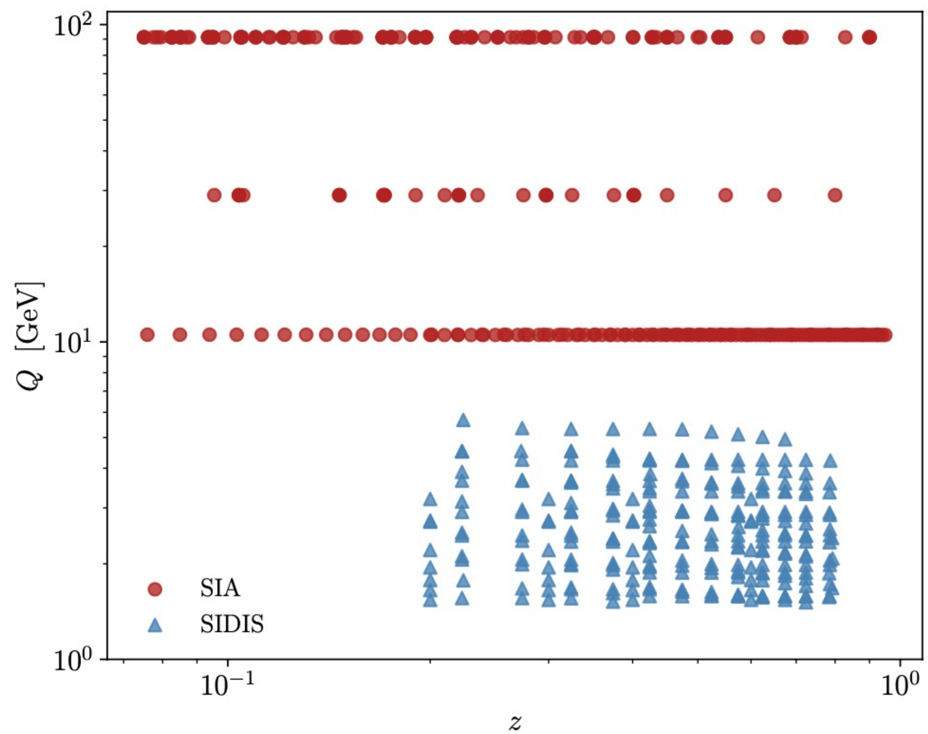


(recently also MAP (Khalek, Bertone, Khoudli, Nocera))

- NNLO for e^+e^- : Anderle, Ringer, Stratmann
  Abdolmaleki et al.
  Salajeghhej, Kniehl, et al.

EXPERIMENTAL DATA

experiment		data type
TPC	29 GeV	incl.
		uds, c, b tag
SLD	91.2 GeV	incl.
		uds, c, b tag
ALEPH	91.2 GeV	incl.
DELPHI	91.2 GeV	incl.
		uds, b tag
OPAL	91.2 GeV	incl.
		u, d, s, c, b tag
BABAR	10.54 GeV	incl.
BELLE	10.52 GeV	incl.
SIA data (sum)		
HERMES	π^+,π^-	(p-Q^2)
	π^+,π^-	(d-Q^2)
	π^+,π^-	(p-x)
	π^+,π^-	(d-x)
COMPASS	π^+,π^-	(d-z)
SIDIS data (sum)		

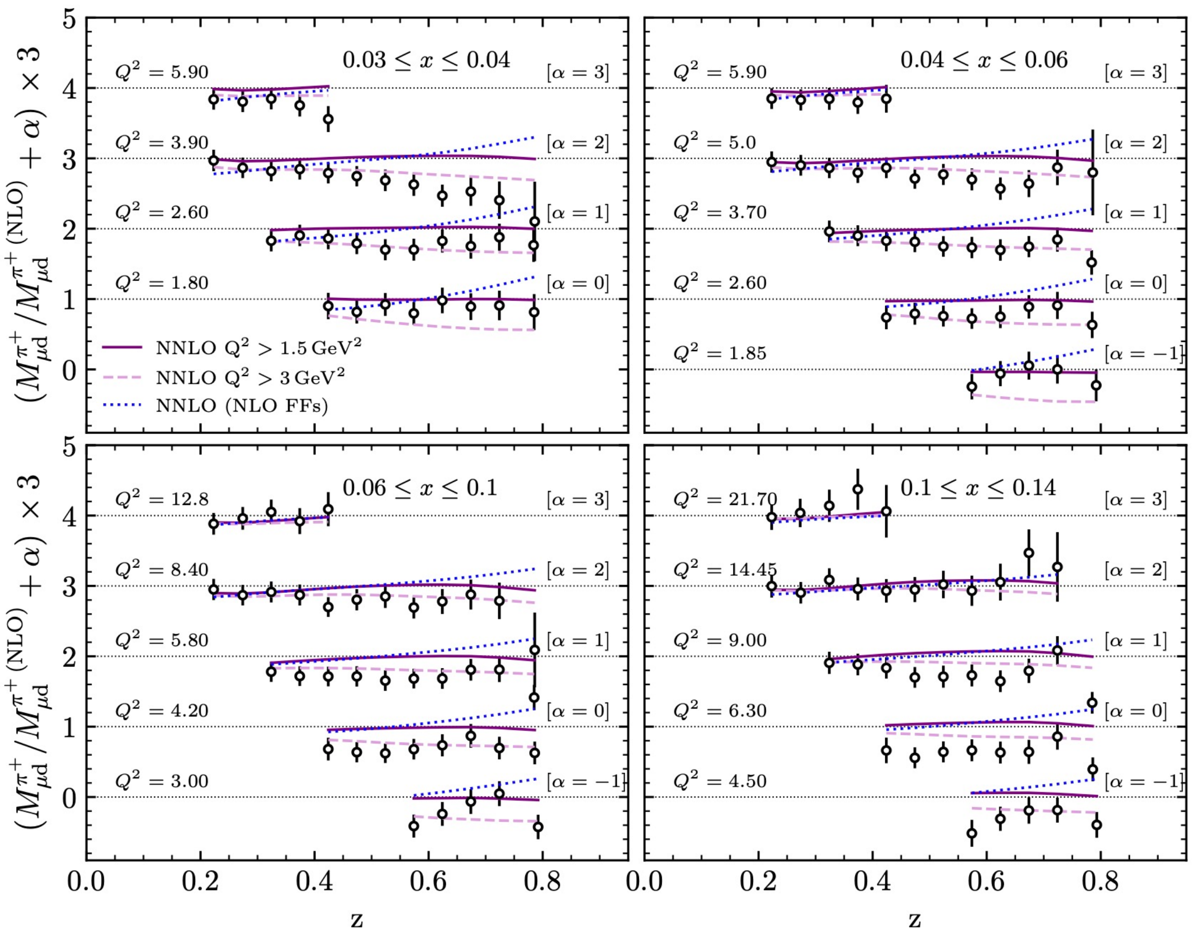


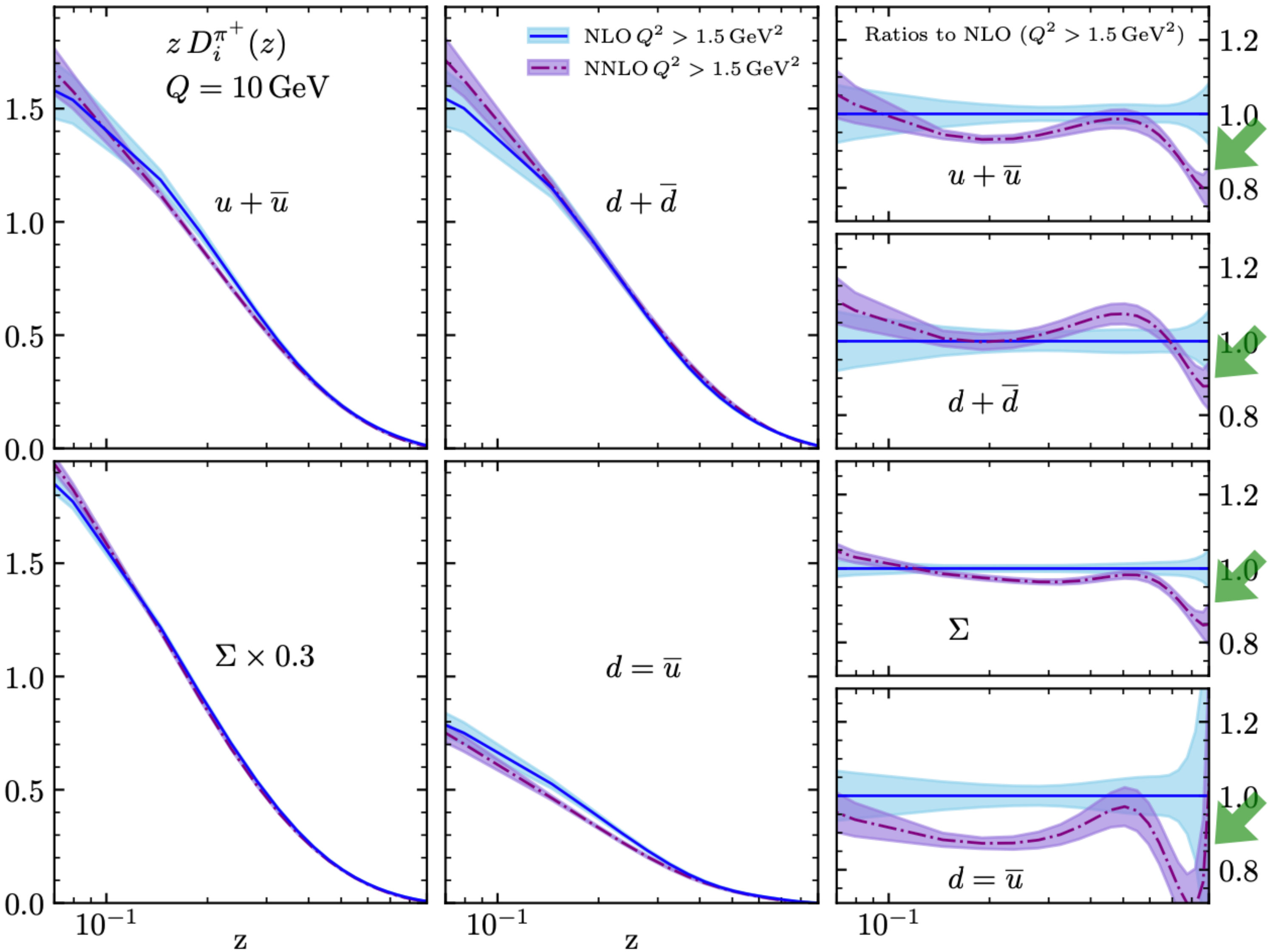
$0.075 < z_{\mathrm{SIA}} < 0.95$

$Q^2_{\mathrm{SIDIS}} > 1.5\,\mathrm{GeV}^2$

χ^2/#data points

Experiment	$Q^2 \geq 1.5\,\mathrm{GeV}^2$			$Q^2 \geq 2.0\,\mathrm{GeV}^2$			$Q^2 \geq 2.3\,\mathrm{GeV}^2$			$Q^2 \geq 3.0\,\mathrm{GeV}^2$		
	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO
SIA	288	1.05	0.96	288	0.91	0.87	288	0.90	0.91	288	0.93	0.86
COMPASS	510	0.98	1.14	456	0.91	1.04	446	0.91	0.92	376	0.94	0.93
HERMES	224	2.24	2.27	160	2.40	2.08	128	2.71	2.35	96	2.75	2.26
TOTAL	1022	1.27	1.33	904	1.17	1.17	862	1.17	1.13	760	1.16	1.07

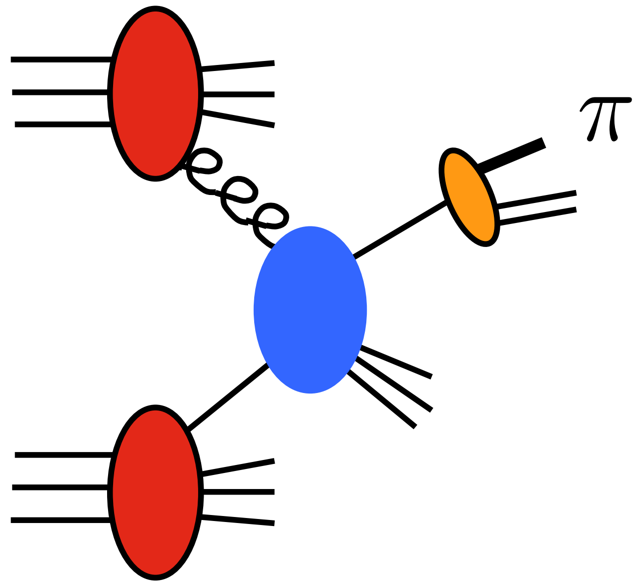




$z\,D_i^{\pi^+}(z)$
$Q = 10\,\mathrm{GeV}$
NLO $Q^2 > 1.5\,\mathrm{GeV}^2$
NNLO $Q^2 > 1.5\,\mathrm{GeV}^2$
Ratios to NLO ($Q^2 > 1.5\,\mathrm{GeV}^2$)
$u + \overline{u}$
$d + \overline{d}$
$\Sigma \times 0.3$
$d = \overline{u}$
Σ
0.0
0.5
1.0
1.5
0.8
1.0
1.2
10^{-1}
z

pp → π+X

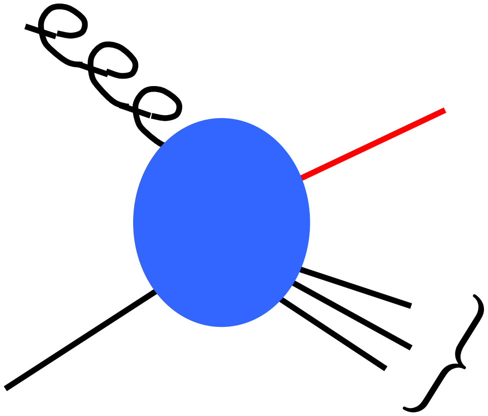
Hadronic high-p_T scattering:



$$\frac{p_T^3 d\sigma}{dp_T d\eta} = \sum_{abc} \int_0^1 dx_a \, dx_b \, dz_c \; f_a(x_a) \, f_b(x_b) \, z_c^2 D_c^{\pi}(z_c)$$

$$\times \, \Omega_{ab \to cX} \left(\hat{x}_T^2, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{\hat{s}} \right)$$

partonic variables: $\hat{x}_T = \dfrac{2p_T}{z_c \sqrt{\hat{s}}}$ $\quad \hat{\eta} = \eta - \dfrac{1}{2} \ln \dfrac{x_a}{x_b}$

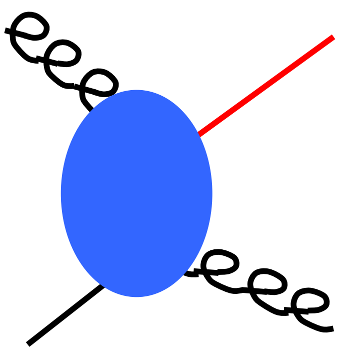


mass^2 $\equiv s_4$

$$\zeta \equiv 1 - \frac{s_4}{\hat{s}}$$

$$= \hat{x}_T \cosh \hat{\eta}$$

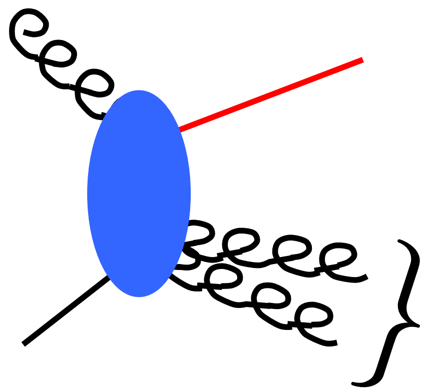
LO:



$$\hat{s}_4 = 0 \quad \Leftrightarrow \quad \zeta = 1$$

$$\Omega^{(\mathrm{LO})}_{ab\to cX}(\zeta, \hat{\eta}) = \delta(1-\zeta)\, \omega^{(0)}_{ab\to cd}(\hat{\eta})$$

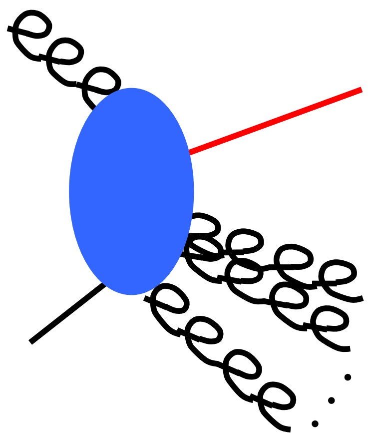
NLO:



$\zeta \neq 1$

$$\alpha_s \left(\frac{\log(1-\zeta)}{1-\zeta} \right)_+ + \ldots$$

N^{k}LO:

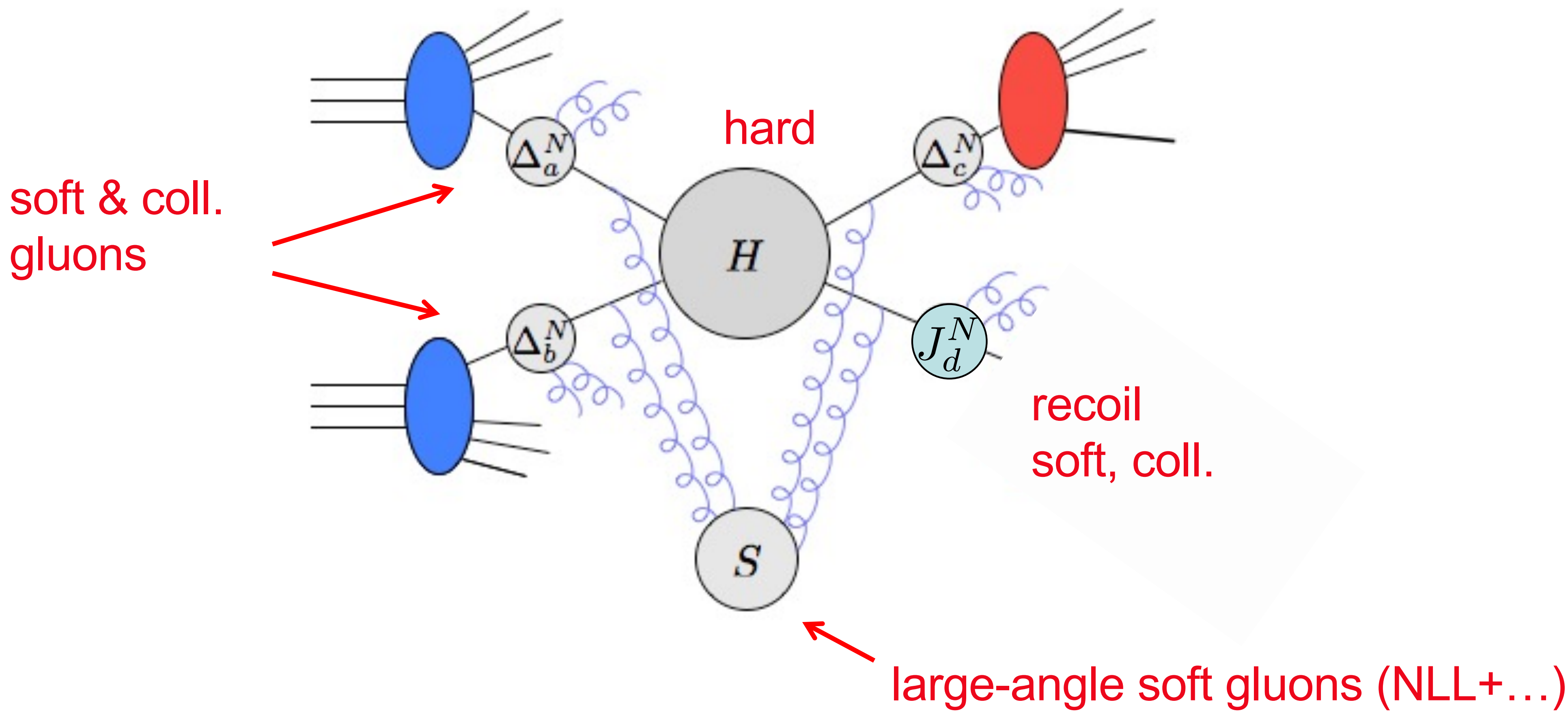


$$\alpha_s^k \left(\frac{\log^{2k-1}(1-\zeta)}{1-\zeta} \right)_+ + \ldots$$

- resummation formulated in terms of suitable Mellin moments:

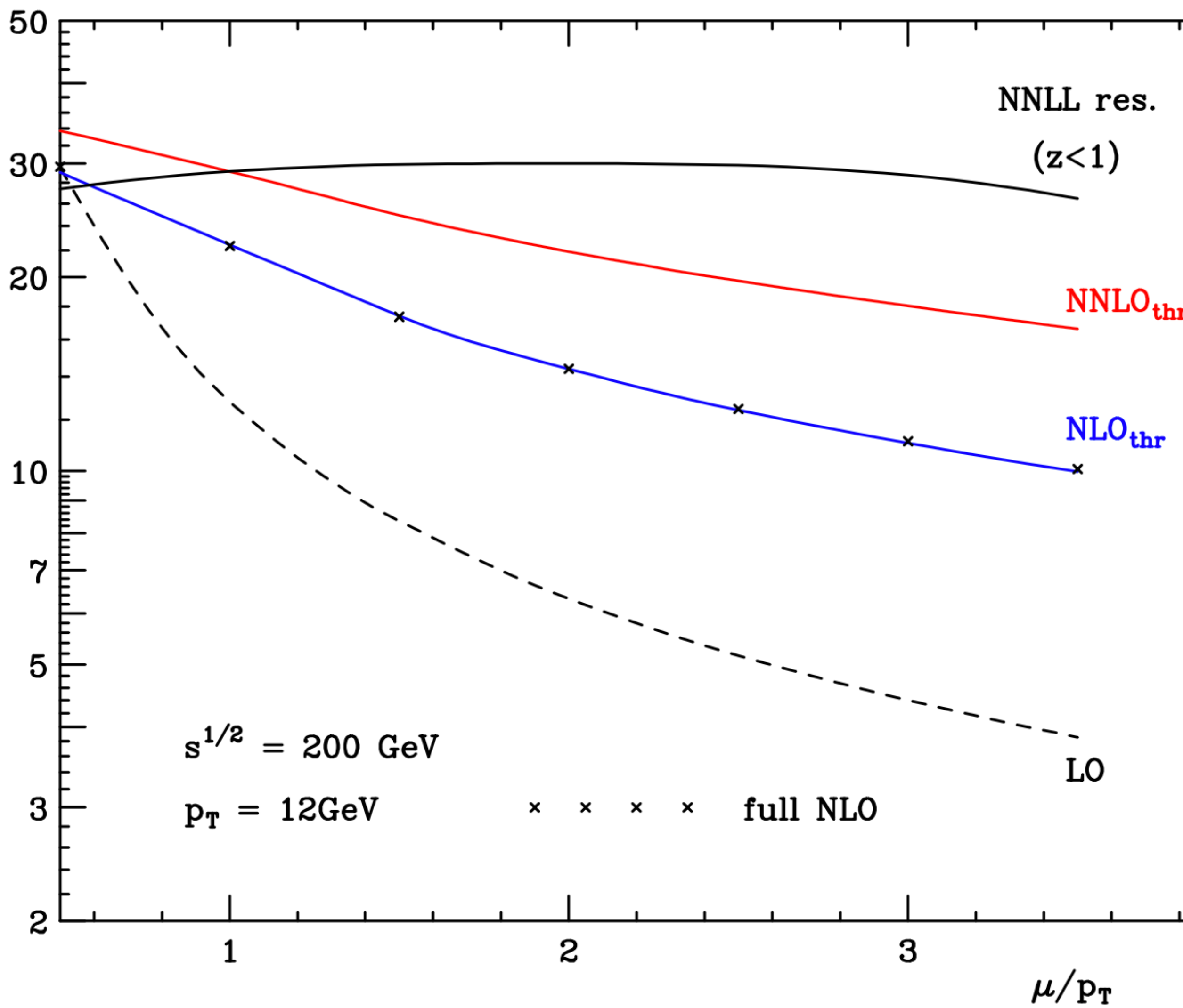
$$\tilde{\Omega}_{ab\to cX}\left(N, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{\hat{s}}\right) \equiv \int_0^1 d\zeta\, \zeta^{N-1}\, \Omega_{ab\to cX}\left(\zeta, \hat{\eta}, \alpha_s(\mu), \frac{\mu^2}{\hat{s}}\right)$$

near threshold:



becomes matrix problem in color space: $\sum_{IL} H_{IL}\, S_{LI}$

Kidonakis, Oderda, Sterman
Mert Aybat, Dixon, Sterman
Bonciani, Catani, Mangano, Nason
Hinderer, Ringer, Sterman, WV
Broggio et al., Balytskyi, Gao



Hinderer, Ringer, Sterman, WV

Conclusions:

- SIDIS: reduction in scale dependence

- early NNLO phenomenology of FFs

- benchmark for future NNLO calculations

- not the full story: further extensions
  F_L, qg contribution, 1/N corrections, power corrections

- polarized case: same near-threshold corrections → PDFs

- HO corrections harder to control for pp → π+X