Factorization for Quasi-TMD correlators of sub-leading twist

Alexey Vladimirov

INT-program 2022: Parton Distributions and Nucleon Structure

IPARCOS
Comunidad de Madrid

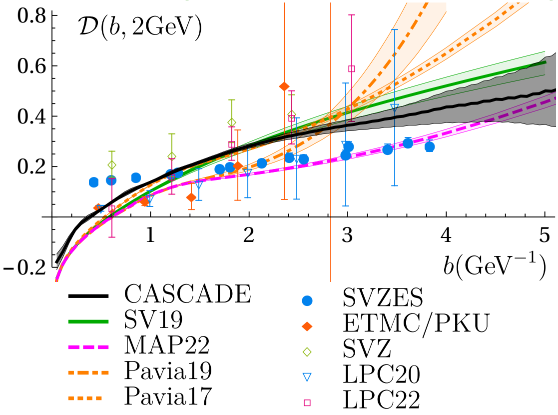
This work has been done mainly by **Simone Rodini**.
It is not yet complete but main results are ready.

Outline

- Motivation
- Quasi-TMD correlators
- TMD Factorization in the background field approach
- Factorization for qTMDs at next-to-leading power

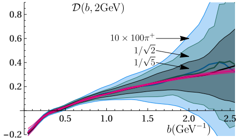
Collins-Soper kernel

[A. Bermudez Martinez, AV,2206.01105]

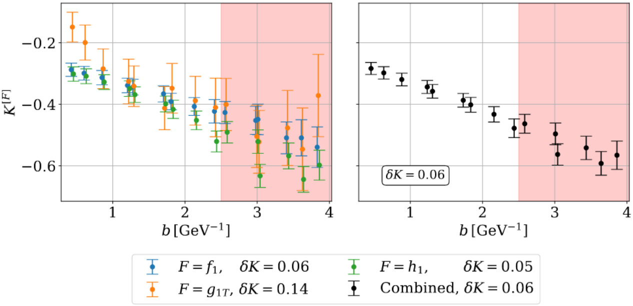


- Direct fits
  - Model bias
- Lattice
  - Huge systematics
- Direct extraction
  - Poor data (?)

precision at EIC



SVZES [=2103.16991] lattice computation used 1-moments of TMDs to extract CS kernel.
It has been done using three TMDs (f_1, h_1, g_{1T}).



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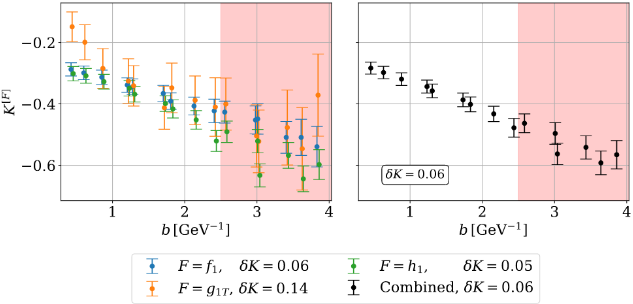

However, the actual lattice computation was done for three more TMDs ($f_{1T}^{\perp}$, $h_1^{\perp}$, $h_{1T}^{\perp}$), but they were excluded from the study because too large lattice artifacts.

In fact, the method used it SVZES [B.Musch, et al, 1111.4249]=MHENS simultaneously determines many other qTMDs.

$$W^{[\Gamma]} = \langle P, S|\bar{q}(y)[staple]\frac{\Gamma}{2}q(0)|P, S\rangle$$

used: $\Gamma_+ = \{\gamma^+, \gamma^+\gamma^5, \sigma^{\alpha+}\}$
computed but not-used: $\Gamma_T = \{1, \gamma^5, \gamma_T^\mu, \gamma_T^\mu\gamma^5, \sigma^{+-}, \sigma^{\mu\nu}\}$
These are qTMDs of higher twist/sub-leading counting/???

Some of them have quite good signal/noise ratio
can we extract any information from them?

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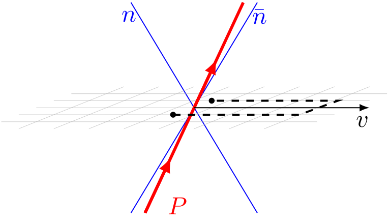
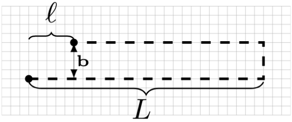
Answer: Yes & No
Some of them can be used to extract CS-kernel (and TMDs?), some NOT.





$$W^{[\Gamma]}(y) = \langle P,S|\bar{q}(y)[staple]\frac{\Gamma}{2}q(0)|P,S\rangle$$

Theory assumptions

- $L \gg b, \ell$
- $(vP) \gg \Lambda, M$
- $(bv) = (bP) = 0$

TMD factorization

- $\{\ell, b\} \lesssim \{1, \lambda^{-1}\}$

$$\lambda \sim M/(vP)$$

Alike hadron tensor with "instant"-to-light current

$$W^{[\Gamma]}(y;P,S,v;L) = \langle P,S|J^{\dagger}(y)\frac{\Gamma}{2}J(0)|P,S\rangle$$

$$J(y;v,L) = [\infty_T + vL, vL+y][vL+y,y]q(y)$$

($L \sim \infty$ i.e. we ignore effects due to it, but divergence is regulated)

SCET

- Modes by method of regions
- Effective action
- Overlap of modes
- Dim.reg.+rap.reg.
- ...

TMD operator expansion

- Modes by parton model
- Background QCD
- Overlap of modes
- Dim.reg.+rap.reg.
- ...

High-energy expansion [Balitsky, et al]

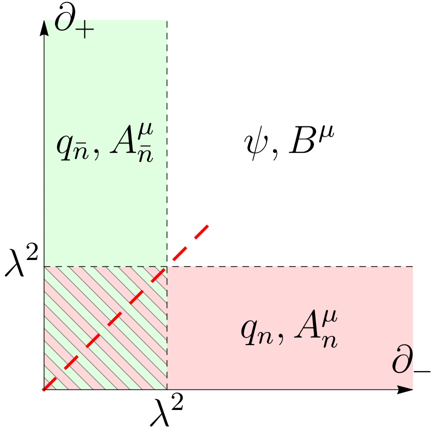
- Modes by parton model
- Background QCD
- No-overlap (?)
- Cut reg.
- ...

Collinear fields (hadron)

$$\{\partial_+, \partial_-, \partial_T\}\, q_{\bar n} \lesssim Q\{1, \lambda^2, \lambda\}\, q_{\bar n},$$
$$\{\partial_+, \partial_-, \partial_T\}\, A^\mu_{\bar n} \lesssim Q\{1, \lambda^2, \lambda\}\, A^\mu_{\bar n},$$

Anti-Collinear fields
(to tame the NP part of contour)

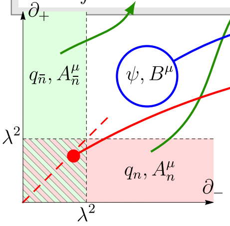
$$\{\partial_+, \partial_-, \partial_T\}\, q_n \lesssim Q\{\lambda^2, 1, \lambda\}\, q_n,$$
$$\{\partial_+, \partial_-, \partial_T\}\, A^\mu_n \lesssim Q\{\lambda^2, 1, \lambda\}\, A^\mu_n.$$



$$W^{[\Gamma]} = \int [D\bar{q} Dq DA] e^{iS_{\rm QCD}} \Psi^*(P,S) J^\dagger(y)\frac{\Gamma}{2}J(0)\Psi(P,S),$$

$$q=\psi+q_{\bar n}+q_n, \qquad A^\mu = B^\mu + A^\mu_{\bar n}+A^\mu_n,$$

$$W^{[\Gamma]} = \int [D\bar{q}_{\bar n} Dq_{\bar n} DA_{\bar n}][D\bar{q}_n Dq_n DA_n] e^{iS_{\rm QCD}[q_{\bar n},A_{\bar n}]+iS_{\rm QCD}[q_n,A_n]}\Psi^*(P,S)\frac{\mathcal{J}^{[\Gamma]}_{eff}(y)}{S(y)}\Psi(P,S)$$

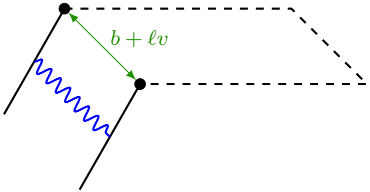


$$\mathcal{J}^{[\Gamma]}_{eff}(y) = \int [D\bar{\psi} D\psi DB] e^{iS_{\rm back.}}\left(J^\dagger(y)\frac{\Gamma}{2}J(0)\right)[\psi+q_{\bar n}+q_n, B+A_{\bar n}+A_n]$$

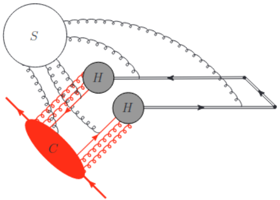
Making life simpler

- Light-cone gauge for background
- Background+Feynman gauge for “fast” components
- Ignore renormalization of staple contour (multiplicative)

Main simplification



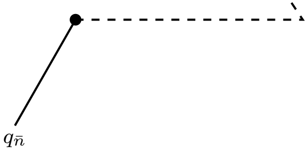
$$\sim \frac{1}{b^2} \sim \lambda^2$$



picture from [1911.03840]

- Exchange diagrams are (at least) NNLP
- The LP and NLP are given by the current factorization (like in ordinary TMD)

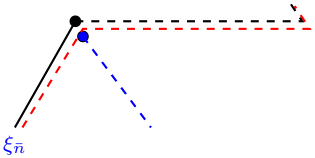
[Ebert, Stewart, Zhao, 19]
[Ji, Liu, Liu, 19]



EOMs fix counting for components

$$q_{\bar n} = \underbrace{\xi_{\bar n}}_{\sim 1} + \underbrace{\eta_{\bar n}}_{\sim \lambda}$$

$$\xi_{\bar n} = \frac{\gamma^- \gamma^+}{2} q_{\bar n}, \qquad \eta_{\bar n} = \frac{\gamma^+ \gamma^-}{2} q_{\bar n}$$



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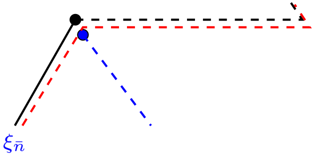
$$J(0) = V_v^\dagger(0)\xi_{\bar n}(0) + ...$$

$$\mathcal{W}_{\text{eff}}^{[\Gamma]} = J^\dagger(\ell v + b)\frac{\Gamma}{2}J(0) = \frac{1}{N_c}\ \bar\xi_{\bar n}(\ell v^- + b)\frac{\Gamma}{2}\xi_{\bar n}(0)\ \text{tr}(V^\dagger(0)V(\ell v^+ + b)) + ...$$



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Γ can be only $\Gamma \in \Gamma_+ = \{\gamma^+, \gamma^+\gamma^5, i\sigma^{\alpha+}\gamma^5\}$
For $\Gamma \in \Gamma_T = \{1, \gamma^5, \gamma^\alpha, \gamma^\alpha\gamma^5, \sigma^{\alpha\beta}, i\sigma^{+-}\gamma^5\}$ we should consider contributions $\sim \lambda$

Factorization theorem for leading-counting qTMDs

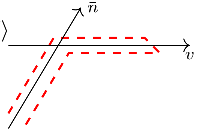
$$W^{[\Gamma]}(\ell,b;S,P;v,L,\mu)=\frac{\langle P,S|\mathcal{W}^{[\Gamma]}_{\text{eff}}|P,S\rangle}{S_{\text{TMD}}(b)}=\underbrace{\Phi^{[\Gamma]}(\ell,b;S,P,\mu,\zeta)}_{\text{TMD}}\underbrace{\Psi(b;v,L;\mu,\bar{\zeta})}_{\text{“instant-jet”}}$$

$$+\mathcal{O}\Big(\frac{M}{P_+},\frac{1}{bP^+},\frac{b}{L},\frac{\ell}{L},\ell\Lambda_{\text{QCD}}\Big)$$

NP functions

$$\Phi^{[\Gamma]}(\ell,b;S,P;\mu,\zeta)=\frac{1}{\sqrt{S_{\text{TMD}}(b)}}\langle P,S|\bar{q}(\ell n+b)[..]\frac{\Gamma}{2}[..]q(0)|P,S\rangle$$

$$\Psi(b;v,L;\mu,\zeta)=\frac{1}{\sqrt{S_{\text{TMD}}(b)}}\frac{1}{N_c}\langle 0|\text{tr}[\text{contour}]|0\rangle$$



$$\zeta\bar{\zeta}=(2(\hat{p}v))^2\mu^2$$

Factorization theorem for leading-counting qTMDs

$$W^{[\Gamma]}(\ell,b;S,P;v,L,\mu)=\frac{\langle P,S|\mathcal{W}^{[\Gamma]}_{\text{eff}}|P,S\rangle}{S_{\text{TMD}}(b)}=|C_H|^2\underbrace{\Phi^{[\Gamma]}(\ell,b;S,P,\mu,\zeta)}_{\text{TMD}}\underbrace{\Psi(b;v,L;\mu,\bar\zeta)}_{\text{“instant-jet”}}$$

$$+\mathcal{O}\Big(\frac{M}{P_+},\frac{1}{bP^+},\frac{b}{L},\frac{\ell}{L},\ell\Lambda_{\text{QCD}}\Big)$$

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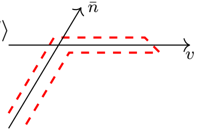
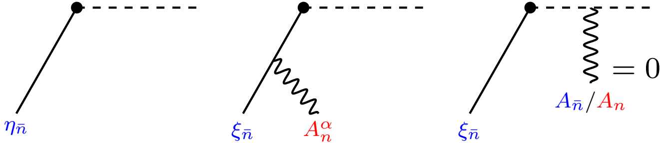


$$\zeta\bar\zeta=(2(\hat p v))^2\mu^2$$

At NLO

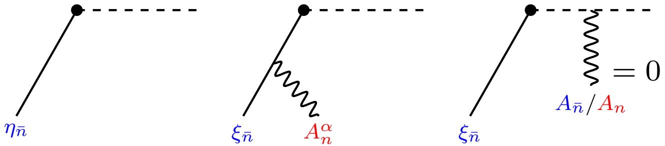
$$|C_H|^2=1+a_sC_F\left(-\mathbf{L}^2+2\mathbf{L}-4+\frac{\pi^2}{6}\right)+a_s^2...$$

[Ebert, Stewart, Zhao, 19]
[Ji, Liu, Liu, 19]
[Schafer, AV, 20]



similar to H-to-L current [Beneke, et al,03]

$$J = V_v^\dagger(0)\xi_{\bar n}(0) - \frac{\gamma^+}{\partial_+}\gamma_\alpha D_T^\alpha[A_n + A_{\bar n}]\xi_n + ...$$



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$$\begin{aligned}
\langle \mathcal{W}_{\text{eff}}^{[\Gamma]}\rangle \quad = \quad & \Phi_{11}^{[\Gamma+]}\Psi \\
& + \frac{i}{2\partial_+}\frac{\partial}{\partial b^\mu}\Phi_{11}^{[\gamma^\mu\gamma^+\Gamma + \Gamma\gamma^+\gamma^\mu]}\Psi \\
& + \frac{i}{2\partial_+}(\Phi_{21}^{\mu[\gamma_\mu\gamma^+\Gamma]} + \Phi_{12}^{\mu[\Gamma\gamma^+\gamma_\mu]})\Psi \\
& - \frac{i}{2\partial_+}\Phi_{11}^{[\gamma^\mu\gamma^+\Gamma + \Gamma\gamma^+\gamma^\mu]}\Psi_\mu + ...
\end{aligned}$$

The $\sim \lambda$ terms are non-vanishing for $\Gamma \in \Gamma_T$

LP = TMD-twist-(1,1)

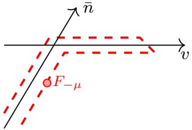
$$\Phi_{11}^{[\Gamma]}(\ell,b) = \langle p,s|\underbrace{\bar{q}[\ell n+b,\infty]}_{\text{tw-1}}\frac{\Gamma}{2}\underbrace{[\infty,0]q}_{\text{tw-1}}|p,s\rangle$$

NLP = TMD-twist-(2,1) & (1,2)

$$\Phi_{21}^{[\Gamma]}(\ell,b) = \int_{-\infty}^{\ell} d\tau \langle p,s|\underbrace{\bar{q}[\ell n+b,\tau n+b]F_{\mu+}[..,\infty]}_{\text{tw-2}}\frac{\Gamma}{2}\underbrace{[\infty,0]q}_{\text{tw-1}}|p,s\rangle$$

$$\Phi_{12}^{[\Gamma]}(\ell,b) = \int_{-\infty}^{0} d\tau \langle p,s|\underbrace{\bar{q}[\ell n+b,\infty]}_{\text{tw-1}}\frac{\Gamma}{2}\underbrace{[\infty,\tau n]F_{\mu+}[\tau n,0]q}_{\text{tw-2}}|p,s\rangle$$

$$\begin{aligned}
\Psi(b) &= \frac{1}{N_c}\langle 0|\mathrm{tr}[\mathrm{contour}]|0\rangle + \mathcal{O}\left(\frac{\ell}{L},\ell\Lambda\right) \\
\Psi^{\mu}(b) &= \frac{1}{N_c}\int_{-\infty}^{0} d\tau \langle 0|\mathrm{tr}[\mathrm{contour}]F^{-\mu}(\tau\bar{n}) + *|0\rangle \\
&\quad + \mathcal{O}\left(\frac{\ell}{L},\ell\Lambda\right)
\end{aligned}$$



TMD distributions of twist-3 $\Rightarrow$ [S.Rodini, AV, 2204.03856]

Some points

- Functions of $(x_1, x_2, x_3, b; \mu, \zeta)$ $[x_1 + x_2 + x_3 = 0]$
- There are 32 TMD distributions (quarks) $\bullet = \oplus, \ominus$

	U	L	$\mathrm{T}_{J=0}$	$\mathrm{T}_{J=1}$	$\mathrm{T}_{J=2}$
U	$f_\bullet^\perp$	$g_\bullet^\perp$		$h_\bullet$	$h_\bullet^\perp$
L	$f_{\bullet L}^\perp$	$g_{\bullet L}^\perp$	$h_{\bullet L}$		$h_{\bullet L}^\perp$
T	$f_{\bullet T}$, $f_{\bullet T}^\perp$	$g_{\bullet T}$, $g_{\bullet T}^\perp$	$h_{\bullet T}^{D\perp}$	$h_{\bullet T}^{A\perp}$	$h_{\bullet T}^{S\perp}$, $h_{\bullet T}^{T\perp}$

16 T-odd, 16 T-even

- UV evolution at LO
  - Mixture between $\oplus$ and $\ominus$ distributions
- Rapidity evolution = Rapidity evolution of twist-2 TMDs

$$\zeta \frac{d}{d\zeta} \Phi_{\text{tw3}} = -\mathcal{D} \Phi_{\text{tw3}}$$

- There is an (integrable) discontinuity at $x_i = 0$ lines $\Rightarrow$ spurious rap.divergences

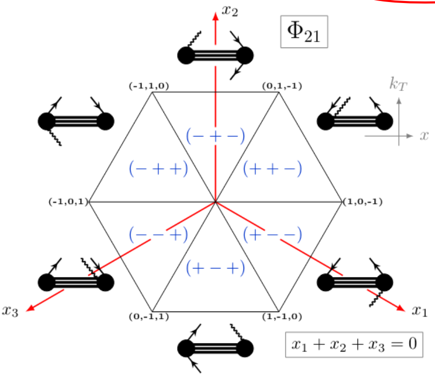
TMD-twist-3 distributions are function of 3 momentum fractions

$$\widetilde{\Phi}_{11}^{[\Gamma]}(z_1,z_2,b) = p^+\int_{-1}^{1} dx e^{ix(z_1-z_2)p^+}\Phi_{11}^{[\Gamma]}(x,b),$$

$$\widetilde{\Phi}_{\mu,21}^{[\Gamma]}(z_1,z_2,z_3,b) = (p^+)^2\int [dx] e^{-i(x_1z_1+x_2z_2+x_3z_3)p^+}\Phi_{\mu,21}^{[\Gamma]}(x_1,x_2,x_3,b),$$

$$\widetilde{\Phi}_{\mu,12}^{[\Gamma]}(z_1,z_2,z_3,b) = (p^+)^2\int [dx] e^{-i(x_1z_1+x_2z_2+x_3z_3)p^+}\Phi_{\mu,12}^{[\Gamma]}(x_1,x_2,x_3,b),$$

$$\boxed{\int [dx] = \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1} dx_3\, \delta(x_1+x_2+x_3)}$$



Support domain $|x_i| < 1$
momentum-fractions
could be **positive or negative**

- important for divergences-cancelation
- agreement with collinear evolution
- evolution mixture

Example equation ($\mathbb{P}^A$-case)

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} \Phi_\oplus \\ \Phi_\ominus \end{pmatrix} = \left(\frac{\Gamma_{\text{cusp}}}{2} \ln\left(\frac{\mu^2}{\zeta}\right) + \Upsilon_{x_1x_2x_3} \right) \begin{pmatrix} \Phi_\oplus \\ \Phi_\ominus \end{pmatrix} + \begin{pmatrix} 2\mathbb{P}^A_{x_2x_1} & 2\pi s \Theta_{x_1x_2x_3} \\ -2\pi s \Theta_{x_1x_2x_3} & 2\mathbb{P}^A_{x_2x_1} \end{pmatrix} \begin{pmatrix} \Phi_\oplus \\ \Phi_\ominus \end{pmatrix},$$

- $\begin{pmatrix} \Phi_\oplus \\ \Phi_\ominus \end{pmatrix}$'s is a pair twist-3 TMDs e.g $\begin{pmatrix} h_{\oplus,L} \\ h_{\ominus,L} \end{pmatrix}$.
- $s = \pm$ depending on DY/SIDIS definition

$\mathbb{P}$ is a construct of BFLK quasi-partonic kernels [Bukhvostov, Frolov, Lipatov, Kuraev, 1985] [Braun, Manashov, 2009]

$$\begin{aligned}
\mathbb{P}^A_{x_2x_1} \otimes \Phi(x_1,x_2,x_3) = -\frac{a_s}{2}\Big\{ & \delta_{x_20}\, C_A \Phi(x_1,0,x_3) \\
& + C_A \int_{-\infty}^{\infty} dv \Big[\frac{x_2}{v}[(v+x_2)\Phi(x_1,x_2,x_3) - x_2\Phi(x_1-v,x_2+v,x_3)] \frac{\theta(v,x_2)-\theta(-v,-x_2)}{(v+x_2)^2} \\
& \quad + \frac{x_1}{v}(\Phi(x_1,x_2,x_3) - \Phi(x_1-v,x_2+v,x_3)) \frac{\theta(v,-x_1)-\theta(-v,x_1)}{v-x_1} \Big] \\
& - C_A \int_{-\infty}^{\infty} dv \Big[\frac{x_2^2(v+2x_2+x_1)}{(x_1+x_2)^2} \frac{\theta(v,x_2)-\theta(-v,-x_2)}{(v+x_2)^2} \\
& \quad + \frac{x_1(2x_2+x_1)}{(x_1+x_2)^2} \frac{\theta(v,-x_1)-\theta(-v,x_1)}{v-x_1} \Big] \Phi(x_1-v,x_2+v,x_3) \\
& + 2\left(C_F - \frac{C_A}{2}\right) \int_{-\infty}^{\infty} dv \Big[\frac{-x_2^2}{(x_1+x_2)^2} \frac{\theta(v,x_2)-\theta(-v,-x_2)}{v+x_2} \\
& \quad + \frac{x_1(x_1x_2-2vx_2-vx_1)}{(x_1+x_2)^2} \frac{\theta(v,-x_1)-\theta(-v,x_1)}{(v-x_1)^2} \Big] \Phi(x_2+v,x_1-v,x_3) \Big\} \\
& + \mathcal{O}(a_s^2),
\end{aligned} \tag{3.15}$$

Example equation ($\mathbb{P}^A$-case)

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} \Phi_\oplus \\ \Phi_\ominus \end{pmatrix} = \left(\frac{\Gamma_{\text{cusp}}}{2} \ln\left(\frac{\mu^2}{\zeta}\right) + \Upsilon_{x_1x_2x_3} \right) \begin{pmatrix} \Phi_\oplus \\ \Phi_\ominus \end{pmatrix} + \begin{pmatrix} 2\mathbb{P}^A_{x_2x_1} & 2\pi s \Theta_{x_1x_2x_3} \\ -2\pi s \Theta_{x_1x_2x_3} & 2\mathbb{P}^A_{x_2x_1} \end{pmatrix} \begin{pmatrix} \Phi_\oplus \\ \Phi_\ominus \end{pmatrix},$$

- $\begin{pmatrix} \Phi_\oplus \\ \Phi_\ominus \end{pmatrix}$'s is a pair twist-3 TMDs e.g $\begin{pmatrix} h_{\oplus,L} \\ h_{\ominus,L} \end{pmatrix}$.
- $s = \pm$ depending on DY/SIDIS definition

$$\Upsilon_{x_1x_2x_3} = a_s \left[3C_F + C_A \ln\left(\frac{|x_3|}{|x_2|}\right) + 2\left(C_F - \frac{C_A}{2}\right) \ln\left(\frac{|x_3|}{|x_1|}\right) \right] + \mathcal{O}(a_s^2).$$

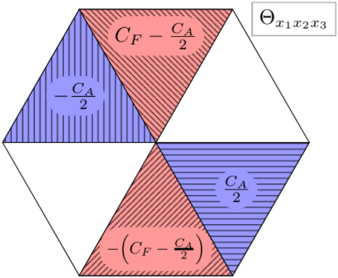
Remnants of cancellation of collinear divergences between SF and TMD

Example equation ($\mathbb{P}^A$-case)

$$\mu^2\frac{d}{d\mu^2}\begin{pmatrix}\Phi_\oplus\\ \Phi_\ominus\end{pmatrix} = \left(\frac{\Gamma_{\text{cusp}}}{2}\ln\left(\frac{\mu^2}{\zeta}\right)+\Upsilon_{x_1x_2x_3}\right)\begin{pmatrix}\Phi_\oplus\\ \Phi_\ominus\end{pmatrix} + \begin{pmatrix}2\mathbb{P}^A_{x_2x_1} & 2\pi s\Theta_{x_1x_2x_3}\\ -2\pi s\Theta_{x_1x_2x_3} & 2\mathbb{P}^A_{x_2x_1}\end{pmatrix}\begin{pmatrix}\Phi_\oplus\\ \Phi_\ominus\end{pmatrix},$$

- $\begin{pmatrix}\Phi_\oplus\\ \Phi_\ominus\end{pmatrix}$'s is a pair twist-3 TMDs e.g $\begin{pmatrix}h_{\oplus,L}\\ h_{\ominus,L}\end{pmatrix}$.
- $s=\pm$ depending on DY/SIDIS definition

- Complex-parts of remnants of cancellation of collinear divergences between SF and TMD
- Discontinuous
- Mixes T-odd and T-even TMDs



Cancellation of spurious rapidity divergences

$$\langle \mathcal{W}^{[\Gamma]}_{\text{eff}} \rangle = \Phi_{11}^{[\Gamma+]}\Psi + \frac{i}{2\partial_+}\frac{\partial}{\partial b^\mu}\Phi_{11}^{[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu]}\Psi + \frac{i}{2\partial_+}\left(\Phi_{21}^{\mu[\gamma_\mu\gamma^+\Gamma]}\Psi + \boxed{\Phi_{12}^{\mu[\Gamma\gamma^+\gamma_\mu]}\Psi}\right) - \frac{i}{2\partial_+}\left(\Phi^{[\gamma^\mu\gamma^+\Gamma]}\Psi_\mu + \boxed{\Phi_{11}^{[\Gamma\gamma^+\gamma^\mu]}\Psi_\mu}\right) + \dots.$$

$$\boxed{\int_{-\infty}^{0} d\tau \bar{q}(\ell+b)F_{\mu+}(\tau)q(0)\Psi}$$

$$\boxed{\Phi\int_{-\infty}^{0} d\tau[contour]F_{\mu-}(\tau\bar{n})}$$

Cancellation of spurious rapidity divergences

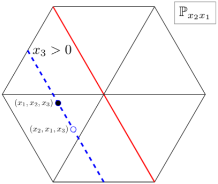
$$\begin{aligned}\langle \mathcal{W}^{[\Gamma]}_{\text{eff}}\rangle &= \Phi^{[\Gamma+]}_{11}\Psi \\ &+\frac{i}{2\partial_+}\frac{\partial}{\partial b^\mu}\Phi_{11}^{[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu]}\Psi \\ &+\frac{i}{2\partial_+}\left(\Phi_{21}^{\mu[\gamma_\mu\gamma^+\Gamma]}\Psi+\boxed{\Phi_{12}^{\mu[\Gamma\gamma^+\gamma_\mu]}\Psi}\right) \\ &-\frac{i}{2\partial_+}\left(\Phi^{[\gamma^\mu\gamma^+\Gamma]}\Psi_\mu+\boxed{\Phi_{11}^{[\Gamma\gamma^+\gamma^\mu]}\Psi_\mu}\right)+\dots\end{aligned}$$

$$\sim\int[dx]\delta(x+x_1)\frac{\Psi_{12}(x_1,x_2,x_3)}{x_2+i0}\Psi$$

$$\int_{-\infty}^{0} d\tau\bar{q}(\ell+b)F_{\mu+}(\tau)q(0)\Psi$$

$$\Phi\int_{-\infty}^{0} d\tau[contour]F_{\mu-}(\tau\bar{n})$$

$$\sim\Phi(x)\int dy\frac{\Psi^\mu(y)}{y+i0}$$



Cancellation of spurious rapidity divergences

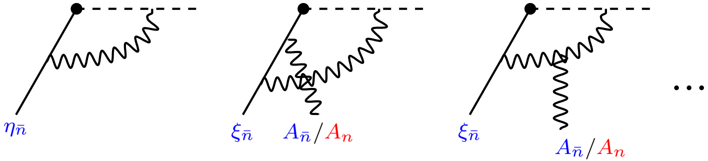
Divergences at x_2 can be regularized & computed.

$$\int_{-\infty}^{0} d\tau \widetilde{\Phi}_{12}^{\mu[\Gamma]}(z,\tau,0) = \ln\left(\frac{\delta^+}{\partial^+}\right) \partial_\mu \mathcal{D}(b) \widetilde{\Phi}_{11}^{[\Gamma]}(z) + \text{fin.terms}$$

$$\int_{-\infty}^{0} d\tau \widetilde{\Psi}_\mu(\tau) = \ln\left(\frac{\delta^+}{\mu v^+}\right) \partial_\mu \mathcal{D}(b) \widetilde{\Psi} + \text{fin.terms}$$

- Lowers the TMD-twist
- Cancel in the sum $\Phi_{12}^{\mu[\Gamma]}\Psi - \Phi_{11}^{[\Gamma]}\Psi^\mu$
- One can define "physical" distributions which have finite at $x_2 = 0$ integrals.

NLO coefficient function



$$
\begin{aligned}
W^{[\Gamma]} \quad = \quad & |C_H|^2 \Phi_{11}^{[\Gamma+]}\Psi \\
& +|C_H|^2 \frac{i}{2\partial_+}\frac{\partial}{\partial b^\mu}\Phi_{11}^{[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu]}\Psi \\
& +\frac{i}{2\partial_+}(C_{H2}^* C_H \Phi_{21}^{\mu[\gamma_\mu\gamma^+\Gamma]}\Psi + C_H^* C_{H2}\Phi_{12}^{\mu[\Gamma\gamma^+\gamma_\mu]}\Psi) \\
& -\frac{i}{2\partial_+}\left(C_H^* C_\Psi \Phi_{11}^{[\gamma^\mu\gamma^+\Gamma]} + C_\Psi^* C_H \Phi_{11}^{[\Gamma\gamma^+\gamma^\mu]}\right)\Psi_\mu + \ldots.
\end{aligned}
$$

Warning: Non-trivial complex parts (depend on direction of v).

$$\lim_{x_2\to 0} C_{H2} = C_\Psi$$

Factorization for qTMD of sub-leading counting

$$
\begin{aligned}
W^{[\Gamma]} &= |C_H|^2 \Phi_{11}^{[\Gamma+]} \Psi && \longleftarrow \text{0. known LP term} \\
&+ |C_H|^2 \frac{i}{2\partial_+} \frac{\partial}{\partial b^\mu} \Phi_{11}^{[\gamma^\mu \gamma^+ \Gamma + \Gamma \gamma^+ \gamma^\mu]} \Psi && \longleftarrow \text{1. kinematic power corr.} \\
&+ (1 + a_s ..) \ln\left(\frac{\zeta}{\bar{\zeta}}\right) \frac{i}{2\partial_+} \Phi_{11}^{[\gamma^\mu \gamma^+ \Gamma + \Gamma \gamma^+ \gamma^\mu]} \Psi \frac{\partial \mathcal{D}}{\partial b^\mu} && \longleftarrow \text{2. remnant of rap.div} \\
&+ \frac{i}{2\partial_+} (C_{H2}^* C_H \Phi_{21}^{\mu[\gamma_\mu \gamma^+ \Gamma]} \Psi + C_H^* C_{H2} \Phi_{12}^{\mu[\Gamma \gamma^+ \gamma_\mu]} \Psi) && \longleftarrow \text{3. actual TMDs of tw3} \\
&- \frac{i}{2\partial_+} \left(C_H^* C_\Psi \Phi_{11}^{[\gamma^\mu \gamma^+ \Gamma]} + C_\Psi^* C_H \Phi_{11}^{[\Gamma \gamma^+ \gamma^\mu]} \right) \Psi_\mu && \longleftarrow \text{4. tw3 interaction with WL} \\
&+ \ldots && \longleftarrow \text{5. higher power terms}
\end{aligned}
$$

Cancellation of divergences

- Spurious divergences cancel between (3 and 4) $\to$ 2
- Usual divergences cancel within each line
- Checked at NLO

Explicit example

$\Gamma = \gamma^\alpha$, $S = 0$

$$\frac{W^{[\gamma^\alpha]}(\ell,b)}{\Psi(b)} = -iMb^\alpha \int_{-1}^{1}\frac{dx}{x}e^{-ix\ell}\Bigg[\frac{|C_H|^2}{M^2}\left(2\frac{\partial}{\partial b^2}f_1(x,b) + \ln\left(\frac{\mu v^+}{xP^+}\right)f_1(x,b)\frac{\partial}{\partial b^2}\mathcal{D}(b)\right)$$
$$+\int\frac{[dx]}{x_2}\left(C_H^* C_{H2}\delta(x-x_3) + C_{H2}^* C_H\delta(x+x_1)\right)[f_\ominus^\perp + g_\oplus^\perp](x_{1,2,3},b)$$
$$+\left(C_H^* C_\Psi + C_\Psi^* C_H\right)f_1(x,b)\frac{b_\mu \Psi^\mu(b)}{\Psi(b)}$$
$$+s\pi\int[dx](\delta(x_2)+a_s\ldots)\left(\delta(x-x_3)+\delta(x+x_1)\right)[f_\oplus^\perp + g_\ominus^\perp](x_{1,2,3},b)\Bigg]$$

- A lot of trash terms.
- Dependence on direction of v
- Explicit divergence at $x = 0$ (!!!)

Explicit example
$\Gamma = 1,\ S = 0$

$$\frac{W^{[1]}(\ell,b)}{\Psi(b)} = M\int_{-1}^{1}\frac{dx}{x}e^{-ix\ell}\Big[2\int\frac{[dx]}{x_2}\left(C_H^* C_{H2}\delta(x-x_3)+C_{H2}^* C_H\delta(x+x_1)\right)e_{\oplus}(x_{1,2,3},b) +2s\pi\int[dx]\left(\delta(x-x_3)+\delta(x+x_1)\right)\left(\delta(x_2)+a_s..\right)e_{\ominus}(x_{1,2,3},b)\Big]$$

- Can be used to extract CS-kernel just like twist-2 case.
- Dependence on direction of v
- Explicit divergence at $x = 0$ (!!!)

Conclusion

- We derived the factorization expression for quasi-TMD correlators of "sub-leading" counting
  - Structurally the same as for DY or SIDIS (but less symmetric)
- Checked the cancellation of all divergences at NLO
- Some of these correlators can be used to determine CS kernel
- Some represent a complicated mixture of different terms, which (most possibly) cannot be used in practice
  - However, one can think about extra subtraction

$$\sim W^{[\gamma^\alpha]} - \frac{1}{p^+}\frac{\partial}{\partial b_\alpha} W^{[\gamma^+]}$$

Coefficient functions

$$
\begin{aligned}
C_{H1}(x) &= 1 + 2a_s C_F |2x(vP)|^{-2\varepsilon} f_x \frac{\Gamma(2-\varepsilon)\Gamma(-1+2\varepsilon)}{\varepsilon} \\
C_{H2}(-x_3) &= 1 - 4a_s C_F \varepsilon \Gamma(1-\varepsilon)\Gamma(2\varepsilon) I_\varepsilon f_{-x_3} |2x_3(vP)|^{-2\varepsilon} \\
&\quad + 4a_s \left(C_F - \frac{C_A}{2}\right) \Gamma(1-\varepsilon)\Gamma(2\varepsilon) I_\varepsilon f_{-x_3} |2x_3(vP)|^{-2\varepsilon} \left(1 - 2\varepsilon + \frac{x_3}{x_2}\left(1 - \left(\frac{x_3 + i0}{x_2 + x_3 + i0}\right)^{2\varepsilon}\right)\right) \\
&\quad + 4a_s \frac{C_A}{2} \Gamma(1-\varepsilon)\Gamma(2\varepsilon) I_\varepsilon f_{-x_3} |2x_3(vP)|^{-2\varepsilon} \left(1 - 2\varepsilon + 2\varepsilon \frac{x_3}{x_2 + x_3}\left(1 - \left(\frac{x_2 + i0}{x_1 + x_2 + i0}\right)^{-2\varepsilon}\right)\right) \\
C_{H2n}(x) &= -2a_s |2x(vP)|^{-2\varepsilon} f_x \Gamma(-\varepsilon)\Gamma(-1+2\varepsilon) C_F (1-\varepsilon)
\end{aligned}
$$