Nuclear structure models for double-beta NMEs: HF, TDA, RPA, beyond-RPA

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+ SuperNEMO team.

Workshop “Recommended Values for Neutrinoless Double Beta Decay NMEs“,
Institute for Nuclear Theory, Seattle, USA, 01 May 2026

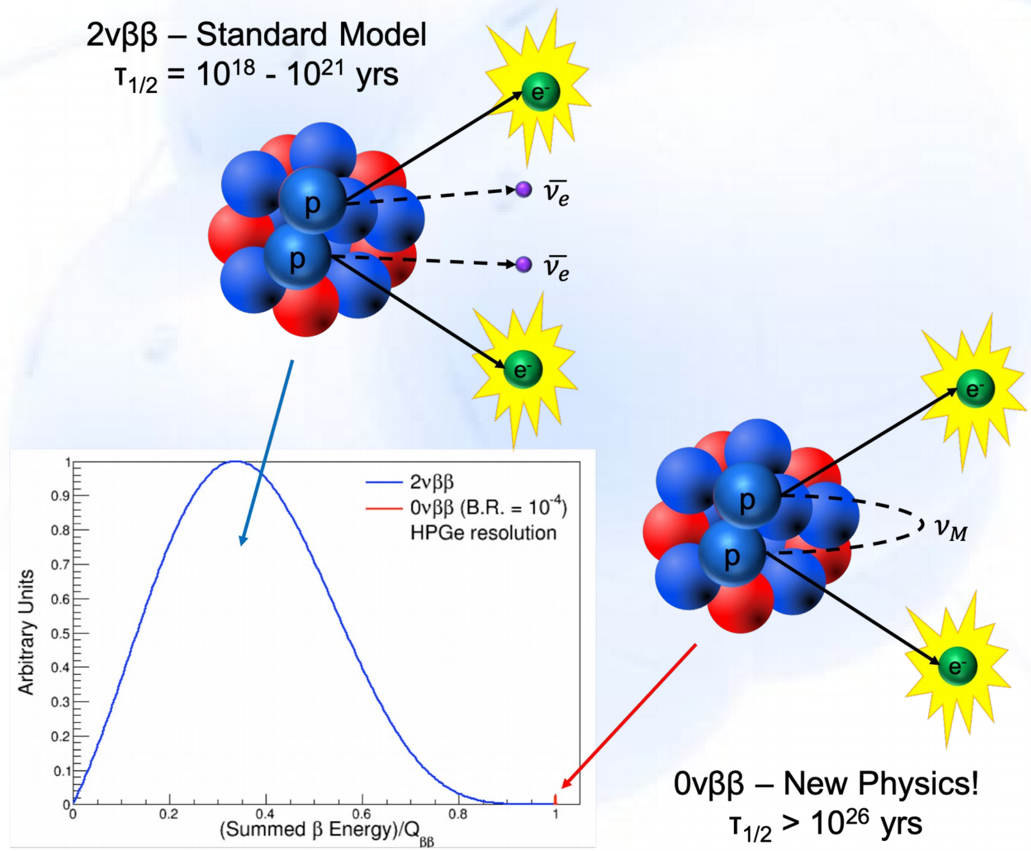
Motivation

Double Beta Decay is so interesting physical phenomenon because: **Particle** & **Nuclear** & **Atomic** physics meet here. It is perfect laboratory to test validity of **SM** ($2\nu\beta\beta$) and search for effects **beyond SM** physics ($0\nu\beta\beta$). It can provide us more information about least understood elementary particle (**neutrino**). It is **challenge** for **experimental physics** – the **slowest decay** process in the universe.

How **theoretical nuclear physics** can contribute to the effort of understanding the Double Beta Decay?

Outline of the Seminar:

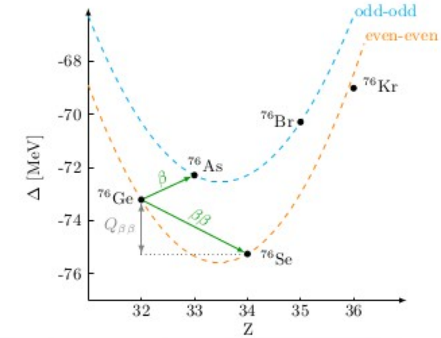
- **Double β Decay.**
- **Nuclear methods (generally).**
- **EMPM & STDA methods.**
- **NMEs and GT resonances.**
- **Summary & Outlook.**



Double β decay & neutrino physics

Existence of double β decay proposed by **Maria Goeppert Mayer** in 1935. In its standard variant, **2 electrons + 2 (anti)neutrinos** are emmitted (**2νββ**).
Decay occurs in nuclei where single β decay is not energetically allowed.

There are 35 naturally occuring isotopes capable of **2νββ**.
2νββ or double electron capture confirmed experimentaly in 14 isotopes.



Isotope	$T_{1/2}(2\nu)$, yr	$\mid M_{2\nu}^{eff} \mid$ ($G_{2\nu}$ from [24])	$\mid M_{2\nu}^{eff} \mid$ ($G_{2\nu}$ from [25])	Recommended Value
$2\nu\beta\beta$:				
^{48}Ca	$5.3^{+1.2}_{-0.8} \cdot 10^{19}$	$0.0348^{+0.0030}_{-0.0034}$	$0.0348^{+0.0030}_{-0.0034}$	0.035 ± 0.003
^{76}Ge	$(1.88 \pm 0.08) \cdot 10^{21}$	$0.1051^{+0.0023}_{-0.0024}$	$0.1074^{+0.0024}_{-0.0022}$	0.106 ± 0.004
^{82}Se	$0.87^{+0.02}_{-0.01} \cdot 10^{20}$	$0.0849^{+0.0005}_{-0.0010}$	$0.0855^{+0.0005}_{-0.0010}$	0.085 ± 0.001
^{96}Zr	$(2.3 \pm 0.2) \cdot 10^{19}$	$0.0798^{+0.0037}_{-0.0032}$	$0.0804^{+0.0038}_{-0.0033}$	0.080 ± 0.004
^{100}Mo	$7.06^{+0.15}_{-0.13} \cdot 10^{18}$	$0.2071^{+0.0019}_{-0.0022}$	$0.2096^{+0.0020}_{-0.0022}$	
		$0.1852^{+0.0017}_{-0.0019}$ (a)		0.185 ± 0.002
^{100}Mo-^{100}Ru(0^+_1)	$6.7^{+0.5}_{-0.4} \cdot 10^{20}$	$0.1571^{+0.0048}_{-0.0056}$	$0.1619^{+0.0050}_{-0.0058}$	
		$0.1513^{+0.0047}_{-0.0053}$ (a)		0.151 ± 0.005
^{116}Cd	$(2.69 \pm 0.09) \cdot 10^{19}$	$0.1160^{+0.0020}_{-0.0019}$	$0.1176^{+0.0020}_{-0.0019}$	
		$0.1084^{+0.0024}_{-0.0019}$ (a)		0.108 ± 0.003
^{128}Te	$(2.25 \pm 0.09) \cdot 10^{24}$	$0.0406^{+0.0008}_{-0.0008}$	$0.0454^{+0.0009}_{-0.0009}$	0.043 ± 0.003
^{130}Te	$(7.91 \pm 0.21) \cdot 10^{20}$	$0.0288^{+0.0004}_{-0.0004}$	$0.0297^{+0.0004}_{-0.0004}$	0.0293 ± 0.0009
^{136}Xe	$(2.18 \pm 0.05) \cdot 10^{21}$	$0.0177^{+0.0002}_{-0.0002}$	$0.0184^{+0.0002}_{-0.0002}$	0.0181 ± 0.0006
^{150}Nd	$(9.34 \pm 0.65) \cdot 10^{18}$	$0.0543^{+0.0020}_{-0.0018}$	$0.0550^{+0.0020}_{-0.0018}$	0.055 ± 0.003
^{150}Nd-^{150}Sm(0^+_1)	$1.2^{+0.3}_{-0.2} \cdot 10^{20}$	$0.0438^{+0.0042}_{-0.0046}$	$0.0450^{+0.0043}_{-0.0048}$	0.044 ± 0.005
^{238}U	$(2.0 \pm 0.6) \cdot 10^{21}$	$0.1853^{+0.0361}_{-0.0227}$	$0.0713^{+0.0139}_{-0.0088}$	$0.13^{+0.09}_{-0.07}$
ECEC(2ν):				
^{78}Kr (b)	$1.9^{+1.3}_{-0.8} \cdot 10^{22}$	$0.2882^{+0.0829}_{-0.0706}$ [105]	$0.3583^{+0.1126}_{-0.0822}$	$0.32^{+0.15}_{-0.11}$
^{124}Xe (b)	$(1.8 \pm 0.5) \cdot 10^{22}$	$0.0568^{+0.0101}_{-0.0650}$ [105]	$0.0607^{+0.0107}_{-0.0070}$	$0.059^{+0.013}_{-0.009}$
^{130}Ba	$(2.2 \pm 0.5) \cdot 10^{21}$	$0.1741^{+0.0239}_{-0.0170}$ [105]	$0.1754^{+0.0241}_{-0.0171}$	$0.175^{+0.024}_{-0.017}$

2νββ half-lifes

Table from **A. Barabash**, *Universe 6*, 159, (2020).

$$T^{-1}_{1/2} = G_{2\nu} \cdot g^4_A \cdot (m_e c^2 \cdot M_{2\nu})^2,$$

phase space integration

effective nuclear transition amplitude

Double β decay & nuclear physics

Nuclear transition amplitudes (**NME**):
2νββ

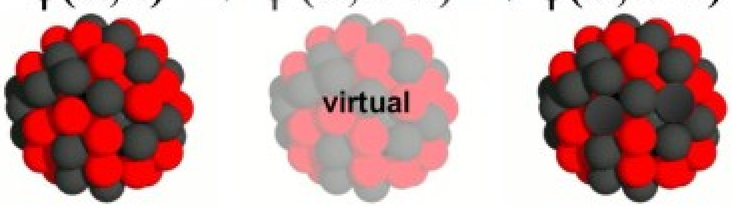
daughter **intermediate** **mother** nucleus

$$M^{2\nu}_{F,GT} = \sum_n \frac{\langle f \parallel \mathcal{O}_{F,GT} \parallel J_n^+\rangle\langle J_n^+ \parallel \mathcal{O}_{F,GT} \parallel i\rangle}{E_n - (E_i + E_f)/2}$$

Fermi op. **Gamow-Teller** op.

$$\mathcal{O}_F = \sum_{k=1}^{A} \tau_k^+, \qquad \mathcal{O}_{GT} = \sum_{k=1}^{A} \tau_k^+ \sigma_{\mathbf{k}}.$$

$\psi(A,Z) \Rightarrow \psi(A,Z+1) \Rightarrow \psi(A,Z+2)$



NME's calculated within various* nuclear structure methods vary a lot among each other. It is not very satisfactory for predictions of half-lifes.

* **EDF, IBM, QRPA, SRPA, Shell Model, In-Medium SRG, Coupled Cluster**

0νββ

$$M^{0\nu} = M^{0\nu}_{GT} - M^{0\nu}_F / g_A^2 + M^{0\nu}_T$$

closure approximation is assumed here
we work with **2-body NME**.

$$M^{0\nu}_F = \sum_{r,s} \langle f | h_F(r_-, \bar{E}_n) \tau_r^+ \tau_s^+ | i \rangle,$$

$$M^{0\nu}_{GT} = \sum_{r,s} \langle f | h_{GT}(r_-, \bar{E}_n) \tau_r^+ \tau_s^+ \vec{\sigma}_r \cdot \vec{\sigma}_s | i \rangle,$$

$$M^{0\nu}_T = \sum_{r,s} \langle f | h_T(r_-, \bar{E}_n) \tau_r^+ \tau_s^+ (3(\vec{\sigma}_r \cdot \hat{\mathbf{r}}_-)(\vec{\sigma}_s \cdot \hat{\mathbf{r}}_-) - \vec{\sigma}_r \cdot \vec{\sigma}_s) | i \rangle$$

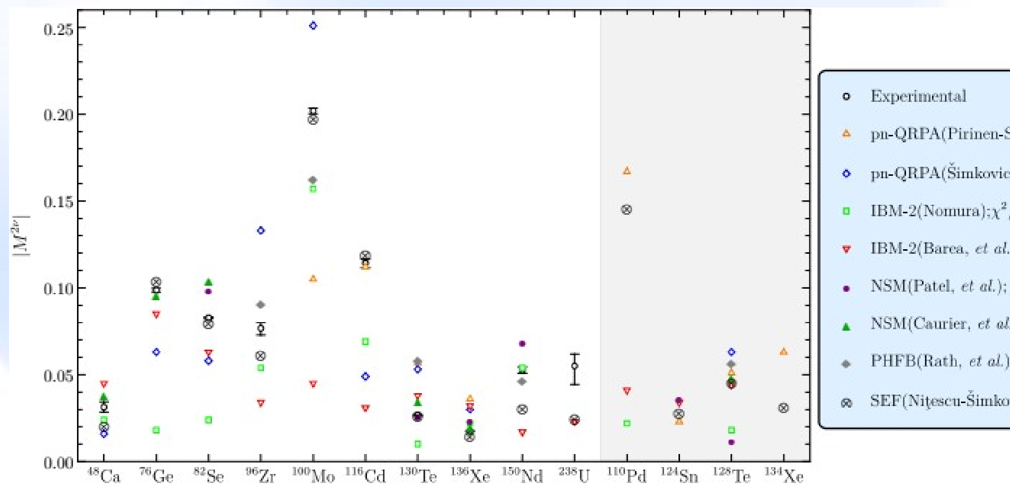


Fig. by *Ovidiu Nitescu*

Double β decay & nuclear physics

Nuclear transition amplitudes (**NME**):

2νββ

daughter **intermediate** **mother** nucleus

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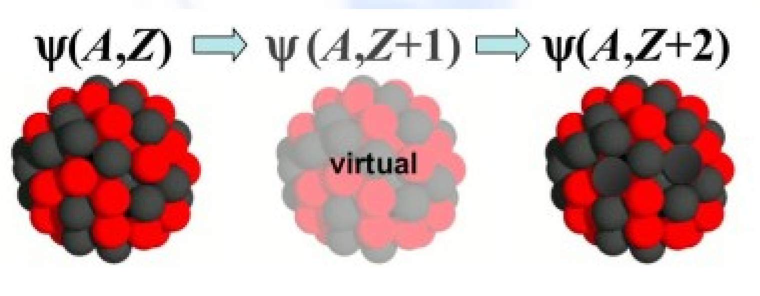

NME's calculated within various* nuclear structure methods vary a lot among each other. It is not very satisfactory for predictions of half-lifes.

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0νββ

$$M^{0\nu} = M^{0\nu}_{GT} - M^{0\nu}_F / g_A^2 + M^{0\nu}_T$$

closure approximation is assumed here

$$M^{0\nu}_F = \sum_{r,s} \langle f | h_F(r_-, \bar{E}_n) \tau_r^+ \tau_s^+ | i \rangle ,$$

$$M^{0\nu}_{GT} = \sum_{r,s} \langle f | h_{GT}(r_-, \bar{E}_n) \tau_r^+ \tau_s^+ \vec{\sigma}_r \cdot \vec{\sigma}_s | i \rangle ,$$

$$M^{0\nu}_T = \sum_{r,s} \langle f | h_T(r_-, \bar{E}_n) \tau_r^+ \tau_s^+ (3(\vec{\sigma}_r \cdot \hat{\mathbf{r}}_-)(\vec{\sigma}_s \cdot \hat{\mathbf{r}}_-) - \vec{\sigma}_r \cdot \vec{\sigma}_s) | i \rangle$$

In **0νββ** we work with **2-body NME**.

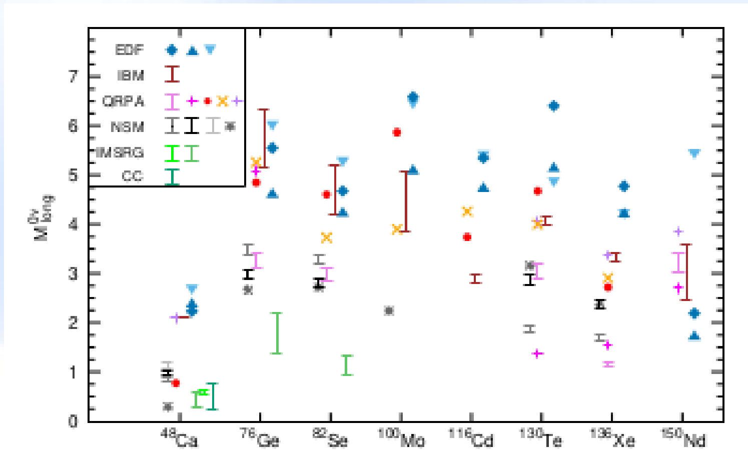
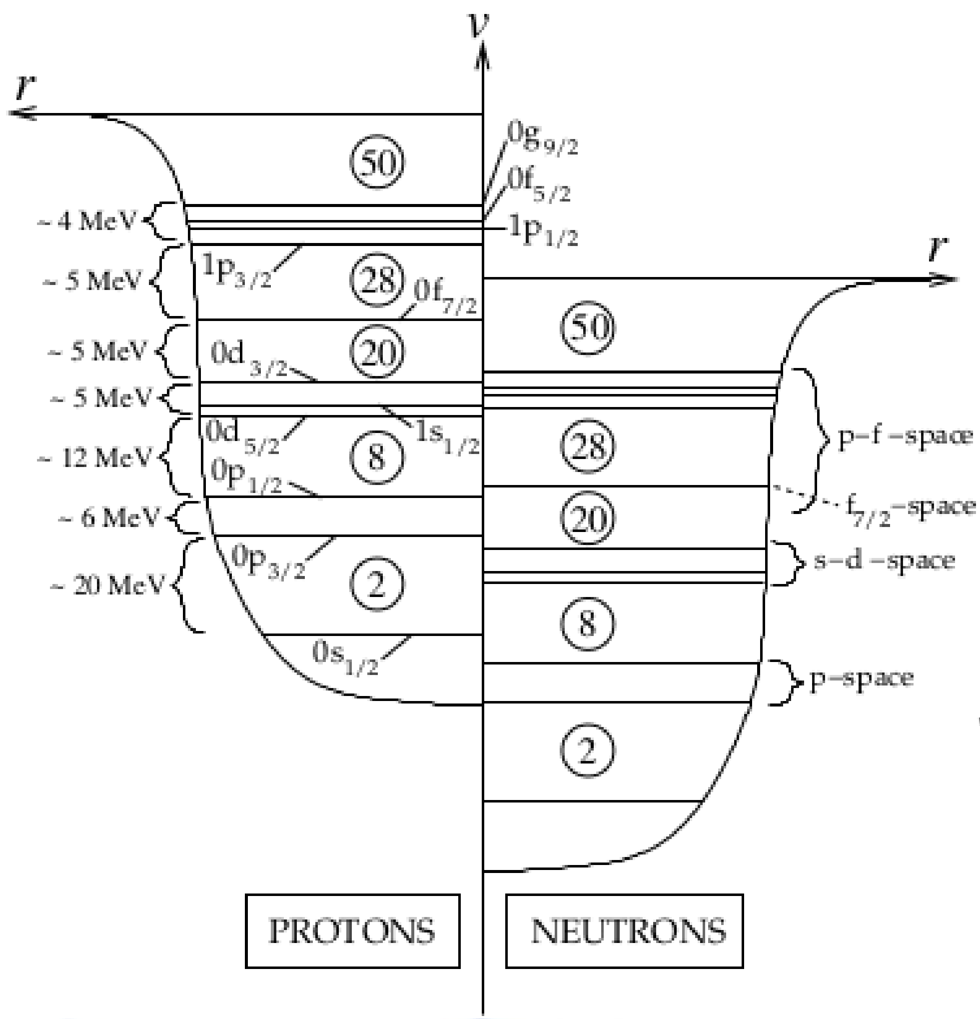


Fig. from *M. Agostini et al.*, **ArXiv:2202.01787v2**

Mean Field

Mean Field: Single-particle levels (protons/neutrons).
Can be either **phenomenological** or result of a **Hartree-Fock** calculation.



Single-particle states

$$|\phi_\alpha\rangle \equiv |\alpha\rangle \equiv |a\, m_\alpha\rangle\,, \quad a = n_a l_a j_a\,,$$

$$\langle \boldsymbol{x}|\alpha\rangle = \phi_\alpha(\boldsymbol{x}) = \eta_a g_{n_a l_a}(r)\left[Y_{l_a}(\Omega)\chi_{\frac{1}{2}}\right]_{j_a m_\alpha}\,, \quad \Omega = (\theta,\varphi)\,.$$

$$g_{nl}(r) = \sqrt{\frac{2n!}{b^3\Gamma(n+l+\frac{3}{2})}}\left(\frac{r}{b}\right)^l \mathrm{e}^{-r^2/2b^2} L_n^{(l+\frac{1}{2})}(r^2/b^2)$$

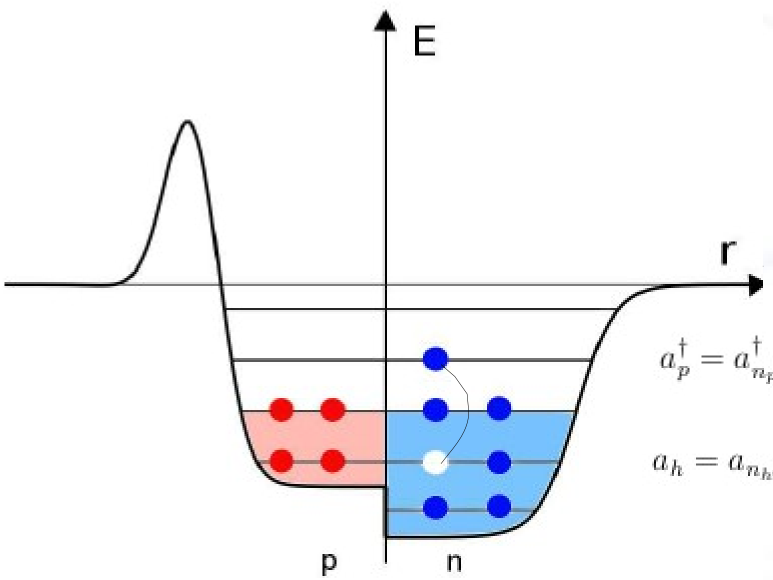
Many-body wave function is represented as a **Slater determinant** of occupied levels

$$\Psi_Z = \Psi_{\pi_1\pi_2\cdots\pi_Z}(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_Z) = $$

$$= \frac{1}{\sqrt{Z!}}\begin{vmatrix} \phi_{\pi_1}(\boldsymbol{x}_1) & \phi_{\pi_1}(\boldsymbol{x}_2) & \cdots & \phi_{\pi_1}(\boldsymbol{x}_Z) \\ \phi_{\pi_2}(\boldsymbol{x}_1) & \phi_{\pi_2}(\boldsymbol{x}_2) & \cdots & \phi_{\pi_2}(\boldsymbol{x}_Z) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{\pi_Z}(\boldsymbol{x}_1) & \phi_{\pi_Z}(\boldsymbol{x}_2) & \cdots & \phi_{\pi_Z}(\boldsymbol{x}_Z) \end{vmatrix}$$

Mean Field & 1particle-1hole exc.

Mean Field: Obtaining mean field by solving **Hartree-Fock (HF) method**.
The **residual interaction** must be taken back into accout.



How to represent **excitations**?

particle-hole configurations.

$$a_p^\dagger a_h|HF\rangle = a_p^\dagger a_h|\Psi\rangle$$

$a_p^\dagger = a_{n_p l_p m_p s_p}^\dagger$ … creation in “particle“ level

$a_h = a_{n_h l_h m_h s_h}$ … annihilation in “hole“ level

Methods to describe **nuclear excitation spectra** which are based on **1particle-1hole** configurations:

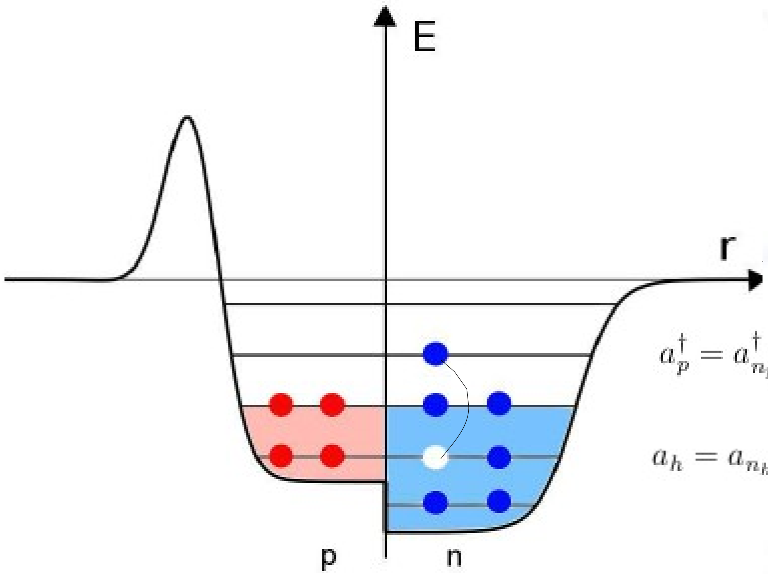
Tamm Dancoff Approximation (**TDA**) & Random Phase Approximation (**RPA**)

from **HF** state

from (implicitly) **correlated** state

Tamm Dancoff Approximation

TDA excitations: We diagonalize nuclear Hamiltonian in the space of $a_p^\dagger a_h|HF\rangle = a_p^\dagger a_h|\Psi\rangle$



$$a_p^\dagger = a^\dagger_{n_p l_p m_p s_p}$$

$$a_h = a_{n_h l_h m_h s_h}$$

Intrinsic nuclear **Hamiltonian**

$$H = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j}^{A} V_{ij} = H_{int} + \frac{\boldsymbol{P}^2}{2Am}$$

Equation of Motion

$$[H_{intr}, Q_\nu^\dagger]|0\rangle \equiv \hbar\omega_\nu Q_\nu^\dagger|0\rangle$$

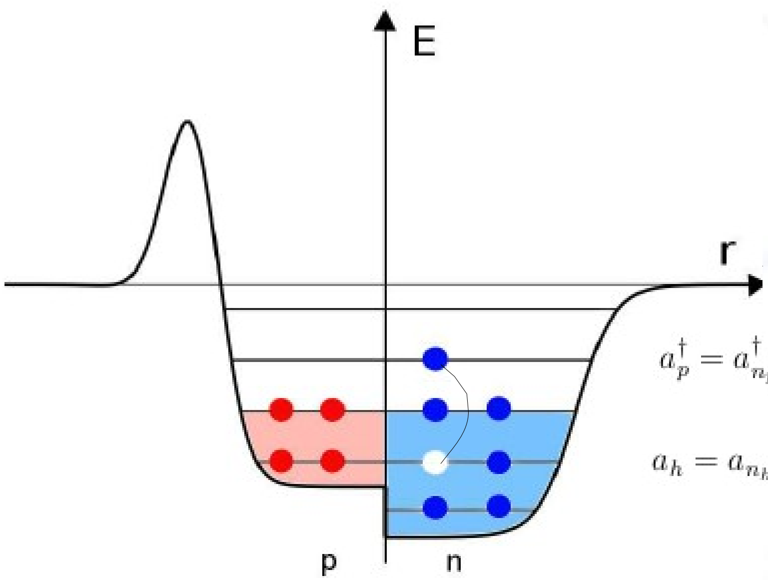
$$|\nu\rangle = Q_\nu^\dagger|0\rangle, \quad Q_\nu|0\rangle = 0$$

TDA phonons

$$O_\nu^\dagger = \sum_{ph} c^\nu_{ph} a_p^\dagger a_{\bar{h}}$$

Phonons are superpositions of various **1particle-1hole excitations**.

Random Phase Approximation



$$a_p^\dagger = a_{n_p l_p m_p s_p}^\dagger$$

$$a_h = a_{n_h l_h m_h s_h}$$

Intrinsic nuclear **Hamiltonian**

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j}^{A} V_{ij} = H_{int} + \frac{P^2}{2Am}$$

Correlated ground state

$$|\mathrm{RPA}\rangle = \mathcal{N}_0 \mathrm{e}^S |\mathrm{HF}\rangle$$

RPA phonons

$$Q_\omega^\dagger = \sum_{ab} \left[X_{ab}^\omega \mathcal{A}_{ab}^\dagger(JM) - Y_{ab}^\omega \tilde{\mathcal{A}}_{ab}(JM) \right]$$

Forward Amplitude

Backward Amplitude

Quasiboson approximation:
We keep RPA phonons with backward amplitude but EoM is derived with |RPA> = |HF> assumption...

$$\begin{pmatrix} \mathsf{A} & \mathsf{B} \\ -\mathsf{B}^* & -\mathsf{A}^* \end{pmatrix} \begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} = E_\omega \begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} .$$

Renormalized Random Phase Approximation

Intrinsic nuclear **Hamiltonian**

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j}^{A} V_{ij} = H_{int} + \frac{P^2}{2Am}$$

RPA phonons

$$Q_\omega^\dagger = \sum_{ab} \left[X_{ab}^{\omega} A_{ab}^\dagger (JM) - Y_{ab}^{\omega} \tilde{A}_{ab}(JM) \right]$$

Forward Amplitude

Backward Amplitude

- 1p1h & 1h1p operators are renormalized ... to eliminate the overlap matrix $\mathbb{N}$

$$a_p^\dagger a_h \rightarrow D_{ph}^{-1/2} a_p^\dagger a_h, \qquad D_{ph} \equiv (n_h - n_p).$$

The RRPA matrices $\mathbb{A}$ & $\mathbb{B}$ ...

$$\mathbb{A}_{phqg} = \left(\sqrt{D_{ph}/D_{qg}} + \sqrt{D_{qg}/D_{ph}}\right)(\delta_{hg} h_{pq} - \delta_{pq} h_{hg}) + \sqrt{D_{ph} D_{qg}} V_{pggh},$$

$$\mathbb{B}_{phqg} = \sqrt{D_{ph} D_{qg}} V_{pqhg}.$$

- Evaluation of 1-body densities ... using the Catara's expansion ...

$$\langle a_p^\dagger a_{p'} \rangle = \sum_{h,\nu,\mu} \left(\delta_{\nu\mu} - \frac{1}{2} \sum_{q,g} X_{qg}^{\mu} X_{qg}^{\nu *}\right) Y_{ph}^{\nu} Y_{p'h}^{\mu *} + O(|Y|^4),$$

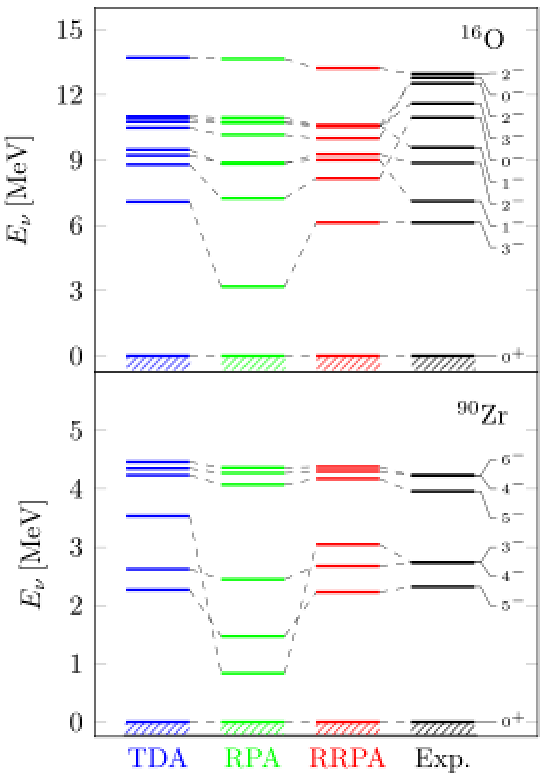
$$\langle a_h^\dagger a_{h'} \rangle = \delta_{hh'} - \sum_{p,\nu,\mu} \left(\delta_{\nu\mu} - \frac{1}{2} \sum_{q,g} X_{qg}^{\mu} X_{qg}^{\nu *}\right) Y_{ph}^{\nu} Y_{ph'}^{\mu *} + O(|Y|^4)$$

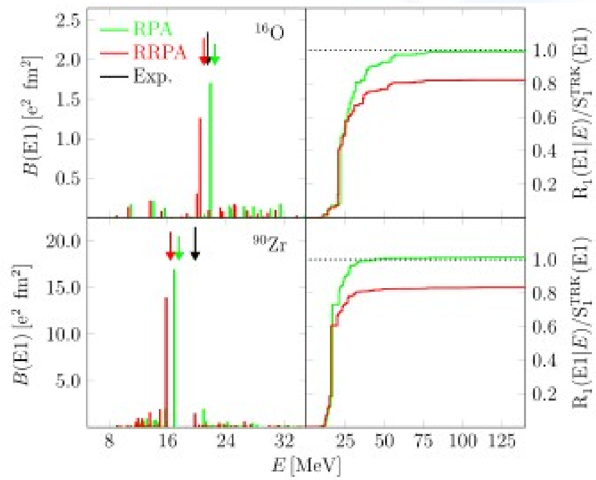
Ground state wave functions kept **explicitelly correlated**.
The **RRPA equations** solved **iterativelly**.
Elements of **EoM** are calculated in iterations.

$$\begin{pmatrix} \mathsf{A} & \mathsf{B} \\ -\mathsf{B}^* & -\mathsf{A}^* \end{pmatrix} \begin{pmatrix} \mathsf{X}^\omega \\ \mathsf{Y}^\omega \end{pmatrix} = E_\omega \begin{pmatrix} \mathsf{X}^\omega \\ \mathsf{Y}^\omega \end{pmatrix} .$$

Renormalized Random Phase Approximation

Figs. *R. Folprecht* et al., **Phys. Rev. C 113 L041302 (2026).**





Intrinsic nuclear **Hamiltonian**

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RPA phonons

$$Q_\omega^\dagger = \sum_{ab} \left[X_{ab}^\omega A_{ab}^\dagger(JM) - Y_{ab}^\omega \tilde{A}_{ab}(JM) \right]$$

Forward Amplitude

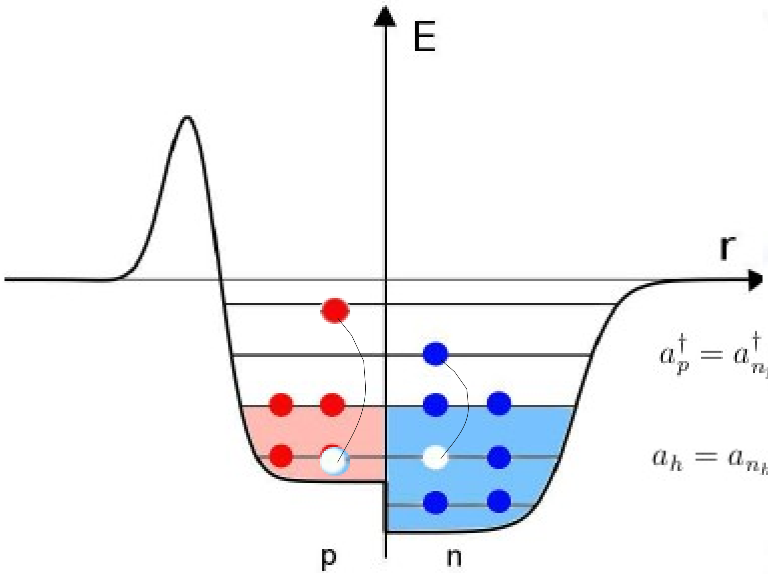
Backward Amplitude

Ground state wave functions kept **explicitelly correlated**.
Elements of **EoM** are calculated in iterations.

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} = E_\omega \begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} .$$

Second Tamm Dancoff Approximation

Second TDA: Direct diagonalization of nuclear Hamiltonian in the space spanned by **(1p-1h + 2p-2h)** excitations.



$$a_p^\dagger = a_{n_p l_p m_p s_p}^\dagger$$

$$a_h = a_{n_h l_h m_h s_h}$$

$$[\boldsymbol{H_{intr}}, \boldsymbol{Q_\nu^\dagger}]|0\rangle \equiv \hbar\omega_\nu \boldsymbol{Q_\nu^\dagger}|0\rangle$$

$$|\nu\rangle = \boldsymbol{Q_\nu^\dagger}|0\rangle, \quad \boldsymbol{Q_\nu}|0\rangle = 0$$

$$Q_\nu^{\dagger(\mathrm{STDA})} = \sum_{ph} X_{ph}^{\nu(1)} a_p^\dagger a_h + \sum_{p_1 p_2 h_1 h_2} X_{p_1 p_2 h_1 h_2}^{\nu(2)} a_{p_1}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2}$$

HF – Hartree-Fock state (nucleons occupy lowest single-particle levels)
1p-1h = 1**particle** – 1**hole** excitation of HF
2p-2h = 2**particle** – 2**hole** excitation of HF

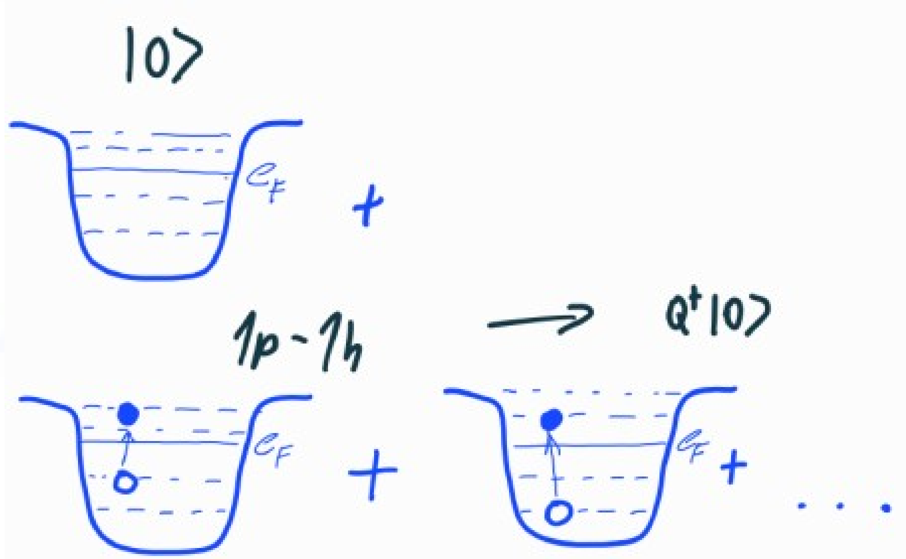
In **Phys. Rev. C 107**, *014305, (2023)* we showed equivalence of **STDA** & **EMPM** (up to 2-phonon).

Equation of Motion Phonon Method

Intrinsic nuclear **Hamiltonian**

$$H = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j}^{A} \boldsymbol{V}_{ij} = \boldsymbol{H}_{int} + \frac{\boldsymbol{P}^2}{2Am}$$

In **Tamm-Dancoff Approximation** (TDA) phonons are linear superpositions of **1particle-1hole** excitation on top of mean-field **Slater determinant** (HF)



$n=0$ → **HF**

$n=1$ → **TDA**

$$[\boldsymbol{H}_{intr}, \boldsymbol{Q}_\nu^\dagger]|0\rangle \equiv \hbar\omega_\nu \boldsymbol{Q}_\nu^\dagger|0\rangle$$

$$|\nu\rangle = \boldsymbol{Q}_\nu^\dagger|0\rangle, \quad \boldsymbol{Q}_\nu|0\rangle = 0$$

$$O_\nu^\dagger = \sum_{ph} c_{ph}^\nu a_p^\dagger a_{\tilde{h}}$$

Hilbert space – divided into subspaces

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n$$

$$\begin{aligned}
\mathcal{H}_0 &= \{|HF>\} \\
\mathcal{H}_1 &= \{O_{\nu_1}^\dagger|HF>\} \\
\mathcal{H}_2 &= \{O_{\nu_1}^\dagger O_{\nu_2}^\dagger|HF>\} \\
&\cdot \\
&\cdot \\
\mathcal{H}_n &= \{O_{\nu_1}^\dagger O_{\nu_2}^\dagger \ldots O_{\nu_n}^\dagger|HF>\}
\end{aligned}$$

Equation of Motion Phonon Method

Equation of Motion (EoM) – recursive eq. to solve **eigen-energies** on each **n**-phonon subspace while knowing the (**n-1**)-phonon eigen-energies

$$\begin{aligned} \mathcal{H}_0 &= \{|HF>\} \\ \mathcal{H}_1 &= \{O^\dagger_{\nu_1}|HF>\} \\ \mathcal{H}_2 &= \{O^\dagger_{\nu_1}O^\dagger_{\nu_2}|HF>\} \\ &\vdots \\ \mathcal{H}_n &= \{O^\dagger_{\nu_1}O^\dagger_{\nu_2}...O^\dagger_{\nu_n}|HF>\} \end{aligned}$$

$$< n,\beta \,|\, \left\{ \left[H, O^\dagger_\lambda\right] \times |\, n-1, \alpha > \right\}^\beta = \left(E^{(n)}_\beta - E^{(n-1)}_\alpha\right) < n,\beta \,|\, \left\{ O^\dagger_\lambda \times |\, n-1,\alpha > \right\}^\beta$$

n-phon state | (**n-1**)-phon state | **n**-phon state | (**n-1**)-phon state

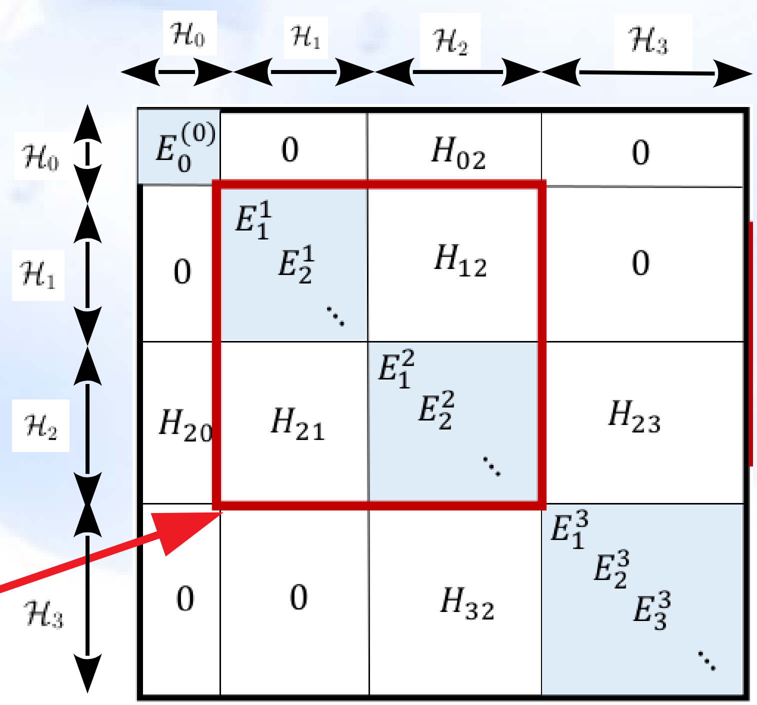
At the end we **diagonalize** nuclear **Hamiltonian** in **(0+1+2+...+n)** - phonon **basis**

$$\boldsymbol{H} = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j}^{A} \boldsymbol{V}_{ij} = \boldsymbol{H}_{int} + \frac{\boldsymbol{P}^2}{2Am}$$

Hamiltonian represented in multiphonon basis

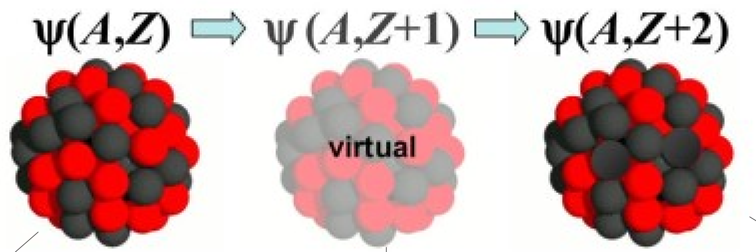
$$\boldsymbol{H}_{intr} = \sum_{n,\alpha} E^n_\alpha |n,\alpha\rangle\langle n,\alpha| + \sum_{nn',\alpha\alpha'} |n',\alpha'\rangle\langle n',\alpha'|\boldsymbol{H}_{intr}|n,\alpha\rangle\langle n,\alpha|$$

equiv. to STDA



Double Beta Decay & EMPM/STDA

“Mother“ nucleus

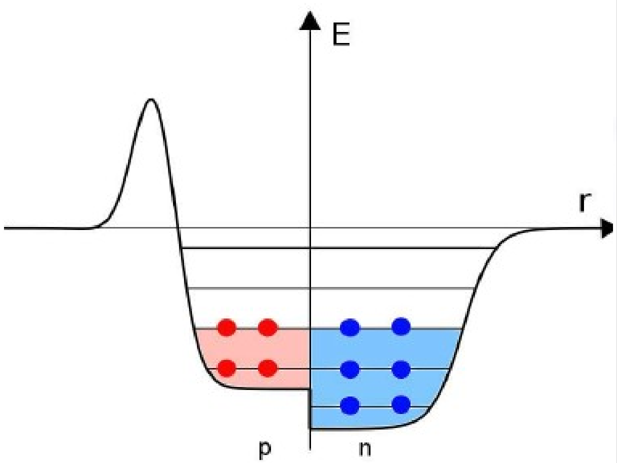


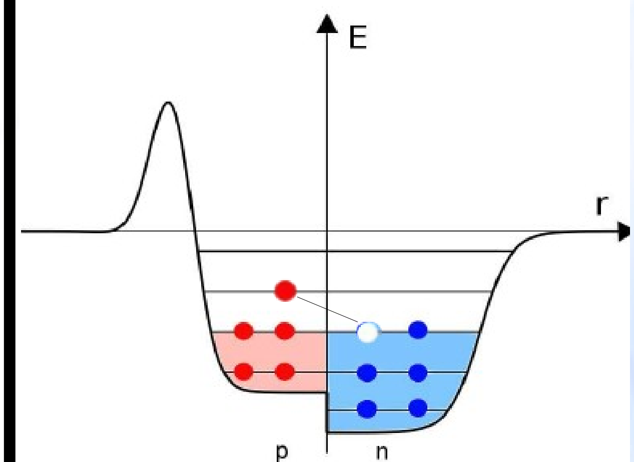
“Daughter“ nucleus

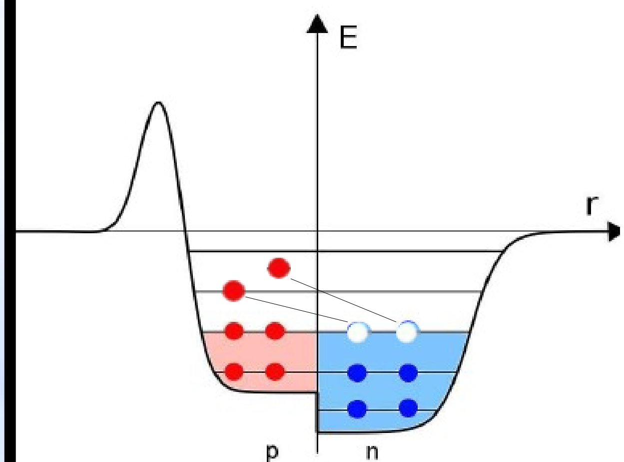
^{48}Ca

^{48}Sc

^{48}Ti







Ground state described within the mean-field |HF>

1p-1h proton-neutron proton-neutron TDA

2p-2h proton-neutron EMPM/STDA

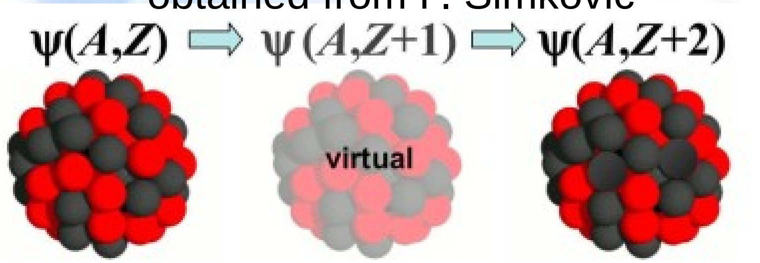
2νββ: $\left(T^{2\nu}_{1/2}\right)^{-1} = g_A^4 m_e^2 \left|M_{GT} - \frac{M_F}{g_A^2}\right|^2 G^{2\nu}$

$$M^{2\nu}_{F,GT} = \sum_n \frac{\langle f \| \mathcal{O}_{F,GT} \| J_n^+\rangle\langle J_n^+ \| \mathcal{O}_{F,GT} \| i\rangle}{E_n - (E_i+E_f)/2}$$

NME in ^{48}Ca – G-matrix of ArgonneV18

P.V. et al., *EPJ Web of Conf.* **342**, 01030 (2025).

^{48}Ca: mean-field **s.p. energies*** taken from phenomenological Wood-Saxon
NN interaction*: **G-matrix** on **Argonne V18** potential (**N$_{max}$=6**); $V^{NN} = V^{pp} + V^{nn} + g_{ph} {*} V^{pn}$
* obtained from F. Šimkovic



^{48}Ca → ^{48}Sc → ^{48}Ti
|HF> 1p-1h(pnTDA) 2p-2h(SecTDA) (EMPM)

2νββ: $$\left(T_{1/2}^{2\nu}\right)^{-1} = g_A^4 m_e^2 \left| M_{GT} - \frac{M_F}{g_A^2} \right|^2 G^{2\nu}$$

$$M_{F,GT}^{2\nu} = \sum_n \frac{\langle f \parallel \mathcal{O}_{F,GT} \parallel J_n^+ \rangle \langle J_n^+ \parallel \mathcal{O}_{F,GT} \parallel i \rangle}{E_n - (E_i + E_f)/2}$$

$$\mathrm{NME(total)} = g_A^2 m_e \left| M_{GT} - \frac{M_F}{g_A^2} \right|$$

$$\mathcal{O}_F = \sum_{k=1}^{A} \tau_k^+, \qquad \mathcal{O}_{GT} = \sum_{k=1}^{A} \tau_k^+ \sigma_{\mathbf{k}}.$$

$g_A = 1.2701$

^{48}Sc ground state:

pn TDA : $E_{g.s.}(6^+) = -1.219$ MeV

experiment: $Q_\beta = 0.2795$ MeV

^{48}Ti ground state:

STDA : $E_{g.s.}(0^+) = -4.4253$ MeV

EMPM: $E_{g.s.}(0^+) = -4.4253$ MeV

experiment: $Q_{\beta\beta} = 4.268$ MeV

NME 2νββ elements:

Isospin symmetry restoration required

STDA* :	$M_F = -0.0408$ MeV^{-1} ;	$M_{GT} = 0.1102$ MeV^{-1} ;	NME(total) = 0.1116
EMPM*:	$M_F = -0.0407$ MeV^{-1} ;	$M_{GT} = 0.1112$ MeV^{-1} ;	NME(total) = 0.1124
experiment**:			NME(total) = 0.0348

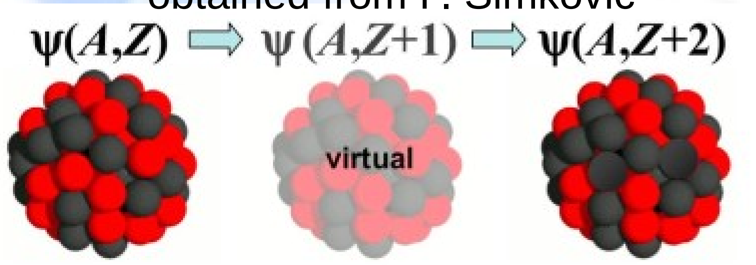
Quenching q = 0.40 (tot); **q = 0.62** (GT)

*Energies shifted to exper. **A. Barabash, **Universe 6**, 159, (2020).

NME in ^{48}Ca – G-matrix of ArgonneV18

P.V. et al., *EPJ Web of Conf.* **342**, 01030 (2025).

^{48}Ca: mean-field **s.p. energies*** taken from phenomenological Wood-Saxon
NN interaction*: **G-matrix** on **Argonne V18** potential (**N_{max}=6**); $V^{NN} = V^{pp} + V^{nn} + g_{ph} * V^{pn}$
* obtained from F. Šimkovic



^{48}Ca → ^{48}Sc → ^{48}Ti
|HF> 1p-1h(pnTDA) 2p-2h(SecTDA) (EMPM)

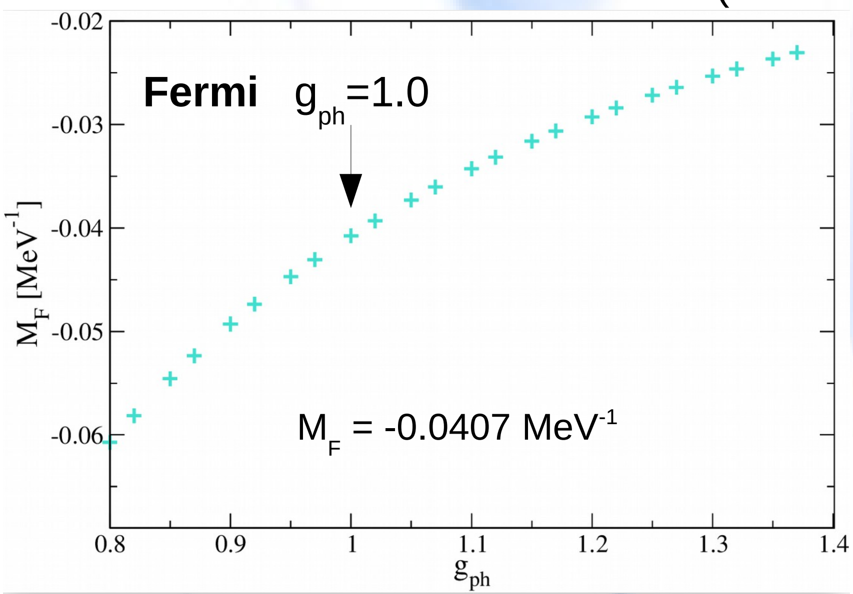
2νββ: $\left(T^{2\nu}_{1/2}\right)^{-1} = g_A^4 m_e^2 \left| M_{GT} - \frac{M_F}{g_A^2} \right|^2 G^{2\nu}$

$$M^{2\nu}_{F,GT} = \sum_n \frac{\langle f \| \mathcal{O}_{F,GT} \| J_n^+ \rangle \langle J_n^+ \| \mathcal{O}_{F,GT} \| i \rangle}{E_n - (E_i + E_f)/2}$$

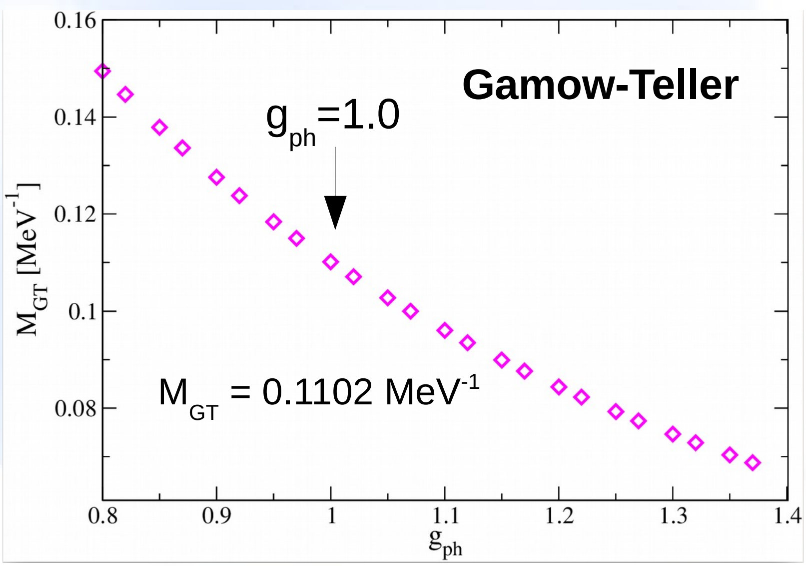
$$\mathrm{NME(total)} = g_A^2 m_e \left| M_{GT} - \frac{M_F}{g_A^2} \right|$$

$$\mathcal{O}_F = \sum_{k=1}^{A} \tau_k^+, \qquad \mathcal{O}_{GT} = \sum_{k=1}^{A} \tau_k^+ \sigma_{\mathbf{k}}.$$

$g_A = 1.2701$

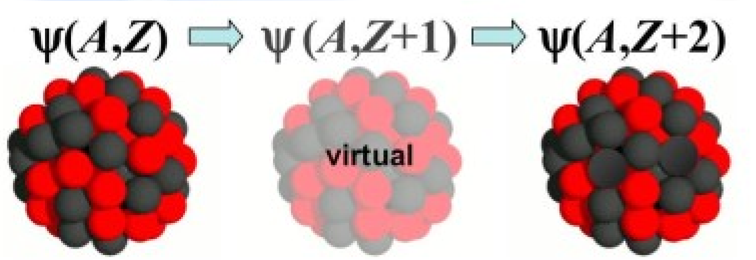


Parameter **g_{ph}** – scaling of the **proton-neutron (particle-hole)** interaction



NME in ^{48}Ca – G-matrix of ArgonneV18

^{48}Ca: mean-field **s.p. energies*** taken from phenomenological Wood-Saxon
NN interaction*: **G-matrix** on **Argonne V18** potential (**N_{max}=6**); $V^{NN} = V^{pp} + V^{nn} + g_{ph} * V^{pn}$
* obtained from F. Šimkovic



^{48}Ca → ^{48}Sc → ^{48}Ti
|HF> 1p-1h(pnTDA) 2p-2h(SecTDA) (EMPM)

2νββ: $$\left(T^{2\nu}_{1/2}\right)^{-1} = g_A^4 m_e^2 \left| M_{GT} - \frac{M_F}{g_A^2} \right|^2 G^{2\nu}$$

$$M^{2\nu}_{F,GT} = \sum_n \frac{\langle f \parallel \mathcal{O}_{F,GT} \parallel J_n^+ \rangle \langle J_n^+ \parallel \mathcal{O}_{F,GT} \parallel i \rangle}{E_n - (E_i + E_f)/2}$$

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$$\mathcal{O}_F = \sum_{k=1}^{A} \tau_k^+, \qquad \mathcal{O}_{GT} = \sum_{k=1}^{A} \tau_k^+ \sigma_{\mathbf{k}}.$$

g_A = 1.2701

^{48}Sc ground state:

pn TDA : $E_{g.s.}(6^+)$ = -1.219 MeV

experiment: Q_β = 0.2795 MeV

NME 2νββ elements:

STDA* : NME(total) = 0.1116

EMPM*: NME(total) = 0.1124

experiment**: NME = 0.0348

^{48}Ti ground state:

STDA : $E_{g.s.}(0^+)$ = -4.4253 MeV

EMPM: $E_{g.s.}(0^+)$ = -4.4253 MeV

experiment: $Q_{\beta\beta}$ = 4.268 MeV

Half-life:

Experiment**: $T_{1/2}(^{48}\mathrm{Ca}) = 5.3 \times 10^{19}$ years

Theory (EMPM): $T_{1/2}(^{48}\mathrm{Ca}) = 0.508 \times 10^{19}$ years

Quenching of **g_A** ~ 0.40 (NMEtot) necessary!!!

*Energies shifted to exper. **A. Barabash, **Universe 6**, 159, (2020).

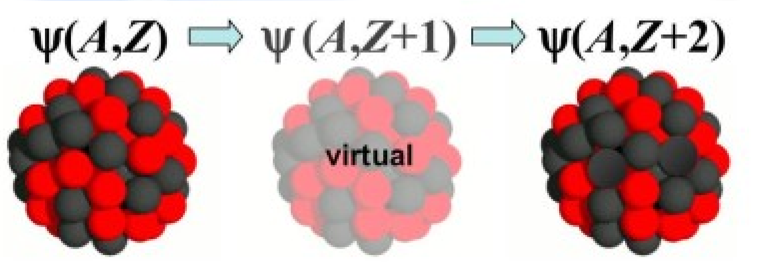
NME in ^{48}Ca – NNLO$_{opt}$

-900 -600 [MeV.fm^6]

P.V. et al., *Nuclear Theory* **42**, 183 (2025).

^{48}Ca: mean-field **s.p. energies** from **Hartree-Fock** calculation

NN interaction: **NNLO$_{opt}$** potential + V^{DD} (**N_{max}=6**); $V^{DD} = g_1 {*} V^{(S=1,T=0)} + g_0 {*} V^{(S=0,T=1)}$



^{48}Ca → ^{48}Sc → ^{48}Ti

|HF> 1p-1h(pnTDA) 2p-2h(SecTDA)

2νββ: $$\left(T_{1/2}^{2\nu}\right)^{-1} = g_A^4 m_e^2 \left| M_{GT} - \frac{M_F}{g_A^2} \right|^2 G^{2\nu}$$

$$M_{F,GT}^{2\nu} = \sum_n \frac{\langle f \parallel \mathcal{O}_{F,GT} \parallel J_n^+ \rangle \langle J_n^+ \parallel \mathcal{O}_{F,GT} \parallel i \rangle}{E_n - (E_i + E_f)/2}$$

$$\mathrm{NME(total)} = g_A^2 m_e \left| M_{GT} - \frac{M_F}{g_A^2} \right|$$

$$\mathcal{O}_F = \sum_{k=1}^{A} \tau_k^+, \qquad \mathcal{O}_{GT} = \sum_{k=1}^{A} \tau_k^+ \sigma_{\mathbf{k}}.$$

g_A = 1.2701

^{48}Sc ground state:

pn TDA : $E_{g.s.}(6^+)$ = -2.190 MeV

adjusted s.p. energies

experiment: Q_β = 0.2795 MeV

^{48}Ti ground state:

STDA : $E_{g.s.}(0^+)$ = -3.627 MeV

adjusted s.p. energies

experiment: $Q_{\beta\beta}$ = 4.268 MeV

NME 2νββ elements:

Isospin symmetry kind of OK

STDA* : M_F =0.00854 MeV^{-1} ; M_{GT} =0.172 MeV^{-1} ; NME(total) = 0.1374

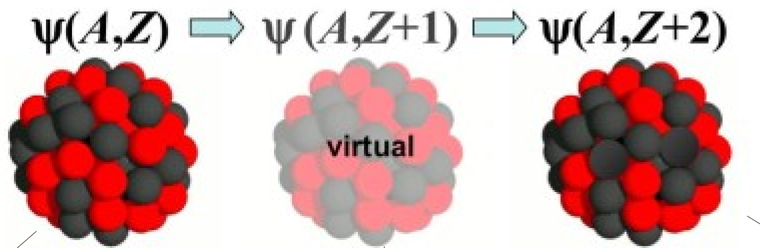
experiment**: NME(total) = 0.0348

*Energies shifted to exper. **A. Barabash, **Universe 6**, 159, (2020).

Quenching: q = 0.51 (GT)

Double Beta Decay & EMPM/STDA

"Mother" nucleus

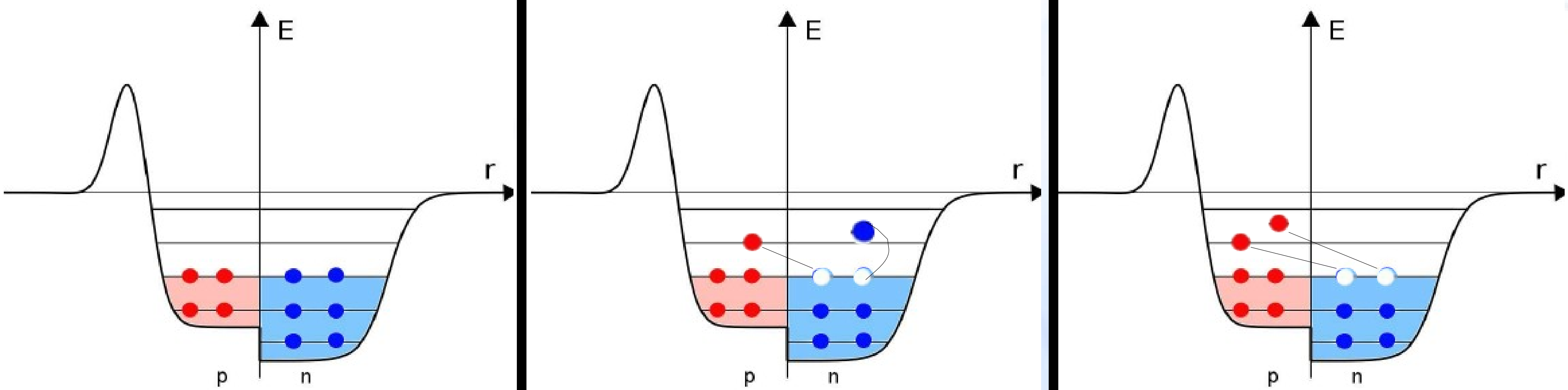


"Daughter" nucleus

^{48}Ca

^{48}Sc

^{48}Ti



Ground state described within the mean-field |HF>

2p-2h (proton-neutron & nucl-nucl) EMPM/STDA

2p-2h proton-neutron EMPM/STDA

Quenching of g_A is supposed to be effect of many-body correlations → **2p-2h** configurations in ^{48}**Sc**.

2νββ: $\left(T^{2\nu}_{1/2}\right)^{-1} = g_A^4 m_e^2 \left| M_{GT} - \frac{M_F}{g_A^2} \right|^2 G^{2\nu}$ $\qquad M^{2\nu}_{F,GT} = \sum_n \frac{\langle f \parallel \mathcal{O}_{F,GT} \parallel J_n^+ \rangle \langle J_n^+ \parallel \mathcal{O}_{F,GT} \parallel i \rangle}{E_n - (E_i + E_f)/2}$

Partial Summary

- We discussed **double beta decay (DBD)** within **nuclear** physics. Precise theoretical calculations of **nuclear matrix elements** (**NME**) of beta transitions crucial for study of **0νββ** & **2νββ DBD.**

- We discussed computational **nuclear structure methods** which start and/or go **beyond mean-field** approximation: Hartree-Fock (**HF**), Tamm Dancoff (**TDA**), Random-Phase Approximation (**RPA**), **Renormalized RPA**, Second Tamm Dancoff (**STDA**), Equation of Motion Phonon Method (**EMPM**).

- Description of **DBD** in ^{48}**Ca** within:

  Hamiltonian = Wood-Saxon s.p. + G-matrix of **Argonne V18** potential;

  Methods = **STDA/EMPM**

- Description of **DBD** in ^{48}**Ca** within:

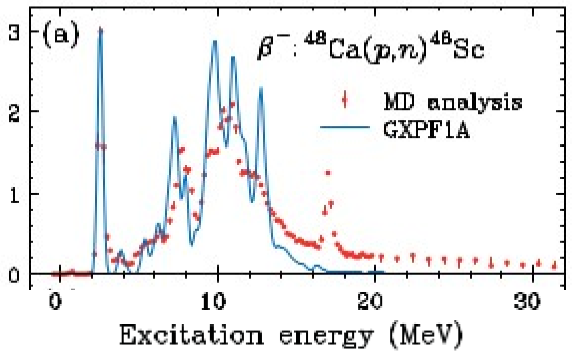
  Hamiltonian = **HF** s.p. + $\mathbf{NNLO_{opt}}$ + $\mathbf{V^{DD}}$ potential;

  Methods = **HF**+**STDA**

What next?

GT resonances in ^{48}Ca – NNLO$_{opt}$

Experiment:
Phys. Rev. Lett. 103, 012503 (2009).
^{48}Ca(p,n)^{48}Sc

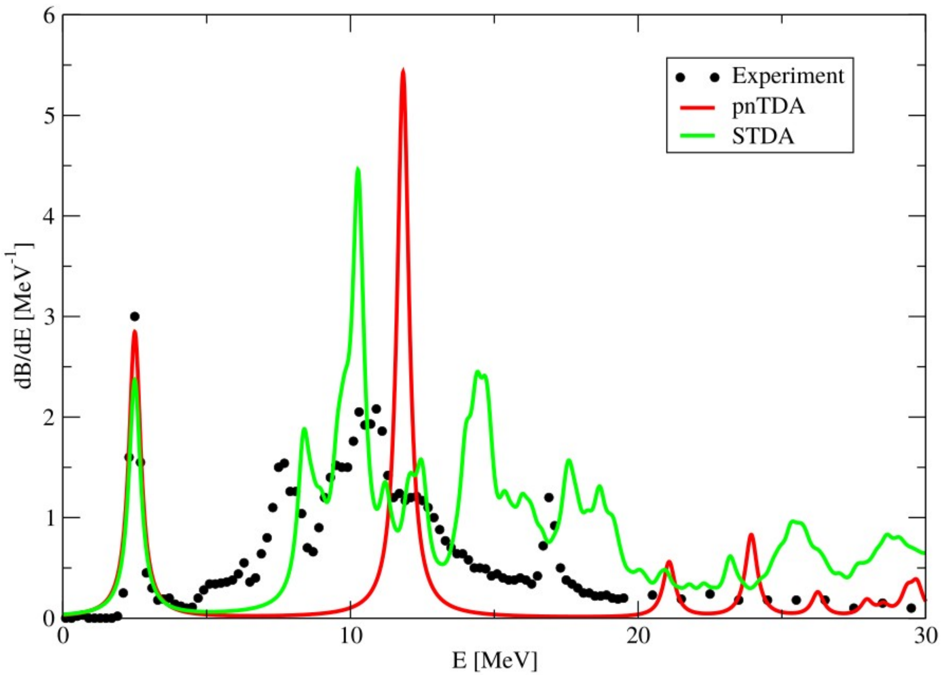


$E_{g.s.}(^{48}Sc)$ = 5.24 MeV
in **pnTDA**
$E_{g.s.}(^{48}Sc)$ = 0.809 MeV
in **STDA**

$E_{g.s.}(^{48}Sc)$ =-0.791 MeV
by **experiment**

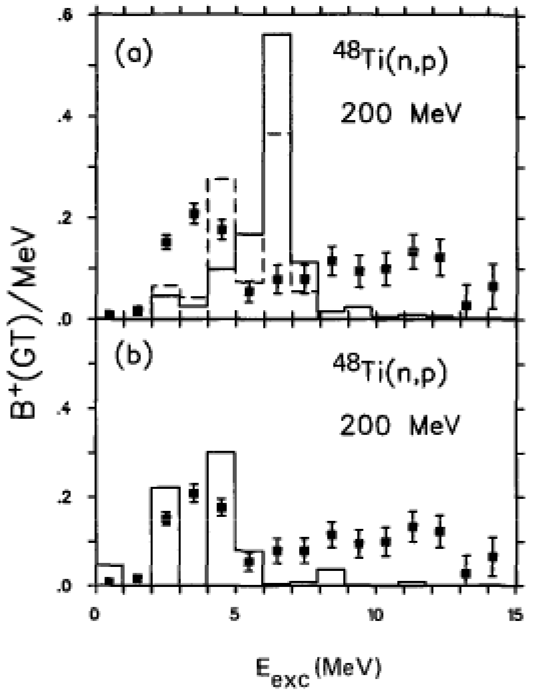
HF + pnTDA + STDA
NN interaction: **NNLO$_{opt}$** potential + V^{DD};
$V^{DD} = g_1 * V^{(S=1,T=0)} + g_0 * V^{(S=0,T=1)}$
$g_1 = 0.0$; $g_0 = 0.0$

NNLO$_{opt}$ – **Phys. Rev. Lett.** 110,192502 (2013).
DD term – **J. Phys. G.** 41, 025109 (2014).

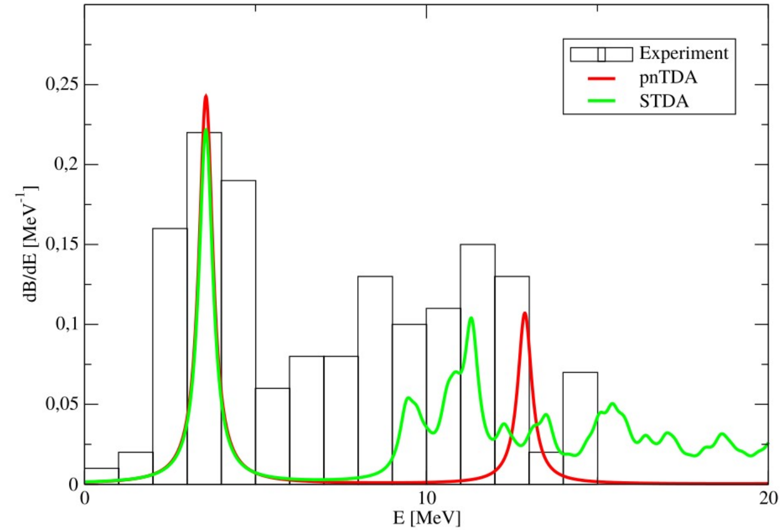


GT resonances in ^{48}Ti – NNLO$_{opt}$

Experiment:
Nucl. Phys. A 514, 49 (1990).
^{48}Ti(n,p)^{48}Sc



HF + pnTDA + STDA
NN interaction: **NNLO$_{opt}$** potential + V^{DD};
$V^{DD} = g_1 * V^{(S=1,T=0)} + g_0 * V^{(S=0,T=1)}$
$g_1 = 0.0$; $g_0 = 0.0$



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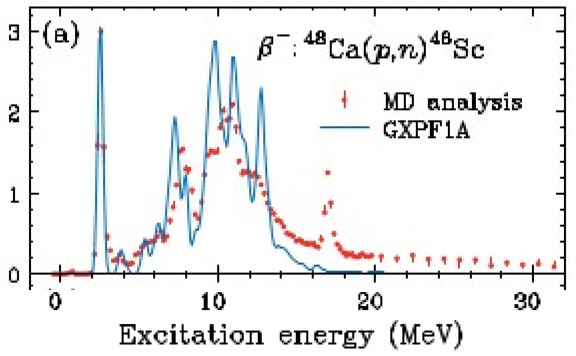
$E_{g.s.}(^{48}Sc) = 0.809$ MeV
in **STDA**

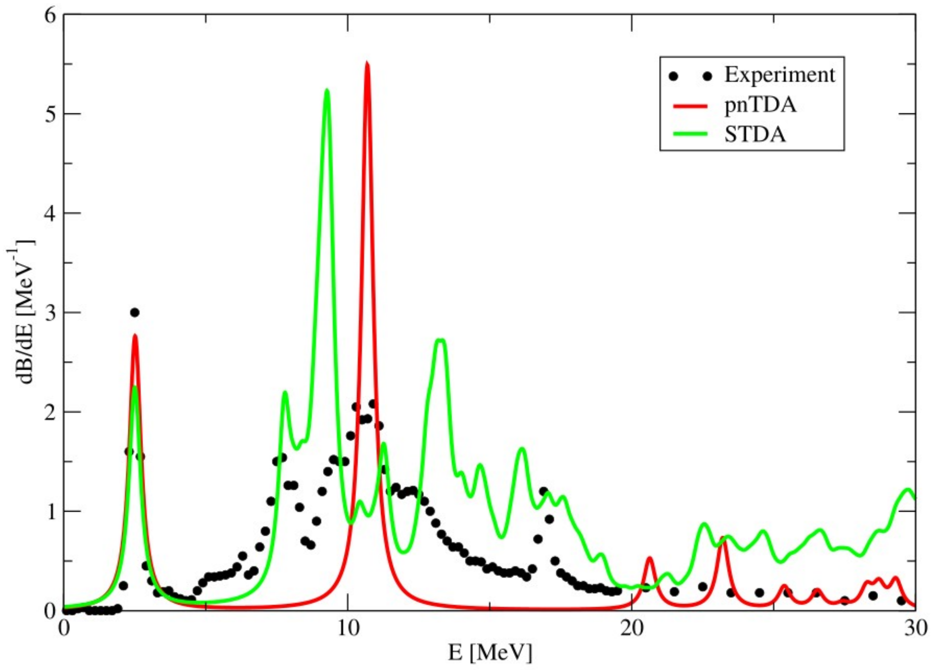
$E_{g.s.}(^{48}Sc) = -0.791$ MeV
by **experiment**

GT resonances in ^{48}Ca – NNLO$_{opt}$

Experiment:
Phys. Rev. Lett. 103, 012503 (2009).
^{48}Ca(p,n)^{48}Sc



HF + pnTDA + STDA
NN interaction: **NNLO$_{opt}$** potential + V^{DD};
$V^{DD} = g_1 * V^{(S=1,T=0)} + g_0 * V^{(S=0,T=1)}$
$g_1 = 200.0$; $g_0 = 0.0$

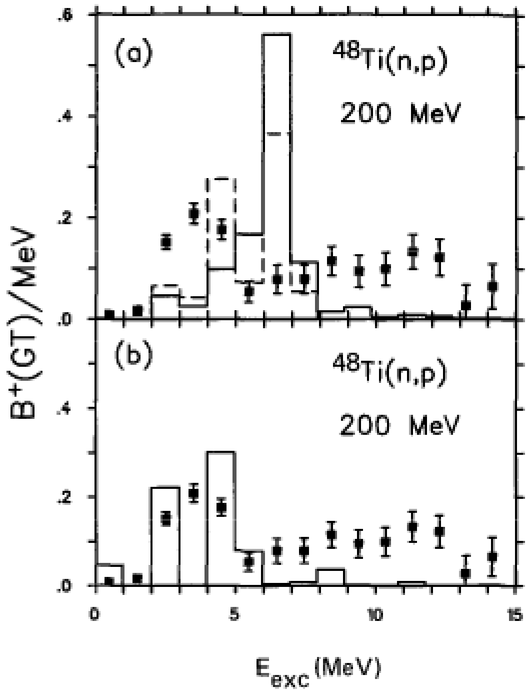


$E_{g.s.}(^{48}Sc) = 5.36$ MeV
in **pnTDA**
$E_{g.s.}(^{48}Sc) = 1.06$ MeV
in **STDA**

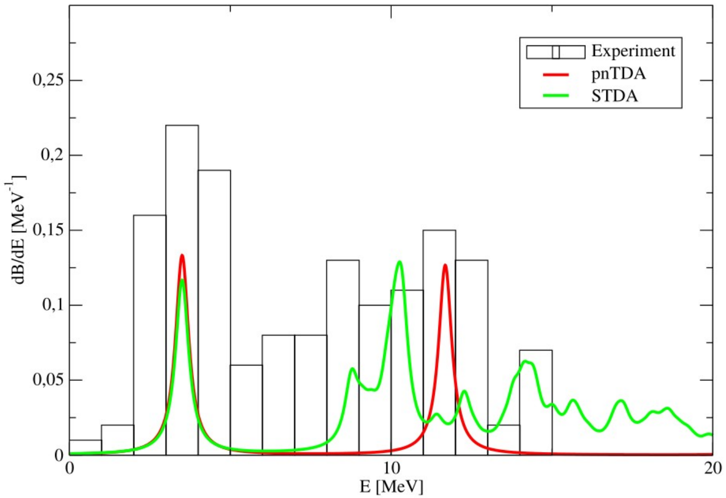
$E_{g.s.}(^{48}Sc) = -0.791$ MeV
by **experiment**

GT resonances in ^{48}Ti – NNLO$_{opt}$

Experiment:
Nucl. Phys. A 514, 49 (1990).
^{48}Ti(n,p)^{48}Sc



HF + pnTDA + STDA
NN interaction: **NNLO$_{opt}$** potential + V^{DD};
$V^{DD} = g_1 {*} V^{(S=1,T=0)} + g_0 {*} V^{(S=0,T=1)}$
$g_1 = 200.0$; $g_0 = 0.0$



$E_{g.s.}(^{48}Sc) = 5.36$ MeV
in **pnTDA**
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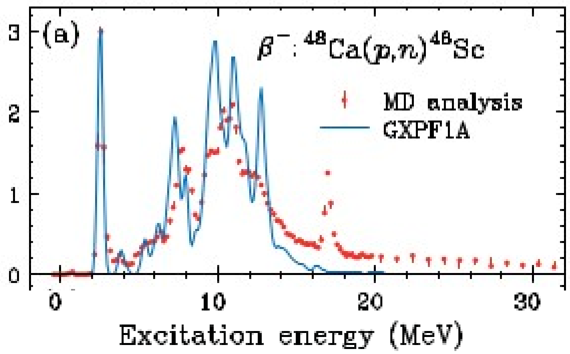
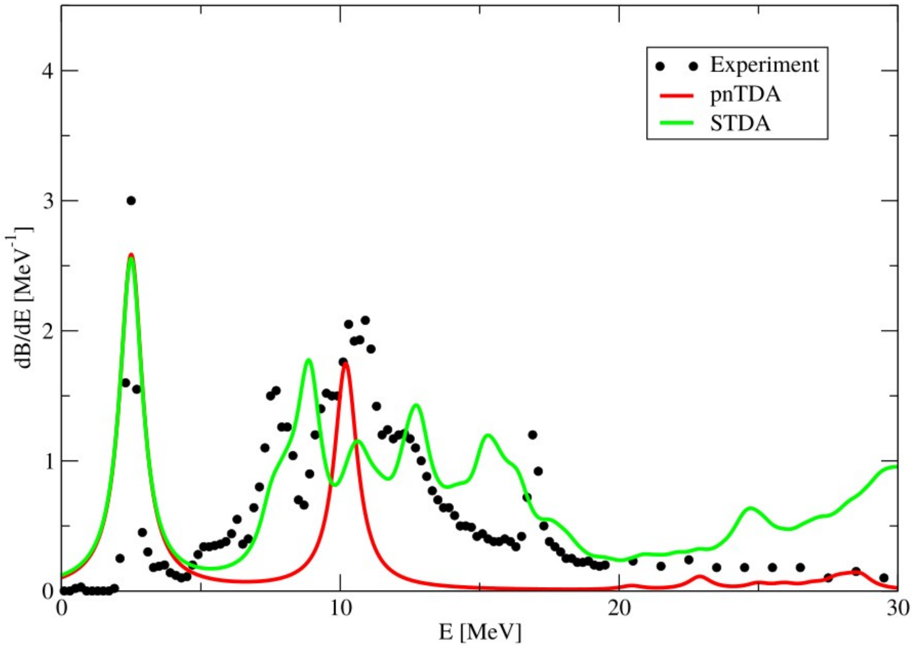
$E_{g.s.}(^{48}Sc) = -0.791$ MeV
by **experiment**

GT resonances in ^{48}Ca – NNLO$_{opt}$

Experiment:
Phys. Rev. Lett. 103, 012503 (2009).
^{48}Ca(p,n)^{48}Sc



HF + pnTDA + STDA
NN interaction: **NNLO$_{opt}$** potential + V^{DD};
$V^{DD} = g_1 {}^*V^{(S=1,T=0)} + g_0 {}^*V^{(S=0,T=1)}$
$g_1 = 300.0$; $g_0 = 0.0$

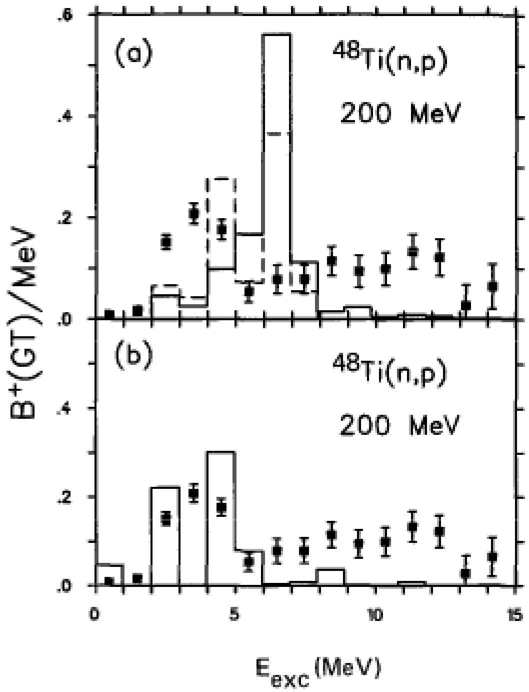


$E_{g.s.}$(^{48}Sc) = 5.40 MeV
in **pnTDA**
$E_{g.s.}$(^{48}Sc) = 1.15 MeV
in **STDA**

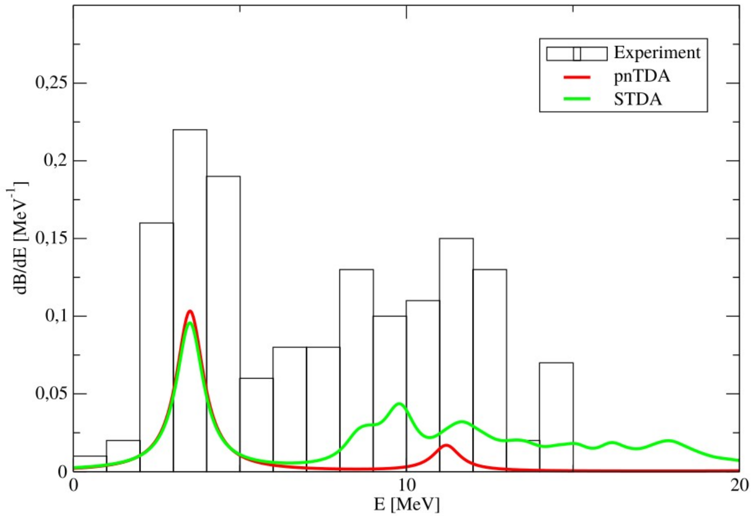
$E_{g.s.}$(^{48}Sc) =-0.791 MeV
by **experiment**

GT resonances in ^{48}Ti – NNLO$_{opt}$

Experiment:
Nucl. Phys. A 514, 49 (1990).
^{48}Ti(n,p)^{48}Sc



HF + pnTDA + STDA
NN interaction: **NNLO$_{opt}$** potential + V^{DD};
$V^{DD} = g_1 {*} V^{(S=1,T=0)} + g_0 {*} V^{(S=0,T=1)}$
$g_1 = 300.0$; $g_0 = 0.0$



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in **pnTDA**
$E_{g.s.}(^{48}Sc) = 1.15$ MeV
in **STDA**

$E_{g.s.}(^{48}Sc) = -0.791$ MeV
by **experiment**

2ν DBD NMEs in ^{48}Ca – NNLO$_{opt}$

HF + pnTDA + STDA

NN interaction: **NNLO$_{opt}$** potential + V^{DD};

$V^{DD} = g_1 * V^{(S=1,T=0)} + g_0 * V^{(S=0,T=1)}$

$g_1 = 300.0$; $g_0 = 0.0$

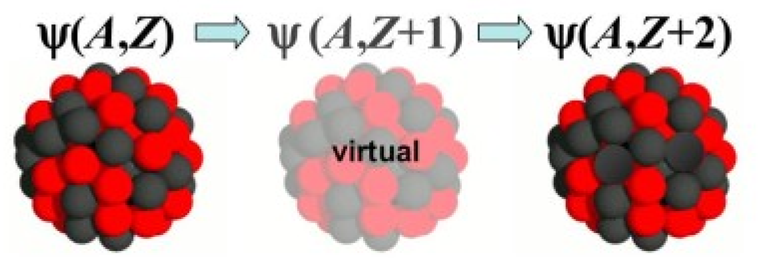
2νββ: $$\left(T_{1/2}^{2\nu}\right)^{-1} = g_A^4 m_e^2 \left| M_{GT} - \frac{M_F}{g_A^2} \right|^2 G^{2\nu}$$

$$M_{F,GT}^{2\nu} = \sum_n \frac{\langle f \parallel \mathcal{O}_{F,GT} \parallel J_n^+ \rangle \langle J_n^+ \parallel \mathcal{O}_{F,GT} \parallel i \rangle}{E_n - (E_i + E_f)/2}$$

$$\mathrm{NME(total)} = g_A^2 m_e \left| M_{GT} - \frac{M_F}{g_A^2} \right|$$

$$\mathcal{O}_F = \sum_{k=1}^{A} \tau_k^+, \qquad \mathcal{O}_{GT} = \sum_{k=1}^{A} \tau_k^+ \sigma_{\mathbf{k}}.$$

$g_A = 1.2701$



^{48}Ca	→	^{48}Sc	→	^{48}Ti
I) |HF>		1p-1h(pnTDA)		2p-2h(SecTDA)
II) |HF>		2p-2h(STDA)		2p-2h(SecTDA)

$M_F \neq 0.0$ => **isospin symmetry** violation for $g_1 = 300.0$; $g_0 = 0.0$

Most of the strength of M_F carried by first 0^+ state in ^{48}Sc...

NME 2νββ elements:

I) STDA* : $M_F = 0.0476$ MeV^{-1} ; $M_{GT} = 0.135$ MeV^{-1} ; NME(total) = 0.0546;

II) STDA* : $M_F = 0.0321$ MeV^{-1} ; $M_{GT} = 0.120$ MeV^{-1} ; NME(total) = 0.0510;

experiment**: NME(total) = 0.0348

*Energies shifted to exper. **A. Barabash, **Universe 6**, 159, (2020).

2ν DBD NMEs in ^{48}Ca – NNLO$_{opt}$

HF + pnTDA + STDA

NN interaction: **NNLO$_{opt}$** potential + V^{DD};

$V^{DD} = g_1 * V^{(S=1,T=0)} + g_0 * V^{(S=0,T=1)}$

$g_1 = 300.0$; $g_0 = 0.0$

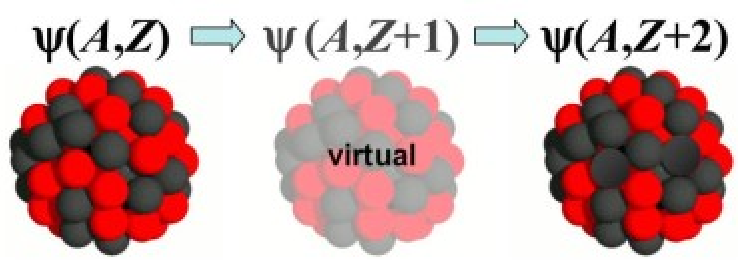
2νββ: $\left(T_{1/2}^{2\nu}\right)^{-1} = g_A^4 m_e^2 \left| M_{GT} - \frac{M_F}{g_A^2} \right|^2 G^{2\nu}$

$$M_{F,GT}^{2\nu} = \sum_n \frac{\langle f \parallel \mathcal{O}_{F,GT} \parallel J_n^+ \rangle \langle J_n^+ \parallel \mathcal{O}_{F,GT} \parallel i \rangle}{E_n - (E_i + E_f)/2}$$

$$\mathrm{NME(total)} = \mathrm{g_A^2 m_e} \left| \mathrm{M_{GT}} - \frac{\mathrm{M_F}}{\mathrm{g_A^2}} \right|$$

$$\mathcal{O}_F = \sum_{k=1}^{A} \tau_k^+, \qquad \mathcal{O}_{GT} = \sum_{k=1}^{A} \tau_k^+ \sigma_{\mathbf{k}}.$$

$g_A = 1.2701$



^{48}Ca	→ ^{48}Sc	→ ^{48}Ti
I) |HF>	1p-1h(pnTDA)	2p-2h(SecTDA)
II) |HF>	2p-2h(STDA)	2p-2h(SecTDA)

$M_F \approx 0.0$ => **isospin symmetry** restored if we **orthogonalize** the pn TDA spectrum of 0+ with respect to the state $|\lambda_0\rangle = \frac{1}{N_0} \sum_r C_{rr}^{\lambda_0} |(r \times s)^{0^+}\rangle,$ $C_{rr}^{\lambda_0} = \sqrt{2[r]}(u_r v_r)$

In analogy to the orthogonalization method for CM spuriosity in **J. Phys. G.** 41, 025109 (2014).

NME 2νββ elements:

I) STDA* : $M_F \approx 0.0$ MeV^{-1} ; $M_{GT} = 0.135$ MeV^{-1} ; NME(total) = 0.135; *quenching* q = 0.560

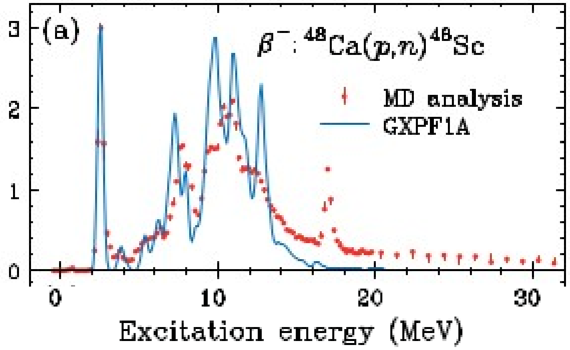
II) STDA* : $M_F \approx 0.0$ MeV^{-1} ; $M_{GT} = 0.120$ MeV^{-1} ; NME(total) = 0.120; *quenching* q = 0.594

experiment**: NME(total) = 0.0348

*Energies shifted to exper. **A. Barabash, **Universe 6**, 159, (2020).

GT resonances in ^{48}Ca – $\Delta N^2LO_{GO}(394)$

Experiment:
Phys. Rev. Lett. 103, 012503 (2009).
^{48}**Ca(p,n)**^{48}**Sc**

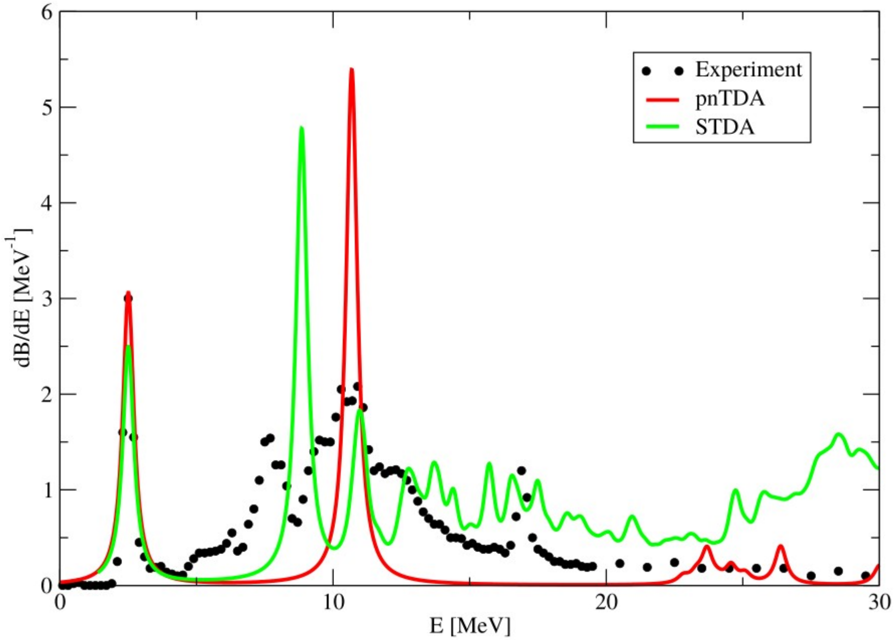


$E_{g.s.}(^{48}Sc)$ = 3.94 MeV
in **pnTDA**
$E_{g.s.}(^{48}Sc)$ = -0.917 MeV
in **STDA**

$E_{g.s.}(^{48}Sc)$ =-0.791 MeV
by **experiment**

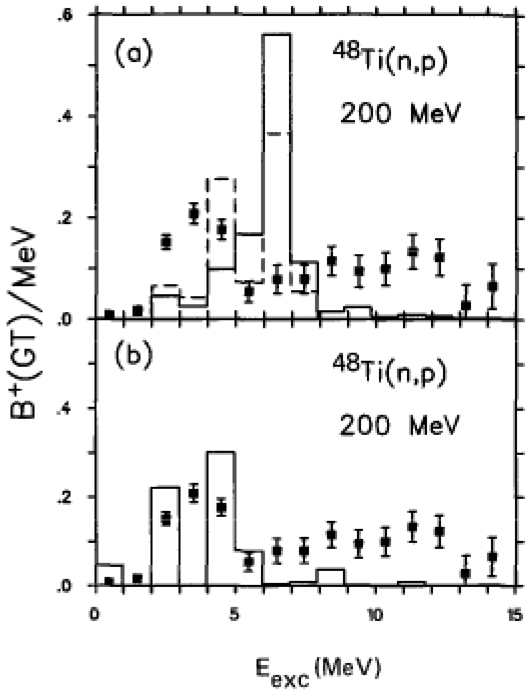
HF + pnTDA + STDA
NN+NNN interaction: $\Delta N^2LO_{GO}(394)$

$\Delta N^2LO_{GO}(394)$ – **Phys. Rev. C** 102, 054301 (2020).



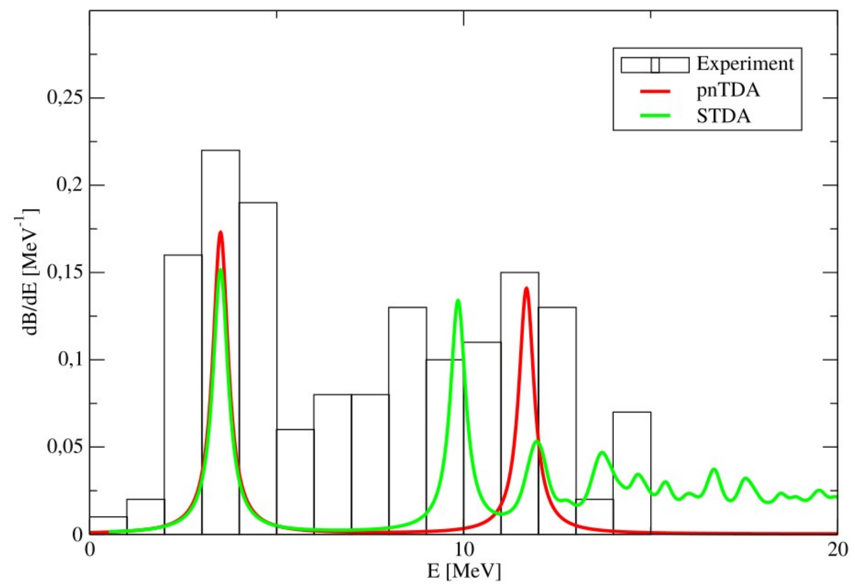
GT resonances in ^{48}Ti – ΔN^2LO_{GO}(394)

Experiment:
Nucl. Phys. A 514, 49 (1990).
^{48}Ti(n,p)^{48}Sc



HF + pnTDA + STDA
NN+NNN interaction: **ΔN^2LO_{GO}(394)**

ΔN^2LO_{GO}(394) – **Phys. Rev. C** 102, 054301 (2020).



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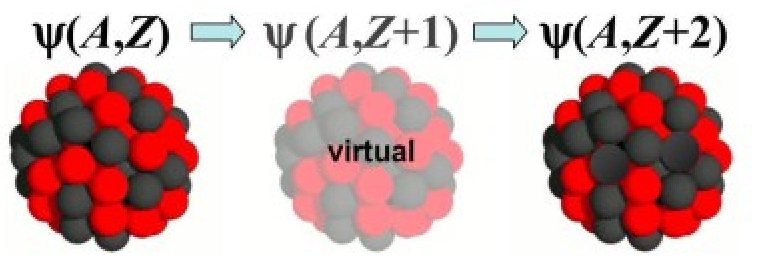
by **experiment**

2ν DBD NMEs in ^{48}Ca – $\Delta N^2LO_{GO}(394)$

HF + pnTDA + STDA
NN+NNN interaction: **$\Delta N^2LO_{GO}(394)$**

$\Delta N^2LO_{GO}(394)$ – **Phys. Rev. C** 102, 054301 (2020).



^{48}Ca →	^{48}Sc →	^{48}Ti
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2νββ: $$\left(T^{2\nu}_{1/2}\right)^{-1} = g_A^4 m_e^2 \left| M_{GT} - \frac{M_F}{g_A^2} \right|^2 G^{2\nu}$$

$$M^{2\nu}_{F,GT} = \sum_n \frac{\langle f \parallel \mathcal{O}_{F,GT} \parallel J_n^+ \rangle \langle J_n^+ \parallel \mathcal{O}_{F,GT} \parallel i \rangle}{E_n - (E_i + E_f)/2}$$

$$\mathrm{NME(total)} = \mathrm{g_A^2 m_e} \left| \mathrm{M_{GT}} - \frac{\mathrm{M_F}}{\mathrm{g_A^2}} \right|$$

$$\mathcal{O}_F = \sum_{k=1}^{A} \tau_k^+, \qquad \mathcal{O}_{GT} = \sum_{k=1}^{A} \tau_k^+ \sigma_{\mathbf{k}}.$$

$g_A = 1.2701$

$M_F \approx 0.0$ => **isospin symmetry** restored if we **orthogonalize** the pn TDA spectrum of 0+ with respect to the state

$$|\lambda_0\rangle = \frac{1}{N_0} \sum_r C^{\lambda_0}_{rr} |(r \times s)^{0^+}\rangle,$$

$$C^{\lambda_0}_{rr} = \sqrt{2[r]}(u_r v_r)$$

In analogy to the orthogonalization method for CM spuriosity in **J. Phys. G.** 41, 025109 (2014).

NME 2νββ elements:

I) STDA* : $M_F \approx 0.0$ MeV^{-1} ; $M_{GT} = 0.124$ MeV^{-1} ; NME(total) = 0.124; *quenching* q = 0.583

II) STDA* : $M_F \approx 0.0$ MeV^{-1} ; $M_{GT} = 0.104$ MeV^{-1} ; NME(total) = 0.104; *quenching* q = 0.638

experiment**: NME(total) = 0.0348

*Energies shifted to exper. **A. Barabash, **Universe 6**, 159, (2020).

Summary & Outlook

- **Double beta decay (DBD)** within **particle (neutrino)** physics. Possible exotic modes (**$0\nu\beta\beta$**) of **DBD** is important for search of physics beyond **Standard Model**.

- **Double beta decay (DBD)** within **nuclear** physics. Precise theoretical calculations of **nuclear matrix elements** (**NME**) of beta transitions crucial for study of **$0\nu\beta\beta$** & **$2\nu\beta\beta$ DBD.**

- We discussed computational **nuclear structure methods** which start and/or go **beyond mean-field** approximation: Hartree-Fock (**HF**), Tamm Dancoff (**TDA**) & Second Tamm Dancoff (**STDA**), Equation of Motion Phonon Method (**EMPM**).

- Possible application of STDA, SRPA, EMPM to calculate **NME** in **Double Beta Decay**. Preliminary results of **NME** in **^{48}Ca**.

- Study of **Gamow-Teller resonances** in ^{48}Ca and ^{48}Ti.

- **Next Goals**:

  - Generalization of our methods to **quasiparticle formalism** (**^{76}Ge, ^{82}Se, ...**).

Thank you for attention!!!