Chaos and Topology in the Sachdev-Ye-Kitaev Model

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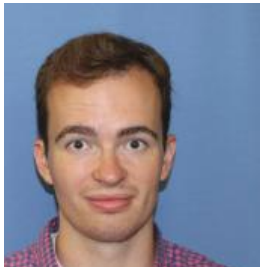

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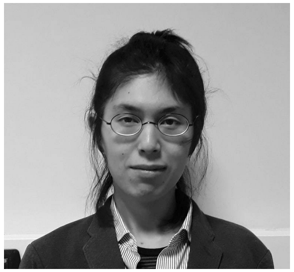

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Contents

I. Introduction

I. Topology of Random Matrrix Theories

II. Classification of Random Matrix Theories

Basics of Topological Insulators

Second quantized Hamiltonian

$$H = \sum_{\alpha\beta} H_{\alpha\beta}\psi_\alpha\psi_\beta,$$

where $H_{\alpha\beta}$ is a lattice Hamiltonian with disorder.

The idea is that the d-dimensional bulk is disordered so that the wave functions are localized but the edge states are not localized

The edge states are described by a nonlinear σ model of Goldstone bosons based on a coset G/H.

Nontrivial ground states exists if the homotopy group

$$\pi_{d-1}(G/H) \neq 0.$$

Kaplan-1992, Golterman-Jansen-Kaplan-1992,
Schnyder-Ryu-Fursaku-Ludwig-2008, Kitaev-2008, Ludwig-2012

Possible Issues

- The many-body Hamiltonian may have different localization properties than the single particle Hamiltonian. At best the particle-hole excitations might be in the universality class of the single-particle Hamiltonian
- Interactions may change the universality class Fidkowski-Kitaev-2011
- Fermi-liquid theory underlies the theory of (topological) insulators. We are interested non-Fermi liquids where concepts like a Fermi-sphere and single-particle excitations are invalid
- Nonlinear sigma models can only be realized if the microscopic theory is chaotic or disordered
- The low energy sector of a physical system may be a transition between two different sigma models

Random Matrix Theories

Nonlinear sigma models are in one-to-one correspondence to random matrix theories which can be classified according to anti-unitary and involutive symmetries:

- 3 Wigner-Dyson ensembles $\begin{pmatrix} H = H^\dagger \end{pmatrix}$
- 3 Chiral ensembles JV-Shuryak-1991, JV-1994 $\begin{pmatrix} 0 & H \\ H^\dagger & 0 \end{pmatrix}$
- Anti-symmetric and anti-selfdual Hermitian matrices Mehta-Rosenzweig-1968, Mehta-Pandey-1983
- Chiral ensembles where the off-diagonal block is complex symmetric or complex anti-symmetric Altland-Zirnbauer, PRL 76 (1996) 3420

The classification also corresponds to the classification of Dirac operators in three dimensions for the Wigner-Dyson ensembles, in four dimensions for the chiral ensembles, and in two or five dimensions for

RMT and Topology

Each of the RMT's has a corresponding nonlinear sigma model based the large symmetric spaces in the Cartan classification. For the complex random matrix theories we have:

RMT	Physics	G/H	$\Pi_2(G/H)$	$\Pi_3(G/H)$
GUE (A)	QCD_3	U(2n)/U(n)$\times$U(n)$^{(*)}$	$\mathcal{Z}$	0
chGUE (AIII)	QCD_4	U(n)$\times$ U(n)/U(n)	0	$\mathcal{Z}$

$^{(*)}$ Other symmetry breaking patterns are possible in the presence of a Chern-Simons term, U(2n) $\to$ U(n+p)$\times$ U(n-p).
Komargodski-Seiberg-2017, Kanazawa-Kieburg-JV-2021

- A non-linear sigma model based on G/H requires the presence of disorder or chaos.
- The nonlinear sigma model of the chiral ensembles first entered the literature in the context of lattice QCD Kogut-etal-1983 and sublattice symmetry Gade 1993

Phase Diagram of QCD$_3$

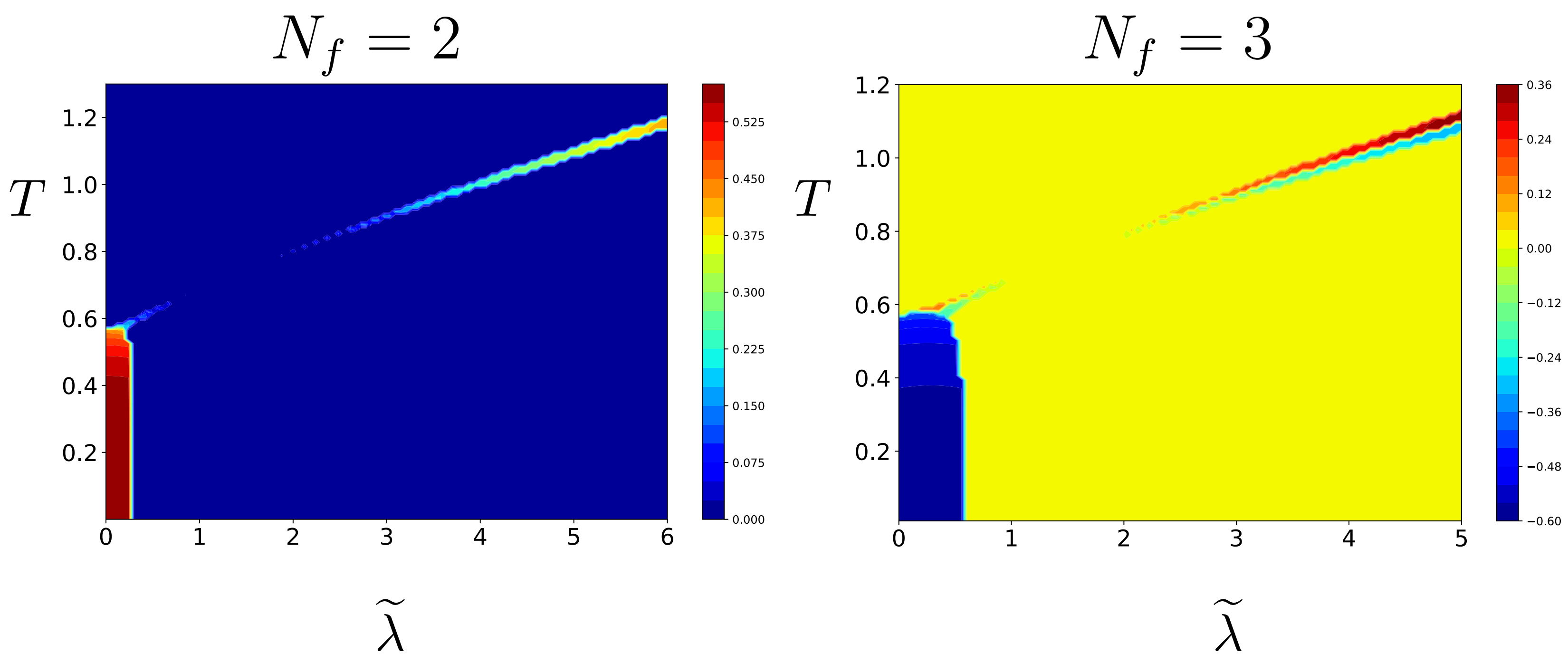


Figure 1: The phase diagram for $N_f = 2$ and $N_f = 3$ at $\mu = 0$ with $\widetilde{g}_1 = 1$ and $\widetilde{g}_2 = 3.75$ in the large-N limit. The magnitude of $|e_1 - e_2|$ is plotted. The strip of first order transitions for high temperature is interrupted roughly between $\widetilde{\lambda} = 1$ and $\widetilde{\lambda} = 2$, but is present close to the broken phase around the origin and at high temperatures.

Kanazawa-Kieburg-JV-2021

Goals and Questions

- We have seen that topological insulators can be classified according to random matrix theories. This requires the presence of disorder or chaos.
- We will now study how the classification is constrained for non-Fermi liquid many-body systems.
- When are many-body systems fully quantum chaotic?
- How does dissipation affect the possible phases of matter?

I. Introduction

The SYK Model

Variants of the SYK Model

Coupled SYK Models

The Sachdev-Ye-Kitaev (SYK) Model

The SYK model is a model of N interacting Majorana fermions. For $q = 4$ the model is given Kitaev-2015

$$H = \sum_{\alpha<\beta<\gamma<\delta} W_{\alpha\beta\gamma\delta}\chi_\alpha\chi_\beta\chi_\gamma\chi_\delta, \qquad q = 4.$$

The Majorana operators satisfy the commutation relations

$$\{\chi_\alpha, \chi_\beta\} = \frac{1}{2}\delta_{\alpha\beta}, \quad \chi_{2k} = \frac{1}{\sqrt{2}}(a_k + a_k^\dagger), \qquad \chi_{2k-1} = \frac{i}{\sqrt{2}}(a_k - a_k^\dagger).$$

The two-body matrix elements are taken to be Gaussian distributed with variance that is chosen such that the ground state energy scales with N. The model does not conserve particle number.

The Hilbert space is $2^{N/2}$ dimensional.

The Complex SYK Model

A variant of the SYK model introduced by French and Wong (1970) and Bohigas and Flores (1971) in the context of nuclear physics was generalized by Mon and French (1975) to the complex SYK model,

$$H = \sum_{\alpha\beta\gamma\delta} W_{\alpha\beta\gamma\delta} a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\delta .$$

The labels of the fermionic creation and annihilation operators run over N single particle states. The Hilbert space is given by all many particle states containing m particles with $m = 0, 1, \cdots, N$.

- $W_{\alpha\beta\gamma\delta}$ is Gaussian random.
- The Hamiltonian is particle number conserving.
- The matrix elements of the Hamiltonian are strongly correlated.

Brody-et-al-1981, Brown-Zelevinsky-Horoi-Frazier-1997, Izrailev-1990,Kota-2001,Benet-Weidenmüller-2002,Zelevinsky-Volya-2004, Borgonovi-Izrailev-Santos-Zelevinsky-2016

Motivation for Studying the SYK Model

- Model of compound nuclei
- Model for many-body quantum chaos
- It is a non-Fermi liquid with a zero temperature entropy that is extensive
- At low temperatures, it saturates the MSS bound for the Lyapunov exponent

  $$\lambda_L < \frac{2\pi\beta}{\hbar}$$

  which is Lyapunov exponent for shockwaves in a black hole.
- The low temperature limit of the SYK model has a reparameterization invariance which leads to the Schwatzian action which is also the low energy limit of Jackiw-Teitelboim gravity.

Classification of the SYK Model

- Follows from the anti-unitary and chiral symmetries symmetries of the gamma matrices

$$C_1 = \prod_k i\gamma_{2k} K, \quad C_2 = \prod_k i\gamma_{2k+i} K. \quad \gamma_c = i^{N^2/4} \prod_k \gamma_{2k},$$

$$C_1^2 = \pm 1, \quad C_2^2 = \pm 1, \quad \gamma_c C_1 \pm C_1 \gamma_c = 0.$$

- This gives 8 possibilities and is the reason that we have Bott periodicity.
- For the SYK model with $q = 4$ only three possibilities are realized. The reason is that γ_c is a unitary symmetry, and C_1 may not commute with the chiral projector.
- E.g. For $N = 2$, $\chi_1 = \sigma_1$, $\chi_2 = \sigma_2$, $\gamma_c = i\sigma_1\sigma_2$ and $P_c = \frac{1}{2}(1 \pm i\sigma_1\sigma_2)$ but $C_1 = \sigma_2 K$ and $[P_c, C_1] \neq 0$.

III. Chaos in the SYK Model

Bohigas Conjecture

OTOC

Spectral Statistics

Spectral Form Factor

Bohigas Conjecture

According to the Bohigas-Giannoni-Schmidt conjecture the eigenvalues of a classically chaotic quantum system are correlated according to Random Matrix theory Bohigas-Giannoni-Schmidt-1984, Seligman-JV-Zirnbauer-1984, JV-Seligman-1985.

Classically, chaoticity is measured by the Lyapunov exponent, the exponential rate of divergence of classical trajectories. The quantum equivalent of that is the Out of Time Order Correlator (OTOC)

$$\langle \mathrm{Tr} e^{-\beta H} \chi(t)\chi(0)\chi(t)\chi(0)\rangle \sim 1 - \alpha e^{2\lambda t}.$$

Maldacena-Shenker-Stanford upper limit (2014)

$$\lambda < \frac{2\pi\beta}{\hbar}$$

This upper limit is saturated in the low-temperature limit of the SYK model.

Time Scales

- Lyapunov Time: $t_L \sim 1/\lambda_L$
- Ehrenfest Time: Time scale at which quantum fluctuations become macroscopic, $t_E \sim (\log N)/\lambda_L$. Also known as the scrambling time.
- Thouless Time: The time scale beyond which eigenvalue correlations are given by Random Matrix Theory.
- The Heisenberg Time: The inverse level spacing.

Altland-Sonner-2020

Correlations of eigenvalues

Pair correlation function

$$\langle \rho(\lambda)\rho(\lambda')\rangle - \langle \rho(\lambda)\rangle\langle \rho(\lambda')\rangle = \langle \rho(\lambda)\rangle \delta(\lambda-\lambda') - \langle \rho(\lambda)\rangle^2 \frac{\sin^2(\pi\langle \rho(\lambda)\rangle(\lambda-\lambda'))}{(\pi\langle \rho(\lambda)\rangle(\lambda-\lambda'))^2},$$

where we gave the result for the simplest random matrix theory.

- This behavior has been found in systems all over physics ranging from the hydrogen in a strong magnetic field to nuclei and black holes. It is most likely the strongest universality property in nature, even much stronger than the universality in critical phenomena.

Spectral Statistics

Random Matrix correlations can be measured by different statistics

- Level spacing distribution Wigner-1955
- The number variance which is the variance of the of the number of eigenvalues in an interval containing n eigenvalues on average. It can be obtained from the pair correlation function.
- The spectral form factor which is the Fourier transform of the pair correlation function. It is a global observable involving all eigenvalues in the spectrum and the universal RMT result is only recovered after local rescaling of the average eigenvalue spacing to a constant (unfolding).

Today I will only discuss the spectral form factor.

Spectral Form Factor

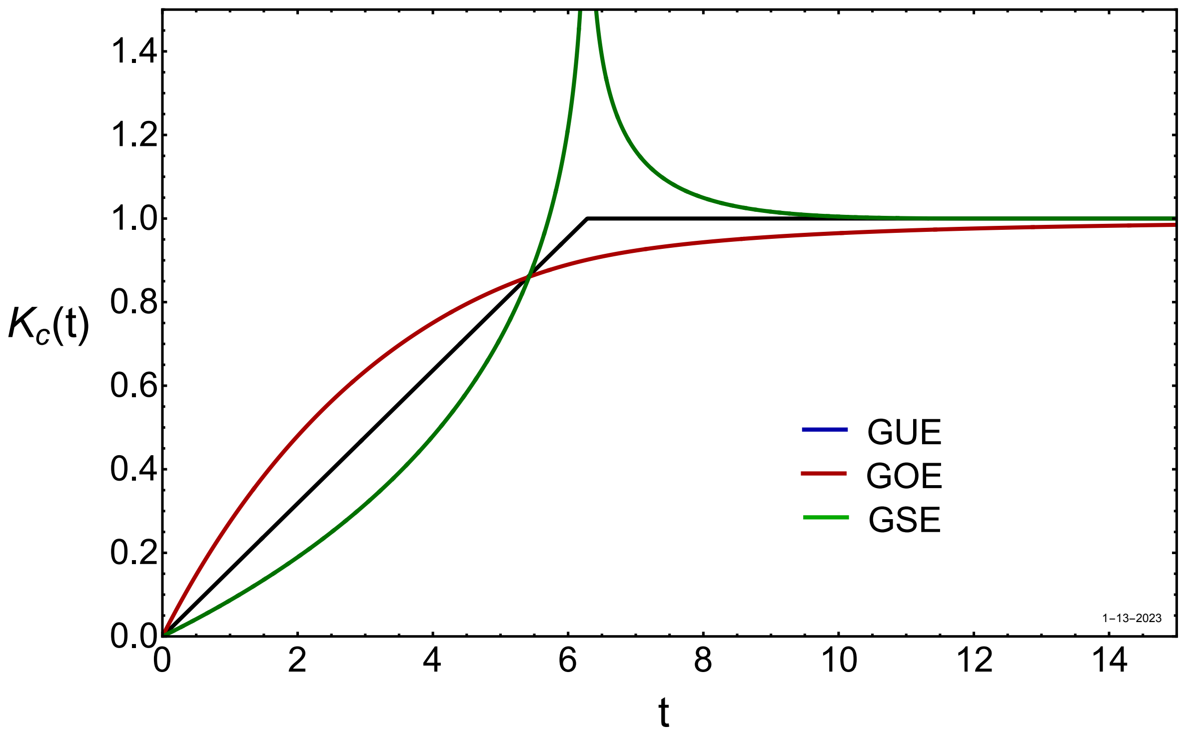
$$
\begin{aligned}
K_c(t) &= \frac{1}{D}\left[\langle \mathrm{Tr} e^{iHt} \mathrm{Tr} e^{-iHt}\rangle - \langle \mathrm{Tr} e^{iHt}\rangle\langle \mathrm{Tr} e^{-iHt}\rangle\right] \\
&= \frac{1}{D}\langle \sum_{kl} e^{it(E_k - E_1)}\rangle_c \\
&= \frac{1}{D}\int dE dE' \rho_{2c}(E,E') e^{it(E-E')}.
\end{aligned}
$$

The connected two-point correlation function is given by

$$
\begin{aligned}
\rho_{2c}(E,E') &= \sum_{kl}\langle \delta(E-E_k)\delta(E'-E_l)\rangle - \sum_{kl}\langle \delta(E-E_k)\rangle\langle \delta(E'-E_l)\rangle. \\
&= \left\langle \sum_{k \neq l}\langle \delta(E-E_k)\delta(E'-E_l)\right\rangle_c + \left\langle \sum_k \delta(E-E')\delta(E-E_k)\right\rangle.
\end{aligned}
$$

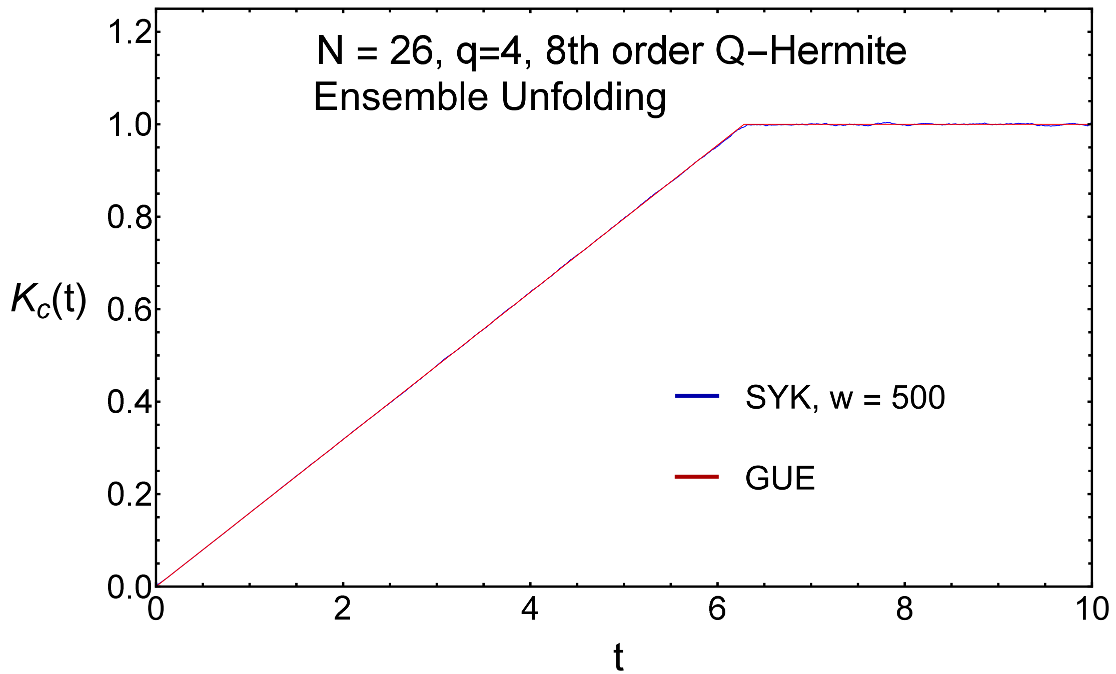
For large t only the second term survives.

Spectral Form Factor of RMT



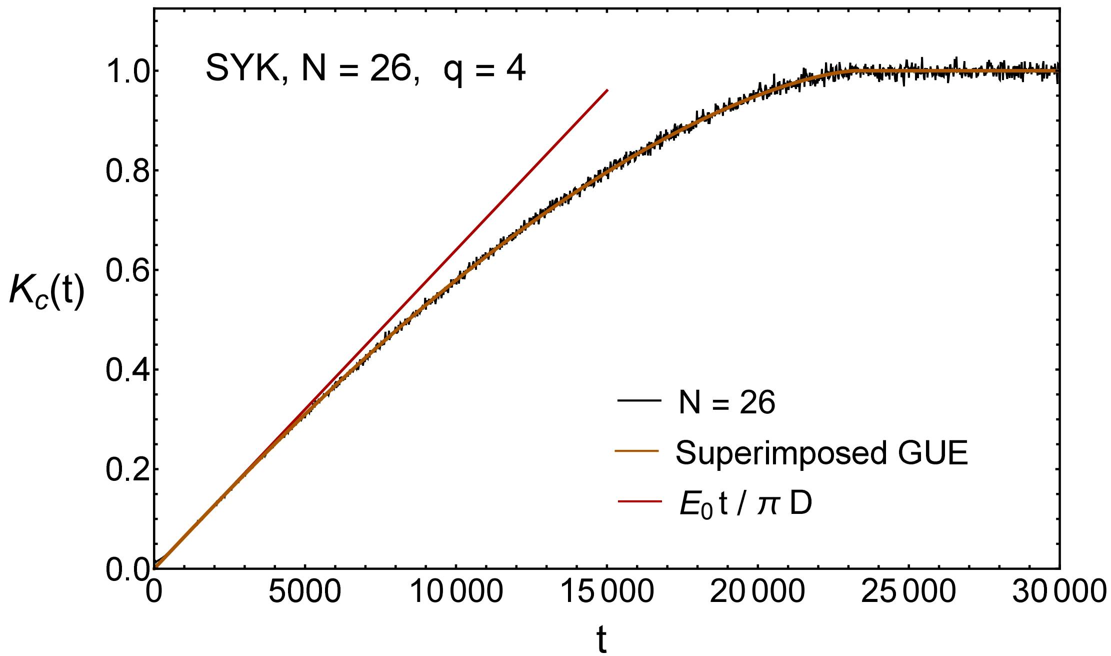
Spectral form factor for the Wigner-Dyson ensembles. The average level density is unfolded to unit average spacing.

Spectral Form Factor of the SYK Model



The spectral form factor of the SYK model after rescaling the eigenvalues according to the local average level spacing (unfolding). The contribution of the collective spectral fluctuations has been filtered out. JV-Jia-2020

Folding in the Average Spectral Density



Spectral form factor of the SYK model using the raw eigenvalues with the average spectral density folded into the universal RMT result

$$K_c(\tau) = \int dE \rho(E) K_{\rm univ.}(\tau/\rho(E)) = \sum_g a_g \frac{\tau^{2g-1}}{\rho^{2g}(E)}.$$

For the above figure we have used the Q-Hermite spectral density for $\rho(E)$, no fitting.

García-García-JV-2017, JV-2021

IV. Coupled SYK Models

The Maldacena-Qi Model

Constraints on Classification

The Maldacena-Qi Model

$$H = \sum_{\alpha<\beta<\gamma<\delta} \left(W_{\alpha\beta\gamma\delta}\chi_\alpha^L\chi_\beta^L\chi_\gamma^L\chi_\delta^L + W_{\alpha\beta\gamma\delta}\chi_\alpha^R\chi_\beta^R\chi_\gamma^R\chi_\delta^R\right) + i\mu\sum_\alpha \chi_\alpha^L\chi_\alpha^R.$$

- the χ_L and χ_R are from the same Clifford algebra.
- Ground state is gapped, and is almost a TFD state, $|TFD\rangle = \sum e^{-\beta E_k}|k\rangle|k\rangle$.
- The model is chaotic for μ not very small or very large.
- A first order phase transition separates a gapped Wormhole phase from a 2 Black-Hole phase.

Maldacena-Qi-2018, García-García-Jia-Rosa-JV-2019

Symmetries of the MQ Model

- Unitary symmetries

$$[\gamma_c, H] = 0, \qquad [Q, H] = 0,$$

with $Q\chi_L = \chi_R \quad \text{and} \quad Q\chi_R = -\chi_L$

- Take a representation of gamma matrices where the even gamma matrices are real and the odd gamma matrices purely imaginary
- Assign the even gamma matrices to L and the odd gamma matrices to R
- Then the Hamiltonian is invariant under complex conjugation, and this is the only anti-unitary symmetry
- Since K is the only anti-unitary symmetry we have that for all even N the symmetry class is AI (GOE).

V. Chaos and Disspation in the SYK Model

Lindblad evolution

Vectorization

Dissipative Form Factor

Anomalous Dissipation

Dissipation in the SYK Model

The Lindblad operator that describes the evolution of the density matrix $\rho = \sum_{k,l} A_{kl}|k\rangle\langle l|$ coupled to a bath is given by

$$\frac{d}{dt}\rho = -i[H_{SYK}, \rho] + \mu \sum_k \psi_k \rho \psi_k - \mu \frac{N}{2}.$$

For large times the density matrix relaxes to the identity operator or a the TFD state in the doubled Hilbert space after a Choi-Jamielkowski transformation

$$\rho = \sum_{k,l} A_{kl}|k\rangle\langle l| \to \sum_{k,l} A_{kl}|k\rangle|l\rangle.$$

$$\sum_k |k\rangle\langle k| \to \sum_k |k\rangle|k\rangle.$$

García-García-Sá-JV-Zheng-2022

Vectorized Lindblad operator

After the Choi-Jamielkowski transformation the Lindblad operator becomes Kulkarni-Numasawa-Ryu-2021, Sá-Ribewiro-Prosen-2021, García-García-Sá-JV-Zheng-2022, Kawabata-Kulkarni-Li-Numasawa-Ryu-2022

$$\mathcal{L} = iH_{SYK}^L - iH_{SYK}^R + i\mu \sum_k \psi_k^L \psi_k^R + \mu \frac{N}{2}.$$

This is non-Hermitian Hamiltonian. Its structure is similar to the Maldacena-Qi Hamiltonian

$$H_{MQ} = H_{SYK}^L + H_{SYK}^R + i\mu \sum_k \psi_k^L \psi_k^R.$$

Vectorized Lindblad operator

This raise the question of the classification on nonhermitian SYK Hamiltonian. It turns out that they are also given by the tenfold classification – just replace complex conjugation by transposition. García-Barcía-Sá-JV-PRX-2022, Kawabata-Kulkarni-Li-Numasawa-Ryu-2022, Sá-Riebeiro-Prosen-2022. For the general classification on non-Hermitian random matrix models see Bernard-LeClair-2002, Kawabata-Shiozaki-Ueda-Sato-2019

MQ type Hamiltonians and Lindblad based Hamiltonian further restrict possible non-linear sigma models. Can-García-Garía-Sá-JV-2023.

Dissipative Form Factor

Introduced by Tankut Can (2018),

$$\left\langle \mathrm{Tr} e^{\mathcal{L}t} \right\rangle = \left\langle \mathrm{Tr} e^{(iH_L - iH_R + i\mu\psi_k^L\psi_k^R)t + \mu\frac{N}{2}} \right\rangle .$$

For $\mu = 0$ this becomes the spectral form factor

$$\left\langle \mathrm{Tr} e^{\mathcal{L}t} \right\rangle = \left\langle \mathrm{Tr} e^{iHt} \mathrm{Tr} e^{-iHt} \right\rangle ,$$

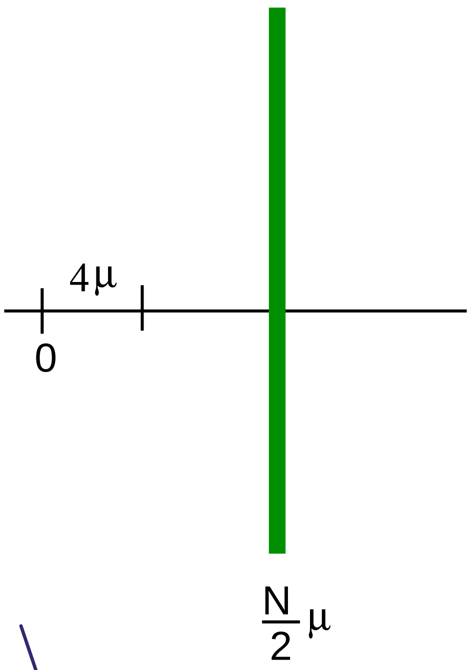
where we used that $H_L = H \otimes \mathbb{I} + \mathbb{I} \otimes H$. For large $t > t_H$ it is given by the expectation value of the TFD state, $|TDF\rangle = \sum |k\rangle|k\rangle$.

$$\left\langle \mathrm{Tr} e^{iHt} \mathrm{Tr} e^{-iHt} \right\rangle \approx D\langle TFD|e^{it(H_L - H_R)}|TFD\rangle = D.$$

Spectrum of the Lindblad SYK Model

Let us look at the Hamiltonian

$$H = iH_L \otimes 1 - i1 \otimes \; H_R + i\mu \sum_k \psi_L^k \psi_R^k + \mu \frac{N}{2}$$



$$H|TFD\rangle = H \sum |k\rangle|k\rangle = 0.$$

For large τ we have that

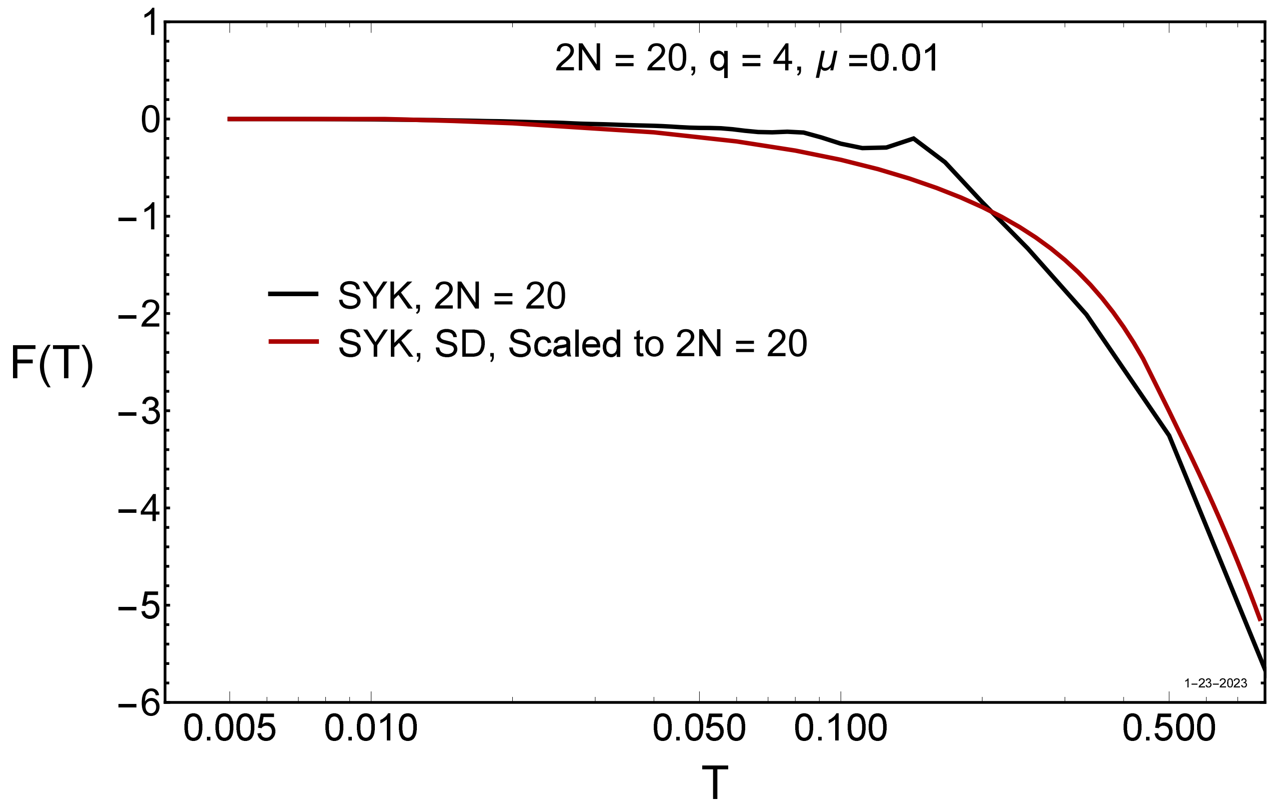
$$e^{-\tau F} = 1 + e^{-\frac{\mu N}{2}\tau} \left\langle \sum_{k,l} e^{i\tau(E_k - E_l)} \right\rangle ,$$

or reversely,

$$K(\tau) = e^{\frac{\mu N}{2}\tau} \left(e^{-\tau F} - 1 \right) .$$

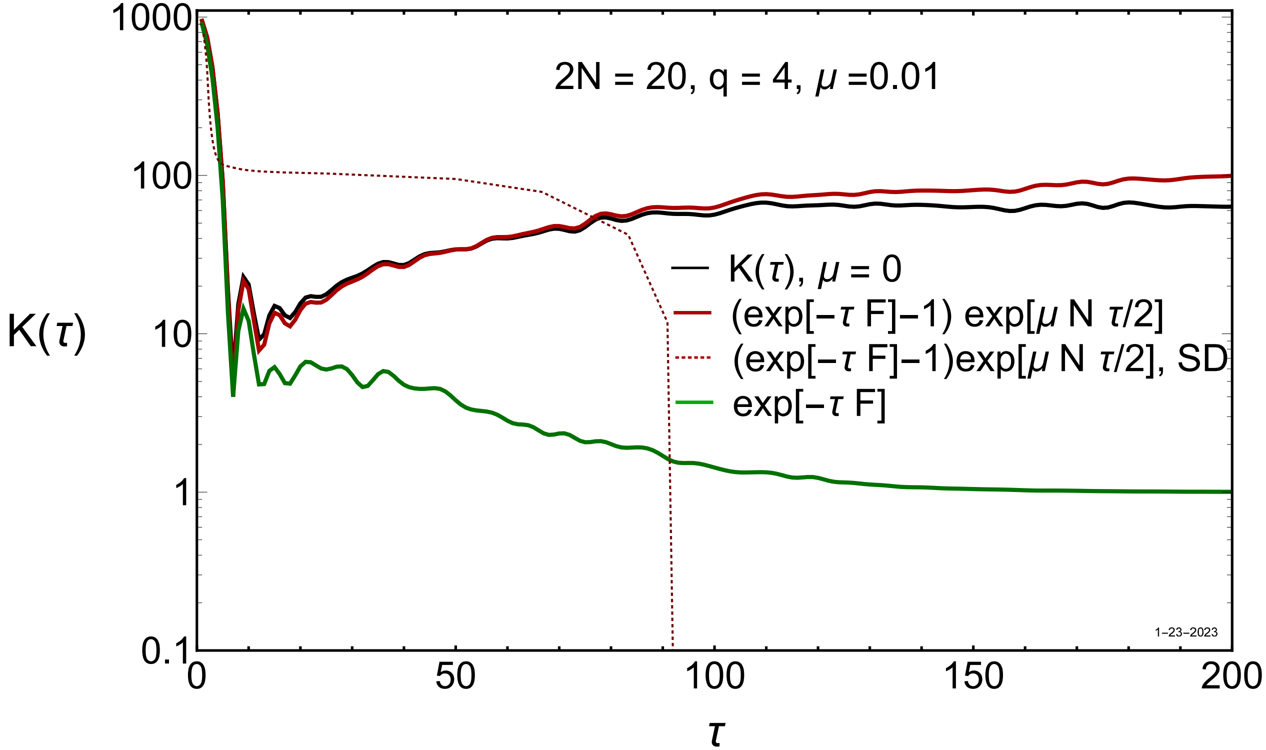
Using that the form factor of one SYK asymptotes to $2^{N/2}$, a phase transition occurs for $\mu\tau = 1$.

Phase Transition



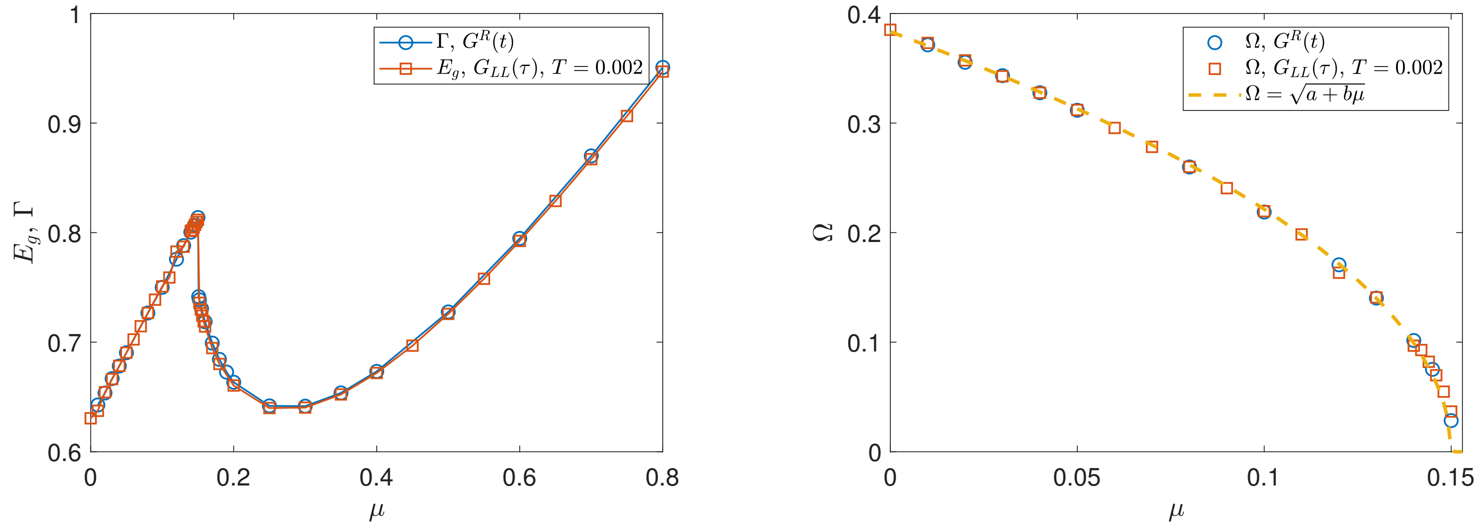
The free energy obtained from the solution of the Schwinger-Dyson equation. A phase transition takes place at $T = 1/\tau = \mu$.

Spectral Form Factor From Free Energy



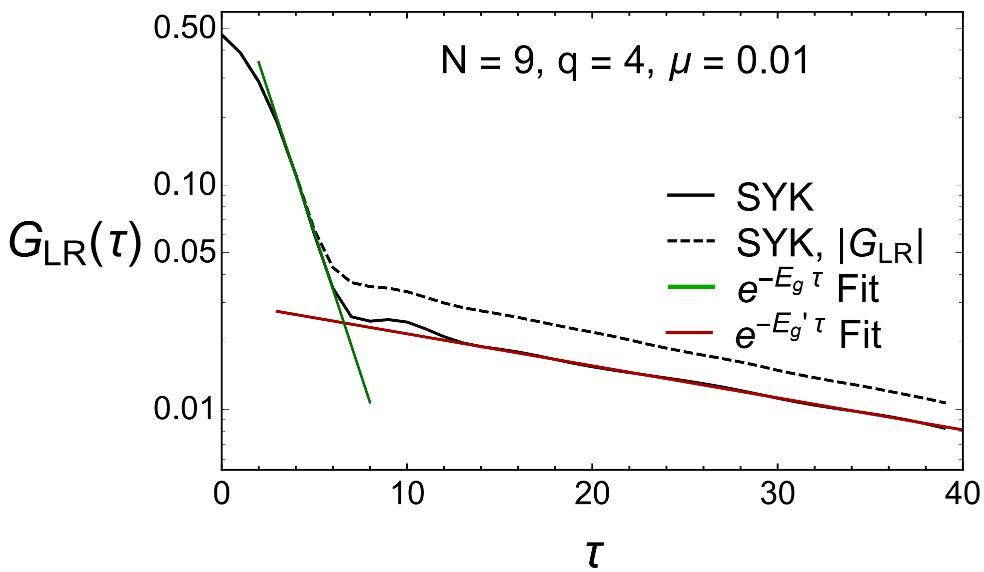
The spectral form factor obtained from the free energy at non zero μ compared to the exact spectral form factor.

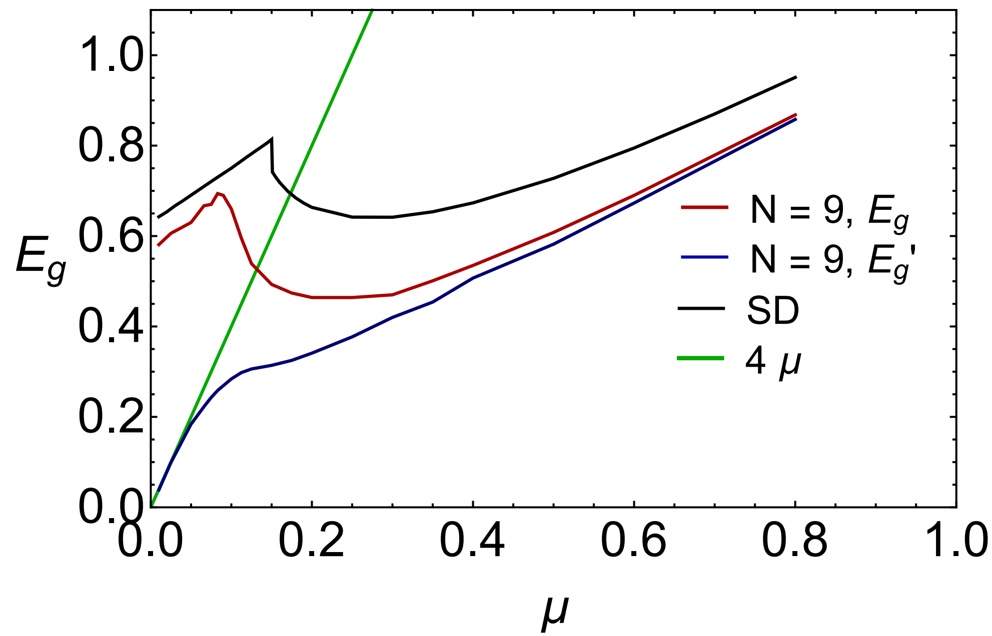
Anomalous Dissipation



Decay rate of Green's functions, $G(t) \sim e^{-\Gamma t} \cos(\Omega t)$. What is the origin of this phase transition?

Finite N Results





The $\exp[-4\mu t]$ tail is absent in the solutions of the Schwinger-Dyson equations. This is consistent with the tail of the dissipative form factor for finite N, which is not there for the solution of the Schwinger-Dyson equations.

VI. Conclusions

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- This model has a wormhole phase with and exact TFD ground state that is separated by a finite gap from the rest of the spectrum.
- This model describes the spectral from factor of a single SYK as can be seen at finite N, but it requires subleading corrections to the solution of the Schwinger-Dyson equations.

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- Spectral correlations of the SYK model are given by the universal RMT results after either unfolding the spectrum of folding in the average spectral density, and after filtering out the scale fluctuations.
- The Lindblad equation describing dissipation to a bath can be mapped to a non-hermitian two-site SYK model by vectorization of the density matrix.
- This model has a wormhole phase with and exact TFD ground state that is separated by a finite gap from the rest of the spectrum.
- This model describes the spectral from factor of a single SYK as can be seen at finite N, but it requires subleading corrections to the solution of the Schwinger-Dyson equations.