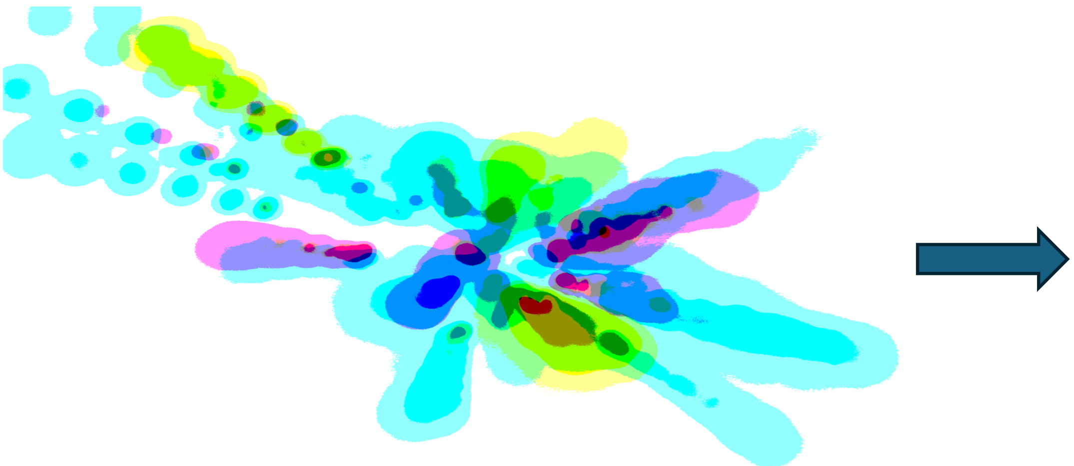
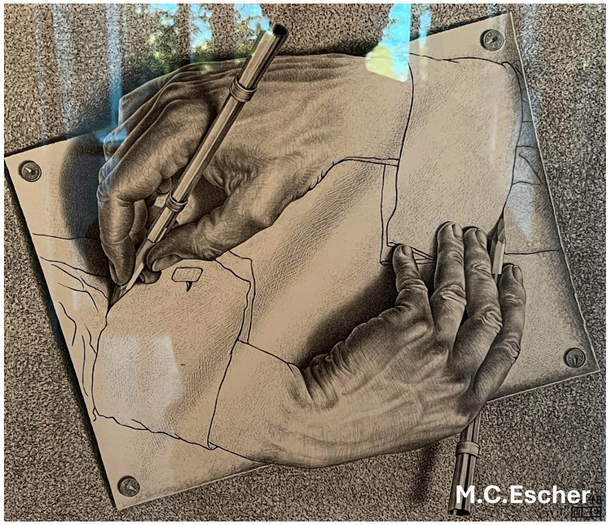
# Chiral anomaly in polarized DIS:
# WZW terms and finite mass effects







Raju Venugopalan

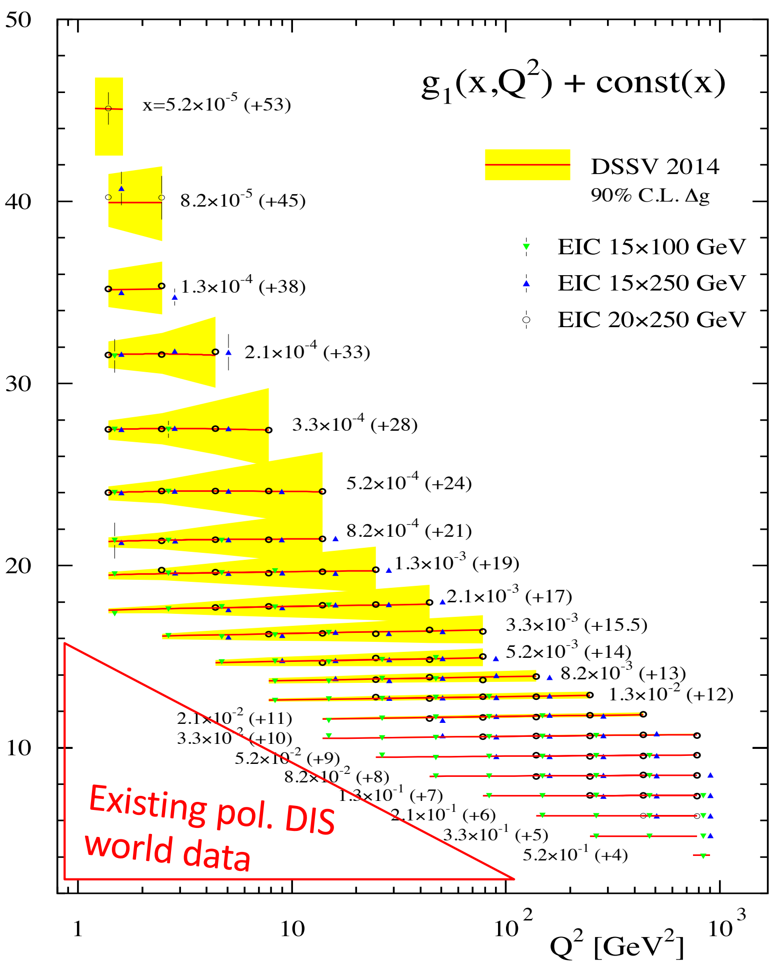
Brookhaven National Laboratory


INT workshop, June 2-6, 2025

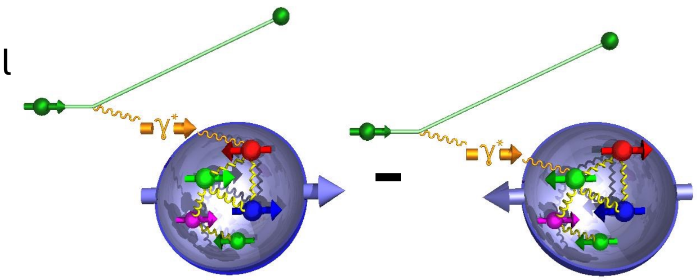
Work* in collaboration with Andrey Tarasov (NCSU and CFNS Stony Brook)

* Based on: https://arxiv.org/abs/2008.08104, https://arxiv.org/abs/2109.10370 and https://arxiv.org/abs/2501.10519

# Interplay of perturbative and non-perturbative dynamics in polarized DIS




$g_1$ extracted from longitudinal spin asymmetry



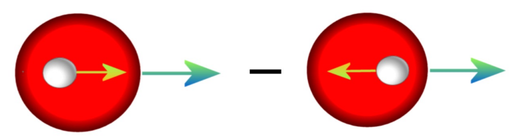
First moment $\Delta\Sigma(Q^2) \propto \int_0^1 dx\, g_1(x,Q^2)$

In the parton model, it is the net quark helicity

$$\Sigma(Q^2) = \sum_f \int_0^1 dx_B\; \underbrace{\left(\Delta q_f(x_B,Q^2) + \Delta\bar{q}_f(x_B,Q^2)\right)}$$

$\Delta q_f$ = Diff. in parton densities of left and right handed quarks of flavor f

$\Delta\bar{q}_f$ = Ditto for anti-quarks



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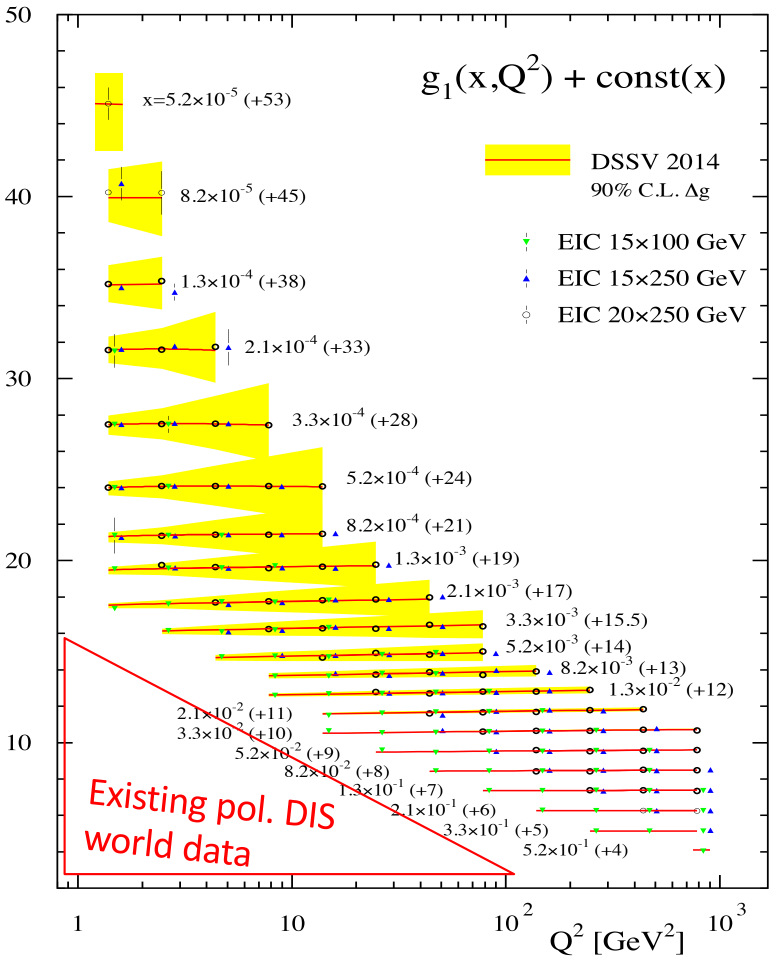


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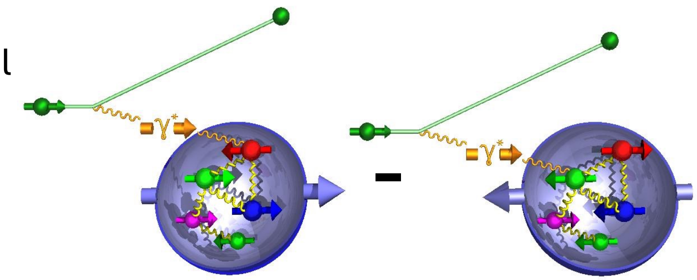
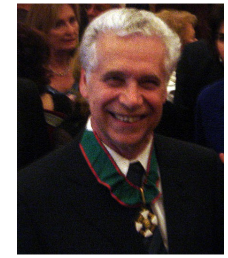



Iso-singlet axial charge: $\Delta\Sigma(Q^2) \propto \int_0^1 dx\, g_1(x,Q^2)$

In QCD, the physics is far more subtle and rich – elements include

- The chiral anomaly and validity of QCD factorization theorems
- Anomalous Ward identities, top. screening of ΔΣ, large N and lattice
- Novel axion-like dynamics, and sphaleron-like transitions at small x

# Remarkable result by Veneziano et al.


G. Veneziano

$$\Delta\Sigma(Q^2) = \sqrt{\frac{2}{3}}\,\frac{N_f}{M_N}\,g_{\bar{\eta}NN}\,\sqrt{\chi'_{QCD}(0)}$$

coupling of "primordial $\eta'$ to proton

Forward slope of QCD topological susceptibility in chiral limit

$\chi_{QCD}(l^2) = \int d^4x\, e^{il\cdot x} < \Omega(x)\Omega(0) >$ with the top. charge density $\Omega(x) = F\tilde{F}$

Inclusive polarized DIS directly sensitive to the topology of the QCD vacuum – at any $Q^2$ !

Not entirely surprising if $\Delta\Sigma(Q^2) \propto < P,S|\, J_\mu^5\, |P,S >$,

*since $J_\mu^5$ is not a conserved current – satisfies the anomaly eqn.*

$$\partial^\mu J^5_{\mu,f} = 2im_f\,\bar{q}_f\gamma_5 q_f + \frac{\alpha_S}{2\pi}\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

In the chiral limit, $\chi_{QCD}(0) = 0$

Topological screening of quark helicity

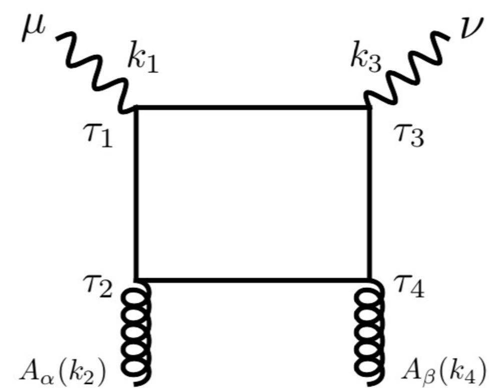

Veneziano, Mod. Phys. Lett. A4 (1989) 1605

Shore, Veneziano, PLB244 (1990) 75; NPB 381(1992)23

Narison, Shore, Veneziano, NPB546 (1999) 235

Review: Shore, hep-ph/0701171

# Worldline approach to polarized DIS: box diagram




Anti-symmetric piece of hadron tensor

$$\tilde{W}_{\mu\nu}(q,P,S) = \frac{2M_N}{P\cdot q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\Big\{S^\beta g_1(x_B,Q^2) + \Big[S^\beta - \frac{(S\cdot q)P^\beta}{P\cdot q}\Big]g_2(x_B,Q^2)\Big\}$$

$$i\tilde{W}^{\mu\nu}(q,P,S) = \frac{1}{2\pi e^2}\,\mathrm{Im}\int d^4x\, e^{-iqx}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_3}{(2\pi)^4}e^{-ik_1\frac{x}{2}}e^{ik_3\frac{x}{2}}\langle P,S|\tilde{\Gamma}_A^{\mu\nu}[k_1,k_3]|P,S\rangle$$

with the polarization tensor $\Gamma_A^{\mu\nu}[k_1,k_3] = \int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k_4}{(2\pi)^4}\;\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\;\mathrm{Tr_c}(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$

Antisymmetric piece of box diagram

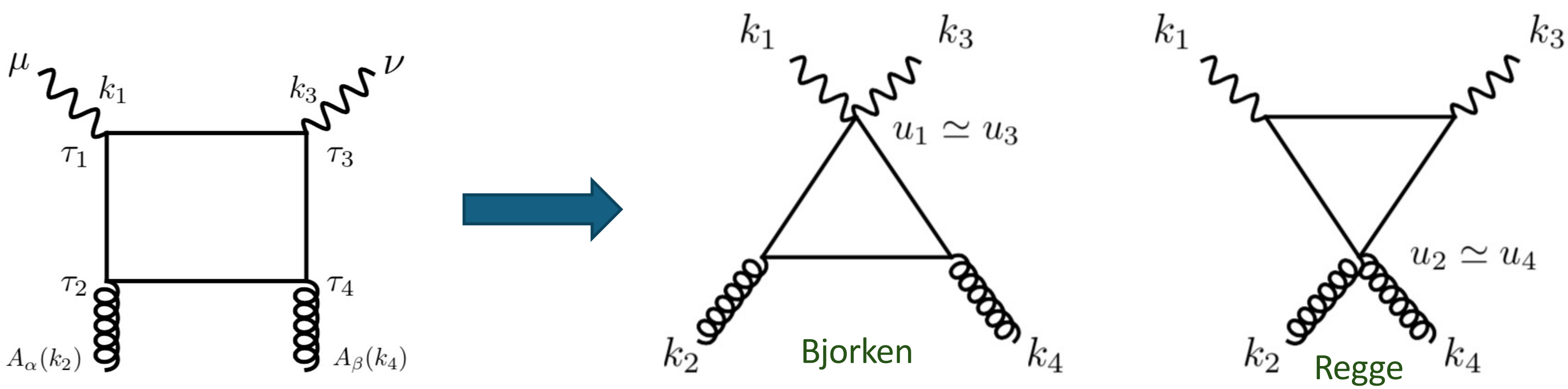
$$\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4] \equiv -\frac{g^2e^2e_f^2}{2}\int_0^\infty\frac{dT}{T}\int\mathcal{D}x\int\mathcal{D}\psi\;\exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\cdot\dot{\psi}\Big)\Big\}$$
$$\times\Big[V_1^\mu(k_1)V_3^\nu(k_3)V_2^\alpha(k_2)V_4^\beta(k_4) - (\mu\leftrightarrow\nu)\Big]$$

Product of boson and Grassmann worldline currents $V_i^\mu(k_i) \equiv \int_0^T d\tau_i(\dot{x}_i^\mu + 2i\psi_i^\mu k_j\cdot\psi_j)e^{ik_i\cdot x_i}$


"Pert. Theory without Feynman diagrams", M. Strassler, NPB 385 (1992) 145
Review: C. Schubert, Phys. Repts. (2001)

A. Tarasov, RV, PRD (2019, 2021, 2022)

# Finding triangles in boxes in Bjorken and Regge asymptotics




$$S^\mu g_1(x_B, Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int\frac{d\xi}{2\pi}e^{-i\xi x}\lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P', S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle$$ + non-pole

$$S^\mu\, g_1(x_B, Q^2)|_{x_B\to 0} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\int\frac{d\xi}{2\pi}e^{-i\xi x}\lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P', S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle$$ + non-pole

This is the “anomaly pole obtained by integrating over x and inverting the anomaly equation

We discovered it is not just a property of first moment but in $g_1(x,Q^2)$ itself

How is the anomaly pole regulated ?

A. Tarasov, RV, PRD (2021, 2022)

Also obtained in a Feynman diagram analysis Bhattacharya, Hatta, Vogelsang, arxiv:2210.13419, arXiv:2305.09431

Bhattacharya,Hatta,Schloenleber, arXiv:2411.07024

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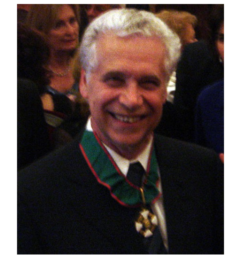

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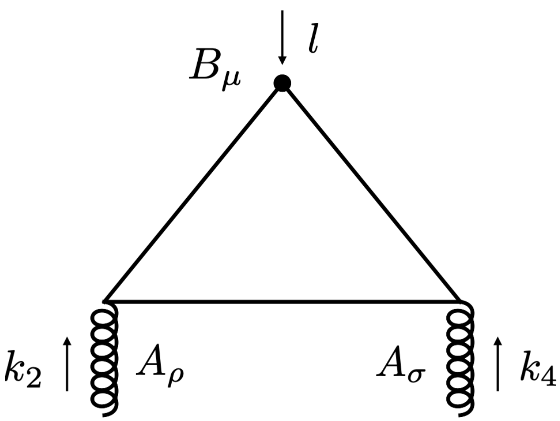


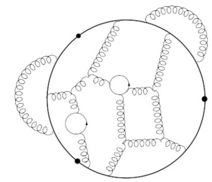

Axial Vector-Vector-Vector (AVV) triangle operator

> The authors of refs. [12, 13] suggest that the triangle diagram provides a *local* probe of the gluon distribution in the target. If this were true, $\Delta\Gamma$ would be protected from infrared problems and the calculation would be reliable in the usual sense. However, we believe there are strong arguments that the triangle is not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the $\eta'$ a mass*.

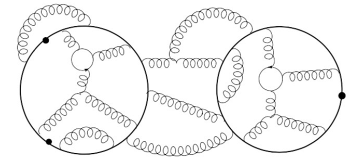
Jaffe, Manohar, NPB337 (1990) 509

Veneziano, Mod. Phys. Lett. A4 (1989) 1605
Shore, Veneziano, PLB244 (1990) 75; NPB 381(1992)23
Narison, Shore, Veneziano, NPB546 (1999) 235
Review: Shore, hep-ph/0701171

# $\Delta\Sigma$ and the $U_A(1)$ problem



OZI-allowed



OZI-suppressed

Veneziano et al showed that starting from the Wess-Zumino action for QCD (and an OZI variant of the Veneziano topological expansion in $N_f$ and $N_c$) one could derive not just the anomaly equation for $J_\mu^5$ but also higher point correlators of $J_\mu^5$ that involve the $\eta'$

Wess,Zumino, PLB 37 (1971) 95

In particular, the corresponding anomalous chiral Ward identities also explain why the $\eta'$ is so massive - more than the proton's mass !

The result of manipulating these Ward identities is that the anomaly $\frac{l^\mu}{l^2} \rightarrow \frac{l^\mu}{l^2 - m_{\eta'}^2}$ where the $\eta'$ mass is given by the Witten-Veneziano formula: $m_{\eta'}^2 \equiv -\frac{2n_f}{F_\eta^2}\chi_{\rm YM}(0)$

Witten, NPB 156 (1979)269;
Veneziano, NPB 159 (1979) 213

We will show that much of these results can be recovered in the language of Wess-Zumino-Witten (WZW) terms in the worldline framework – that allows a further study of dynamical pert. - nonpert. interplay

# Anomalies in the worldline formulation of QFT

Fermion action in background of scalar, pseudoscalar, vector and axial vector fields:

$$S_{\text{fermion}}[\bar{\Psi},\Phi,\Pi,A,B,\Psi] = \int d^4x\,\bar{\Psi}^I\left[i\partial\!\!\!/ - \Phi + i\gamma^5\Pi + A\!\!\!/ + \gamma^5 B\!\!\!/\right]^{IJ}\Psi^J$$

Effective action: $-\mathcal{W}[A,B,\Phi,\Pi] = \mathrm{Ln}\,\mathrm{Det}\,[\mathcal{D}]$ with $\mathcal{D} = p\!\!\!/ - i\Phi(x) - \gamma_5\,\Pi - A\!\!\!/ - \gamma_5 B\!\!\!/$

Split into real and imaginary parts: $\mathcal{W}_R = -\frac{1}{2}\mathrm{Ln}\left(\mathcal{D}^\dagger\mathcal{D}\right)$ ; $\mathcal{W}_I = \frac{1}{2}\mathrm{Arg}\,\mathrm{Det}\left(\mathcal{D}^2\right)$

Entire dynamics of the anomaly comes from $W_I$ - the phase of the Dirac determinant

# Heat kernel regularization of the phase as a worldline path integral

$W_I$ can also be expressed as a worldline Lagrangian of 0+1- bosonic (coordinate) and Grassmann fields

D'Hoker, Gagne, hep-th/9508131, hep-th/9512080

Jacobian for zero modes multiplied by G-parity factor

$$W_{\mathcal{I}} = -\frac{i}{32}\int_{-1}^{1} d\alpha \int_{0}^{\infty} dT\,\mathcal{N}\int_{\mathrm{PBC}} \mathcal{D}x\,\mathcal{D}\psi\ \mathrm{tr}\ \chi\,\bar{\omega}(0)\exp\left[-\int_0^T d\tau \mathcal{L}_{(\alpha)}(\tau)\right]$$

Worldline Lagrangian with chiral symmetry breaking interpolating parameter $\alpha$

$$\mathcal{L}_{(\alpha)}(\tau) = \mathcal{L}(\tau)\Big|_{\Phi\to\alpha\Phi, B\to\alpha B} \quad \text{with} \quad \mathcal{L}(\tau) = \frac{\dot{x}^2}{2\mathcal{E}} + \frac{1}{2}\psi\dot{\psi} - i\dot{x}^\mu \mathcal{A}_\mu + \frac{\mathcal{E}}{2}\mathcal{H}^2 + i\mathcal{E}\psi^\mu\psi_5\mathcal{D}_\mu\mathcal{H} + \frac{i\mathcal{E}}{2}\psi^\mu\psi^\nu\mathcal{F}_{\mu\nu}$$

in the chiral basis $\mathcal{A}_\mu \equiv \begin{pmatrix} A_\mu^L & 0 \\ 0 & A_\mu^R \end{pmatrix} = \begin{pmatrix} A_\mu + B_\mu & 0 \\ 0 & A_\mu - B_\mu \end{pmatrix}$ and $\mathcal{H} \equiv \begin{pmatrix} 0 & iH \\ -iH^\dagger & 0 \end{pmatrix} = \begin{pmatrix} 0 & i\Phi + \Pi \\ -i\Phi + \Pi & 0 \end{pmatrix}$

Similar to so-called $\eta$ -invariant parametrization of the Dirac operator in chiral gauge theories

Alvarez-Gaume, della Peitra^2, PLB 166 (1986) 177; Alvarez-Gaume, Ginsparg, Ann. Phys. 161 (1986) 423
Ball, Osborn, PLB 165 (1985) 410; Kaplan and Schmaltz, PLB 368 (1986) 44; Witten, Yonekura, arXiv:1909.08775

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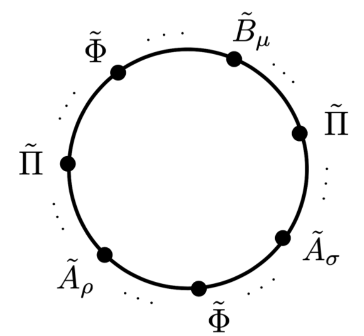
Worldline Lagrangian with chiral symmetry breaking interpolating parameter $\alpha$

Can combine real and imaginary parts in a " perturbative" expansion (consistent anomaly)

$$W = \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^D p_1}{(2\pi)^D} \cdots \frac{d^D p_n}{(2\pi)^D} (2\pi)^D \delta^{(D)}(p_1 + \cdots + p_n) \int \frac{d^D q}{(2\pi)^D}\ \text{tr} \frac{1}{\not{q} - im}$$
$$\times \left(i\tilde{\varphi}_1 + \gamma_5 \tilde{\Pi}_1 + \tilde{\not{A}}_1\right) \cdots \frac{1}{\not{q} - \not{p}_1 - \cdots - \not{p}_{n-1} - im} \left(i\tilde{\varphi}_n + \gamma_5 \tilde{\Pi}_n + \tilde{\not{A}}_n\right)$$

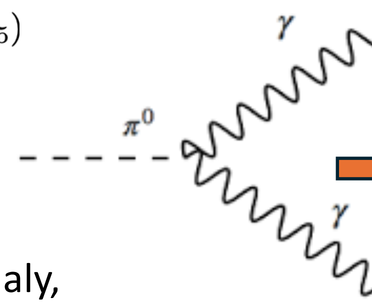


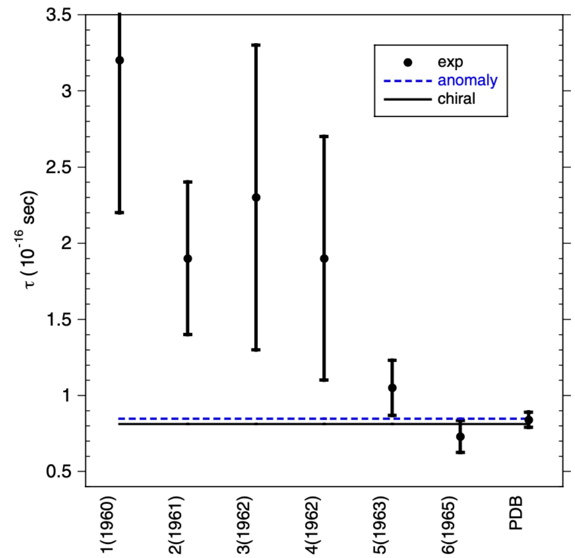

Quark loop with external sources

A systematic expansion in these external sources also generate WZW terms
- agree exactly with "nonet" chiral pert. theory Kaiser,Leutwyler, EPJ C17 (2000)623

# What’s in a phase? WZW terms and “QCD axion” from worldline action

$$W_{\Im}[\Pi^5] = -\frac{i}{5}\int_{p^1,\ldots,p^5} (2\pi)^4\delta^{(4)}(p^1+\cdots+p^5)\,(-4im)\,\mathrm{Tr_c}(\tilde{\Pi}_1\cdots\tilde{\Pi}_5)$$
$$\times\ \varepsilon_{\mu_1\cdots\mu_4}p^1_{\mu_1}\cdots p^4_{\mu_4}I'(p^i)$$




A.Bernstein,B.Holstein, RMP (2013)

Most precise computation in QCD – without the anomaly, the pion decay rate would be off by a factor of 1000 …

Likewise, expanding to O($\Phi\ \Pi\ A^2$),

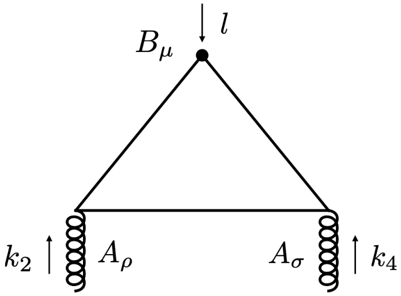
$$S^{\bar{\eta}}_{\rm WZW} = -i\frac{\sqrt{2\,n_f}}{F_{\bar{\eta}}}\int d^4x\,\bar{\eta}\,\Omega$$

*$\bar{\eta}$ is the primordial $\eta'$, $\Omega$ the topological charge density, $F_{\eta}$ the $\bar{\eta}$ decay const.*

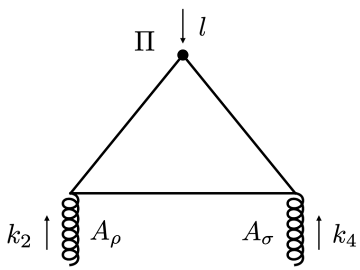
*It is the QCD axion in the chiral limit and at large Nc*

# Triangles in QED and QCD and the forward limit

Tarasov,RV, arXiv:2501.10519




$$\left.\frac{\delta W_{\mathcal{I}}}{\delta B_\mu(x)}\right|^{\text{singlet}}_{B_\mu=0,\,\Pi=0,\,\Phi=m} = -\frac{i\mathcal{E}}{64}\int_{-1}^{1} d\alpha \int_0^\infty dT\mathcal{N}\int_P \mathcal{D}x\mathcal{D}\psi\ \exp\Big[-\int_0^T d\tau\mathcal{L}_{(\alpha)}(\tau)\Big]\Big[\mathrm{tr}\ \chi\frac{\delta\bar\omega(0)}{\delta B_\mu(x)} - \int_0^T d\tau_1\ \mathrm{tr}\ \chi\bar\omega(0)\frac{\delta\mathcal{L}_{(\alpha)}(\tau_1)}{\delta B_\mu(x)}\Big]$$




$$\left.\frac{\delta W_{\mathcal{I}}}{\delta \Pi(x)}\right|^{\text{singlet}}_{B_\mu=0,\,\Pi=0,\,\Phi=m} = -\frac{i\mathcal{E}}{64}\int_{-1}^{1} d\alpha \int_0^\infty dT\mathcal{N}\int_P \mathcal{D}x\mathcal{D}\psi\ \exp\Big[-\int_0^T d\tau\mathcal{L}_{(\alpha)}(\tau)\Big]\Big[\mathrm{tr}\ \chi\frac{\delta\bar\omega(0)}{\delta \Pi(x)} - \int_0^T d\tau_1\ \mathrm{tr}\ \chi\bar\omega(0)\frac{\delta\mathcal{L}_{(\alpha)}(\tau_1)}{\delta \Pi(x)}\Big]$$

Check explicitly that one recovers the anomaly equation

$$\int d^4x\, e^{ilx}\partial_\mu \left.\frac{\delta W_{\mathcal{I}}}{\delta B_\mu(x)}\right|^{\text{singlet}}_{B_\mu=0;\Pi=0;\Phi=m} = -\frac{1}{4\pi^2}\int d^4x\, e^{ilx}F_{\alpha\rho}(x)\tilde{F}^{\alpha\rho}(x)\ -2m\int d^4x\, e^{ilx}\left.\frac{\delta W_{\mathcal{I}}}{\delta \Pi(x)}\right|^{\text{singlet}}_{B_\mu=0;\Pi=0;\Phi=m}$$

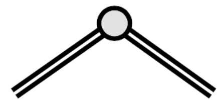
In QED $(\Phi = m)$ the two terms on r.h.s exactly cancel for $l^2$=0. Corollary: massless QED is anomalous

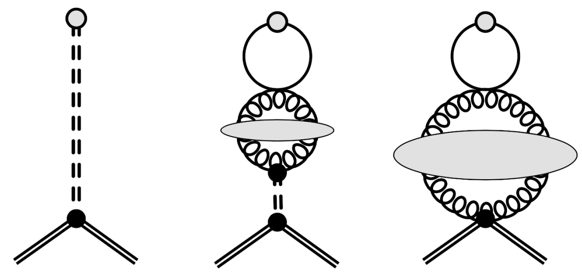
Adler, hep-th/0405040
Recently, Castelli et al., PLB 857, 138999 (2024)

Au contraire, massless QCD is anomaly free ($N_f \geq 2$). Hint: setting $\Phi = \mathcal{S}(x)$ introduces the chiral condensate

# Isosinglet form factors and chiral Ward identities

$$\langle P', S|J_5^\mu|P, S\rangle = \bar{u}(P', S)\Big[\gamma^\mu\gamma_5 G_A(l^2) + l^\mu\gamma_5 G_P(l^2)\Big]u(P, S)$$ with $G_A(0) = \Delta\Sigma$





From the Wess-Zumino action

$$S[A, \bar{\Psi}, \Psi, B, \mathcal{S}, \Pi, \Theta] = \int d^4x \left(\mathcal{L}_{QCD} + B_\mu\bar{\Psi}\gamma^\mu\gamma_5\Psi + \mathcal{S}\bar{\Psi}\Psi + \Pi\bar{\Psi}i\gamma_5\Psi + \Theta\Omega\right)$$

obtain

$$G_P(l^2) = -i\frac{l_\mu}{l^2}\Big[\frac{\delta^2 Z}{\delta B_\mu \delta\Theta}(l^2)\, g_{\Omega NN}(l^2) + \frac{\delta^2 Z}{\delta B_\mu \delta\Pi}(l^2)\, g_{\phi_5 NN}(l^2)\Big]$$

$$\frac{\delta^2 Z}{\delta B_\mu \delta\Theta}(l^2) = i\int d^4x\, e^{ilx}\, \langle 0|T J_5^\mu(x)\Omega(0)|0\rangle$$

$$\frac{\delta^2 Z}{\delta B_\mu \delta\Pi}(l^2) = i\int d^4x\, e^{ilx}\, \langle 0|T J_5^\mu(x)\phi_5(0)|0\rangle$$

with $\phi_5 = i\bar{\Psi}\gamma_5\Psi$

Ward identity for 1-point function:

$$\partial_\mu\frac{\delta Z}{\delta B_\mu} - 2N_f\frac{\delta Z}{\delta\Theta} - 2m\frac{\delta Z}{\delta\Pi} + 2\mathcal{S}\frac{\delta Z}{\delta\Pi} - 2\Pi\frac{\delta Z}{\delta\mathcal{S}} = 0$$

Gives anomaly equation when sources are put to zero

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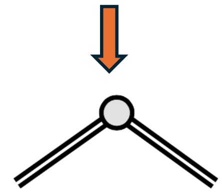

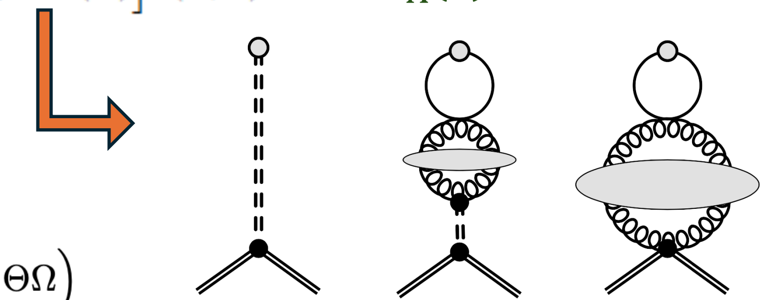

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$$S[A, \bar{\Psi}, \Psi, B, \mathcal{S}, \Pi, \Theta] = \int d^4x \left(\mathcal{L}_{QCD} + B_\mu \bar{\Psi}\gamma^\mu \gamma_5 \Psi + \mathcal{S}\bar{\Psi}\Psi + \Pi \bar{\Psi} i\gamma_5 \Psi + \Theta\Omega\right)$$

obtain

$$G_P(l^2) = -i\frac{l_\mu}{l^2}\left[\frac{\delta^2 Z}{\delta B_\mu \delta\Theta}(l^2)\, g_{\Omega NN}(l^2) + \frac{\delta^2 Z}{\delta B_\mu \delta\Pi}(l^2)\, g_{\phi_5 NN}(l^2)\right]$$

$$\frac{\delta^2 Z}{\delta B_\mu \delta\Theta}(l^2) = i\int d^4x\, e^{ilx}\, \langle 0|T J_5^\mu(x)\Omega(0)|0\rangle$$

$$\frac{\delta^2 Z}{\delta B_\mu \delta\Pi}(l^2) = i\int d^4x\, e^{ilx}\, \langle 0|T J_5^\mu(x)\phi_5(0)|0\rangle$$

with $\phi_5 = i\bar{\Psi}\gamma_5\Psi$

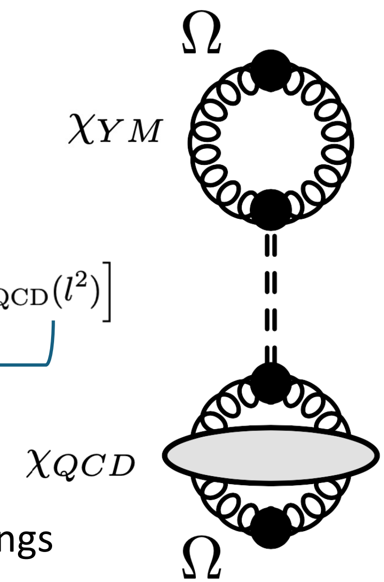
Ward identities for 2-point functions:

$$il_\mu \frac{\delta^2 Z}{\delta B_\mu \delta\Theta}(l^2) - 2N_f \frac{\delta^2 Z}{\delta\Theta\delta\Theta}(l^2) - 2m\frac{\delta^2 Z}{\delta\Pi\delta\Theta}(l^2) = 0$$

$$il_\mu \frac{\delta^2 Z}{\delta B_\mu \delta\Pi}(l^2) - 2N_f \frac{\delta^2 Z}{\delta\Theta\delta\Pi}(l^2) - 2m\frac{\delta^2 Z}{\delta\Pi\delta\Pi}(l^2) - 2\langle\phi\rangle = 0$$

Chiral condensate

# Topology, polology and large N



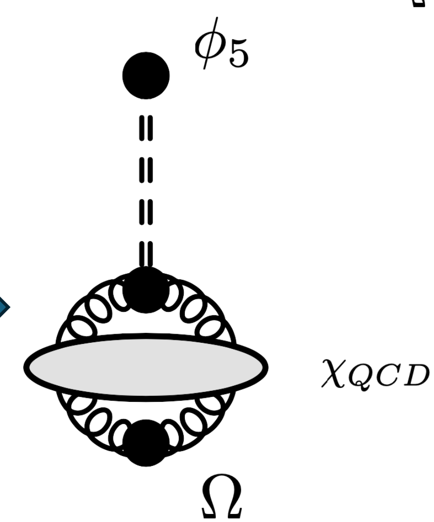

Non-pert. Green functions:

WZW WZW

$$\frac{\delta^2 Z}{\delta\Theta\delta\Theta}(l^2)\Big|_{\theta=0} \equiv \chi_{\rm QCD}(l^2) \;= \chi_{\rm YM}(l^2) + i\Big[-i\chi_{\rm YM}(l^2)\Big]\Big[-i\frac{\sqrt{2N_f}}{F_{\bar\eta}}b(l^2)\Big]\frac{i}{l^2-m_{\bar\eta}^2}\Big[-i\frac{\sqrt{2N_f}}{F_{\bar\eta}}b(l^2)\Big]\Big[-i\chi_{\rm QCD}(l^2)\Big]$$

We have assumed here pole dominance and smooth off-forward behavior of WZW couplings

$$\frac{\delta^2 Z}{\delta\Pi\delta\Theta}(l^2) = i\int d^4x\, e^{ilx}\,\langle 0|T\phi_5(x)\Omega(0)|0\rangle = \frac{1}{l^2-m_{\bar\eta}^2}\frac{\sqrt{2N_f}}{F_{\bar\eta}}b(l^2)\chi_{\rm QCD}(l^2)\,c(l^2)$$




$$\frac{\delta^2 Z}{\delta\Pi\delta\Pi}(l^2) = i\int d^4x\, e^{ilx}\,\langle 0|T\phi_5(x)\phi_5(0)|0\rangle = \frac{-1}{l^2-m_{\eta'}^2}c^2(l^2)$$

# Putting it all together…

Employing, a) the anomaly equation, b) the Dirac equation, c) chiral Ward identities, d) WZW and e) pole dominance, we have sufficient info to establish the following:

A Goldberger-Treiman relation: $2M_N\, G_A(0) = \sqrt{2N_f}\, F_{\bar\eta}\, g_{\bar\eta NN}$ Relates isosinglet axial-vector and pseudo-scalar form factors ($\widetilde{H}\, and\, \widetilde{E}$)

$$\Delta\Sigma = \frac{\sqrt{2N_f}}{2M_N} F_{\bar\eta}\, g_{\bar\eta NN} = \frac{2N_f}{2M_N}\sqrt{\chi'_{\rm QCD}(0)}\left(1 - m_{\bar\eta}^2\,\frac{2\chi'_{\rm QCD}(0) - \chi'_{\rm YM}(0)}{\chi_{\rm YM}(0)}\right)^{1/2} g_{\bar\eta NN}$$

Finite mass correction to result of Veneziano et al. in chiral limit with $m_{\bar\eta}^2 = -\dfrac{2\,m\langle\phi\rangle}{N_f F_{\bar\eta}^2}$

Finite mass correction to Witten-Veneziano formula

$$m_{\eta'}^2 = -\frac{2N_f}{F_{\bar\eta}^2}\chi_{\rm YM}(0) + m_{\bar\eta}^2$$ & the relation $\chi'_{\rm YM}(0) = 4\chi'_{\rm QCD}|_{m=0}(0)$

- The finite mass corrections to $\Delta\Sigma$ are at most a few percent.

Corrections to the Witten-Veneziano formula are on the order of 10%.

# Experiment, Lattice and OZI violations

The OZI rule (Ellis-Jaffe in this context) tells us that the iso-singlet axial charge is proportional to the iso- octet axial charge. The latter is well-known from hyperon decays

$\Delta\Sigma^{\rm OZI} \equiv G_A^{\rm OZI} = 0.579 \pm 0.021$ Likewise, $\sqrt{\chi'_{\rm QCD,OZI}(0)} = F_\pi/\sqrt{6} \approx 32\,{\rm MeV}$ Shore (2007), Narison (2021)

Hence $$\Delta\Sigma^{\rm expt.} = \Delta\Sigma^{\rm OZI}\,\frac{\sqrt{\chi'_{\rm QCD}(0)}}{\sqrt{\chi'_{\rm QCD,OZI}(0)}}$$

HERMES: $\Delta\Sigma^{expt.}(Q^2 = GeV^2) = 0.33 \pm 0.011(th.) \pm 0.025(expt.) \pm 0.028(evol.)$
COMPASS: $\Delta\Sigma^{expt.}(Q^2 \to \infty) = 0.33 \pm 0.03(stat.) \pm 0.05(syst.)$

From the above, the central value of 0.33, gives $\sqrt{\chi'_{\rm QCD}(0)} \approx 18\,{\rm MeV}$ or $\sqrt{\chi'_{\rm YM}(0)} \approx 36\,{\rm MeV}$

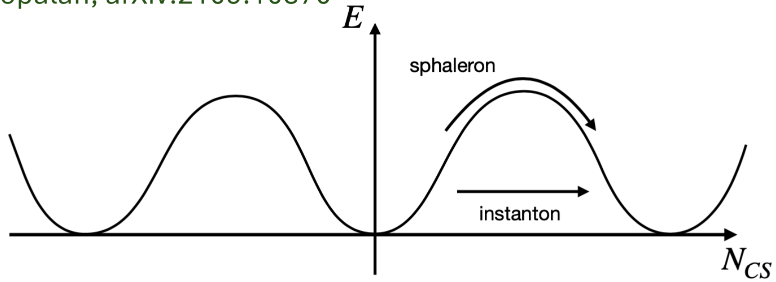
A recent lattice computation has central value of YM slope of 17.1 MeV – not bad, given many uncertainties…

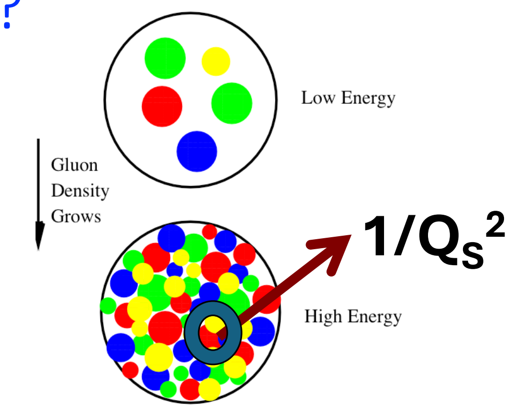
C. Bonanno, JHEP 01 (2024) 116

Further lattice studies at finite m, and refinements of our studies can test OZI violations in more systematically

# What about $g_1$ at small $x_{Bj}$ ?

Tarasov,Venugopalan, arXiv:2109.10370




Distribution of large x color sources

$$g_1^{\text{Regge}}(x_B,Q^2) = \left(\sum_f e_f^2\right) \frac{n_f \alpha_s}{\pi M_N} i \int d^4y \int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi} e^{-i\xi x} \int \mathcal{D}\rho\, W_Y[\rho] \int D\bar\eta\, \tilde{W}_{P,S}[\bar\eta] \int [DA]$$

$$\times \operatorname{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)\,\eta_0(y)\, \exp\left(iS_{\text{CGC}} + i\int d^4x \left[\frac{1}{2}(\partial_\mu\bar\eta)(\partial^\mu\bar\eta) - \frac{\sqrt{2n_f}}{F_{\bar\eta}}\bar\eta\,\Omega\right]\right)$$

Generalization of $g_{\bar\eta NN}$

Axion-like action (DeVecchia-Veneziano)

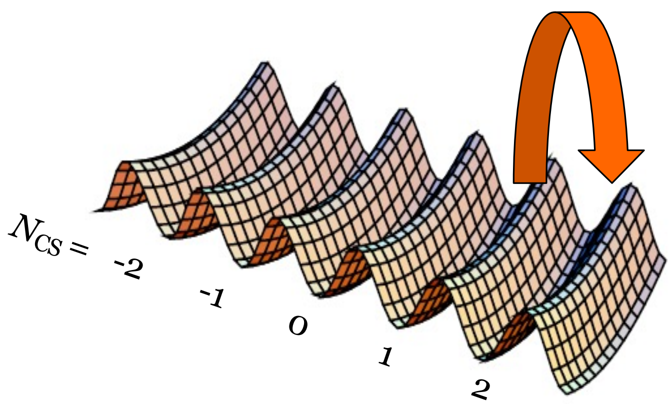
$$S_{\text{CGC}}[A,\rho] = -\frac{1}{4}\int d^4x F_a^{\mu\nu}F_{\mu\nu}^a + \frac{i}{N_c}\int d^2x_\perp\, \operatorname{tr}_c\left[\rho(x_\perp)\ln\left(U_{[\infty,-\infty]}(x_\perp)\right)\right]$$

# Spin diffusion via sphaleron transitions

Two scales – the height of the barrier given by $m_{\eta'}^2 = 2n_f \frac{\chi_{\mathrm{YM}}}{F^2}$

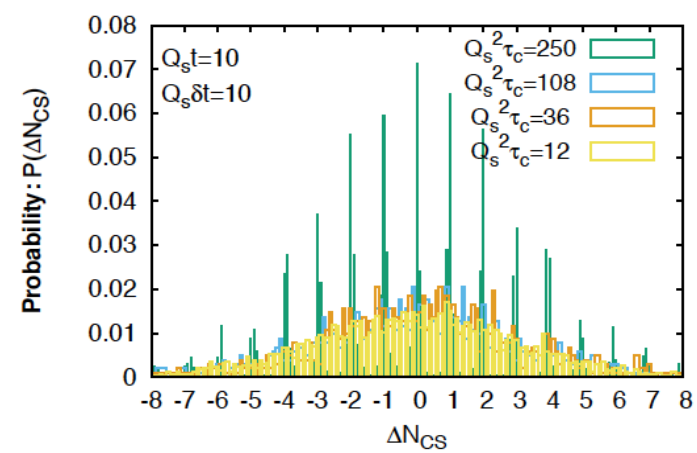
- the gluon saturation scale $Q_S$

When $Q_S{}^2 >> m_{\eta'}{}^2$ over the barrier gauge configurations dominate over instanton configurations




Over the barrier (sphaleron) transitions between different topological sectors of QCD vacuum…
characterized by integer valued Chern-Simons #

**Topological transitions in overoccupied gauge fields**



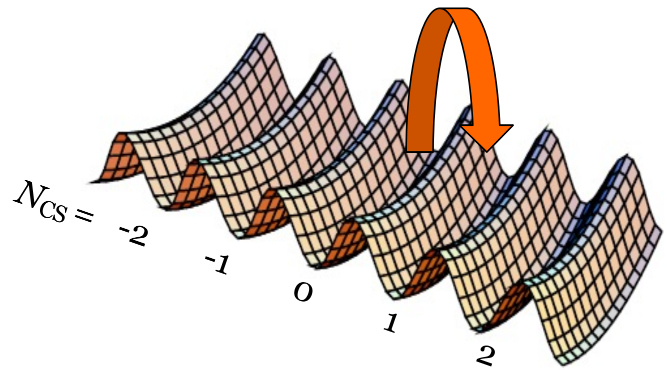

Mace, Schlichting, RV: PRD (2016) 1601.07342

Axion-like dynamics in a hot QCD plasma - McLerran,Mottola,Shaposhnikov (1990)

# Spin diffusion in topologically disordered media

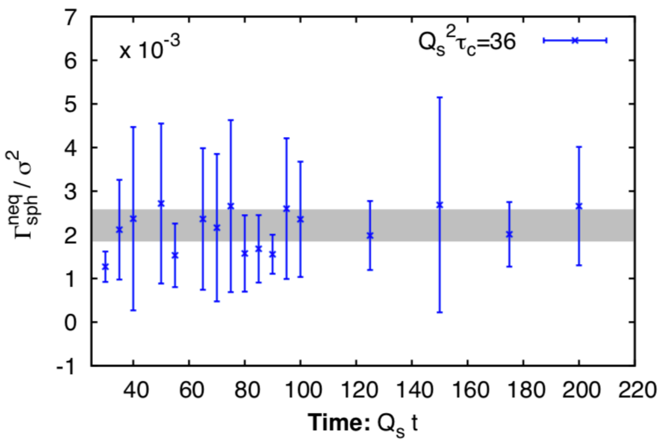
Two scales – the height of the barrier given by $m_{\eta'}^2 = 2n_f \frac{\chi_{\rm YM}}{F^2}$

- the gluon saturation scale $Q_S$




When $Q_S^2 >> m_{\eta'}{}^2$ over the barrier sphaleron-like configurations dominate over instanton configurations

Sphaleron transition rate off-equilibrium

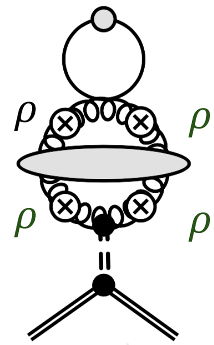



Numerical results of 3+1-D classical Yang-Mills of overoccupied configurations

- Sphaleron transition rate scales as the string tension of spatial Wilson loops
- The rate is large $\propto \ Q_S^4$

# $g_1$ at small $x_{Bj}$ from sphaleron transitions

For $Q_S^2 \ < m_{\eta'}^2$
over the barrier transitions




From our small $x_B$ effective action, $\frac{\partial^2 \eta'}{\partial t^2} = -\gamma \frac{\partial \eta'}{\partial t} - m_{\eta'}^2 \eta'$ $\qquad \gamma = \frac{2n_f \, \Gamma_{sphaleron}}{F^2{}_{\eta'} \, Qs}$

Spin diffusion due to "drag force" on "axion" propagation in the shock wave background
-drag force is proportional to sphaleron transition rate

McLerran,Mottola,Shaposhnikov (1990)

$$g_1^{\mathrm{Regge}}(x_B, Q^2) \propto \mathrm{F}(\mathrm{x_B}) \times \frac{Q_S^2 m_{\eta'}^2}{F_{\tilde{\eta}}^3 M_N} \exp\left(-4\, n_f C \,\frac{Q_S^2}{F_{\tilde{\eta}}^2}\right)$$

Very rapid quenching of spin diffusion at small $x_{Bj}$

Opportunity for discovery of sphaleron transitions at the EIC