Ab initio No-Core Shell Model: Recent Results and Plans

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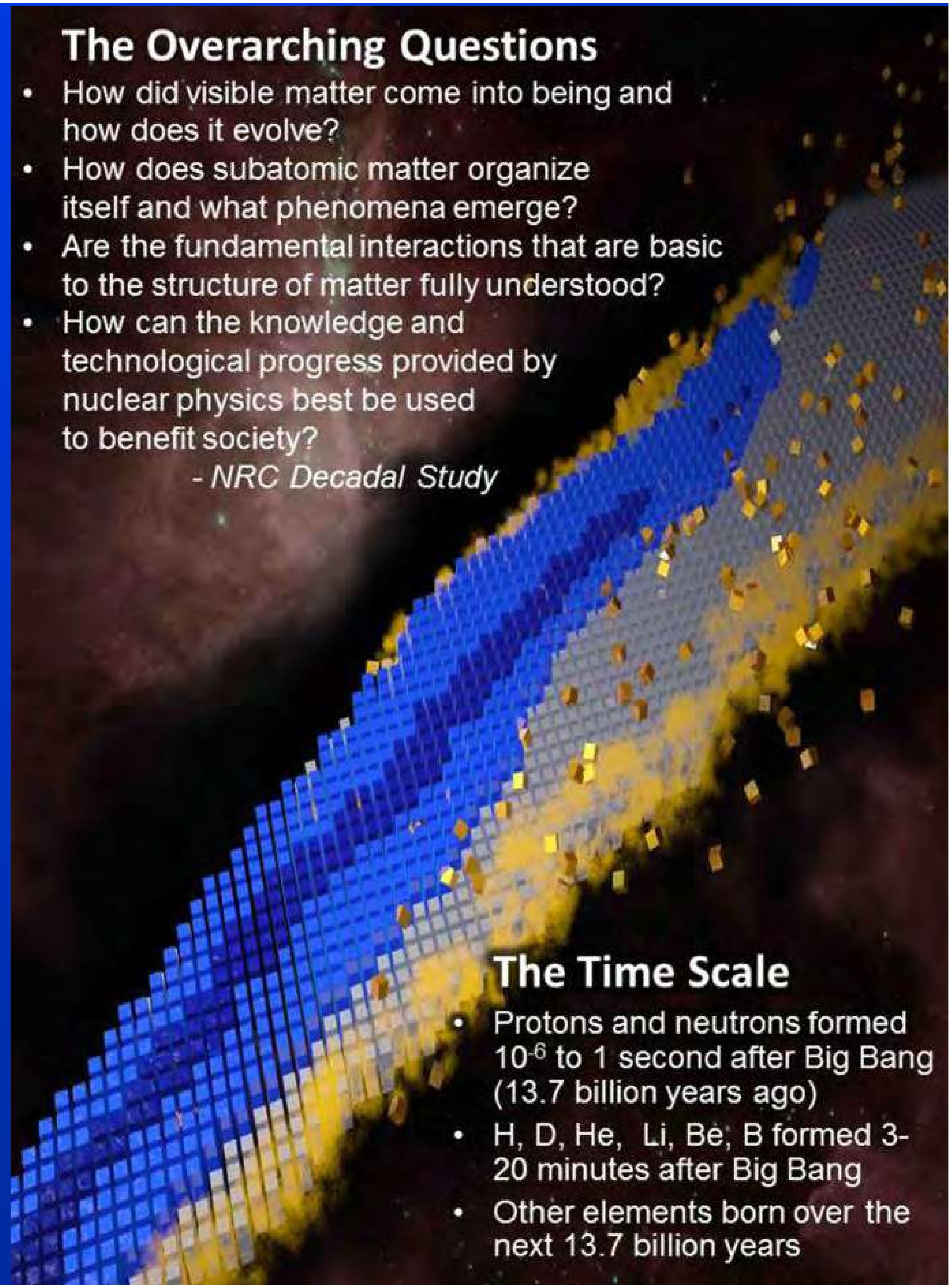
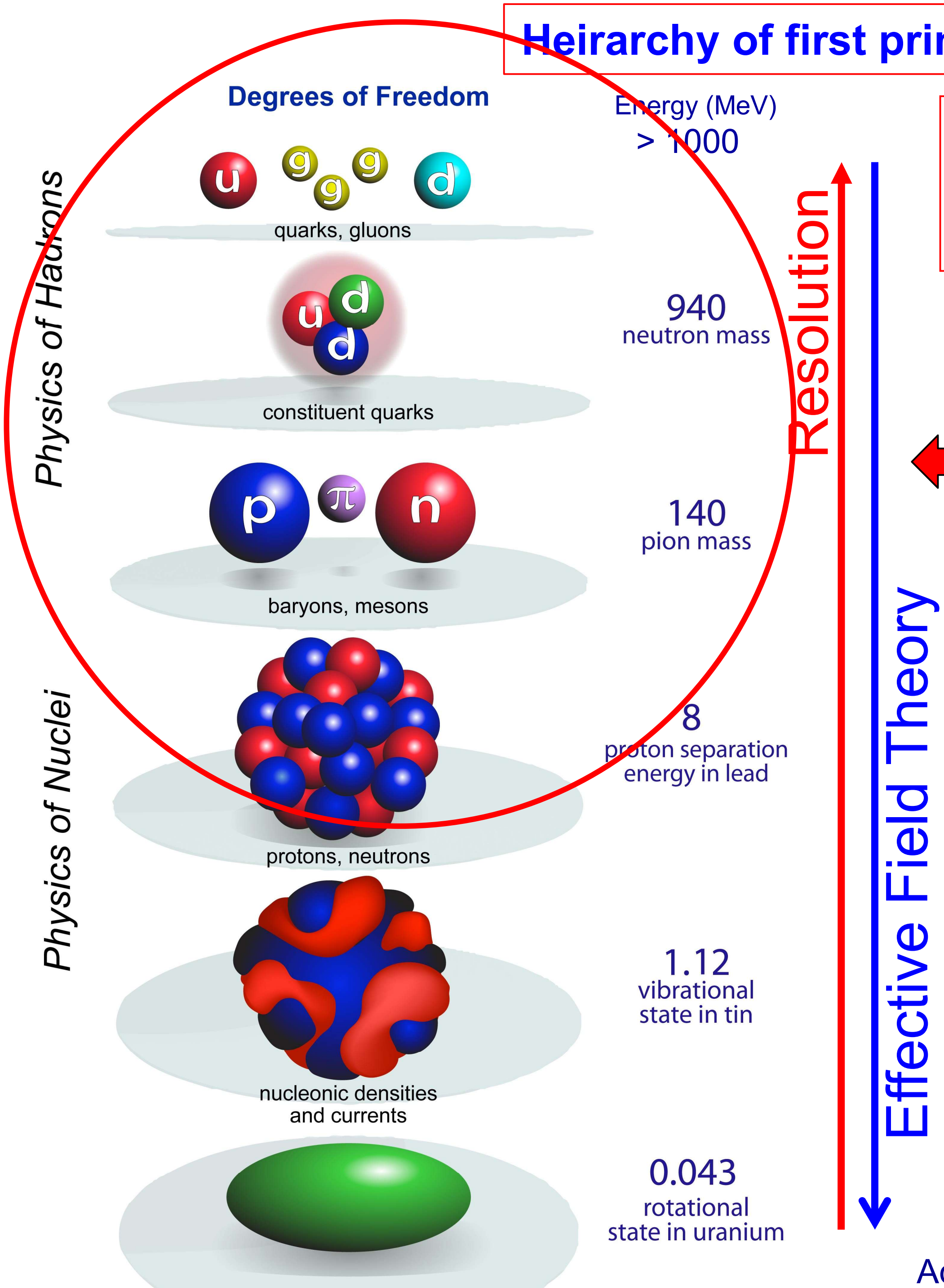
Institute for Nuclear Theory
Program INT 26-1
“Nuclear Hamiltonians
for Advancing Nuclear Physics
and Beyond"

May 29, 2026



Heirarchy of first principles problems



Hot and/or dense quark-gluon matter
Quark-gluon percolation
Hadron structure

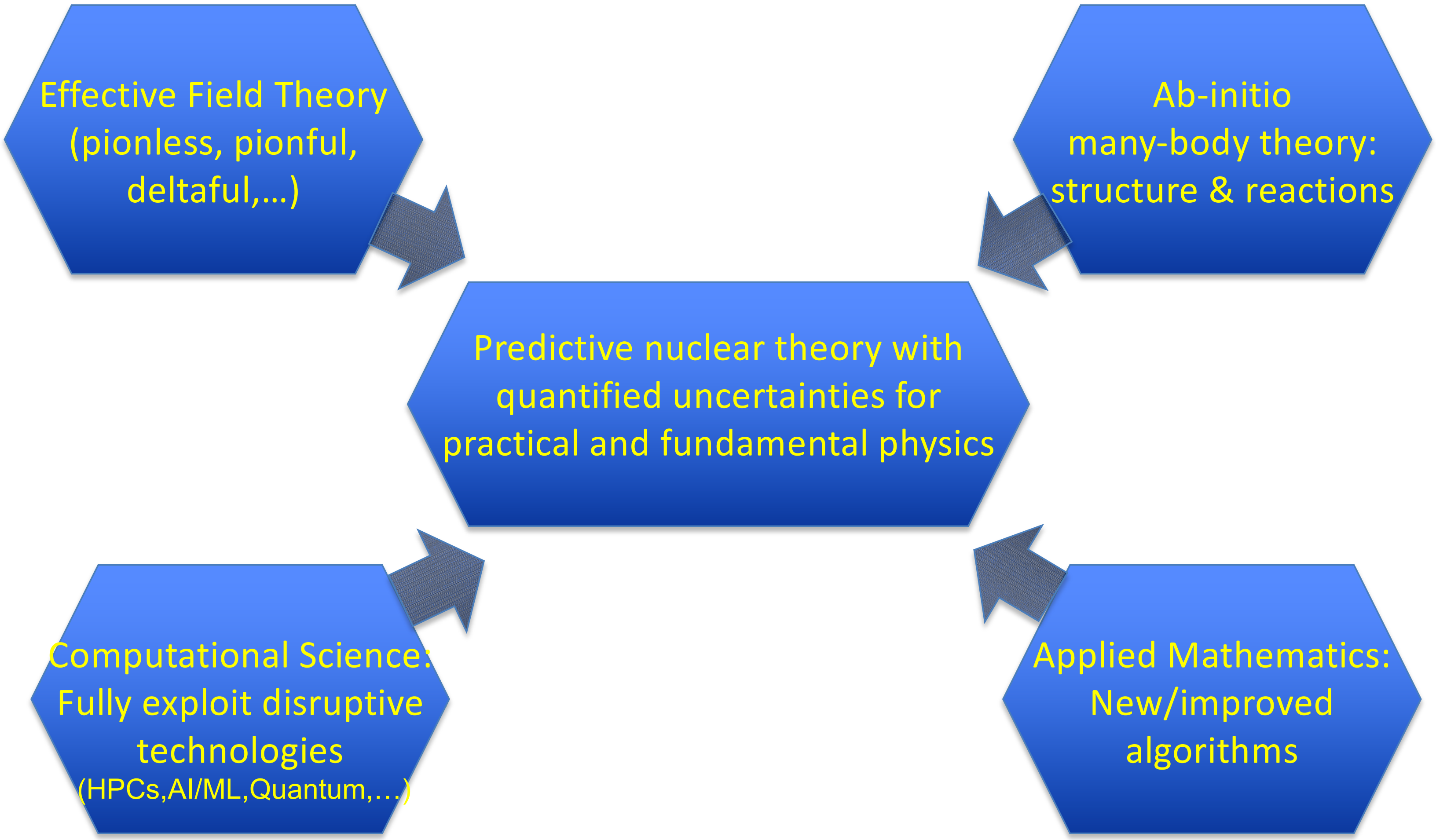
Hadron-Nuclear interface

Nuclear structure
Nuclear reactions

Nuclear astrophysics

Applications of nuclear science

Adapted from W. Nazarewicz, JPG 43, 044002, 2016



DOE supports these multi-disciplinary collaborations
SciDAC/NUCLEI: https://nuclei.mps.ohio-state.edu/

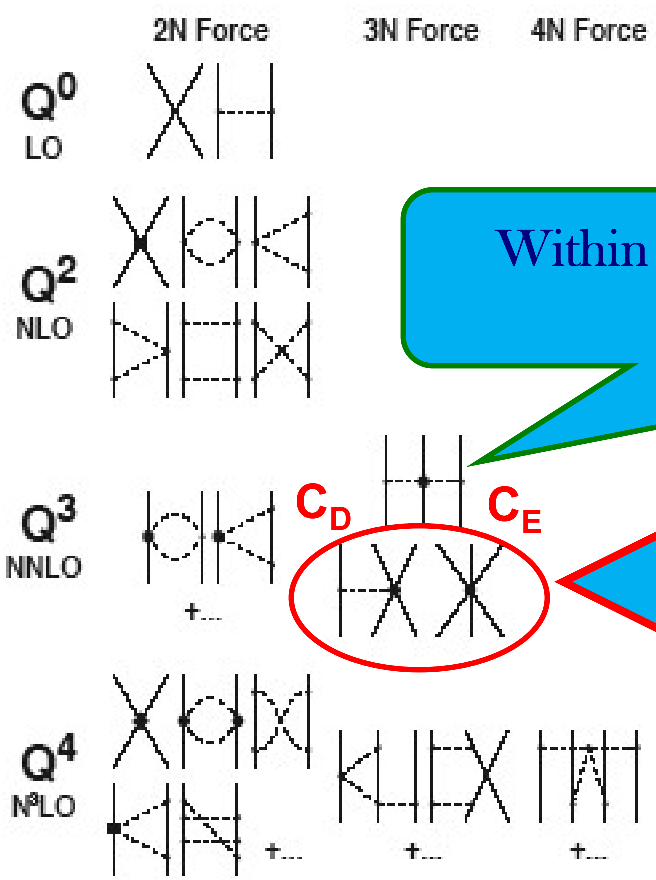
Effective Nucleon Interaction

Chiral Perturbation Theory (χPT)

Weinberg's *χ*PT allows for controlled power series expansion

$$\text{Expansion parameter}: \left(\frac{Q}{\Lambda_\chi}\right)^{\upsilon},\ Q - \text{momentum transfer},$$

$$\Lambda_\chi \approx 1\ GeV,\ \chi\text{ - symmetry breaking scale}$$



Within ***χ*PT 2□**-NNN Low Energy Constants (LEC) are related to the NN-interaction LECs $\{c_i\}$

Additional terms from χPT with LECs specific to NNN systems

Regularization is essential, which is also implicit within the Harmonic Oscillator (HO) wave function basis (see below)

R. Machleidt and D.R. Entem, Phys. Rep. 503, 1 (2011);
E. Epelbaum, H. Krebs, U.-G Meissner, Eur. Phys. J. A51, 53 (2015); Phys. Rev. Lett. 115, 122301 (2015)

Adapted from P. Navratil slide

No Core Shell Model (NCSM)
A large sparse matrix eigenvalue problem

$$H = T_{rel} + V_{NN} + V_{3N} + \bullet\bullet\bullet$$

$$H\left|\Psi_i\right\rangle = E_i\left|\Psi_i\right\rangle$$

$$\left|\Psi_i\right\rangle = \sum_{n=0}^{\infty} A_n^i \left|\Phi_n\right\rangle$$

$$\text{Diagonalize } \left\{\left\langle\Phi_m\right|H\left|\Phi_n\right\rangle\right\}$$

P. Navratil, J. P. Vary and B.R. Barrett, *Phys. Rev. Lett*. **84**, 5728 (2000); *Phys. Rev. C* **62**, 054311 (2000)

Review:
B.R. Barrett, P. Navratil and J.P. Vary, *Prog. Part. Nucl. Phys*. **69**, 131 (2013)

- Adopt realistic NN (and NNN) interaction(s) & renormalize as needed - retain induced many-body interactions: Chiral Effective Field Theory (Chiral EFT) interactions
- Adopt the 3-D Harmonic Oscillator (HO) for the single-nucleon basis states, 〈, ®,…
- Evaluate the nuclear Hamiltonian, H, in basis space of HO (Slater) determinants (each determinant manages the bookkeeping of anti-symmetrization)
- Diagonalize this sparse many-body H in its "m-scheme" basis where [〈 =(n,l,j,$m_{j,}$ | z)]

HO basis space (configurations)

$$\left|\Phi_n\right\rangle = [a_\alpha^+ \bullet\bullet\bullet a_\varsigma^+]_n \left|0\right\rangle$$

$$n = 1,2,\ldots,10^{10} \text{ or more!}$$

Now at ~10^{11}

- Evaluate observables and compare with experiment

Comments

- Computationally demanding => needs new algorithms & high-performance computers
- Requires convergence assessments and extrapolation tools to retain predictive power
- Achievable for nuclei up to atomic number of about 20 with largest computers available

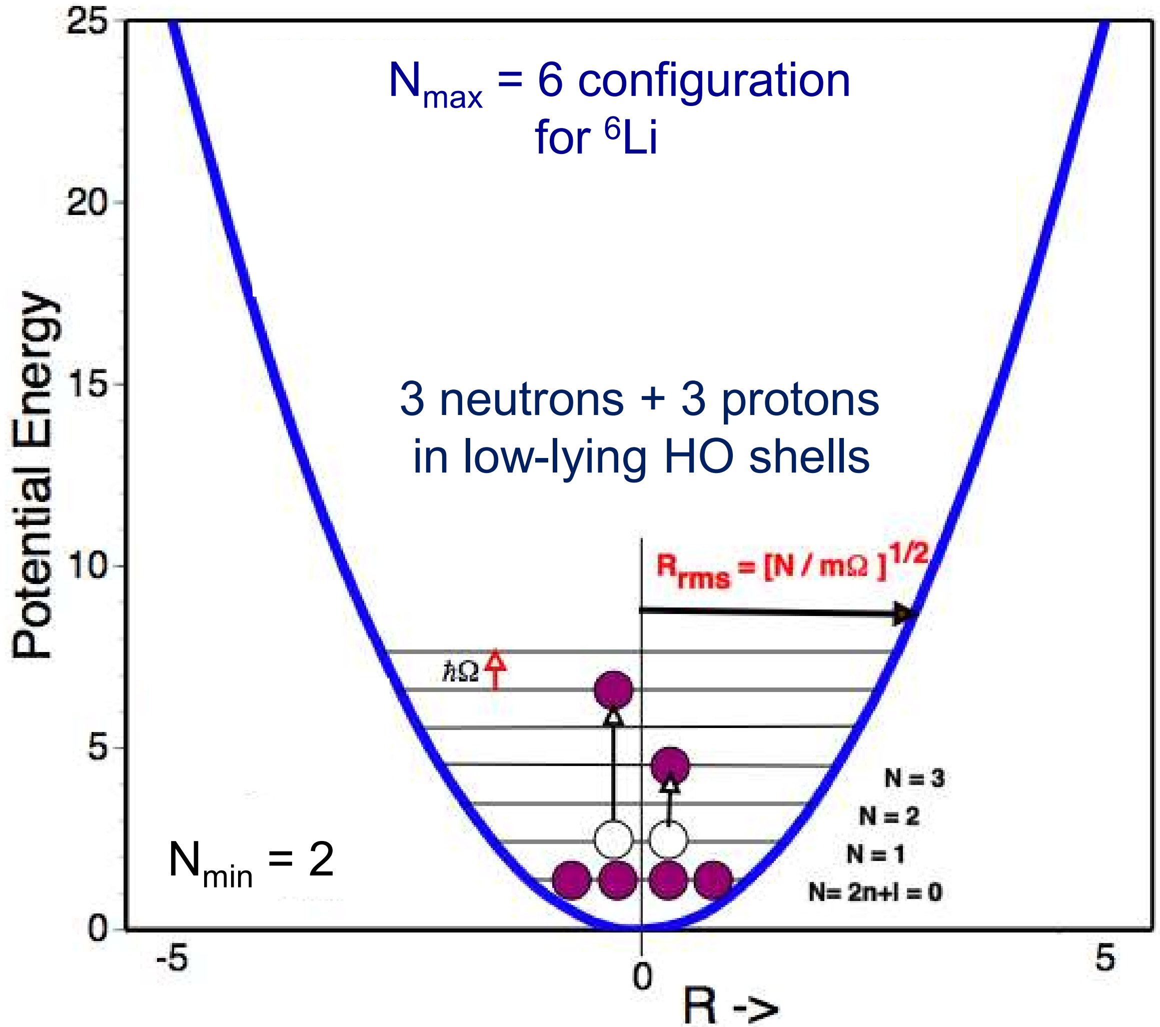
$N_{\min} \equiv$ HO quanta of lowest configuration

$N_{\max} \equiv$ maximum HO quanta above the lowest configuration

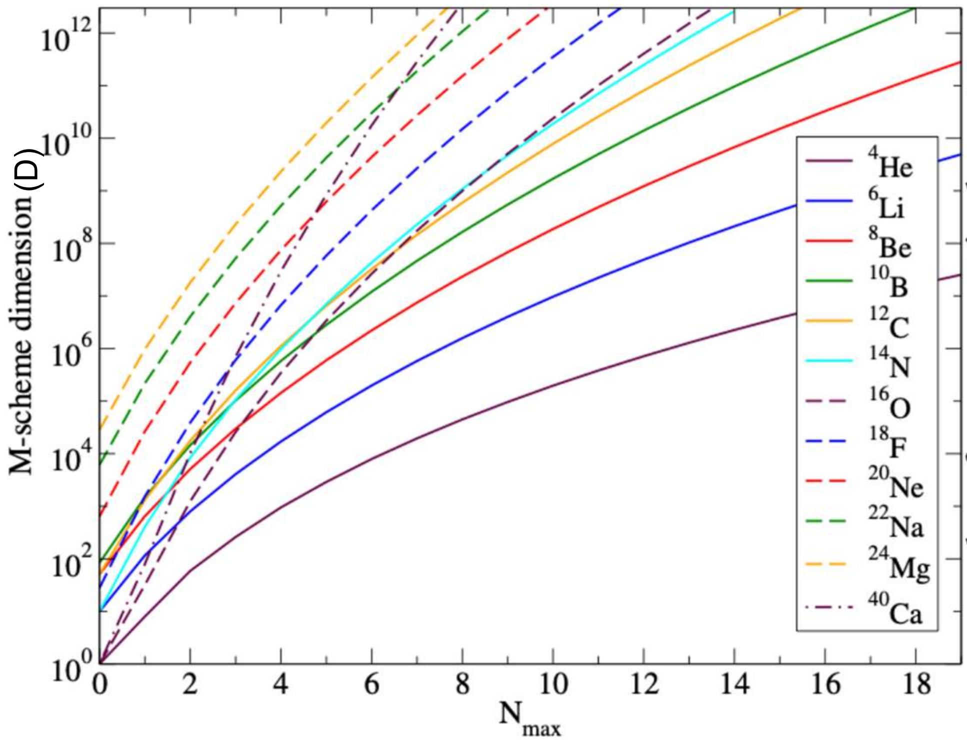
Retain configurations with $N_{\min} \leq \sum_{i=1}^{A}(2n_i + l_i) \leq N_{\min} + N_{\max}$

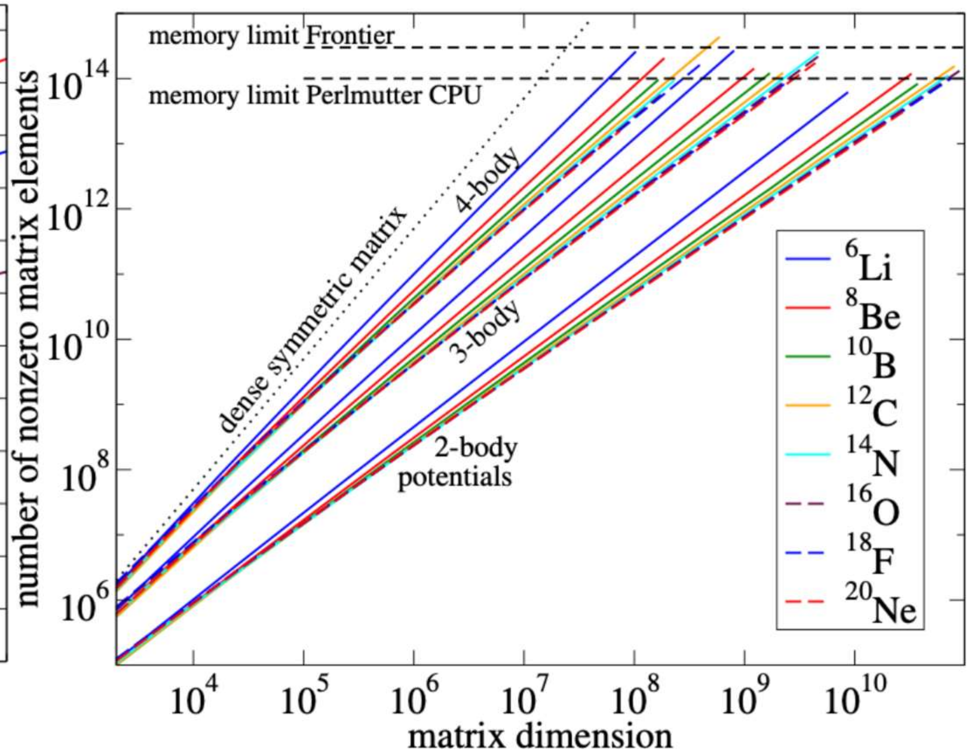
consistent with symmetry constraints (parity, M_J,...)

extrapolate: N_{max} -> infinity



Challenge
Exponential increase in Matrix Dimension (D)





Opportunities

- ➢ Memory/cpu time grows only as $D^{3/2}$
- ➢ Algorithm development (SciDAC/NUCLEI) funding)
- ➢ Exaflop machines now available (DOE/INCITE competitive awards)
- ➢ Improved understanding of Chiral EFT
- ➢ Developing methods for extrapolating D->inf (N_{max}->inf)

Coordinating χEFT and NCSM UV regulators makes sense and would reduce demands on computational resources (Discussion topic at INT-26-1)

Some relevant results to consider:
S.A. Coon, M.I. Avetian, M.K.G. Kruse, U. van Kolck, P. Maris and J.P. Vary, Phys. Rev. C 86, 054002 (2012)*

Fit function:

$$E_{gs}(\lambda_{sc}) = a \exp(-b/\lambda_{sc}) + E_{gs}$$

"By taking [the change from reduced mass to nucleon mass] into account, the successful emulation of the Idaho N3LO interaction In a HO basis suggests that Λ^{NN} ~ 780 MeV/c."

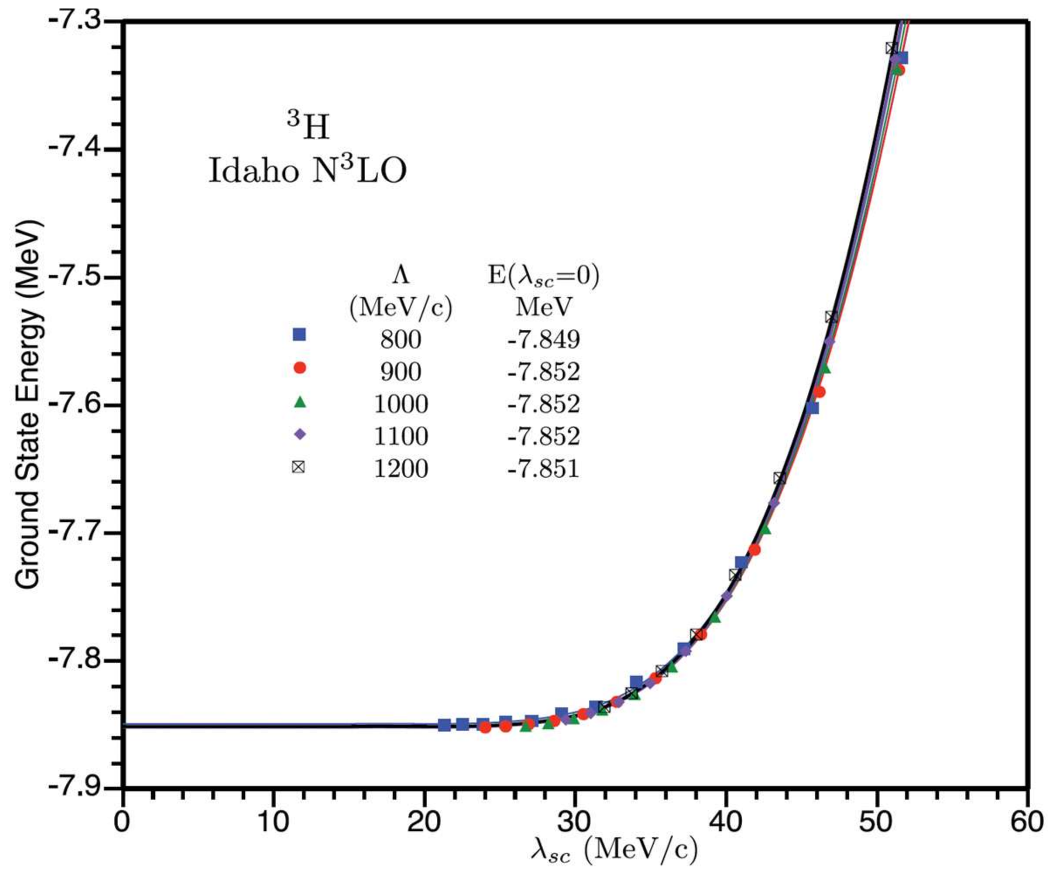
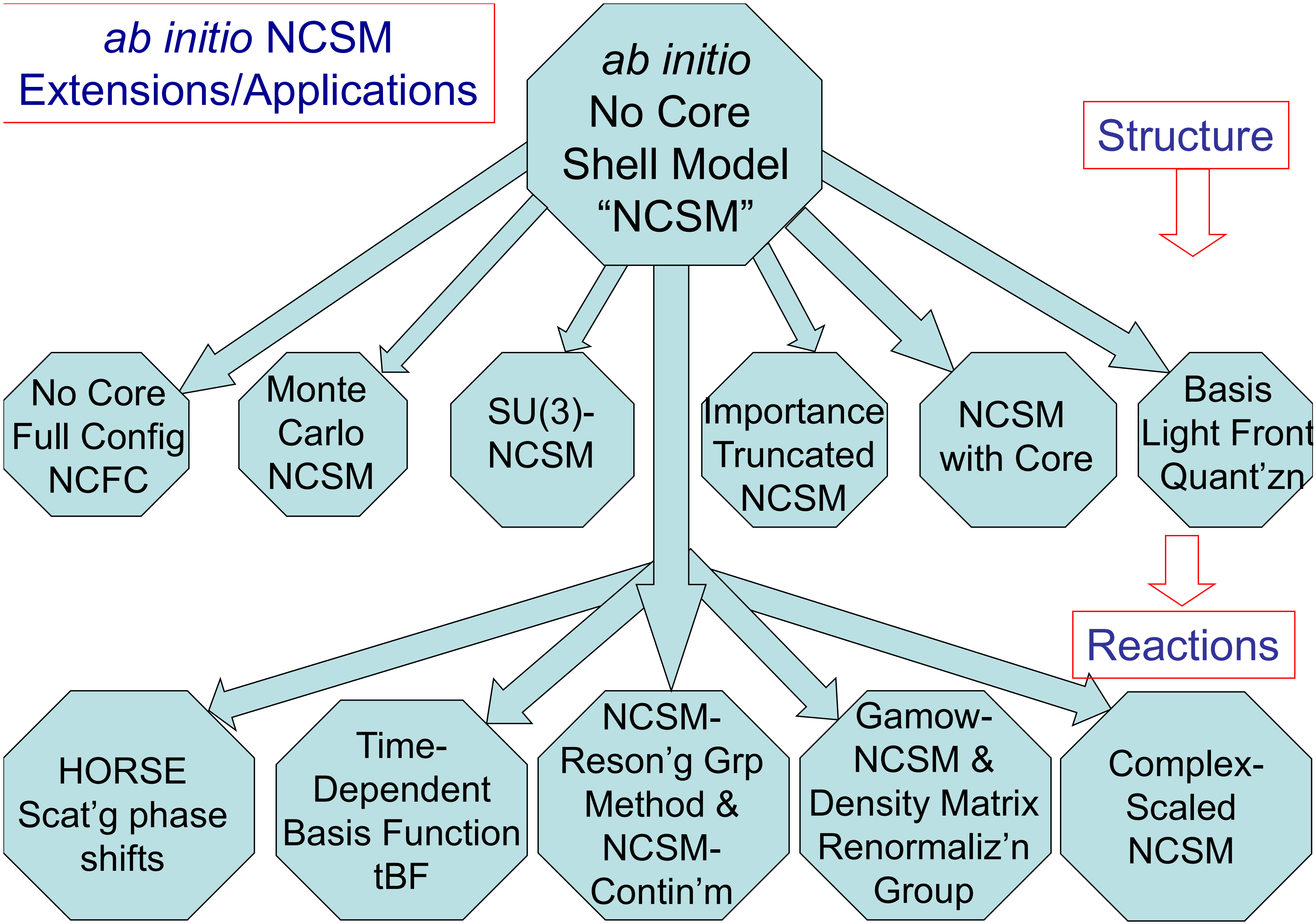


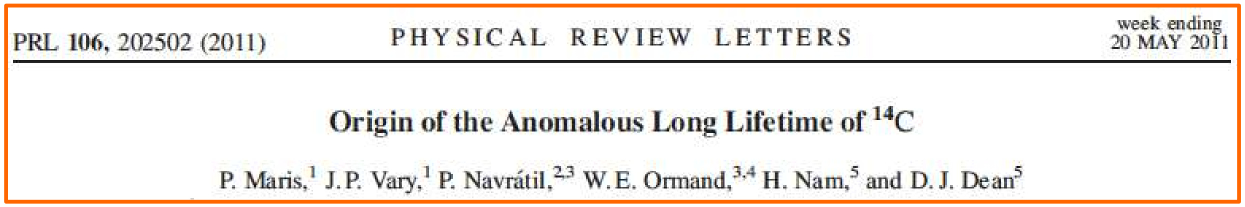
FIG. 7. (Color online) The ground-state energy of ^{3}H calculated at five fixed values of $\Lambda = \sqrt{m_N(N+3/2)\hbar\omega}$ and variable $\lambda_{sc} = \sqrt{(m_N\hbar\omega)/(N+3/2)}$. The curves are fits to the points and the functions fitted are used to extrapolate to the ir limit $\lambda_{sc} = 0$.

* **ACKNOWLEDGMENTS**

This study was conceived and initiated at the National Institute for Nuclear Theory's program "Effective Field Theories and the Many-Body Problem" in the spring of 2009.

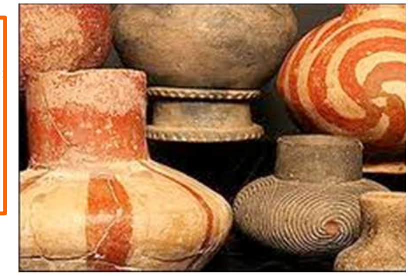
ab initio NCSM Extensions/Applications




PRL 106, 202502 (2011) PHYSICAL REVIEW LETTERS week ending 20 MAY 2011

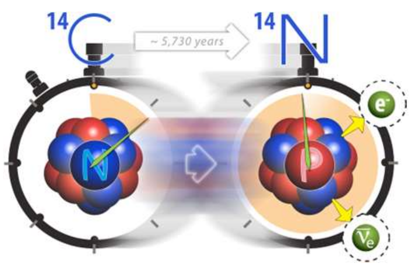
Origin of the Anomalous Long Lifetime of ^{14}C

P. Maris,[1] J.P. Vary,[1] P. Navrátil,[2,3] W.E. Ormand,[3,4] H. Nam,[5] and D. J. Dean[5]



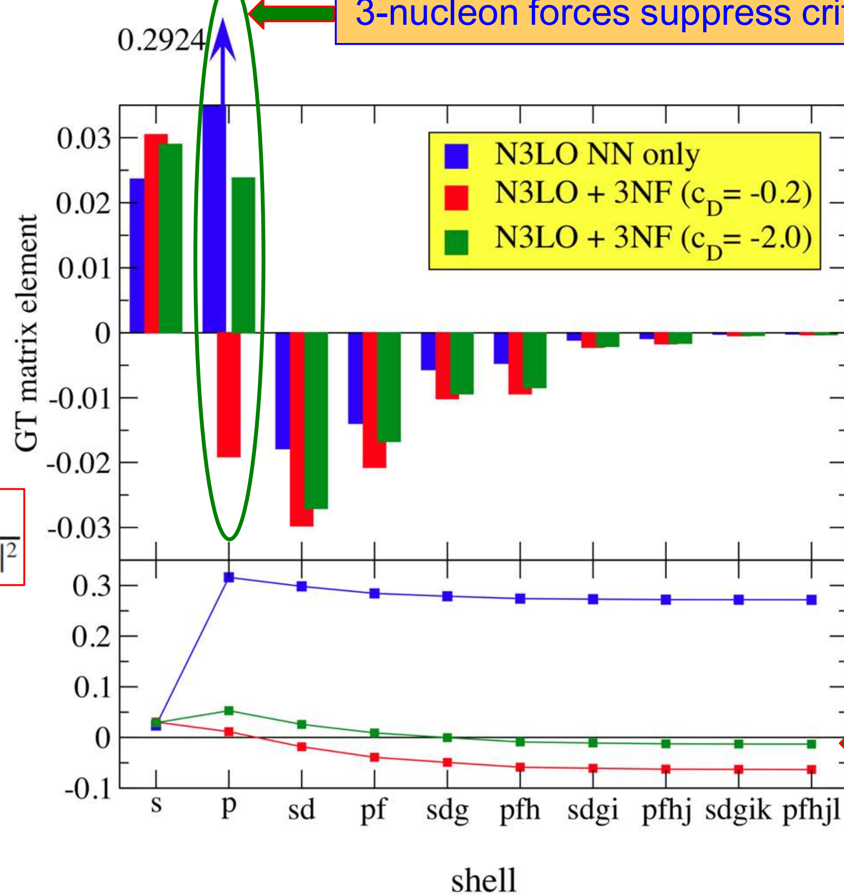
- Solves the puzzle of the long but useful lifetime of ^{14}C
- Establishes a major role for strong 3-nucleon forces in nuclei
- Strengthens foundation for guiding experiments

3-nucleon forces suppress critical component



$$T_{1/2} = \frac{1}{f(Z,E_0)} \frac{2\pi^3\hbar^7 \ln 2}{m_e^5 c^4 G_V^2} \frac{1}{g_A^2 |M_{GT}|^2}$$

$$M_{GT} = \sum_k \langle \Psi_f || \sigma(k)\tau_+(k) || \Psi_i \rangle$$



- Dimension of matrix solved for 8 lowest states ~ 1x10^9
- Each run takes ~ 6 hours on 215,000 cores on Cray XT5 Jaguar at ORNL
- "Scaling of *ab initio* nuclear physics calculations on multicore computer architectures," P. Maris, M. Sosonkina, J. P. Vary, E. G. Ng and C. Yang, 2010 Intern. Conf. on Computer Science, Procedia Computer Science 1, 97 (2010)

net decay rate
Is very small

PHYSICAL REVIEW C **103**, 054001 (2021)

Editors' Suggestion

Light nuclei with semilocal momentum-space regularized chiral interactions up to third order

P. Maris,[1,*] E. Epelbaum,[2] R. J. Furnstahl,[3] J. Golak,[4] K. Hebeler,[5,6] T. Hüther,[5] H. Kamada,[7] H. Krebs,[2] Ulf-G. Meißner,[8,9,10] J. A. Melendez,[3] A. Nogga,[9] P. Reinert,[2] R. Roth,[5] R. Skibiński,[4] V. Soloviov,[4] K. Topolnicki,[4] J. P. Vary,[1] Yu. Volkotrub,[4] H. Witała,[4] and T. Wolfgruber[5]

The LENPIC team:
www.lenpic.org

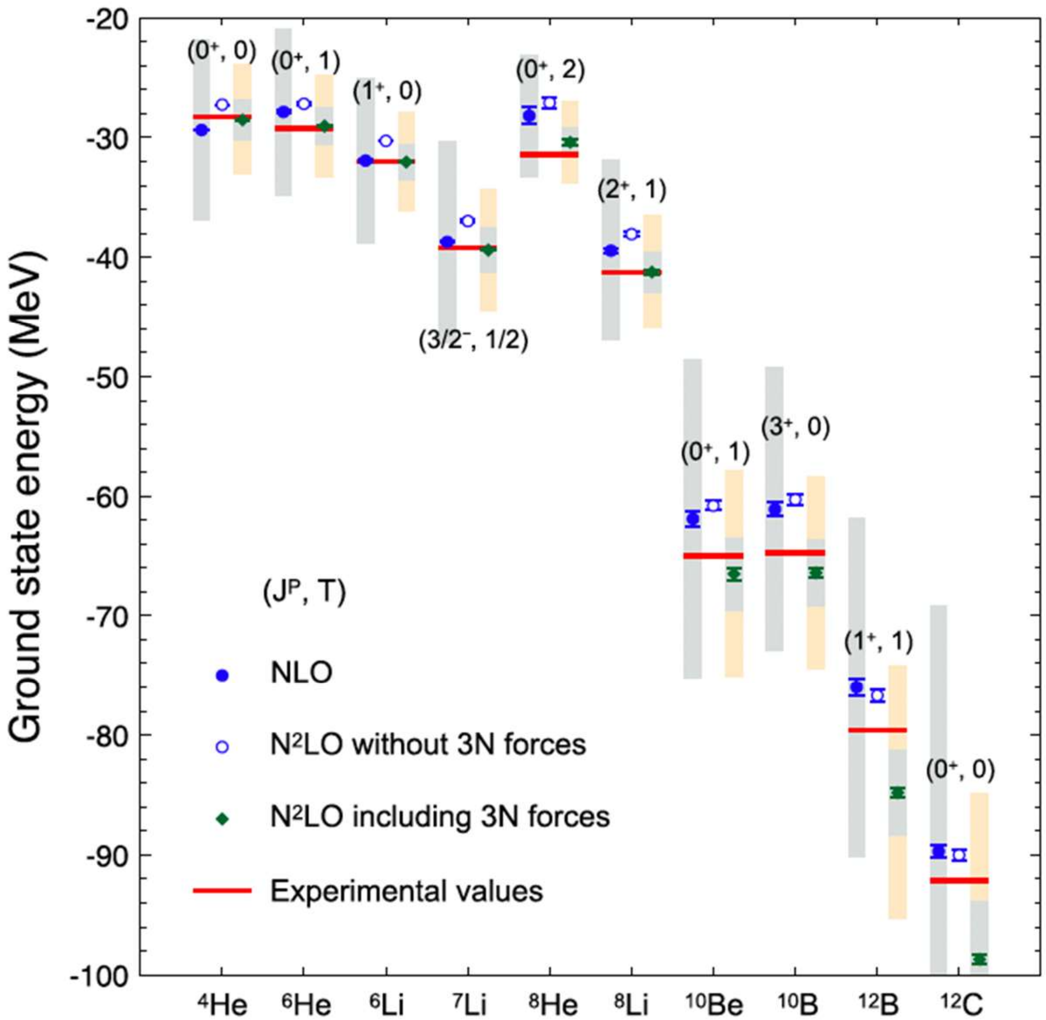


FIG. 8. Calculated ground-state energies in MeV using chiral NLO, and N^2LO interactions at $\Lambda = 450$ MeV (blue and green symbols) in comparison with experimental values (red levels). For each nucleus the NLO, and N^2LO results are the left and right symbols and bars, respectively. The open blue symbols correspond to incomplete calculations at N^2LO using NN-only interactions. Blue and green error bars indicate the NCCI extrapolation uncertainty. All results shown are for $\alpha = 0.08$ fm^4. The light (coral) and dark (gray) shaded bars indicate the 95% and 68% DoB truncation errors, respectively, estimated using the Bayesian model $\bar{C}^{650}_{0.5-10}$ (at NLO we only show the 68% DoB truncation errors because the 95% errors would be off one or even both ends of the scale).

Excitation energies from effective field theory with quantified uncertainties

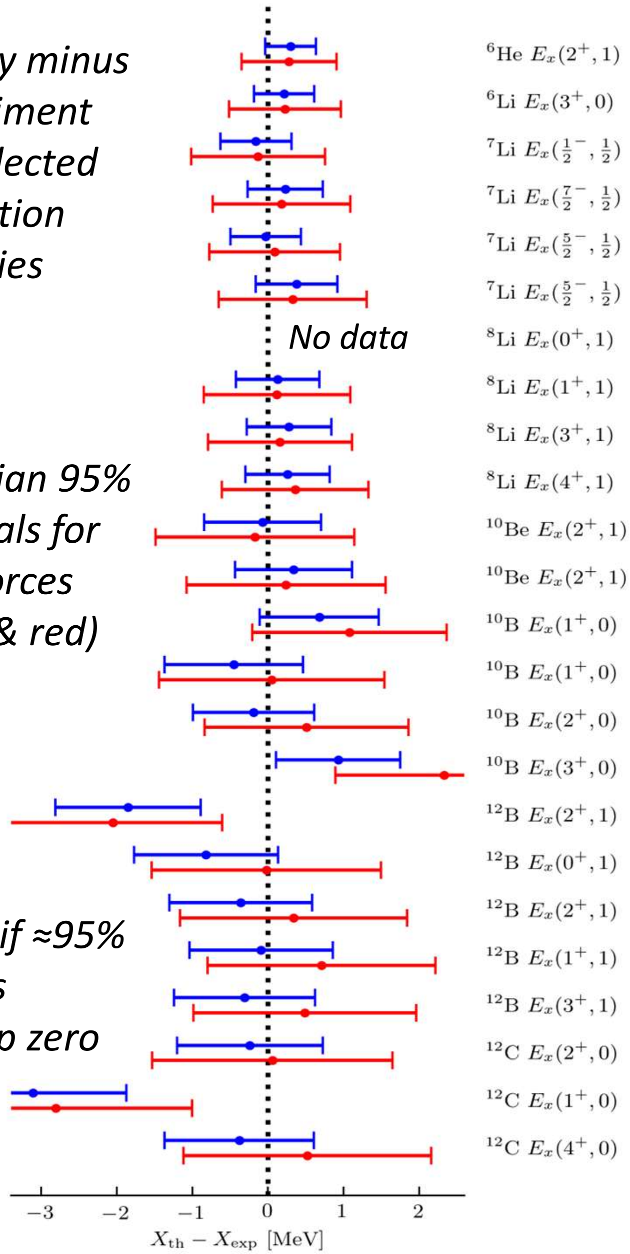
U.S. DEPARTMENT OF ENERGY | Office of Science

NUCLEI Nuclear Computational Low-Energy Initiative

Theory minus experiment for selected excitation energies

Bayesian 95% intervals for two forces (blue & red)

Check if ≈95% of bars overlap zero



Objectives

- Predict properties of ground and excited states of light nuclei with robust theoretical error estimates.
- Test consistent LENPIC chiral effective field theory (EFT) interactions with 2- and 3-nucleon forces.
- Extend and test a Bayesian statistical model that learns from the order-by-order EFT convergence pattern to account for correlated excitations.

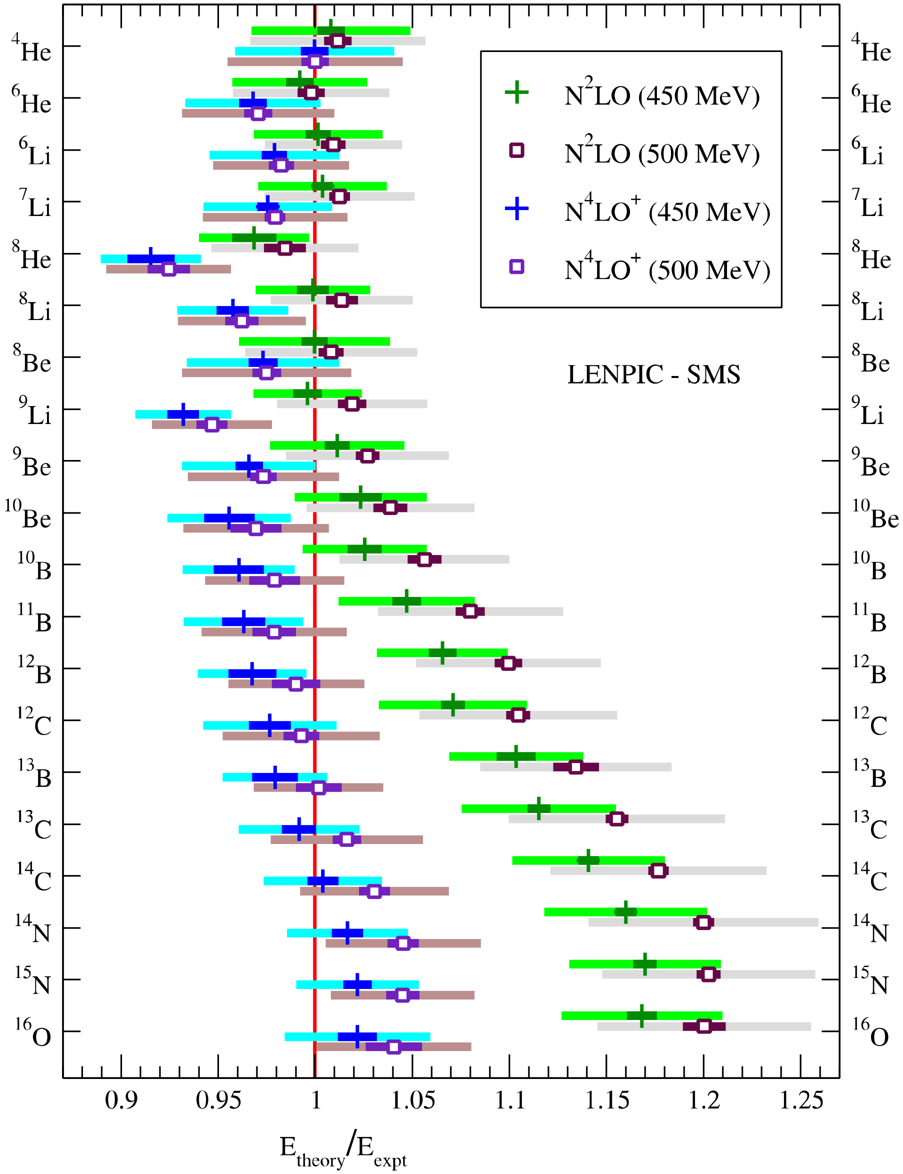
Impact

- First test of novel chiral nucleon-nucleon potentials with consistent three-nucleon forces.
- Demonstrates understanding of theoretical uncertainties due to chiral EFT expansion.
- Accounting for correlations produces agreement with experimental excitation energies (see figure).
- Exceptions in ^{12}C and ^{12}B indicate different theoretical correlations in the nuclear structure.

Accomplishments

P. Maris et al, Phys. Rev. C **103**, 054001 (2021); Editors' Suggestion; arXiv: 2012.12396 [nucl-th]

Binding Energies with LENPIC-SMS chiral EFT



P. Maris, H. Le, A. Nogga, R. Roth, J.P. Vary
Front. Phys. 11, 1098262 (2023)

- NN potential up to N^{4}LO$^+$
- 3NFs at N^{2}LO
- SRG evolved to $\alpha = 0.08$ fm^4
- LECs fitted to
  - NN scattering data
  - ^{3}H binding energy
  - Nd scattering
- Parameter-free predictions
- Error bars
  - numerical uncertainty
  - chiral EFT uncertainty from Bayesian analysis

Adapted from P. Maris, LENPIC Annual Meeting, Bonn, March 11-13, 2024

Daejeon16 NN interaction

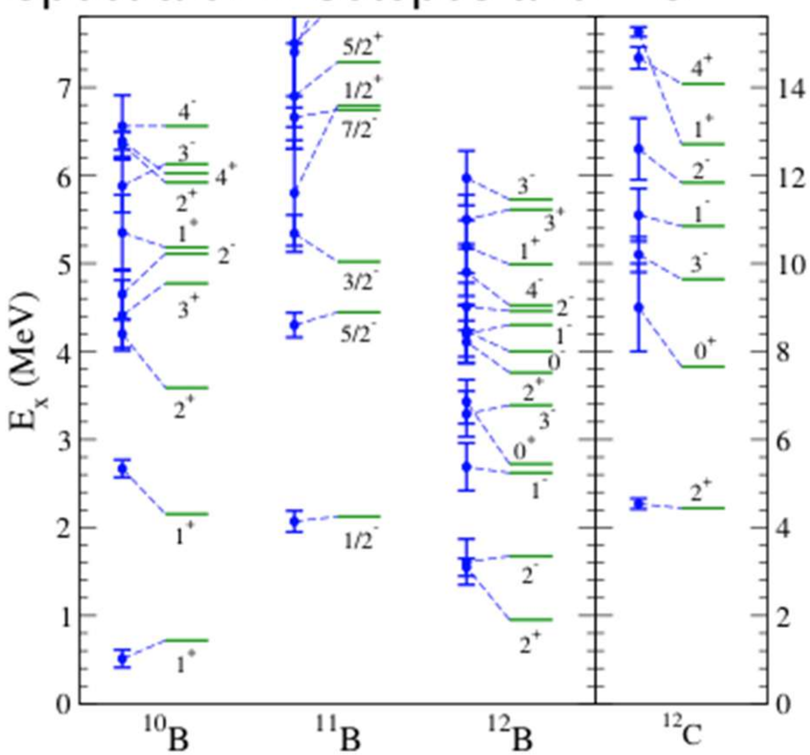
Based on SRG evolution of Entem-Machleidt "500" chiral N3LO to $\lambda = 1.5\ \mathrm{fm}^{-1}$ followed by Phase-Equivalent Transformations (PETs) to fit selected properties of light nuclei.

A.M. Shirokov, I.J. Shin, Y. Kim, M. Sosonkina, P. Maris and J.P. Vary,
"N3LO NN interaction adjusted to light nuclei in ab exitu approach,"
Phys. Letts. B 761, 87 (2016); arXiv: 1605.00413

Application to excited states of p-shell nuclei

(Maris, Shin, Vary, in preparation)

Spectra of B isotopes and ^{12}C



- difference of extrapolated E_b
- extrapolation uncertainties: max of E_b uncertainties
- good agreement with positive and negative parity spectra
- need large bases for 'intruder' and 'non-normal parity' states
- spectrum ^{10}B
  - correct gs 3^+ and excited 1^+
  - third 1^+ 'intruder' state
- excited 0^+ state in ^{12}C
  - Hoyle state?
  - see MCNCSM results below

Alpha clusters in Carbon-12 from ab initio theory & statistical learning

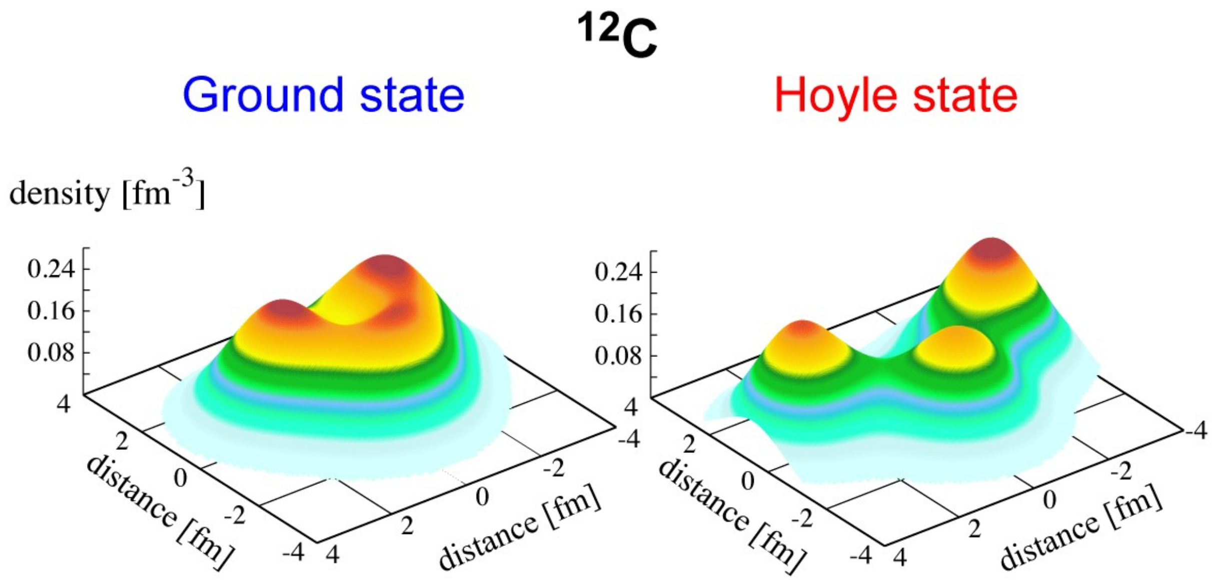
Objectives

- *Ab initio* nuclear theory aims for parameter-free predictions of critical nuclear properties with controlled uncertainties using supercomputer simulations
- Specfic goal is to determine extent of alpha clustering in the Ground state and the Hoyle state of Carbon-12 (^{12}C)

Impact

- Ground state found to have 6% alpha clustering while Hoyle state discovered to be 3-alphas 61% of the time
- With this high percentage of 3-alphas, the Hoyle state is confirmed as a natural gateway state for the cosmic formation of ^{12}C, the key element for organic life
- Statistical learning confirms 3-alpha feature of Hoyle state

Ab initio Monte-Carlo Shell Model results for density contours of 12C Ground state and first excited 0^+ (Hoyle) state using the Daejeon16 two-nucleon potential. Simulations were performed on Fugaku in Japan, the world's largest supercomputer at the time.



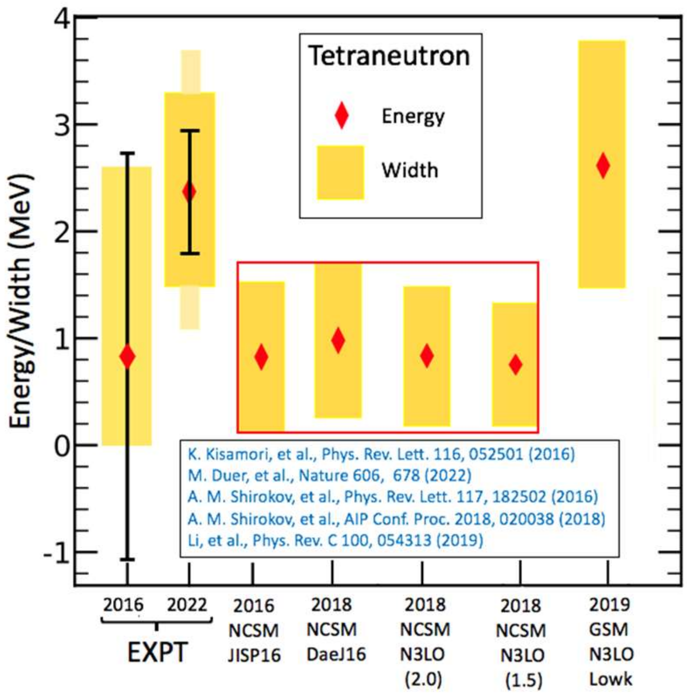
Accomplishments

T. Otsuka, T. Abe, T. Yoshida, Y. Tsunoda, N. Shimizu, N. Itagaki, Y. Utsuno, J. Vary, P. Maris and H. Ueno, "Alpha-Clustering in Atomic Nuclei from First Principles with Statistical Learning and the Hoyle State Character," Nature Communications 13:2234 (2022)

Tetraneutron discovery confirms prediction

Objectives

- *Ab initio* nuclear theory aims for parameter-free predictions of nuclear properties with controlled uncertainties using supercomputer simulations
- Specific goal is to predict if the tetraneutron (4-neutron system) has a bound state, a low-lying resonance or neither



Experiment and theory for the tetraneutron's resonance energy and width. *Ab initio* No-Core Shell Model (NCSM) and Gamow Shell Model (GSM) predictions use different neutron-neutron interactions and different basis function techniques.

Impact

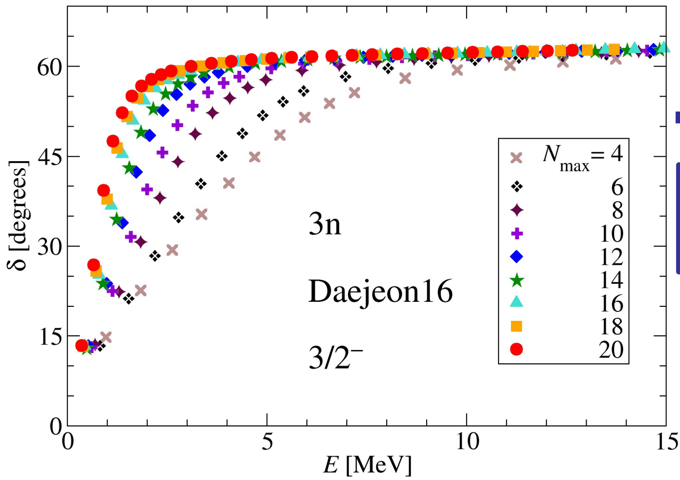
- Discovery in 2022 announced in Nature [1] confirms *ab initio* theory predictions from 2016 [2] of a short-lived tetraneutron resonance at low energy and the absence of a tetraneutron bound state
- Demonstrates the predictive power of *ab initio* nuclear theory since theory and experiment are within their combined uncertainties
- Sets stage for further experimental and theoretical research on new states of matter formed only of neutrons
- Shows need to anticipate a long wait time for experimental confirmation of such an exotic phenomena, ~ 6 years in this case
- Emphasizes the value of DOE supercomputer allocations (NERSC) and support for multi-disciplinary teamwork (SciDAC/NUCLEI)

Accomplishments

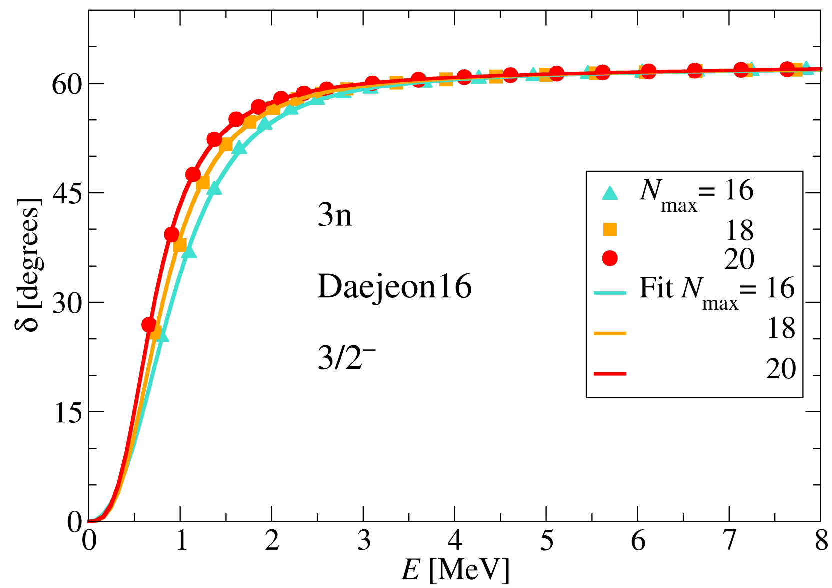
[1] M. Duer, et al., Nature 606, 678 (2022)

[2] A.M. Shirokov, G. Papadimitriou, A.I. Mazur, I.A. Mazur, R. Roth and J.P. Vary, "Prediction for a four-neutron resonance," Phys. Rev. Lett. 117, 182502 (2016)

3n Results with Daejeon16 NN interaction



Selection of points, parameterization



Extraction of S-matrix pole position

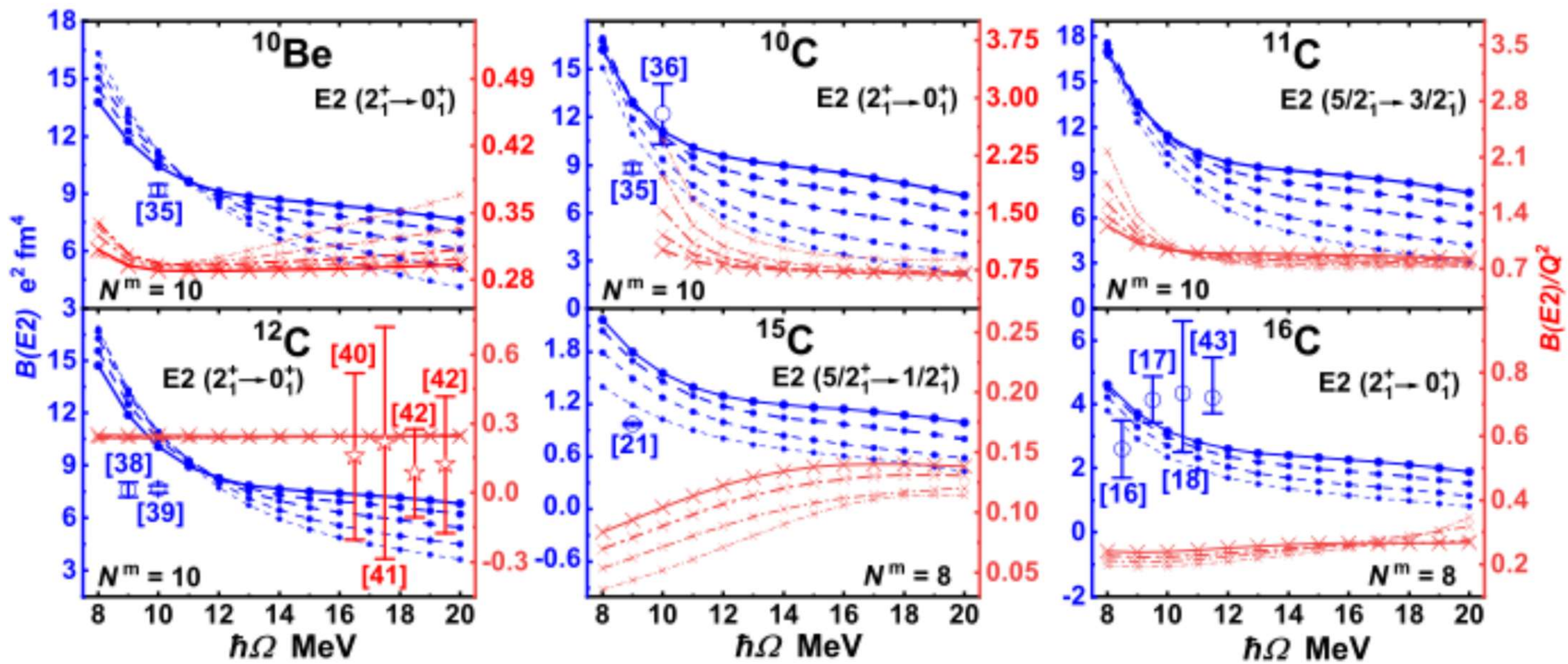
N_{max}	$3/2^-$		$1/2^-$	
	E_r [MeV]	Γ [MeV]	E_r [MeV]	Γ [MeV]
16	0.607	1.524	0.606	1.604
18	0.537	1.176	0.531	1.133
20	0.481	0.963	0.481	0.962

I.A. Mazur, M. K. Efimenko, A. I. Mazur, I. J. Shin, V. A. Kulikov, A. M. Shirokov and J. P. Vary, Rev. C 110, 014004 (2024); arXiv: 2403.18232

$$\frac{B(E2)}{Q_p^2} = \frac{\langle J_f || \sum_{i\in p} r_i^2 Y_2(\hat{r}_i) || J_i \rangle^2}{\langle J_i || \sum_{i\in p} r_i^2 Y_2(\hat{r}_i) || J_i \rangle^2},$$

Ratios of observables converge better

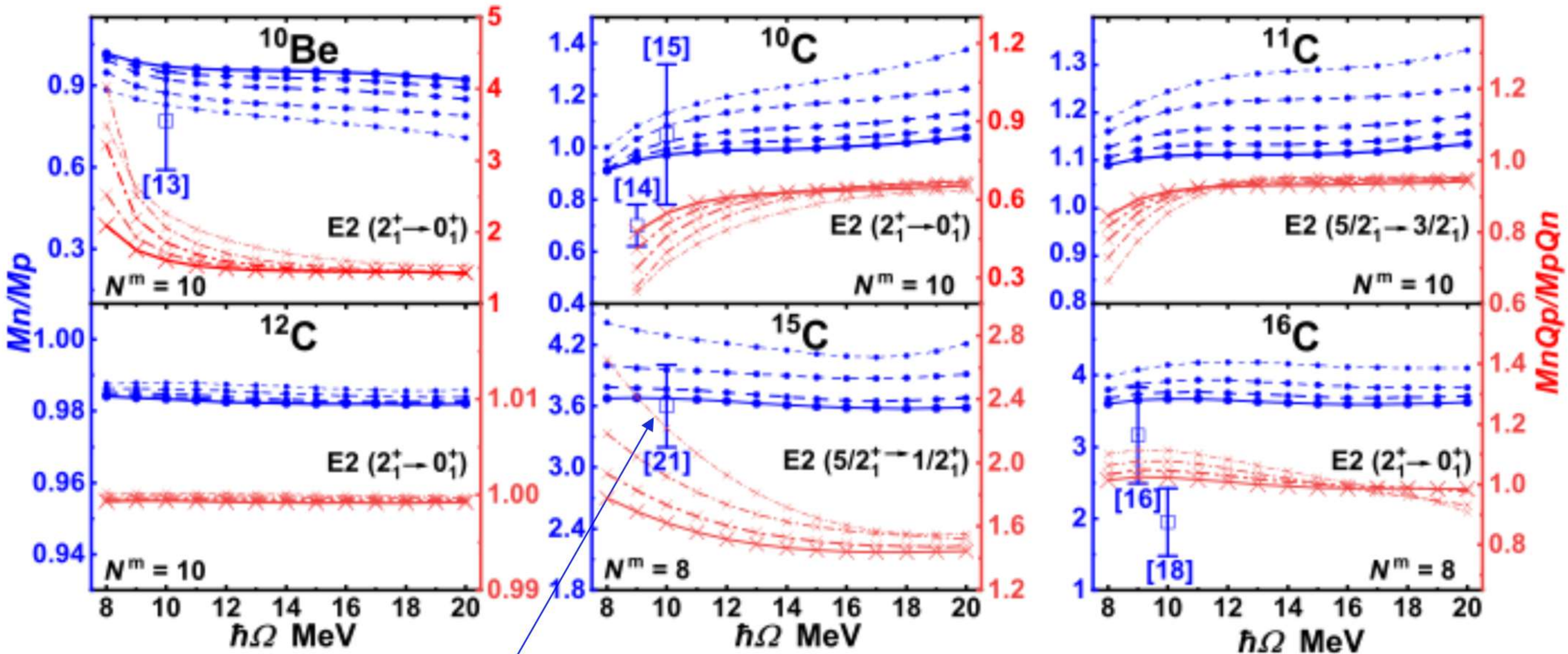
He Li, et al., Phys. Rev. C 110, 064325 (2024); arXiv: 2401.05776



$$\frac{M_n}{M_p} = \frac{\langle J_f || \sum_{i\in n} r_i^2 Y_2(\hat{r}_i) || J_i \rangle}{\langle J_f || \sum_{i\in p} r_i^2 Y_2(\hat{r}_i) || J_i \rangle},$$

Ratios of observables and
ratios of ratios converge better

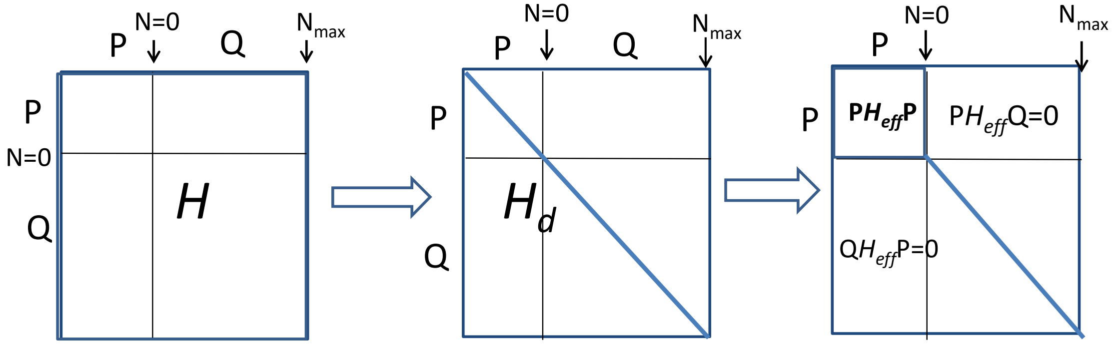
He Li, et al., Phys. Rev. C 110, 064325 (2024);
arXiv: 2401.05776



Jie Chen et al., PRC 106, 064312 (2022)

Ab-initio effective interaction from the NCSM

Okubo-Lee-Suzuki (OLS) similarity transformation
of the NCSM solution



$$H_d = U H U^\dagger \qquad H_{eff} = \frac{U_P^\dagger}{\sqrt{U_P U_P^\dagger}} H_d \frac{U_P}{\sqrt{U_P U_P^\dagger}}$$

Flow

- NCSM for ^{18}F at N_{max}
- H_{eff} for ^{18}F at $N = 0$ (OLS)
- ^{16}O at N_{max} (core energy)
- ^{17}O, ^{17}F at N_{max} (one-body terms)
- ϵ_j, $\langle ij|V_{eff}|kl\rangle_{JT}$

Okubo, Progr. Theor. Phys. 12 (1954); Suzuki, Lee, Prog. Theor. Phys. 68 (1980)

Dikmen, Lisetskiy, Barrett, Maris, Shirokov, Vary, PRC91, 064301 (2015)

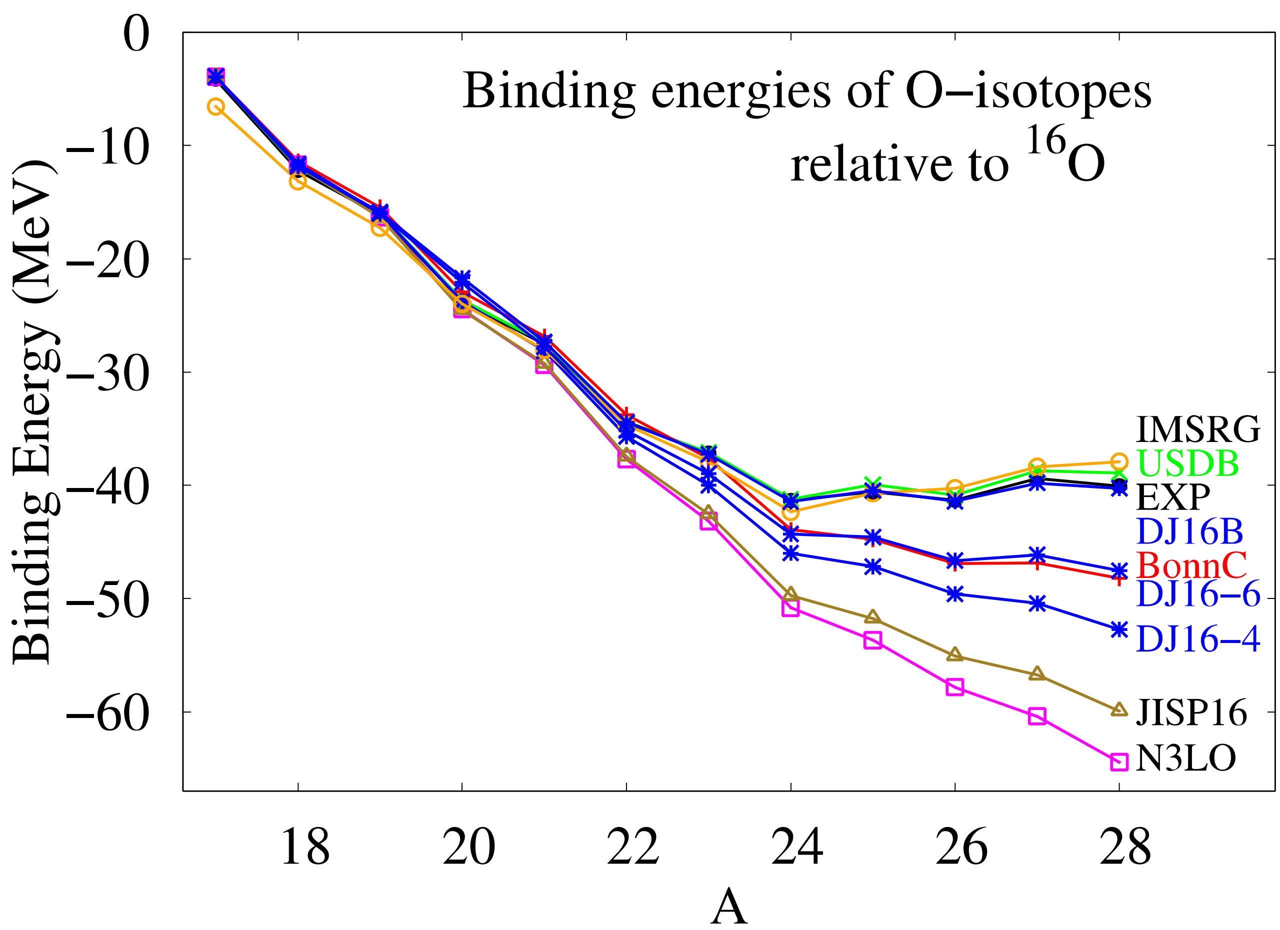
Vary, Basili, Du, Lockner, Maris, Pal, Sarker, PRC98, 065502 (2018)

Smirnova, Barrett, Kim, Shin, Shirokov, Dikmen, Maris, Vary, PRC100, 054329 (2019) (Nmax = 4)

Shin, Smirnova, Shirokov, Yang, Barrett, Li, Kim, Maris, Vary, Phys. Rev. C 110, 034306 (2024) (Nmax=6)

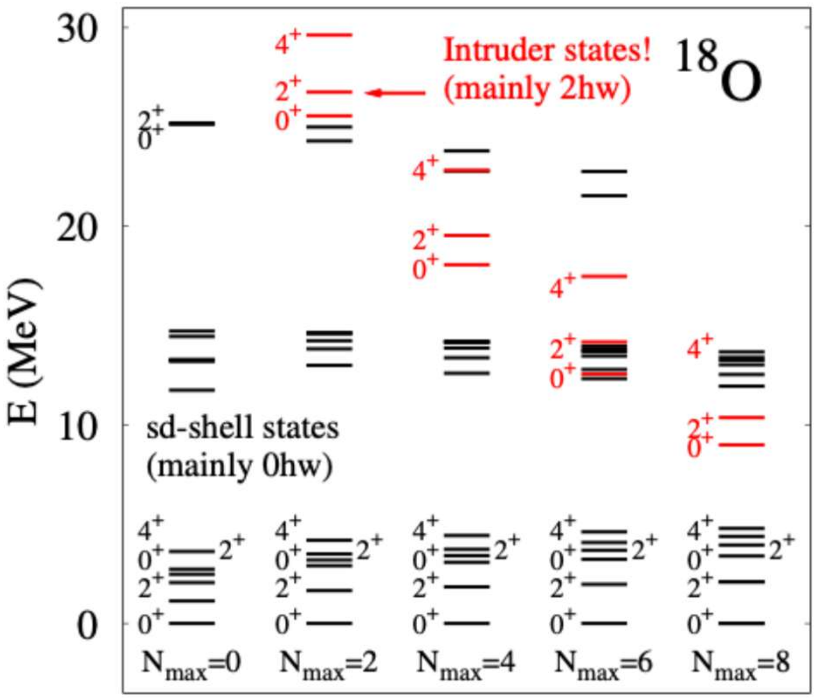
Binding energies of O-isotopes

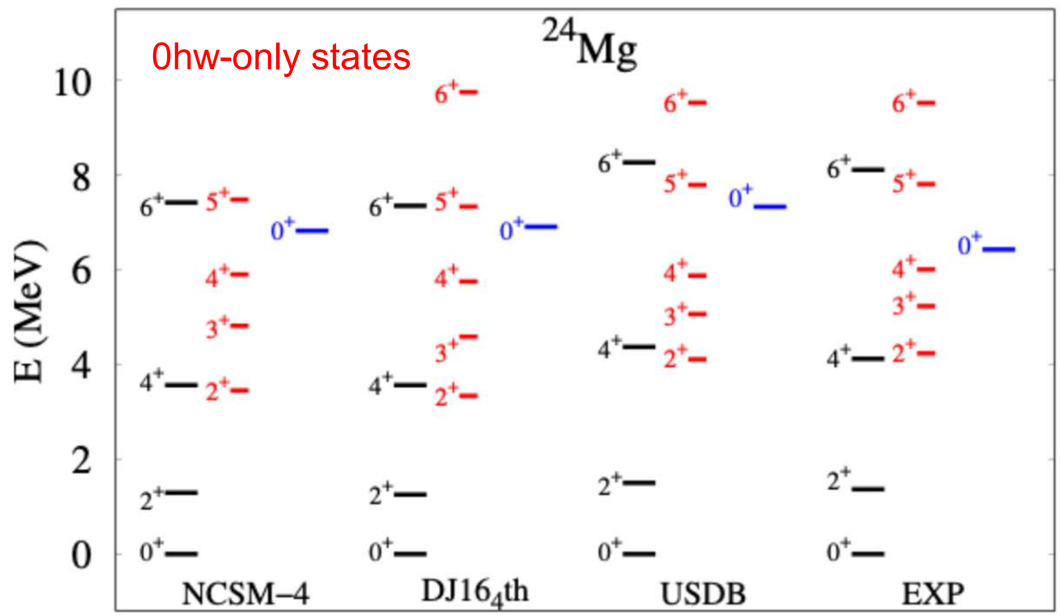
rms(DJ16-6) ≈ 3671 keV;   rms(DJ16B) ≈ 235 keV; rms(USDB) ≈ 467 keV



Shin, Smirnova, Shirokov, Yang, Barrett, Li, Kim, Maris and Vary, Phys. Rev. C 110, 034306 (2024)

Comparing NCSM (NCSM-4)
with derived **charge-dependent** Valence Effective SM ($DJ16_4$-th)
at N_{max} = 4, $\hbar\omega$ = 14 MeV, using Daejeon16 plus Coulomb.



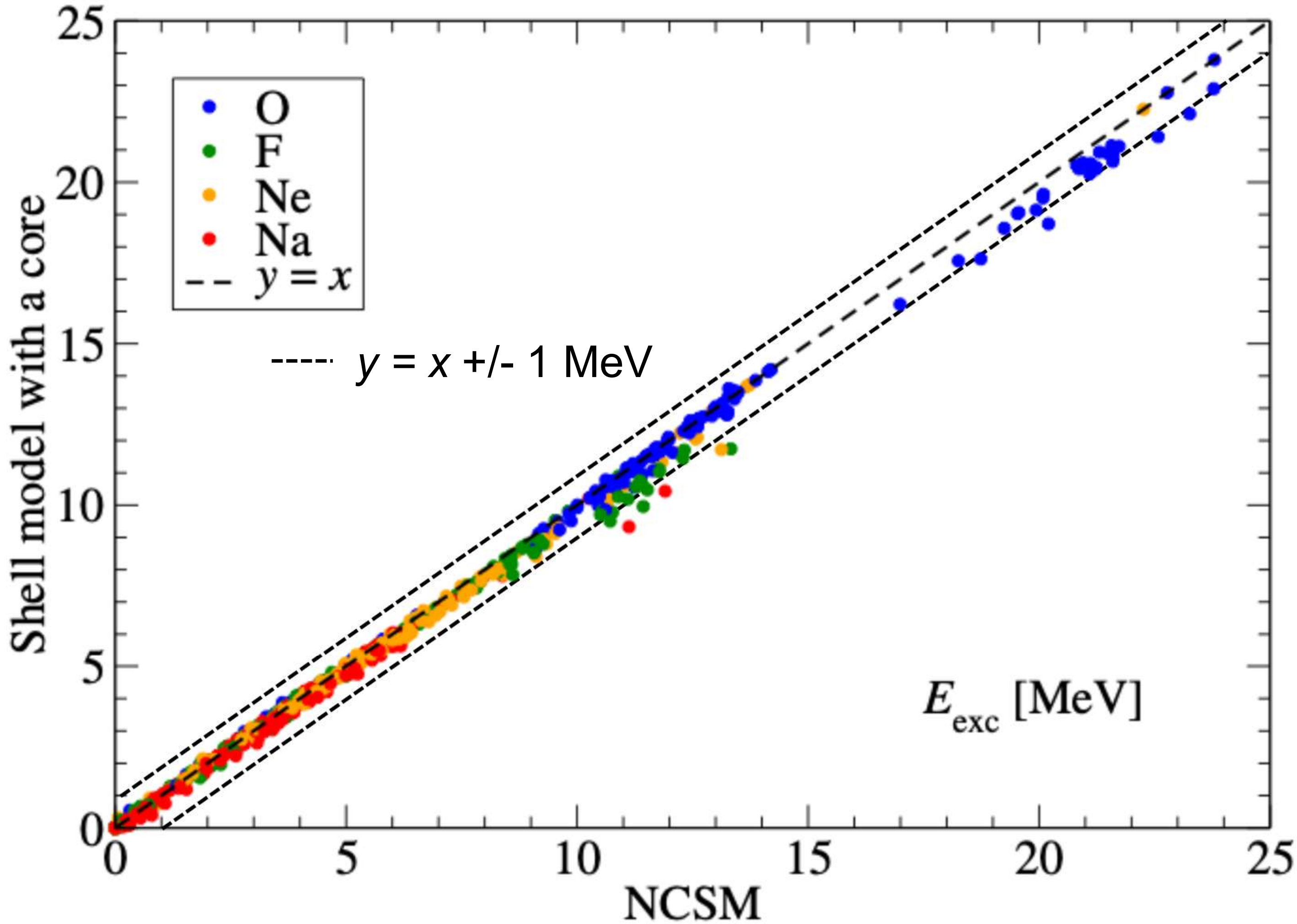


- Dimension at N_{max} = 8 is 12.2 billion
- Low-lying "0hw states" well converged
- Intruders not yet converged at N_{max} = 8
- 0hw and 2hw state spacings ~Nmax indep.

- Low-lying "0hw-states" well reproduced
- Favorable comparison with USDB and
  and with EXP

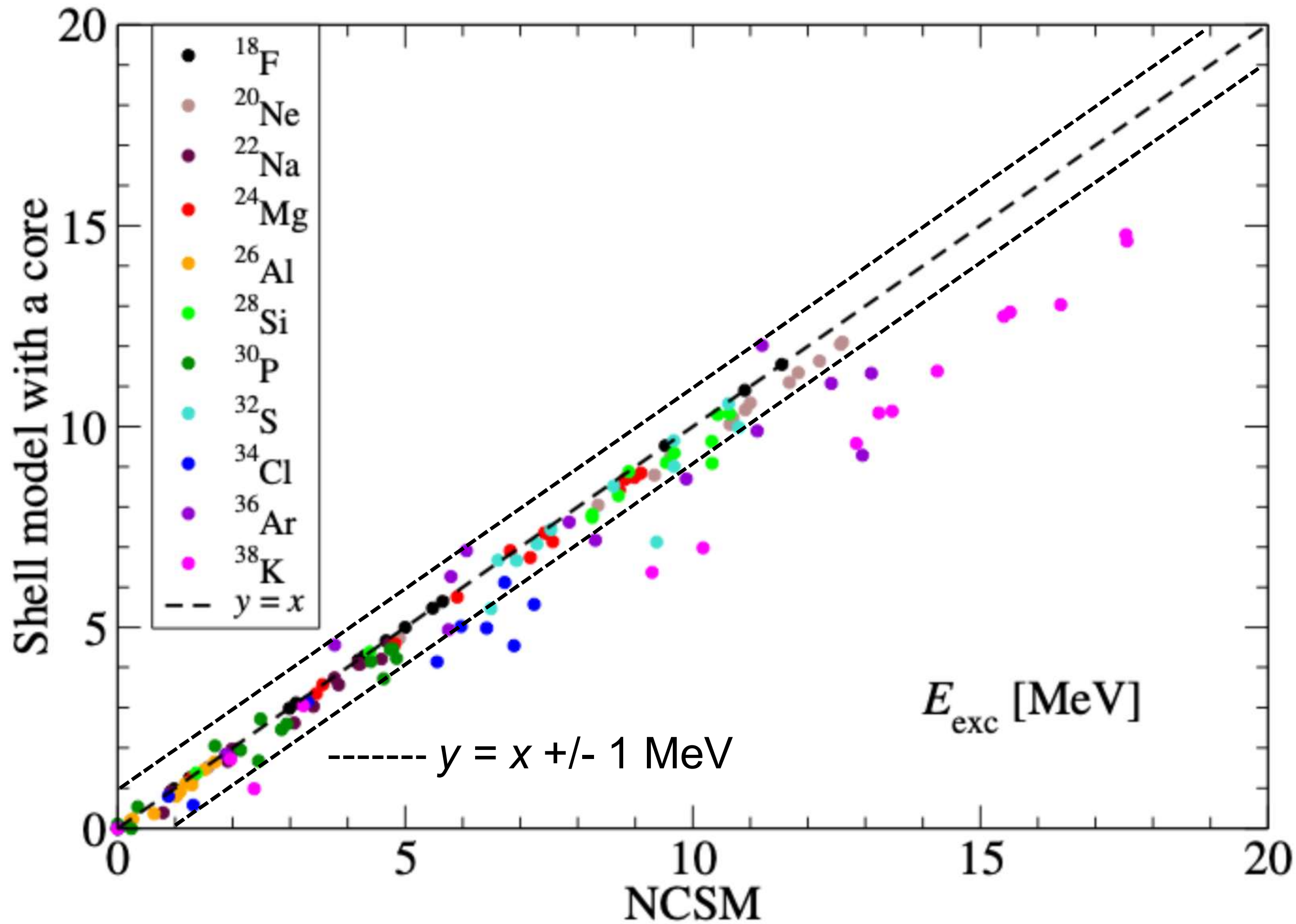
I.J. Shin, I Mazur, N.A. Smirnova, A.M. Shirokov and Y. Kim et al, in preparation

E_{exc} correlation between Full NCSM and derived charge-dependent Valence Effective SM Deajeon16 NN interaction and NCSM at N_{max} = 4



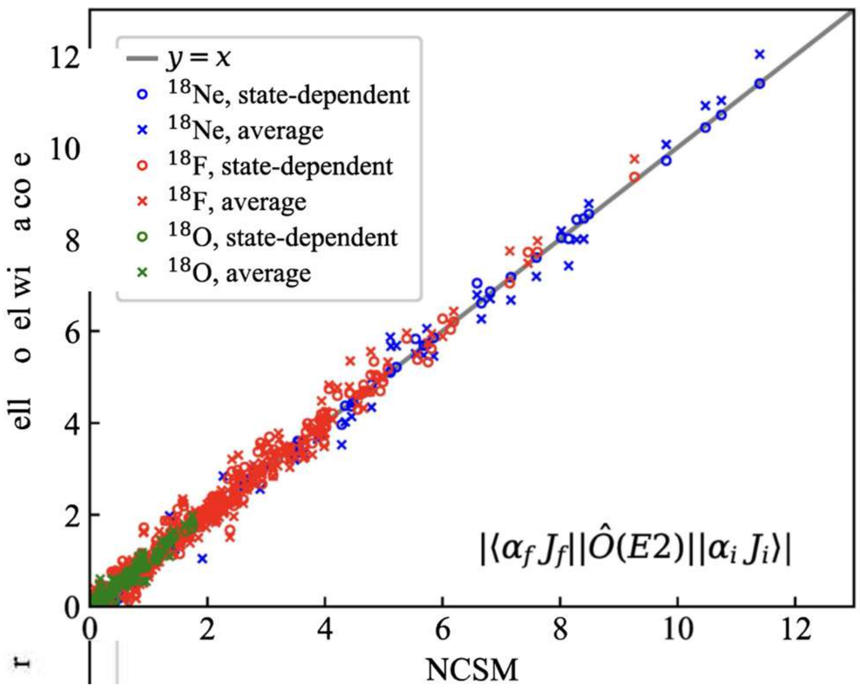
I.J. Shin, I Mazur, N.A. Smirnova, A.M. Shirokov and Y. Kim, in preparation

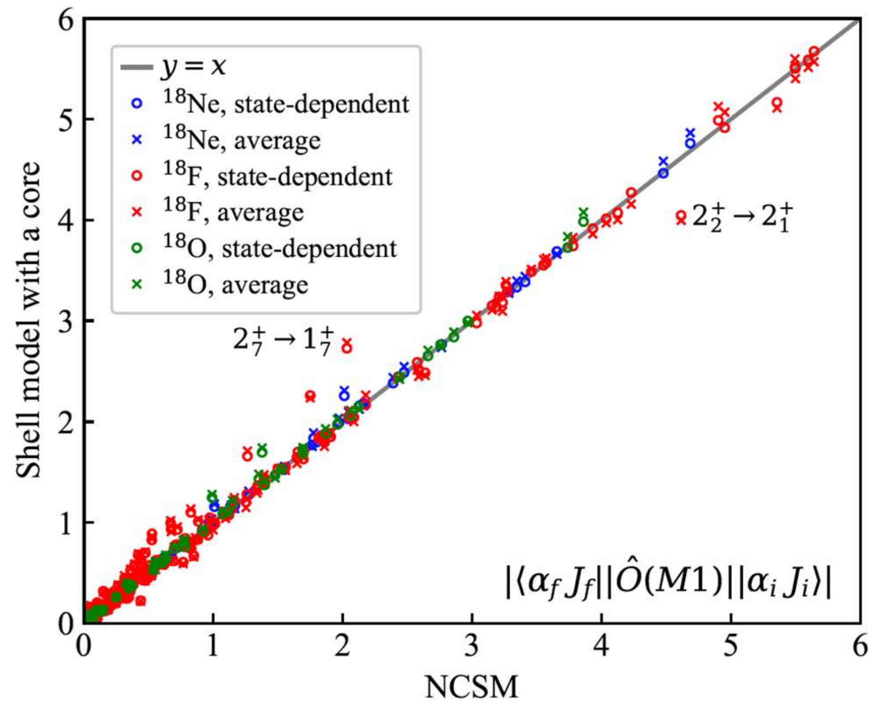
E_{exc} correlation between Full NCSM and derived charge-dependent Valence Effective SM Deajeon16 NN interaction and NCSM at N_{max} = 4



I.J. Shin, I Mazur, N.A. Smirnova, A.M. Shirokov and Y. Kim, in preparation

Full NCSM E2 and M1 reduced transition matrix elements vs charge-dependent effective valence space interaction results with effective charges derived from A=17 NCSM calculations. Employs Daejeon 16 in N_{max} = 4 space with $\hbar\omega$ = 14 MeV





Zhen Li, N.A. Smirnova, A.M. Shirokov, I.J. Shin, B.R. Barrett, P. Maris and J.P. Vary, in Akito Arima Memorial Volume, T.T.S. Kuo, T. Otsuka, K.K. Phua and J. Vary, eds., World Scientific (Singapore), 135 (2025); arXiv: 2205.15939

Future plans

- Adopt LENPIC Chiral EFT interaction complete through N3LO (arriving 2026)
- Employ SRG and solve NCSM at N_{max} = 4 followed by N_{max} = 6 for A=16 – 20
- Develop charge-dependent valence NN Veff at N_{max} = 4 followed by N_{max} = 6
- Solve for sd-shell nuclei (see example cases presented above)
- Compare with expt & previous thy results to assess role/need for valence NNN Veff
- Develop charge-dependent NN Veff for the fp-shell with NCSM solutions at N_{max} = 4 for A = 40, 41, 42

Many outstanding nuclear physics
discoveries & opportunities remain

Origin of the successful nuclear shell model
Collective motion & Clustering phenomena
Nuclear reactions and breakup
Astrophysical processes & drip lines
New opportunities: AI and Quantum Computing
Predictive Nuclear Theory will provide
a window on Physics beyond the Standard Model

A comprehensive program of theory & experiment
is needed to maximize discovery potential

Thank you for your attention

I welcome your questions

Backup Slides and References

Extrapolating to the infinite matrix limit i.e. to the "continuum limit"

Results with both IR and UV extrapolations

Early References:
*S.A. Coon, M.I. Avetian, M.K.G. Kruse, U. van Kolck, P. Maris, and J.P. Vary, Phys. Rev. C 86, 054002 (2012); arXiv: 1205.3230
R.J. Furnstahl, G. Hagen, T. Papenbrock, Phys. Rev. C 86 (2012) 031301
E.D. Jurgenson, P. Maris, R.J. Furnstahl, P. Navratil, W.E. Ormand, J.P. Vary, Phys. Rev. C 87, 054312(2013); arXiv 1302.5473
S.N. More, A. Ekstroem, R.J. Furnstahl, G. Hagen and T. Papenbrock, Phys. Rev. C87, 044326 (2013); arXiv 1302.3815

* **ACKNOWLEDGMENTS**

This study was conceived and initiated at the National Institute for Nuclear Theory's program "Effective Field Theories and the Many-Body Problem" in the spring of 2009.

Convergence and Uncertainty Assessments

Convergence properties of *ab initio* calculations of light nuclei in a harmonic oscillator basis

Phys. Rev. C 86, 054002 (2012); arXiv:1205.3230

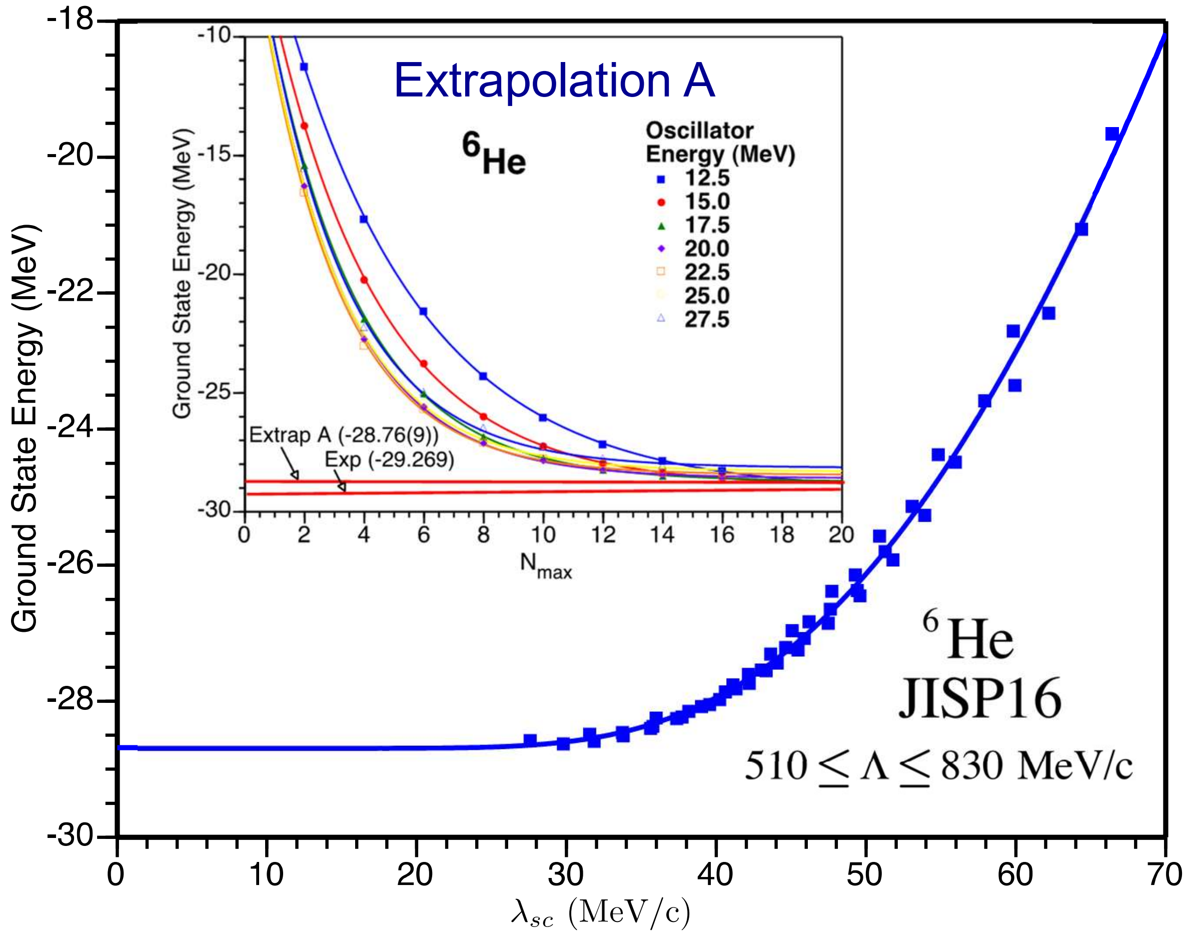
S. A. Coon[a], M. I. Avetian[a], M. K. G. Kruse[a], U. van Kolck[a,b], P. Maris[c], J. P. Vary[c]

UV regulator:

$$\Lambda = \sqrt{(N + 3/2) m\hbar\Omega}$$

IR regulator:

$$\lambda_{sc} = \sqrt{\frac{m\hbar\Omega}{(N + 3/2)}}$$



Coordinating χEFT and NCSM UV regulators makes sense and would reduce demands on computational resources (Discussion topic at INT-26-1)

Some relevant results to consider:
S.A. Coon, M.I. Avetian, M.K.G. Kruse, U. van Kolck, P. Maris and J.P. Vary, Phys. Rev. C 86, 054002 (2012)*

Fit function:

$$E_{gs}(\lambda_{sc}) = a\exp(-b/\lambda_{sc}) + E_{gs}$$

"By taking [the change from reduced mass to nucleon mass] into account, the successful emulation of the Idaho N3LO interaction In a HO basis suggests that Λ^{NN} ~ 780 MeV/c."

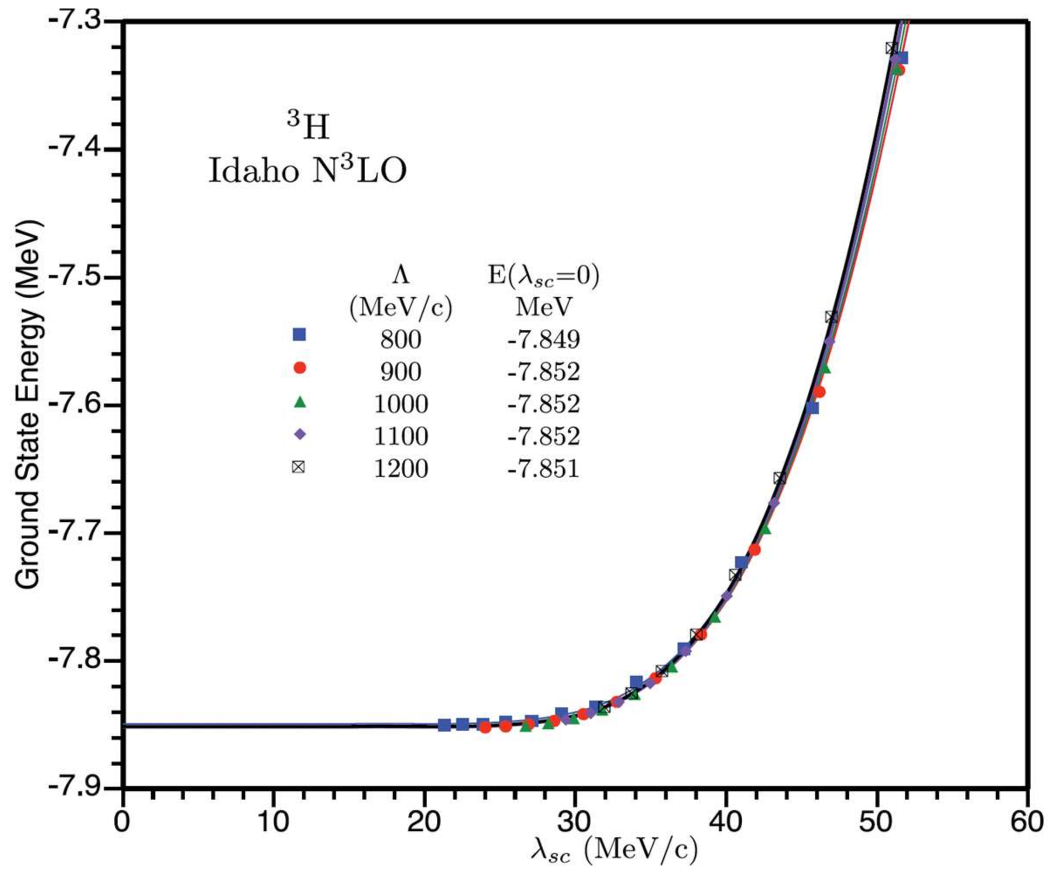


FIG. 7. (Color online) The ground-state energy of ^{3}H calculated at five fixed values of $\Lambda = \sqrt{m_N(N+3/2)\hbar\omega}$ and variable $\lambda_{sc} = \sqrt{(m_N\hbar\omega)/(N+3/2)}$. The curves are fits to the points and the functions fitted are used to extrapolate to the ir limit $\lambda_{sc} = 0$.

* **ACKNOWLEDGMENTS**

This study was conceived and initiated at the National Institute for Nuclear Theory's program "Effective Field Theories and the Many-Body Problem" in the spring of 2009.

Combined IR and UV extrapolation: HO-basis regulator definitions

	Ref. 1	Ref. 2	Ref. 3
UV: Λ	$\sqrt{(N+3/2)m\hbar\Omega}$	$\sqrt{2(N+3/2)m\hbar\Omega}$	$\sqrt{2(N+3/2)m\hbar\Omega}$
IR: λ	$\sqrt{\frac{m\hbar\Omega}{(N+3/2)}}$	$\sqrt{\frac{m\hbar\Omega}{2(N+3/2)}}$	$\sqrt{\frac{m\hbar\Omega}{2(N+3/2)}}$
N (p-shell)	N_{max} + 1	N_{max} + 2	N_{max} + 3

$$E(\Lambda,\lambda) \approx E_\infty + B_0 e^{-2\Lambda^2/B_1^2} + B_2 e^{-2k_\infty/\lambda}$$

[1]S.A. Coon, M.I. Avetian, M.K.G. Kruse, U. van Kolck, P. Maris, and J.P. Vary, Phys. Rev. C 86, 054002 (2012); arXiv: 1205.3230

[2]R.J. Furnstahl, G. Hagen, T. Papenbrock, Phys. Rev. C 86 (2012) 031301

[3]E.D. Jurgenson, P. Maris, R.J. Furnstahl, P. Navratil, W.E. Ormand, J.P. Vary, Phys. Rev. C 87, 054312(2013); arXiv 1302.5473

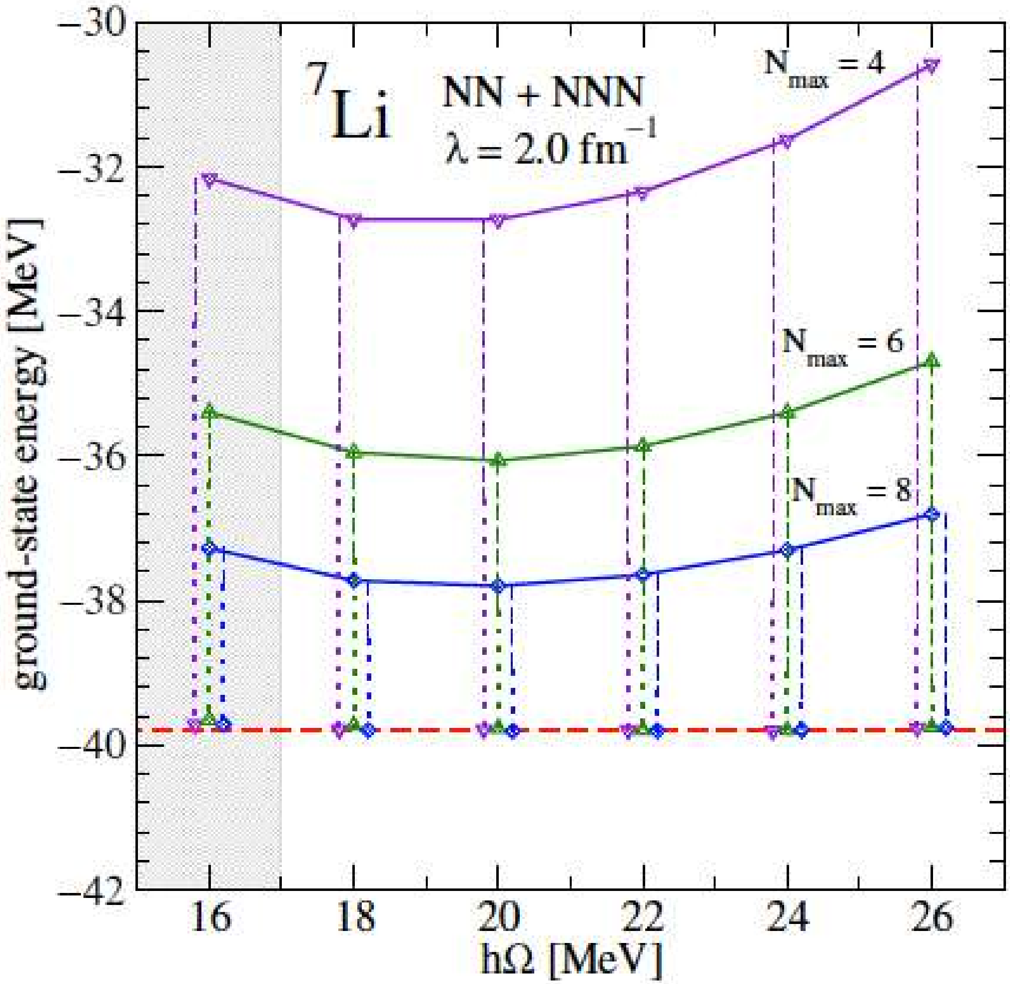


FIG. 17. (color online) Ground-state energy of ^{7}Li for the NN+NNN evolved Hamiltonians at $\lambda = 2.0\,\mathrm{fm}^{-1}$, with IR (vertical dashed) and UV (vertical dotted) corrections from Eq. (5) that add to predicted E_∞ values (points near the horizontal dashed line, which is the global E_∞).

E.D. Jurgenson, P. Maris, R.J. Furnstahl, P. Navratil, W.E. Ormand, J.P. Vary, Phys. Rev. C. 87, 054312 (2013); arXiv: 1302:5473