ASTROMETRY WITH INTENSITY INTERFEROMETRY

↳ measurement of positions, motions of "stars" ↳ Hanbury Brown, Twiss 1954, 1956

Hipparchus 190 BC 850 Ptolemy Almagest 1,022 M. Baryakhtar, M. Galanis, K. Van Tilburg, N. Weiner

Ibn Yunus, Tycho Brahe, Friedrich Bessel → Hipparcos (10^6, 20 mas), Gaia (10^9, 20 μas)
10^{-7} rad 10^{-9} rad

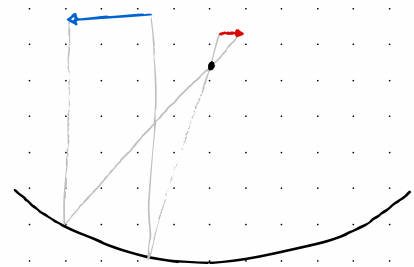
Outline

- Astrometric observatories: imager, amplitude interferometer, intensity interferometer
- Intensity interferometry 102: atmosphere | spectrum | source spread | aperture spread | timing | SNR
- Time-delay II: $\sigma_{\Delta\theta}^{rel} \sim 10^{-12}$ rad $\sim 0.2\,\mu$as for $\Delta\theta \lesssim 10^{-6}$ rad and $m < 15$
  $\sigma_\theta^{abs} \sim 10^{-8}$ rad ~ 2 mas
- Applications: apparent size | Galactic lensing | exoplanets | binaries | quasar microlensing
  distance ladder DM substructure

Astrometric Observatories

<u>Imager</u> (HST, JWST, Gaia, Roman, Keck, ELT, ...)

- $\sigma_\theta \sim \dfrac{\lambda}{\min\{D, r_0\}} \max\left\{\dfrac{1}{\sqrt{N_\gamma}}, \delta(\mathrm{PSF})\right\}$ ($r_0 \to$ 20 cm)
  $\sim 10^{-6}$ rad $\times$ $10^{-4} \sim 10^{-10}$ rad $\sim 20\,\mu$as



- pro: high info, high SNR, large FOV
- con: atmosphere, tight tolerances, cost

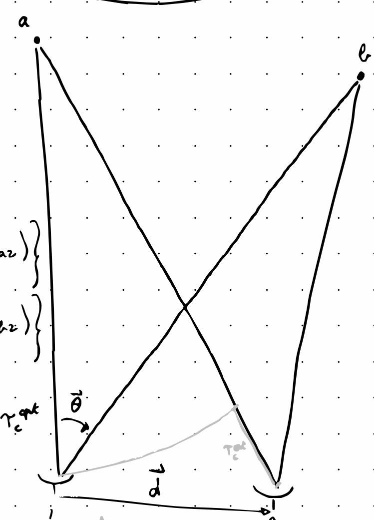
<u>Amplitude Interferometer</u> (VLBI, CHARA, SIM)

$$E_1 = \sqrt{F_a}\, e^{i\phi_{a1}} + \sqrt{F_b}\, e^{i\phi_{b1}} \qquad E_2 = \sqrt{F_a}\, e^{i\phi_{a2}} + \sqrt{F_b}\, e^{i\phi_{b2}}$$

$$\phi_{cj} = \omega t + k r_{cj} + \tilde{\phi}_c^{em}(t - r_{cj}) + \tilde{\phi}_{cj}^{atm} \qquad c = a, b; \; j = 1, 2$$

$$\begin{aligned}\langle E_1(t) E_2(t+\tau)\rangle = F_a \exp\Big\{ & ik(r_{a1} - r_{a2}) + \tilde{\phi}_a^{em}(t - r_{a1}) - \tilde{\phi}_a^{em}(t + \tau - r_{a2}) \\ & + \tilde{\phi}_{a1}^{atm} - \tilde{\phi}_{a2}^{atm}\Big\} \\ + F_b \exp\Big\{ & ik(r_{b1} - r_{b2}) + \tilde{\phi}_b^{em}(t - r_{b1}) - \tilde{\phi}_b^{em}(t + \tau - r_{b2}) \\ & + \tilde{\phi}_{b1}^{atm} - \tilde{\phi}_{b2}^{atm}\Big\}\end{aligned}$$

$$r_{c1} - r_{c2} = |\vec{r}_{c1}| - |\vec{r}_{c1} + \vec{d}| = r_{c1} - \sqrt{r_{c1}^2 + 2\vec{r}_{c1}\cdot\vec{d}} = -\hat{\theta}_c \cdot \vec{d} = \tau_c^{opt}$$



- $\sigma_\theta \sim \dfrac{\lambda}{d} \max\left\{\dfrac{1}{\sqrt{N_\gamma}}, \sigma_{\tilde{\phi}}\right\}$
  pro: high SNR, medium FOV
  con: low info, ultratight tolerance, atmosphere

VLBI: 10^{-9} rad and ↳ 10^{-8} rad CHARA, Keck

Intensity Interferometer

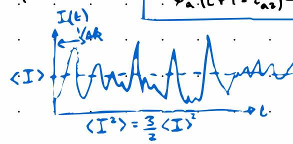
$$\frac{\langle I_1(t)\, I_2(t+\tau)\rangle}{\langle I_1\rangle\langle I_2\rangle} = \frac{\langle E_1(t)E_1^*(t)E_2(t+\tau)E_2^*(t+\tau)\rangle}{\langle E_1E_1^*\rangle\langle E_2E_2^*\rangle}$$

$$= 1 + \frac{2F_a^2F_b^2}{(F_a+F_b)^2}\cos\left\{k\,\vec{d}\cdot(\hat{\theta}_b-\hat{\theta}_a) \boxed{\begin{matrix}+\tilde{\phi}_a^{em}(t-\tau_{a1}) + \tilde{\phi}_b^{em}(t+\tau-\tau_{b2}) \\ -\tilde{\phi}_a^{em}(t+\tau-\tau_{a2}) - \tilde{\phi}_b^{em}(t-\tau_{a2})\end{matrix}} \underbrace{\boxed{\begin{matrix}+\tilde{\phi}_{a1}^{atm} + \tilde{\phi}_{a2}^{atm} \\ -\tilde{\phi}_{b1}^{atm} - \tilde{\phi}_{b2}^{atm}\end{matrix}}}_{\equiv\,\tilde{\Delta\phi}^{atm}}\right\}$$

$$\equiv 1 + C$$

$$\sigma_\theta \sim \frac{\lambda}{d}\,\frac{\sigma_C}{|C|}$$

$\hookrightarrow 10^{-12}\text{rad} = 0.2\,\mu\text{as}$



- pro: ultimate resolution | offline correlation | loose tolerances | atmosphere aberration
- con: bright sources, low SNR | ~~ul~~tra-narrow FOV: $\Delta\theta_{FOV} \gtrsim \sigma_\theta$

  new technology! time-delay II: $\Delta\theta_{FOV} \lesssim 10^{-6}$

Intensity Interferometry 102

$$\langle[n(\vec{r}\,'+\vec{r}) - n(\vec{r}\,')]^2\rangle = C_n^2(h)\, r^{2/3}$$

$C_n^2 = 10^{-16}\,m^{-2/3}$

1 atmospheric aberrations: $\langle(\tilde{\Delta\phi}^{atm})^2\rangle = \Delta\theta^{5/3}\,\underbrace{2.9\,k^2\int ds\, C_n^2(s\cos\gamma)\,s^{5/3}}_{\theta_{\text{isoplanatic patch}}^{-5/3}} \approx \left(\frac{\Delta\theta}{10^{-5}}\right)^{5/3}$

2 narrow spectrum: $\Delta k \ll k$ w/ grisms $\Rightarrow t_{coh} = \frac{2\pi}{\Delta k}$

3 finite source size: $\theta_s \equiv \frac{R_s}{D_s} = \begin{cases} 3\times10^{-11} & \text{blue giant in LMC} \quad R_s \approx 70\,R_\odot \quad D_s \approx 50\,\text{kpc} \\ 3\times10^{-13} & 10^{15}\text{cm} = 3\times10^{-4}\text{pc} \quad \text{quasar @ } D_s = \text{Gpc}\end{cases}$

4 aperture spread: light collection areas A_1, A_2

5 timing resolution: $t_{res} \lesssim \begin{cases} 3\text{ ps} & \text{SNSPDs} \\ 50\text{ ps} & \text{SPADs}\end{cases}$

$$C = \frac{\langle I_1 I_2\rangle}{\langle I_1\rangle\langle I_2\rangle} - 1 = \underbrace{\frac{2\pi/\Delta k}{t_{res}}}_{5}\underbrace{\int\frac{d^2b_1}{A_1}\int\frac{d^2b_2}{A_2}}_{4}\left|\underbrace{\int d^2\theta}_{3}\underbrace{\int dk}_{2}\, \overbrace{f(k,\vec{\theta})}^{\text{flux distribution}}\exp\left\{i\,k(\vec{d}+\vec{\Delta b})\cdot\vec{\theta} + \underbrace{\tilde{\Delta\phi}^{atm}}_{1}\right\}\right|^2$$

C suppressed by product of:

5 $\frac{2\pi/\Delta k}{t_{res}} \sim 1$ @ $\frac{k}{\Delta k} = 5000$; $t_{res} = 10\text{ps}$

4 $\frac{\lambda^2}{A(\Delta\theta)^2} \sim 1$ @ $A = 1\,m^2$; $\Delta\theta = 10^{-6}$rad

3 $\left(\frac{\lambda}{d\,\theta_s}\right)^{\#}$ once source is resolved: # = 3 for star

fix $\longrightarrow$ 2 $\exp\left\{-(\Delta k)^2(\vec{d}\cdot\vec{\Delta\theta})^2\right\}$ ultranarrow FOV: $\Delta\theta_{FOV} \sim \frac{\lambda}{d}\,\frac{k}{\Delta k}$

1 $\exp\left\{-\left(\frac{\Delta\theta}{10^{-5}}\right)^{5/3}\right\}$

SNR

$$\sigma_c = \frac{1}{\sqrt{N_{\gamma\gamma}}}\frac{1}{\sqrt{N_{channels}}}\frac{1}{\sqrt{N_1 N_2}}$$

$$N_{\gamma\gamma} = \left(\frac{F_a + F_b}{k^2}\right)^2 \left(\frac{\Delta k}{k}\right)^2 A_1 A_2\, t_{res}\, t_{int}$$ number of coincident photons

$$N_{channels} \simeq \frac{k}{\Delta k}$$ number of Δk spectral bins

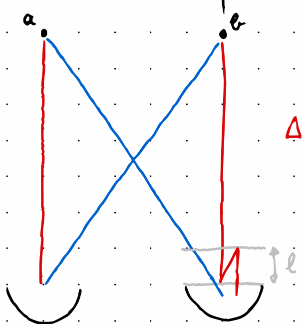
N_1, N_2 number of apertures at sites 1 and 2

$$\sigma_c \sim 0.05 \underbrace{\left(\frac{3\times10^{-14}\,W\,m^{-2}}{F}\right)}_{m=15}\left(\frac{5000}{k/\Delta k}\right)^{1/2}\left(\frac{10\,ps}{t_{res}}\right)^{1/2}\left(\frac{10^4\,s}{t_{int}}\right)^{1/2}\left(\frac{1\,m^2}{A}\right)\left(\frac{10}{\sqrt{N_1 N_2}}\right)$$

Time Delay Intensity Interferometry

increase $\Delta\theta_{FOV}$

$$C \equiv \frac{\langle I_1 I_2\rangle}{\langle I_1\rangle\langle I_2\rangle} - 1 = \frac{2\pi/\Delta k}{t_{res}}\left|\int d^2\theta \int dk\, f(k,\vec{\theta})\exp\left\{ik\left(\vec{d}\cdot\vec{\theta} - 2n\ell\right)\right\}\right|^2$$



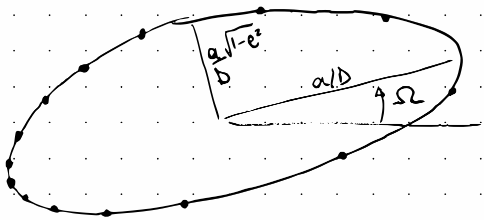
$$\Delta\theta_{ref} \simeq \frac{2\ell}{d} \approx 10^{-6}\,rad\left(\frac{\ell}{50\,um}\right)\left(\frac{10^3\,km}{d}\right)$$

Applications

- Stellar size + "imaging"
- Exoplanet detection
- Galactic microlensing

Binaries

blue giants in LMC



Suppose inclination = 0

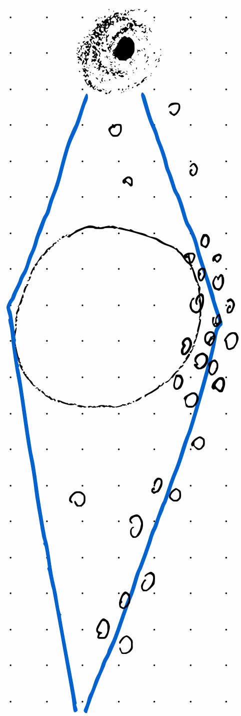
→ 10 AU/50 kpc ~ 10^{-9} rad to 10^{-3} precision

$\frac{a}{D}, e, \Omega, i, \omega, \nu$

period: $T = 2\pi\sqrt{\frac{a^3}{G(M_a+M_b)}}$

radial velocity: $v_r = \sqrt{\frac{GM}{a}} \approx 60\,\mathrm{km/s}$

Astrometric Microlensing of Strongly Lensed Quasars



$$\delta\theta_{rms} \sim \underbrace{\frac{GM_s}{r_s}}_{2\times10^{-15}} \underbrace{\mu}_{10} \underbrace{\sqrt{\frac{S_G}{\rho_s}\frac{R_G}{r_s}}}_{30} \sim \mathcal{O}(10^{-12}\,\mathrm{rad})$$

$$t_L \sim \frac{r_s}{v_L \mu} \sim 1\,\mathrm{yr}$$