Tensor networks for asymptotic free QFTs

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with:
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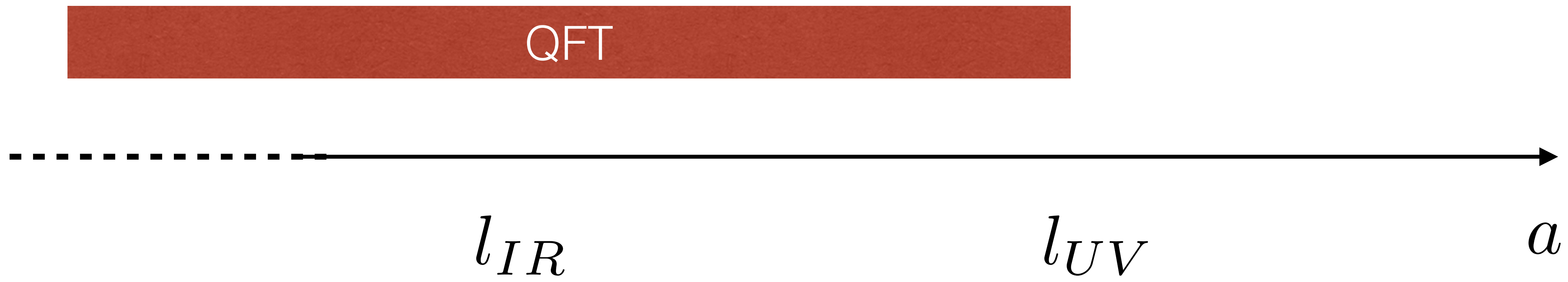
INT Workshop
tensor networks in many-body and quantum field theory
Seattle, 06/04/2023



(*talk Shialesh)

We want to recover the QFT from the IR all the way to the UV

For tensor networks this is achieved via entanglement scaling

Entanglement (full entanglement structure) can also be used as a probe for both the IR and UV QFT physics we will consider the following **d=1+1** systems:

1. The Gross-Neveu model

JHEP 2021, 207 (2021)
G. Roose, N. Bultinck, L. Vanderstraeten, F. Verstraete, KVA, J. Haegeman

2. $\lambda\phi^4$ - theory

Phys. Rev. D. 106, L071501 (2022)
B. Vanhecke, F. Verstraete, KVA

Phys. Rev. Lett. 123(25), 250604 (2019)
B. Vanhecke, J. Haegeman, KVA, L.Vanderstaeten, F.Verstraete

2. Gross-Neveu model

PHYSICAL REVIEW D VOLUME 10, NUMBER 10 15 NOVEMBER 1974

Dynamical symmetry breaking in asymptotically free field theories*

David J. Gross† and André Neveu
Institute for Advanced Study, Princeton, New Jersey 08540

(Received 21 March 1974)

Two-dimensional massless fermion field theories with quartic interactions are analyzed. These models are asymptotically free. The models are expanded in powers of $1/N$, where N is the number of components of the fermion field. In such an expansion one can explicitly sum to all orders in the coupling constants. It is found that dynamical symmetry breaking occurs in these models for any value of the coupling constant. The resulting theories produce a fermion mass dynamically, in addition to a scalar bound state and, if the broken symmetry is continuous, a Goldstone boson. The resulting theories contain no adjustable parameters. The search for symmetry breaking is performed using functional techniques, the new feature here being that a composite field, say, $\bar{\psi}\psi$, develops a nonvanishing vacuum expectation value. The "potential" of composite fields is discussed and constructed. General results are derived for arbitrary theories in which all masses are generated dynamically. It is proved that in asymptotically free theories the dynamical masses must depend on the coupling constants in a nonanalytic fashion, vanishing exponentially when these vanish. It is argued that

$$\frac{dg(\mu)}{d\log\mu} = -(\beta_0 g^2 + \beta_1 g^3 + \ldots) \qquad M = \mu g^{\beta_1/\beta_0} \exp^{-\frac{1}{2\beta_0 g^2}}(1 + \mathcal{O}(g^2))$$

$$\psi \to \gamma^5 \psi \qquad <\bar{\psi}\psi> \sim M$$

The Gross-Neveu Lagrangian and symmetries:

$$\mathcal{L} = \sum_{c \in N} \bar{\psi}_c i \partial\!\!\!/ \psi_c + \frac{g^2}{2} \left(\sum_{c \in N} \bar{\psi}_c \psi_c \right)^2$$

Manifest symmetries:

$$\psi \to \gamma_5 \psi \ (\text{with } \gamma^5 = \gamma^0 \gamma^1 = \alpha)$$

$$U(N): \ \psi_c \to U_{cc'} \psi_{c'}$$

$$\gamma^0 = \sigma_y \text{ and } \gamma^x = i\sigma_z$$

$$\mathcal{L} = \sum_{m \in 2N} \bar{\lambda}_m \mathrm{i} \partial\!\!\!/ \lambda_m + \frac{g^2}{2} \left(\sum_{m \in 2N} \bar{\lambda}_m \lambda_m \right)^2$$

Extra symmetry:

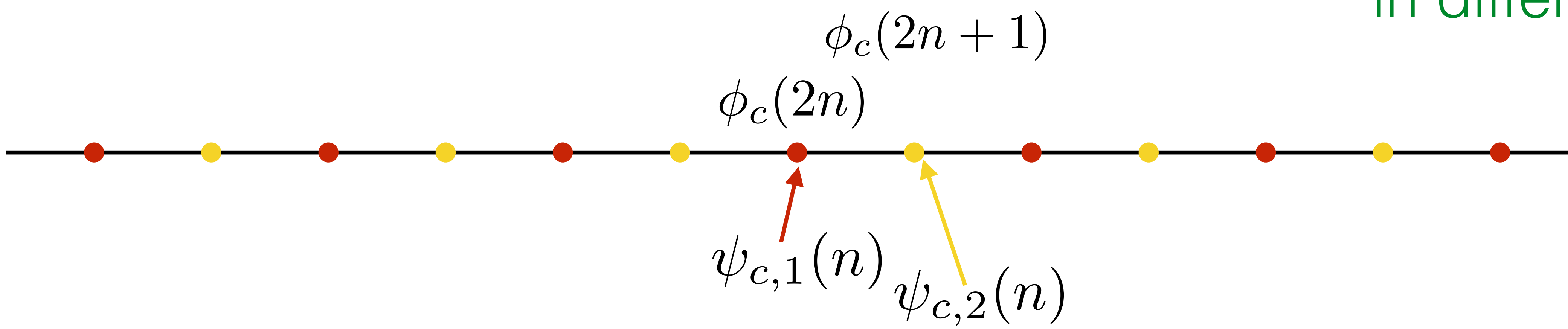
$$\psi_c = (\lambda_{2c-1} + \mathrm{i}\lambda_{2c})/\sqrt{2}$$

$$O(2N): \ \lambda_m \to O_{mn} \lambda_n$$

Prohibits other marginal operators like: $(\bar{\psi}\gamma^5\psi)^2$ $\quad (\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)$

Hamiltonian formulation:

kogut-susskind but in different basis



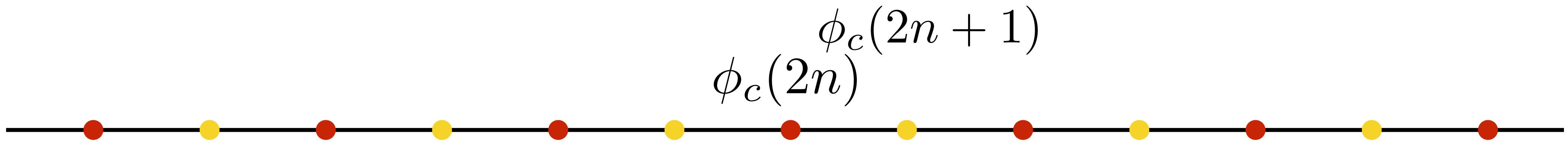
$$H = \sum_n \left(K_{n,n+1} + \frac{g^2}{2} \left(\frac{K_{n,n+1} - K_{n+1,n+2}}{2} \right)^2 \right)$$

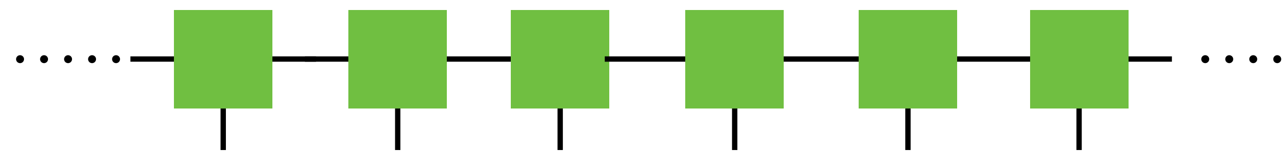
$$K_{n,n+1} = \sum_{c \in N} \mathrm{i}(\phi^\dagger_{c,n}\phi_{c,n+1} - \phi^\dagger_{c,n+1}\phi_{c,n})$$

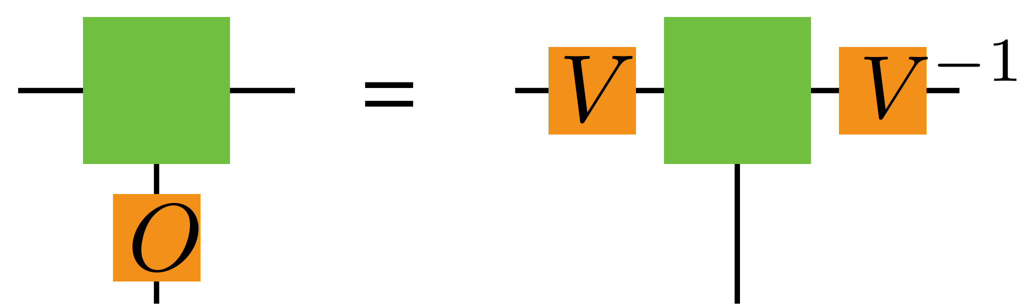
chiral symmetry: $\gamma^5 \to T$

mass term: $\frac{m}{2} \sum_n (-1)^n K_{n,n+1}$

MPS, O(2N), SPT and chiral symmetry breaking:

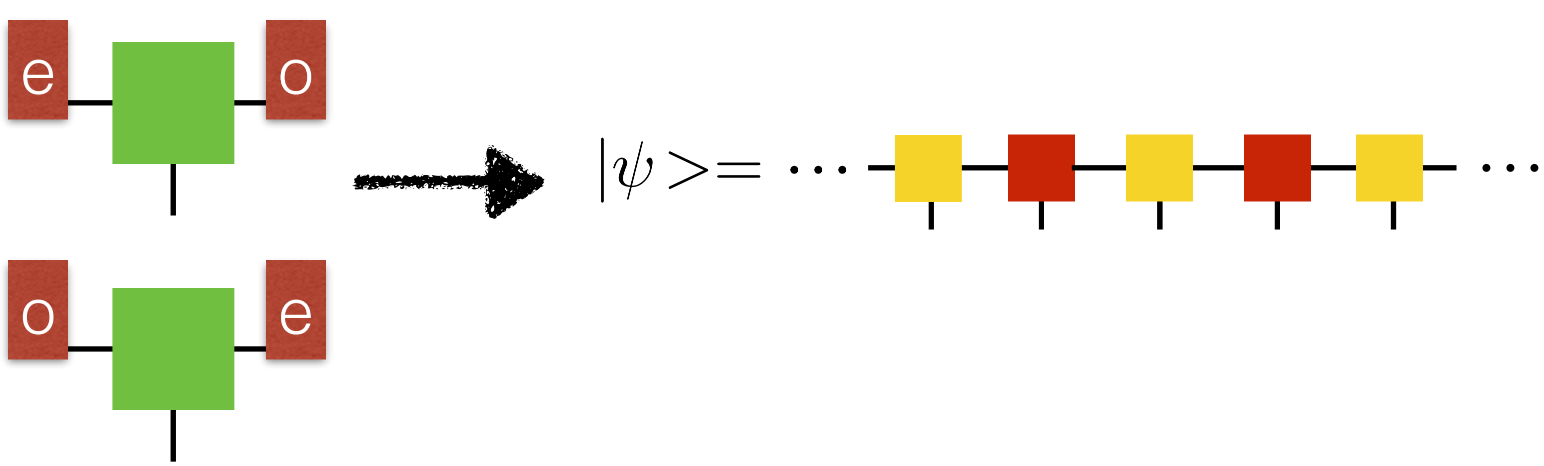


$|\psi>=$ 

$|\psi>= O|\psi>\rightarrow$ 

$SO(4) = SU(2) \otimes SU(2)$ $\quad (\frac{1}{2},0) \oplus (0,\frac{1}{2})$

two options: $\big((j_1, j_2) \quad 2(j_1+j_2) = even/odd\ \big)$



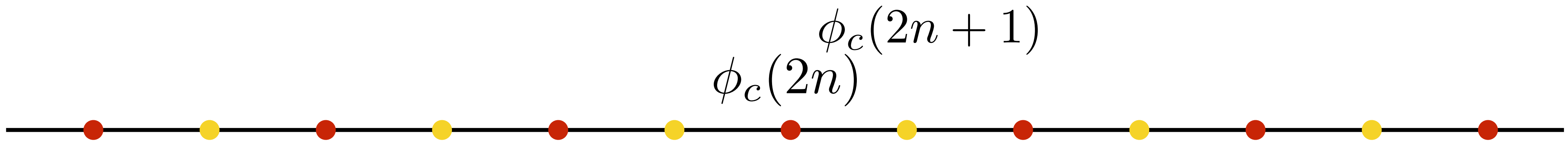
(Explicitly imposing the symmetry structure)

tensor product structure of the indices:

$$
\begin{aligned}
i &= [(m_1, j_1 = \frac{1}{2}), (0,0)] \oplus [(0,0), (m_2, j_2 = \frac{1}{2})] \\
\alpha &= [(m'_1, j'_1), (m'_2, j'_2), \alpha_{(j'_1, j'_2)}] \\
\beta &= [(m''_1, j''_1), (m''_2, j''_2), \beta_{(j''_1, j''_2)}]
\end{aligned}
$$

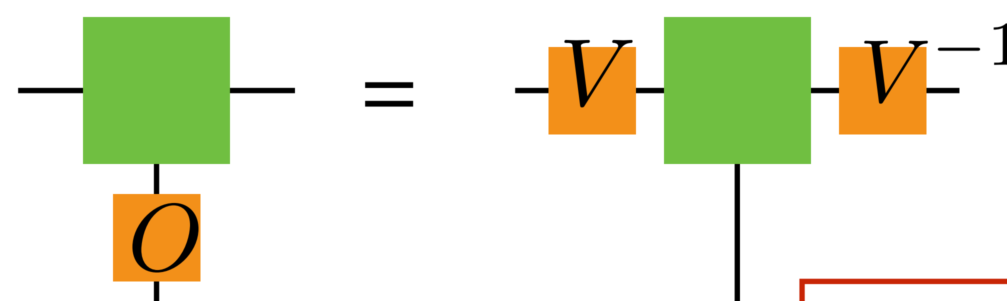
$$
\begin{aligned}
A^i_{\alpha,\beta} &= \begin{pmatrix} \frac{1}{2} & j'_1 & j''_1 \\ m_1 & -m'_1 & m''_1 \end{pmatrix} \times \begin{pmatrix} 0 & j'_2 & j''_2 \\ 0 & -m'_2 & m''_2 \end{pmatrix} [a]_{\alpha_{(j'_1,j'_2)}, \beta_{(j''_1,j''_2)}} \\
&\oplus \begin{pmatrix} 0 & j'_1 & j''_1 \\ 0 & -m'_1 & m''_1 \end{pmatrix} \times \begin{pmatrix} \frac{1}{2} & j'_2 & j''_2 \\ m_2 & -m'_2 & m''_2 \end{pmatrix} [b]_{\alpha_{(j'_1,j'_2)}, \beta_{(j''_1,j''_2)}}
\end{aligned}
$$

MPS, O(2N), SPT and chiral symmetry breaking:



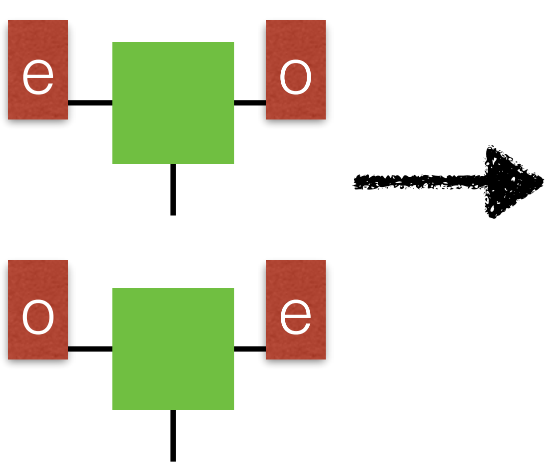
$|\psi>= \cdots\cdots$

$|\psi>= O|\psi> \rightarrow$

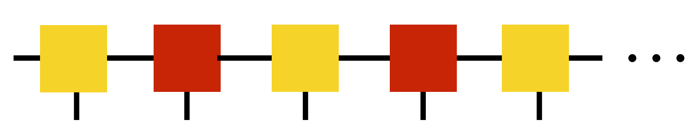


$$SO(4) = SU(2) \otimes SU(2) \qquad (\frac{1}{2},0) \oplus (0,\frac{1}{2})$$

two options: $\left((j_1, j_2) \quad 2(j_1 + j_2) = even/odd \right)$



$|\psi>= \cdots$



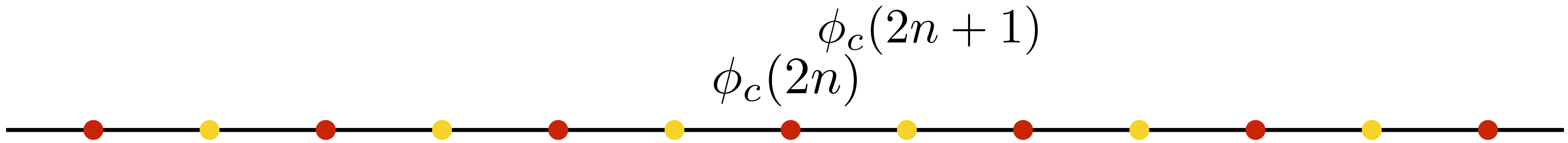
Coleman,Mermin-Wagner

\+

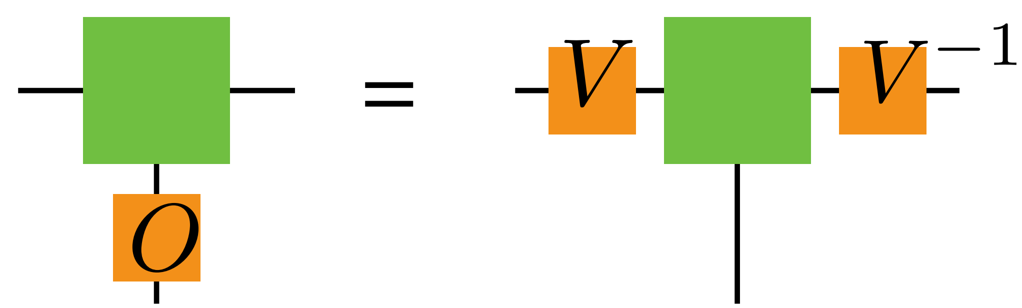
Lieb-Schultz-Mattis

chiral symmetry (T) has to be broken if the groundstate is gapped

MPS, O(2N), SPT and chiral symmetry breaking:



$|\psi>= O|\psi> \rightarrow$



$SO(4) = SU(2) \otimes SU(2)$ $\quad (\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$

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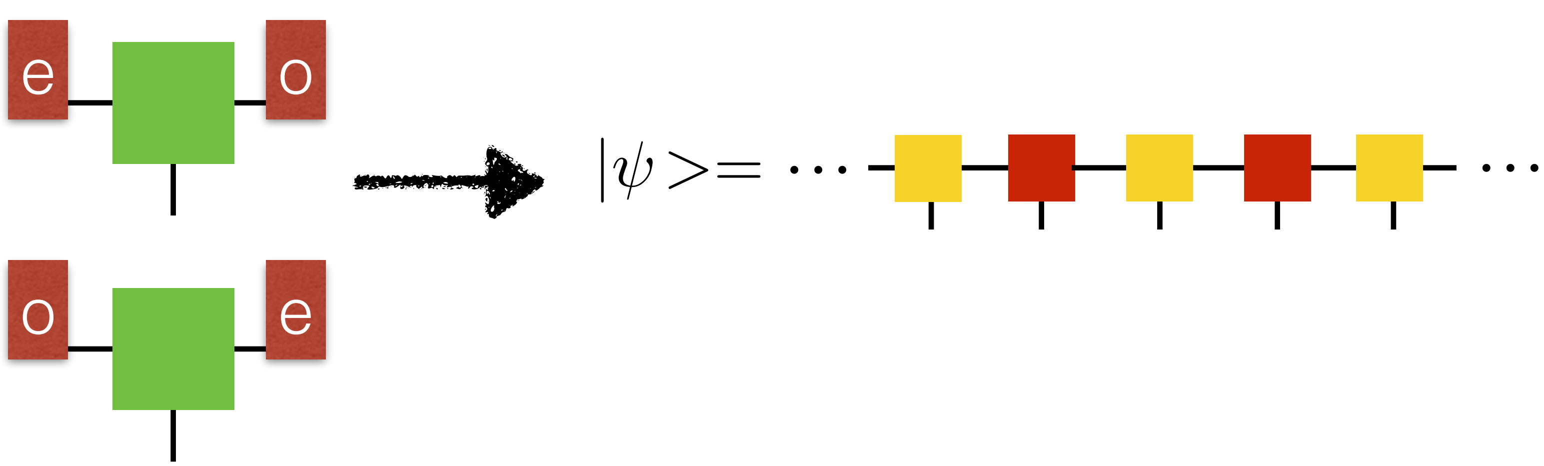
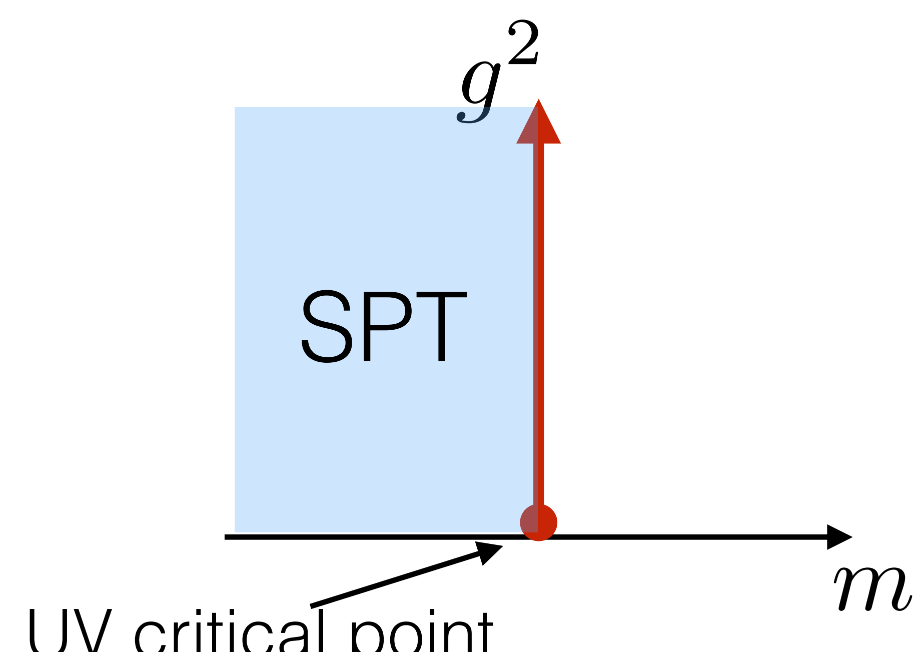
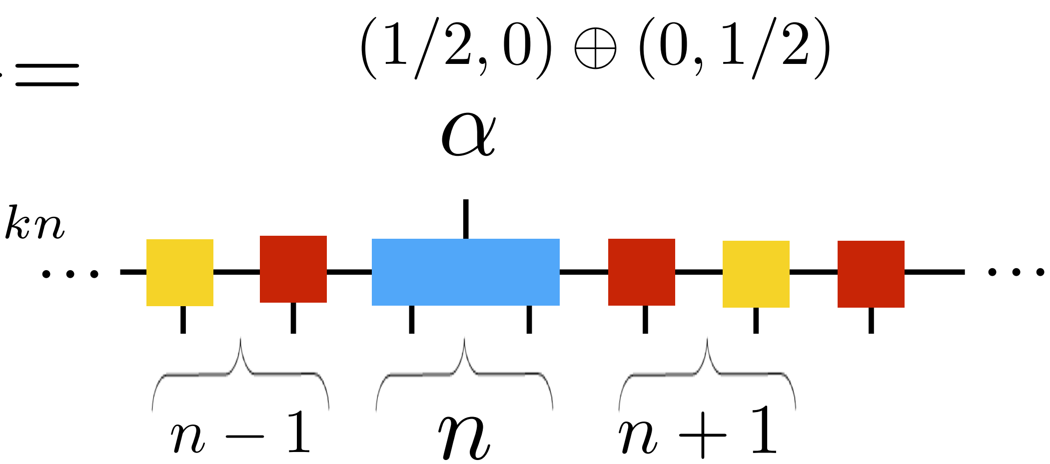




Recovering the QFT in the IR

kink-excitations:

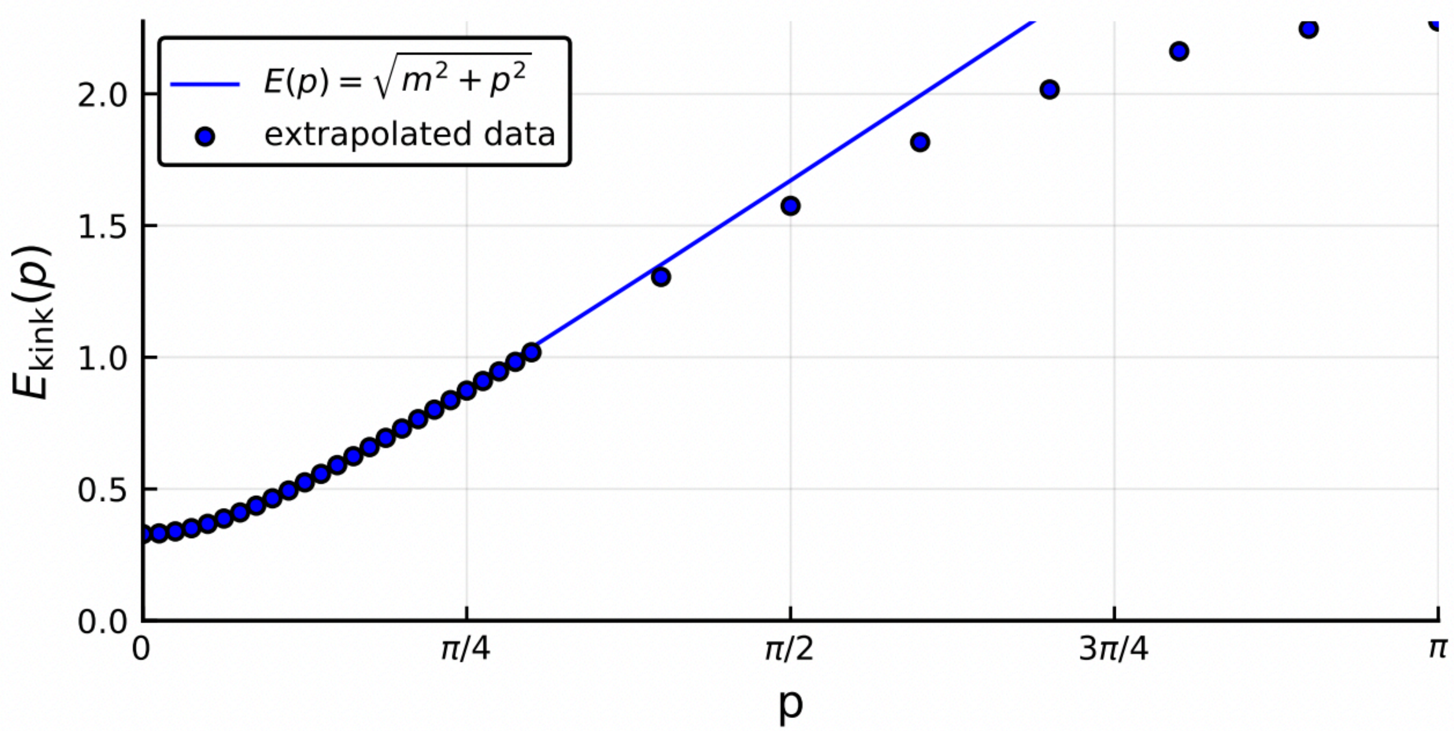
$$|k,\alpha> = \sum_n exp^{ikn} \cdots$$



allows us to extract two mass-scales:

$$E_K(p) = m_{K,1}\left(1+\frac{p^2}{2m_{K,2}^2}+\cdots\right)$$

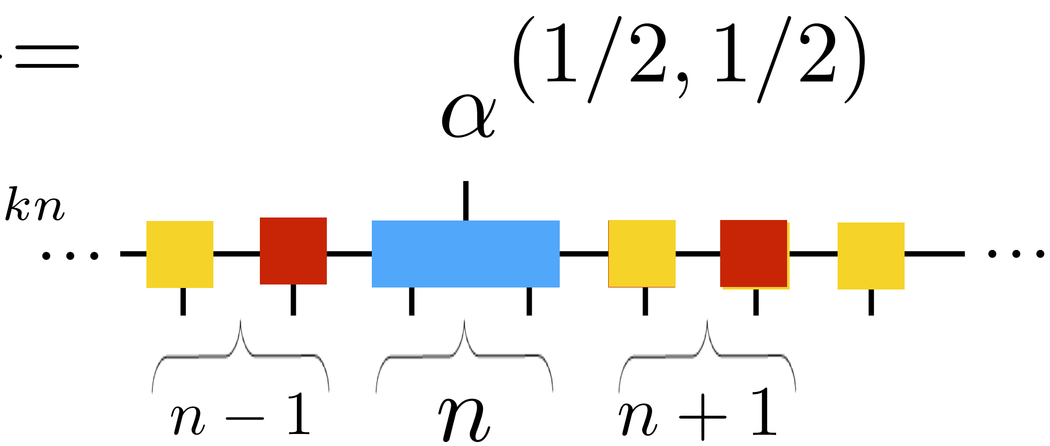
QFT prediction: $m_{K,2} \to m_{K,1}$



Recovering the QFT in the IR

fermion-excitations:

$$|k, \alpha> = \sum_n exp^{ikn} \cdots$$



third mass scale:

$$E(0) = m_F$$

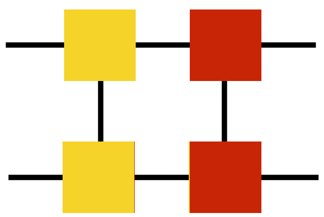
QFT prediction (bethe-anzats):

$$m_F \to 2m_K$$

Recovering the QFT in the IR

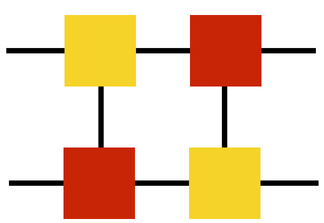
correlation lengths from transfer operator: $< O(n_1)O(n_2) > \propto T^{(n_2-n_1)}$

$$\sim \exp^{-(n_2-n_1)/\xi}$$

$T =$ 

$$|T| = \exp^{-\epsilon_i} \quad \epsilon_i < \epsilon_{i+1} \quad \epsilon_0 = 0 \quad \xi = 1/\epsilon_1$$

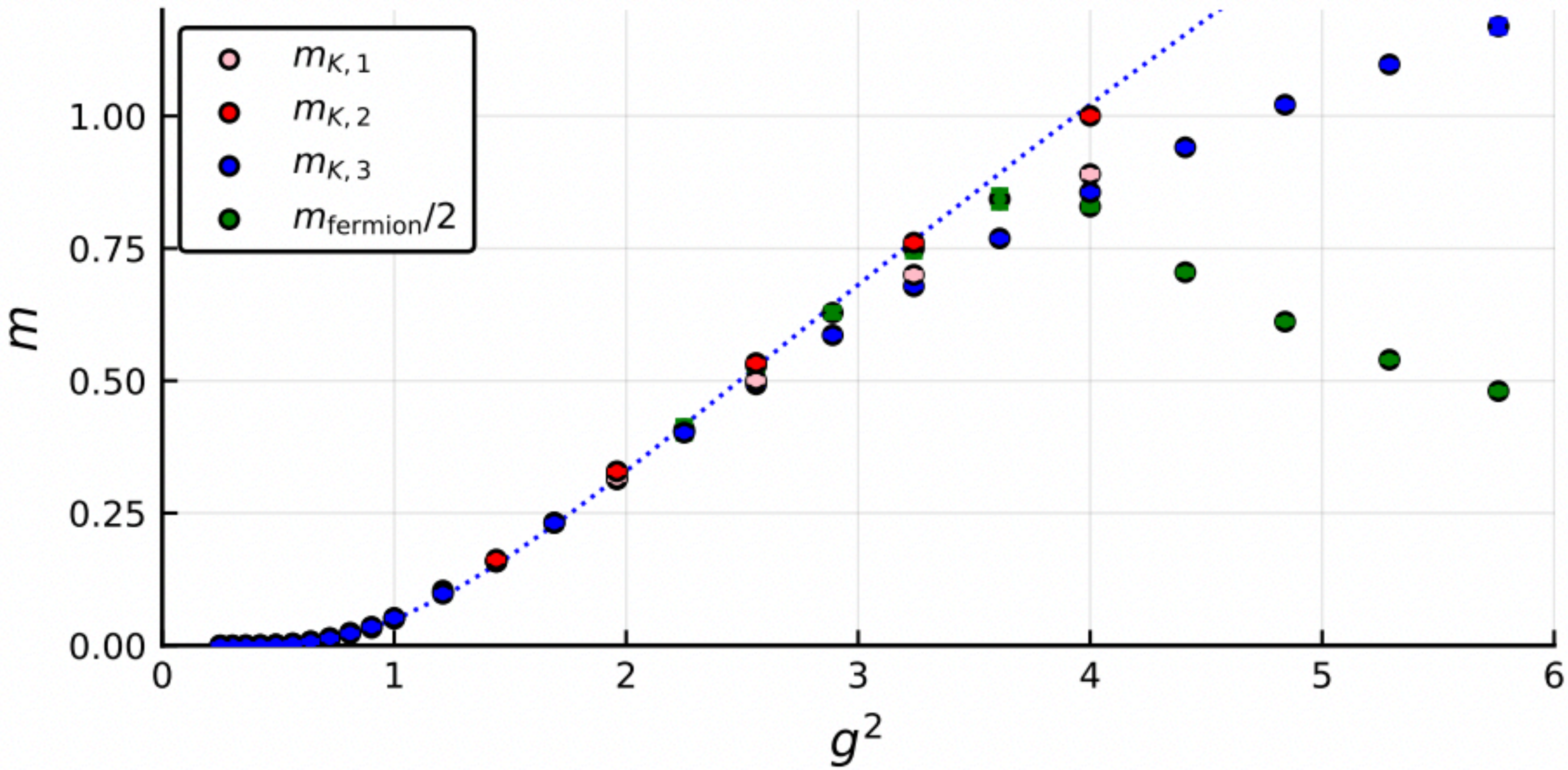
correlations for kink-operators follow from mixed transfer operator:

T_{mixed} = 

$$|T| = \exp^{-\rho_i} \qquad m_{K,3} = 1/|\rho_0|$$

QFT prediction: (Källén-Lehmann) $m_{K,3} \to m_{K,1}$

Recovering the QFT in the IR **and UV**

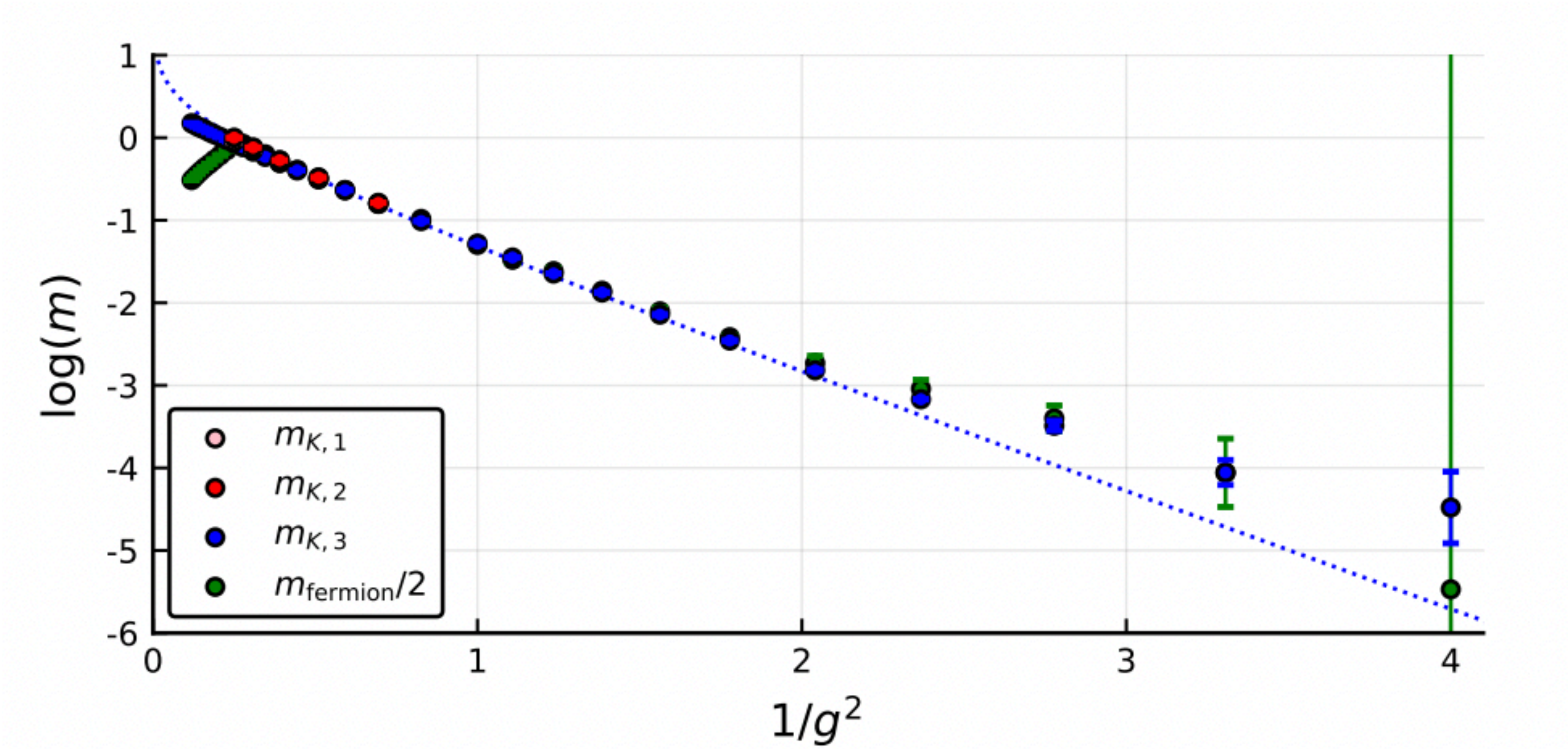


QFT prediction: (bethe anzats+perturbation theory)

$$m_K = C \times g \times \exp^{-\frac{\pi}{g^2}}(1 + a_1 g^2 + \ldots)$$

$$C = \frac{8}{e}\sqrt{\frac{e}{\pi}}\frac{1}{\sqrt{2\pi}} = 1.092$$

Recovering the QFT in the IR **and UV**



fit: $\log(m_K) = -\frac{\pi}{g^2} + \log(g) + \log(C)$ $\quad C \approx 1.121$

$$\left(C = \frac{8}{e}\sqrt{\frac{e}{\pi}}\frac{1}{\sqrt{2\pi}} = 1.092 \right)$$

Extrapolating in the bond dimension

M.M. Rams, P. Czarnik and L. Cincio,
Phys. Rev. X 8 (2018) 041033.

extract a finite entanglement length scale (similar to system size L)

$$< O(n_1)O(n_2) >= \sum_i f_i \exp^{-(n_2-n_1)\epsilon_i} \overset{\lim D\to\infty}{\approx} \int_{u_0}^{+\infty} du f(u) \exp^{-(n_2-n_1)u}$$

$$|T| = \exp^{-\epsilon_i}$$

$$\delta = \epsilon_2 - \epsilon_1 = 1/L$$

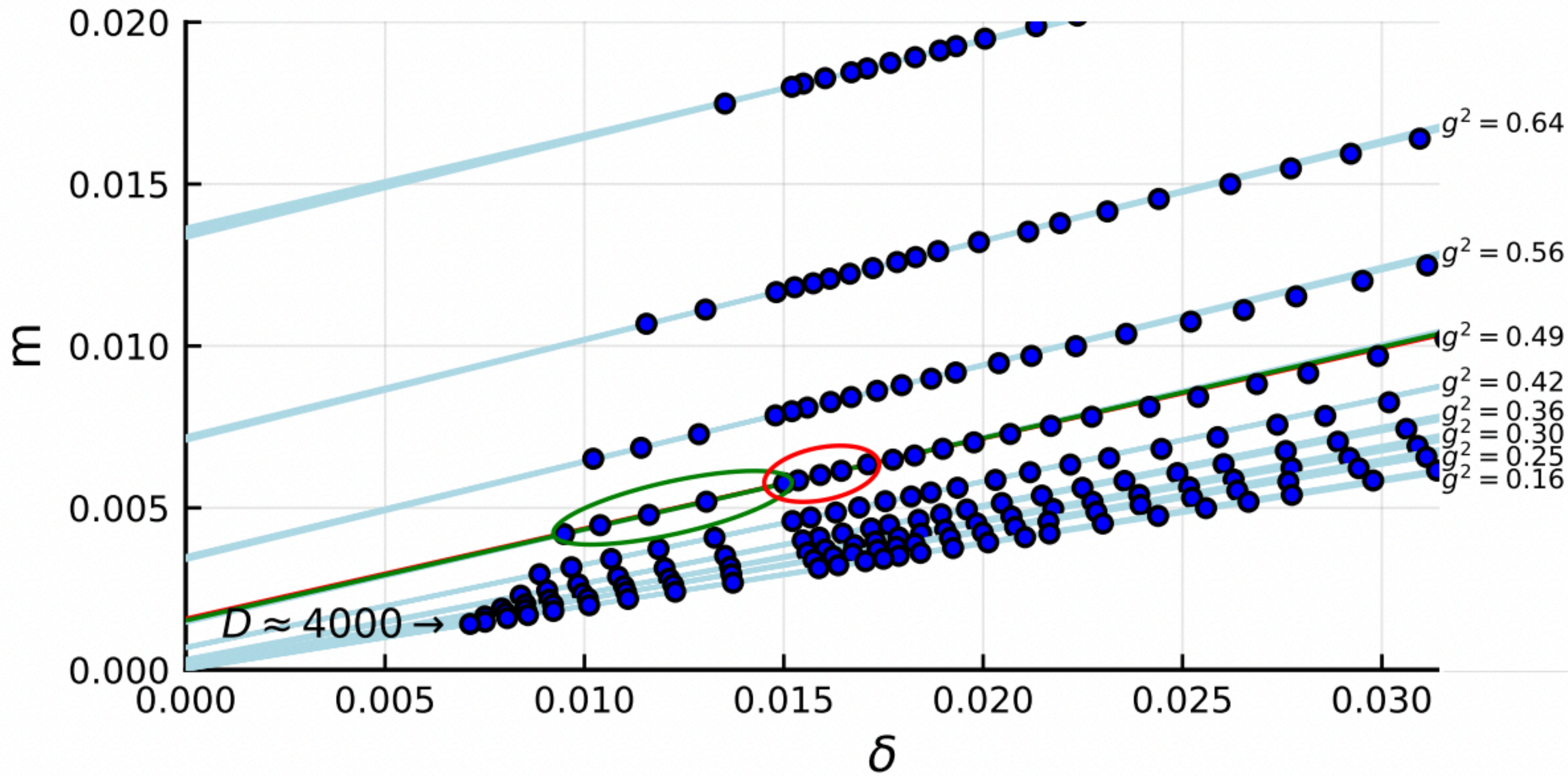
or: $\delta = \sum_{i=1} c_i \epsilon_i \quad (\sum_{i=1} c_i = 0)$

for gapped theory we then expect:

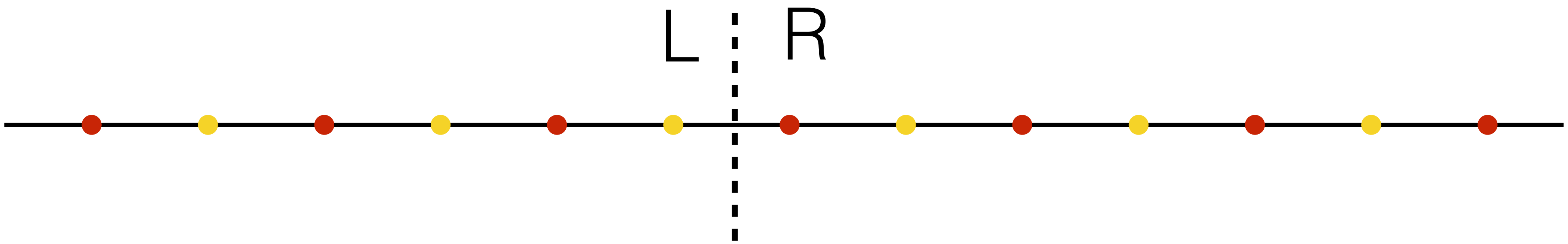
$$m(\delta) = m(0) + \delta \times m'(0) + \frac{1}{2} m''(0)\delta^2 + \ldots$$

Extrapolating in the bond dimension

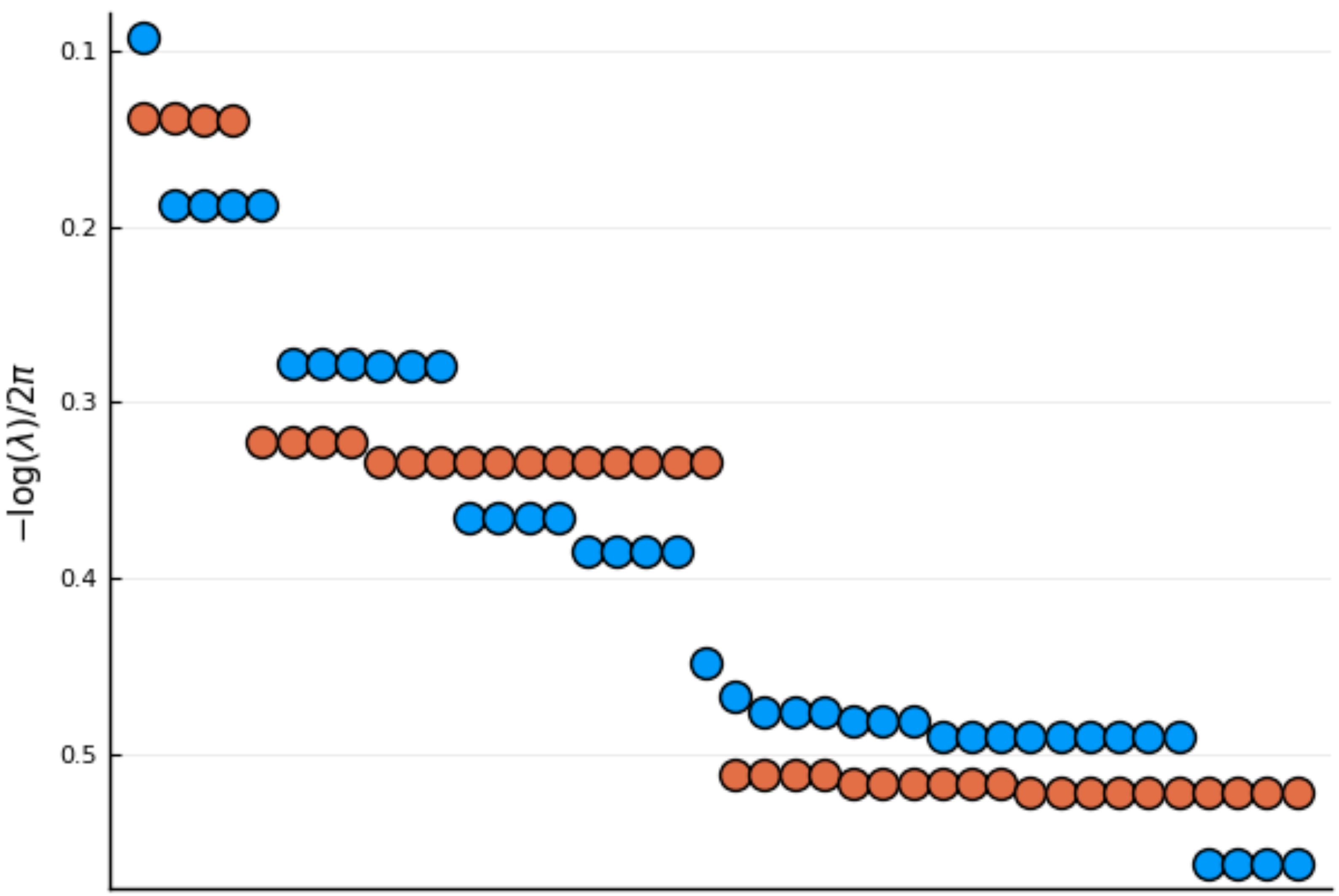
inverse correlation length $m_{K,3}$ in kink-sector:



Entanglement spectrum near the UV critical point



Schmidt decompostion: $|\psi >= \sqrt{\lambda_i}|\psi_i >_L \; |\phi_i >_R$



Entanglement spectrum near the UV critical point

see also:
Cho et al, Phys. Rev. B95 (2017)

relativistic QFT, Unruh result:

$$\rho = \exp^{-2\pi \int_0^\infty dx\, x T_{00}(x)}$$

→

plane to the strip:

$$z \to w = \log z$$

$$\begin{pmatrix} z = x + it \\ w = s + i\tau \end{pmatrix}$$

↙

$$\rho \to \rho_{strip} = \exp^{-2\pi \int_0^{\log(l_{IR})} ds\, T_{00}(s)} = \exp^{-2\pi H_E} \qquad H_E = \int_0^L \sum_{m\in 2N} 2i\lambda_{m,1}\partial_x \lambda_{m,2}$$

2 types of boundary conditions:

Neveu-Schwarz:

$$\lambda_{m,1}|_0 = \lambda_{m,2}|_L = 0$$

$$\partial_x \lambda_{m,2}|_0 = \partial_x \lambda_{m,1}|_L = 0$$

Ramon:

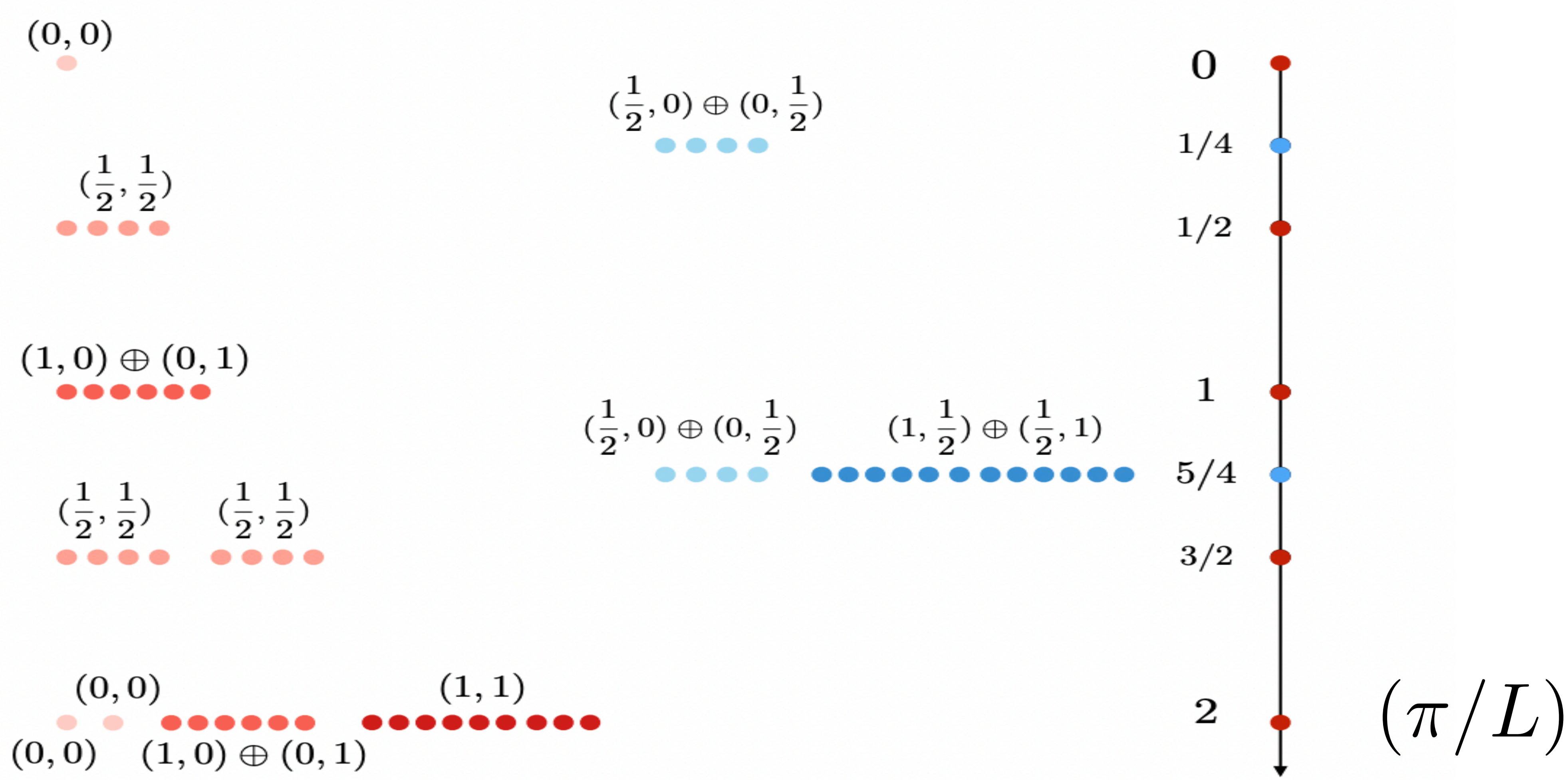
$$\lambda_{m,1}|_0 = \lambda_{m,1}|_L = 0$$

$$\partial_x \lambda_{m,2}|_0 = \partial_x \lambda_{m,2}|_L = 0$$

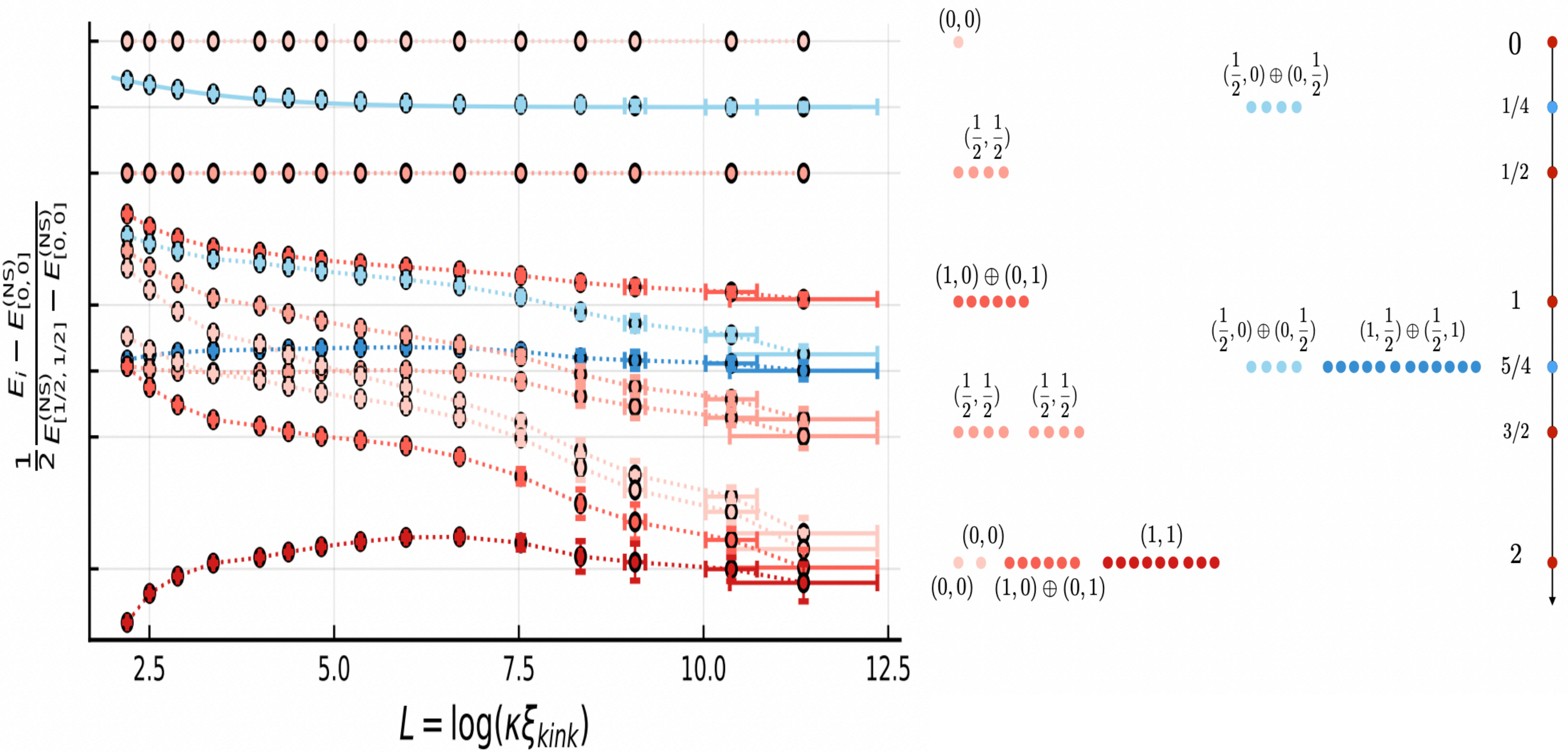
Entanglement spectrum near the UV critical point

$$H^{(\mathrm{NS})} = \sum_{k=1}^{+\infty} \sum_{m=1}^{2N} \frac{\pi(k-1/2)}{L} \phi_m^\dagger(k)\phi_m(k) + \frac{N}{\pi} \sum_{k=1}^{+\infty} \log\left(1 + e^{-2\pi \frac{\pi(k-1/2)}{L}}\right)$$

$$H^{(\mathrm{R})} = \sum_{k=1}^{+\infty} \sum_{m=1}^{2N} \frac{\pi k}{L} \phi_m^\dagger(k)\phi_m(k) + 0 \sum_{c=1}^{N} \alpha_c^\dagger \alpha_c + \frac{N}{\pi} \sum_{k=1}^{+\infty} \log\left(1 + e^{-2\pi \frac{\pi k}{L}}\right) + \frac{N}{2\pi} \log(2)$$

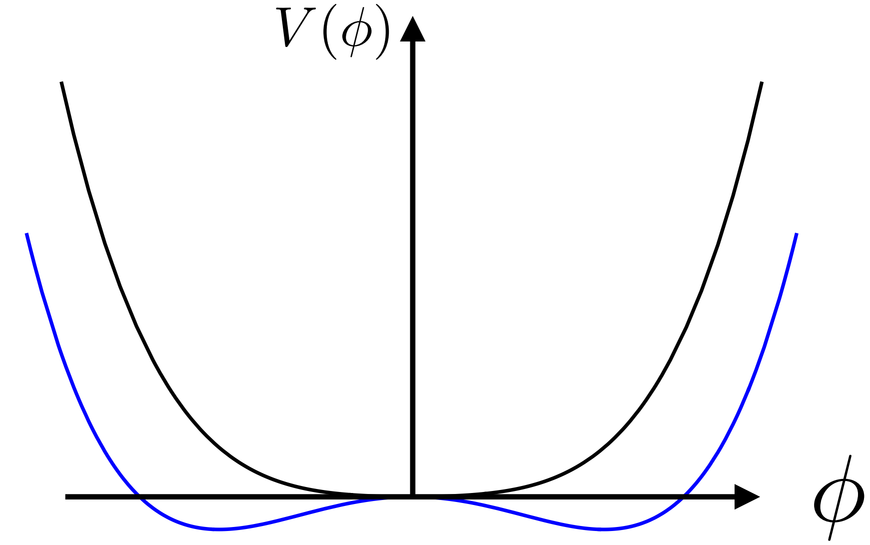


Entanglement spectrum near the UV critical point



2. $\lambda\phi^4$-theory

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\mu_p^2\phi^2 + \frac{1}{4}\lambda_p\phi^4$$



$\mathbb{Z}_2$ - symmetry breaking

<u>literature (non-exhaustive):</u>

<u>Hamiltonian truncation:</u> Rychkov and Vitale,Phys RevD91 (2015)

<u>MPS:</u> Sugihara, JHEP 2004; Milsted et al, Phys. Rev. D 88 (2013).

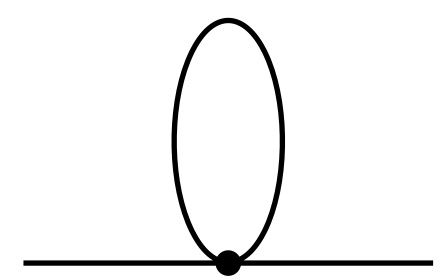
<u>Monte-Carlo:</u> Schaich et al, Phys. Rev. D79 (2009); Bosetti et al, Phys. Rev. D92 (2015); Bronzin et al, Phys. Rev. D99 (2019).

<u>TRG:</u> Kadoh et al, JHEP 2019

<u>TNR:</u> Delcamp, Tilloy, Phys. Rev. Research 2,033278 (2020)

<u>Light-front:</u> J.P. Vary, Mengyao Hang et al, Phys.Rev D 105 (2022)

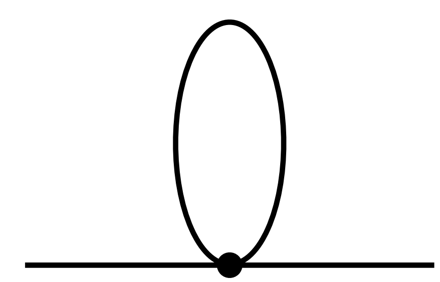
superrenormalizable, a single UV divergence:



$$\mu_p^2 = \mu_{Rp}^2 + \delta\mu_p^2$$

with:

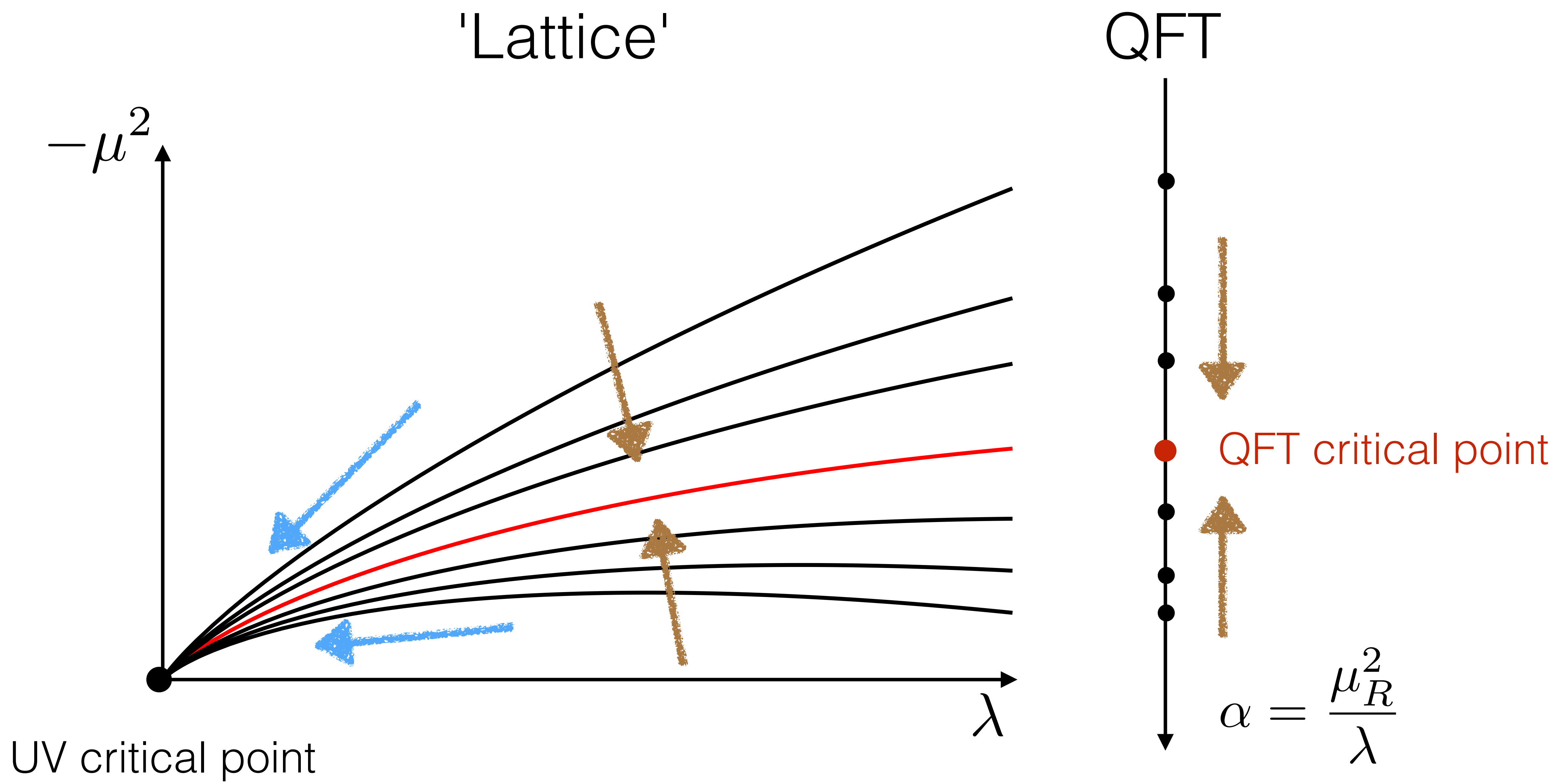
$$\delta\mu_p^2 = -$$



In 'lattice units': $\Lambda_{UV} = \frac{1}{a} = 1$ $\lambda = \lambda_p a^2$ $\mu^2 = \mu_p^2 a^2$

$$\mu^2 = \mu_R^2 - 3\lambda A(\mu_R^2) = \lambda\left(\frac{\mu_R^2}{\lambda} - 3A\left(\frac{\mu_R^2}{\lambda}\lambda\right)\right) \qquad A \sim -\log\lambda$$

e.g. on a 2d lattice: $A(x) = \int_0^\pi \int_0^\pi \frac{\mathrm{d}y\mathrm{d}z}{x + 4\sin(y)^2 + 4\sin(z)^2}$



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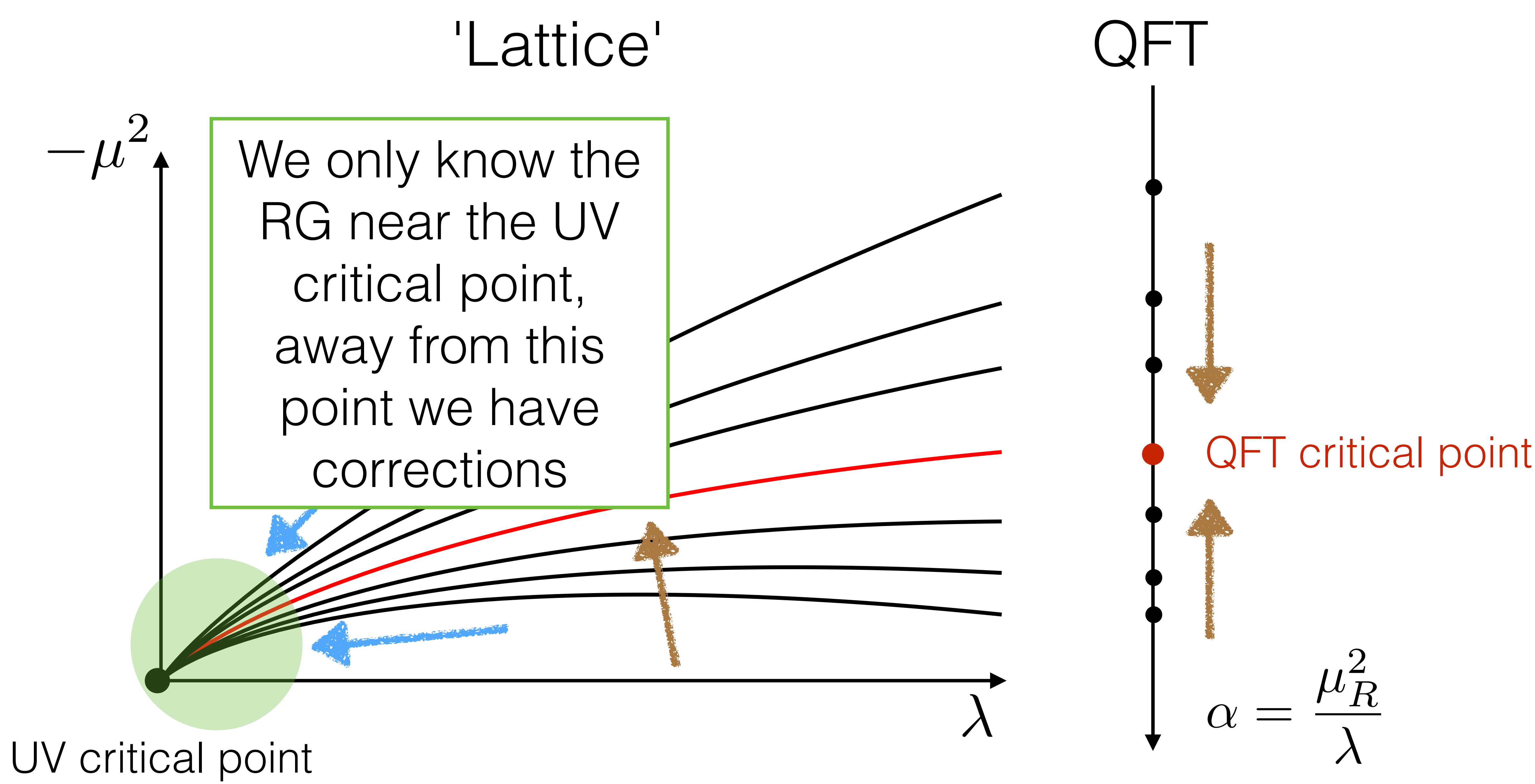

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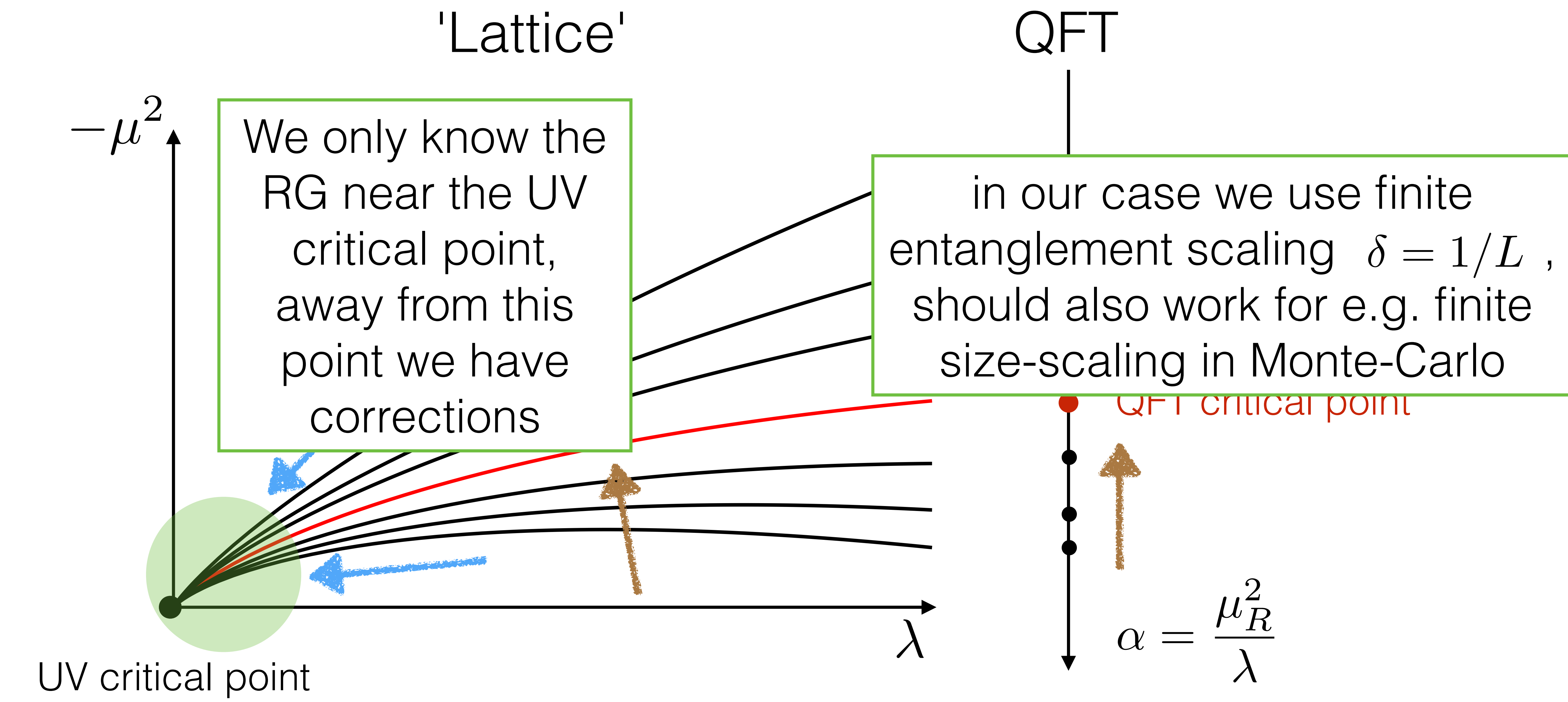


From the path integral to a tensor network:

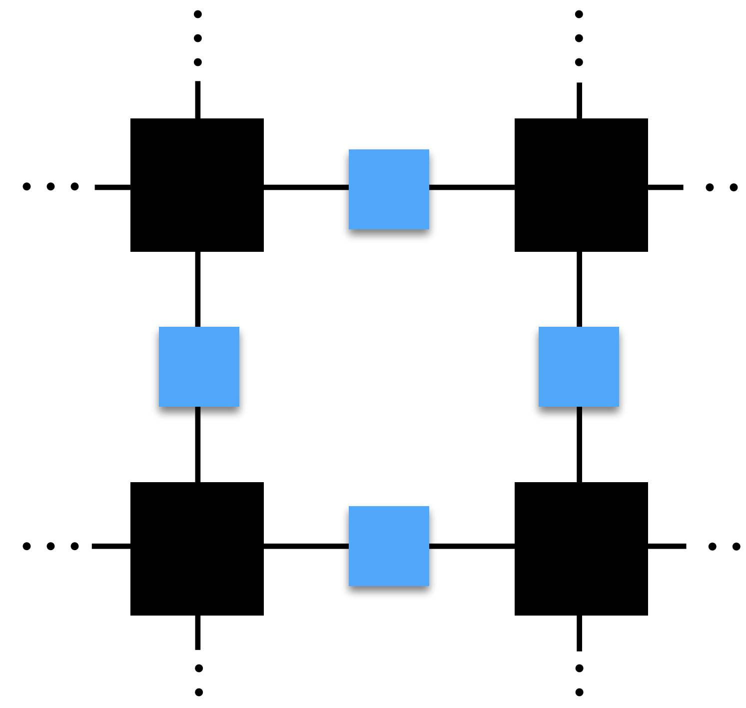
see also:
Kadoh et al, JHEP 2019

1.Discretize space-time:

$$Z = \int \prod_i \mathrm{d}\phi_i e^{-\sum_{\langle i,j \rangle} \frac{1}{2}(\phi_i - \phi_j)^2 - \sum_i \frac{1}{2}\mu^2\phi_i^2 + \frac{1}{4}\lambda\phi_i^4}$$

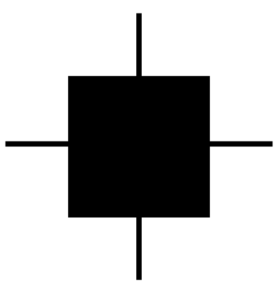
$$T = \int \mathrm{d}\phi \, |\phi\rangle \, |\phi\rangle \, \langle\phi| \, \langle\phi|$$

$$t = \int \mathrm{d}\phi \, \mathrm{d}\phi' e^{\frac{-1}{2}(\phi-\phi')^2 - \frac{\mu^2}{8}(\phi^2+\phi'^2) - \frac{\lambda}{16}(\phi^4+\phi'^4)} \, |\phi\rangle \, \langle\phi'|$$

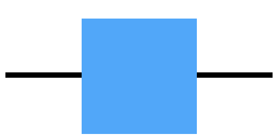


2.Discretize field space:

$$\phi_i = i \times \sqrt{2}\delta\phi$$

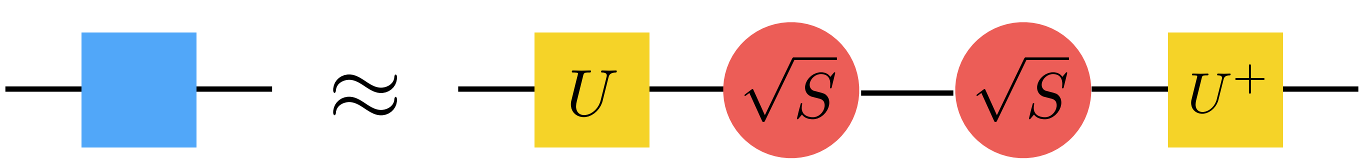


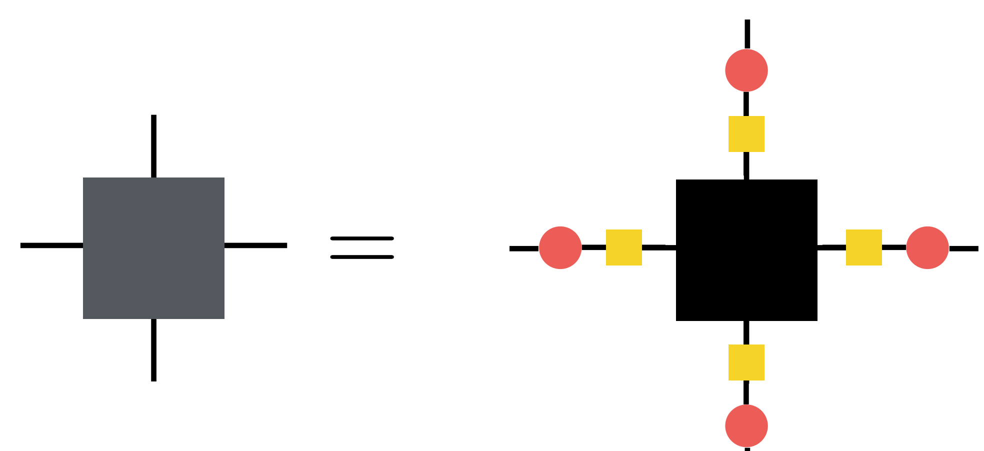
$$T_{ijkl} = (\delta\phi)^{-1}\delta_{i,j}\delta_{i,k}\delta_{i,l}$$



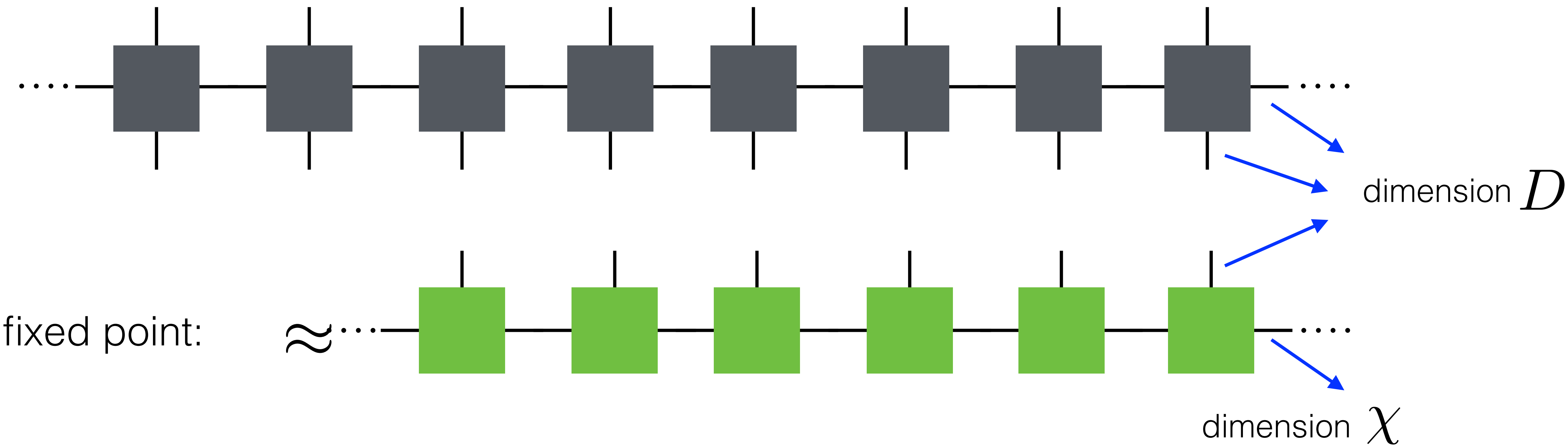
$$t_{ij} = \delta\phi e^{-(\phi_i-\phi_j)^2 - \frac{\mu^2}{4}(\phi_i^2+\phi_j^2) - \frac{\lambda}{4}(\phi_i^4+\phi_j^4)}$$

3. Truncate:

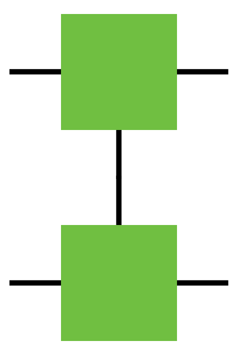




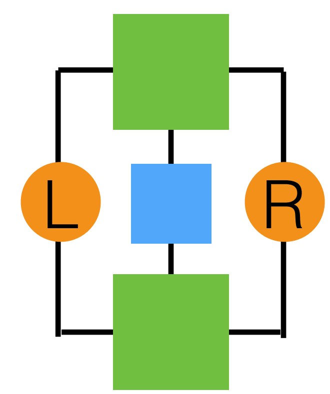
Approximating the transfer **MPO** fixed point with an **MPS**



MPS
transfer matrix T:

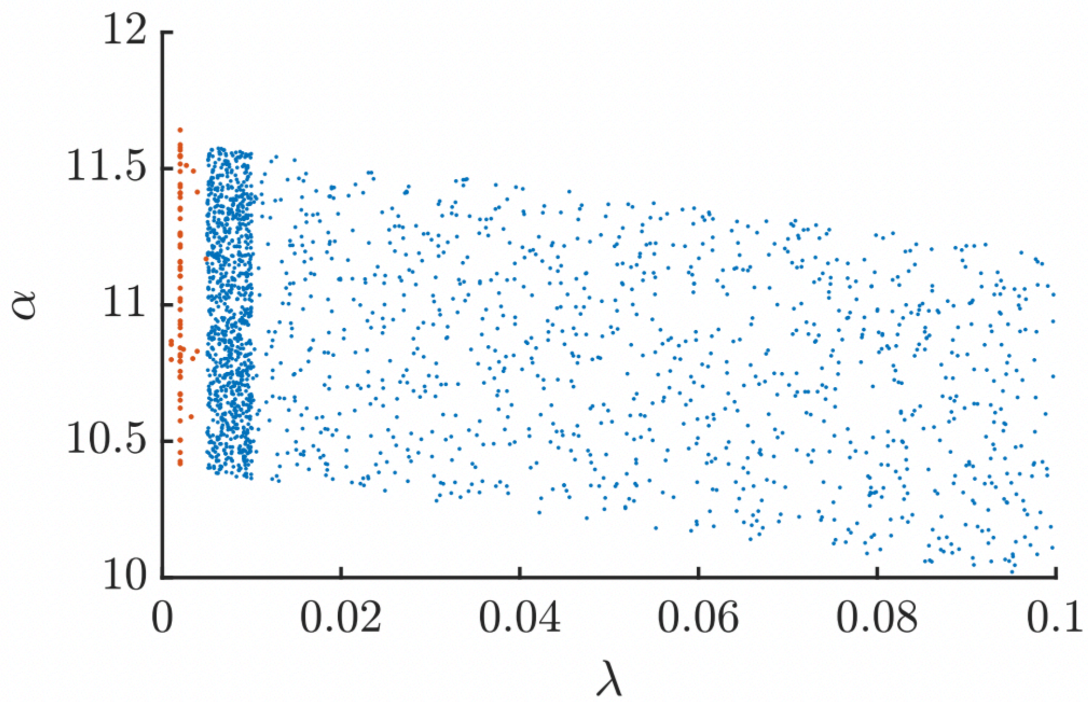


$$< \psi|\hat{O}|\psi >=$$



$$|T| = \exp^{-\epsilon_i} \qquad \delta = \epsilon_2 - \epsilon_1 = 1/L \qquad \delta = \sum_{i=1} c_i \epsilon_i \quad (\sum_{i=1} c_i = 0)$$

double collapse



2081 data-points with random bond dimension

$\chi \in [100, 200]$

4 'observables:'

$$\xi, \phi, \phi^3, S$$

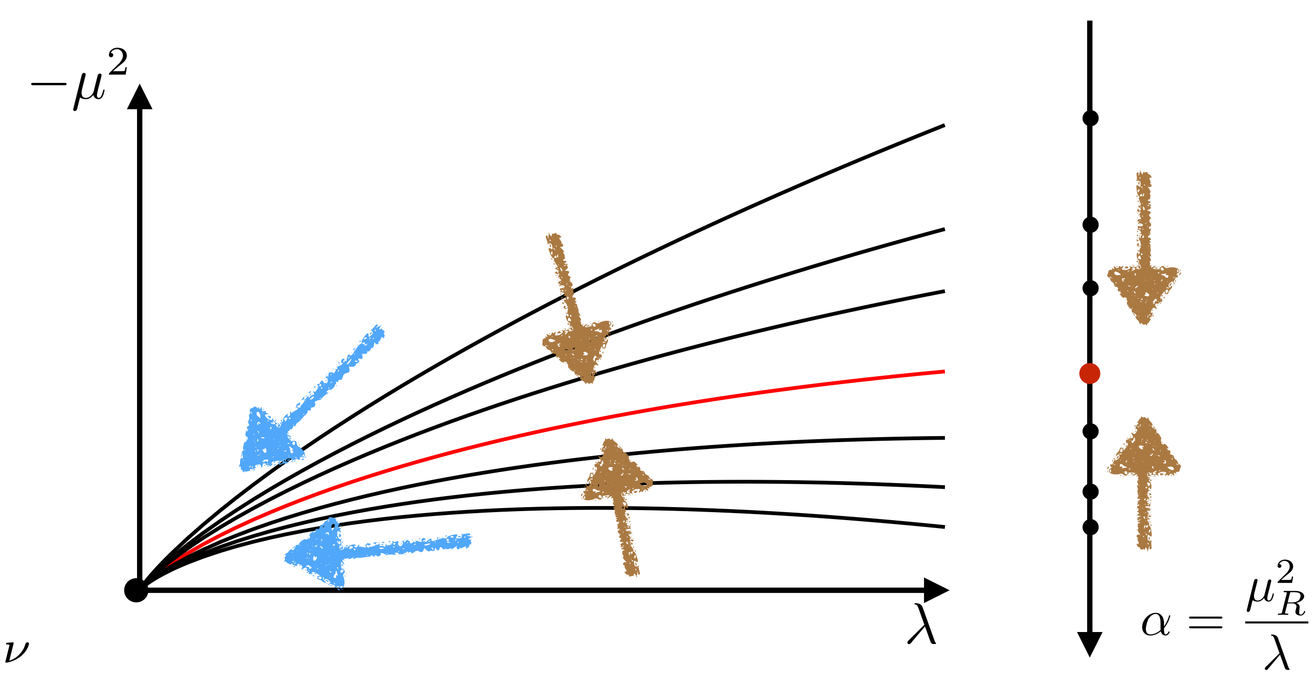
$$\tilde{\phi}^3 = \phi^3 + \frac{3}{4\pi}\log(\lambda)\phi$$

UV scaling:

$$\xi \to \xi/a$$
$$\phi \to \phi$$
$$\tilde{\phi}^3 \to \tilde{\phi}^3$$
$$S \sim -\frac{c_{UV}}{6}\log a$$
$$\lambda \to \lambda a^2$$
$$\delta \to \delta a$$
$$\alpha \to \alpha$$

IR scaling:

$$\xi \to \xi/s$$
$$\phi \to \phi s^{\beta}$$
$$\tilde{\phi}^3 \to \tilde{\phi}^3 s^{\beta}$$
$$S \sim -\frac{c_{IR}}{6}\log s$$
$$\lambda \to \lambda$$
$$\delta \to \delta s$$
$$(\alpha - \alpha_c) \to (\alpha - \alpha_c)s^{1/\nu}$$



double collapse

	UV exponent	IR exponent
λ	2	0
$\alpha - \alpha_c$	0	$1/\nu = 1$
L^{-1}, δ	1	1
ξ	-1	-1
ϕ	0	$\beta = 1/8$
$\phi^3 - \frac{3}{4\pi}\log(\lambda)\phi$	0	$\beta = 1/8$
$\exp(S)$	$-\frac{c_{uv}}{6} = -\frac{1}{6}$	$-\frac{c_{ir}}{6} = -\frac{1}{12}$

$$O \;=\; \xi, \phi, \tilde{\phi}^3, e^S$$

Scale-invariant observables:

$$\mathcal{O} = \lambda^{-d_{uv}/2}\Delta^{-d_{ir}}O \qquad (\Delta = \delta/\sqrt{\lambda})$$

Scale-invariant 'distance': $\Delta^{-1/\nu}(\alpha - \alpha_c)$

$$[\mu^2,\, \lambda,\, \chi,\, O] \rightarrow [\lambda, (\alpha - \alpha_c), \Delta^{-1/\nu}(\alpha - \alpha_c), \lambda^{-d_{uv}/2}\Delta^{-d_{ir}}O]$$

only the first two variable change
under IR/UV scaling

double collapse: 4D data-set should collapse to 2D curve,
use this to find α_c

double collapse

However, scaling gets corrections further from UV/IR fixed point (parameterized ignorance)

$$\alpha \to \alpha + A\lambda\log(\lambda) + B\lambda + C\lambda^2\log(\lambda)^2 + D\lambda^2\log(\lambda) + E\lambda^2 + \dots .$$

$$O \to O\left(A_O + B_O\lambda\log(\lambda) + C_O\lambda + D_O\lambda^2\log(\lambda)^2 + E_O\lambda^2\log(\lambda) + F_O\lambda^2 + \dots\right)$$

$$O \sim (\alpha - \alpha_c)^{\beta/\nu}\left(1 + A(\alpha - \alpha_c)^{\omega} + \dots\right)$$

Procedure:

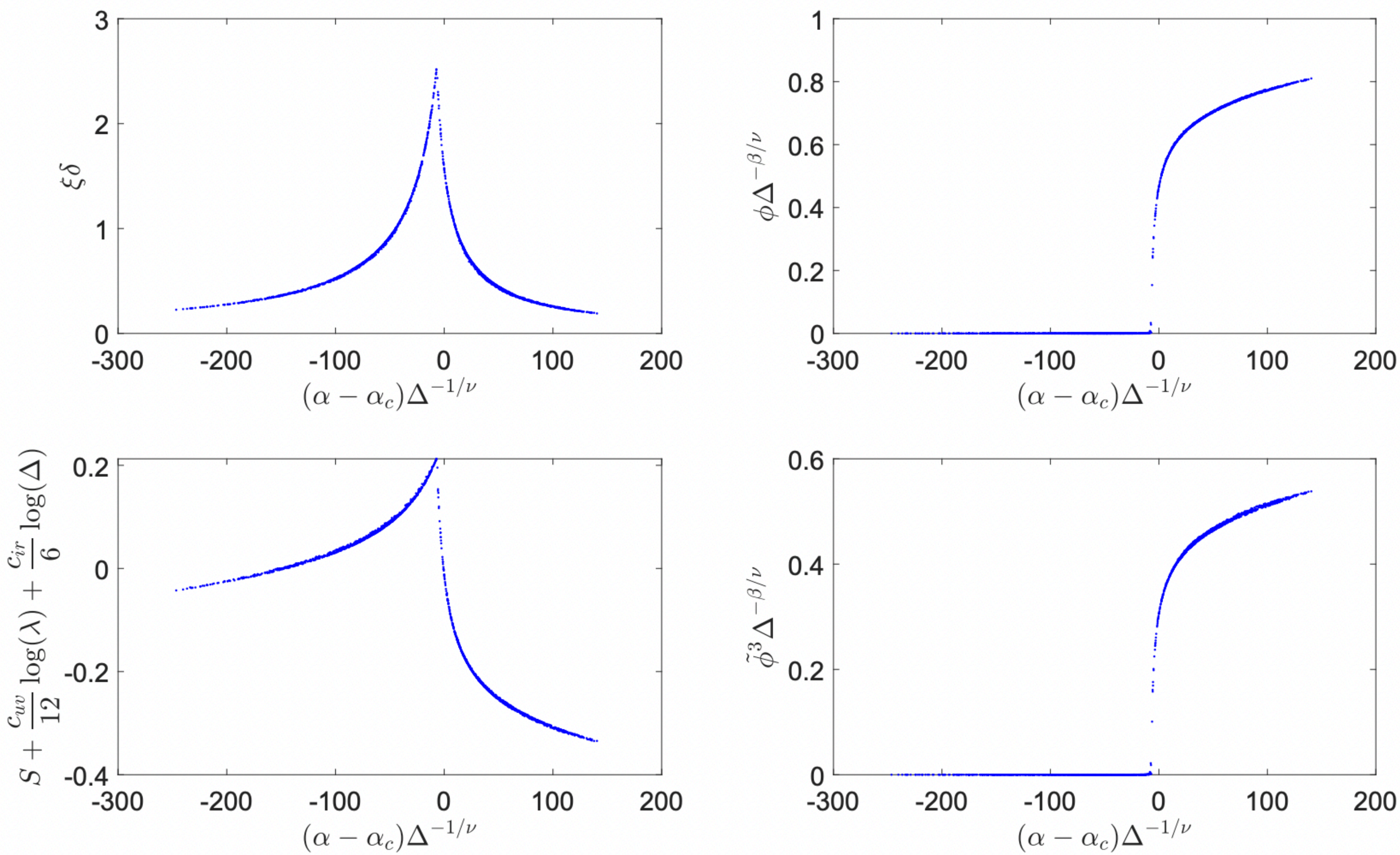
1. Pick value for α_c and some correction parameters

2. Generate 2D plot of the 4 scaled observables

3. Take template scaling function for each observable, and optimize free parameters in this function such that sum of orthogonal distances is minimal

4. Cost-function: sum of orthogonal distance for the 4 observables

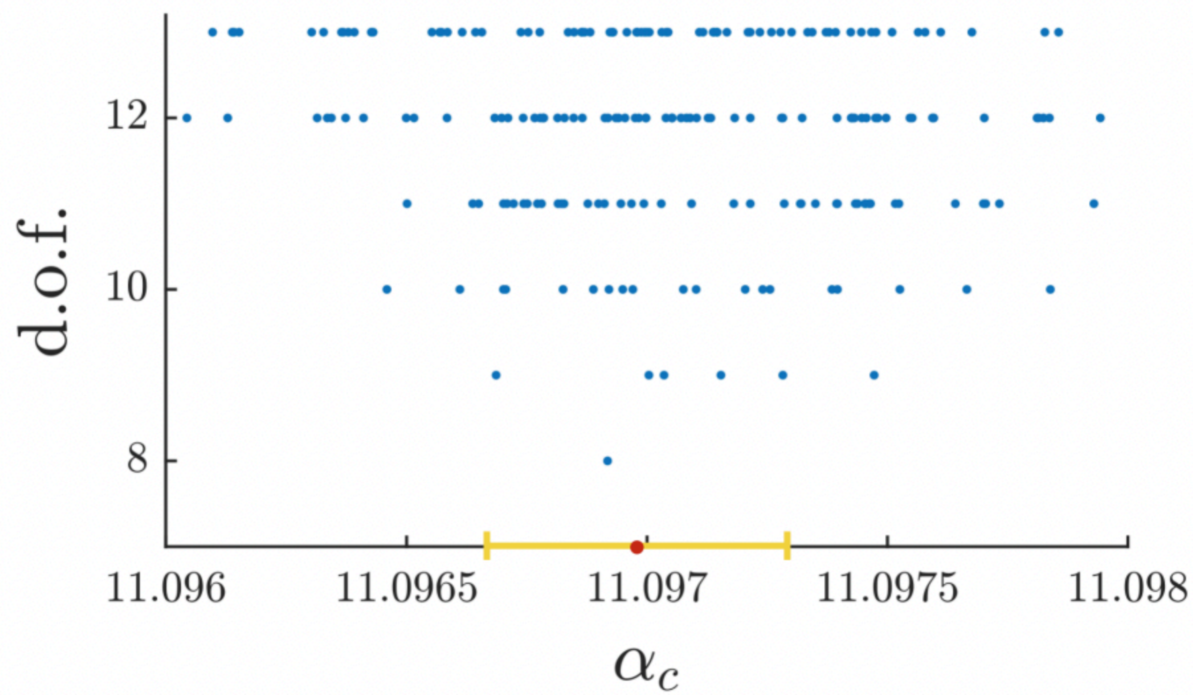
Collapse plots:



$$\begin{aligned}
\alpha \to \quad & \alpha - 3.9\,\lambda\log(\lambda) - 5.9\,\lambda \\
& - 7.1\,\lambda^2\log(\lambda) + 3\,\lambda^2 \\
\xi \to\, & \xi\,(1 + 8.5e^{-4}\,\lambda\log(\lambda) + 1.6e^{-3}\,\lambda\,) \\
\phi \to \quad & \phi\,(1 + 1.1e^{-2}\,\lambda\,)
\end{aligned}$$

$$\begin{aligned}
\tilde{\phi}^3 \to\, & \tilde{\phi}^3\,(1 - 5.9e^{-1}\,\lambda - 1.2\,\lambda^2\log(\lambda)^2 \\
& - 5.8e^{-1}\,\lambda^2\log(\lambda)\,) \\
S \to \quad & S - 2e^{-2}\,(\alpha-\alpha_c)\,,
\end{aligned}$$

value for the critical point



Method	Year	α_c
MPS[19]	2013	11.064(20)
Hamiltonian truncation[29]	2017	11.04(12)
Borel resummation[30]	2018	11.23(14)
Monte Carlo[10]	2018	11.055(20)
TRG[26]	2019	10.913(56)
gilt-TNR[25]	2020	11.0861(90)
This work	2021	11.09698(31)

Leaving other IR parameters free: $\beta = 0.1298,\ \nu = 0.98466,\ c_{ir} = 0.5114$

Leaving UV parameter free: $\phi^3 - c\log(\lambda)\phi$ $\quad c = 0.23888(65)$

$$\frac{3}{4\pi} \approx 0.23873$$

Dates: **11-15 September 2023**
Venue: **Abingdon, UK**
Applications: **OPEN, closing 23.59 on 17 July 2023**.