„Z(3) gauge theory with $N_f = 3$ quarks"

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Tensor networks in many-body
and quantum field theory
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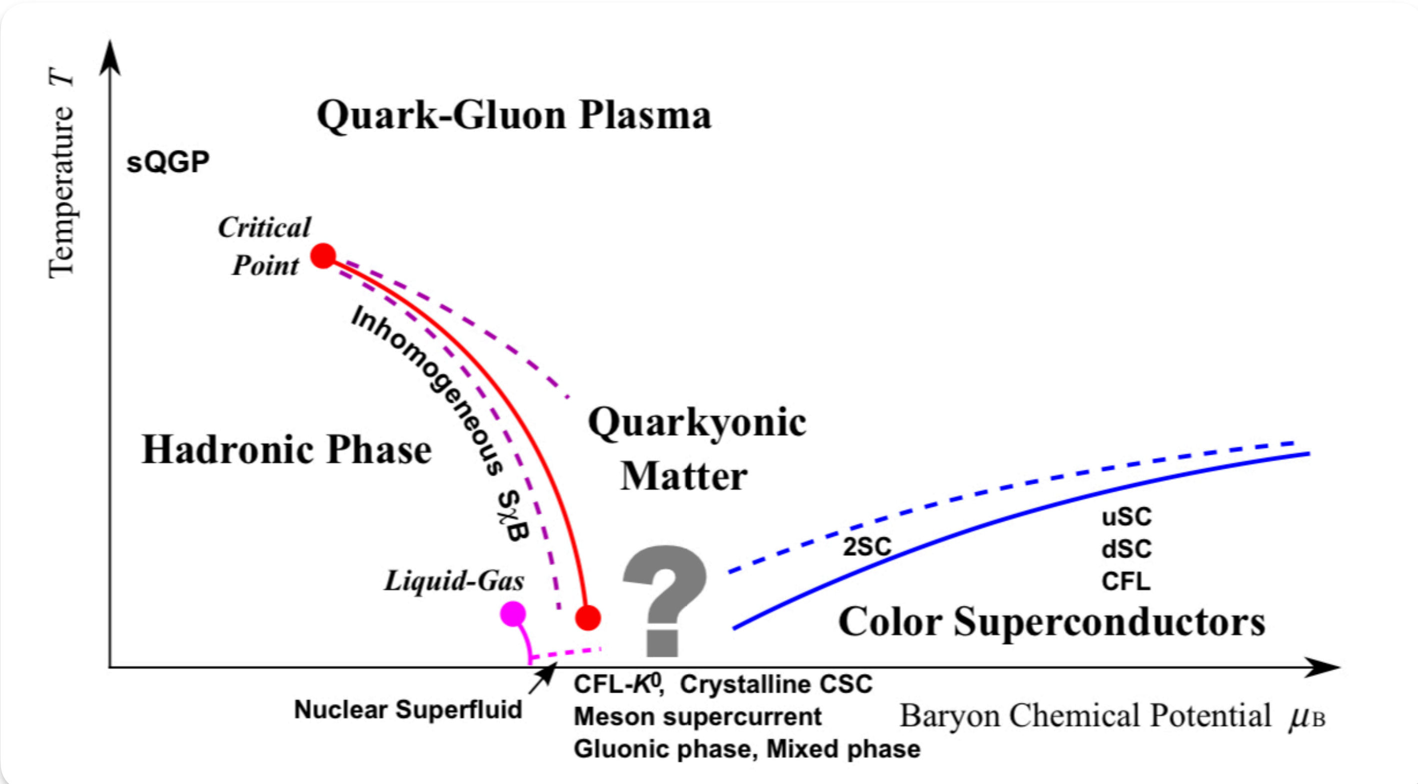
Introduction

Conjectured phase diagram of QCD



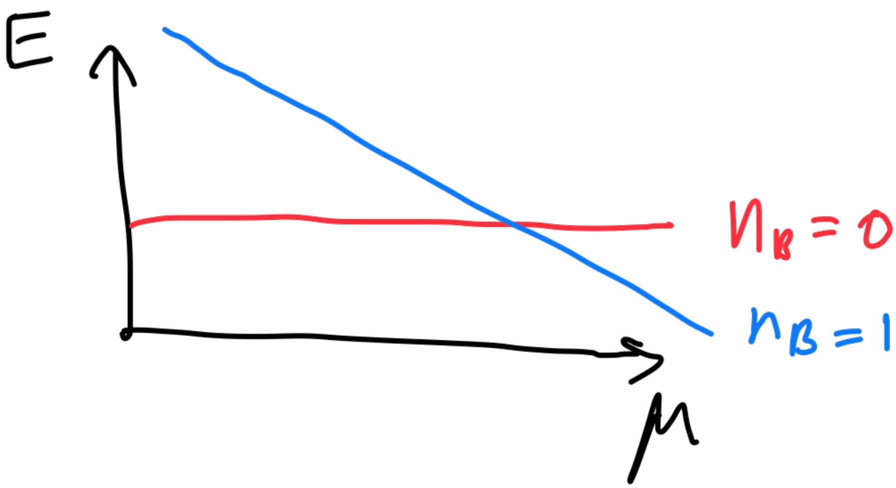
Fukushima & Hatsuda

First-principle calculations are severely limited by the sign problem

Quantum computing for the sign problem

Adiabatic state preparation

a) Level crossing



b) Spontaneous translation symmetry breaking – phonons and other zero modes

VQE

$$|\tilde{\Omega}\rangle = \prod_{i=1}^{N} e^{i\theta \hat{O}_i} |0\rangle$$

> How many operators of certain type are needed in order to achieve a given precision ε?

> How does N scale with system size? Is there a „minimal" subset of operators?

Simple 1+1d model

$Z(3)$ gauge theory coupled to $N_f = 3$ dynamical quarks which shares the following features with QCD:

> Has a sign problem at $\mu_B \neq 0$

> At $N_f = 1$ mesons are $\bar{u}u$, $\bar{u}\,\partial_x u$ ...
Baryons $u(\partial_x u)(\partial_x^2 u)$

> At $N_f = 3$ octet + singlet
$\pi^a = \bar{q}\, t^a q \qquad \eta' = \bar{q} q$

Baryons uds, $u(\partial_x u)d$, ...

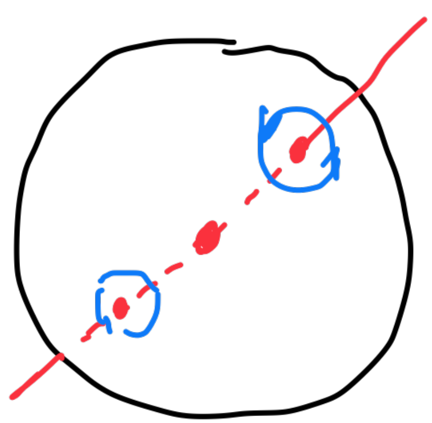
> Fermi surface of Baryons at $\mu \neq 0$

Continuum $Z(3)$ gauge theory

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2 + \bar{q} \not{D}^1 q + |D_\mu^3 \phi|^2 + V(|\phi|^2)$$

$$D_\mu^n = \partial_\mu - ien A_\mu$$

If $\langle \phi \rangle \neq 0$ then $U(1)$ is spontaneously broken to $Z(3)$.



Gas of vortices would confine quarks over large distances

QED in 1+1 dimensions

$$\mathcal{H} = \sum_{\substack{x=1..N \\ f=u,d,s}} \left(-\frac{i}{2} \psi^{\dagger}_{x+1,f} U^{\dagger}_{x} \psi_{x,f} + h.c. \right) +$$

$$+ \sum_{x,f} (-1)^{x} m \, \psi^{\dagger}_{x,f} \psi_{x,f} + \frac{g^2}{2} \sum_{x} E_x^2 + \mu \sum_{x,f} q_{x,f}$$

- staggered fermions of $N_f = 3$ „flavors":

u d s

- coupled to $\mathbb{Z}(3)$ gauge field

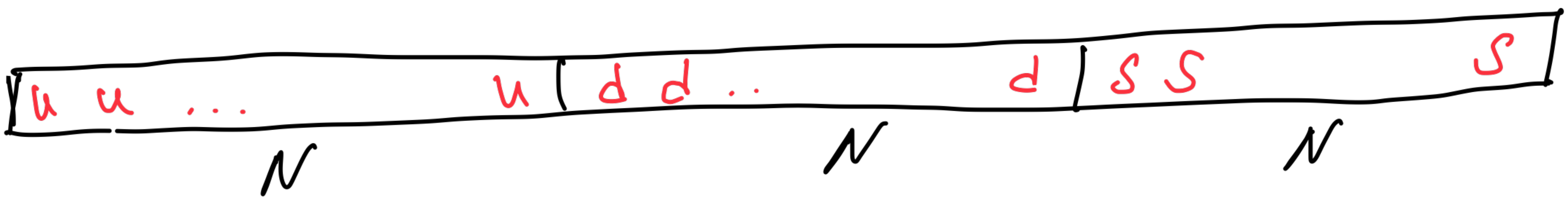
Fermions

$$\{\psi_{x,f}, \psi^{+}_{y,f'}\} = \delta_{ff'}\,\delta_{xy}$$

- Usual Jordan-Wigner mapping

but $3\times$ JW fermions c_α:

c_α:

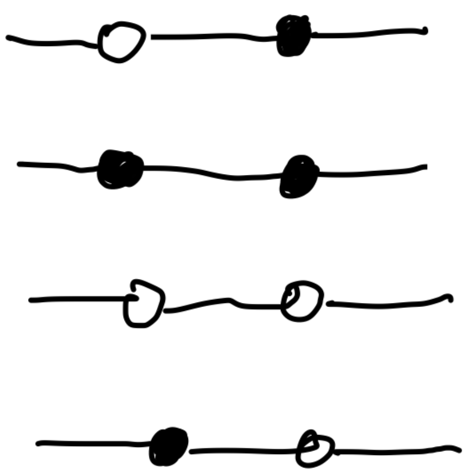


$$c_\alpha = \prod_{\beta=1}^{\alpha-1} Z_\beta\, \Sigma^-_\alpha \qquad \Sigma^\pm_\alpha = \frac{1}{2}(X \pm iY)$$

Fermions: charge density

For staggered fermions even and odd sites are different:

staggered | dirac



vacuum

$q=+1$ fermion

$q=-1$ hole

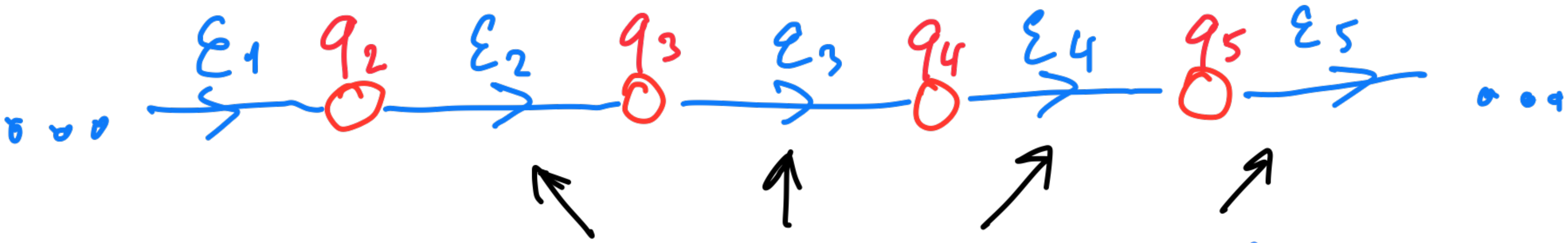
both fermion and hole

Charge density is:

$$q_{x,f} = \psi^{+}_{x,f}\,\psi_{x,f} - \frac{1-(-1)^{x}}{2}$$

$\mathbb{Z}(3)$ gauge field

charges live on sites



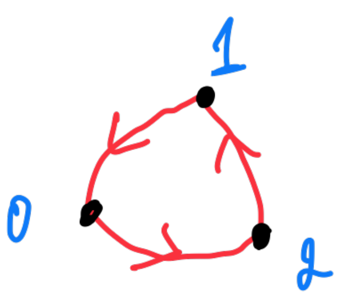
gauge field lives on links

– Electric field takes discrete values:

$$E_x|\varepsilon_x\rangle = \varepsilon_x|\varepsilon_x\rangle, \quad n = 1 \ldots 3$$

– „Ladder" operators U_x:

$$U_x|\varepsilon_x\rangle = |\varepsilon_x - 1 \bmod 3\rangle$$

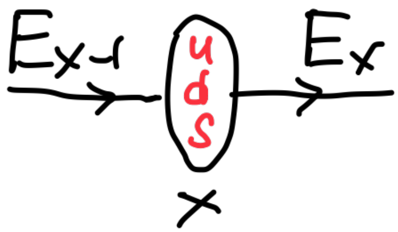


$$U_x\, e^{i\frac{2\pi}{3}E_x} = e^{i\frac{2\pi}{3}}\, e^{i\frac{2\pi}{3}E_x}\, U_x$$

Gauss' law

Hamiltonian commutes with local constraints G_x:

$$G_x = e^{i\frac{2\pi}{3}E_{x-1}} \prod_{f=u,d,s} e^{i\frac{2\pi}{3} q_{x,f}} \, e^{-i\frac{2\pi}{3}E_x}$$



$$E_x - E_{x-1} = \sum_f q_{x,f} \mod 3$$

Eigenvalues of G_x label different Gauss' sectors. Each sector describes states of the system in the presence of static (probe) charges, according to eigenvalues of G_x.

Gauss' law fixing

> In order to fix a particular G.L. sector we introduce a constraint term:

$$H_{fix} = \lambda \sum_x \left(1 - \mathrm{Re}\, G_x\right)$$

where λ sets an energy penalty for undesired sectors.

> Alternatively, one can solve G.L:

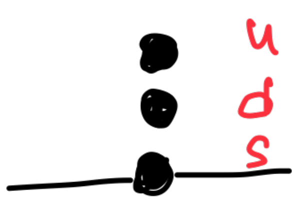
$$E_x = E_{x-1} + q_x = \sum_{y \leq x} q_y$$

and eliminate gauge fields.

Baryon and meson states

Baryon

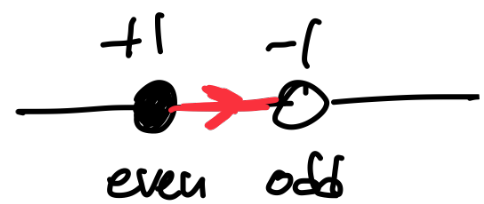
uds



$q = +3$

Meson

$\bar{q} q$



$q = 0$

$u(\partial_x u)\, d$

$q = +3$

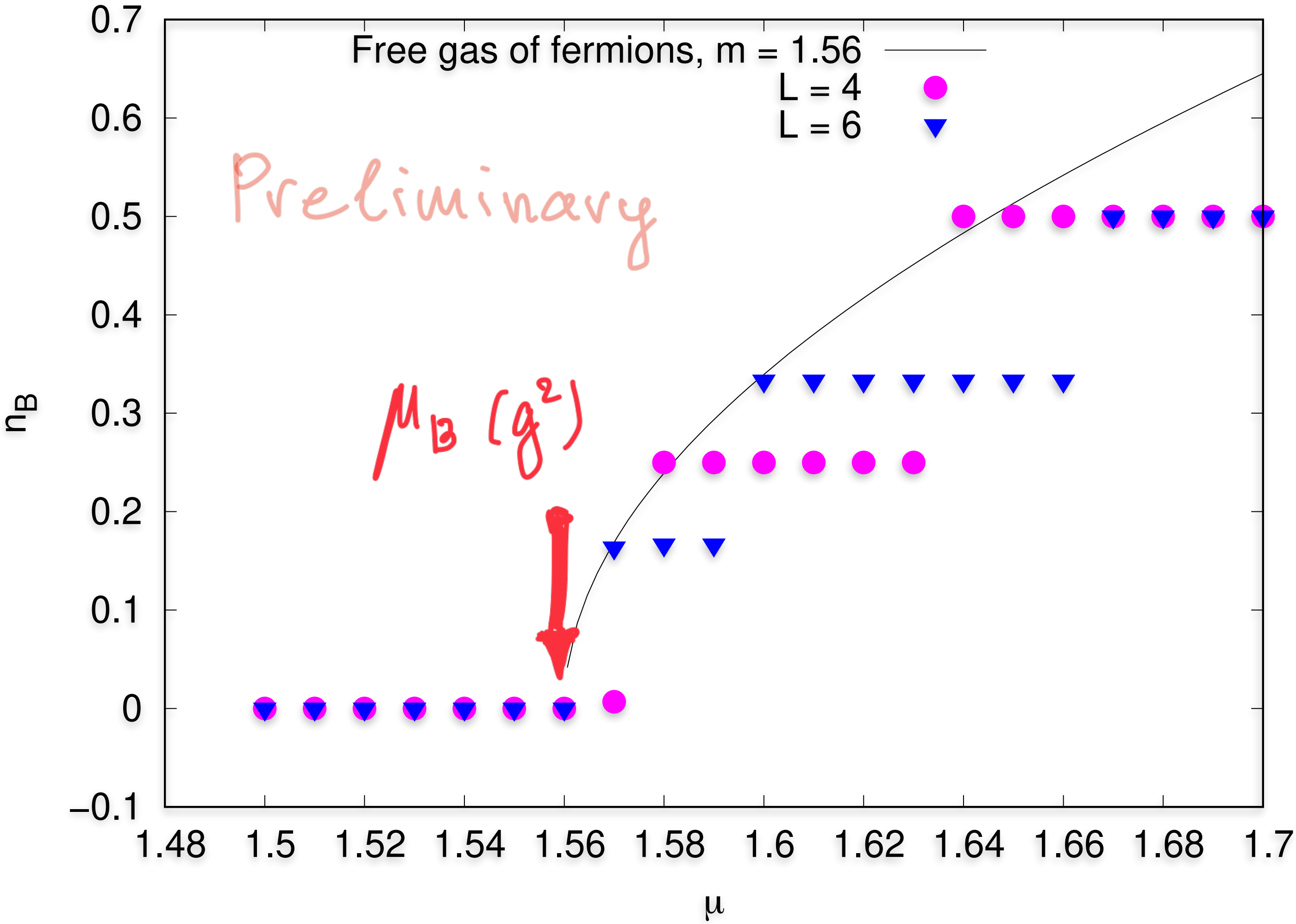
In strong coupling limit $g \gg m$:

$$M_B = 3m + \frac{g}{2\pi}\frac{1}{g^2}$$

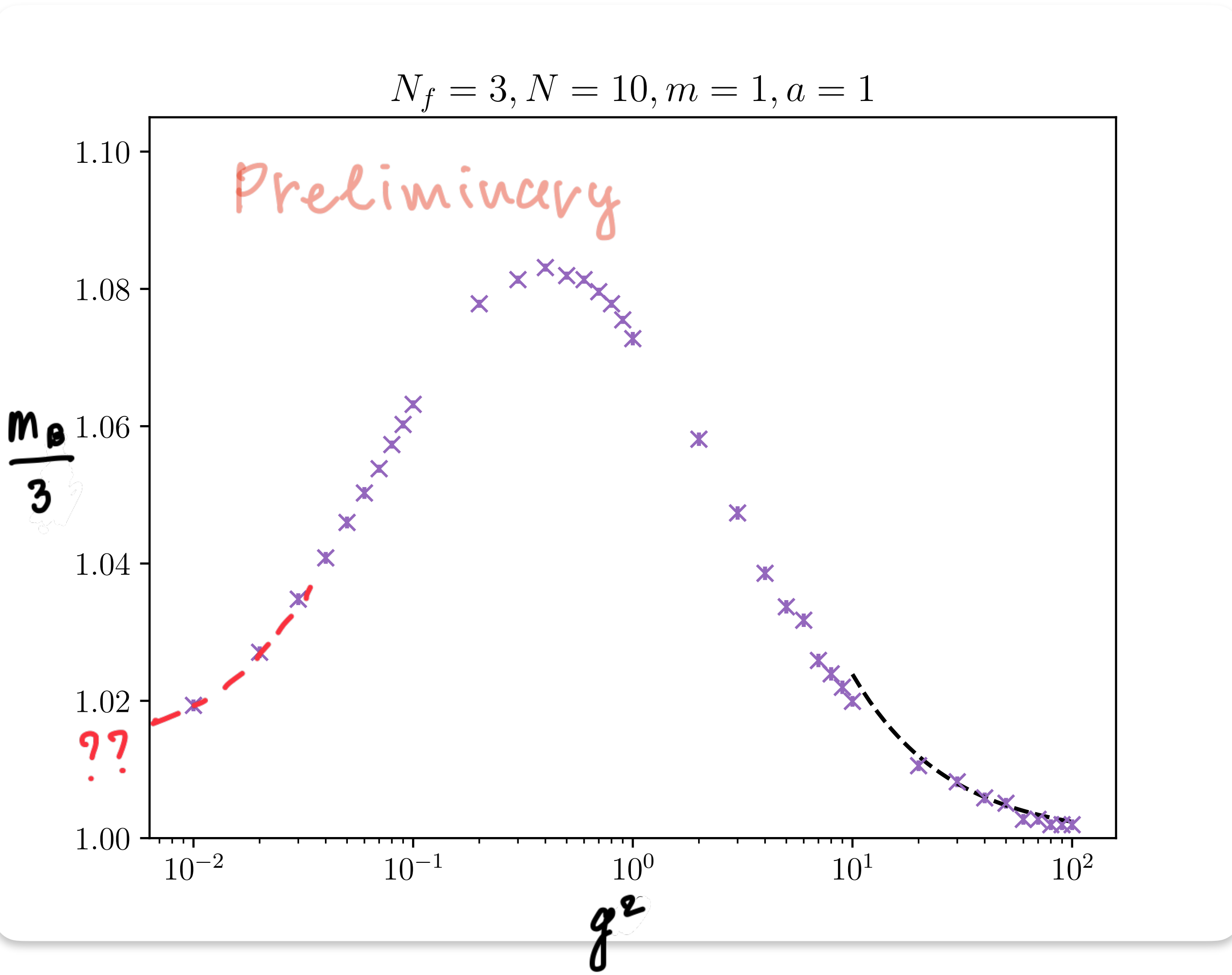
Simulation details

- We use DMRG with open B.C. In particular, we use iTensors and write the code in Julia.
- We find ground state $|\Omega\rangle$ and energy ε_0 which we use in order to compute the baryon charge n_B

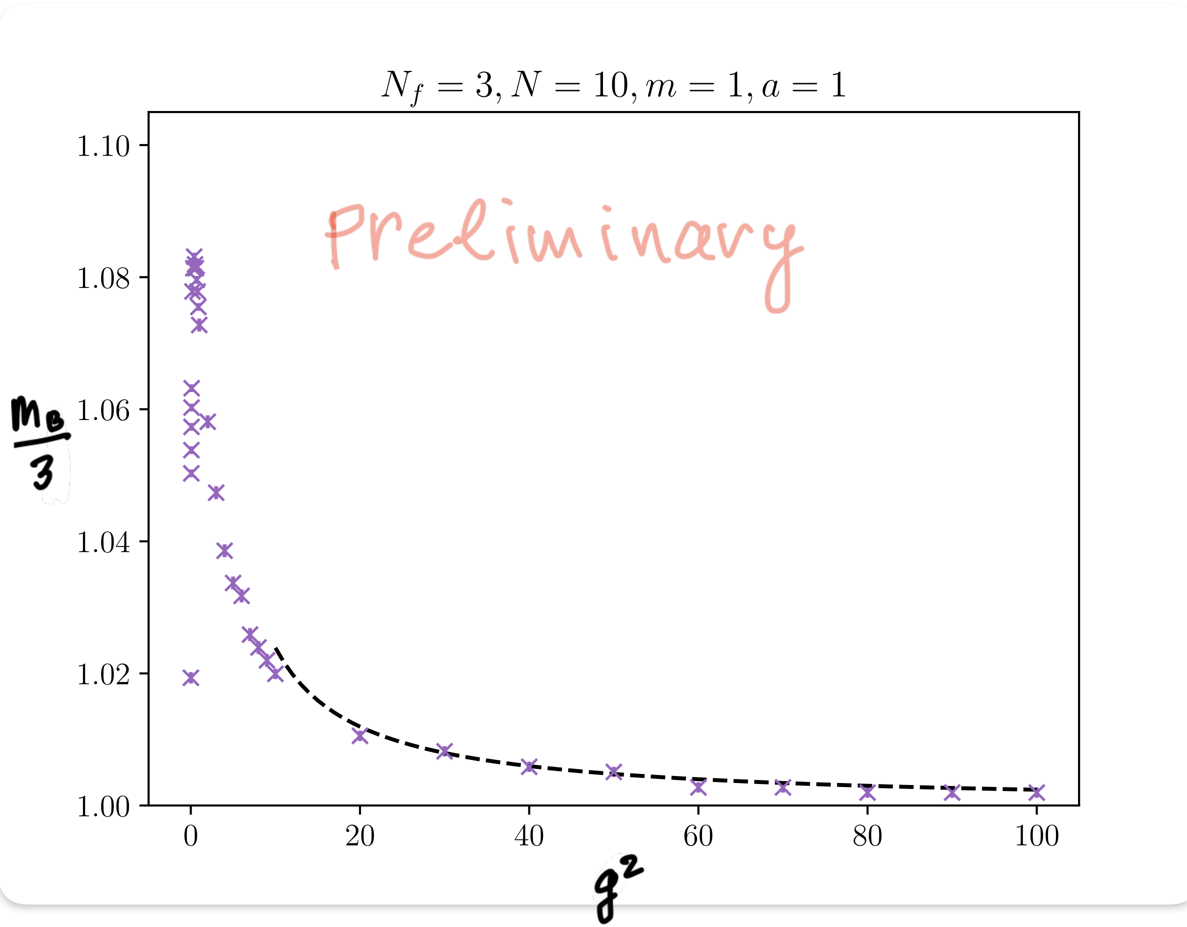
Silver blaze on the lattice



Baryon mass vs g^2

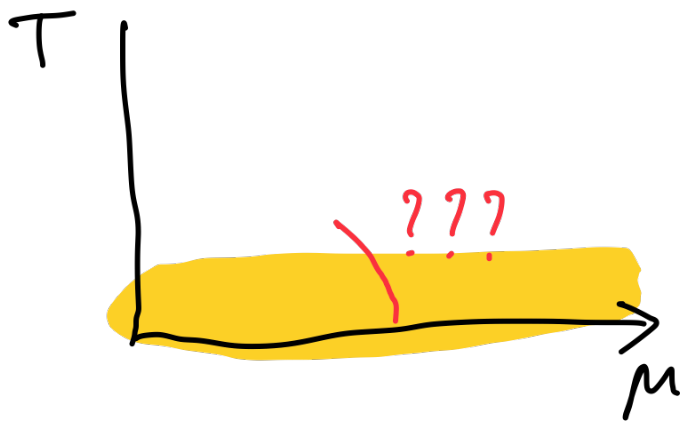


Baryon mass vs g^2



Plan of future work

1) Finish computation for the phase diagram



2) Compute mass spectrum with tensor networks

3) Study quantum algorithms