Factorization for Jet propagation in Heavy Ion collisions

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EFT/Factorization is ubiquitous for pp jets

\[ p + p \rightarrow \text{jet}(R, p_T) + X \]

\[ \text{jet } p_T \rightarrow \text{UV scale.} \rightarrow \text{jet } p_T \]

\[ p_T R \]

\[ p_T \theta_E \]

Hadron Mass \( \Lambda_{QCD} \rightarrow \text{IR scale} \rightarrow \Lambda_{QCD} \)

The semi-inclusive jet function in SCET and small radius resummation for inclusive jet production

Zhong-bo Kang, Felix Ringer and Ivan Vitev

\[ J_{c}(z_c, p_T, R, \mu) \]

\[ d\sigma^{pp\rightarrow \text{jet}X}_{dpTd\eta} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_{a_{\text{min}}}^{1}}^{1} \frac{dx_a}{x_a} f_a(x_a, \mu) \int_{x_{b_{\text{min}}}^{1}}^{1} \frac{dx_b}{x_b} f_b(x_b, \mu) \times \int_{z_{\text{cmin}}}^{1} \frac{dz_c}{z_c^2} \frac{d\hat{c}_{ab}}{d\hat{c}_{ab}}(\hat{s}, \hat{p}_T, \hat{\eta}) \times J_{c}(z_c, p_T, R, \mu) + O(R^2) + O\left(\frac{\Lambda_{QCD}^2}{(p_TR)^2}\right) \]

Perturbatively calculable jet function at \( p_T R \)

Obeys time-like DGLAP equation

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EFT/Factorization is ubiquitous for pp jets

- The EFT captures the leading contribution in the ratio of well separated scales (expansion parameters).
- The EFT automatically identifies all contributions consistent with a given expansion parameter.
- It helps to isolate and parametrize the universal non-perturbative physics in terms of matrix elements of gauge invariant operators (PDFs).
- It allows us to resum large logarithms in a systematically improvable manner.
- Systematically keep track of errors.

Goal of this talk:

- To reorganize jet-in-medium an an EFT calculation.
- Explicitly isolate the universal, observable and model independent physics of the medium in terms of a gauge invariant operator.
Identifying two distinct regimes

Define a Bjorken $x$ for the “QGP DIS”

$$x = \frac{(p_T R)^2}{T p_T} \sim \frac{Q^2}{M_p \sqrt{s}}$$

Two regimes

• $x \sim 1$ → The three scales are equally spaced → large $x$ physics

• $x \ll 1$ → $p_T R$ is much closer to $T$ than $p_T$ → small $x$ physics.

• The EFT is distinct for the two hierarchies and so are the dominant radiative corrections.

• Exploit the hierarchy of scales to make an expansion at the level of the Lagrangian, i.e before computing any diagrams.
The EFT at $p_{TR}$

- $x \sim 1 \rightarrow 200$ GeV jet with $p_{TR} \sim 20$ GeV $\rightarrow t_F \sim 0.1$ fermi $\ll L \rightarrow$ not-so-small $\theta_E \sim 0.1$
- $x \ll 1 \rightarrow 200$ GeV jet with $p_{TR} \sim 5$ GeV $\rightarrow t_F \sim 1.6$ fermi $\sim L \rightarrow$ small $\theta_E \sim 0.05$

- In the small $x$ regime, the interaction of the jet with the medium is completely coherent.
An EFT for Small $x$ physics
Soft Collinear Effective Theory (SCET)

Step 1: Identify the relevant degrees of freedom.

- The jet is made up of collinear partons
  \[ p_c \sim \frac{p_TR}{x} (1,x^2,x) \]
- QGP is a bath made of soft partons
  \[ p_s \sim \frac{p_TR}{x} (x,x,x) \]

Step 2: Write down an effective Lagrangian at leading power in x (expansion parameter)

- Interaction between d.o.f s is dominated by forward (small angle) scattering mediated by the Glauber mode.

\[ L_{QCD} = L_{collinear} + L_{soft} + L_{Glauber} + O(x^2) \]
We assume $q_B$ is initially unentangled from the partons that are involved in the hard interaction.

$$\rho(0) = |e^+\rangle\langle e^-| \otimes \rho_B$$

The QGP density matrix

$$\Sigma = \lim_{t \to \infty} \text{Tr} \left[ e^{-i \int dt H_{\text{eff}}(t)} \rho(0) e^{i \int dt H_{\text{eff}}(t)} \Theta_{\text{alg}} \right]$$

$H_{\text{eff}} = H_c + H_s + H_G$

$H_G(t) \equiv H_G \Theta(t_M - t)$  \hspace{1cm} Forward scattering is restricted for time $t_M$: Temporal extent of medium

Step 3: Prove factorization of soft physics from collinear order by order in the interactions.

$$\Sigma(t) = \text{Tr} \left[ \rho(t) M \right] = \Sigma^{(0)}(t) + \Sigma^{(1)}(t) + \Sigma^{(2)}(t) + \ldots$$

$O(H_G^0) \quad O(H_G^1) \quad O(H_G^2)$
Factorization for $O(n)$ interaction

Path Ordered in $y^-$

\[ \Sigma_R^{(2n)} = \frac{|C_G|^{2n}}{Q^4} \left[ \int d^+ p_e \mathcal{M} \right] I_{\mu\nu} \int d\bar{y}^+ \int d^2 \bar{y} \text{Im} \left\{ \prod_{i=1}^{n} \int d\bar{y}_i \Theta(\bar{y}_i - \bar{y}_{i+1}) \right\} \]

\[ \int \frac{d^2 k_{i\perp}}{(2\pi)^2} S(k_{i\perp}, \bar{y}_i^- \bar{y}_i^+, \bar{y}_i^+) J^{(n)}(k_1^\perp, k_2^\perp, \ldots, k_n^\perp, p_T, R) \]

$n$ copies of the Medium Structure (Soft)Function

Jet(collinear) function at order $n$

Process independent Universal medium physics

“A TMDPDF of the medium”

Valid for inhomogeneous medium
The single interaction jet function $J^{(1)}$

- The jet function contains corrections from both vacuum and medium induced radiation.
- The vacuum corrections lead to DGLAP evolution with a logarithm $\ln R$.

- The medium induced corrections lead to a BFKL rapidity logarithm $\ln x \sim \ln \frac{p_T R^2}{T}$.
- There is a finite correction which is interference between the hard interaction and medium evolution.

$$F(w) = \text{CosInt}[w] - \text{Sinc}[w]$$

$$w = \frac{L}{t_F}$$

All radiative corrections consistent with the expansion parameter are automatically captured!
The medium TMDPDF

\[
S^{AB}_{\text{med}}(k_\perp) = \frac{1}{k_\perp^2} \int \frac{dk^-}{2\pi} \int d^4x e^{-ik \cdot x} \text{Tr} \left[ O^A_S(x) O^B_S(0) \rho_{QGP} \right]
\]

Model independent and universal!

- SCET Operator version of color source density function \( \rho^A \) in the CGC

One loop corrections in the thermal medium using Real Time formalism

\[
\nu \frac{d}{d\nu} S(\vec{k}_\perp) = \frac{\alpha_s N_c}{\pi^2} \int d^2q_\perp \left( \frac{S(\vec{q}_\perp)}{(\vec{q}_\perp - \vec{k}_\perp)^2} - \frac{k_\perp^2 S(k_\perp)}{2q_\perp^2 (\vec{q}_\perp - \vec{k}_\perp)^2} \right)
\]

BFKL equation

\[
\mu \frac{d}{d\mu} S(\vec{k}_\perp) = -\frac{\alpha_s \beta_0}{\pi} \mu \sim k_\perp
\]

Running of the QCD coupling
Resumming logs in a single interaction

\[ \mu \sim p_T \]

\[ \mu \sim p_T R \]

\[ \nu \sim p_T R \]

\[ \nu \sim \frac{T}{R} \sim \frac{p_T R}{x} \]

DGLAP ln \( R \)

BFKL ln \( x \)

jet

Hard

Medium

\[ p^+ \sim p_T^2 \]

\[ p^- \sim (p_T R)^2 \]
The regime of multiple scatterings
A definition of the mean free path

\[ \Sigma(t) = Tr[\rho(t)M] = \Sigma^{(0)}(t) + \Sigma^{(1)}(t) + \Sigma^{(2)}(t) + \ldots \]

Leading log sum of the Glauber series to all orders.

\[ \Sigma_{med} = \int d^2b[\sigma(b)]_{\text{vac}} \int_{y \in \text{Med}} d^2y_\perp dy^+ \left[ 1 - P \exp \left\{ - \int \frac{dy^-}{\lambda_{\text{mfp}}(p_TR, b, y)} \right\} \right] + O(R^2) \]

Vacuum evolution of probe → DGLAP

Medium evolution of probe → BFKL

\[ \lambda_{\text{mfp}}^{-1}(R, p_T, y) = H_G(p_T, \mu) \int d^2k_\perp S_{\text{med}}(k_\perp, y, \mu, \nu) J_{\text{med}}(R, p_T, k_\perp) \]

Universal observable independent structure function

Medium jet function

V. Vaidya
An Effective Field Theory for jet substructure in Heavy Ion Collisions. JHEP 11, 064 (2021)
An emergent expansion parameter

\[
\Sigma = \int d^2b[\sigma(b)]_{\text{vac}} \int_{y \in \text{Med}} d^2y_\perp dy^+ \left[ 1 - \mathbf{P} \exp \left\{ -\frac{dy^-}{\lambda_{\text{mfp}}(Q, \vec{b}, y)} \right\} \right] \\
\lambda_1 \sim 1 \rightarrow \text{Multiple interactions need to be resummed}
\]

\[
\int_{\text{Nucleus}} \frac{dy^-}{\lambda_{\text{mfp}}(Q_s, 1/Q_s, y)} = 1
\]

For Heavy ion jet physics, this defines the \textbf{breakdown of opacity} expansion.

For small x DIS, this defines the \textbf{saturation} scale.
A non-perturbative medium

\[ \lambda^{-1}_{\text{mfp}}(R, N, p_T, y) = H_G(p_T, \mu) \int d^2k_\perp S_{\text{med}}(k_\perp, y, \mu, \nu) j_c^{\text{med}}(R, p_T, N, k_\perp) + O(R^2) \]

\( p_T \gg p_T R \gg T \)

• Jet function is perturbative and calculable.

• Can the EFT at scale \( p_T R \sim Q \) be matched onto the medium PDF at \( T \sim \Lambda_{QCD} \)?

• Medium PDF: Non-perturbative but observable independent!

• Only need to recompute the jet function for different jet substructure observables.

\[ \frac{T}{p_T R} \sim \frac{\Lambda_{QCD}}{Q} \]

Expansion parameter

\( R, x \)
Summary

• For a coherent (unresolved) interaction, radiative corrections in medium described by BFKL.
• Universality of a gauge invariant, model independent medium structure function can be derived rigorously.

Open Questions

• How to match and isolate the non-perturbative physics at the scale $\Lambda_{QCD}$.
• Formulate the EFT for $x \sim 1 \rightarrow$ Ongoing work!
• Extract the universal non-perturbative structure functions from data.
An EFT in a (slightly) boosted frame

- Work in a frame where the medium is boosted by a factor $\gamma = \frac{p_T R}{T}$,
- Allows us to conveniently use an EFT framework already developed in literature

I. Rothstein, I. Stewart, JHEP 1608 (2016) 025

- Energy of medium partons $\sim T \frac{p_T R}{T} = p_T R$
- Energy of jet partons $\sim p_T \frac{T}{p_T R} = \frac{p_T R}{x}$

$T, m_D \rightarrow$ IR scale $\sim 1$GeV
Renormalization for the medium jet function

\[
J_{\text{Med, BFKL}}^{AB(1)}(\vec{k}_\perp, \vec{q}_T n) = -\frac{\alpha_s N_C}{\pi^2} \int \frac{d^2 q_\perp}{(q_\perp - \vec{k}_\perp)^2} \left\{ J_{\text{Med}}^{AB}(\vec{q}_\perp, \vec{q}_T n) - \frac{1}{2} J_{\text{Med}}^{(AB)}(\vec{k}_\perp, \vec{q}_T n) k_\perp^2 \right\} \ln \frac{\nu}{Q z_c} - F \left[ \frac{(q_\perp^2 + m_D^2)t_M}{Q z_c} \right]
\]

\[
t_F = \frac{Q z_c}{q_\perp^2 + m_D^2}
\]

Formation time for radiation

\( t_F \gg t_f \) : Long Lived medium : \( F \to 0 : \nu \sim Q z_c \)

\( t_F \ll t_f \) : Short Lived medium: \( F(x) \sim \ln x : \nu \sim q_\perp^2 t_f \ll Q z_c \)
An EFT within SCET

Step 2: Write down an effective Lagrangian at leading power in $x$ for these d.o.fs

- Interaction between d.o.f.s is dominated by forward(small angle) scattering mediated by the Glauber mode.

$$L_{QCD} = L_{\text{collinear}} + L_{\text{soft}} + L_{\text{Glauber}} + O(x^2)$$

$$\equiv L_{\text{SCET}} + L_G$$

$$L_G \sim O_{cs}^{qq} = O_n^{q\alpha} \frac{1}{P^2} O_S^{q\alpha}$$

**An effective field theory for forward scattering and factorization violation**

I. Rothstein, I. Stewart, JHEP 1608 (2016) 025

Gauge invariant building blocks

$$O_S^{q\alpha} = \overline{s}_s^n T^{\alpha} \frac{n}{2} S^n_s \psi_s^n$$

$$O_n^{q\alpha} = \overline{\chi}_n W_{n} T^{\alpha} \frac{\bar{n}}{2} W^+_n \chi_n$$