Factorization for Jet propagation in Heavy Ion collisions

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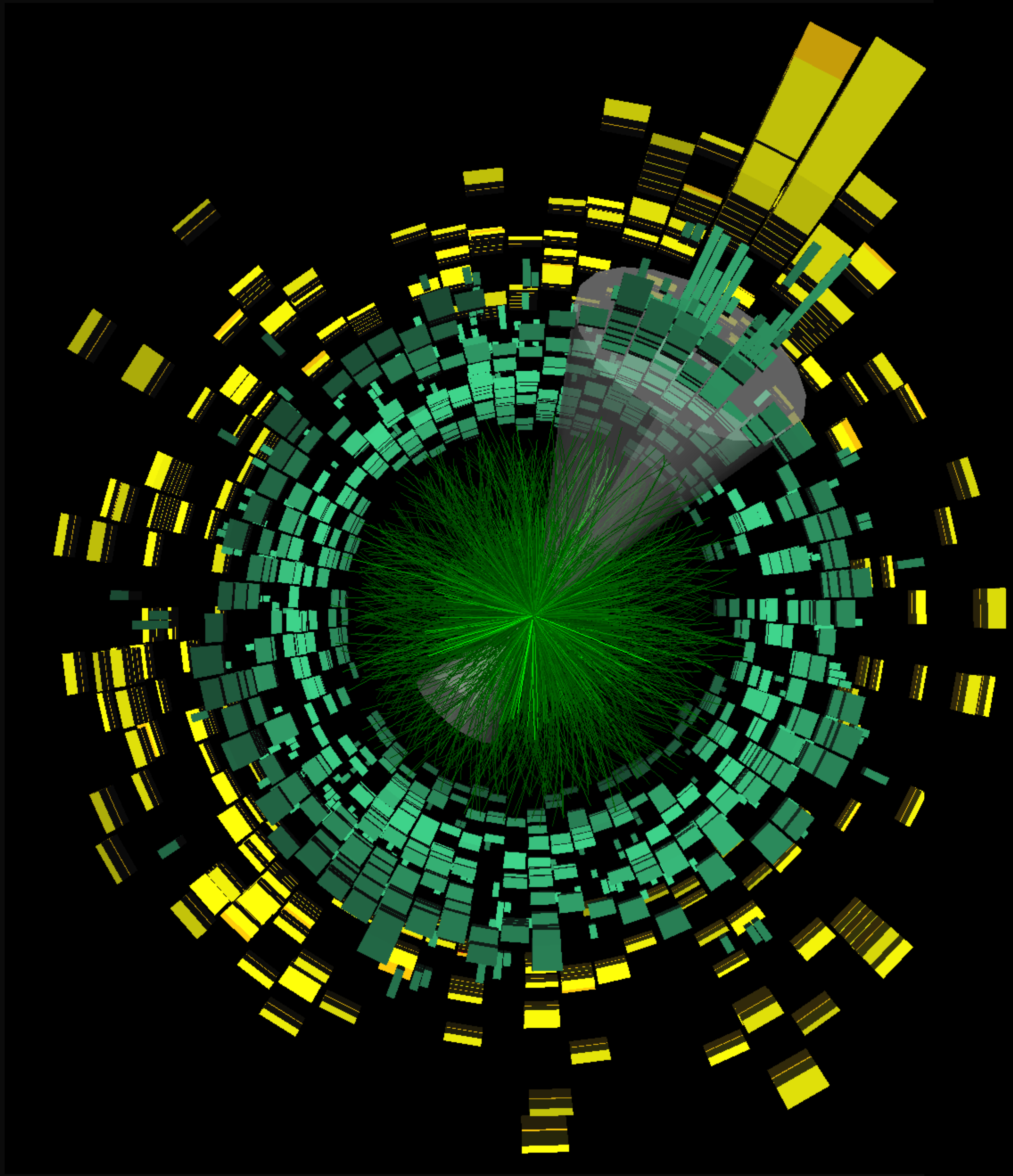
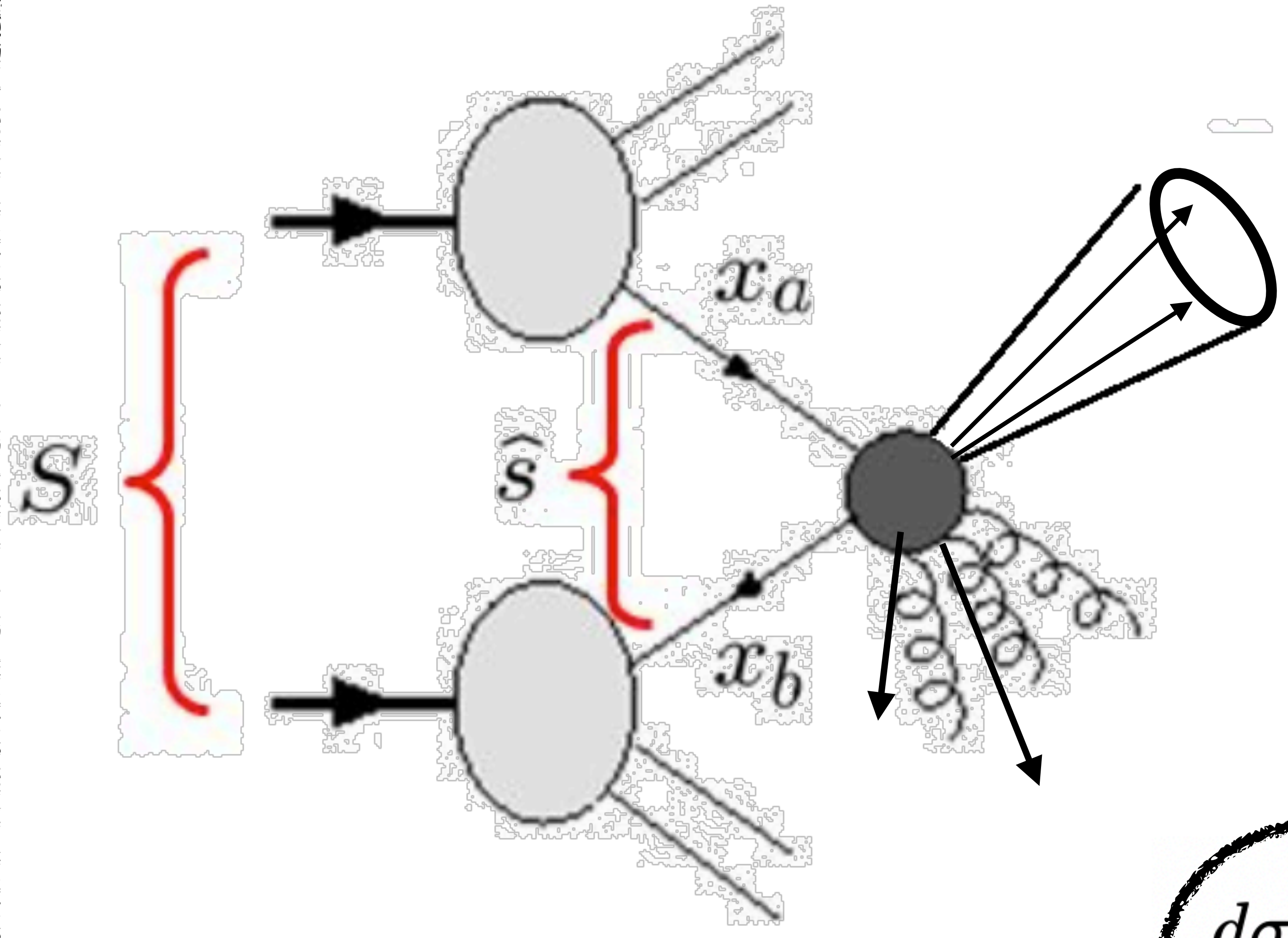


EFT/Factorization is ubiquitous for pp jets



$p + p \to jet(R, p_T) + X$

Energy Energy correlator

jet p_T → UV scale. → jet p_T

$p_T R$ $\quad$ $p_T \theta_E$

Hadron Mass Λ_{QCD} → IR scale → Λ_{QCD}

The semi-inclusive jet function in SCET and small radius resummation for inclusive jet production

Zhong-bo Kang, Felix Ringer and Ivan Vitev
JHEP 10 (2016) 125

$$\frac{d\sigma^{pp\to \text{jet}X}}{dp_T d\eta} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a,\mu) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b,\mu) \times \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s},\hat{p}_T,\hat{\eta},}{dvdz} J_c(z_c, p_T, R, \mu) + O(R^2) + O\left(\frac{\Lambda_{QCD}^2}{(p_T R)^2}\right)$$

Hard function at p_T

Perturbatively calculable jet function at $p_T R$

Obeys time-like DGLAP equation

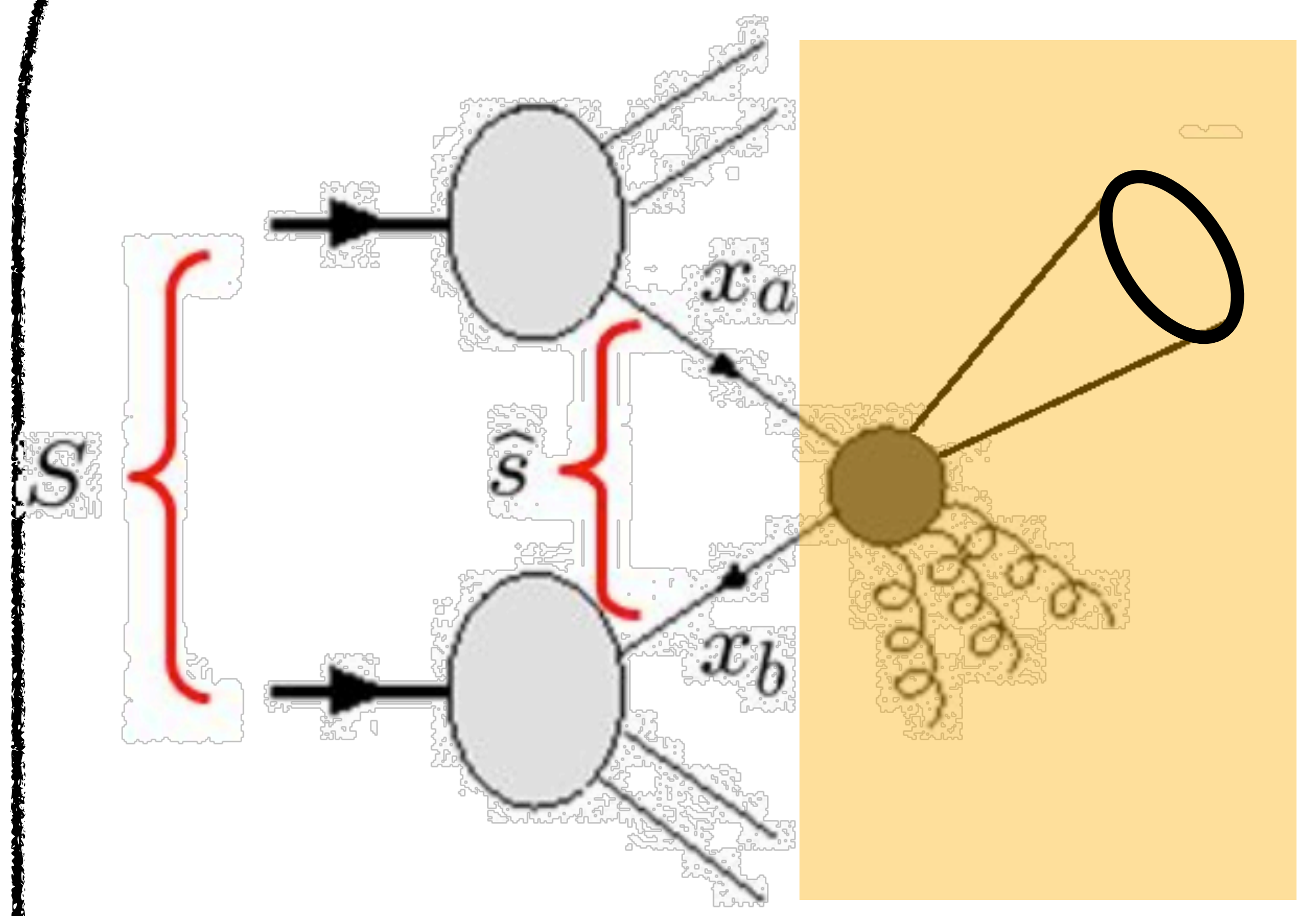
EFT/Factorization is ubiquitous for pp jets

- The EFT captures the leading contribution in the ratio of well separated scales(expansion parameters).
- The EFT automatically identifies all contributions consistent with a given expansion parameter.
- It helps to isolate and parametrize the **universal** non-perturbative physics in terms of matrix elements of gauge invariant operators (PDFs).
- It allows us to resum large logarithms in a systematically improvable manner.
- Systematically keep track of errors.

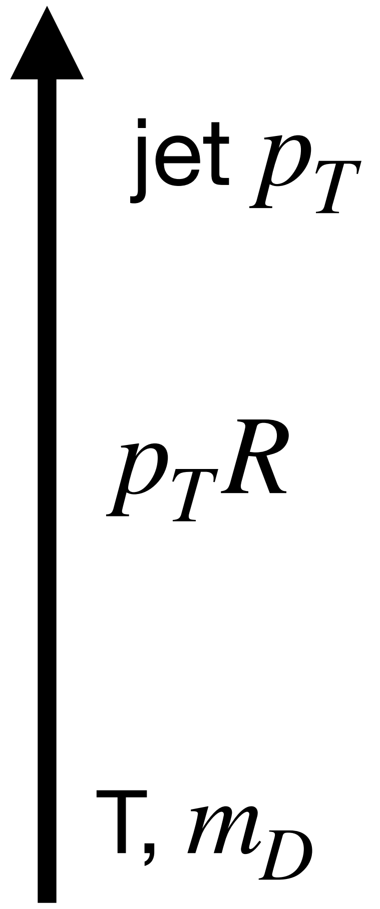
Goal of this talk:

- **To reorganize jet-in-medium an an EFT calculation.**
- **Explicitly isolate the universal, observable and model independent physics of the medium in terms of a gauge invariant operator**

Identifying two distinct regimes



QGP/Nucleus



Two regimes

2203.09407 P. Caucal, Y. Mehtar-Tani

- x ~ 1 → The three scales are equally spaced →large x physics
- x << 1 → $p_T R$ is much closer to T than p_T → small x physics.

Define a Bjorken x for the "QGP DIS"

$$x = \frac{(p_T R)^2}{T p_T} \sim \frac{Q^2}{M_p \sqrt{s}}$$

- The EFT is distinct for the two hierarchies and so are the dominant radiative corrections.
- Exploit the hierarchy of scales to make an **expansion at the level of the Lagrangian,** i.e before computing any diagrams.

The EFT at $p_T R$

jet p_T

$p_T R$

T, $m_D \rightarrow$ IR scale ~ 1GeV

1/L → 1/ Medium size → Size of the Nucleus ~ few fermi

Coherence time of the probe

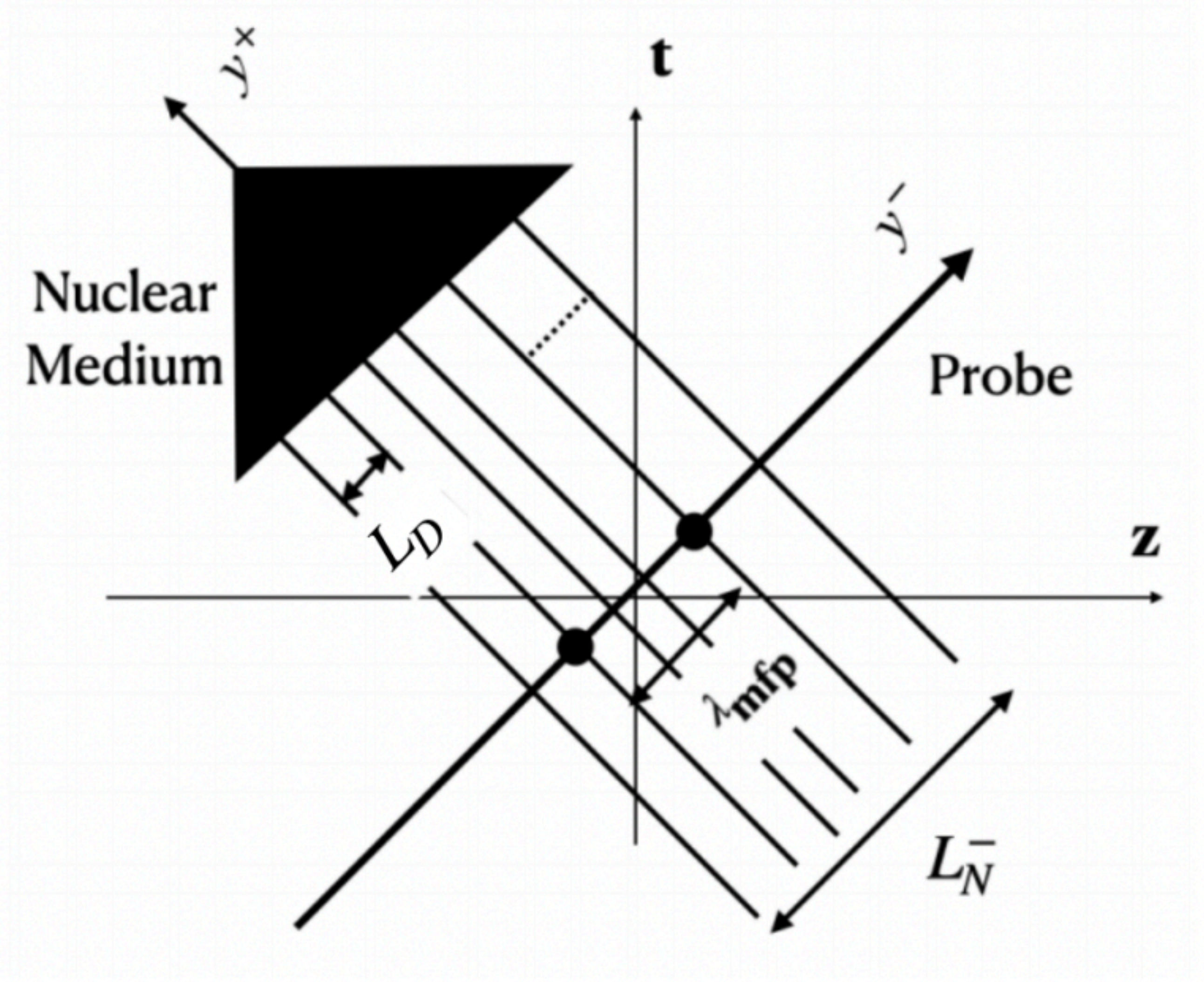
$$t_F \sim \frac{E}{q_T^2} \sim \frac{1}{p_T R^2}$$

Emergent scales

$\lambda_{\text{mfp}}, \ \hat{q}$

Color screening length in the medium

L_D



- x ~ 1 → 200 GeV jet with $p_T R \sim$ 20 GeV → $t_F \sim 0.1$ fermi $\ll L$ →not-so-small $\theta_E \sim 0.1$
- x << 1 →200 GeV jet with $p_T R \sim$ 5 GeV → $t_F \sim 1.6$ fermi $\sim L \rightarrow$ small $\theta_E \sim 0.05$

  - In the small x regime, the interaction of the jet with the medium is completely coherent.

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An EFT for Small x physics

Soft Collinear Effective Theory(SCET)

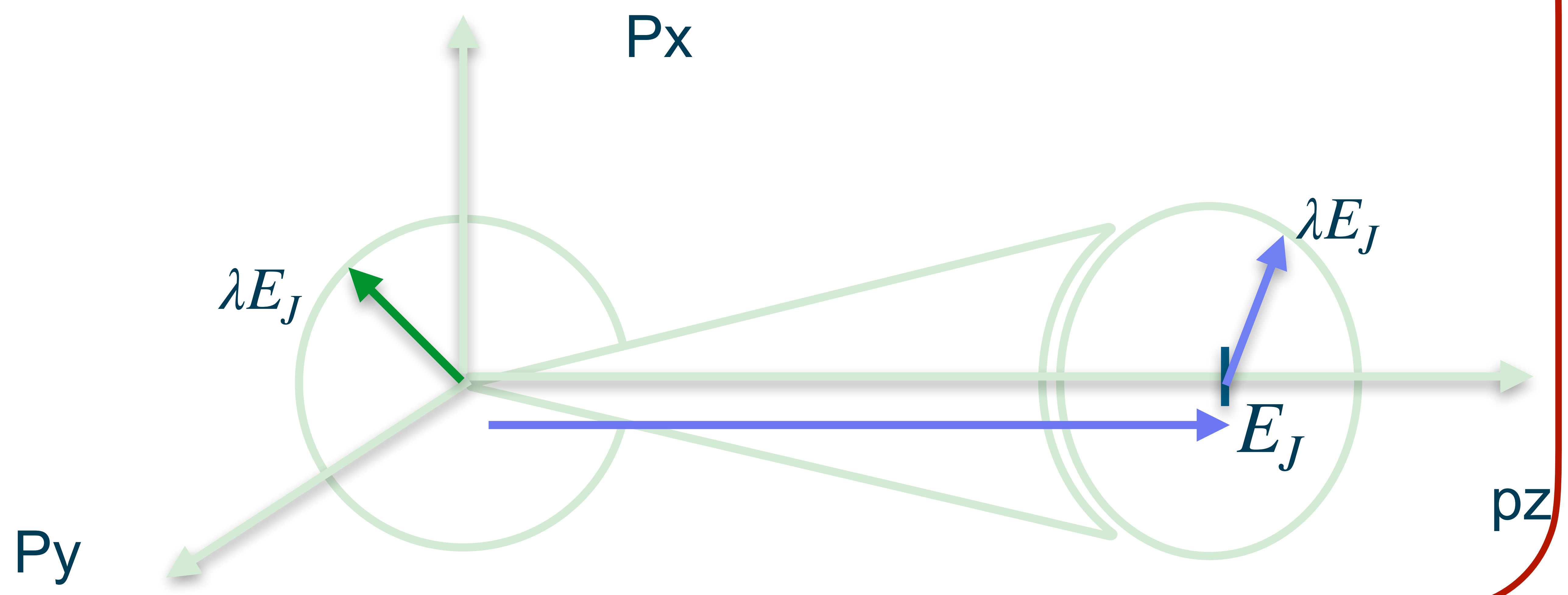
Step 1 : Identify the relevant degrees of freedom.

- The jet is made up of collinear partons

$$p_c \sim \frac{p_T R}{x}\left(1, x^2, x\right)$$

- QGP is a bath made of soft partons

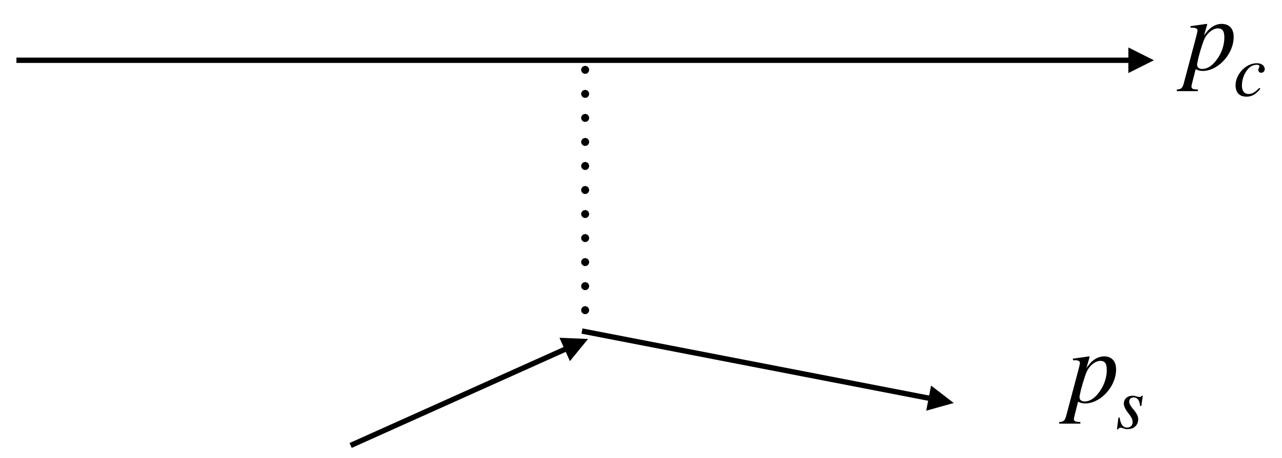
$$p_s \sim \frac{p_T R}{x}(x, x, x)$$



Step 2: Write down an effective Lagrangian at leading power in x(expansion parameter)

- Interaction between d.o.f s is dominated by forward(small angle) scattering mediated by the Glauber mode.

$$L_{QCD} = L_{collinear} + L_{soft} + L_{Glauber} + O(x^2)$$



Tree level Glauber exchange

The probe as an open quantum system

Step 3: Evolve the system with this effective action.

QGP density matrix

$$\rho(0) = |e^+\rangle\langle e^-| \otimes \rho_B$$

We assume ρ_B is initially unentangled from the partons that are involved in the hard interaction.

$$\Sigma = \lim_{t\to\infty} \mathrm{Tr}\left[e^{-i\int dt H_{eff}(t)}\rho(0)e^{i\int dt H_{eff}(t)}\Theta_{\mathrm{alg}}\right]$$

$$H_{\mathrm{eff}} = H_c + H_s + H_G$$

$H_G(t) \equiv H_G\Theta(t_M - t)$ Forward scattering is restricted for time t_M : Temporal extent of medium

Step 3: Prove factorization of soft physics from collinear order by order in the interactions.

$$\Sigma(t) = Tr[\rho(t)M] = \Sigma^{(0)}(t) + \Sigma^{(1)}(t) + \Sigma^{(2)}(t) + \ldots$$

$$O(H_G^0) \qquad O(H_G^1) \qquad O(H_G^2)$$

Factorization for O(n) interaction

Hard function

Path Ordered in y^-

$$\Sigma_R^{(2n)} = \frac{|C_G|^{2n}}{Q^4}\left[\int d^+p_e\mathcal{M}\right] I_{\mu\nu}\int d\bar{y}^+\int d^2\bar{y}_\perp \mathrm{Im}\Bigg\{\Bigg[\Pi_{i=1}^n\int d\bar{y}_i^-\Theta(\bar{y}_i^- - \bar{y}_{i+1}^-)$$

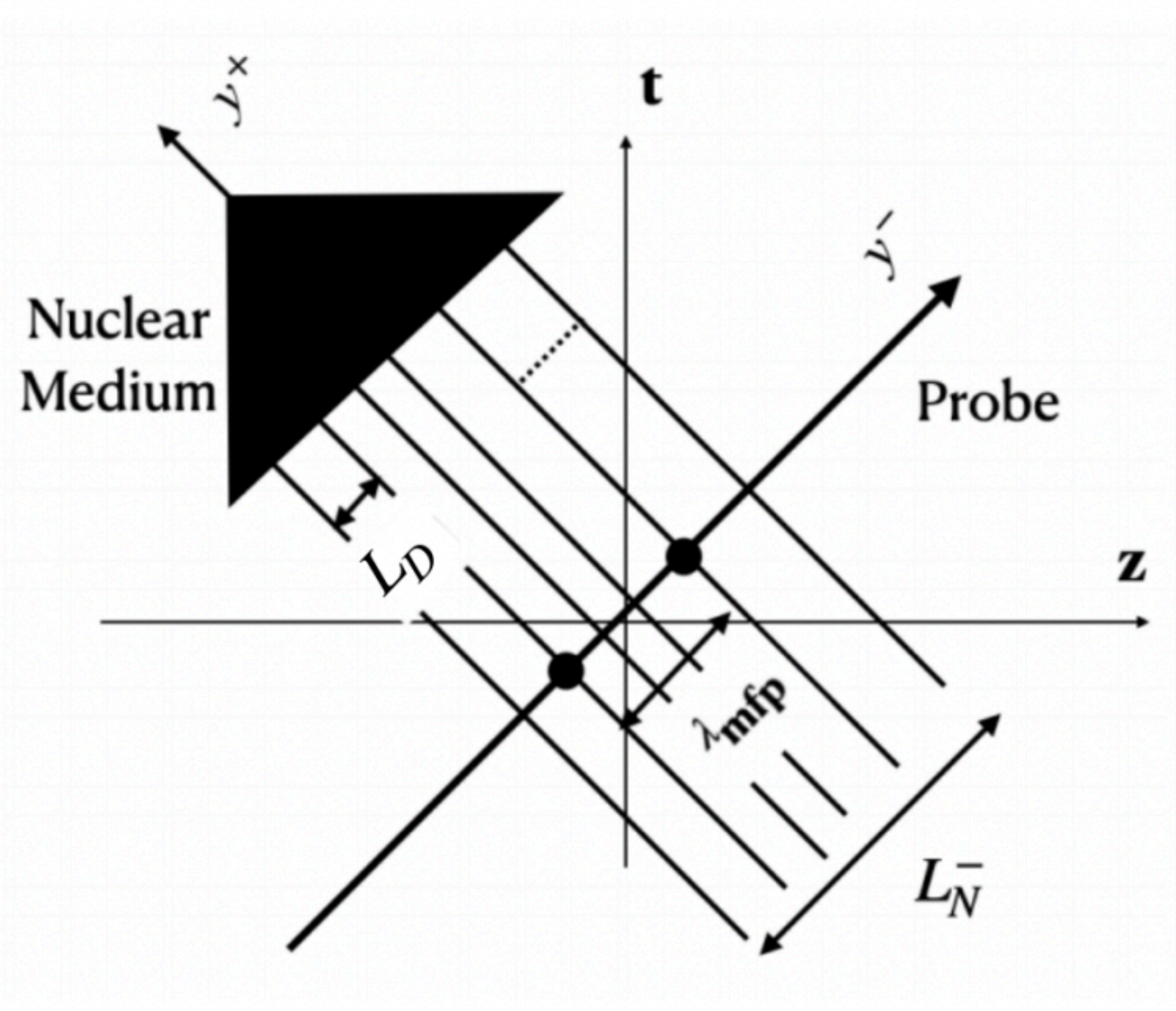
$$\int\frac{d^2k_{i\perp}}{(2\pi)^2} S\left(k_{i,\perp},\bar{y}_i^-,\bar{y}^+,\bar{y}^\perp\right)\Bigg] J^{(n)}(k_1^\perp,k_2^\perp,\ldots,k_n^\perp,p_T,R)\Bigg\}$$

Jet(collinear) function at order n

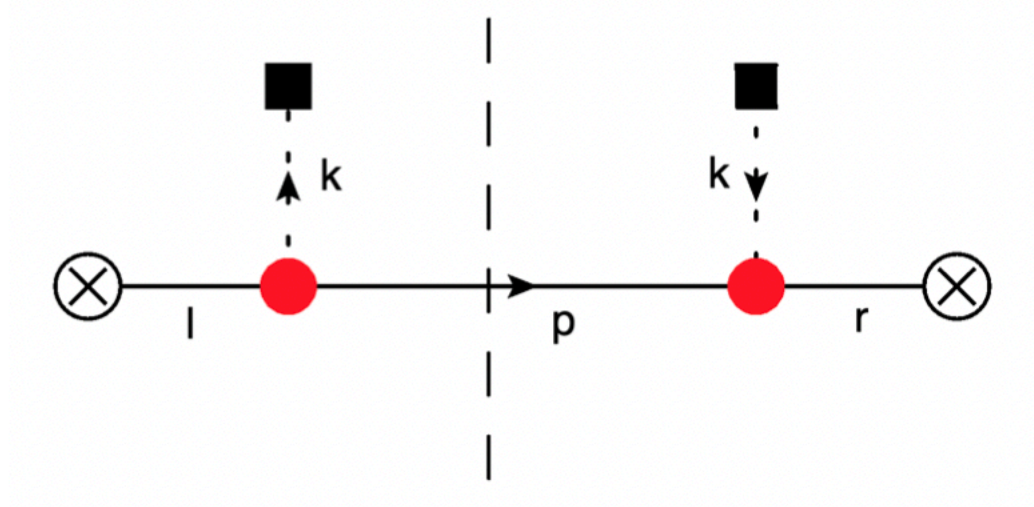
n copies of the Medium Structure (Soft)Function

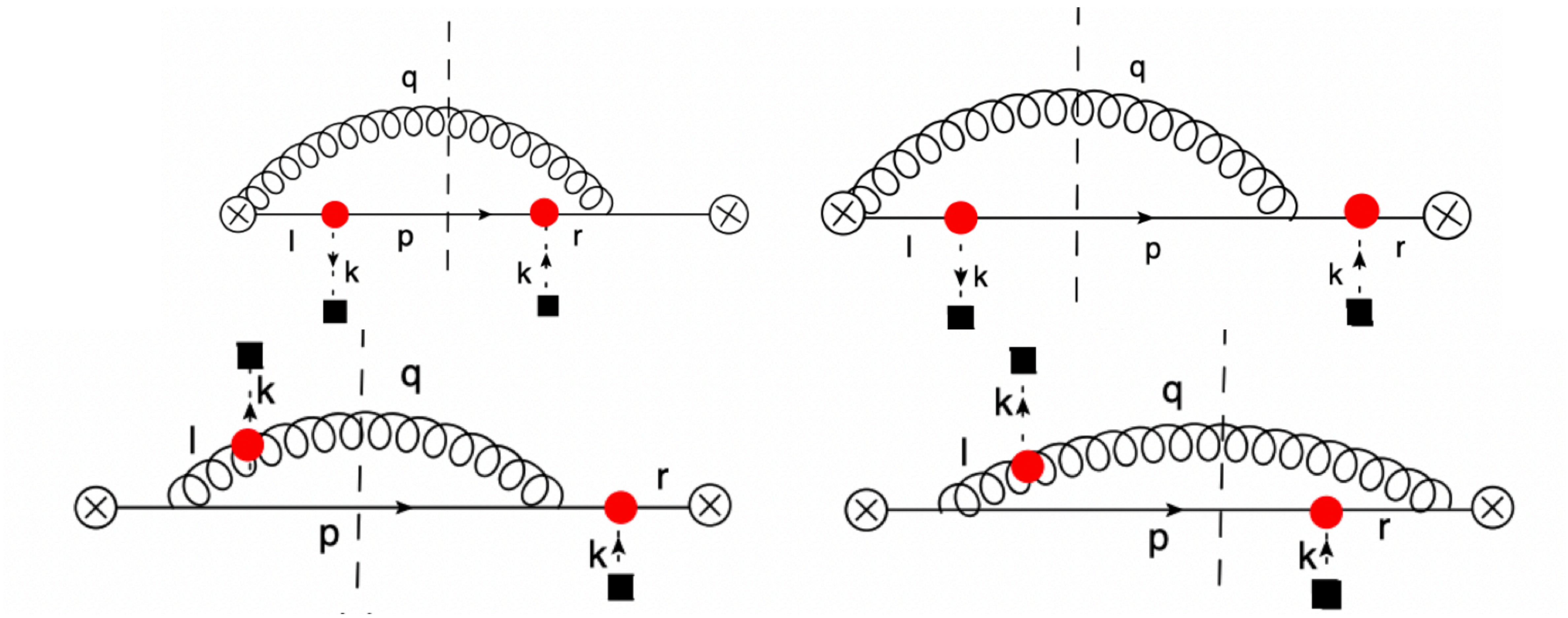
Process independent Universal medium physics
"A TMDPDF of the medium"

Valid for inhomogeneous medium



The single interaction jet function $J^{(1)}$





V.Vaidya 2109.11568

- The jet function contains corrections from both vacuum and medium induced radiation.
- The vacuum corrections lead to DGLAP evolution with a logarithm $\ln R$.
- The medium induced corrections lead to a BFKL rapidity logarithm $\ln x \sim \ln \frac{p_T R^2}{T}$.
- There is a finite correction which is interference between the hard interaction and medium evolution.

$$F(w) = \text{CosInt}[w] - \text{Sinc}[w] \qquad w = \frac{L}{t_F}$$

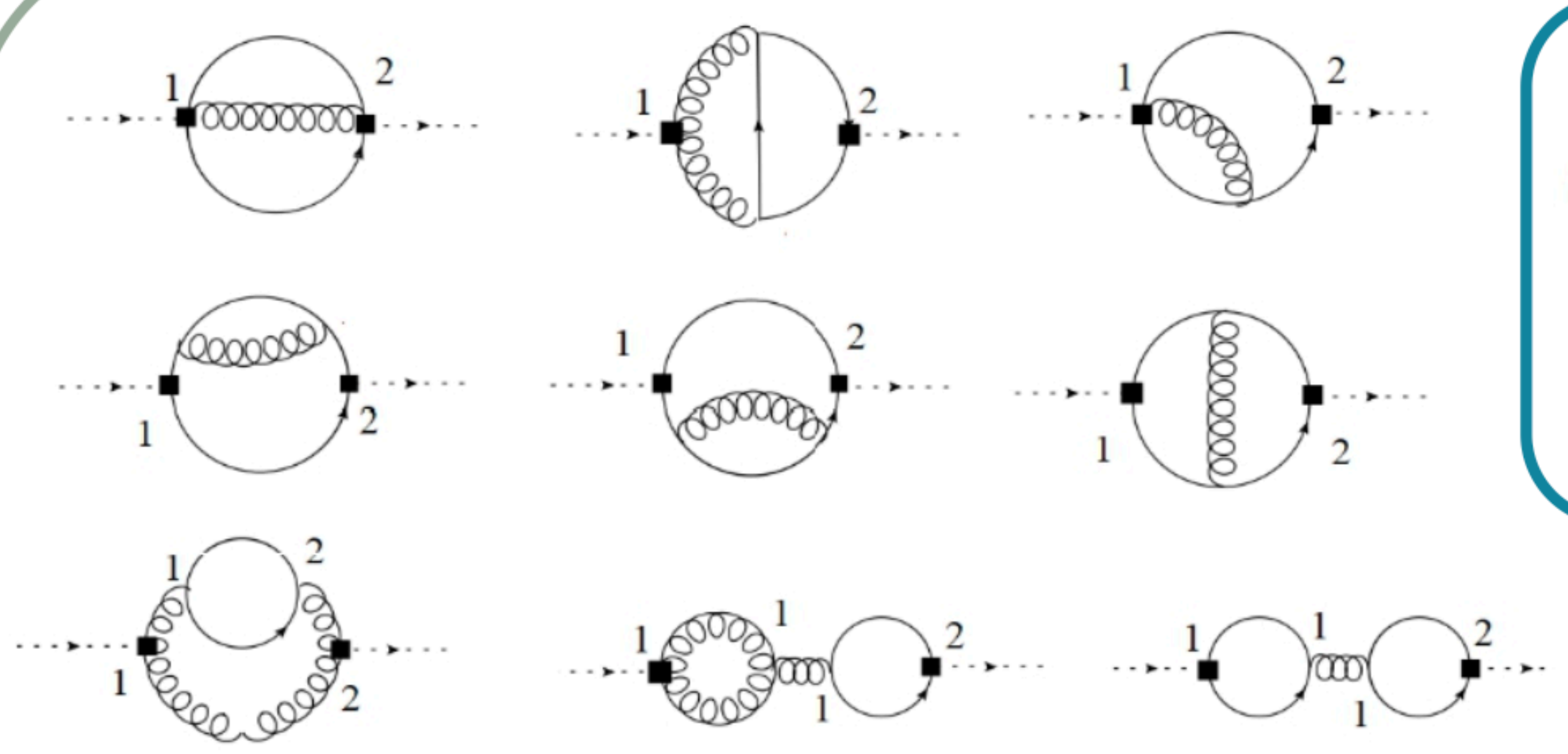
All radiative corrections consistent with the expansion parameter are automatically captured!

The medium TMDPDF

$$S^{AB}_{\rm med}(k_\perp) = \frac{1}{k_\perp^2}\int \frac{dk^-}{2\pi}\int d^4x e^{-ik\cdot x}\mathrm{Tr}\left[O_S^A(x)O_S^B(0)\rho_{QGP}\right]$$

Model independent and universal !

- SCET Operator version of color source density function ρ^A in the CGC



One loop corrections in the thermal medium using Real Time formalism

$$\nu\frac{d}{d\nu}S(\vec{k}_\perp) = \frac{\alpha_s N_c}{\pi^2}\int d^2q_\perp\left(\frac{S(\vec{q}_\perp)}{(\vec{q}_\perp-\vec{k}_\perp)^2} - \frac{k_\perp^2 S(k_\perp)}{2q_\perp^2(\vec{q}_\perp-\vec{k}_\perp)^2}\right)$$

BFKL equation

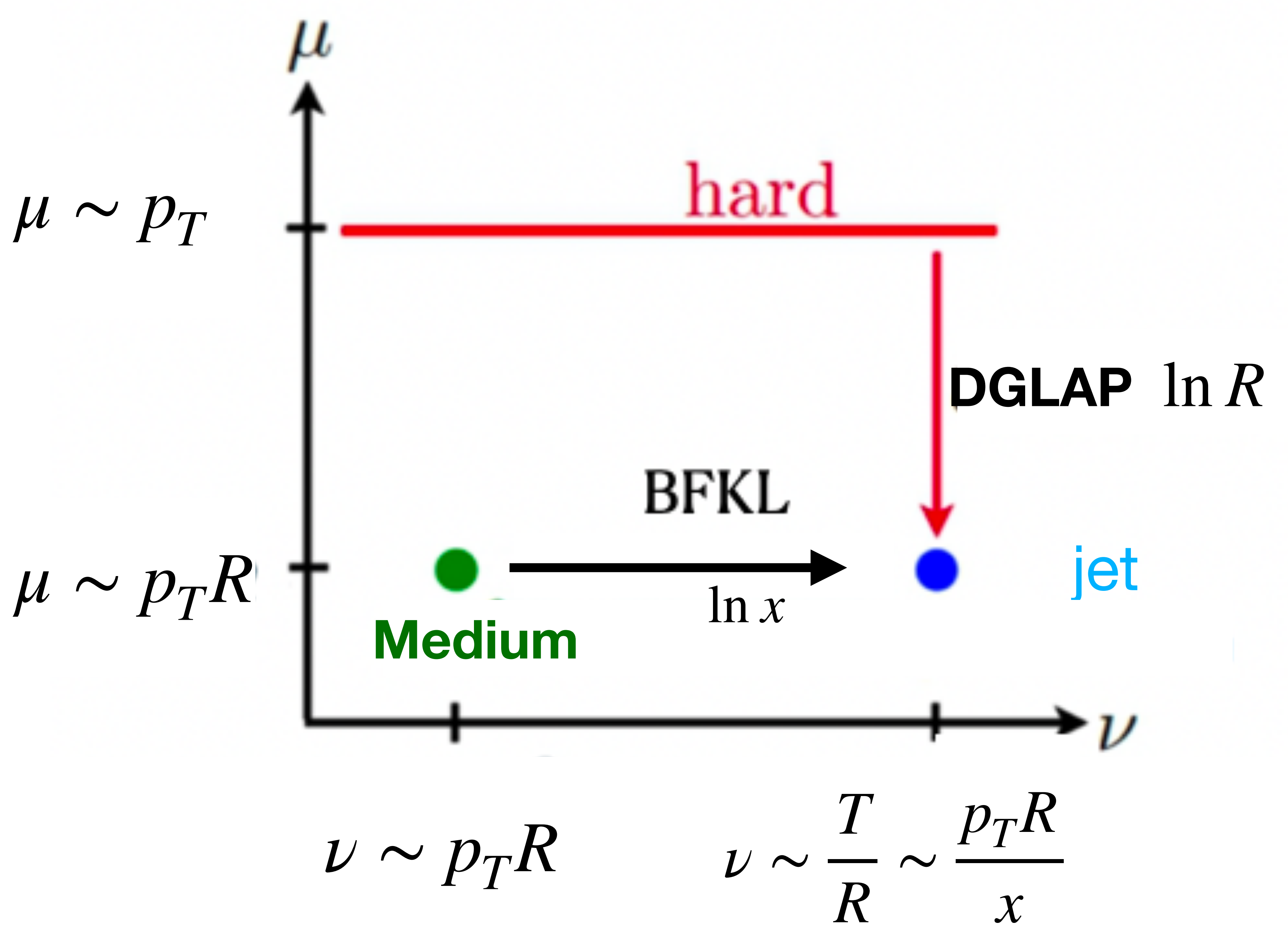
V.Vaidya 2107.00029

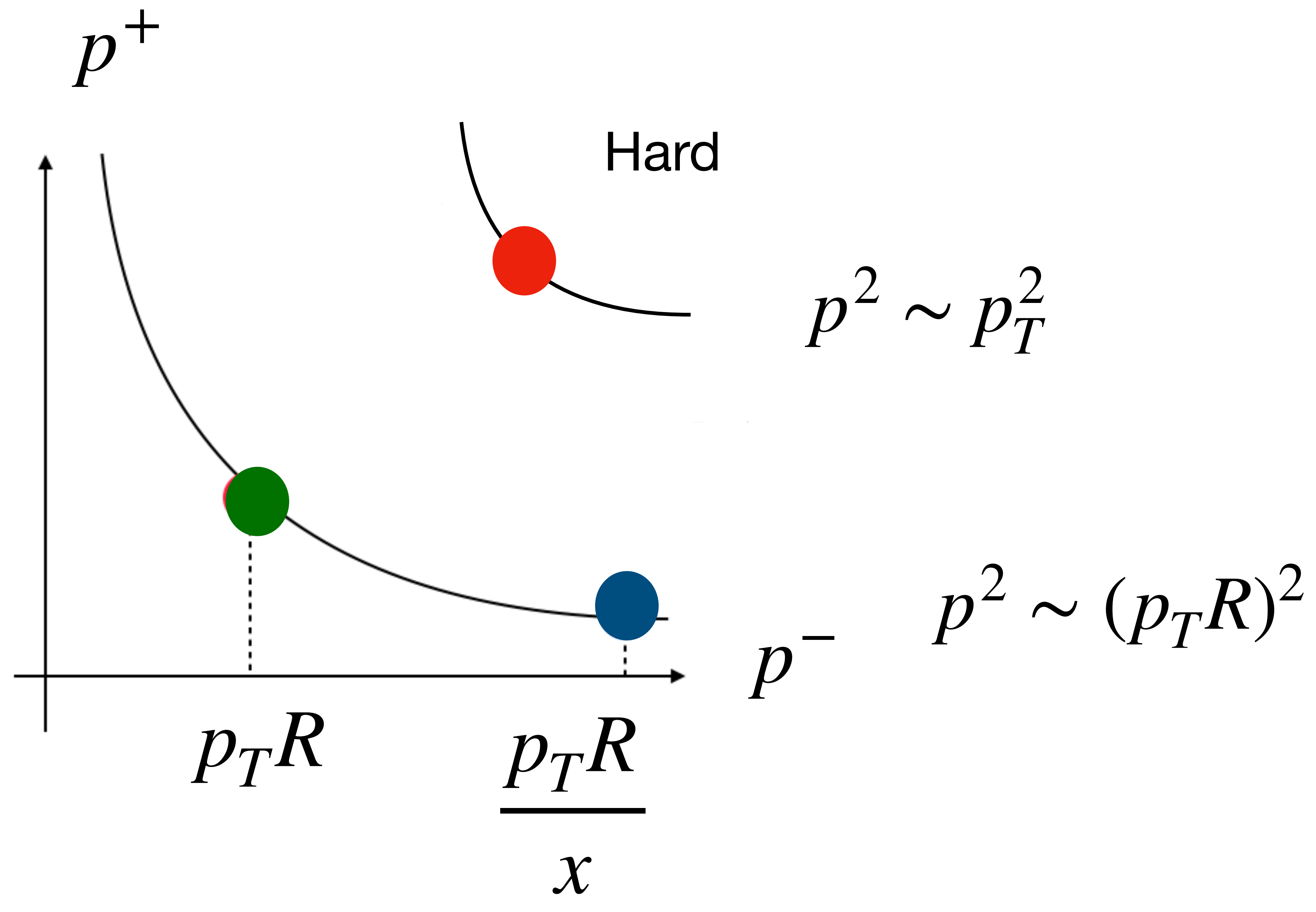
$$\mu\frac{d}{d\mu}S(\vec{k}_\perp) = -\frac{\alpha_s\beta_0}{\pi}$$

Running of the QCD coupling

$$\mu \sim k_\perp$$

Resumming logs in a single interaction





The regime of multiple scatterings

A definition of the mean free path

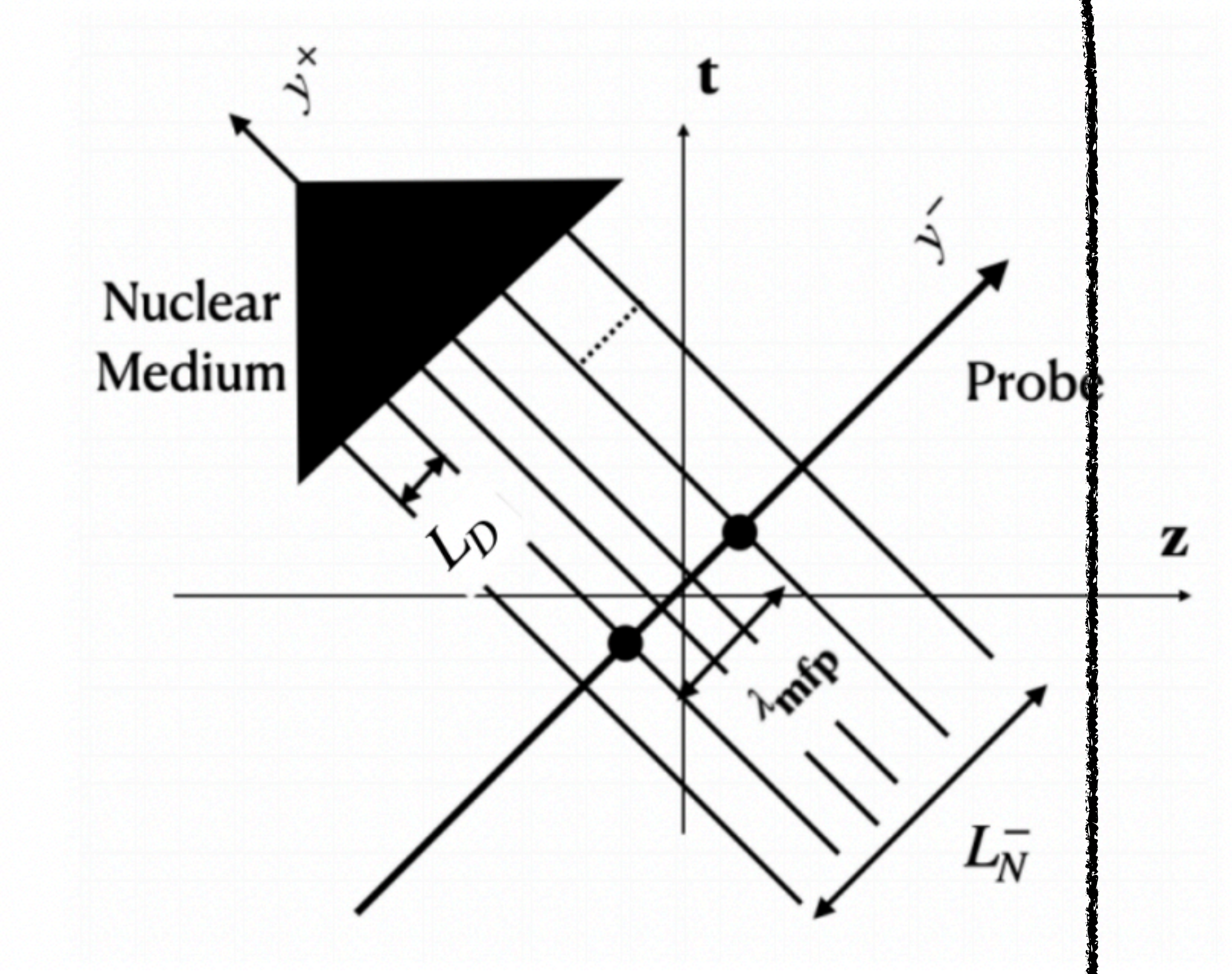
$$\Sigma(t) = Tr[\rho(t)M] = \Sigma^{(0)}(t) + \Sigma^{(1)}(t) + \Sigma^{(2)}(t) + \dots$$

Leading log sum of the Glauber series to all orders.

$$\Sigma_{\rm med} = \int d^2b[\sigma(b)]_{\rm vac}\int_{y\in{\rm Med}} d^2y_\perp dy^+\left[1 - \mathbf{P}\exp\left\{-\int\frac{dy^-}{\lambda_{\rm mfp}(p_TR, \vec{b}, y)}\right\}\right] + O(R^2)$$

Vacuum evolution of probe → DGLAP

Medium evolution of probe →BFKL



$$\lambda^{-1}_{\rm mfp}(R, \quad , p_T, y) = H_G(p_T, \mu)\int d^2k_\perp S_{\rm med}(k_\perp, y, \mu, \nu) J_c^{\rm med}(R, p_T, \quad k_\perp)$$

Universal observable independent structure function

Medium jet function

V. Vaidya
An Effective Field Theory for jet substructure in Heavy Ion Collisions. JHEP 11, 064 (2021)

An emergent expansion parameter

$$\Sigma = \int d^2b[\sigma(b)]_{\text{vac}} \int_{y\in\text{Med}} d^2y_\perp dy^+ \left[1 - \mathbf{P}\exp\left\{ -\int \frac{dy^-}{\lambda_{\text{mfp}}(Q, \vec{b}, y)} \right\}\right]$$

λ_1

$\lambda_1 \sim 1 \rightarrow$ Multiple interactions need to be resummed

$$\int^{\text{Nucleus}} \frac{dy^-}{\lambda_{\text{mfp}}(Q_s, 1/Q_s, y)} = 1$$

For Heavy ion jet physics, this defines the **breakdown of opacity** expansion.

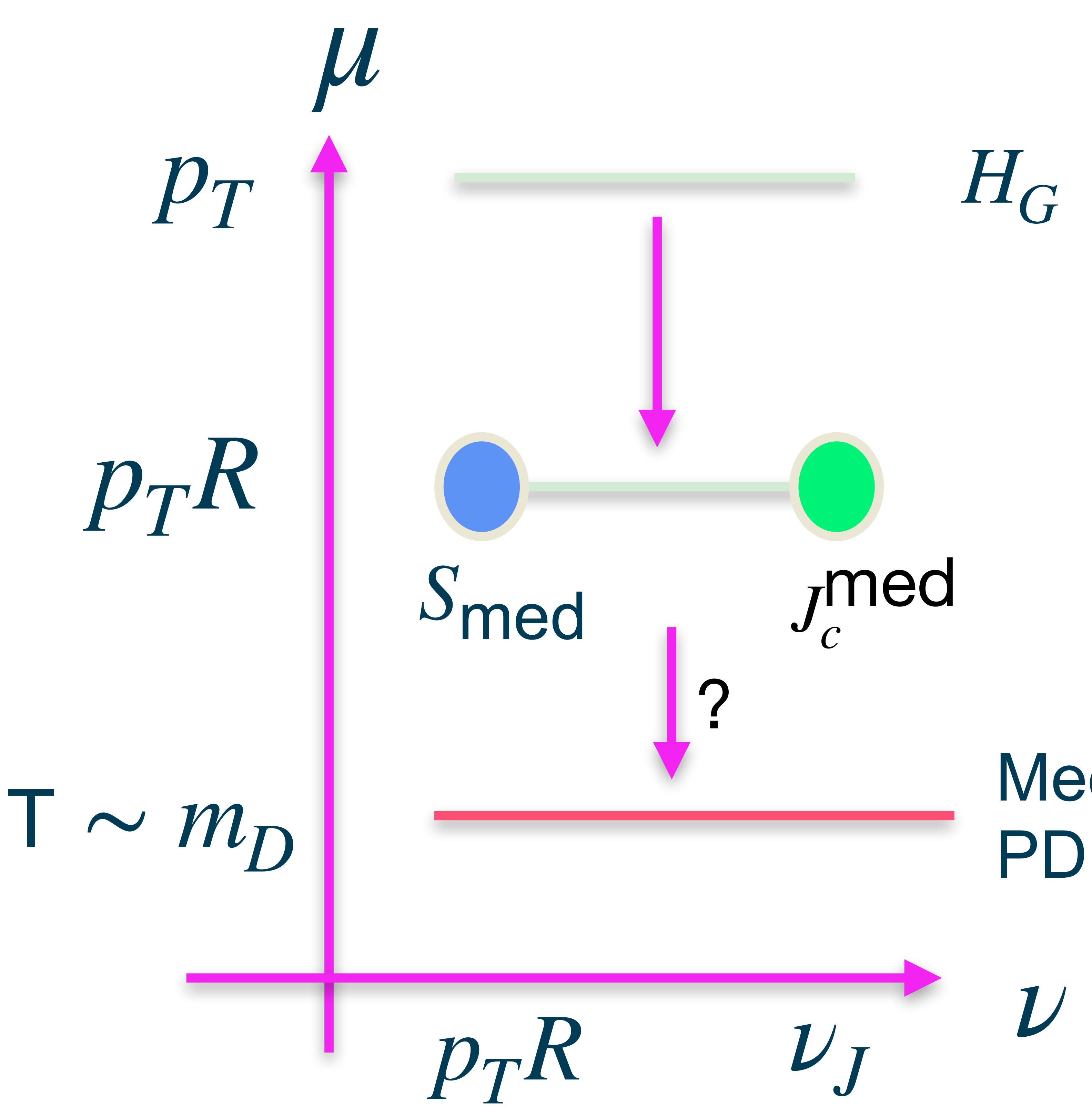
For small x DIS, this defines the **saturation** scale.

A non-perturbative medium

$$\lambda^{-1}_{\text{mfp}}(R,N,p_T,y) = H_G(p_T,\mu)\int d^2k_\perp S_{\text{med}}(k_\perp,y,\mu,\nu)J_c^{\text{med}}(R,p_T,N,k_\perp) + O(R^2)$$

$p_T \gg p_T R \gg T$

- Jet function is perturbative and calculable.
- Can the EFT at scale $p_T R \sim Q$ be matched onto the medium PDF at T $\sim \Lambda_{QCD}$?
- Medium PDF: Non perturbative but observable independent!
- Only need to recompute the jet function for different jet substructure observables.



Expansion parameter

R , x

$$\frac{T}{p_T R} \sim \frac{\Lambda_{QCD}}{Q}$$

Summary

- For a coherent(unresolved) interaction, radiative corrections in medium described by BFKL.
- Universality of a gauge invariant , model independent medium structure function can be derived rigorously.

Open Questions

- How to match and isolate the non-perturbative physics at the scale Λ_{QCD}.
- Formulate the EFT for x $\sim$1 $\rightarrow$Ongoing work !
- Extract the universal non-perturbative structure functions from data.

An EFT in a (slightly) boosted frame

- Work in a frame where the medium is boosted by a factor $\gamma = \frac{p_T R}{T}$,
- Allows us to conveniently use an EFT framework already developed in literature

I. Rothstein, I. Stewart, JHEP 1608 (2016) 025

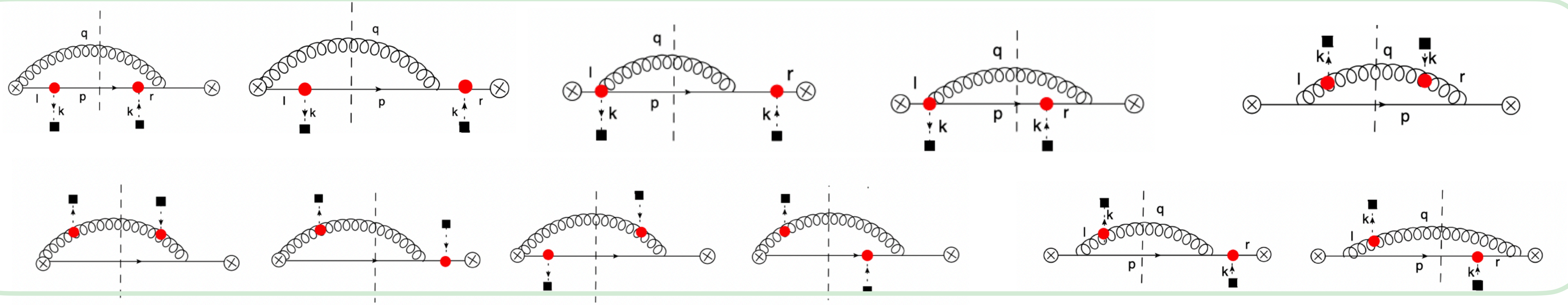
- Energy of medium partons $\sim T\frac{P_T R}{T} = p_T R$
- Energy of jet partons $\sim p_T \frac{T}{p_T R} = \frac{p_T R}{x}$

jet p_T

$p_T R$

T, $m_D \rightarrow$ IR scale ~ 1GeV

Renormalization for the medium jet function



$$J^{AB(1)}_{\text{Med, BFKL}}(\vec{k}_\perp, \vec{q}_{Tn}) = -\frac{\alpha_s N_C}{\pi^2}\int \frac{d^2 q_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2}\left\{J^{AB}_{\text{Med}}(\vec{q}_\perp, \vec{q}_{Tn}) - \frac{1}{2}\frac{J^{(AB)}_{\text{Med}}(\vec{k}_\perp, \vec{q}_{Tn})k_\perp^2}{q_\perp^2}\right\}\left\{\ln\frac{\nu}{Qz_c} - F\left[\frac{(\vec{q}_\perp^{\,2} + m_D^2)t_M}{Qz_c}\right]\right\}$$

$$t_F = \frac{Qz_c}{q_\perp^2 + m_D^2}$$ → Formation time for radiation

$t_M \gg t_F$: Long Lived medium : F → 0 : $\nu_J \sim Qz_c$

$t_M \ll t_F$: Short Lived medium: F(x) → ln x : $\nu_J \sim q_\perp^2 t_M \ll Qz_c$

An EFT within SCET

Step 2: Write down an effective Lagrangian at leading power in x for these d.o.fs

- Interaction between d.o.f s is dominated by forward(small angle) scattering mediated by the Glauber mode.

An effective field theory for forward scattering and factorization violation

I. Rothstein, I. Stewart, JHEP 1608 (2016) 025

$$L_{QCD} = L_{collinear} + L_{soft} + L_{Glauber} + O(x^2)$$

$$\equiv L_{SCET} + L_G$$

$$L_G \sim O_{cs}^{qq} = O_n^{q\alpha} \frac{1}{P_\perp^2} O_S^{q\alpha}$$

$$O_S^{q\alpha} = \bar{\psi}_s S_n T^\alpha \frac{n}{2} S_n^+ \psi_s^n$$

$$O_n^{q\alpha} = \bar{\chi}_n W_n T^\alpha \frac{\bar{n}}{2} W_n^+ \chi_n$$

Gauge invariant building blocks