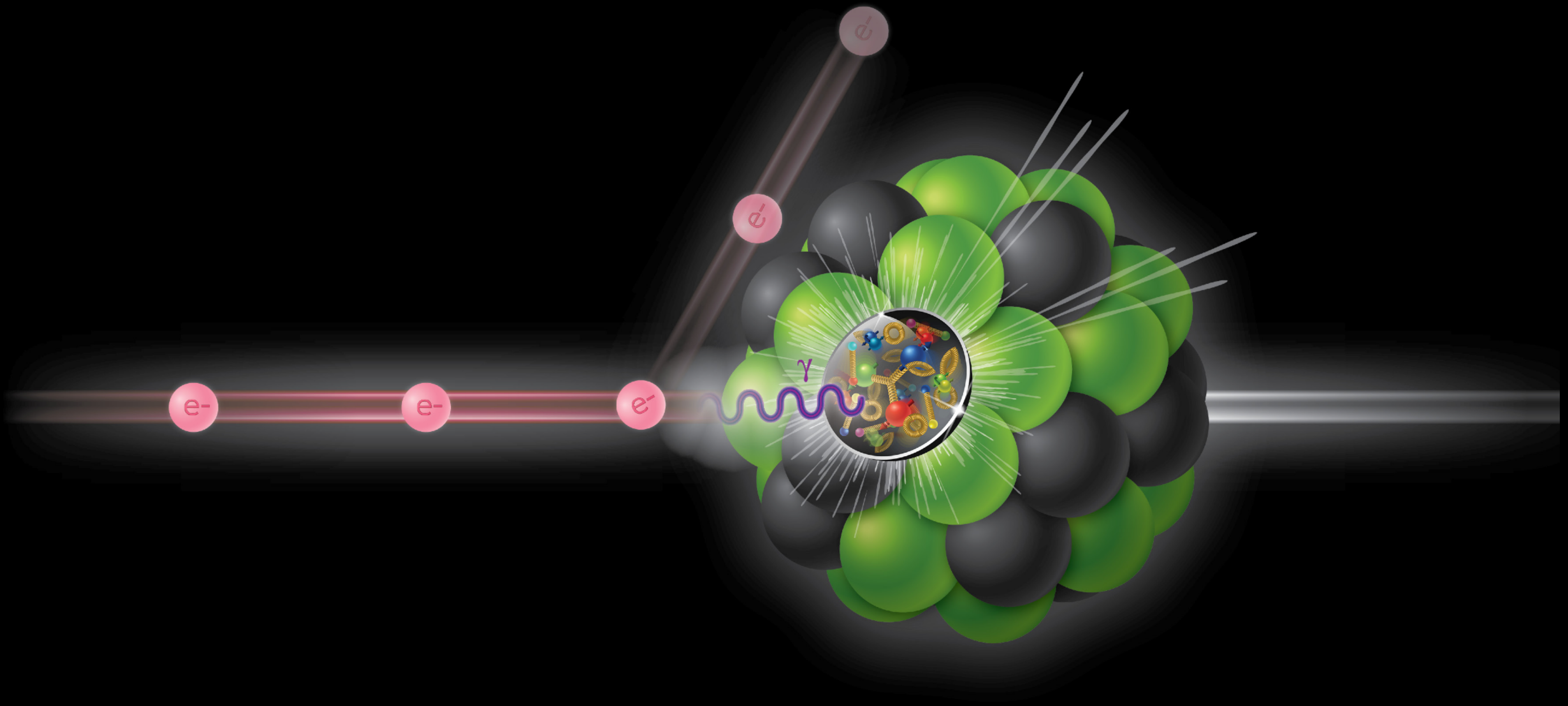

Power Counting to small x

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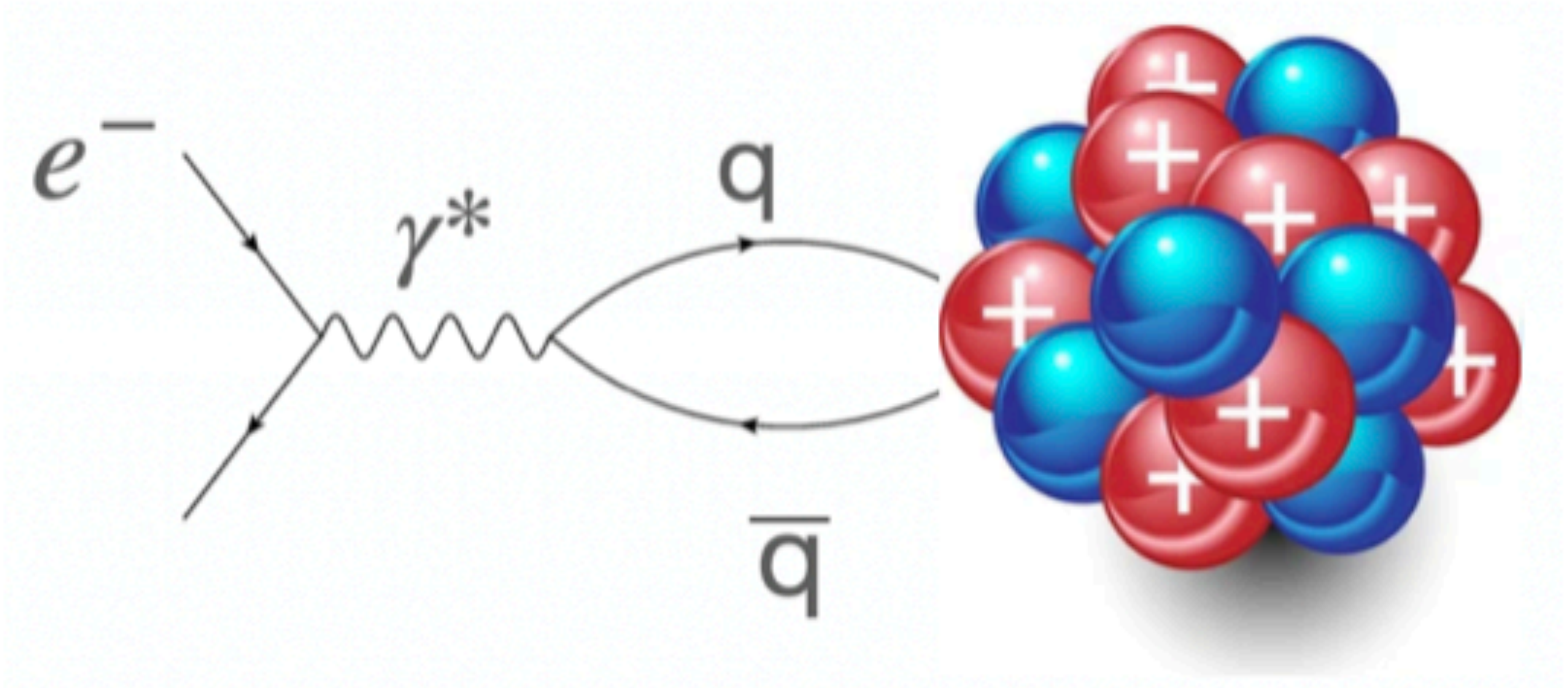
University of South Dakota

In collaboration with Iain Stewart

Sept 14, 2022

Small x DIS

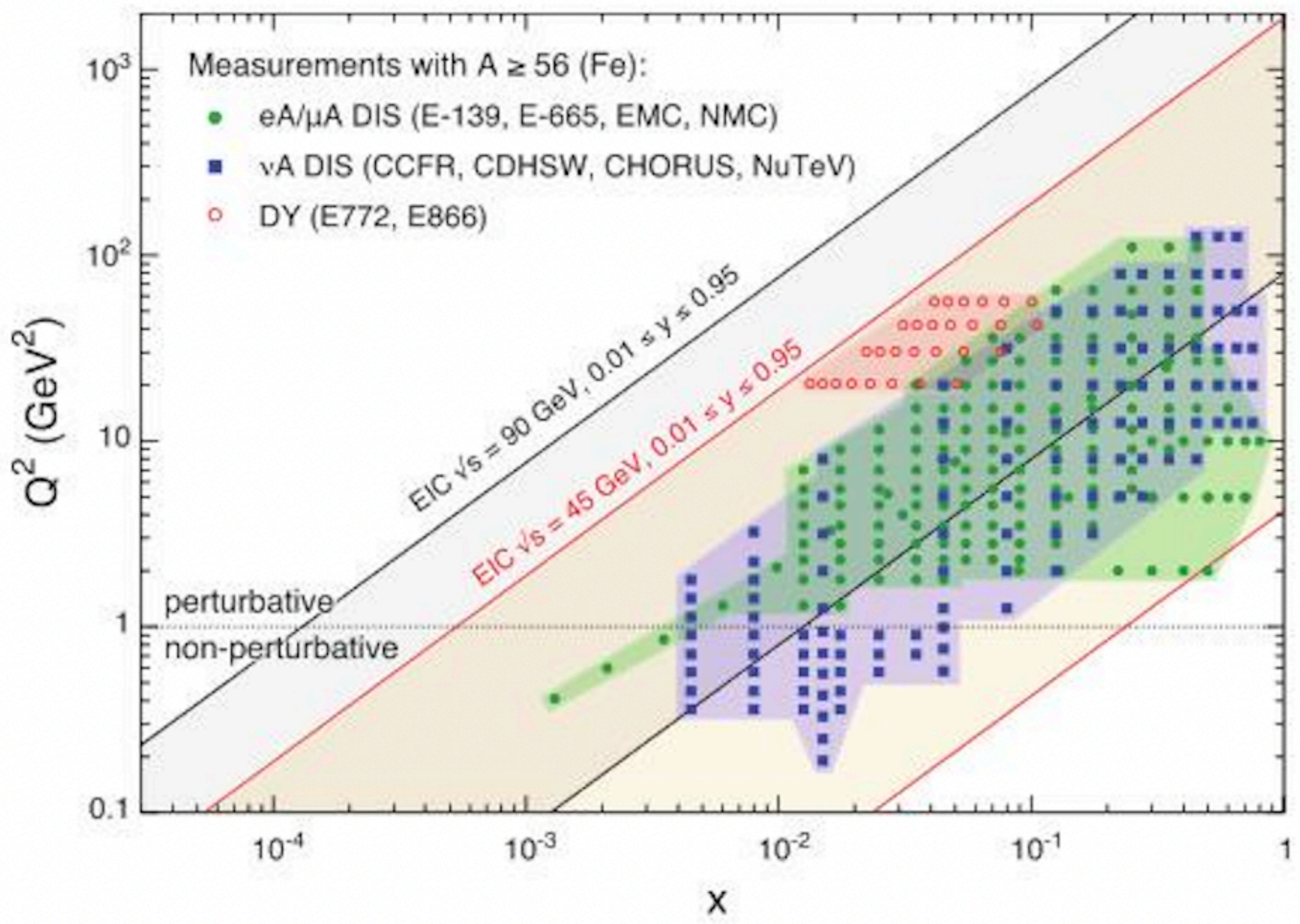


$$e^- + A \rightarrow e^- + X$$

$$x = \frac{Q^2}{s} \ll 1$$

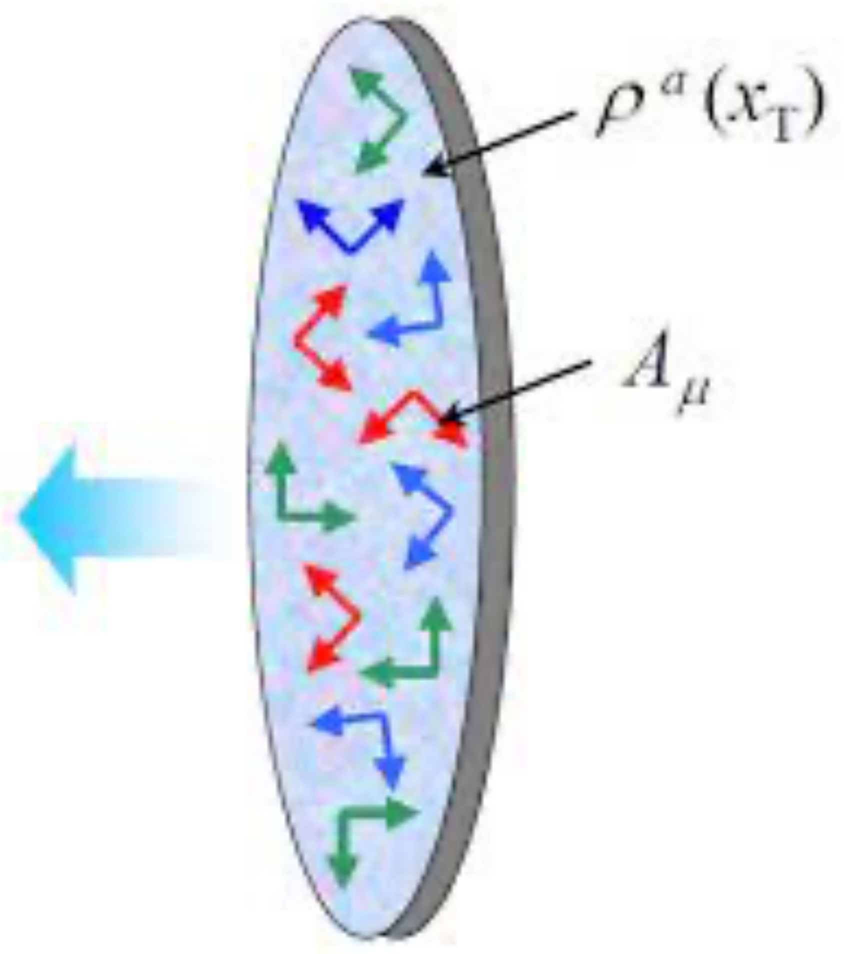
Goal: Understand evolution of a quark-anti quark dipole in a background Nucleus

- Access the one dimensional proton structure at small Bjorken x.
- Glimpse into the novel phenomenology of saturation.



Why revisit this problem?

- Current understanding of small x : A mix of perturbative QCD combined with the model of the Nucleus as a classical source of small x gluons → Color Glass Condensate.



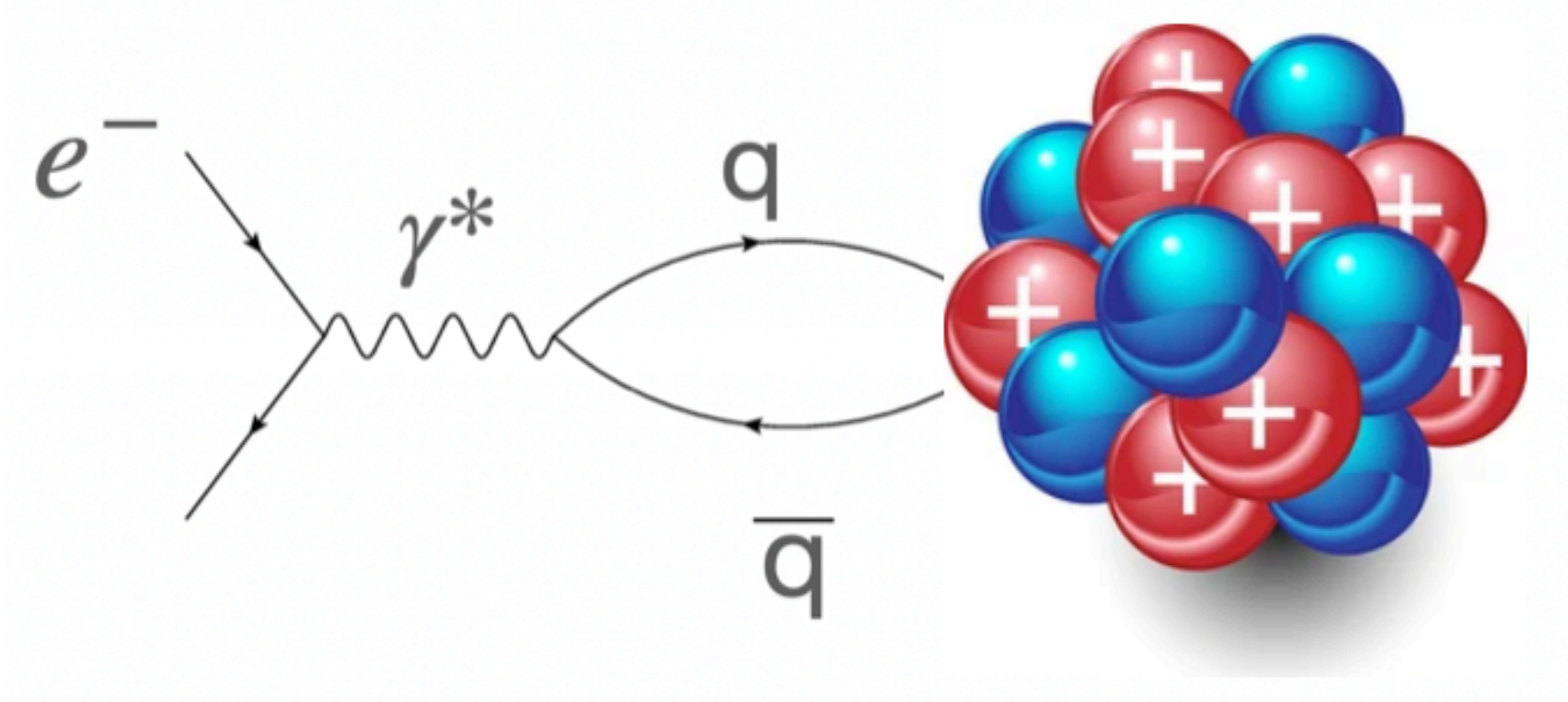
For review, see 1002.0333. F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan

Large x and TMD physics is understood in terms of rigorous factorization of physics at well separated scales.

Can we develop a EFT framework that

- Manifestly factorizes the physics at well separated scales in terms of gauge invariant operators.
- Is systematically improvable
- Gives a power counting argument for different emergent non-linear regimes.

Hierarchy of scales



$$e^- + A \to e^- + X$$

Center of mass energy $\sqrt{s}$

Electron momentum transfer Q

Color Confinement Λ_{QCD}

Size of the Nucleus

Emergent Scales

Mean free path of the probe

λ_{mfp}

Color screening length

L_{D}

Quantum coherence time
of radiation

$$t_c \sim \frac{E}{q_T^2}$$

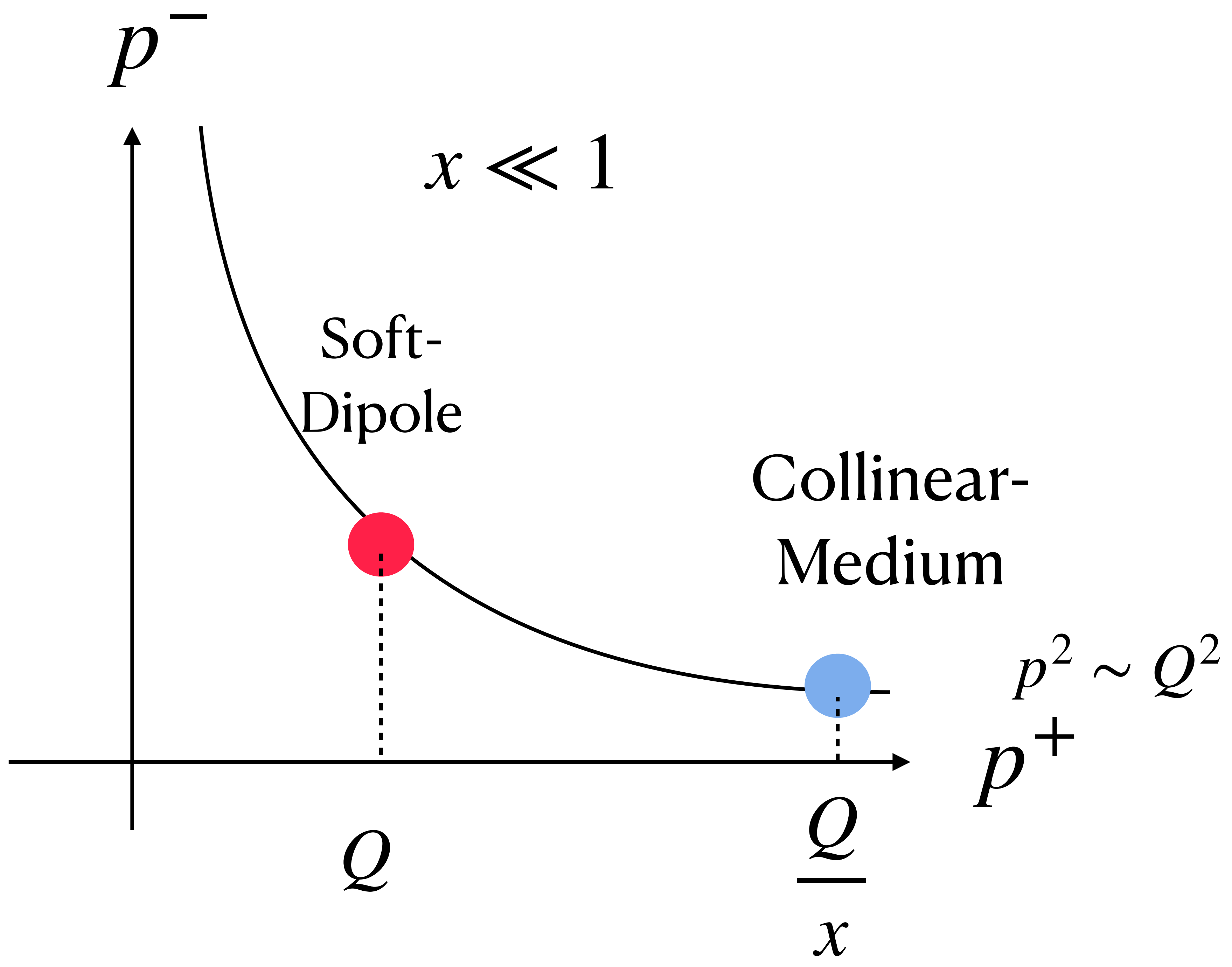
An EFT within SCET

- The (boosted)medium is made up of collinear partons

$$p_{\bar{n}} \sim \frac{s}{Q}\left(x^2, 1, x\right)$$

- The Dipole is made up of soft partons

$$p_s \sim \frac{s}{Q}(x, x, x)$$



Interaction between degrees of freedom is dominated by small angle(forward) scattering

$$L_{QCD} = L_{collinear} + L_{soft} + L_{Glauber}^{c-s} + O(x^2)$$

$x \sim \theta \ll 1$ is the expansion parameter of the EFT

I. Rothstein, I. Stewart, JHEP 1608 (2016) 025

The probe undergoes **multiple** small angle scatterings with the environment

An Open Quantum system monitored by a Nuclear environment

VV , X. Yao, JHEP 10 (2020) 024,

VV , JHEP 11 (2021) 064

$$\rho(0) = |e^-\rangle\langle e^-| \otimes \rho_{\text{Nucleus}}$$

Probe and medium are initially unentangled

$$\rho_{probe}(t) = \text{Tr}_{med}\left[e^{-iH_{eff}t}\rho(0)e^{iH_{eff}t}\right]$$

Only follow the evolution of the probe reduced density matrix

$$\sigma = \lim_{t\to\infty} \text{Tr}\left[\rho(t)_{probe}M\right] = \Sigma^{(0)} + \Sigma^{(1)} + \ldots$$

Expand order by order in the number of interactions to prove factorization and then resum the series.

Factorization at O(2n)

Hard function

Path Ordered in y^-

$$\Sigma_R^{(2n)} = \frac{|C_G|^{2n}}{Q^4}\left[\int d^+p_e\mathcal{M}\right] I_{\mu\nu}\int d\bar{y}^+\int d^2\bar{y}_\perp \mathrm{Im}\left\{\left[\Pi_{i=1}^{n}\int d\bar{y}_i^-\Theta(\bar{y}_i^- - \bar{y}_{i+1}^-)\right.\right.$$

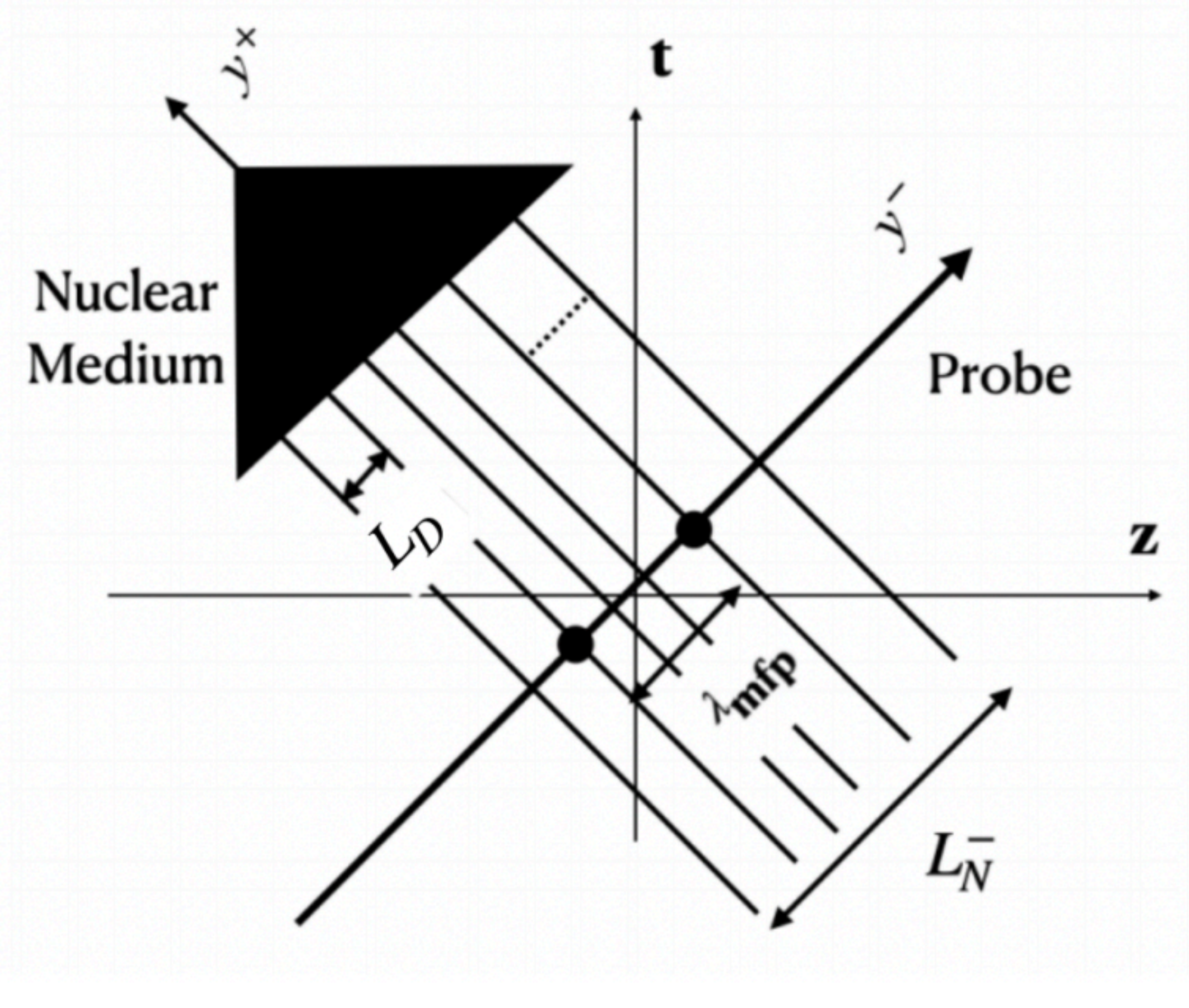
$$\left.\left.\int\frac{d^2k_{i\perp}}{(2\pi)^2}\mathcal{B}_{\bar{n}}(k_{i,\perp},\bar{y}_i^-,\bar{y}^+,\bar{y}^\perp)\right] \quad S_n^{\mu\nu}(k_{1\perp},k_{2,\perp},...k_{n,\perp};\bar{y}_1^-,\bar{y}_2^-,...\bar{y}_n^-)\right\}$$

Dipole function

n copies of the Medium Structure Function

Process independent Universal physics

Obeys BFKL evolution equation



Assuming successive interactions happen with distinct nucleons.

$$\lambda_3 = \frac{L_D}{\lambda_{\mathrm{mfp}}} \ll 1$$

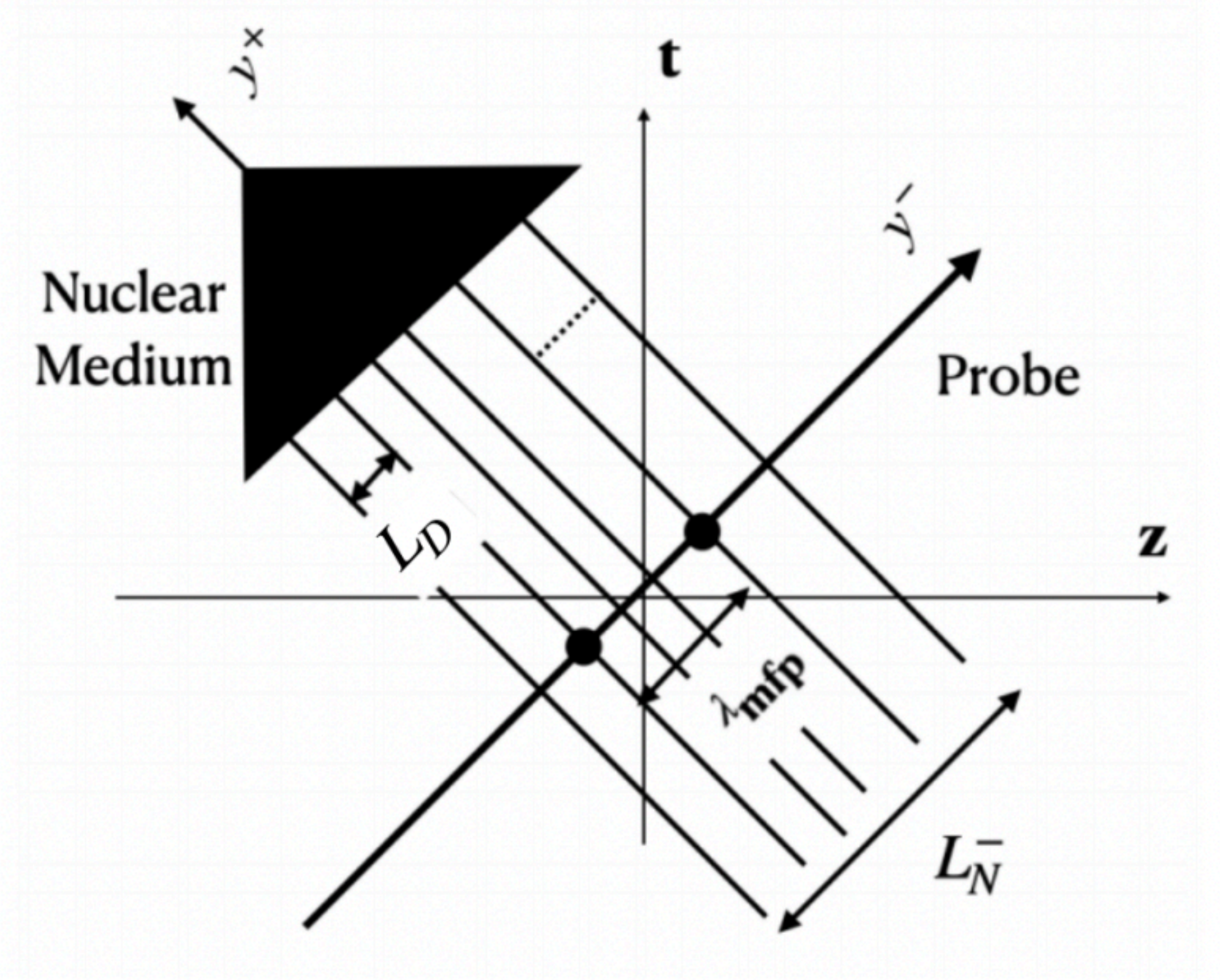
A definition of the mean free path

Leading log sum of the Glauber series to all orders.

$$\Sigma = \int d^2b[\sigma(b)]_{\rm vac} \int_{y\in {\rm Med}} d^2y_\perp dy^+ \left[1 - \mathbf{P}\exp\left\{ -\int \frac{dy^-}{\lambda_{\rm mfp}(Q,\overrightarrow{b},y)}\right\}\right]$$

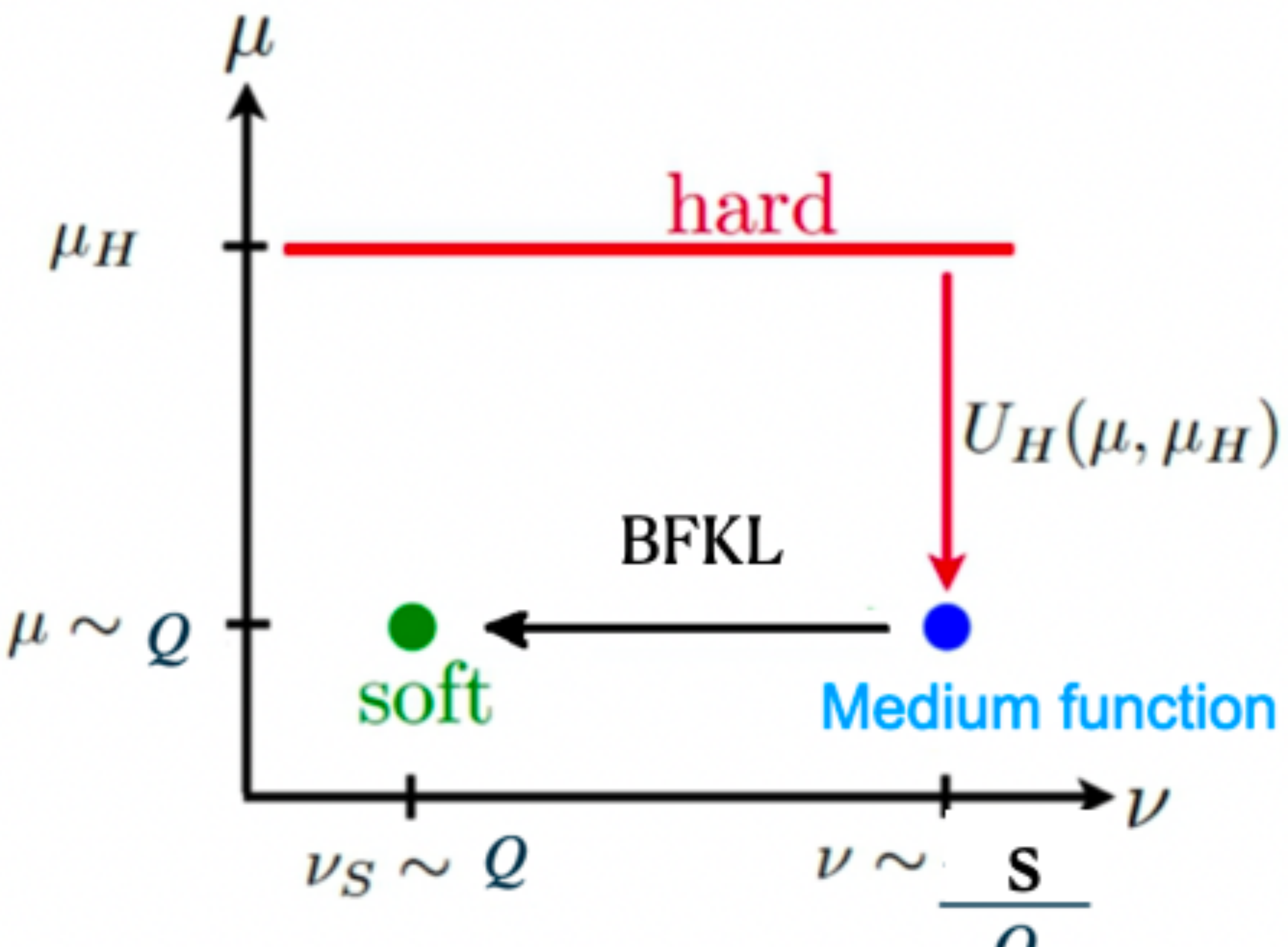
Vacuum evolution of probe

Emergent mean free path of the probe



$$\boxed{\lambda(\vec{b}/Q,\bar{y}) = \frac{1}{|C_G|^2 C_F\left[\mathbf{B}(\vec{b}/Q,\bar{y}) - \mathbf{B}(0,\bar{y})\right]}}$$

$$\mathbf{B}(\vec{b}) = \frac{(N_C^2-1)}{2(2\pi)^2}\mathrm{Tr}\left[\left\{\frac{e^{-i\vec{b}\cdot\mathcal{P}_\perp}}{\mathcal{P}_\perp^4}\delta(\mathcal{P}^+)O_{\bar{n}}^A(0)\right\}O_{\bar{n}}^A(0)\boldsymbol{\rho_A}\right]$$



The onset of saturation

$$\Sigma = \int d^2b[\sigma(b)]_{\text{vac}} \int_{y\in\text{Med}} d^2y_\perp dy^+ \left[1 - \mathbf{P}\exp\left\{-\int \underbrace{\boxed{\frac{dy^-}{\lambda_{\text{mfp}}(Q,\vec{b},y)}}}_{\lambda_1}\right\}\right]$$

$\lambda_1 \sim 1 \rightarrow$ Onset of Saturation $\equiv$ Multiple interactions need to be resummed

$$\int^{\text{Nucleus}} \frac{dy^-}{\lambda_{\text{mfp}}(Q_s, 1/Q_s, y)} = 1$$

The role of the medium size

For complete factorization, we need another mode that "sees" the size of the medium $p^+ \sim 1/L^-_{\text{nucleus}}$

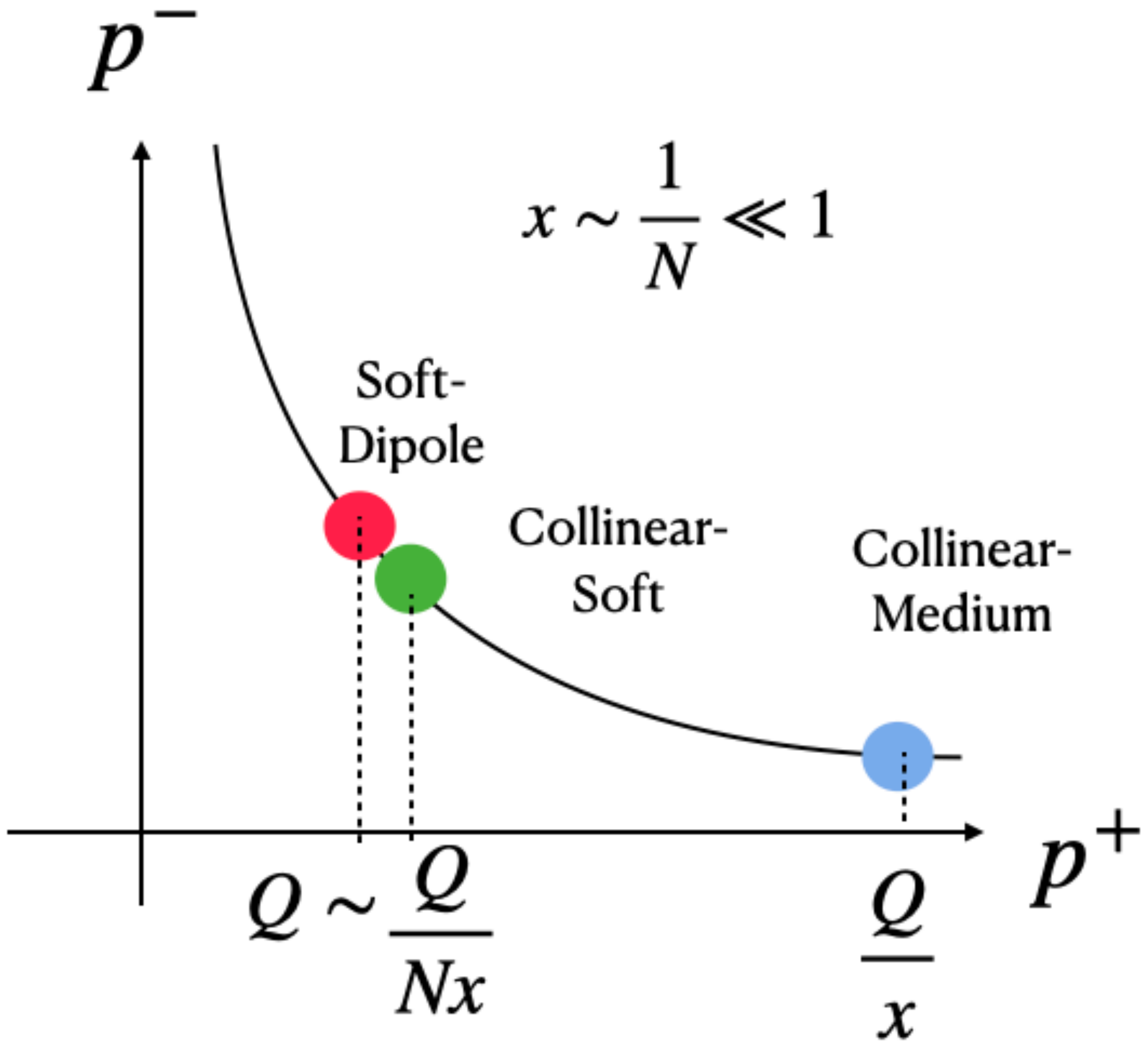
N → Number of Nucleons in the path of the Dipole

A collinear Soft mode $$p_{cs} \sim \frac{s}{Q}\left(Nx^2, \frac{1}{N}, x\right)$$

Key to understanding the transition into the non-linear regime

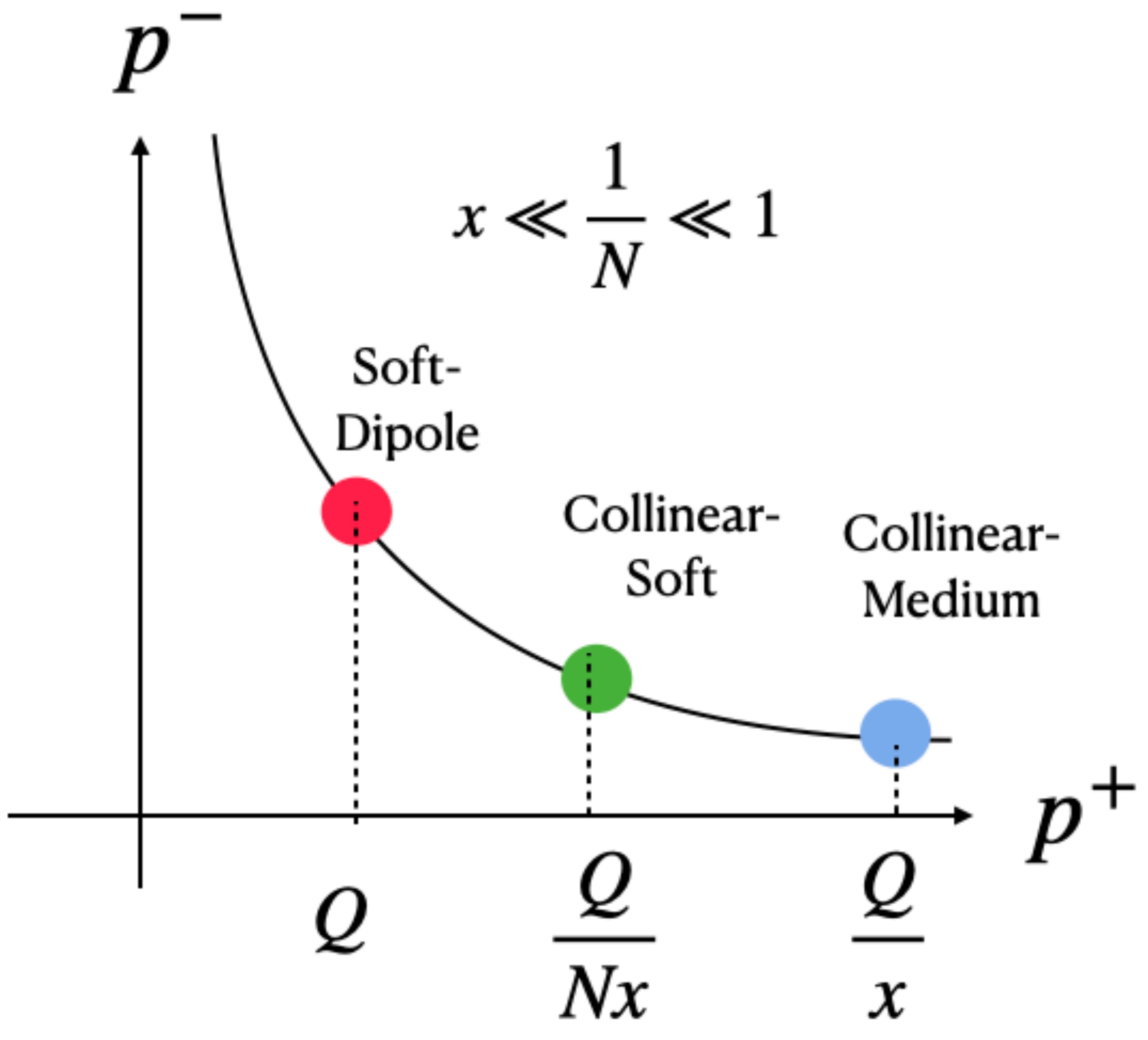
Not so small x

N → Number of Nucleons in the path of the Dipole



Current two mode EFT formulation which obeys linear BFKL describes this regime

Really small x

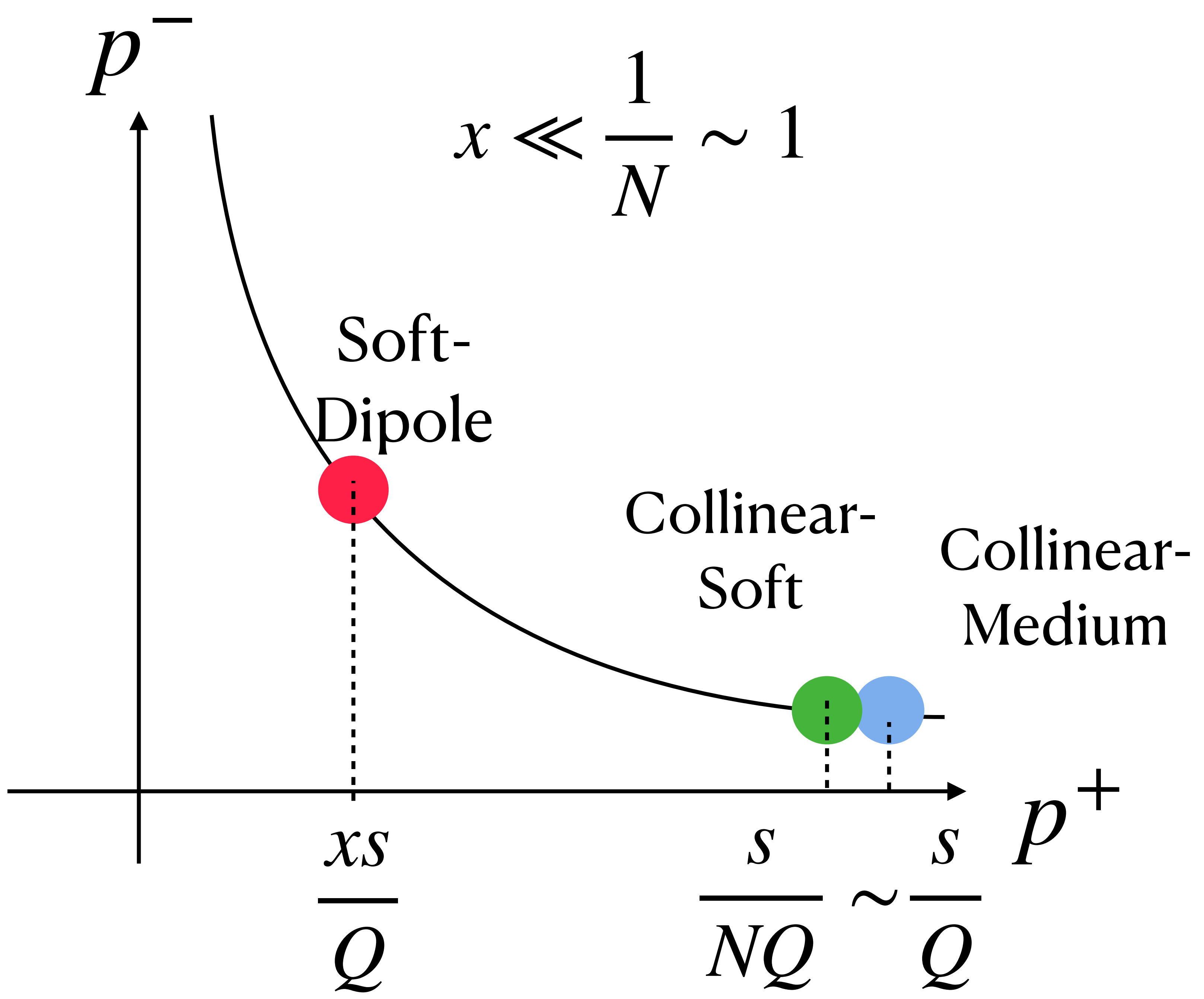


Additional logarithms in $\ln xN$ from the collinear soft function then modify the

linear BFKL evolution → BK/JIMWLK

The march into non-linear small x ≡ Decoupling of the collinear Soft mode from the Soft

The case of a proton



Successive interactions happen with the same scattering center

O(n) interaction described in terms of an n point non-perturbative function in the Proton state - Loss of predictive power?

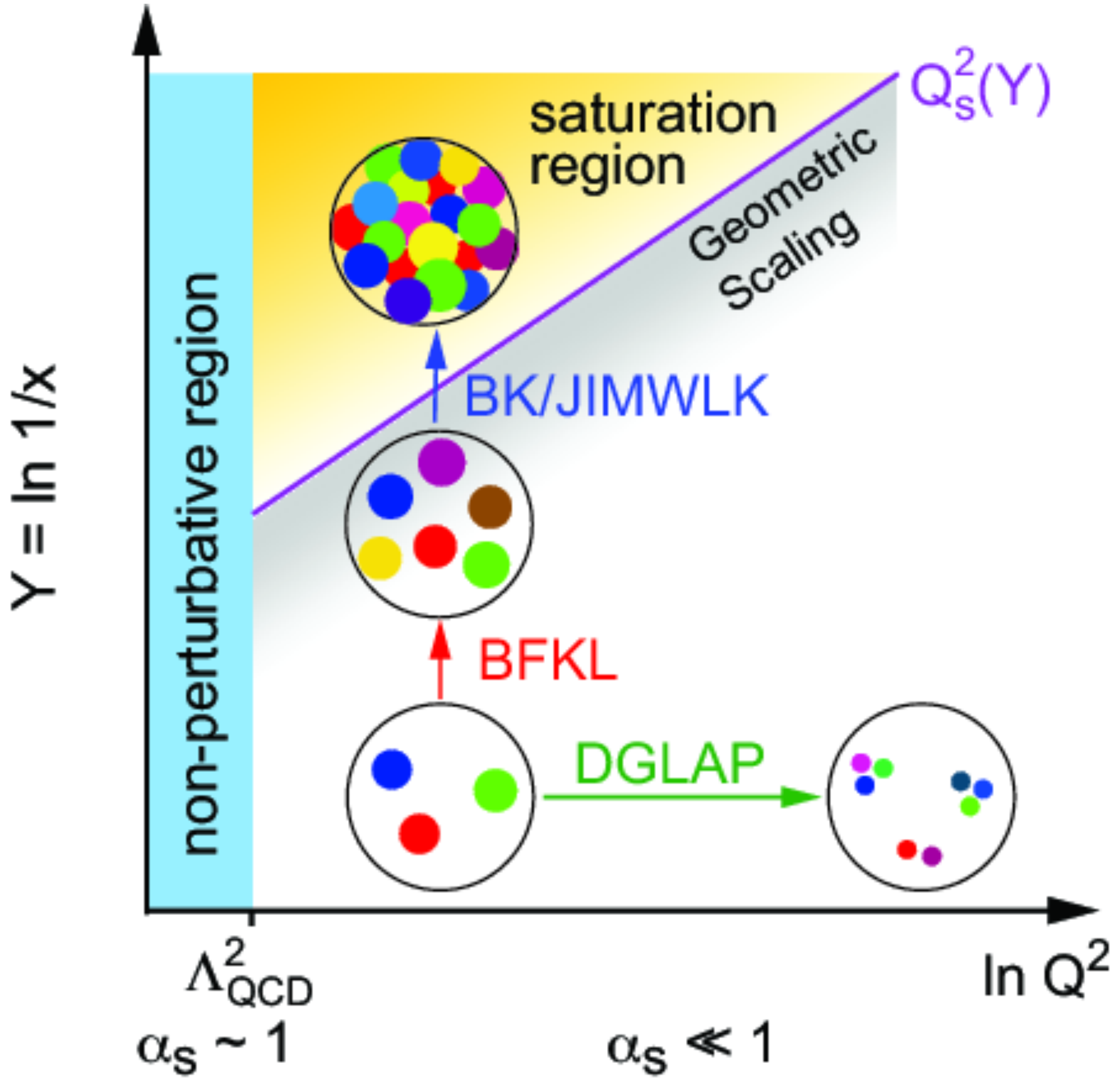
Question: Is there meaningful factorization for a single nucleon ?

Summary

- An EFT and factorization formula for the small x regime in terms of Gauge invariant operators .
- An explanation for distinct regimes in terms of kinematic and emergent power counting parameters.

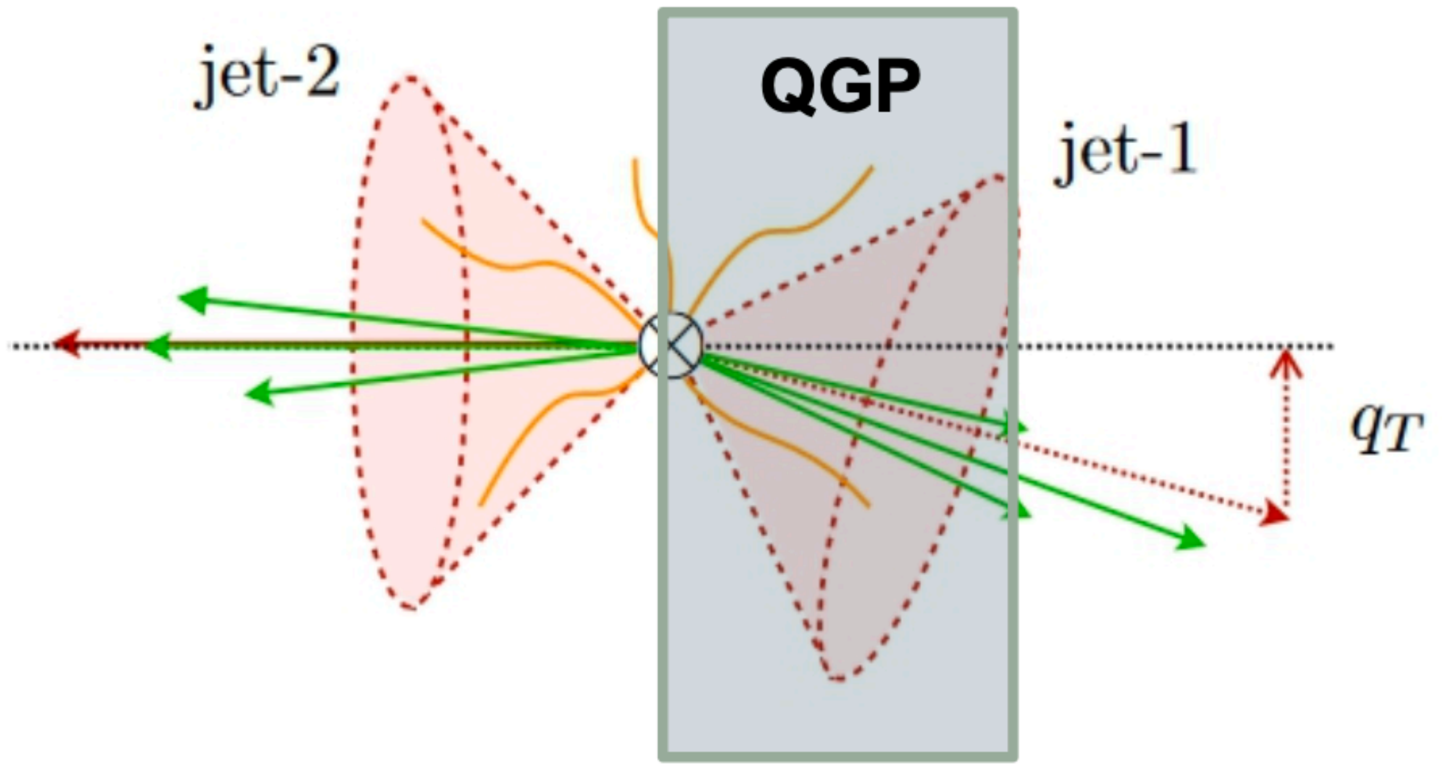
Outlook

- How to implement matching from Q $\rightarrow \Lambda_{QCD}$?
- What happens when we can no longer assume successive interactions with distinct nucleons?
- What to do for N $\rightarrow$ 1 $\rightarrow$ A single Nucleon?

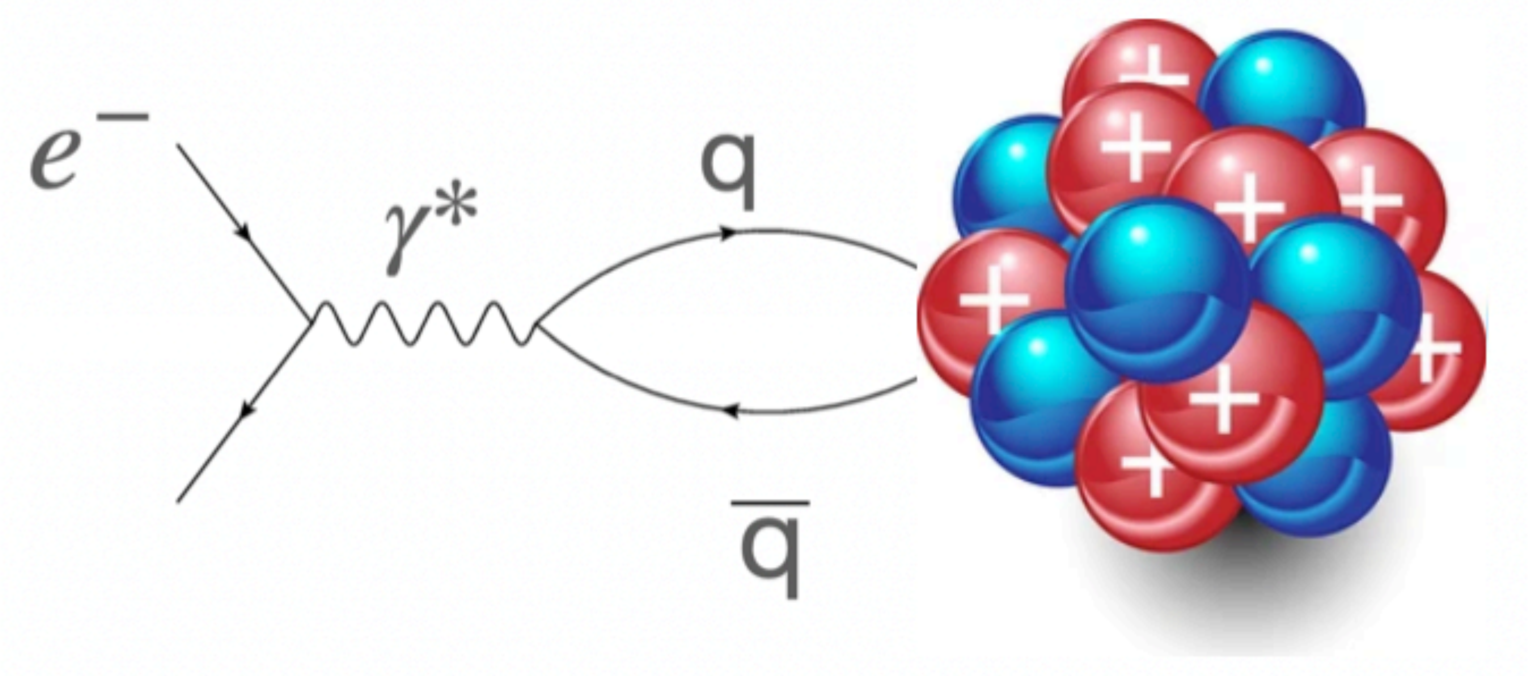


Back up

Jet propagation in QGP



Dipole evolution in a large nucleus



Jet Energy

UV / Hard scale

Center of mass energy s

Jet observable
~ qT , jet mass

Emergent Scales
Quantum Coherence time of radiation

$$t_c \sim \frac{E_J}{q_T^2}$$

Mean free path of the probe

$$\lambda_{mfp}$$

Electron momentum transfer Q

Temperature

Debye Screening Mass

Color Confinement Λ_{QCD}

Lifetime of the QGP

Size of the Nucleus

Glauber Lagrangian

$$L_G \sim O_{cs}^{qq} = O_n^{q\alpha} \frac{1}{P_\perp^2} O_S^{q\alpha}$$

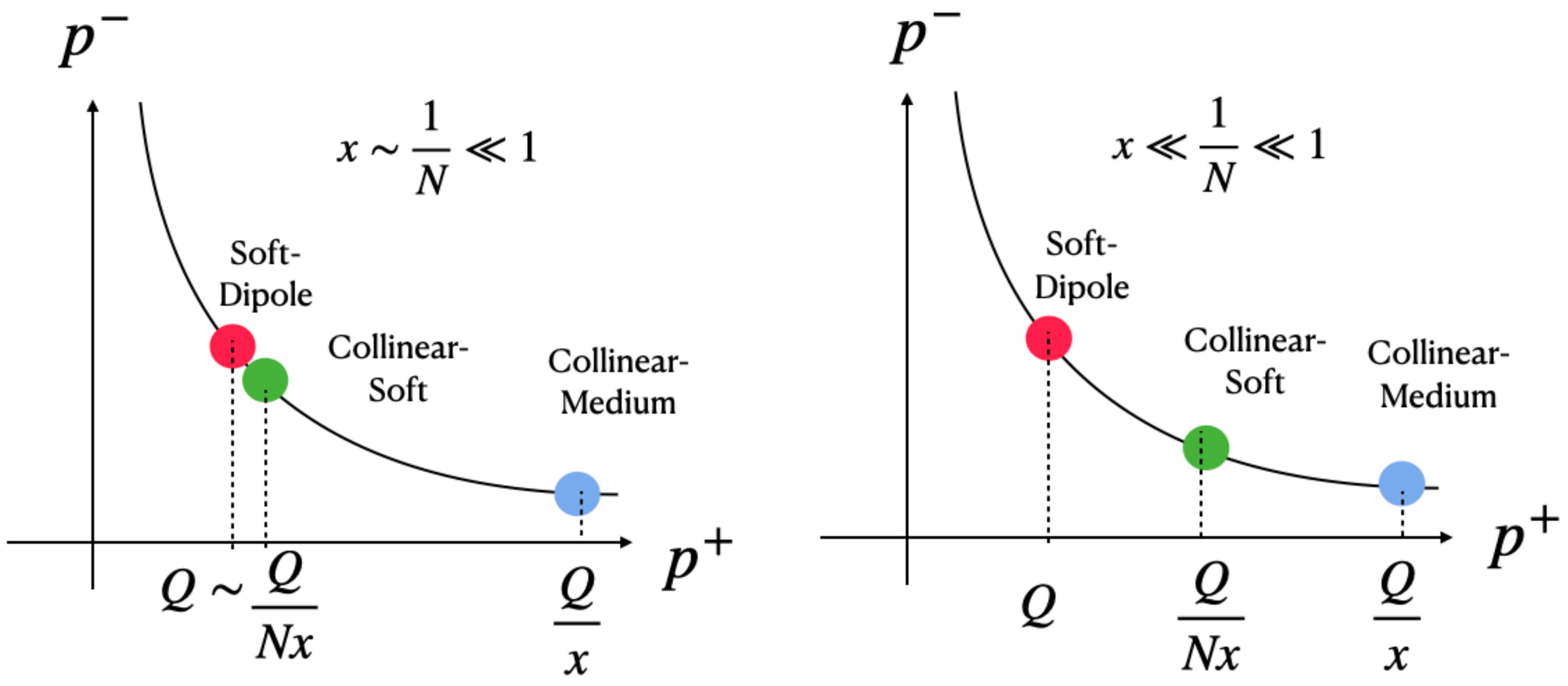
$$O_n^{q\alpha} = \bar{\chi}_n W_n T^\alpha \frac{\bar{n}}{2} W_n^+ \chi_n$$

$$O_S^{q\alpha} = \overline{\psi}_s S_n T^\alpha \frac{n}{2} S_n^+ \psi_s^n$$

Yet another emergent expansion parameter

$$\lambda_3 = \frac{L_D^-}{\lambda_{\mathrm{mfp}}(Q, \vec{b})}$$

$\lambda_3 \sim 1 \rightarrow$ Breakdown of independent scattering


p^-
$x \sim \frac{1}{N} \ll 1$
Soft-Dipole
Collinear-Soft
Collinear-Medium
p^+
$Q \sim \frac{Q}{Nx}$
$\frac{Q}{x}$
p^-
$x \ll \frac{1}{N} \ll 1$
Soft-Dipole
Collinear-Soft
Collinear-Medium
p^+
Q
$\frac{Q}{Nx}$
$\frac{Q}{x}$