Lattice Calculation of Radiative Corrections to Light Meson Leptonic Decays

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In collaboration with Peter Boyle, Norman Christ, Xu Feng,
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(RBC & UKQCD collaborations)

INT workshop, Testing the Standard Model in Charged-Weak Decays
January 15, 2026

Outline

1 Background & Motivation

2 Radiative corrections in leptonic decay $K_{\ell 2}/\pi_{\ell 2}$

3 Radiative leptonic decays $K/\pi \to \ell\nu\gamma$

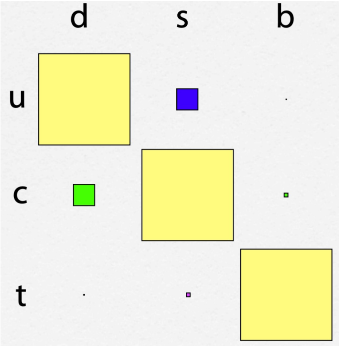
4 Radiative Corrections in $R_{e\mu} = P_{e2}/P_{\mu 2}$

5 Outlook

First row's CKM unitarity

- CKM matrix: fundamental parameters of the SM

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



- Precision test of CKM unitarity: potential BSM contributions to flavor-changing interactions and CP violation
- First row's CKM unitarity: ~ 2.4σ tension

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9983(6)_{V_{ud}}(4)_{V_{us}}$$

Particle Data Group, PRD 110 (2024) 030001

$0^+ \to 0^+$ β decays

$\to V_{ud} = 0.97367(32)$

$$\frac{K_{l2}}{\pi_{l2}} \to \frac{V_{us}}{V_{ud}} = 0.23108(51)$$

$$K_{l3} \to V_{us} = 0.22330(53)$$

Theoretical uncertainty: radiative corrections

- Radiative corrections (RC):

Precision: $\sim O(0.2\%)$

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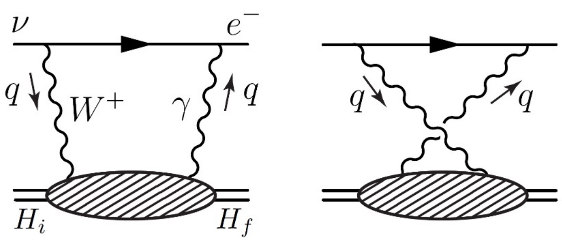
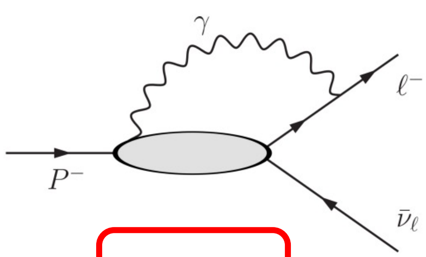
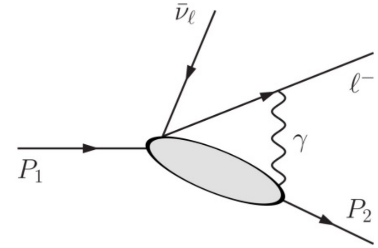


RC in $0^+ \to 0^+$ β decays



RC in K_{l2}

this talk



RC in K_{l3}

$RC \sim O(1\%)$

- RC$\sim O(1\%)$: One of the dominant source of theoretical uncertainties

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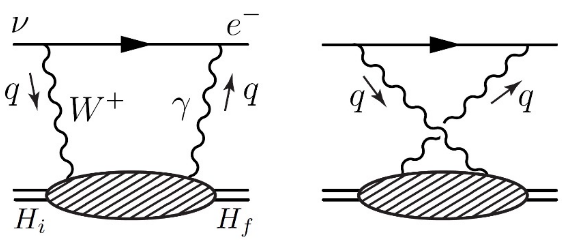

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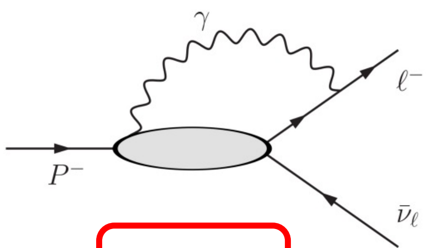

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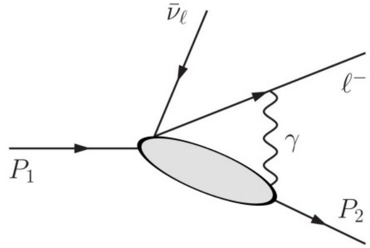

RC in K_{l3}

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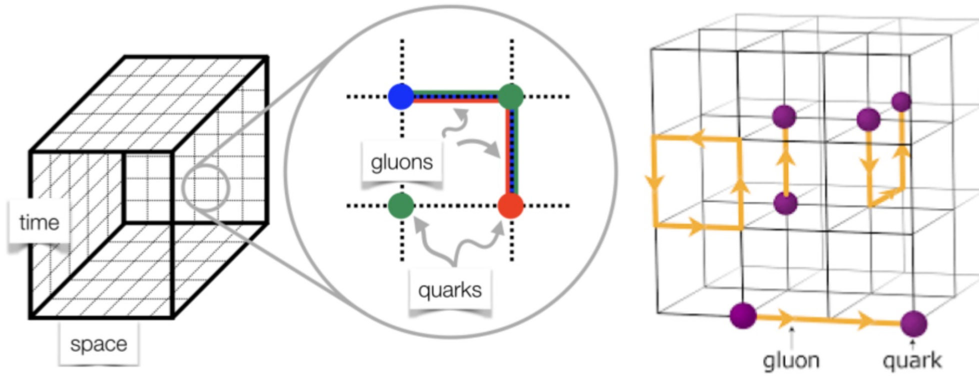
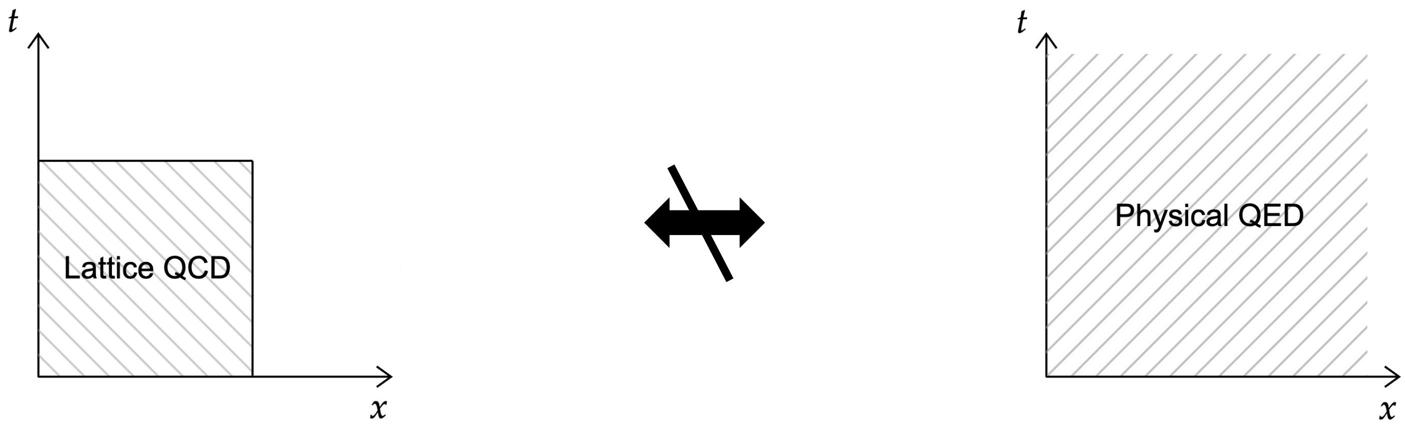
- RC$\sim O(1\%)$: One of the dominant source of theoretical uncertainties

➡ Dispersive method, Chiral PT, lattice QCD



Motivation 1: challenge of QED on lattice



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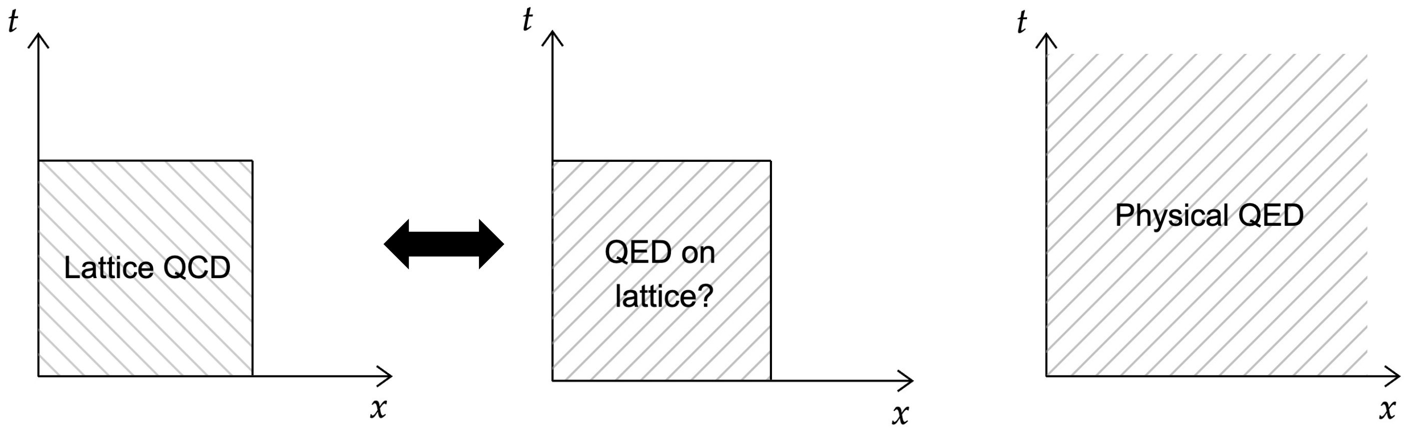

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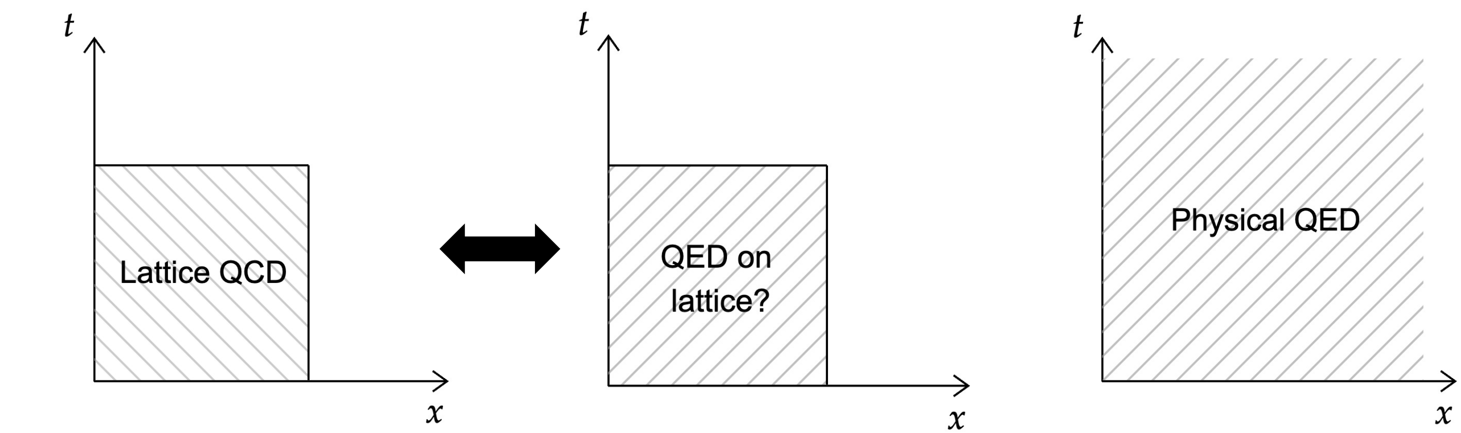

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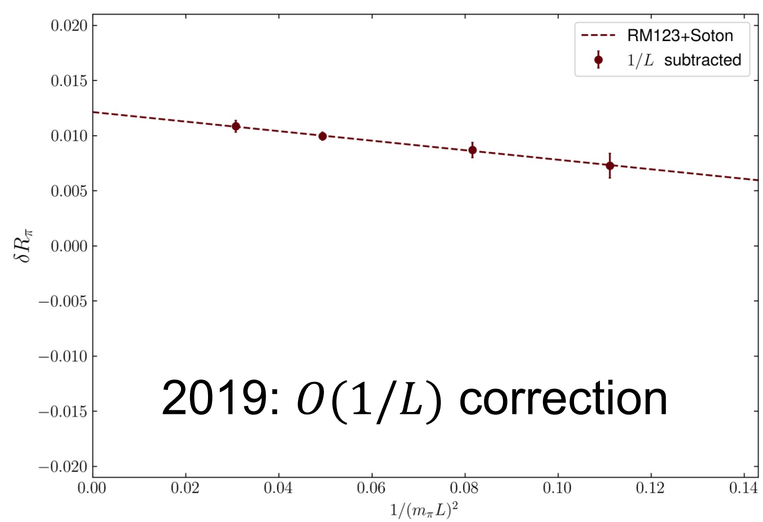

M. Di Carlo, et al. PRD, 2019 [arXiv:1904.08731]

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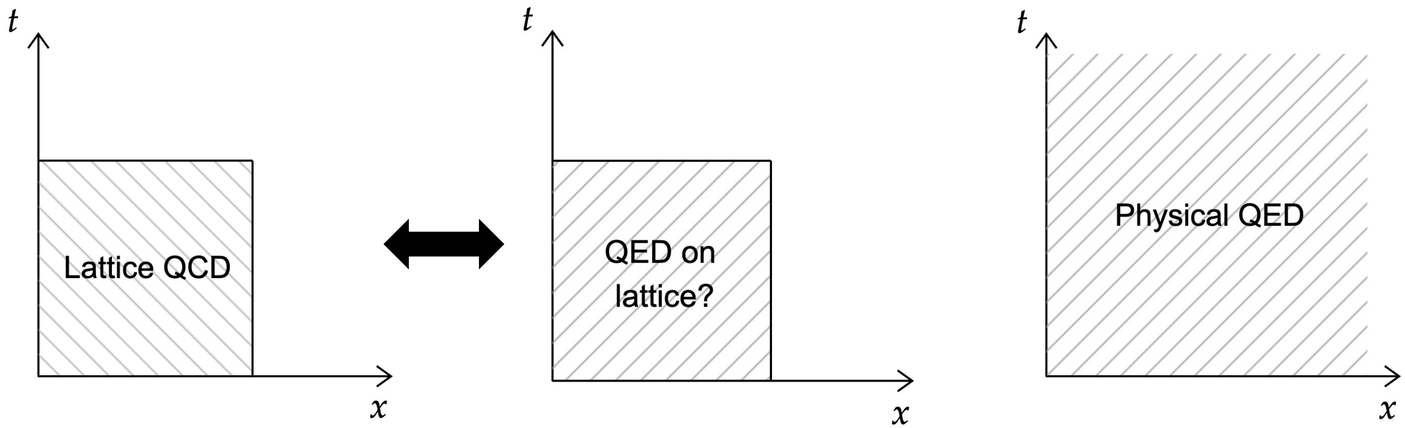

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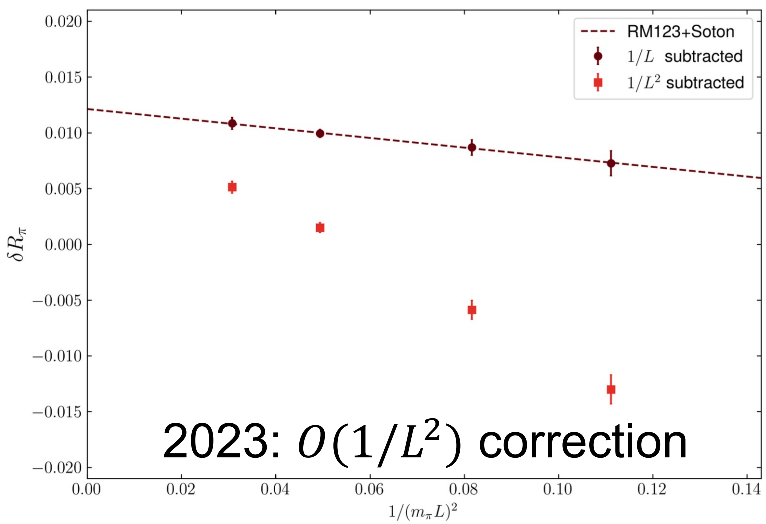

P. Boyle, et al. JHEP, 2023 [arXiv:2211.12865]

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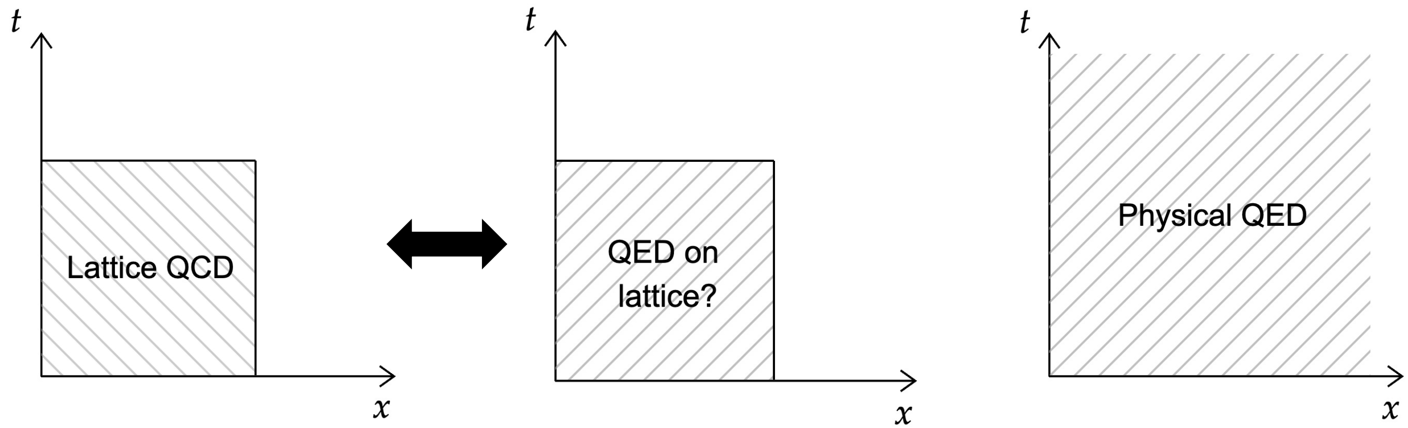

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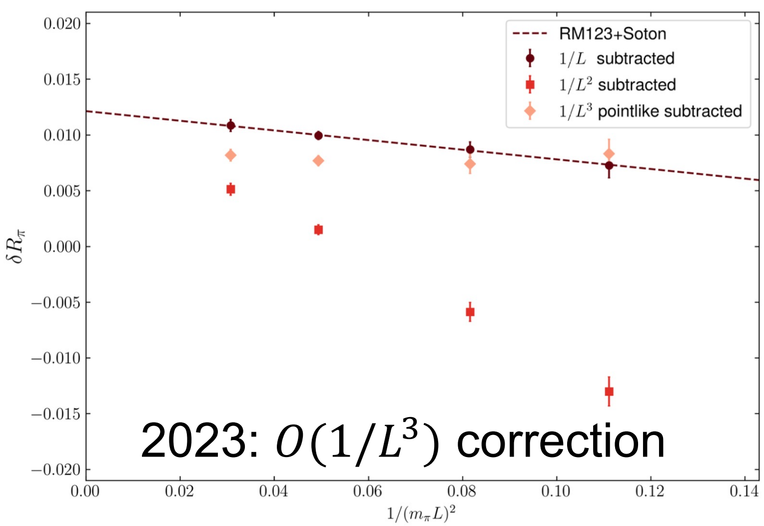

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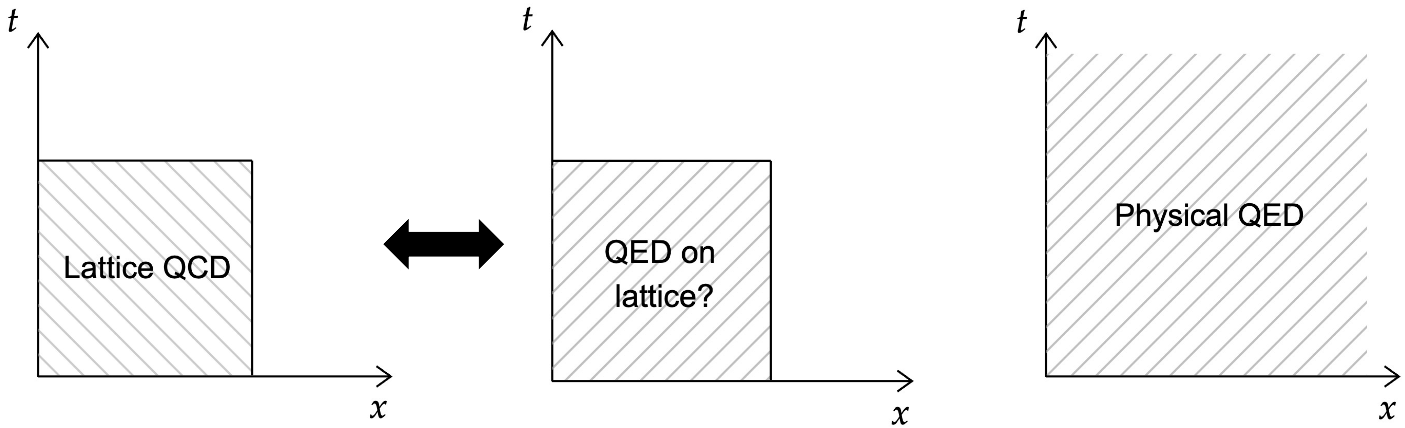

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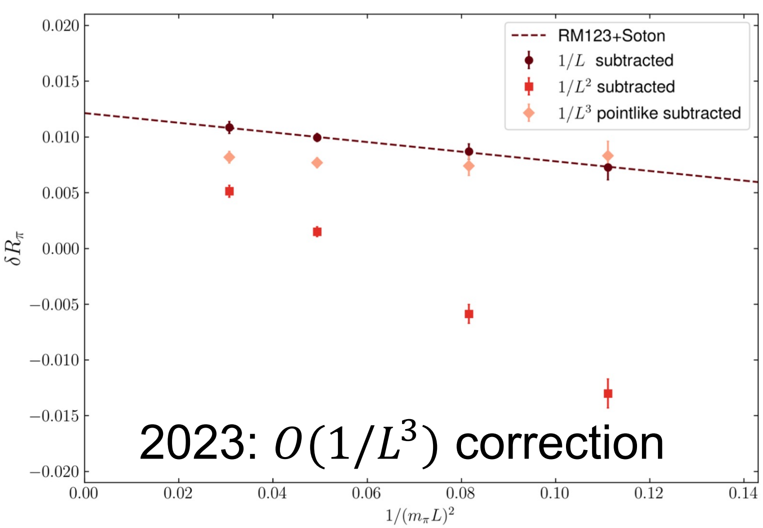


	$O(1/L)$	$O(1/L^2)$	$O(1/L^3)$
$\delta_{\mathrm{FV}}^{\mathrm{QED}_L}/0.01$	-46.8%	$+26.2\%$	-39.3%

Power-law FV correction is not well convergent

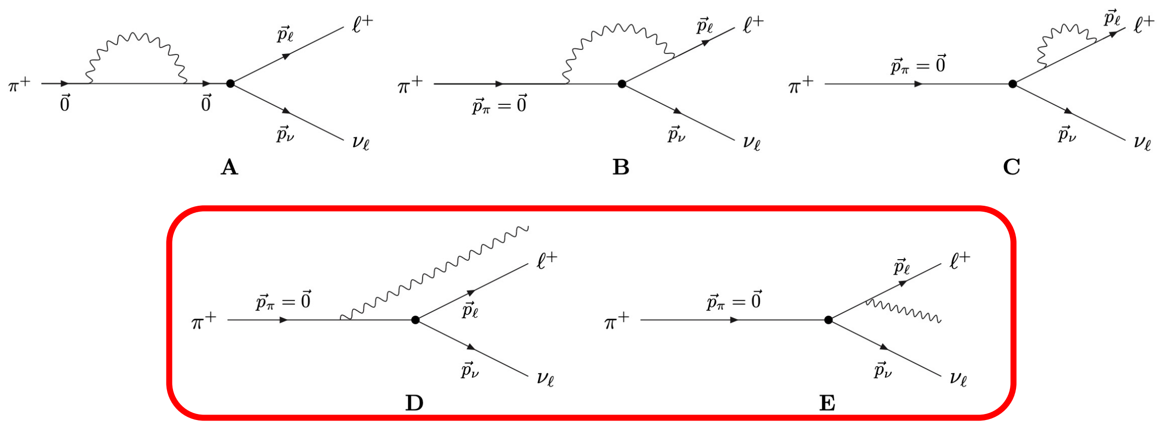
$$\delta R_{K\pi} = -0.0086(3)_{\mathrm{stat}}\left({}^{+11}_{-4}\right)_{\mathrm{fit}}(5)_{\mathrm{disc.}}(5)_{\mathrm{quench.}}(39)_{\mathrm{vol.}}$$

45% FV errors!

P. Boyle, et al. JHEP, 2023 [arXiv:2211.12865]

Motivation 2: Puzzles in radiative leptonic decays

- Bloch-Nordsieck theorem:
  IR-safe quantity = virtual-photon loop + real photon emission



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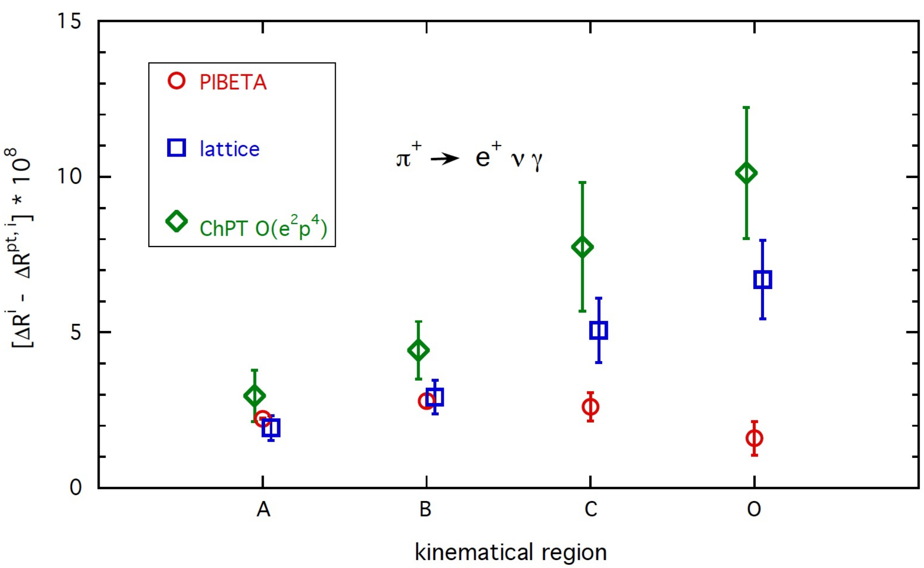

$$\pi \rightarrow e\nu\gamma$$

lattice v.s. PIBETA: 5σ discrepancy

R. Frezzotti, et al. PRD, 2021 [arXiv:2012.02120]

An independent lattice calculation is important to cross check these findings.

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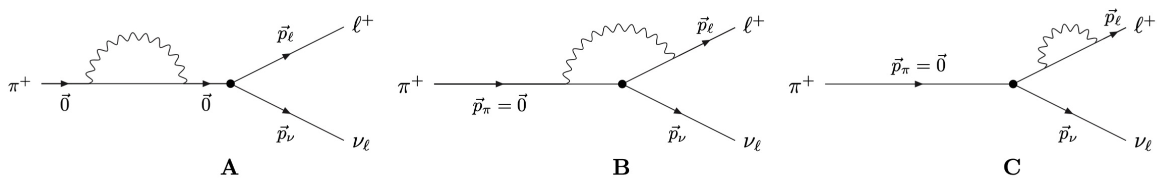

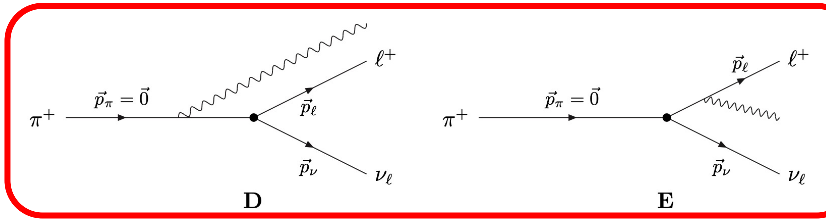

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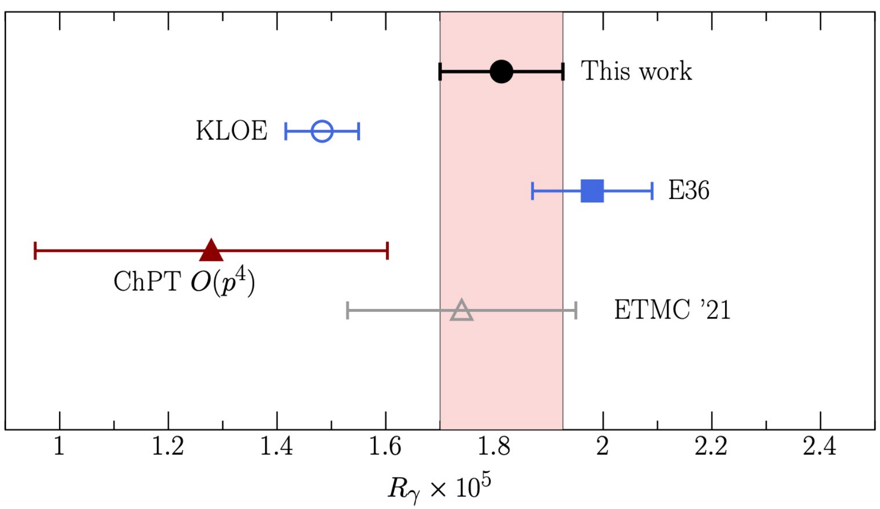

$$K \to e\nu\gamma$$

KLOE v.s. E36: 4σ discrepancy

Previous lattice result close to E36 result

R. Di Palma, et al., PRD, 2025 [arXiv:2504.08680]

An independent lattice calculation is important to cross check these findings.

Motivation 3: $R_{e\mu} = \Gamma(P \to e\nu(\gamma))/\Gamma(P \to \mu\nu(\gamma))$

- Helicity suppression: sensitive to new physics, test of lepton flavor universality

- Theoretical clean: structure-dependent (SD) information enters through small radiative corrections. size~$O(0.1\%)$

		$(P=\pi)$	$(P=K)$
$O(\alpha)$, pt	$\Delta^{(P)}_{e^2p^2}$ (%)	-3.929	-3.786
$O(\alpha)$, SD	$\Delta^{(P)}_{e^2p^4}$ (%)	0.053 ± 0.011	0.135 ± 0.011
	$\Delta^{(P)}_{e^2p^6}$ (%)	0.073	
resumming LLs	Δ_{LL} (%)	0.055	0.055

V. Cirigliano, I. Rosell, PRL, 2007 [arXiv:0707.3439]

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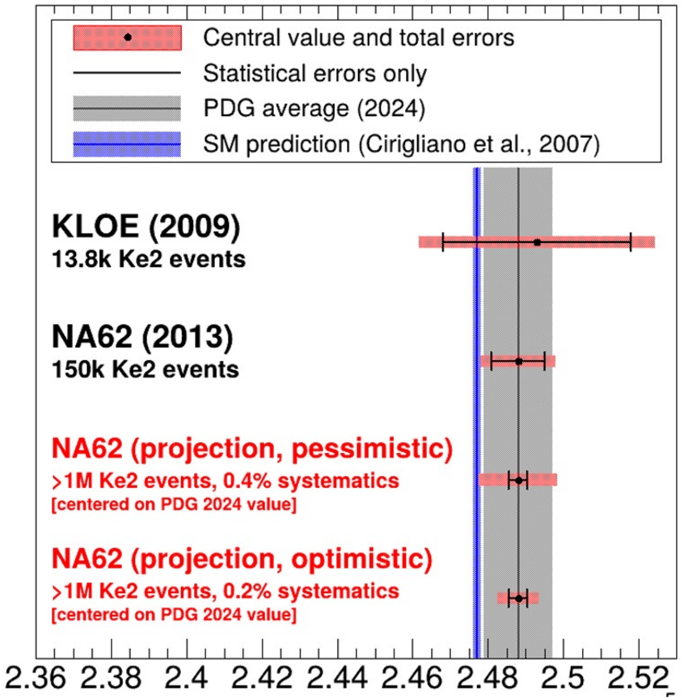
- NA62 future plan: accuracy ~ $O(0.2\%)$



(Figure from Karim Massri's talk on Kaon 2025, Mainz)

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- Lattice QCD: complement ChPT prediction using first-principles input, can improve precision to percent level for SD.

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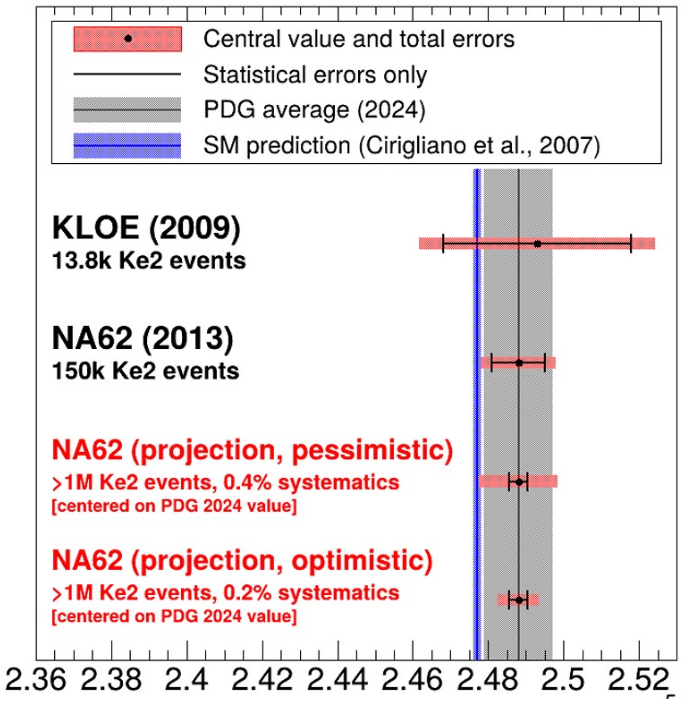

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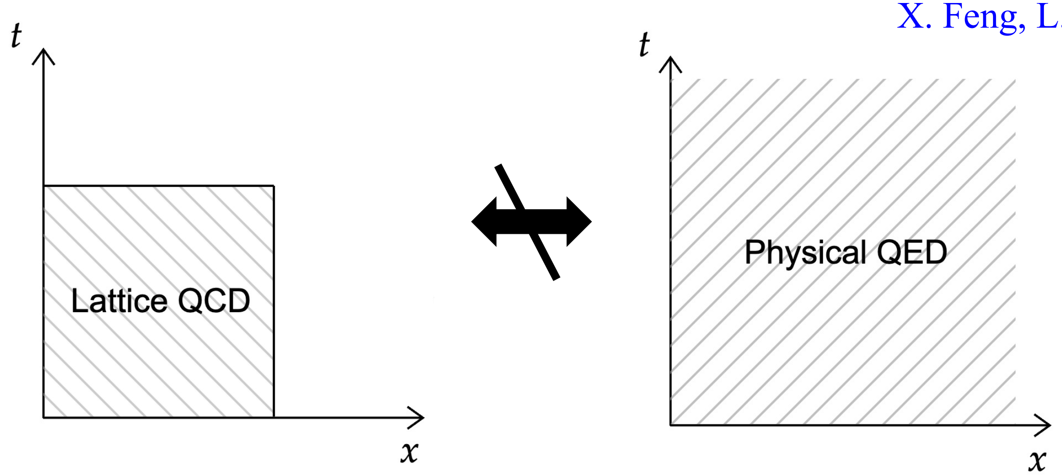
Outline

Infinite-volume reconstruction (IVR)

- Reconstruct hadronic matrix elements outside the lattice box

X. Feng, L. Jin. PRD 2019 [arXiv:1812.09817]



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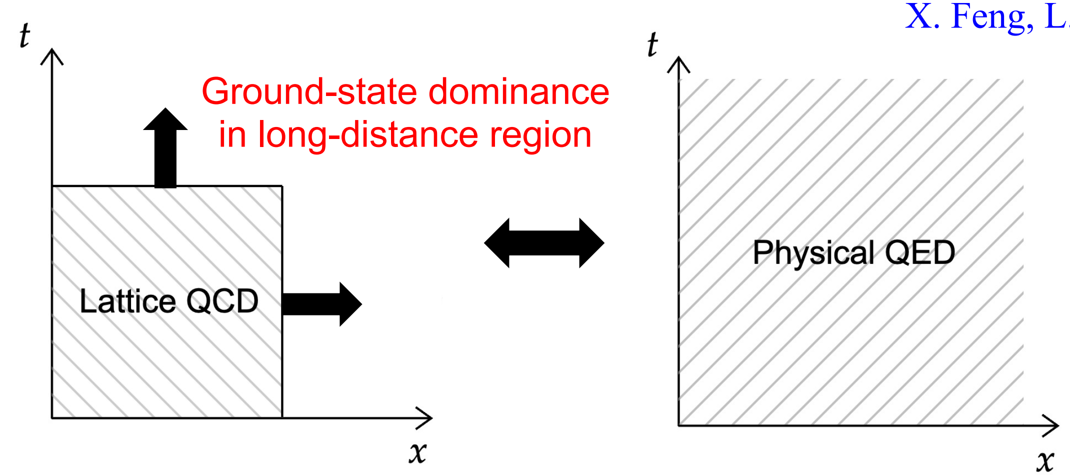
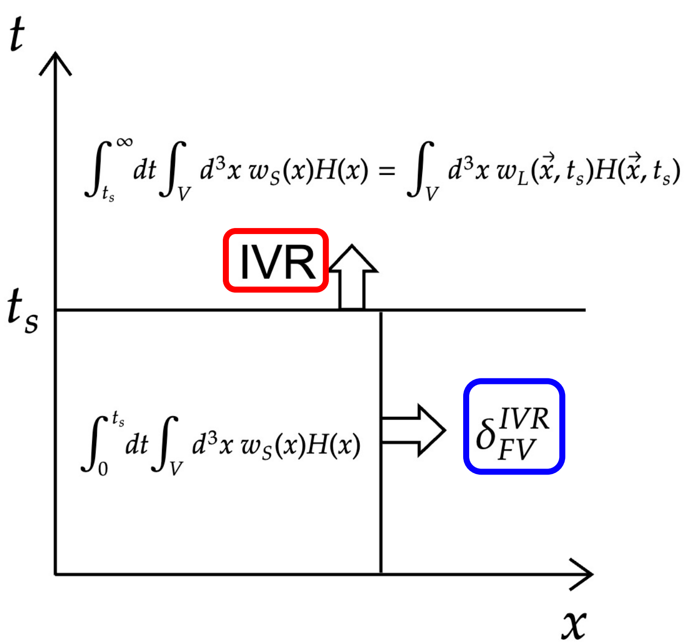


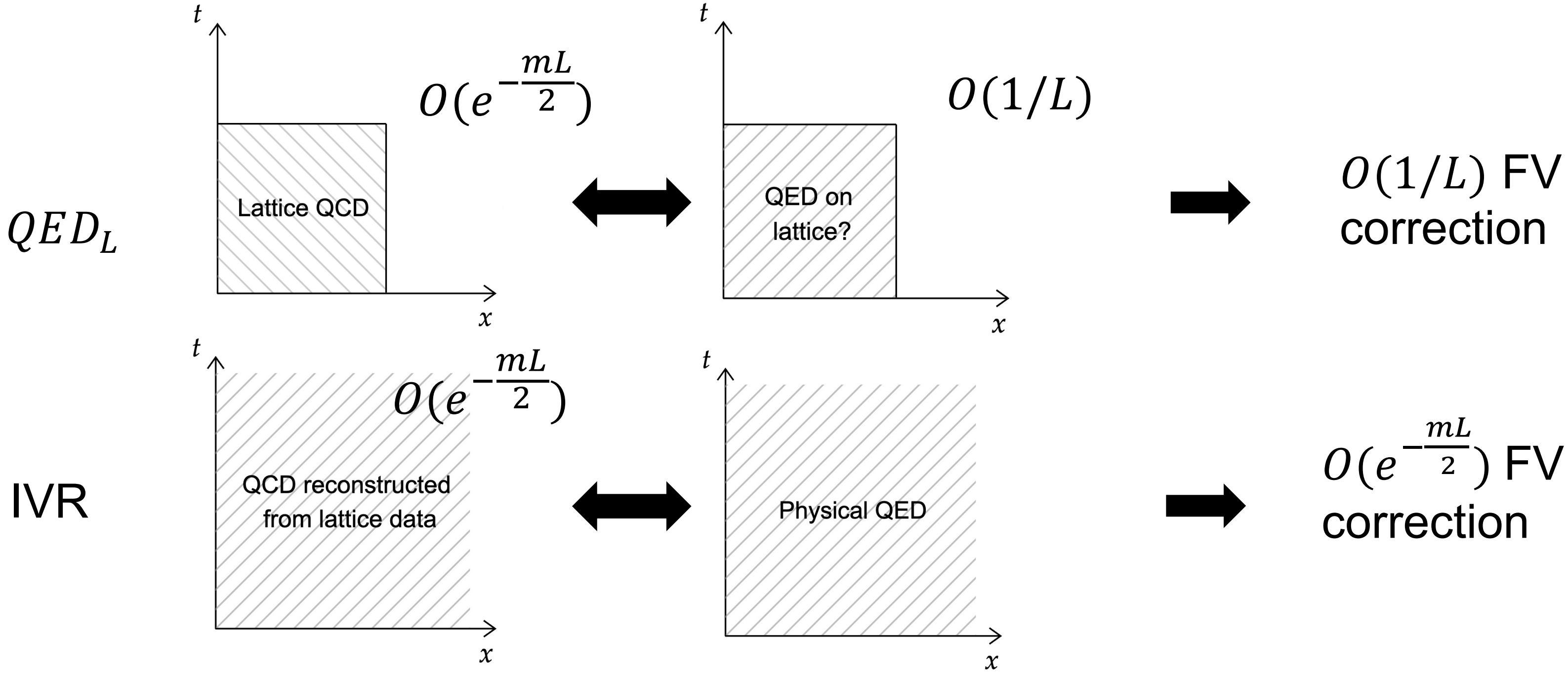


IVR: reconstruction in temporal direction

δ_{FV}^{IVR}: reconstruction in spatial direction

(residue $O(e^{-\frac{mL}{2}})$ FV effects)

IVR Finite-Volume Scaling: $O(1/L) \rightarrow O(e^{-mL})$



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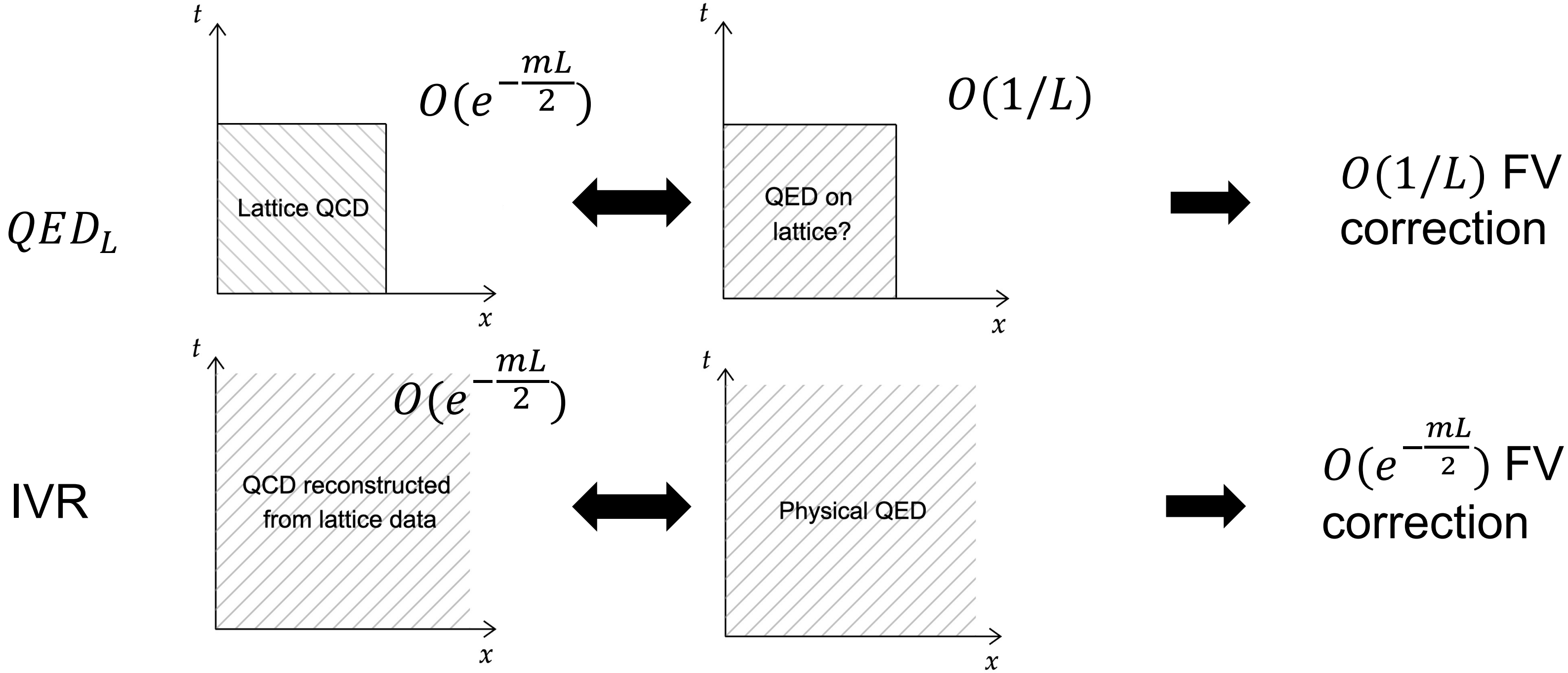
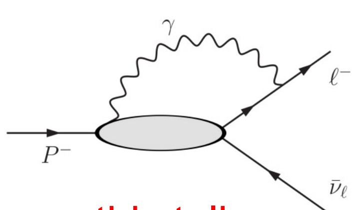
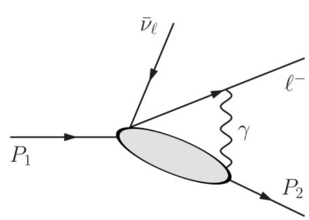


- A general framework for radiative corrections & long-distance contributions: γ/ℓ propagators implemented in infinite volume do not generate $O(1/L)$ FV errors for a broad class of diagrams. Xin-Yu Tuo, Xu Feng, PRD, 2025 [arXiv:2407.16930]



this talk

RC in K_{l2}



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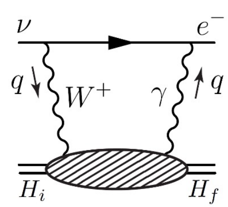
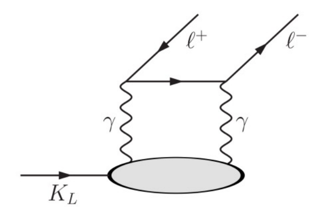


RC in $0^+ \rightarrow 0^+$ β decays

see Xu's talk

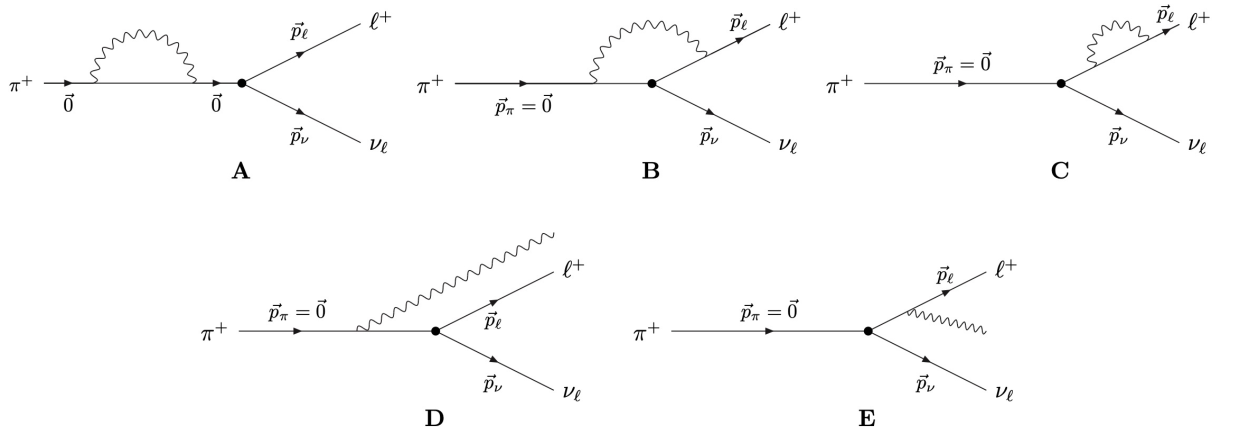


$K_L \rightarrow \ell^+\ell^-$

…

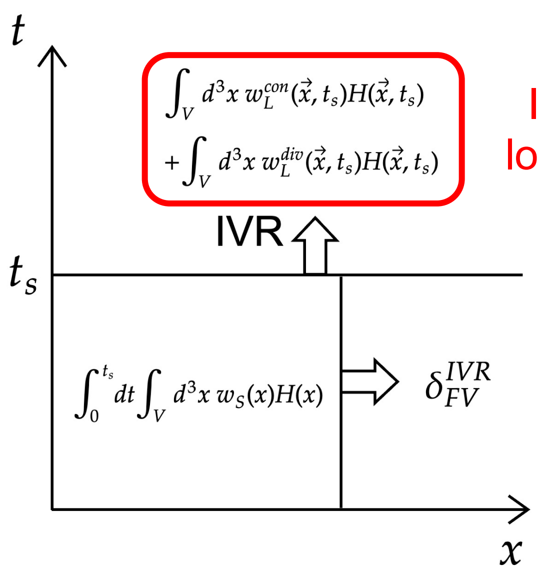
Extending IVR to IR-divergent diagrams

- IR-safe quantity = virtual-photon loop + real photon emission



- Subtraction of IR div. in long-distance part

N. Christ, X. Feng, L. Jin, C. Sachrajda, T. Wang, PRD, 2023 [arXiv:2304.08026]



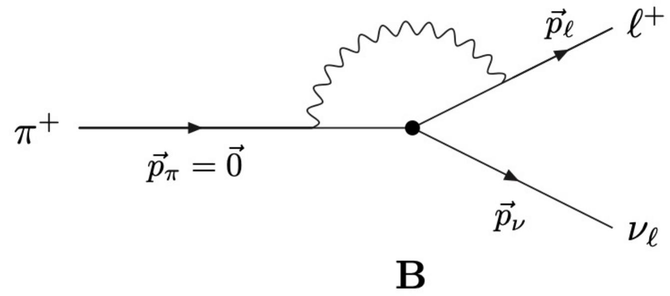
IR div. is related to long-distance physics

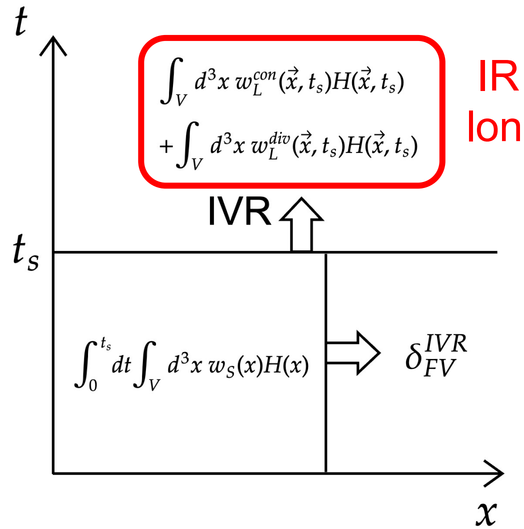
$I_S + I_L^{\mathrm{con}}$: lattice calculation

I_L^{div}: analytical calculation

Extending IVR to IR-divergent decays

- Example: diagram B





IR div. is related to long-distance region

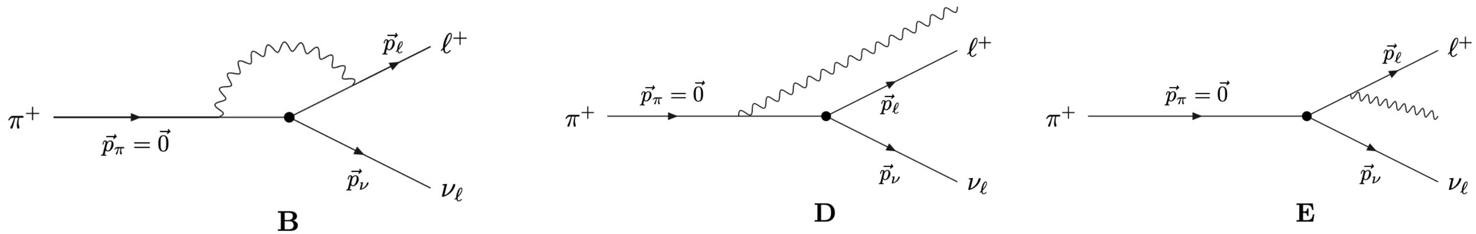
- A special "lattice IR subtraction" : $e^{i\vec{k}\cdot\vec{x}} = (e^{i\vec{k}\cdot\vec{x}} - 1) + 1$ trick

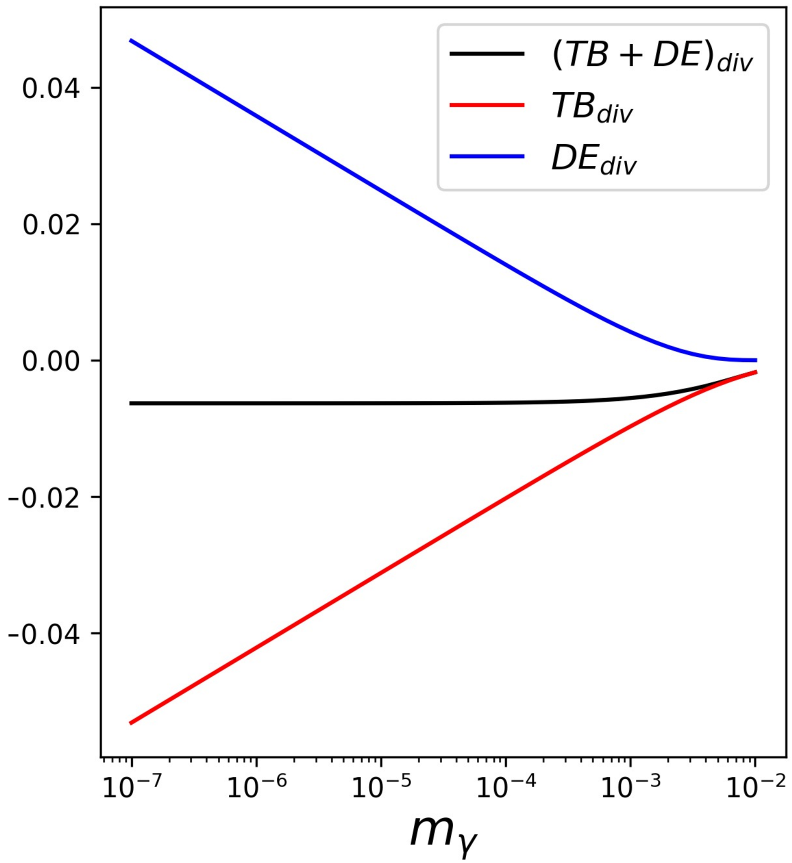
$$w_L^{\mu\rho}(t,\vec{x}) = i\frac{G_F V_{ud}^* e^2}{\sqrt{2}} \int \frac{d^3k}{(2\pi)^3} \int \frac{dk_E^0}{2\pi} \frac{\bar{L}^{\mu\rho}(k,p)}{((k_E^0)^2 + E_\gamma^2 - i\epsilon)((p_{\ell,E}^0 - k_E^0)^2 + E_\ell^2 - i\epsilon)}$$
$$\times \left[\left(e^{-i\vec{k}\cdot\vec{x}} - 1\right) + 1\right] \frac{e^{ik_E^0 t_s}}{-ik_E^0 + E_\pi(\vec{k}) - m_\pi}$$

w_L^{con} w_L^{div}

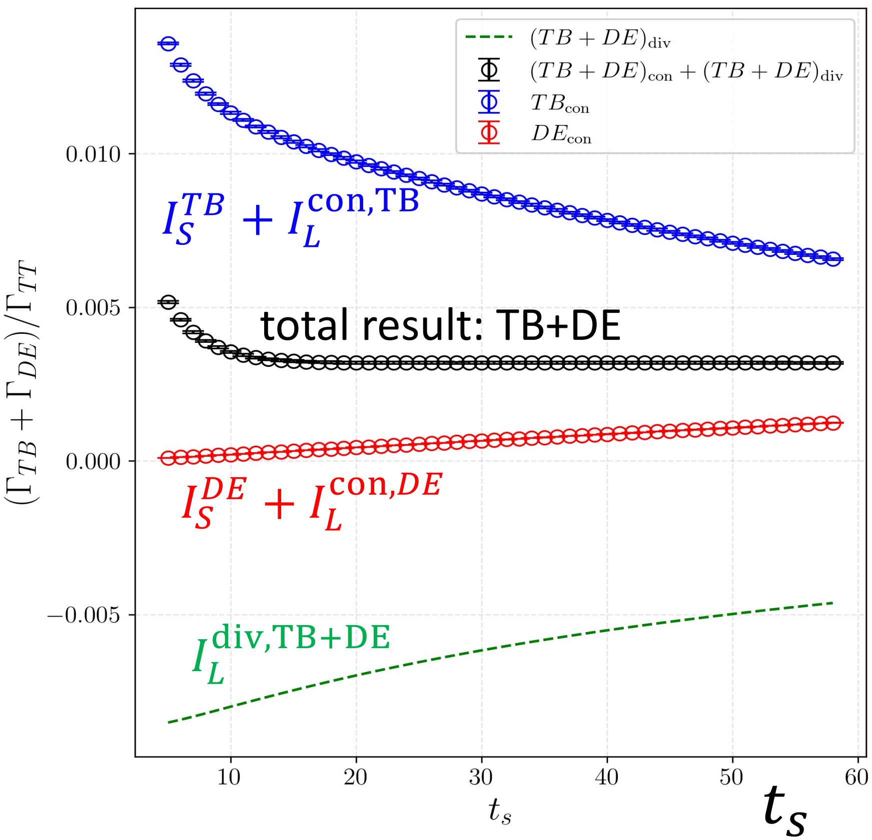
- Similar IR subtraction can be done for the other diagrams.

Example: IR convergent combination TB+DE





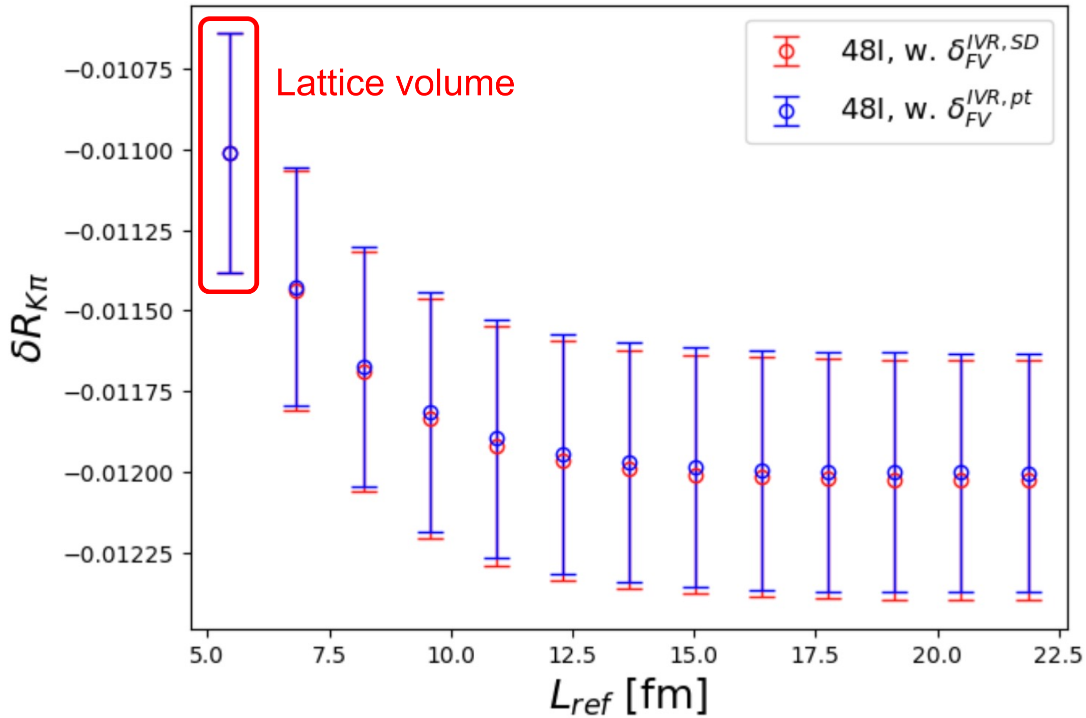
I_L^{div}: analytical calculation



total result for π TB+DE independent of t_s

IVR Finite-Volume Scaling: $O(1/L) \to O(e^{-mL})$

- Convergence with volume $V_{\text{ref}} = L_{\text{ref}}^3$



$\delta_{\text{FV}}^{\text{IVR,pt}}$: point particle

$\delta_{\text{FV}}^{\text{IVR,SD}}$: including structure dependent information

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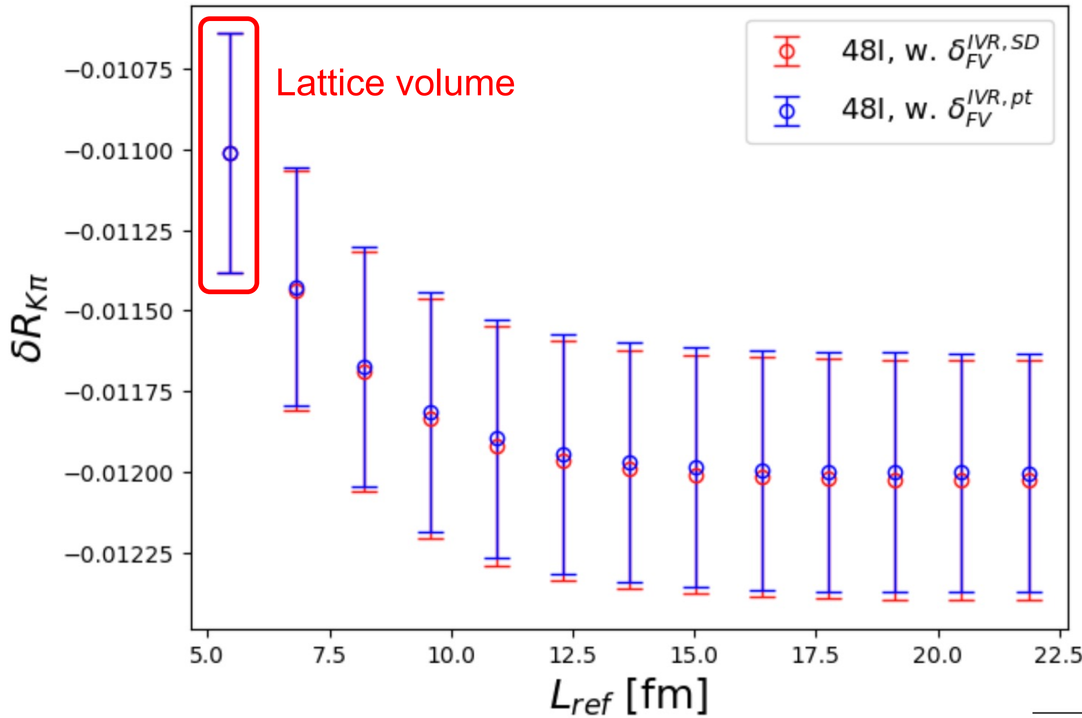


$\delta_{FV}^{IVR,pt}$: point particle

$\delta_{FV}^{IVR,SD}$: including structure dependent information

- Comparison: $O(1/L) \rightarrow O(e^{-mL})$

	$O(1/L)$	$O(1/L^2)$	$O(1/L^3)$
$\delta_{FV}^{QED_L}/0.01$	-46.8%	$+26.2\%$	-39.3%

	π	K
$\delta_{FV}^{IVR,pt}/0.01$	$+7.90\%$	$+0.004\%$
$\delta_{FV}^{IVR,SD}/0.01$	$+7.82\%$	$+0.008\%$

QED_L, pt~60% FV effects ➡

IVR, pt~8% FV effects
IVR, SD~0.08% residue errors

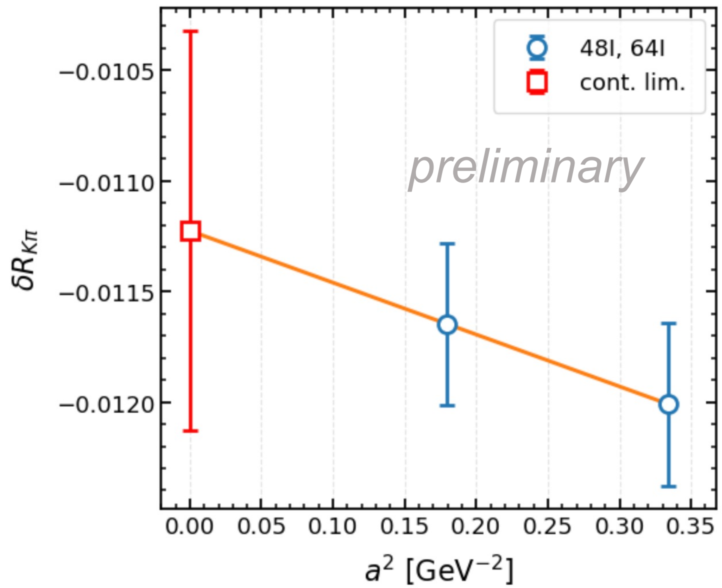
Result for $\delta R_{K\pi}$

P. Boyle, N. Christ, X. Feng, T. Izubuchi, L. Jin, C. Sachrajda, X. Tuo, paper in preparation

- First lattice calculation at physical pion mass with a controlled continuum extrapolation and negligible finite-volume errors.

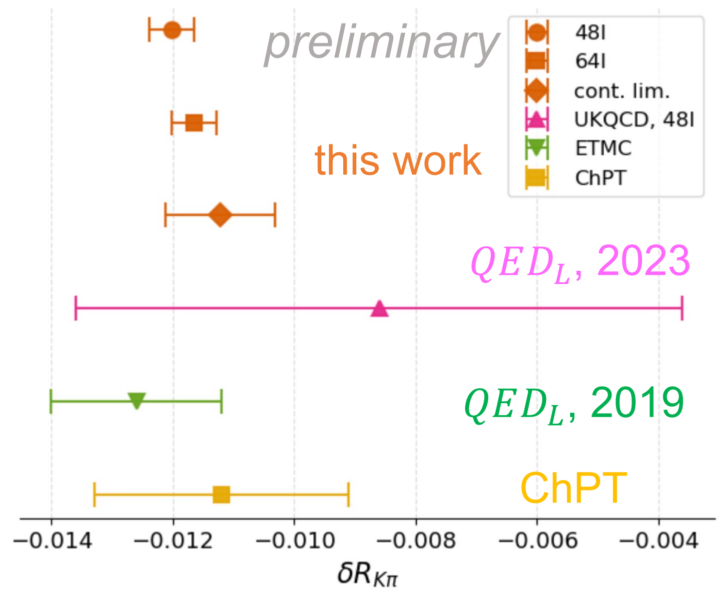
$$\delta R_{K\pi} = -0.01123(91)$$

Continuum extrapolation



Comparison with literature

M. Di Carlo, et al. PRD, 2019 [arXiv:1904.08731]
P. Boyle, et al. JHEP, 2023 [arXiv:2211.12865]
V. Cirigliano, H. Neufeld, PLB, 2011 [arXiv:1102.0563]



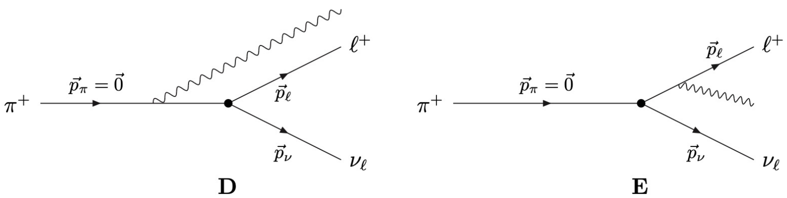
- Impact on $|V_{us}|/|V_{ud}|$ (choose $f_\pi^{\rm iso}/f_K^{\rm iso}$=1.1930(33) from FLAG)

This work: $0.23184(28)_{\rm exp}\,(10)_{\delta R_{K\pi}}(65)_{f_P^{\rm iso}}$

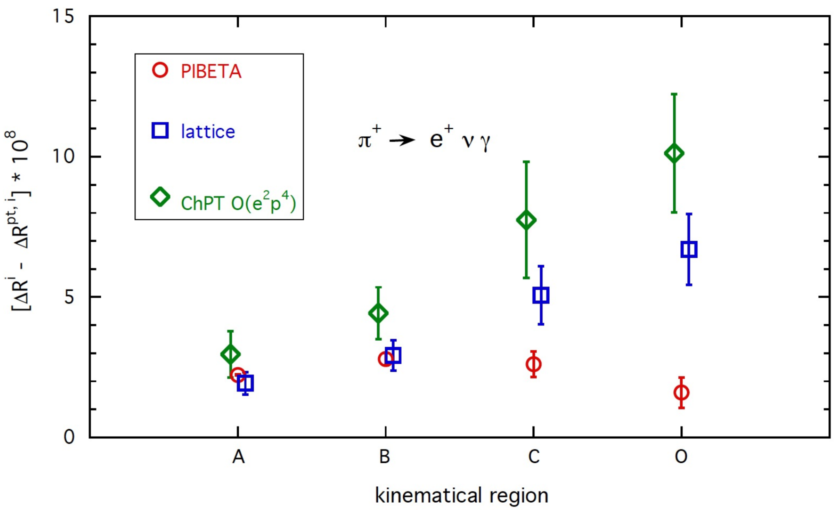
QED_L, 2023: $0.23154(28)_{\rm exp}\,(15)_{\delta R_{K\pi}}(45)_{\delta R_{K\pi},{\rm vol}}(65)_{f_P^{\rm iso}}$

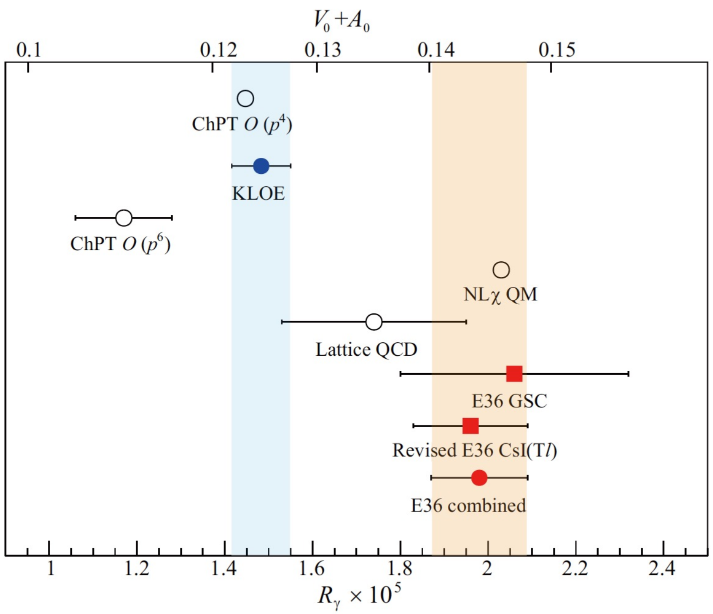
Outline

Puzzles in radiative leptonic decays



- Puzzles in radiative leptonic decays:





$\pi \to e\nu\gamma$

lattice v.s. PIBETA: 5σ discrepancy

R. Frezzotti, et al. PRD, 2021 [arXiv:2012.02120]

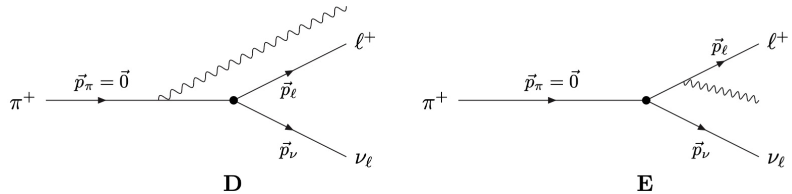
$K \to e\nu\gamma$

KLOE v.s. E36: 4σ discrepancy

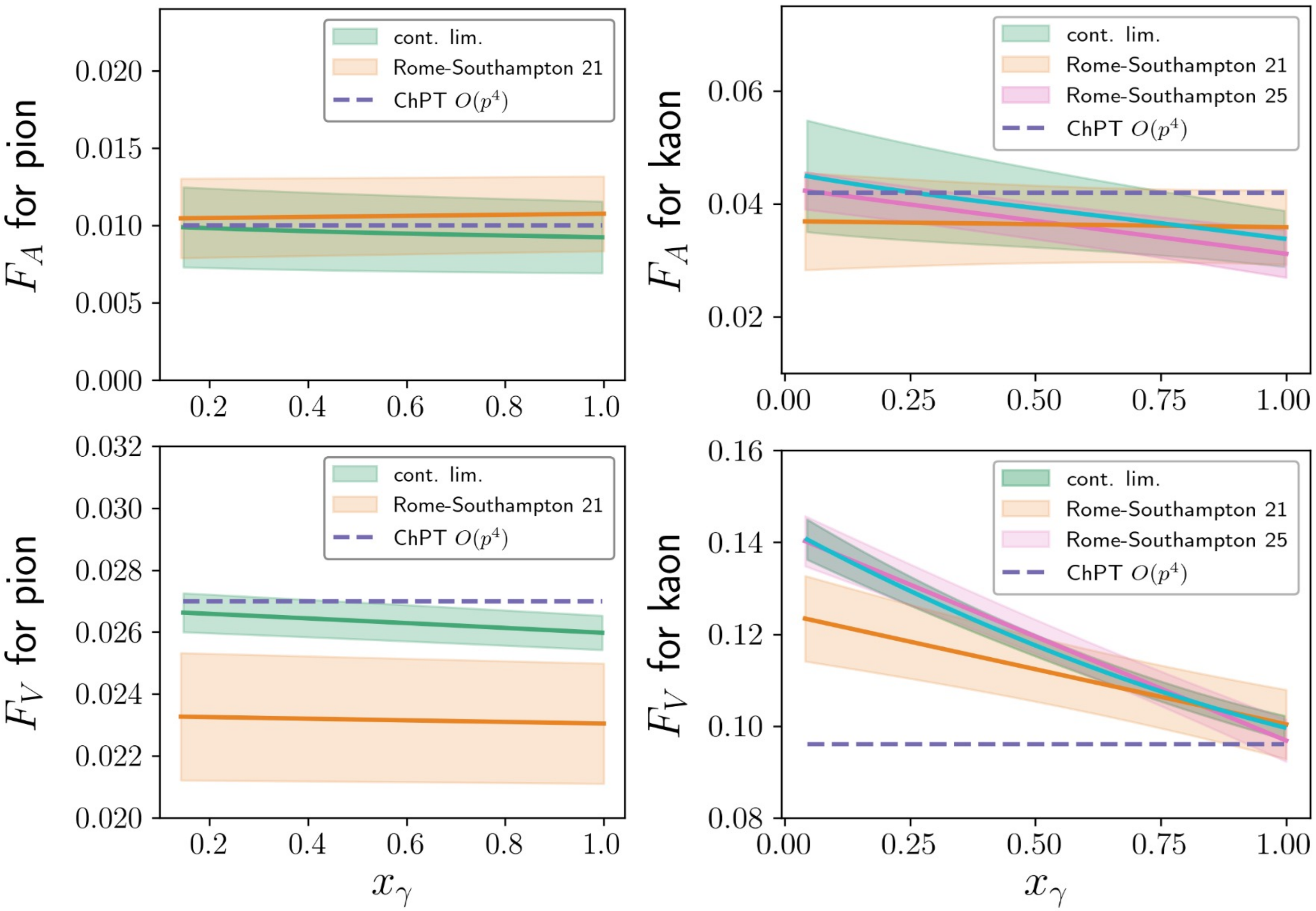
A. Kobayashi, et al. PLB, 2023 [arXiv:2212.10702]

- An independent lattice calculation is important to cross check these findings.

F_V, F_A: consistency between lattice calculations



➢ Nontrivial consistency check: different actions, different methods



green: our results

P. Boyle, et al.
[arXiv: 2510.26993]

orange: ETMC 21

R. Frezzotti, et al. PRD, 2021
[arXiv:2012.02120]

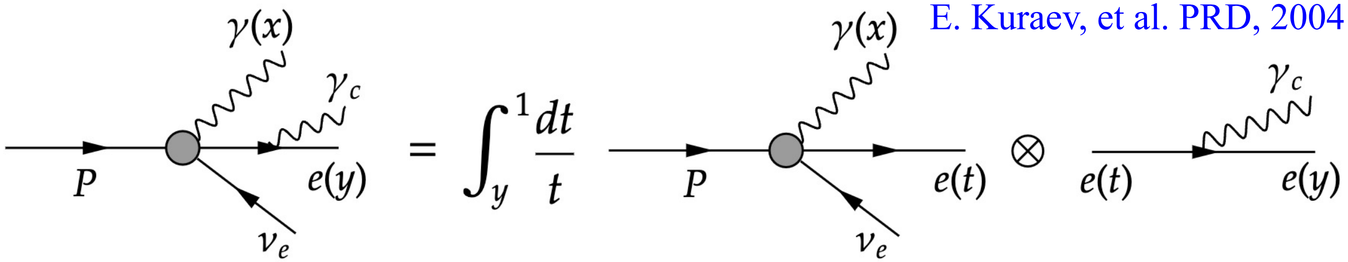
pink: ETMC 25

R. Di Palma, et al., PRD, 2025
[arXiv:2504.08680]

Puzzle 1: solved by collinear photon radiation

- Radiative correction (RC) from additional collinear photon

E. Kuraev, et al. PRD, 2004 [arXiv:0310275]



size ~ $O\left(\alpha^2 \log\frac{m_{\pi/K}^2}{m_e^2}\right)$: enhanced by $\log\frac{m_{\pi/K}^2}{m_e^2} \sim O(10)$

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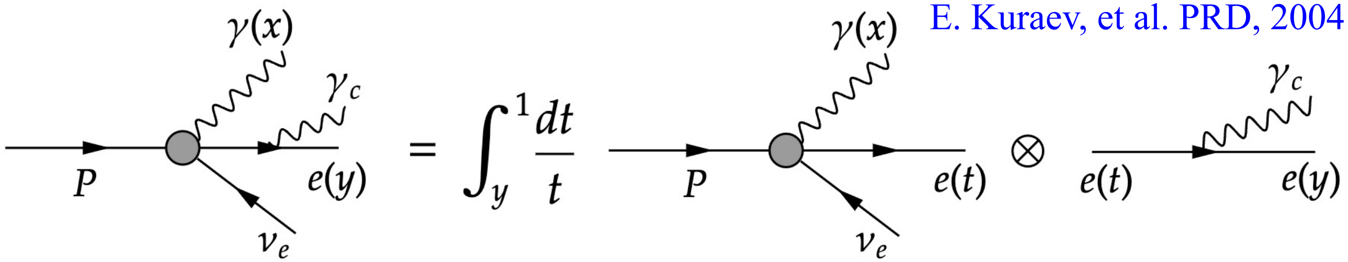
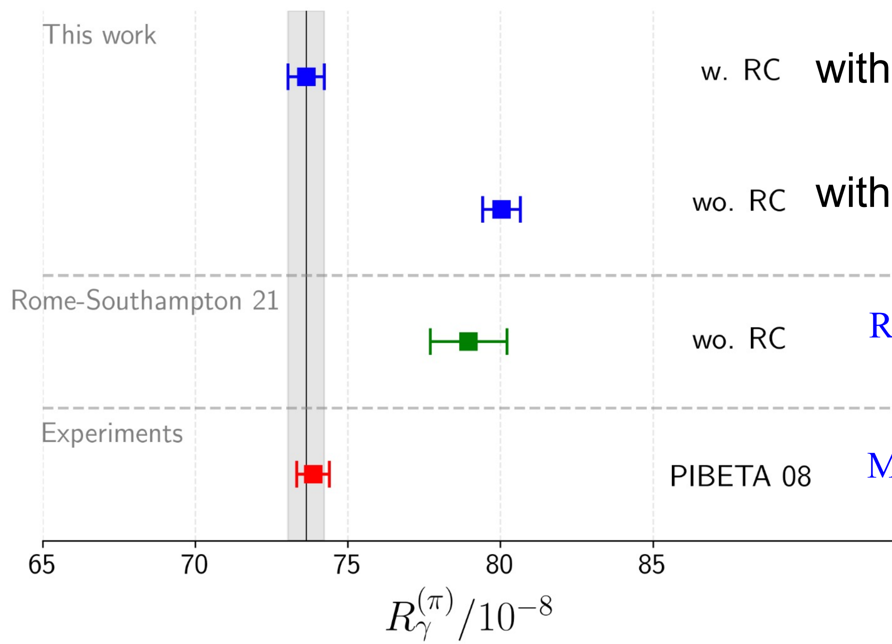


size $\sim O\left(\alpha^2 \log\frac{m_{\pi/K}^2}{m_e^2}\right)$: enhanced by $\log\frac{m_{\pi/K}^2}{m_e^2} \sim O(10)$

- Lattice results

Our work:
P. Boyle, et al.
[arXiv: 2510.26993]



with collinear RC

without collinear RC

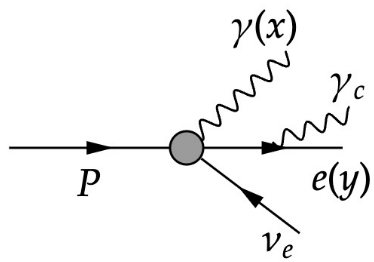
R. Frezzotti, et al. PRD, 2021 [arXiv:2012.02120]

M. Bychkov, et al. PRL, 2009 [arXiv:2012.02120]

- No discrepancy between lattice QCD and PIBETA experiments.

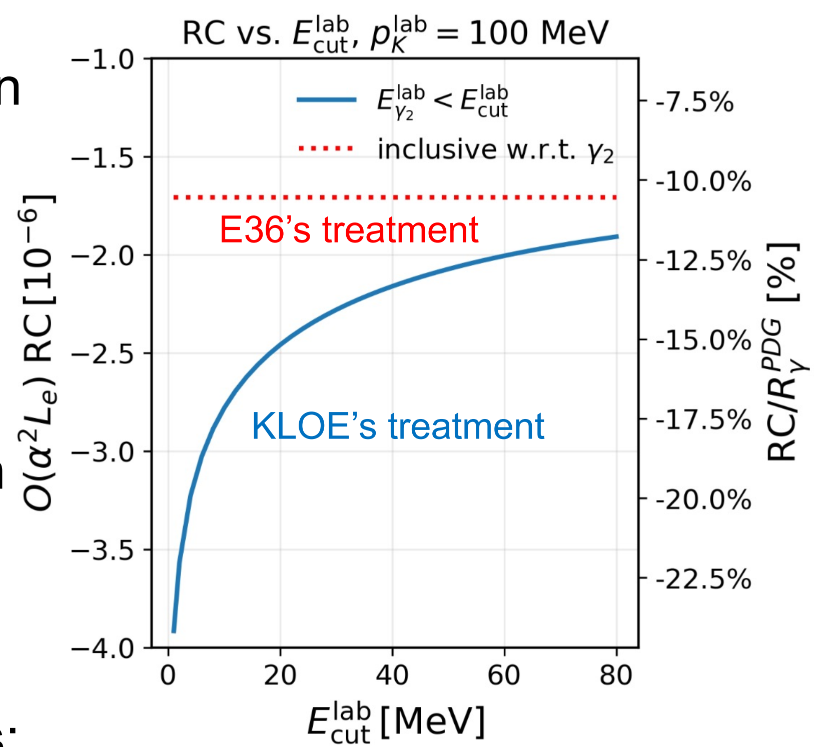
Collinear radiative corrections in $K \to e\nu\gamma$

- The event selection method for the bremsstrahlung photons are different in KLOE and E36:



- KLOE require exactly one photon, with condition $E_\gamma^{\mathrm{lab}} > 20$ MeV

- E36 measurements are inclusive with respect to the bremsstrahlung photons: $K \to e\nu\gamma(\gamma)$

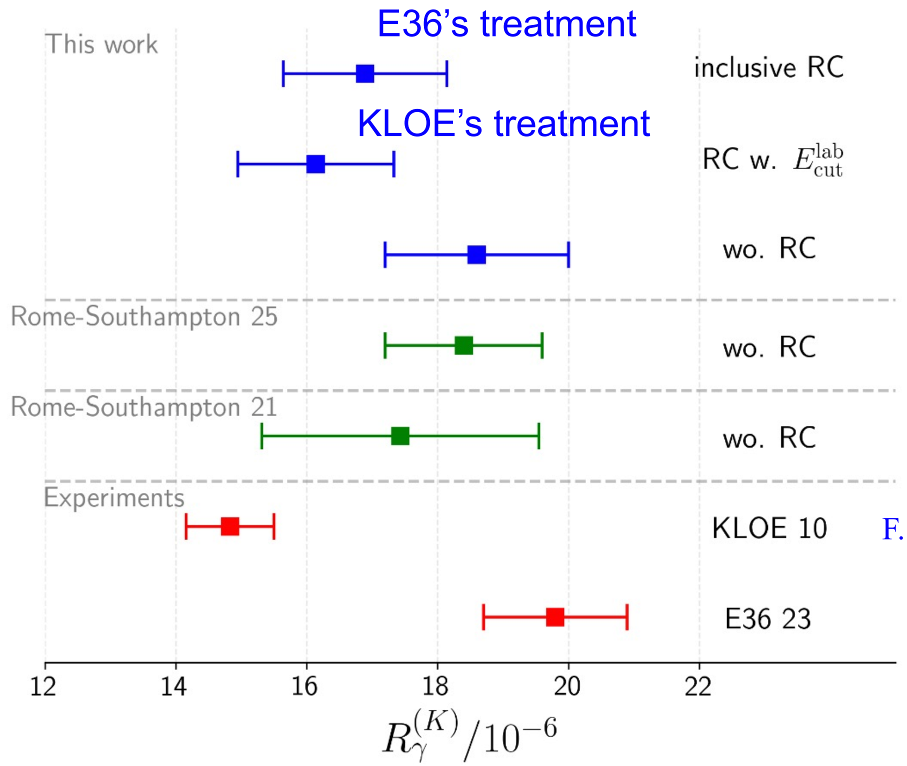


P. Boyle, et al. [arXiv: 2510.26993]

- The radiative corrections as large as $O(10\%)$ are important for a meaningful comparison between KLOE's and E36's measurements.

Lattice result: $K \to e\nu\gamma$

➢ Lattice results with different treatments



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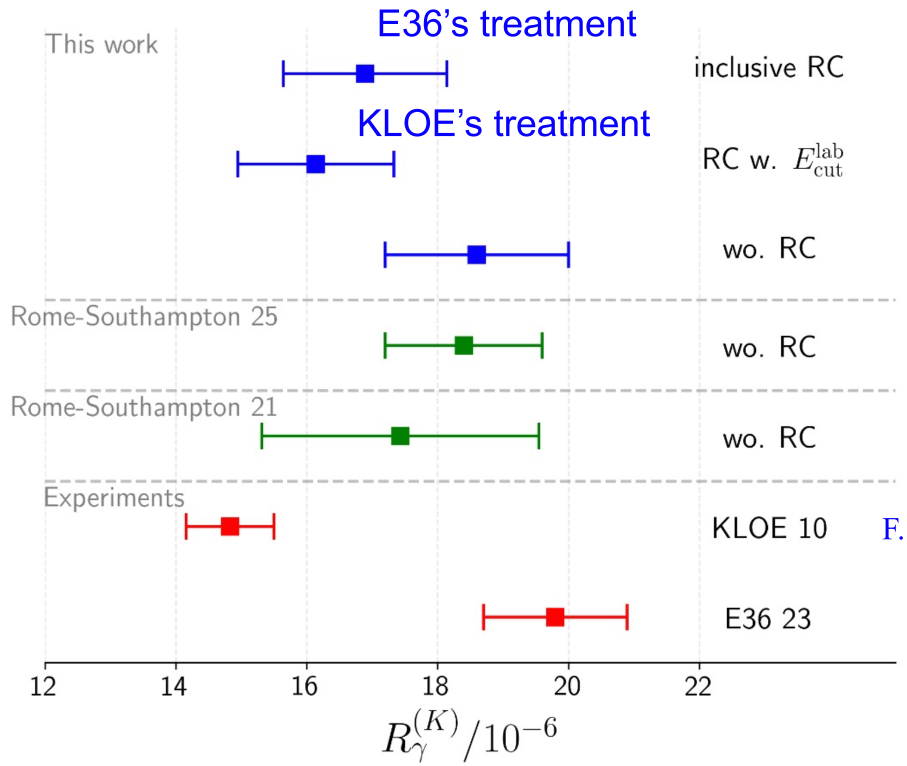

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A. Kobayashi, et al. PLB, 2023 [arXiv:2212.10702]

- New experiment ongoing: NA62.
- Following recent discussions between us and NA62 group, they plan to include collinear radiative corrections in Monte Carlo simulations of $K \to e\nu\gamma$ to ensure more direct comparison with theory.

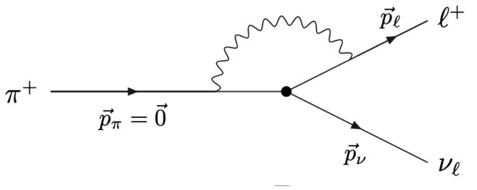
Outline

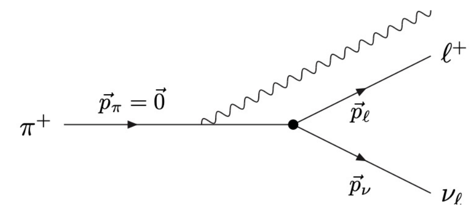
Ongoing work: lattice calculation of SD part in $R_{e\mu}$

- Hadronic structure-dependent (SD) information enters through small radiative corrections. size~$O(0.1\%)$

		$(P=\pi)$	$(P=K)$
$O(\alpha)$, pt	$\Delta^{(P)}_{e^2p^2}$ (%)	-3.929	-3.786
$O(\alpha)$, SD	$\Delta^{(P)}_{e^2p^4}$ (%)	0.053 ± 0.011	0.135 ± 0.011
$O(\alpha)$, SD	$\Delta^{(P)}_{e^2p^6}$ (%)	0.073	
resumming LLs	Δ_{LL} (%)	0.055	0.055

V. Cirigliano, I. Rosell, PRL, 2007 [arXiv:0707.3439]





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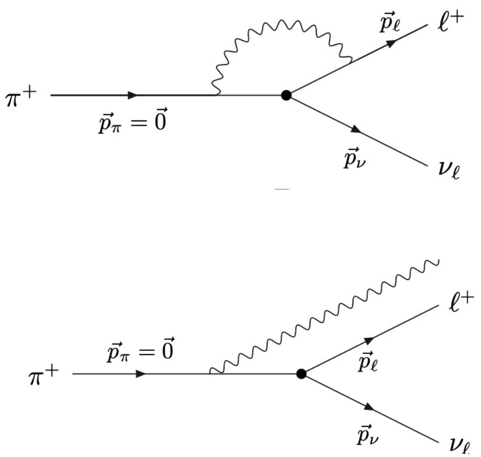

- Based on F_V and F_A calculated on lattice, SD contributions from real-photon emission can be determined with 3% relative error:

meson	ChPT	Lattice QCD
$P=\pi$	0.073%	0.0596(20)%
$P=K$	1.289	1.271(20)

- $\Gamma_{SD}(K \to e\nu\gamma)$ has similar size compared to $\Gamma(K \to e\nu)$ without radiative correction. Its effects is excluded in $R^{(K)}_{e\mu}$.

Ongoing work: SD part in $R_{e\mu}$ from virtual loop

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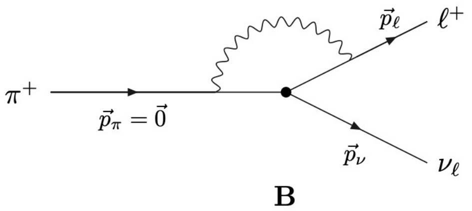

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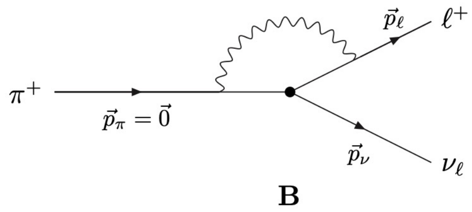


- Challenge on lattice: errors dominated by point-particle contribution. Naïve calculation: $O(10\%)$ relative error for SD, similar to ChPT precision.

- Improvement by gauge invariance: compared to Feynman gauge, point-particle contribution for TB is greatly reduced in Coulomb gauge (no IR divergence).

→ Ongoing work: lattice can improve precision to percent level for SD, complement ChPT prediction using first-principles input.

Conclusions and Outlook

- A general lattice toolbox for QED & long-distance contribution with only exponentially small finite-volume effects: IVR method.
- For radiative corrections in leptonic decay $K_{\ell 2}/\pi_{\ell 2}$, our work reduces the theoretical uncertainty in $|V_{us}|/|V_{ud}|$:

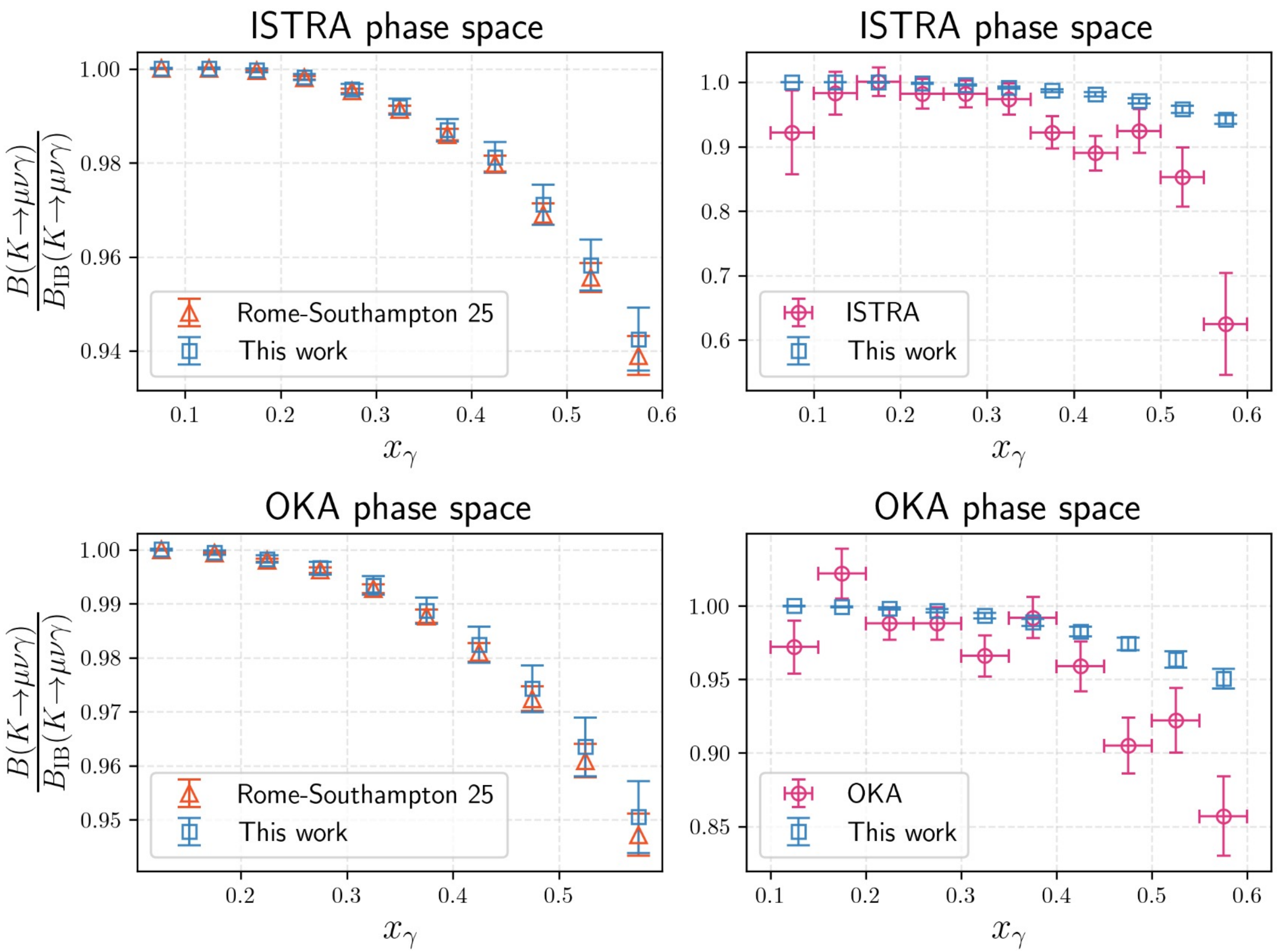
  This work: $0.23184(28)_{\text{exp}}\ (10)_{\delta R_{K\pi}}(65)_{f_P^{\text{iso}}}$

  QED_L, 2023: $0.23154(28)_{\text{exp}}\ (15)_{\delta R_{K\pi}}(45)_{\delta R_{K\pi},\text{vol}}(65)_{f_P^{\text{iso}}}$

- For $K/\pi \to e\nu\gamma$, the collinear radiative corrections are the key to understand current puzzles in lattice and experiments.
- For radiative corrections to $R_{e\mu}$, we plan to calculate SD contributions with percent level precision.
- Outlook: A key next step is the lattice determination of radiative corrections in semileptonic decays, $K_{\ell 3}$ and $\pi_{\ell 3}$.

Backup Slides

Results for $K \to \mu\nu\gamma$: OKA, ISTRA

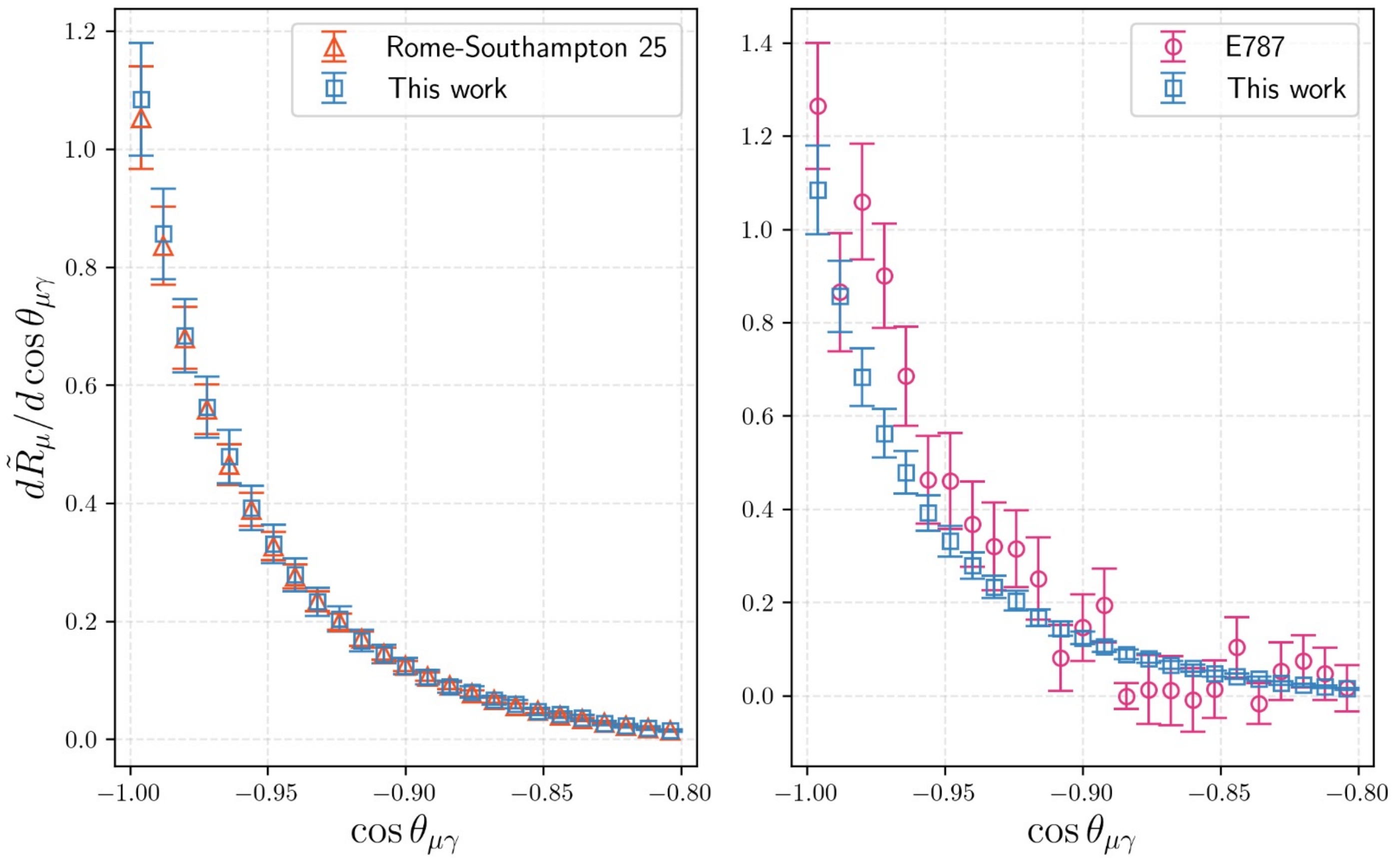


R. Di Palma, et al. [arXiv:2504.08680]

V. I. Kravtsov et al. (OKA), Eur. Phys. J. C, 2019 [arXiv:1904.10078]

V. A. Duk, et al. (ISTRA+), PLB, 2011 [arXiv:1005.3517]

Results for $K \to \mu\nu\gamma$: E787



R. Di Palma, et al. [arXiv:2504.08680]

S. Adler, et al. (E787), PRL, 2000 [arXiv:0003019]