COLLISIONAL AND RADIATIVE ENERGY LOSS IN PRESENCE OF CHIRAL MAGNETIC CURRENT

KIRILL TUCHIN

IOWA STATE UNIVERSITY
OF SCIENCE AND TECHNOLOGY

INT Workshop "Probing QCD at high energy and density with jets"

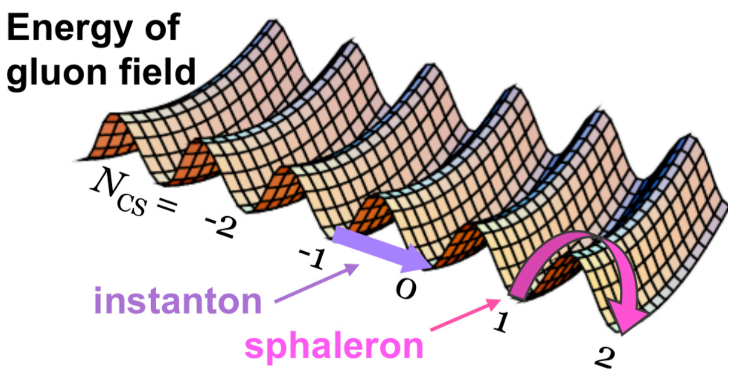
October 19th, 2023

OUTLINE

- Chiral matter, systems with chiral fermions
- Collisional energy loss: from classical Fermi model to QFT
- Radiative energy loss
- Boundary effects

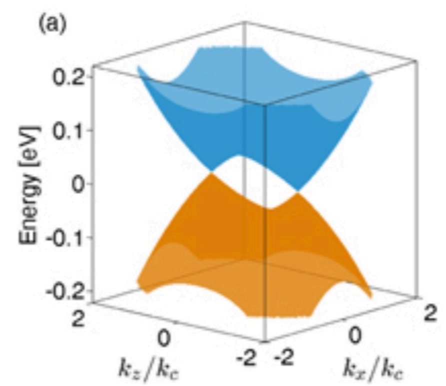
CHIRAL MEDIA

Source of chirality:	**Quark-gluon plasma**	**Dirac & Weyl semimetals**	**Dark matter**
	Sphaleron transitions	*Emergent chiral quasiparticles*	*Axion field*



Chiral magnetic effect in $ZrTe_5$

Qiang Li[1]*, Dmitri E. Kharzeev[2,3]*, Cheng Zhang[1], Yuan Huang[4], I. Pletikosi
R. D. Zhong[1], J. A. Schneeloch[1], G. D. Gu[1] and T. Valla[1]*



$\mathcal{P}$-odd fluctuations and long range order in heavy ion collisions. Deformed QCD as a toy model

Ariel R. Zhitnitsky

Do dark matter axions form a condensate with long-range correlation?

Alan H. Guth,[1,*] Mark P. Hertzberg,[1,2,†] and C. Prescod-Weinstein[3,‡]

Low-dimensional long-range topological charge structure in the QCD vacuum

I. Horváth,[1] S. J. Dong,[1] T. Draper,[1] F. X. Lee,[2,3] K. F. Liu,[1] N. Mathur,[1] H. B. Thacker,[4] and J. B. Zhang[5]

CHIRAL MAGNETIC EFFECT

Sikivie (84), Wilczek (87), Carroll et al (90)

Maxwell-Chern-Simons theory $\mathcal{L}_{\mathrm{MCS}} = \mathcal{L}_{\mathrm{QED}} + c_A \theta(x) \vec{E} \cdot \vec{B}$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0\,,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho - c\,\boldsymbol{\nabla}\theta \cdot \boldsymbol{B}\,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B}\,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j} + c(\partial_t \theta\, \boldsymbol{B} + \boldsymbol{\nabla}\theta \times \boldsymbol{E})\,,$$

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Anomalous Hall Effect

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(assume for now ∇θ=0)

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Chiral magnetic effect:

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Anomalous Hall Effect

Chiral magnetic effect: $\boldsymbol{j} = \sigma_\chi \boldsymbol{B} \equiv b_0 \boldsymbol{B}$

P-odd, T-odd

P-even, T-odd

⇒Breaks Parity!

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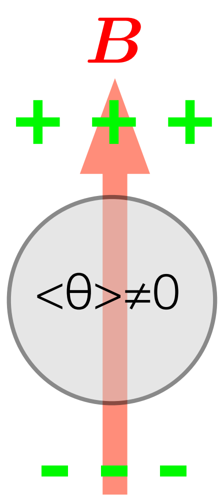
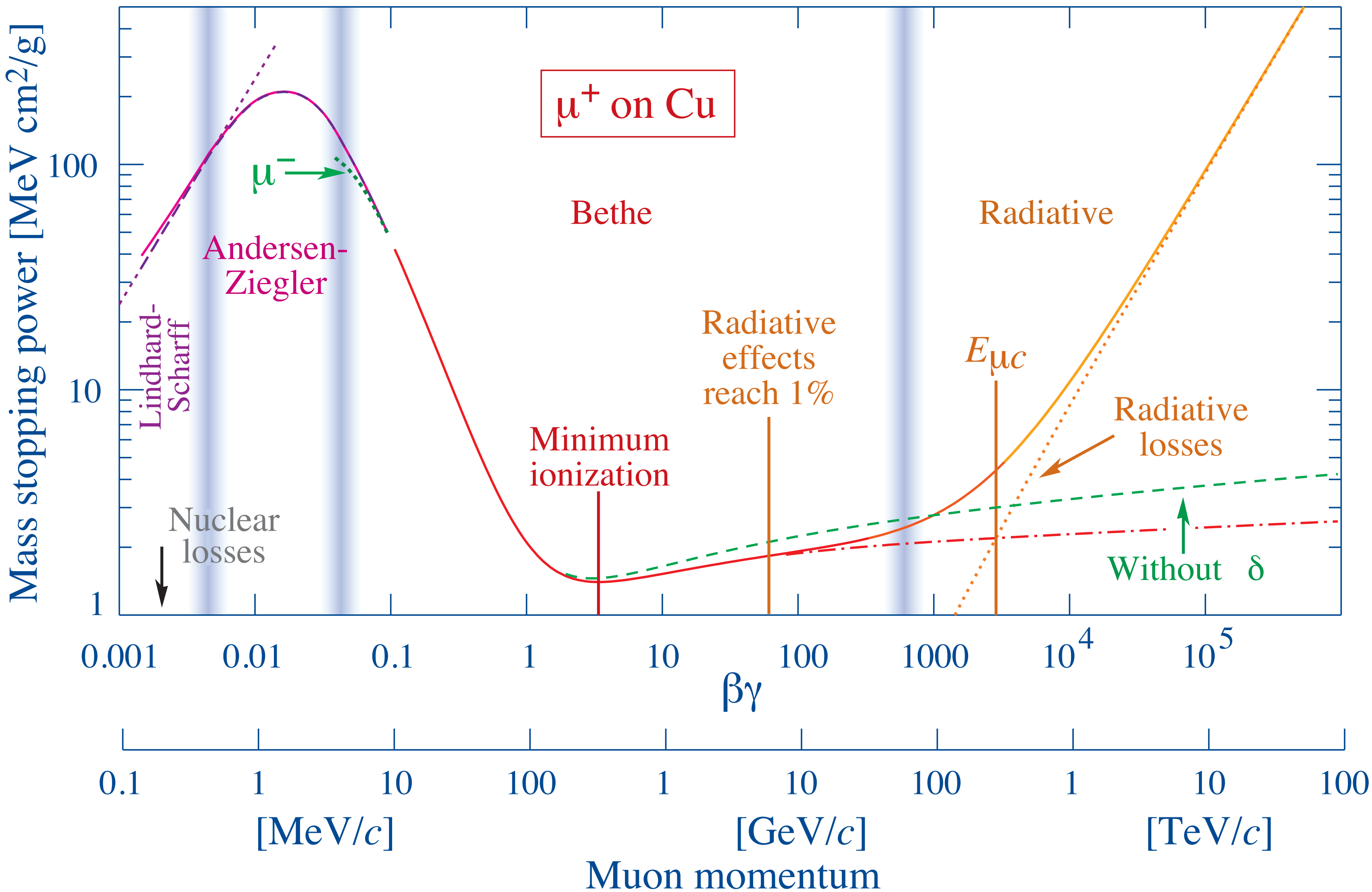
Kharzeev, Zhitnitsky (2007),
Kharzeev, McLerran, Warringa (2008)

Charge separation:

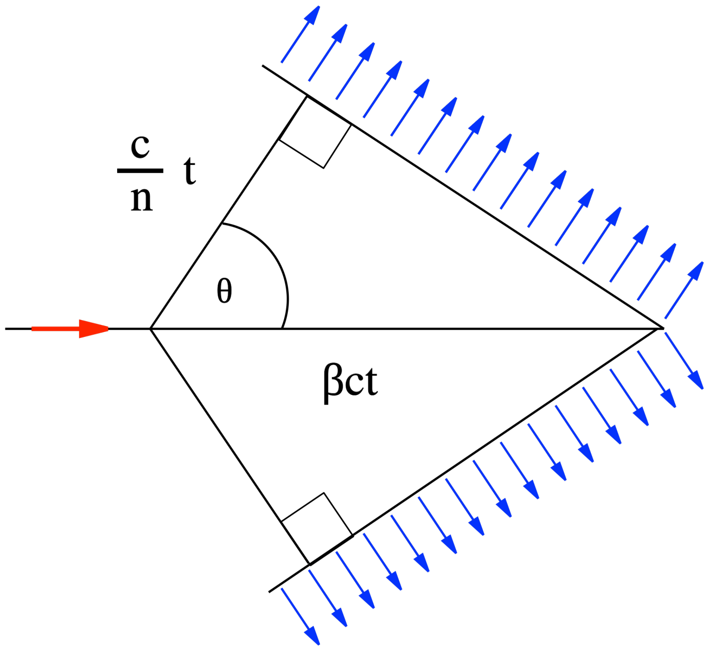


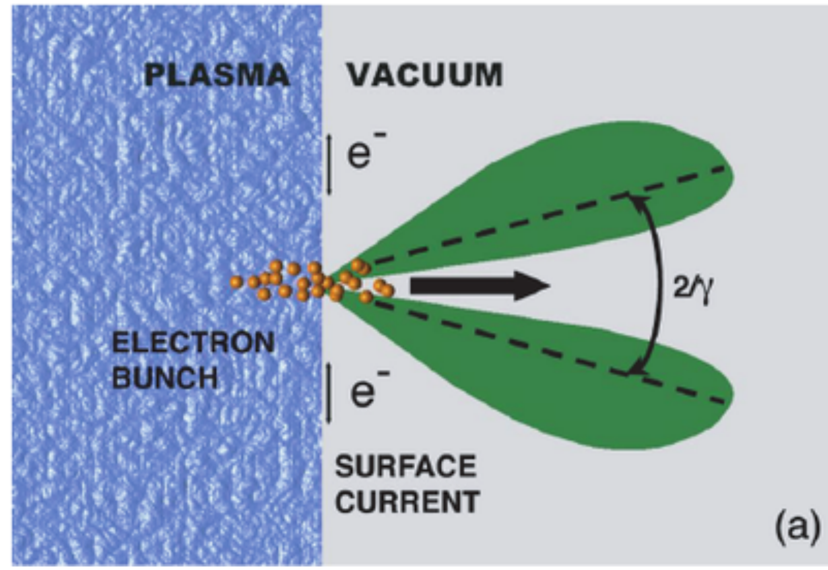
Critical assumption:
existence of chiral domains.

PROBING MATTER: ENERGY LOSS



PROBING MATTER: CHERENKOV AND TRANSITION RADIATION





Classical Cherenkov radiation is emitted by a charged particle that moves faster than the phase velocity of light: $vn>1$

$$\cos\theta = \frac{1}{\beta\sqrt{\epsilon}} = \frac{1}{\beta n}$$

Classical transition radiation is emitted by a charged particle that moves through *inhomogeneous* matter.

FERMI'S MODEL OF COLLISIONAL ENERGY LOSS

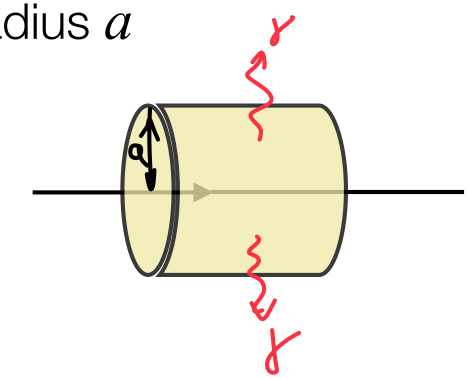
MARCH 15, 1940 PHYSICAL REVIEW VOLUME 57

The Ionization Loss of Energy in Gases and in Condensed Materials*

ENRICO FERMI
Pupin Physics Laboratories, Columbia University, New York, New York
(Received January 22, 1940)

The energy loss rate = flux of the Poynting vector out of cylinder of radius a coaxial with the particle path:

$$-\frac{d\varepsilon}{dz} = 2\pi a \int_{-\infty}^{\infty} (E_\phi B_z - E_z B_\phi) dt = 2a\,\mathrm{Re} \int_0^{\infty} (E_{\phi\omega} B^*_{z\omega} - E_{z\omega} B^*_{\phi\omega}) d\omega$$



Maxwell equations $\nabla \times \boldsymbol{B}_\omega = -i\omega \boldsymbol{D}_\omega + j_\omega$ etc.

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

Energy loss: $a \to 0$ UR limit: $-\frac{d\varepsilon}{dz} = \frac{q^2}{4\pi v^2}\omega_p^2 \ln\frac{v}{a\omega_p}$

(small) Cherenkov radiation contribution emerges at $a \to \infty$ if $v > 1/\sqrt{\epsilon(0)}$

EM FIELDS OF POINT CHARGE IN CHIRAL MEDIUM 1

Hansen, KT, 2012.06089

EM field of a point charge with large enough constant velocity v

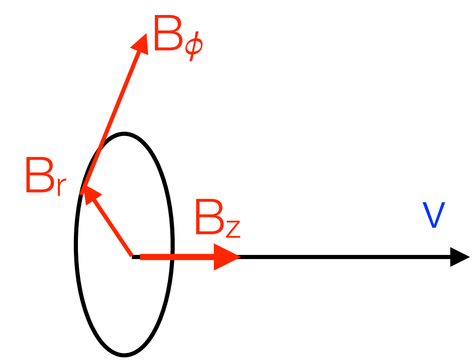
$$\nabla \times \boldsymbol{B} = \partial_t \boldsymbol{D} + \sigma_\chi \boldsymbol{B} + qv\hat{\boldsymbol{z}}\delta(z - vt)\delta(\boldsymbol{b}) \,,$$

$$\nabla \cdot \boldsymbol{D} = q\delta(z - vt)\delta(\boldsymbol{b}) \,,$$

$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B} \,,$$

$$\nabla \cdot \boldsymbol{B} = 0 \,,$$

impact parameter



$$\boldsymbol{B}(\boldsymbol{r}, t) = \int \frac{d^2k_\perp d\omega}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t} \sum_\lambda \boldsymbol{\epsilon}_{\lambda\boldsymbol{k}} \frac{q\hat{\boldsymbol{z}} \cdot \boldsymbol{\epsilon}^*_{\lambda\boldsymbol{k}} \lambda k}{k_\perp^2 + \omega^2(1/v^2 - \epsilon) - \lambda\sigma_\chi k} \,,$$

$$\boldsymbol{E}(\boldsymbol{r}, t) = \int \frac{d^2k_\perp d\omega}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t} \left(\sum_\lambda \boldsymbol{\epsilon}_{\lambda\boldsymbol{k}} \frac{iq\omega\hat{\boldsymbol{z}} \cdot \boldsymbol{\epsilon}^*_{\lambda\boldsymbol{k}}}{k_\perp^2 + \omega^2(1/v^2 - \epsilon) - \lambda\sigma_\chi k} + \hat{\boldsymbol{k}} \frac{q}{ivk\varepsilon} \right) \,,$$

High energy approximation:

$$B_\phi = \frac{eb}{8\pi x_-^2} e^{-\frac{b^2\sigma}{4x_-}} \left[\sigma \cos\left(\frac{b^2\sigma_\chi}{4x_-} \right) + \sigma_\chi \sin\left(\frac{b^2\sigma_\chi}{4x_-} \right) \right]$$

Finite $\sigma_\chi \Rightarrow$ field oscillations

EM FIELDS OF POINT CHARGE IN CHIRAL MEDIUM 2

Need ω-Fourier components:

$$\boldsymbol{B}_\omega(\boldsymbol{r}) = \int \frac{d^2k_\perp}{(2\pi)^2} \frac{q\,k\,e^{i\omega z/v + i\boldsymbol{k}_\perp\cdot\boldsymbol{b}}}{[k_\perp^2 + \omega^2(1/v^2-\epsilon)]^2 - (\sigma_\chi k)^2}$$
$$\times \left\{ [k_\perp^2 + \omega^2(1/v^2-\epsilon)] \sum_\lambda \lambda \boldsymbol{\epsilon}_{\lambda\boldsymbol{k}}(\hat{\boldsymbol{z}}\cdot\boldsymbol{\epsilon}^*_{\lambda\boldsymbol{k}}) + \sigma_\chi k \sum_\lambda \boldsymbol{\epsilon}_{\lambda\boldsymbol{k}}(\hat{\boldsymbol{z}}\cdot\boldsymbol{\epsilon}^*_{\lambda\boldsymbol{k}}) \right\}$$

can be computed analytically for constant chiral conductivity, e.g.:

$$B_{\phi\omega}(\boldsymbol{r}) = \frac{q}{2\pi}\frac{e^{i\omega z/v}}{k_1^2-k_2^2}\sum_{\nu=1}^{2}(-1)^{\nu+1}k_\nu(k_\nu^2-s^2)K_1(bk_\nu)$$

$$B_{b\omega}(\boldsymbol{r}) = \sigma_\chi \frac{q}{2\pi}\frac{i\omega}{v}\frac{e^{i\omega z/v}}{k_1^2-k_2^2}\sum_{\nu=1}^{2}(-1)^{\nu}k_\nu K_1(bk_\nu)$$

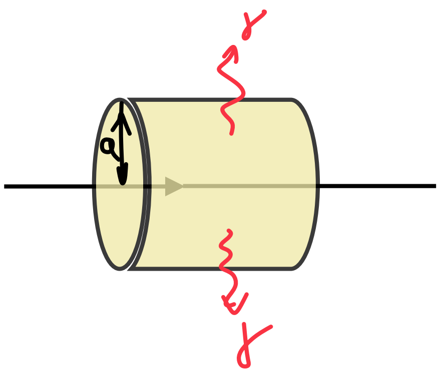
with $k_\nu^2 = s^2 - \frac{\sigma_\chi^2}{2} + (-1)^\nu \sigma_\chi \sqrt{\omega^2\epsilon + \frac{\sigma_\chi^2}{4}}$ and $s^2 = \omega^2\left(\frac{1}{v^2} - \epsilon(\omega)\right)$

- Cherenkov radiation: $s^2<0$, $\sigma_\chi=0$ — small contribution to the total energy loss

FERMI'S MODEL WITH CHIRAL MAGNETIC CURRENT

Hansen, KT, 2012.06089

$$-\frac{d\varepsilon}{dz} = 2\pi a \int_{-\infty}^{\infty} (E_\phi B_z - E_z B_\phi) dt = 2a\,\mathrm{Re} \int_0^{\infty} (E_{\phi\omega} B_{z\omega}^* - E_{z\omega} B_{\phi\omega}^*) d\omega$$



For simplicity consider $\omega_0 = 0$

UR limit $\gamma \gg 1$ at $a \to 0$ gives energy loss

$$-\frac{d\varepsilon}{dz} = \frac{q^2}{4\pi v^2} \left(\omega_p^2 \ln \frac{v}{a\omega_p} + \frac{1}{4} \gamma^2 \sigma_\chi^2 \right)$$

increases as (energy)^2 due to the anomaly

Chiral Cherenkov radiation emerges at $a \to \infty$ even if $\epsilon = 1$

$$\frac{dW}{d\omega} = -\frac{d\varepsilon}{dz\omega d\omega}\bigg|_{a\to\infty} = \frac{q^2}{4\pi} \left\{ \frac{1}{2}\left(1 - \frac{1}{v^2}\right) + \frac{\sigma_\chi}{2\omega} + \frac{(1+v^2)\sigma_\chi^2}{8v^2\omega^2} + \ldots \right\}, \quad \omega < \sigma_\chi \gamma^2$$

Power of chiral Cherenkov radiation $P = \frac{q^2}{4\pi} \frac{\sigma_\chi^2 \gamma^2}{4}$

In the UR limit, energy loss is due to the ***chiral*** *Cherenkov radiation.*

EG: ENERGY LOSS IN A SEMIMETAL (CLASSICAL)

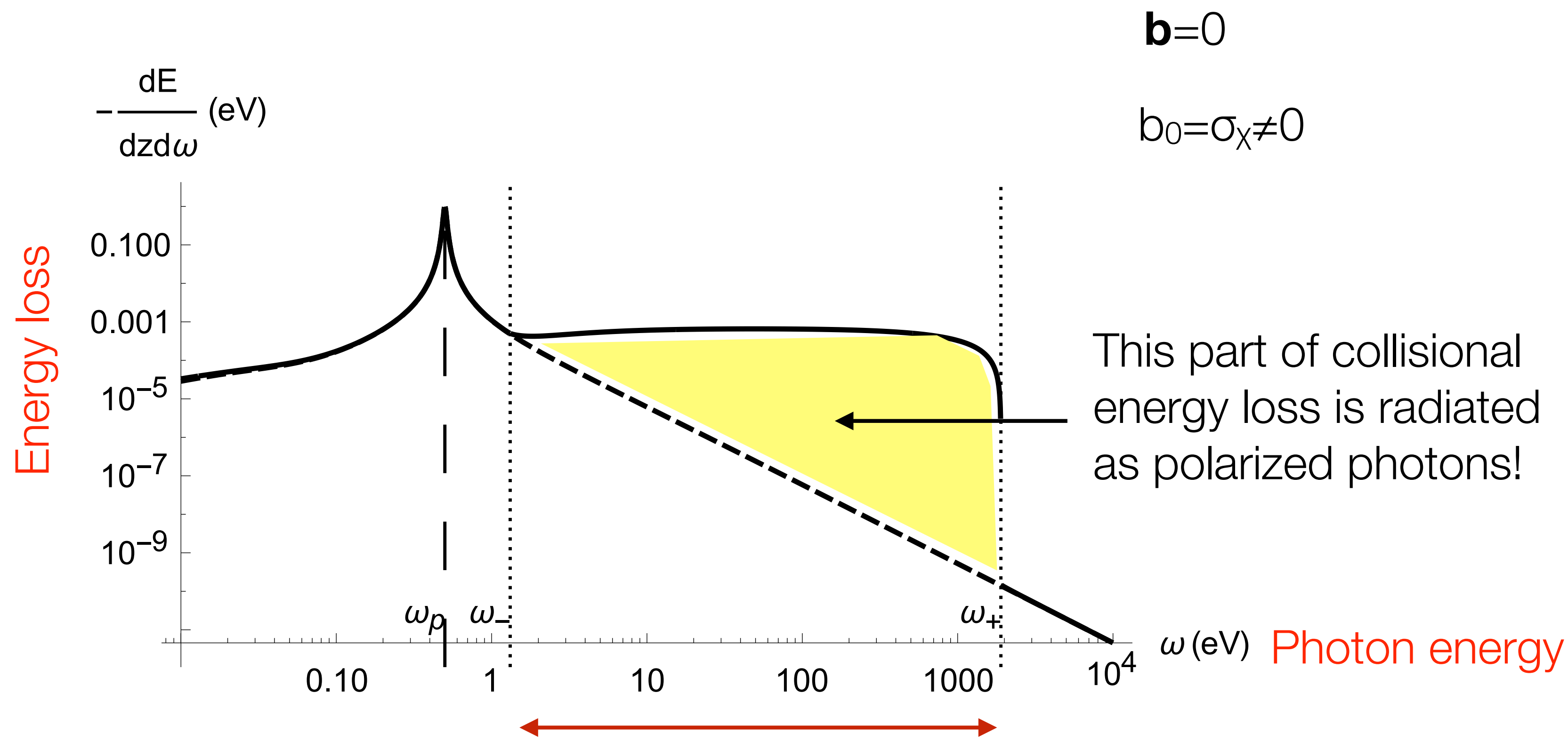


FIG. 2. Collisional energy loss spectrum of electron with $\gamma = 100$ in a semimetal with parameters $\omega_p = 0.5$ eV, $\Gamma = 0.025$ eV (so that its conductivity is 10 eV at room tempearture) [41] and $m = 0.5$ MeV. Solid line: $\sigma_\chi = 0.19$ eV [42, 43], dashed line: $\sigma_\chi = 0$. $\omega_\pm$ are defined in (13). The seeming discontinuity at $\omega = \omega_+$ is a visual artifact.

EG: ENERGY LOSS IN QGP (CLASSICAL)

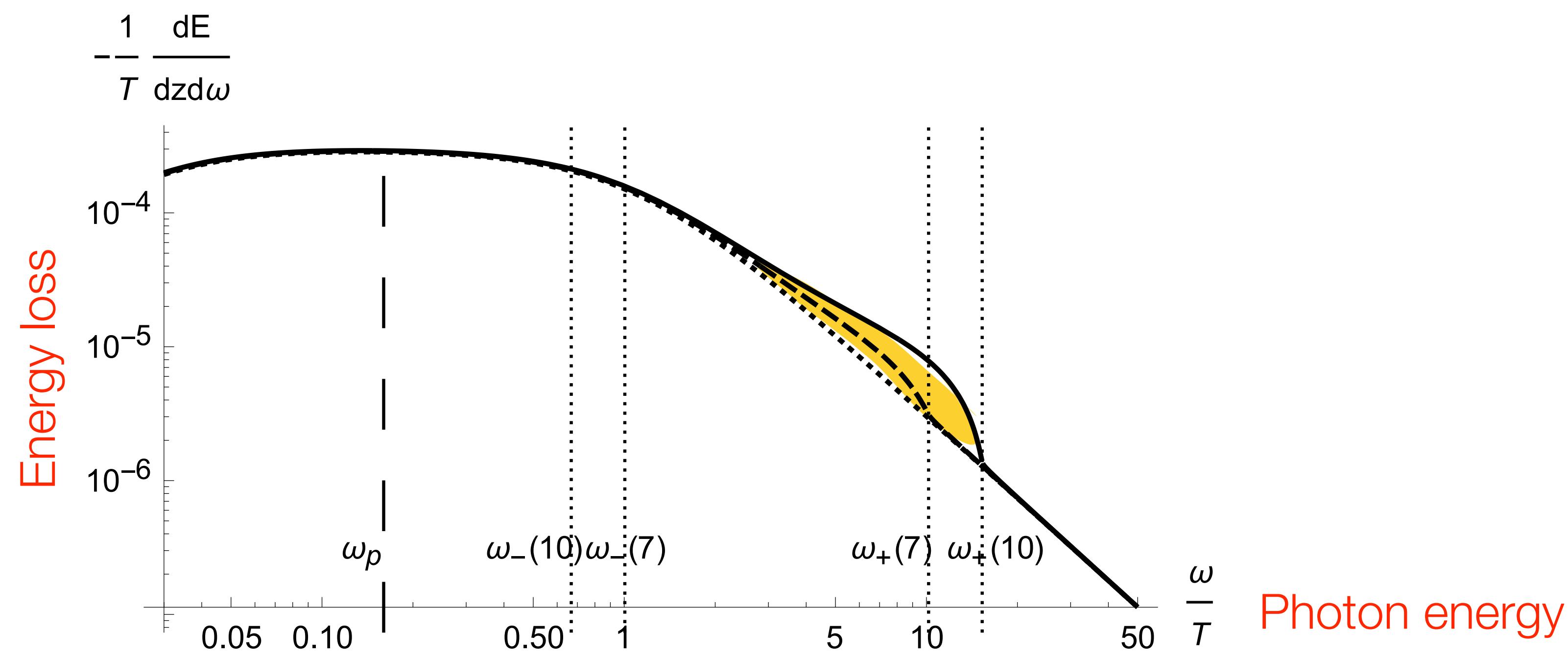


FIG. 1. Electromagnetic part of the collisional energy loss spectrum of a d-quark with $\gamma = 20$ in Quark-Gluon Plasma. Plasma parameters: $\omega_p = 0.16T$, $\Gamma = 1.11T$ [36], $m = T = 250$ MeV. Solid line: $\sigma_\chi = 10$ MeV, dashed line: $\sigma_\chi = 7$ MeV, dotted line: $\sigma_\chi = 0$. $\omega_\pm$ are defined in (13).

The same qualitative picture in **QCD** (after e→g, including color factors etc.)

$$-\left.\frac{d\varepsilon}{dz}\right|_{\text{anom}} = \frac{g^2 C_F}{4\pi}\frac{\tilde{\sigma}_\chi \varepsilon}{3}$$

COLLISIONAL ENERGY LOSS IN QFT: 1→2 PROCESSES

KT, 1702.07329

In radiation gauge: $\nabla^2 \boldsymbol{A} = \partial_t^2 \boldsymbol{A} - \sigma_\chi \boldsymbol{\nabla} \times \boldsymbol{A}$

The dispersion relation $k^2 = -\lambda \sigma_\chi |\boldsymbol{k}|$ $\rightarrow$ photon becomes space- or timelike

λ =helicity

$k^2 = (p \pm p')^2 = 2m(m \pm \varepsilon)$ forbidden in vacuum, but allowed in chiral medium

Pair production: $k^2 > 0 \Rightarrow \lambda \sigma_\chi < 0$

Photon radiation: $k^2 < 0 \Rightarrow \lambda \sigma_\chi > 0$

UR approx.: $$\boldsymbol{A} = \frac{1}{\sqrt{2\omega V}} \boldsymbol{\epsilon}_\lambda \, e^{i\omega z + i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp - i\omega t} \exp\left\{ -i \frac{1}{2\omega} \int_0^z \left[k_\perp^2 - \sigma_\chi(z')\omega\lambda \right] dz' \right\}$$

"m_γ^2"

(Neglect plasma frequency)

CHIRAL CHERENKOV RADIATION IN QED

KT, 1702.07329

$$\mathcal{M} = -eQ\bar{u}(p')\gamma^\mu u(p)\epsilon^*_\mu \times 4\pi\varepsilon x(1-x)\delta(q_\perp^2 + \kappa_\lambda)$$

$$x = \frac{\omega}{\varepsilon} \qquad \kappa_\lambda(z) = x^2 m^2 - (1-x)x\lambda\sigma_\chi\varepsilon$$ can become negative!

Chiral Cherenkov effect: photon radiation at $\vartheta \sim \sqrt{|\sigma_\chi|/\omega}$

Kappa is negative if $\lambda\sigma_\chi > 0$ and $x < x_0 = \dfrac{1}{1 + m^2/(\lambda\sigma_\chi\varepsilon)}$ $\Rightarrow$ $\omega < \omega^* = \dfrac{\lambda b_0 \varepsilon^2}{\lambda b_0\varepsilon + m^2}$

- Photon radiation rate:

$$\frac{dW_+}{dx} = \frac{\alpha Q^2}{2\varepsilon x}\left\{\sigma_\chi\varepsilon\left(\frac{x^2}{2} - x + 1\right) - m^2 x\right\}\theta(x_0 - x)$$

Vanishes as $\hbar \to 0$

Quantum anomaly!

$$\frac{dW_-}{dx} = 0\,.$$

Classical limit: $x \to 0$ (no recoil)

- Total rate of energy loss:

$$-\frac{d\varepsilon}{dz} = \int_0^1 \frac{dW_+}{dx} x\varepsilon dx = \frac{1}{3}\alpha Q^2 \sigma_\chi \varepsilon$$

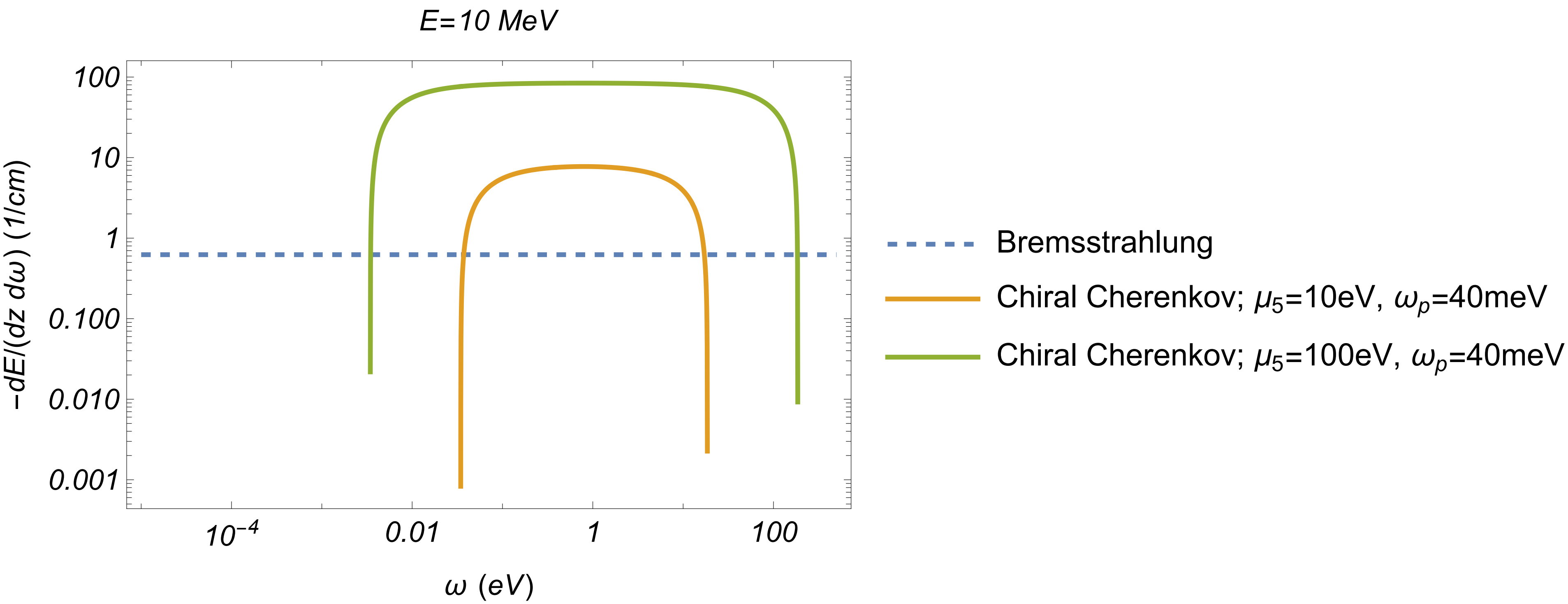
Thus the recoil reduces the energy loss $\gamma^2 \to \gamma$

CHIRAL CHERENKOV RADIATION AT MODERATE ENERGIES

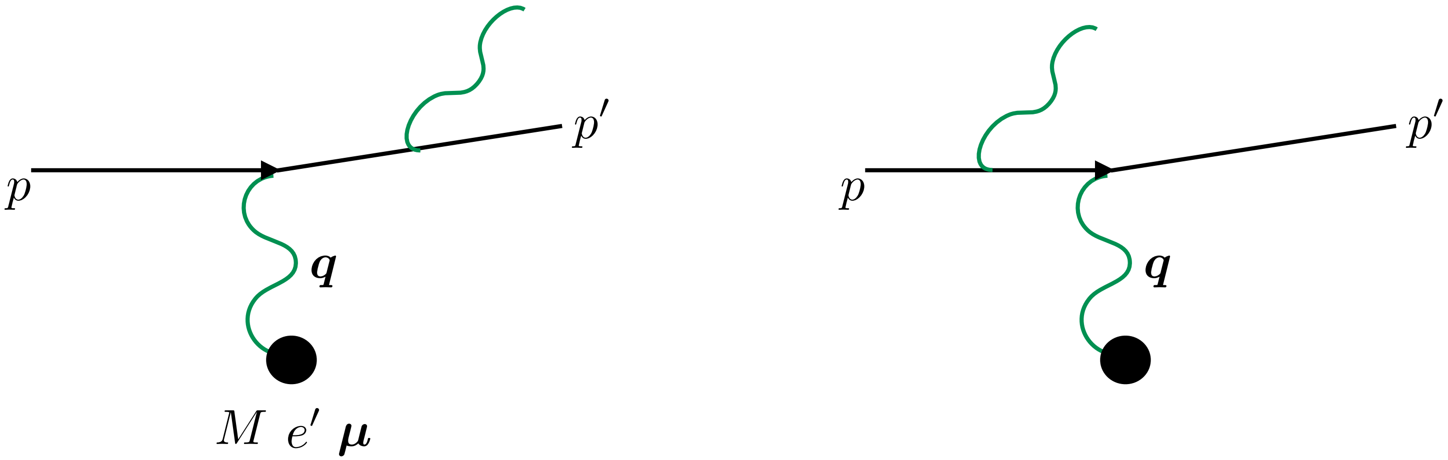
At not very high energies, need to take ω_p into account:

$$\omega^2 - \boldsymbol{k}^2 \approx -\lambda\sigma_\chi\omega + \omega_p^2 \quad \Rightarrow \quad \kappa_\lambda = x^2m^2 - (1-x)x\lambda\sigma_\chi\varepsilon + (1-x)\omega_p^2$$

$$\Rightarrow \quad \omega_- < \omega < \omega_+$$



Hansen, KT, 2022,2023



Photon propagator: $\left[g^{\mu\nu}\partial^2 - (1-1/\xi)\partial^\mu\partial^\nu - \epsilon^{\mu\nu\alpha\beta}b_\alpha\partial_\beta\right] D_{\nu\lambda}(x) = i\delta^\mu{}_\lambda\delta^4(x)$

$$D^{\nu\lambda}(k) = -\frac{i}{k^4 + b^2k^2 - (k\cdot b)^2}\Bigg\{k^2 g^{\nu\lambda} + b^\nu b^\lambda + i\epsilon_{\nu\lambda\alpha\beta}b^\alpha k^\beta$$
$$- \frac{(b\cdot k)}{k^2}(k^\nu b^\lambda + k^\lambda b^\nu) + \left[b^2\xi - (1-\xi)\left(k^2 - \frac{(b\cdot k)^2}{k^2}\right)\right]\frac{k^\lambda k^\nu}{k^2}\Bigg\}$$

Lenhert, Potting

Qiu, Cao, Huang

Static limit: $D_{00}(\boldsymbol{q}) = \frac{i}{\boldsymbol{q}^2}\,,$

$$D_{0i}(\boldsymbol{q}) = D_{0i}(\boldsymbol{q}) = 0\,,$$

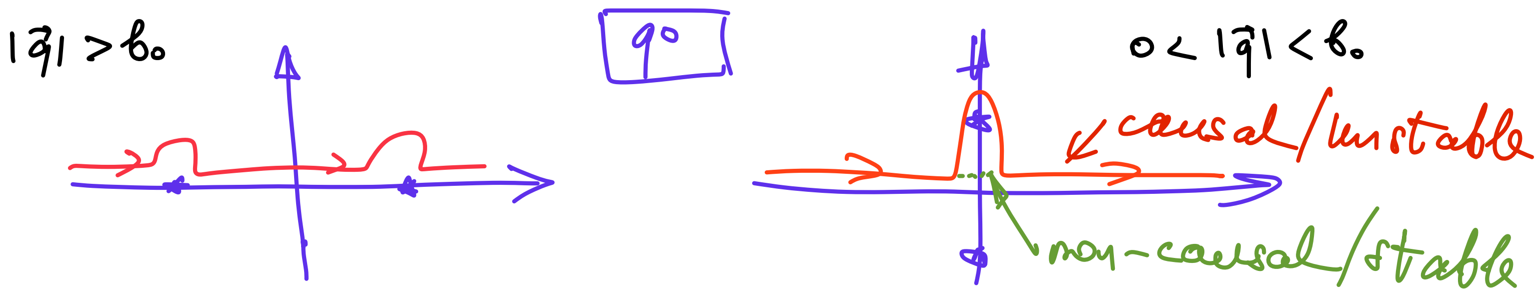
$$D_{ij}(\boldsymbol{q}) = -\frac{i\delta_{ij}}{\boldsymbol{q}^2 - b_0^2} - \frac{\epsilon_{ijk}q^k}{b_0(\boldsymbol{q}^2 - b_0^2)} + \frac{\epsilon_{ijk}q^k}{b_0\boldsymbol{q}^2}$$

Instability!

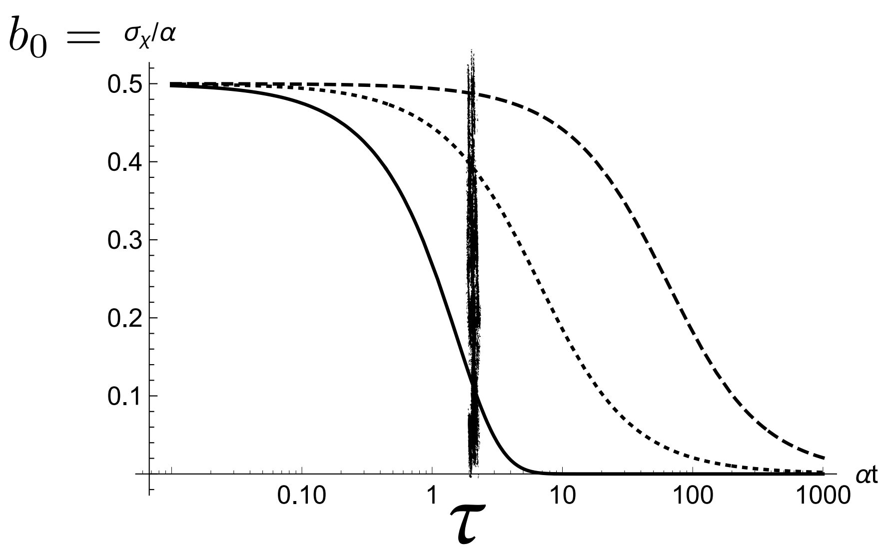
CHIRAL MAGNETIC INSTABILITY

Poles: $(q^0)^2 - \boldsymbol{q}^2 = -\lambda b_0 |\boldsymbol{q}|$ Modes $|\boldsymbol{q}| \leq b_0$ are unstable

Carroll, Field, Jackiw



At $q^0 \to 0$ there's only one unstable mode $|\boldsymbol{q}| = b_0$



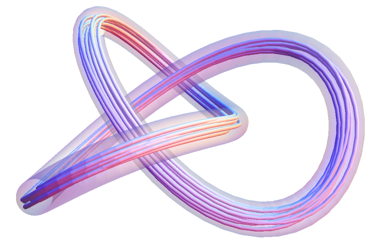
Chirality slowly flows from the chiral medium to the magnetic field until the evolution stops.

KT 1702.07329

MAGNETIC MOMENT CONTRIBUTION

Hansen, KT, 2022

$$D_{ij}(\boldsymbol{q}) = -\frac{i\delta_{ij}}{\boldsymbol{q}^2 - b_0^2} - \frac{\epsilon_{ijk}q^k}{b_0(\boldsymbol{q}^2 - b_0^2)} + \frac{\epsilon_{ijk}q^k}{b_0\boldsymbol{q}^2}$$



- D_{ij} couples only to the magnetic moment of the target

Current of an ion with charge e' and magnetic moment μ:

$$J^0(\boldsymbol{x}) = e'\delta(\boldsymbol{x})\,, \qquad \boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{\nabla} \times (\boldsymbol{\mu}\delta(\boldsymbol{x}))$$

Produces the potentials $A^0(\boldsymbol{q}) = e'/\boldsymbol{q}^2$ $\quad A^\ell(\boldsymbol{q}) = -\frac{1}{\boldsymbol{q}^2 - b_0^2}\left[i(\boldsymbol{\mu} \times \boldsymbol{q})^\ell + \frac{b_0}{\boldsymbol{q}^2}(\boldsymbol{\mu} \cdot \boldsymbol{q} q^\ell - \boldsymbol{q}^2\mu^\ell)\right]$

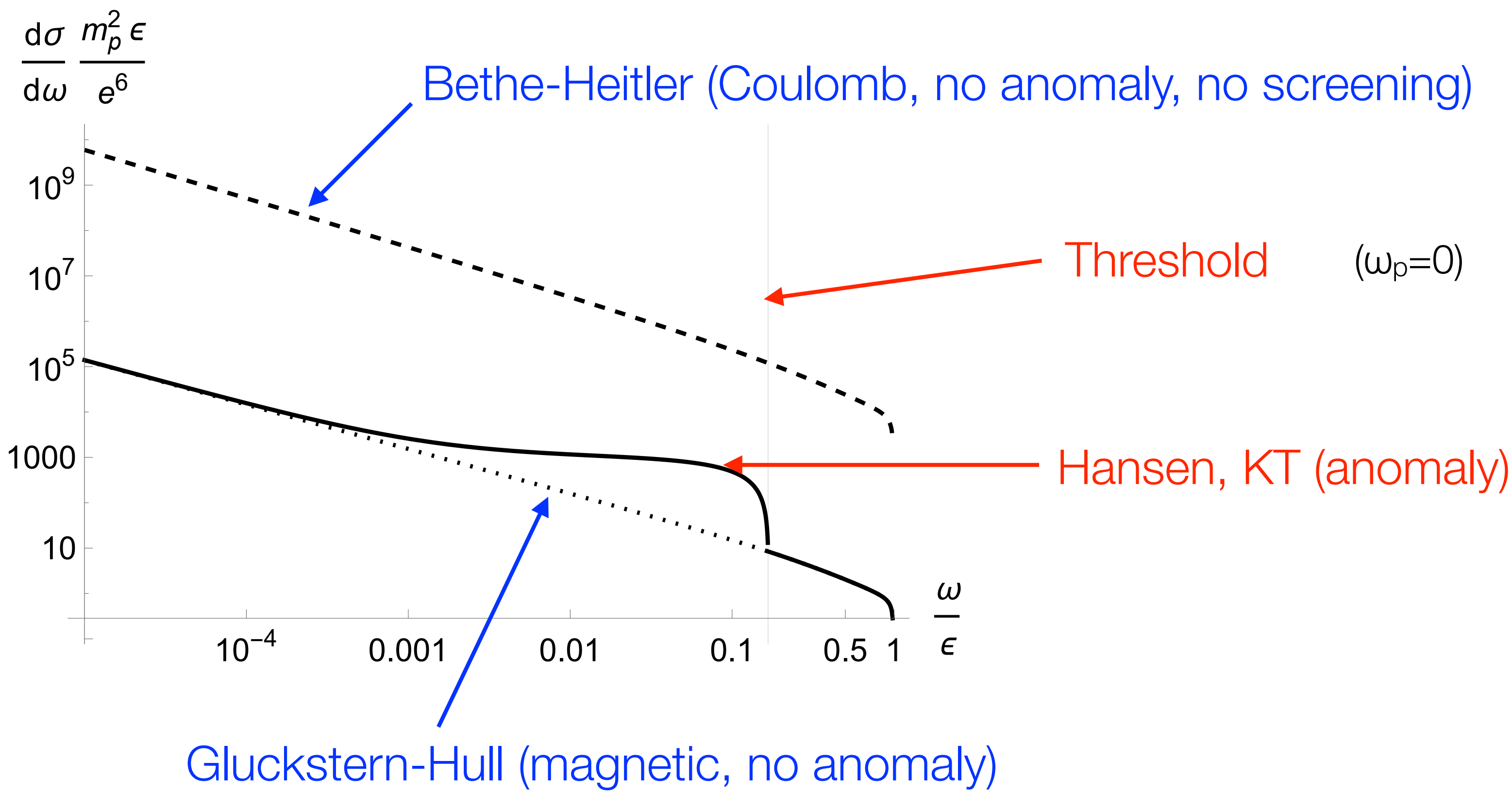
- Averaging over the directions of **μ** decouples the electric and magnetic terms:

$$\langle \mu_i \rangle = 0,\ \langle \mu_i \mu_j \rangle = \frac{\mu^2}{3}\delta_{ij}$$

- Photon x-section:

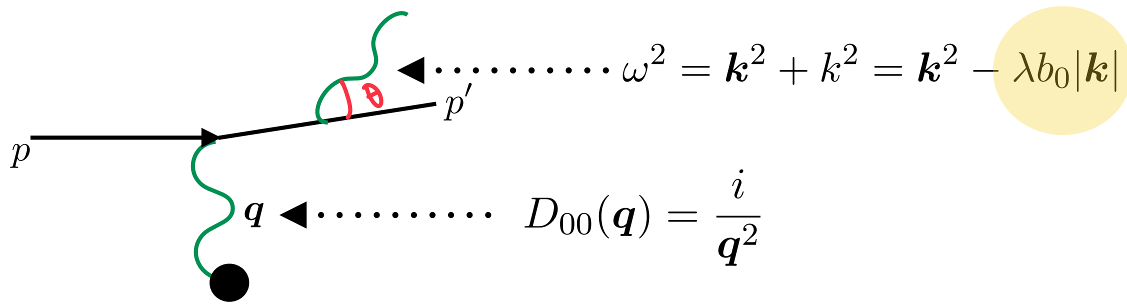
$$\frac{d\sigma_\mu}{d\omega} \approx \frac{2e^4\mu^2}{3(2\pi)^3\omega}\left[\frac{3b_0^2}{m^2}\ln\left(\frac{4\epsilon^4}{m^2\omega^2}\right) + \ln^2\frac{4\epsilon^2}{m^2} + \frac{2b_0^4\pi}{m^2\Gamma^2}\Theta(\omega_0 - \omega)\right]. \qquad \omega_0 = \frac{2\epsilon^2 b_0}{2\epsilon b_0 + m^2}$$

MAGNETIC MOMENT CONTRIBUTION TO PHOTON BREMSSTRAHLUNG



ELECTRIC MONOPOLE CONTRIBUTION TO PHOTON BREMSSTRAHLUNG

Hansen, KT, 2023



- Fermion propagator: $b_0\lambda > 0$

$$\frac{1}{2p\cdot k - k^2 + iE/\tau} = \frac{1}{\omega E\left(\frac{m^2}{E\omega}\frac{\omega-\omega^*}{E-\omega^*} + \theta^2 + \frac{i}{\omega\tau}\right)} \quad \text{with} \quad \omega^* = \frac{\lambda b_0 E^2}{\lambda b_0 E + m^2}$$

- The resonance emerges when $\omega<\omega^*$ due to the anomaly in the photon dispersion relation.

- The photon propagator has similar behavior: $\boldsymbol{q}^2_{\text{min}} = \frac{1}{4}\frac{\omega^2 E^2}{E'^2}\left[\frac{m^2(\omega-\omega^*)}{\omega E(E-\omega^*)} + \theta^2\right]^2$

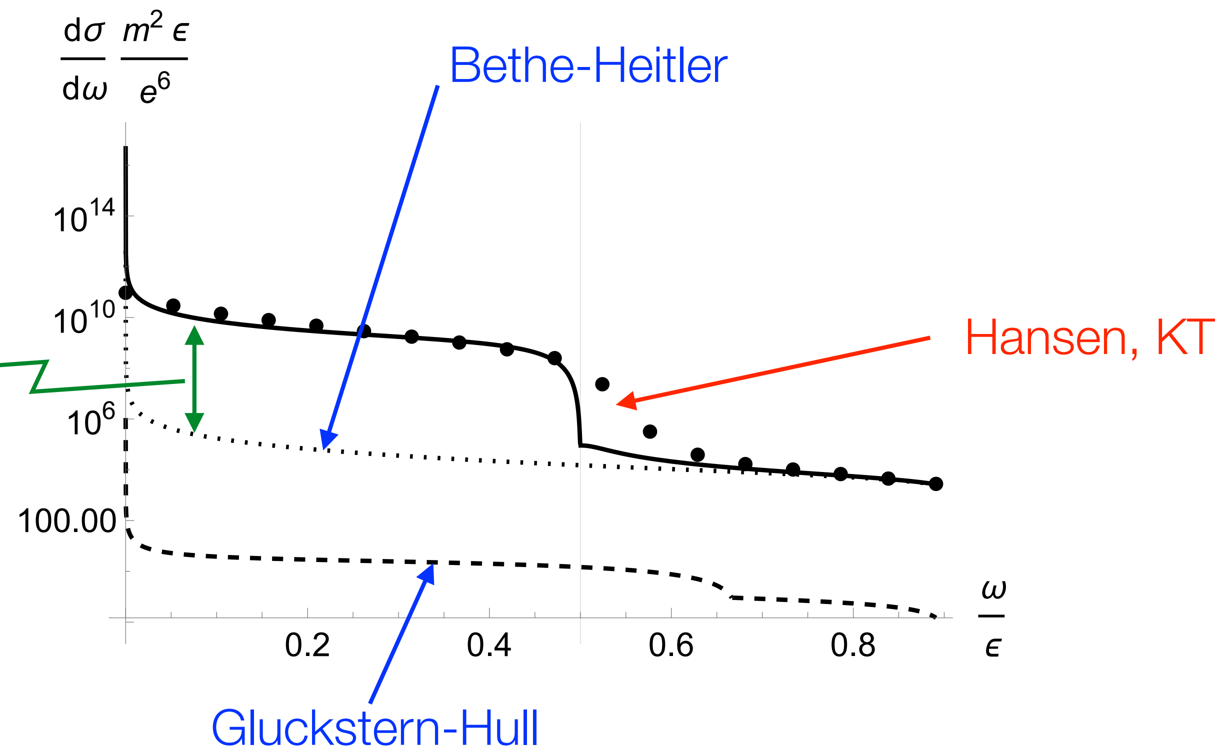
- The divergence is cutoff either by the inelastic rate $1/\tau$ or the Debye mass μ:

$$\boldsymbol{q}^2 \to \boldsymbol{q}^2 + \frac{E^2}{4E'^2\tau^2} + \mu^2$$

LOW TEMPERATURES $\mu \ll m$

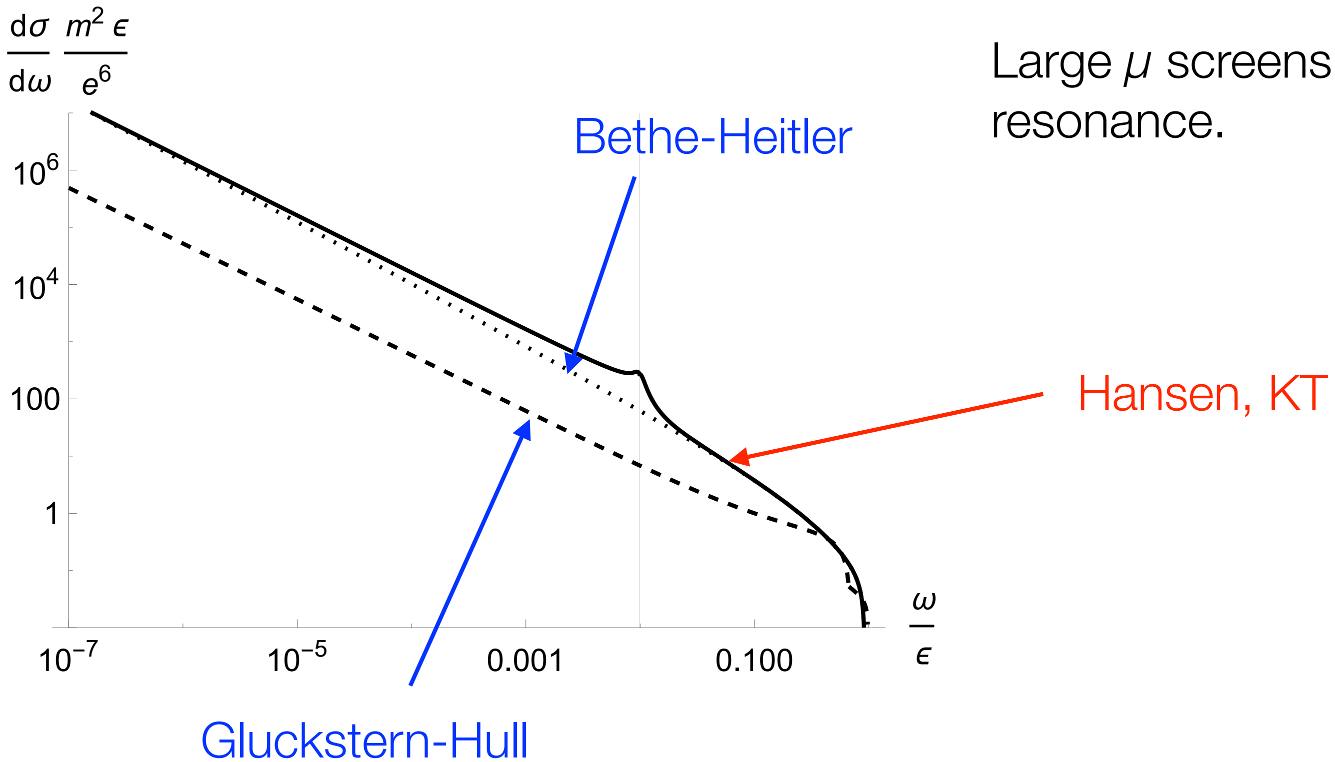
- Only one photon polarization ($b_0\lambda>0$) is enhanced!

$$\frac{d\sigma(b_0\lambda > 0)}{d\omega} \approx \frac{3\pi}{2}\frac{m^2\tau^3 b_0}{\ln\frac{2E^2}{m\omega}}\frac{d\sigma_{\rm BH}}{d\omega}$$



- Energy loss to radiation: $-\frac{dE(b_0\lambda > 0)}{dz} \approx \frac{\pi\tau E}{\ln\frac{2E}{m}}\left(-\frac{dE}{dz}\right)_{\rm BH}$
- At higher energy - must take LPM into account.

HIGH TEMPERATURES $\mu \gg m$



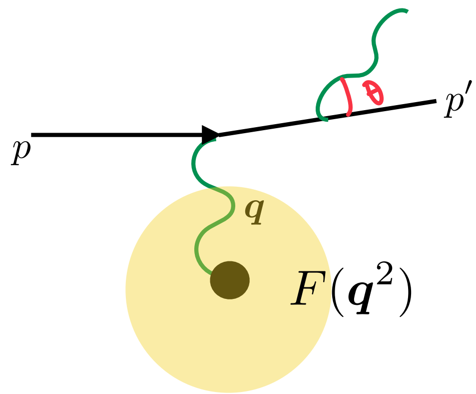
Large μ screens the resonance.

$$-\frac{dE(b_0\lambda > 0)}{dz} \approx \frac{e^2 E}{16\pi^2 \ell}\left\{ \ln\frac{2E}{m} - \frac{1}{3} + \frac{2b_0 E}{9m^2}\left(\pi + 2\ln\frac{E}{b_0}\right) + 2\tau(E-\omega^*)\arctan\frac{2m^2\omega^*\tau}{E(E-\omega^*)}\right\}.$$

- At high temperature the effect of anomaly is much weaker.
- Coherence effects/LPM - same as without anomaly?

SCREENING ISSUES IN SEMIMETALS

- Screening is due to the electron cloud around nucleus. The distances larger than the atomic radius are screened.



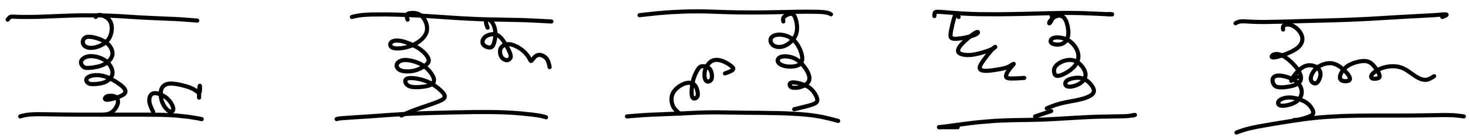
- If the chiral resonance is screened, then there is no anomalous bremsstrahlung.
- It happens when q_{min}<1/(atomic radius).
- Thomas-Fermi model: $q_{\min} \sim \frac{1}{\tau} \ll \alpha m Z^{1/3}$

⇒ no anomalous bremsstrahlung in semimetals.

RADIATIVE ENERGY LOSS IN QCD

- θ couples to the color gauge fields ⇒ color version of CME

- The collisional and radiative energy loss of a fast quark in QGP gets anomalous contributions.



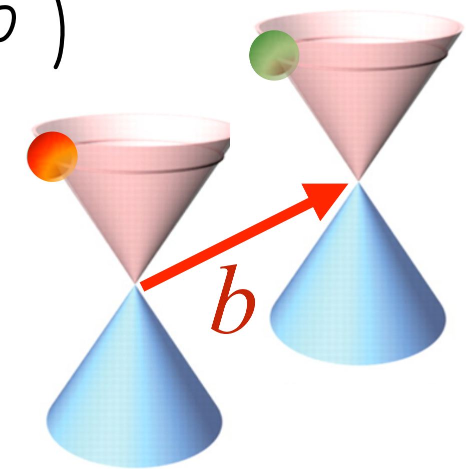
- dN/dy gets y-dependent corrections to the plateau

- Work in progress with Jeremy Hansen

ANISOTROPIC MATTER

$$\nabla\theta = \boldsymbol{b}/c_A \approx \text{const.} \qquad (\dot{\theta} = 0)$$

Qui, Cao, Huang (2017)



$$\begin{cases} \nabla \cdot \boldsymbol{B} = 0\,, \\ \nabla \cdot \boldsymbol{E} = -\boldsymbol{b}\cdot\boldsymbol{B}\,, \\ \nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}\,, \\ \nabla \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{b}\times\boldsymbol{E} \end{cases} \Rightarrow \begin{cases} \nabla \cdot \boldsymbol{B} = 0\,, \qquad \nabla \cdot \boldsymbol{D} = 0 \\ \nabla \times \boldsymbol{E} = i\omega\boldsymbol{B}\,, \qquad \nabla \times \boldsymbol{B} = -i\omega\boldsymbol{D} \end{cases}$$

where $\boldsymbol{D} = \boldsymbol{E} + \frac{i}{\omega}\boldsymbol{b}\times\boldsymbol{E} = 0\,.$ $\quad D_i = \varepsilon_{ij}E_j$

In momentum space: $\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{E}) = \omega\boldsymbol{k}\times\boldsymbol{B} = -\omega^2\boldsymbol{D}$

$$\left[k_ik_j - k^2\delta_{ij} + \omega^2_{\boldsymbol{k}\lambda}\varepsilon_{ij}(\omega_{\boldsymbol{k}\lambda})\right] e_{\boldsymbol{k}\lambda j} = 0$$

Dielectric tensor

$$\varepsilon = \begin{pmatrix} 1 & -ib/\omega_{\boldsymbol{k}\lambda} & 0 \\ ib/\omega_{\boldsymbol{k}\lambda} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Non-trivial solutions iff $\left|k_ik_j - k^2\delta_{ij} + \omega^2_{\boldsymbol{k}\lambda}\varepsilon_{ij}(\omega_{\boldsymbol{k}\lambda})\right| = 0$

The photon dispersion relation $\omega^2 = \boldsymbol{k}^2 + \mu^2(\boldsymbol{k},\lambda)$ $\qquad \mu^2(\boldsymbol{k},\lambda) = \frac{1}{2}b^2 - \lambda\,\mathrm{sgn}(\boldsymbol{k}\cdot\boldsymbol{b})\sqrt{(\boldsymbol{k}\cdot\boldsymbol{b})^2 + \frac{1}{4}b^4}$

QUANTIZATION OF EM FIELD

KT, 1809.08181

$$\boldsymbol{A}(\boldsymbol{x},t) = \underbrace{\sum_{\boldsymbol{k}\lambda}(a_{\boldsymbol{k}\lambda}\boldsymbol{A}_{\boldsymbol{k}\lambda} + a^{\dagger}_{\boldsymbol{k}\lambda}\boldsymbol{A}^{*}_{\boldsymbol{k}\lambda})}_{\text{Transverse EM waves}} + \underbrace{\sum_{\boldsymbol{k}\nu}(a_{\boldsymbol{k}\nu}\boldsymbol{A}_{\boldsymbol{k}\lambda} + a^{\dagger}_{\boldsymbol{k}\nu}\boldsymbol{A}^{*}_{\boldsymbol{k}\lambda})}_{\text{Longitudinal EM waves}}$$

Transverse EM waves

Longitudinal EM waves (exist only if there is spatial dispersion)

$$\boldsymbol{A}_{\boldsymbol{k}\lambda} = \boldsymbol{e}_{\boldsymbol{k}\lambda}\left(\frac{k\,v_{\boldsymbol{k}\lambda}}{2\omega^2_{\boldsymbol{k}\lambda}\varepsilon_{ij}e^{*}_{\boldsymbol{k}\lambda i}e_{\boldsymbol{k}\lambda j}V}\right)^{1/2} e^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega_{\boldsymbol{k}\lambda}t} \qquad \boldsymbol{A}_{\boldsymbol{k}\nu} = \hat{\boldsymbol{k}}\left(\frac{k^2}{\omega^2_{\boldsymbol{k}\nu}k_ik_j\partial\varepsilon_{ij}/\partial\omega_{\boldsymbol{k}\nu}V}\right)^{1/2} e^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega_{\boldsymbol{k}\nu}t}$$

Chiral Cherenkov radiation: $$\frac{dW}{d\Omega d\omega} = \frac{\alpha Q^2}{16\pi}\sum_{\lambda}\delta(\omega+\varepsilon'-\varepsilon)\frac{k^3}{\varepsilon\varepsilon'\omega^2\varepsilon_{ij}e^{*}_{\boldsymbol{k}\lambda i}e_{\boldsymbol{k}\lambda j}}\sum_{ss'}|\mathcal{M}_0|^2$$

At high energies: $$e^{*}_{\boldsymbol{k}\pm i}e_{\boldsymbol{k}\pm j} \to \frac{1}{2}\left(\delta^{ij} - \frac{k_ik_j}{k^2}\right)$$

Photon propagator:

$$D_{00} = \frac{i\boldsymbol{q}^2}{\boldsymbol{q}^4 + (\boldsymbol{b}\times\boldsymbol{q})^2}\,,$$

$$D_{0i} = \frac{(\boldsymbol{b}\times\boldsymbol{q})_i}{\boldsymbol{q}^4 + (\boldsymbol{b}\times\boldsymbol{q})^2}\,,$$

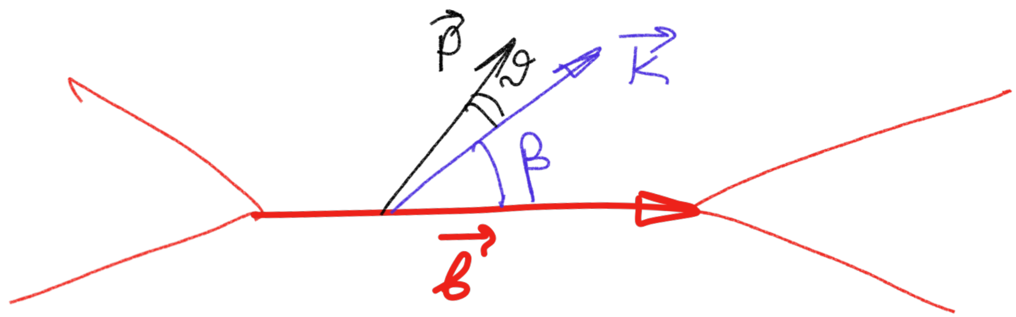
$$D_{ij} = -\frac{i}{\boldsymbol{q}^4 + (\boldsymbol{b}\times\boldsymbol{q})^2}\left\{\boldsymbol{q}^2\delta_{ij} + b_ib_j - \frac{[\boldsymbol{q}^4 - (\boldsymbol{b}\cdot\boldsymbol{q})^2]q_iq_j}{\boldsymbol{q}^4} - \frac{\boldsymbol{b}\cdot\boldsymbol{q}}{\boldsymbol{q}^2}(b_iq_j + b_jq_i)\right\}$$

CHIRAL CHERENKOV RADIATION AT HIGH ENERGY

KT, 1809.08181

$$\delta(\omega+\varepsilon'-\varepsilon)\approx 2x(1-x)\varepsilon\delta\left(k_\perp^2+\underbrace{\mu^2(1-x)+m^2x^2}_{\varkappa}\right) \qquad \mu^2\approx-\lambda\omega b\cos\beta \equiv m_\gamma^2$$

Now the photon "mass" depends now on the photon direction angle β w.r.t. b.

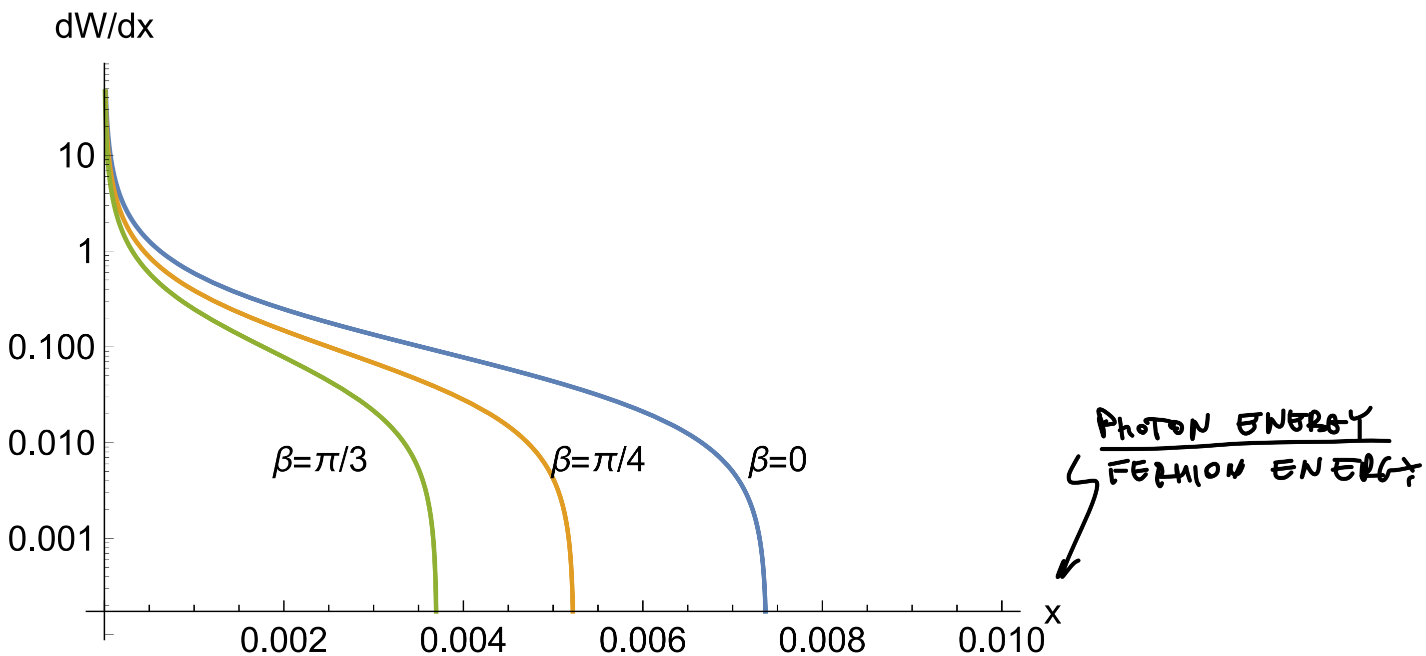


Photon spectrum $\quad \dfrac{dW}{d\Omega d\omega}=\dfrac{\alpha Q^2 x}{2\pi}\delta\left(x^2\varepsilon^2\vartheta^2+\kappa_\lambda\right)\left[\lambda\varepsilon b\cos\beta\left(1-x+\dfrac{x^2}{2}\right)-m^2x\right]\theta\left(-\kappa_\lambda\right)$

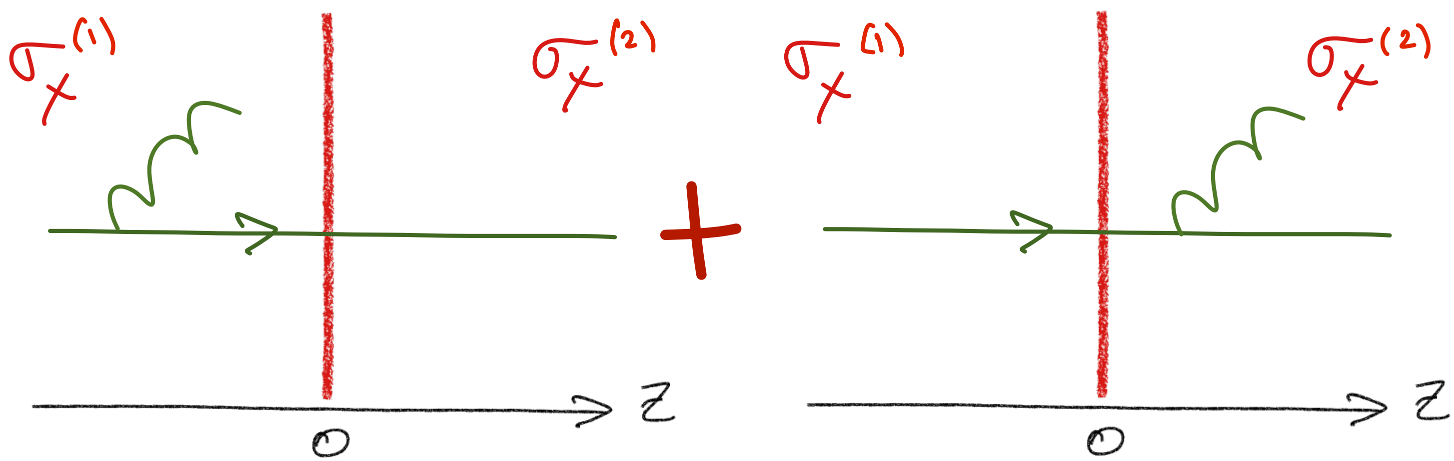
$$\kappa_\lambda=\mu^2(1-x)+m^2x^2=-x(1-x)\lambda\varepsilon b\cos\beta+m^2x^2$$

Kappa is negative if $\quad \lambda\cos\beta>0$ and $x<x_{\max} \qquad x_{\max}=\left(1+\dfrac{m^2}{\lambda\varepsilon b\cos\beta}\right)^{-1}$

CHERENKOV SPECTRUM IN A SEMIMETAL



CHIRAL TRANSITION RADIATION



$$\frac{dN}{d^2q_\perp dx} = \frac{\alpha Q^2}{2\pi^2 x}\left\{\left(\frac{x^2}{2} - x + 1\right) q_\perp^2 + \frac{x^4 m^2}{2}\right\}\sum_\lambda \left|\frac{1}{q_\perp^2 + \kappa_\lambda^{(1)} - i\delta} - \frac{1}{q_\perp^2 + \kappa_\lambda^{(2)} + i\delta}\right|^2$$

(Transition radiation in ordinary materials corresponds to $\kappa_{\rm tr} = m^2x^2 + m_\gamma^2(1-x)$ finite at $\hbar \to 0$)

Contribution of the pole at $q_\perp^2 + \kappa_\lambda = 0$ is the chiral Cherenkov radiation.

The rest is the "chiral transition radiation"

SUMMARY

- Chiral anomaly opens new channels of energy loss.

- Relevant in many areas of physics: nuclear, condensed matter, search for dark matter.

CHIRAL OPTICS: FRESNEL EQUATIONS

Stewart, KT, 1906.04602

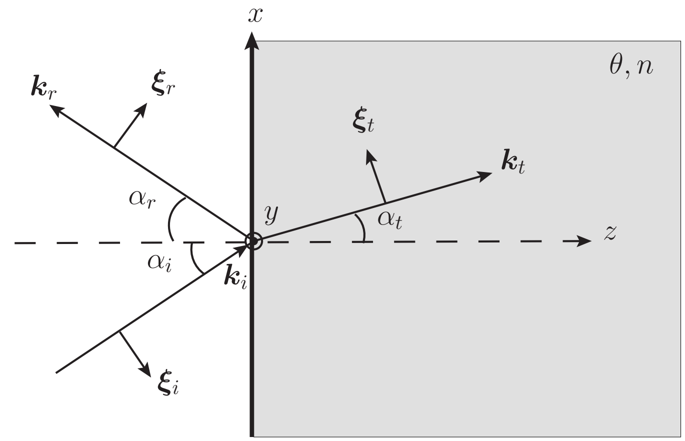
$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0\,,$$

$$\boldsymbol{\nabla} \cdot (\boldsymbol{D} + \tilde{\theta}\boldsymbol{B}) = 0\,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B}\,,$$

$$\boldsymbol{\nabla} \times (\boldsymbol{B} - \tilde{\theta}\boldsymbol{E}) = \partial_t(\boldsymbol{E} + \tilde{\theta}\boldsymbol{B})\,,$$

$$\tilde{\theta} = c_A \theta$$



Equations in the bulk depend only on ∂θ

Boundary conditions
Sikivie (84)

$$\Delta B_\perp = 0\,, \qquad \Delta \boldsymbol{E}_\parallel = 0\,,$$

$$\Delta(E_\perp + \tilde{\theta}B_\perp) = 0\,, \qquad \Delta(\boldsymbol{B}_\parallel - \tilde{\theta}\boldsymbol{E}_\parallel) = 0\,.$$

⇒ *Chiral Fresnel equations* for Incident, Reflected and Transmitted waves

Depend on θ itself!

Example:

$$A_t^+ = \frac{2}{2 - i\tilde{\theta}} A_i^+\,, \quad A_r^- = -\frac{i\tilde{\theta}}{2 - i\tilde{\theta}} A_i^+\,, \quad A_t^- = A_r^+ = 0 \quad \text{if } n = 1\,.$$

1. Amplitudes do not depend on the angle of incidence
2. Effective index of refraction $n_{\rm eff} = 1 - i\tilde{\theta}$
3. The circular polarization is preserved, rather than linear

BREWSTER'S ANGLE

At a certain incidence angle α_B there is no reflection of the incident wave component linearly polarized in xz-plane (plane of incidence)

$$\tan^2\alpha_B = \frac{(n^2-1+\tilde{\theta}^2)\left(n^2-1+\tilde{\theta}^2+\sqrt{(n^2+1)^2+\tilde{\theta}^2[\tilde{\theta}^2+2(n^2-1)]}\right)}{2(n^2-1)}$$

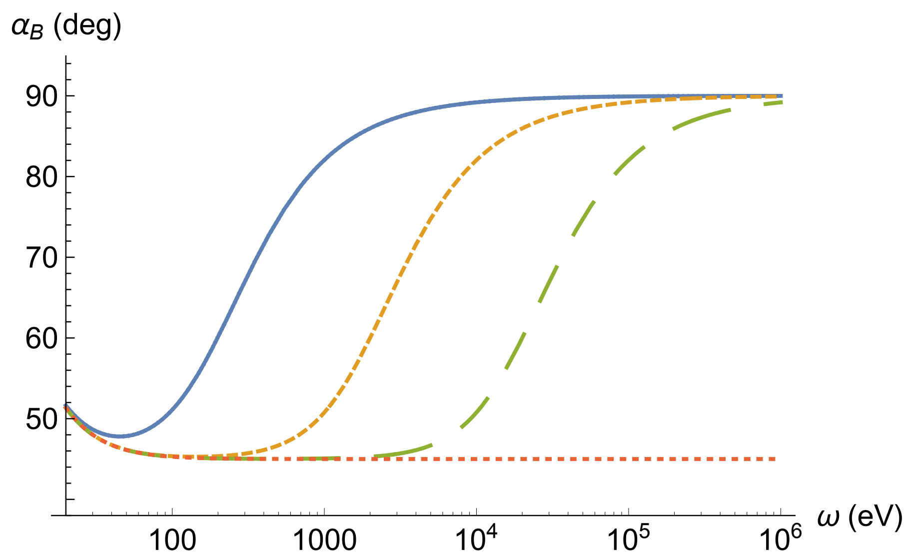


FIG. 2. Brewster's angle as a function of the wave frequency for a typical value of the plasma frequency $\omega_{\rm pl} = 10$ eV at different values of $\tilde{\theta} = 10^{-1}$, 10^{-2}, 10^{-3} (left to right). The dotted line corresponds to $\theta = 0$.