Constraints from the SπRIT experiment and their implications on nuclear matter equation of state

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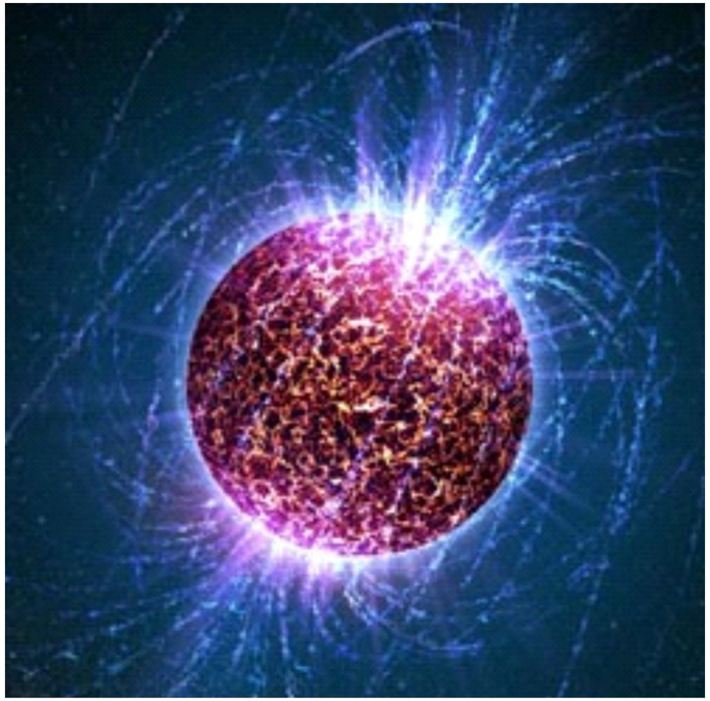
Goals

- Goal 1: Constraint high density part of symmetry energy term with SπRIT experiment.

- Goal 2: Combine multi-messenger constraints for a comprehensive understanding on nuclear matter equation of state (EoS).



Credit: Casey Reed

Common EoS parameters

- $E(\rho,\delta) = E(\rho,\delta=0) + \delta^2 S(\rho), \delta = \frac{\rho_n - \rho_p}{\rho_0}$ is the neutron excess over baryon number density.
- Taylor expands the first term (Symmetric matter term),
  - $\mathrm{E}(\rho,\delta=0) \approx \mathrm{E_{sat}} + \frac{1}{2}\mathrm{K_{sat}x^2} + \frac{1}{6}\mathrm{Q_{sat}x^3} + \frac{1}{24}\mathrm{Z_{sat}x^4} + \cdots$ where $\mathrm{x} = \frac{\rho-\rho_0}{3\rho_0}$,
- Taylor expands the second term (symmetry energy term),
  - $\mathrm{S}(\rho) = \mathrm{S_0} + \mathrm{L_{sym}x} + \frac{1}{2}\mathrm{K_{sym}x^2} + \cdots$
- Rule : L is 1st order, K is 2nd, Q is 3rd, Z is 4th
- Sym for symmetry energy term, Sat for symmetric matter term
- Another parameter in nuclear EoS: effective mass that describes the momentum dependence of potential energy $m^* = m/\left(1 + \frac{m}{p}\frac{\partial V}{\partial p}\right)$.
- In asymmetric matter, effective mass of p and n are expected to be different which give rise to **effective-mass splitting,**

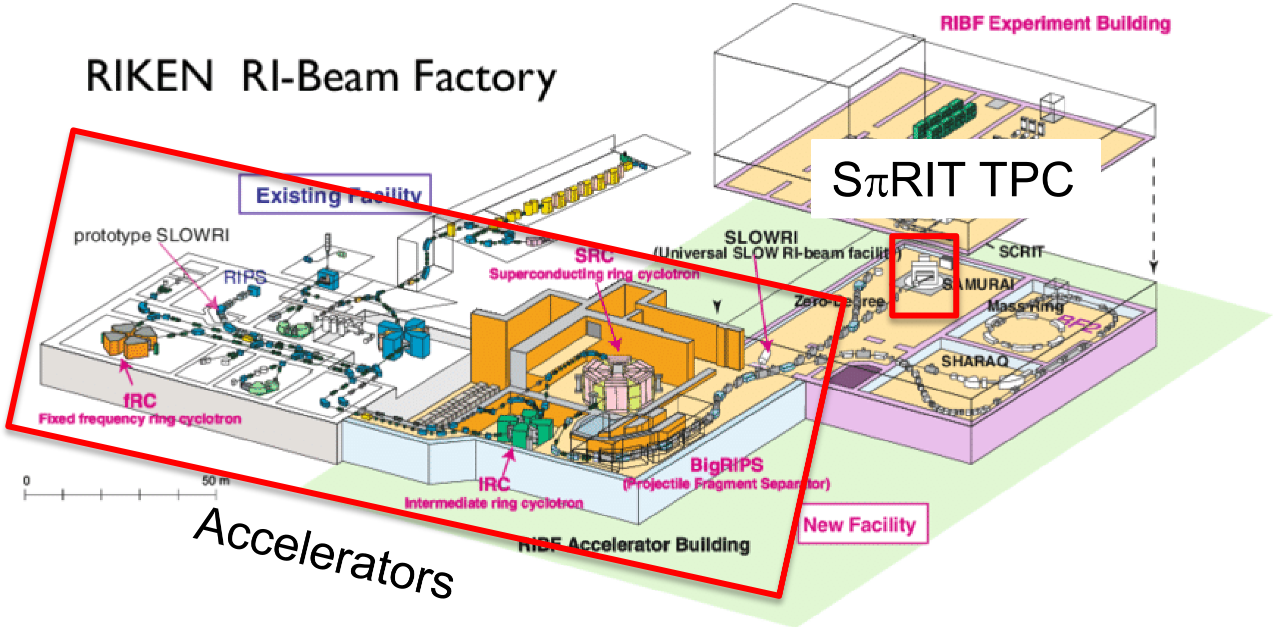
$$\Delta m_{np}^* = \frac{m_n^* - m_p^*}{m_N}$$

Part 1: SπRIT experiment.

SπRIT experiment

- The experiment ran in 2016 in RIKEN, Japan.
- Rare isotope beam is provided by RIKEN in Japan, set to collide with stationary target.
- Collision fragments are detected with ***SπRIT Time Projection Chamber (TPC)***.
- δ_{sys} =(N-P)/(N+P) is called the asymmetry

Beam	Target	E_{beam}/A	δ_{sys}	Purpose
^{132}Sn	**^{124}Sn**	**270**	**0.22**	**Max δ_{sys}**
^{124}Sn	^{112}Sn	270	0.15	Mirror system
^{108}Sn	**^{112}Sn**	**270**	**0.09**	**Min δ_{sys}**
^{112}Sn	^{124}Sn	270	0.15	Mirror system

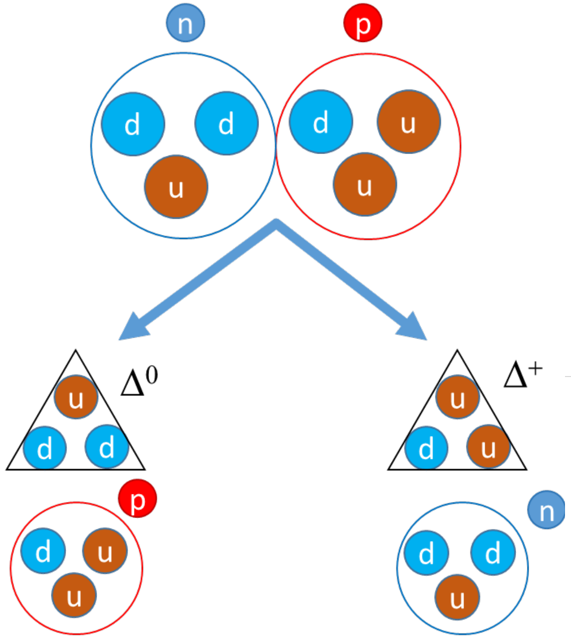


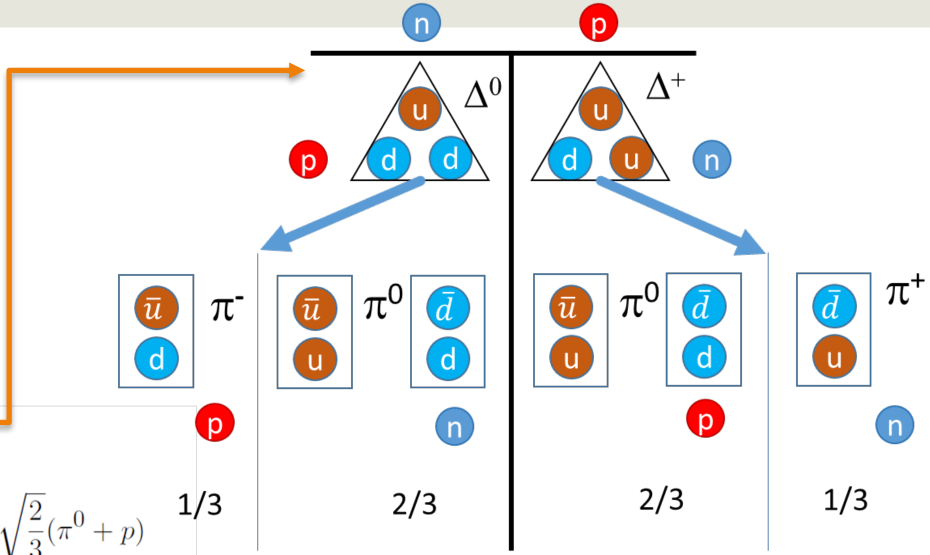
M. Wada: DOI: 10.1007/s10751-007-9552-1

Observable: Pion ratio

- Pion ratio $Y(p^-)/Y(p^+)$ from SπRIT experiment.
- ^{132}Sn + ^{124}Sn and ^{108}Sn + ^{112}Sn @ 270 AMeV
- Sensitive to high density part.

$$p + p \rightarrow \sqrt{\frac{3}{4}}(\Delta^{++} + n) - \sqrt{\frac{1}{4}}(\Delta^{+} + p)$$
$$n + p \rightarrow \sqrt{\frac{1}{2}}(\Delta^{+} + n) - \sqrt{\frac{1}{2}}\left(\Delta^{0} + p\right)$$
$$n + n \rightarrow \sqrt{\frac{1}{4}}\left(\Delta^{0} + n\right) - \sqrt{\frac{3}{4}}(\Delta^{-} + p)$$





$$\Delta^{++} \rightarrow \pi^{+} + p$$
$$\Delta^{+} \rightarrow \sqrt{\frac{1}{3}}(\pi^{+} + n) + \sqrt{\frac{2}{3}}(\pi^{0} + p)$$
$$\Delta^{0} \rightarrow \sqrt{\frac{2}{3}}(\pi^{0} + n) + \sqrt{\frac{1}{3}}(\pi^{-} + p)$$
$$\Delta^{-} \rightarrow \pi^{-} + n$$

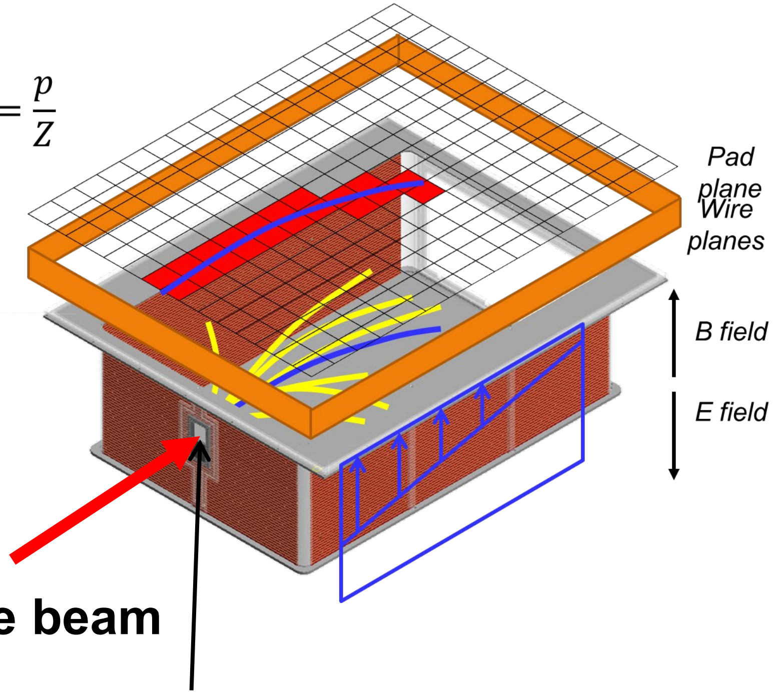
Courtesy Jon Barney

- Effects that acts similarly on different pions are cancelled out to eliminate sensitivity on effects that are not being considered here.
- Symmetry energy effects are magnified due to symmetry forces acting on π$^-$ and π$^+$ with opposite sign.
- Systematic uncertainties due to detector errors (if any) are cancelled out in the division.

SπRIT experiment

$$B\rho = \frac{p}{Z}$$

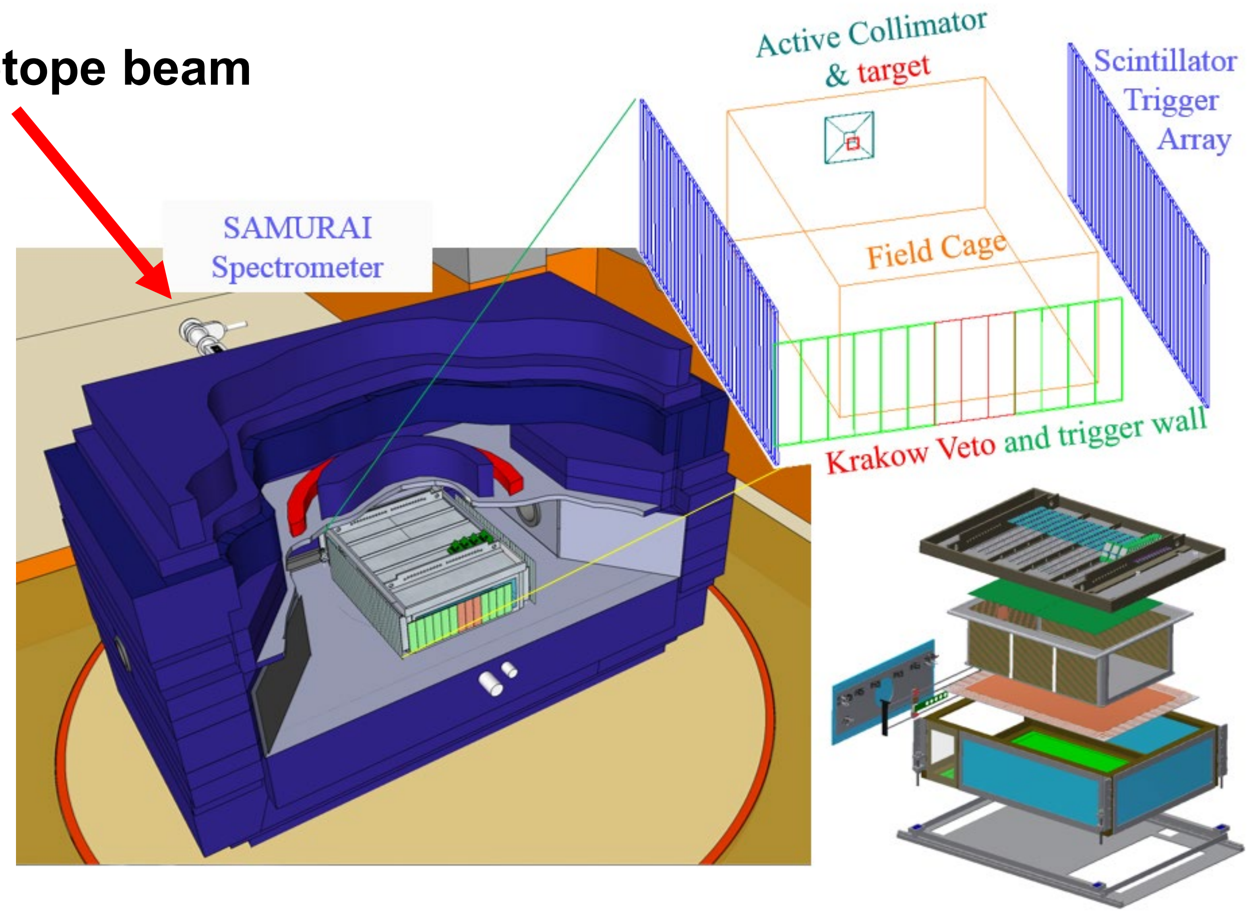
Courtesy J. Barney



Rare Isotope beam

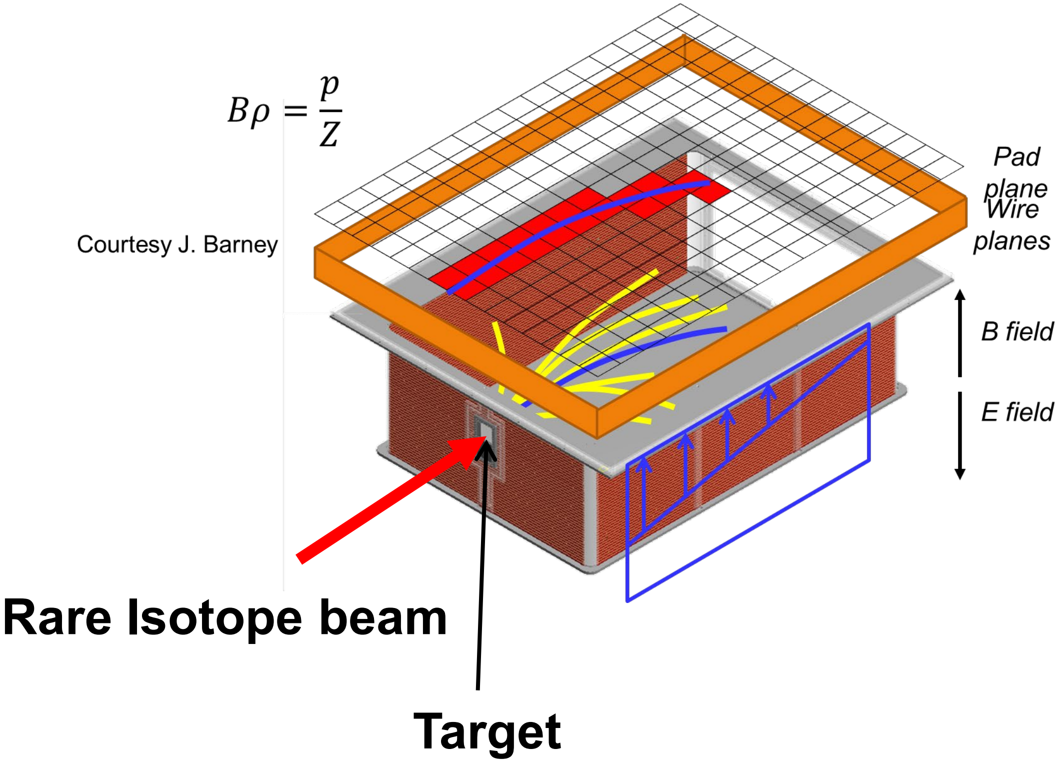
Target

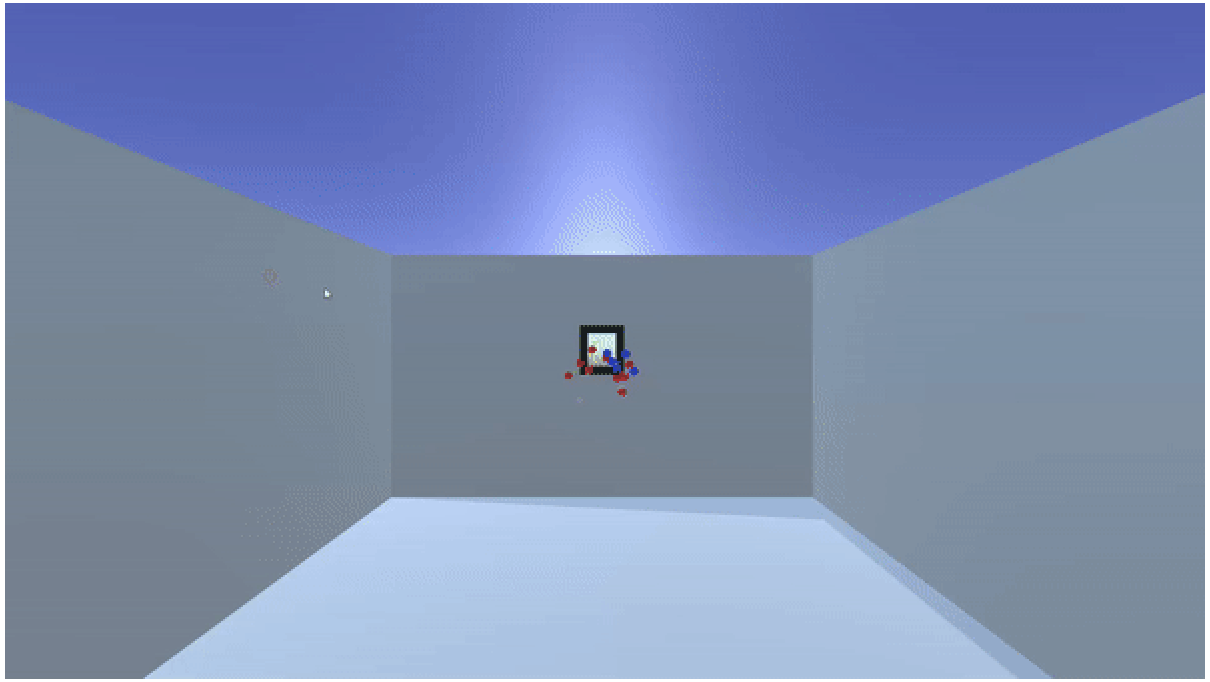
Rare Isotope beam



URL: https://groups.nscl.msu.edu/hira/sepweb/pages/slideshow/samurai.html

SπRIT experiment



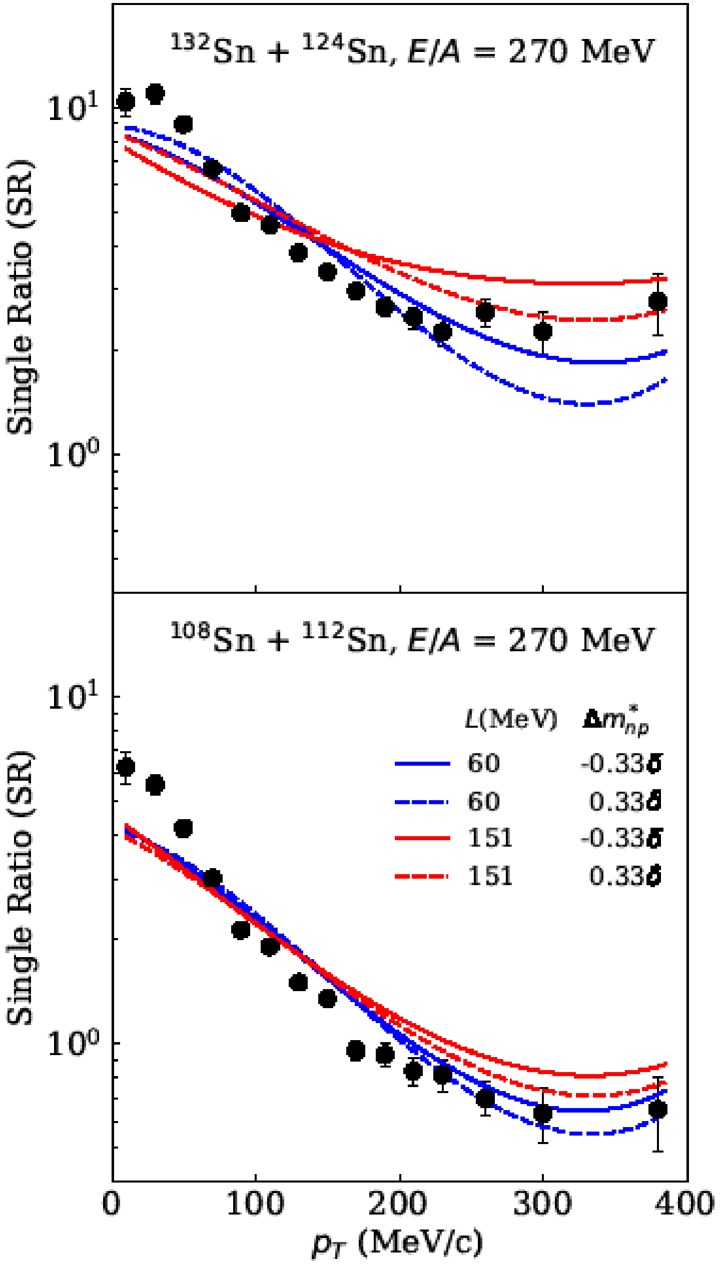


Credit: Jacob Cosby
https://groups.nscl.msu.edu/hira/cosmic/SpiritTPC.html
Beam comes out of the screen.

J. Estee, et al., Phys. Rev. Lett. 126, 162701 (2021).

Pion spectra ratio

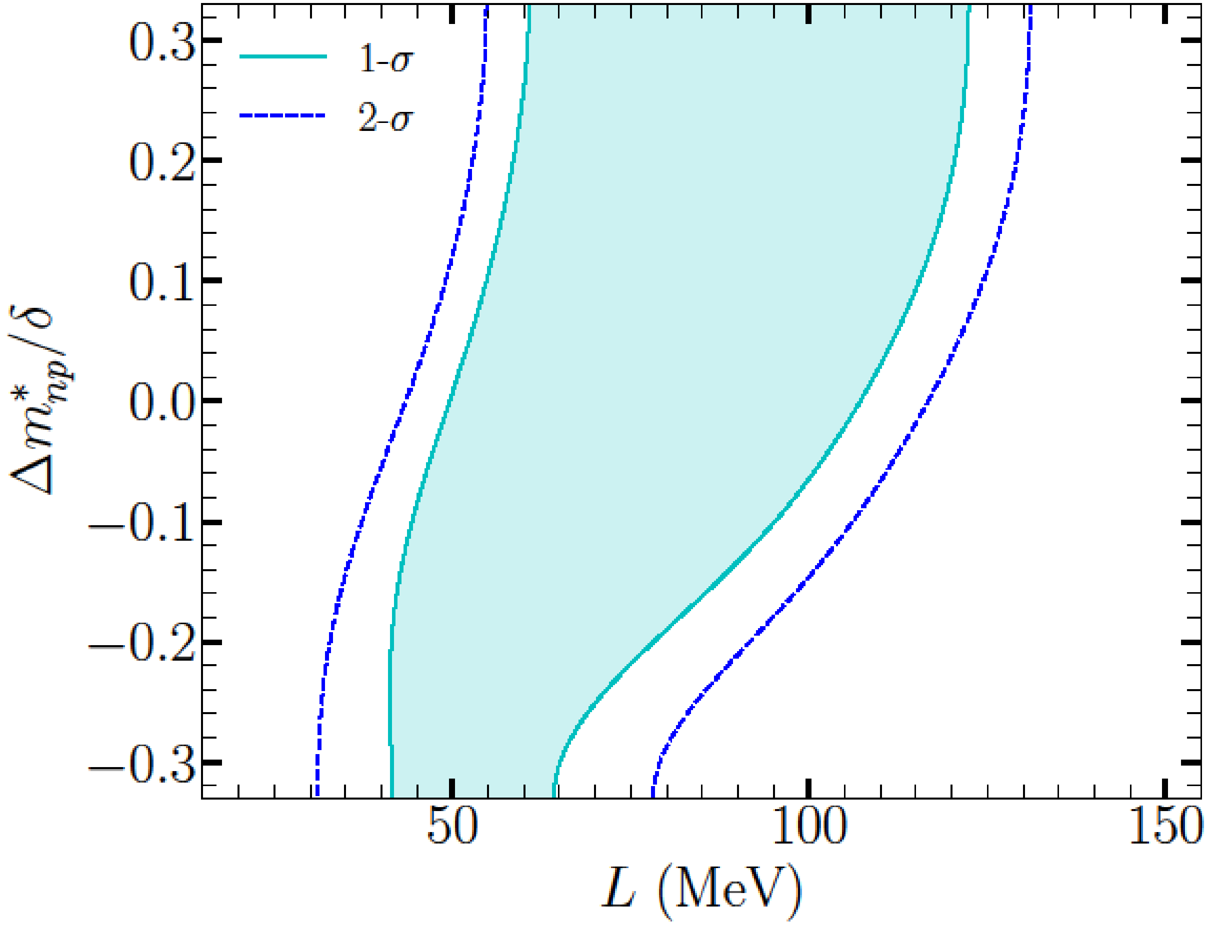
- Ratios of π$^-$ to π$^+$ spectra are shown on the right
  - Centrality selection from multiplicity.
  - <b> = 2.1 fm, pions are generated mostly from central collisions.
- Compared to predictions from dcQMD:
  - Best fitted pion and delta potential are used.
  - Only L and $\Delta m_{np}/\delta$ are allowed to vary. Other parameters are fixed to values from previous studies.
  - K$_{sat}$ = 250 MeV, Q$_{sat}$ = -350 MeV, K$_{sym}$ = -488 + 6.728L and $S(0.67\rho_0) = 25.5\ MeV$
  - L is 1st derivative, K is 2nd derivative, Q is 3rd derivative. "Sat" for symmetric matter and "sym" for symmetry energy term.
- Only spectra at $p_T > 200\ MeV/c$ are compared as low energy effects, such as Coulomb interaction, diminish at high momentum. The cut isolates the effect of symmetry energy.



Results

- Chi-square analysis is performed to constraint nuclear EoS parameters.
- Without constraint on $\Delta m^*_{np}/\delta$, we have L = 79.9 +/- 37.6 MeV and S_0= 35.5+/-2.9 MeV.
- Pions are sensitive to high density part, -> Final constraint:
  - $S(0.232\ \mathrm{fm}^{-3}) = 52 \pm 13$ MeV.
  - $L(0.232\ \mathrm{fm}^{-3}) = 140 \pm 110$ MeV.

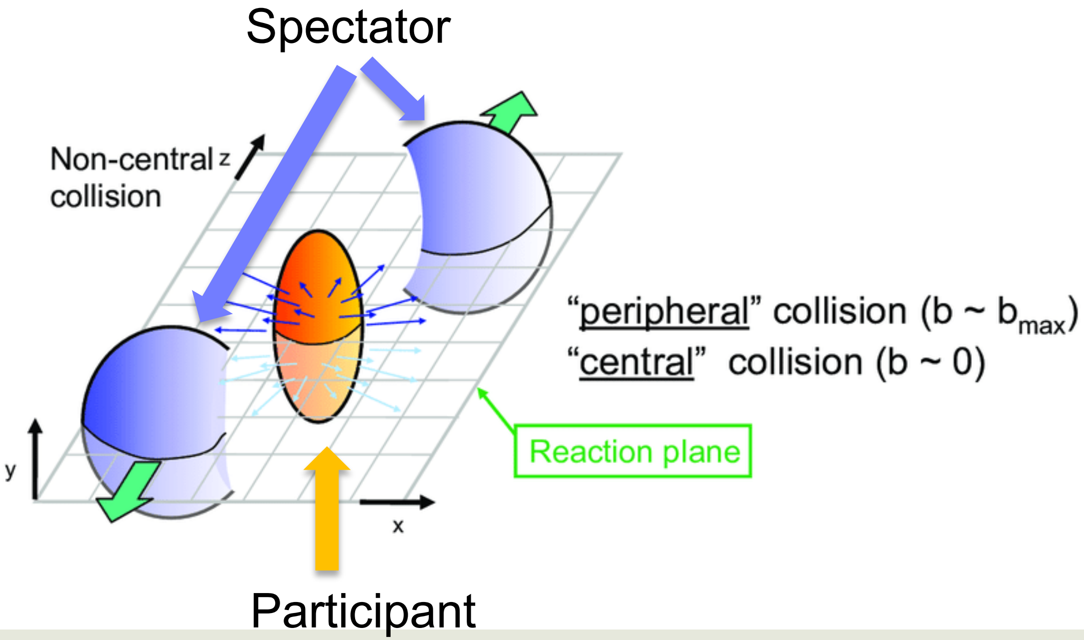
J. Estee, et al., Phys. Rev. Lett. 126, 162701 (2021).



Work in progress: Collective flow

- Collisions are rarely head-on, so spectator nucleons often blocks the emission of participant nucleons along reaction plane.
- If mean field is very repulsive, particles are promptly ejected. Spectators don't have time to clear the path of emission, resulting in strong asymmetry, vice versa.

- Flow is quantified with v_1 (directed flow) and v_2 (elliptical flow) defined as,
  - $\frac{dN}{d\Phi} \propto 1 + 2v_1 cos\phi + 2v_2 cos2\phi + \cdots$
- Higher order terms are not constructed due to statistics



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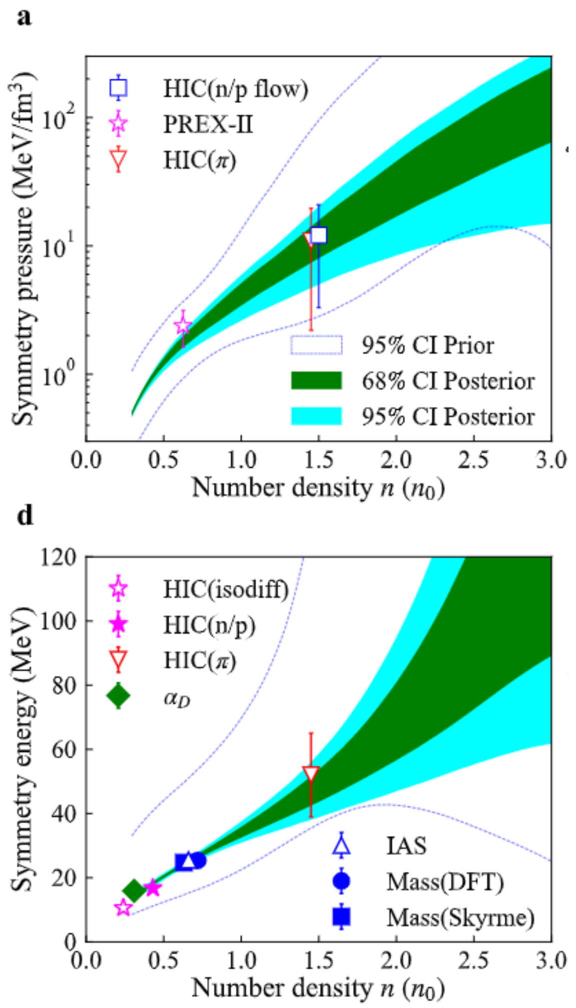
Work in progress: Collective flow

- Compare data to predictions from ImQMD.
- Bayesian analysis will be performed to constrain S_0, L, effective masses and in-medium cross-section with Markov chain Monte Carlo.
- Emulate ImQMD with Gaussian emulator.
- Preliminary results show strong constraining power on effective masses.

Part 2: Multi-messenger constraints on nuclear matter EoS.

Will use n and ρ interchangeably to denote baryon number density

All terrestrial experiment constraints



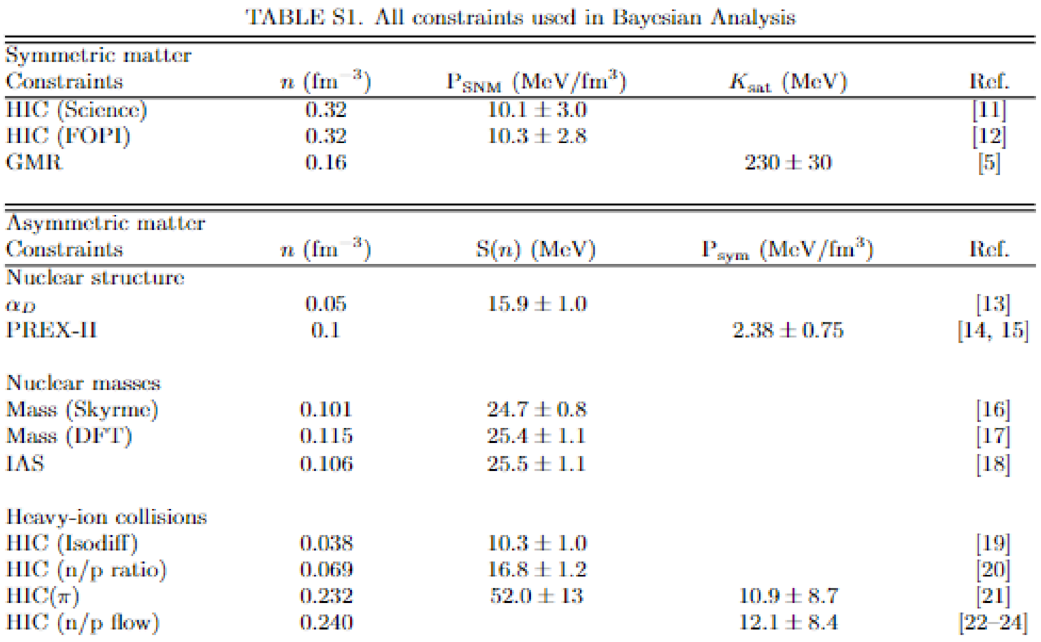


TABLE S1. All constraints used in Bayesian Analysis

Symmetric matter Constraints	n (fm^{-3})	P_{SNM} (MeV/fm^3)	K_{sat} (MeV)	Ref.
HIC (Science)	0.32	10.1 ± 3.0		[11]
HIC (FOPI)	0.32	10.3 ± 2.8		[12]
GMR	0.16		230 ± 30	[5]

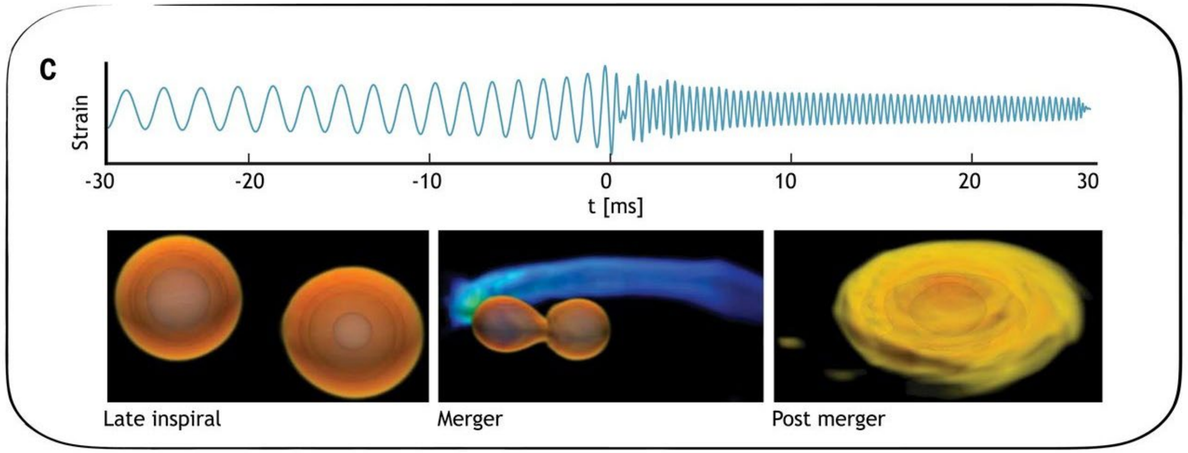
Asymmetric matter Constraints	n (fm^{-3})	$S(n)$ (MeV)	P_{sym} (MeV/fm^3)	Ref.
Nuclear structure				
α_D	0.05	15.9 ± 1.0		[13]
PREX-II	0.1		2.38 ± 0.75	[14, 15]
Nuclear masses				
Mass (Skyrme)	0.101	24.7 ± 0.8		[16]
Mass (DFT)	0.115	25.4 ± 1.1		[17]
IAS	0.106	25.5 ± 1.1		[18]
Heavy-ion collisions				
HIC (Isodiff)	0.038	10.3 ± 1.0		[19]
HIC (n/p ratio)	0.069	16.8 ± 1.2		[20]
HIC(π)	0.232	52.0 ± 13	10.9 ± 8.7	[21]
HIC (n/p flow)	0.240		12.1 ± 8.4	[22–24]

Choice of measurements will be elaborated on Rohit’s talk.

W.G. Lynch, Physics Letters B 137098 (2022).

Astronomical observation of NS properties

	$M(\odot)$	R (km)	Λ
LIGO	1.4		190^{+390}_{-120}
Riley J0030+0451	$1.34^{+0.15}_{-0.16}$	$^{a}12.71^{+1.14}_{-1.19}$	
Miller J0030+0451	$1.44^{+0.15}_{-0.14}$	$^{a}13.02^{+1.24}_{-1.06}$	
Riley J0740+6620	$2.07^{+0.07}_{-0.07}$	$^{b}12.39^{+1.30}_{-0.98}$	
Miller J0740+6620	$2.08^{+0.07}_{-0.07}$	$^{b}13.7^{+2.6}_{-1.5}$	



Deformability from gravitation wave observation of NS merger

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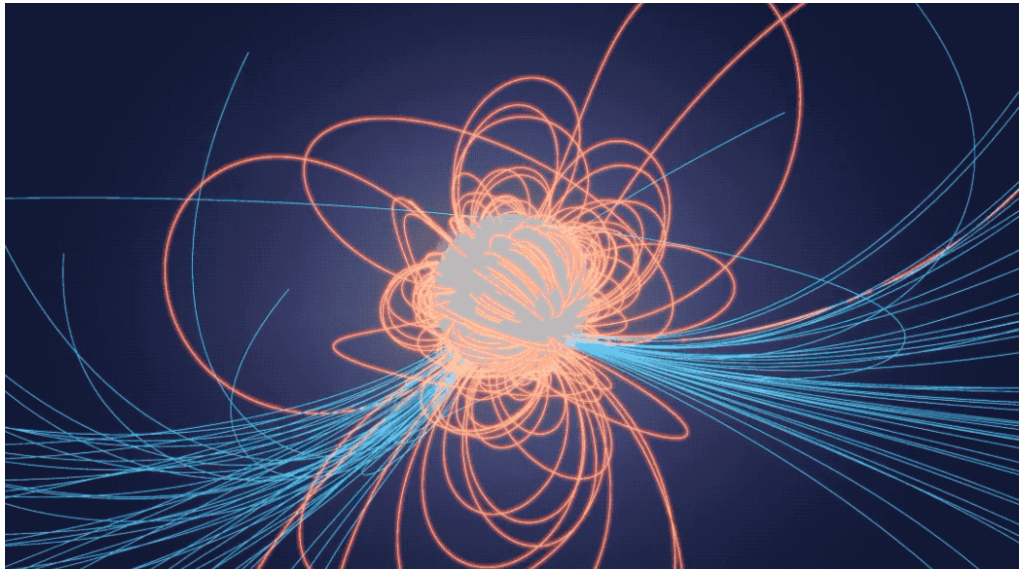

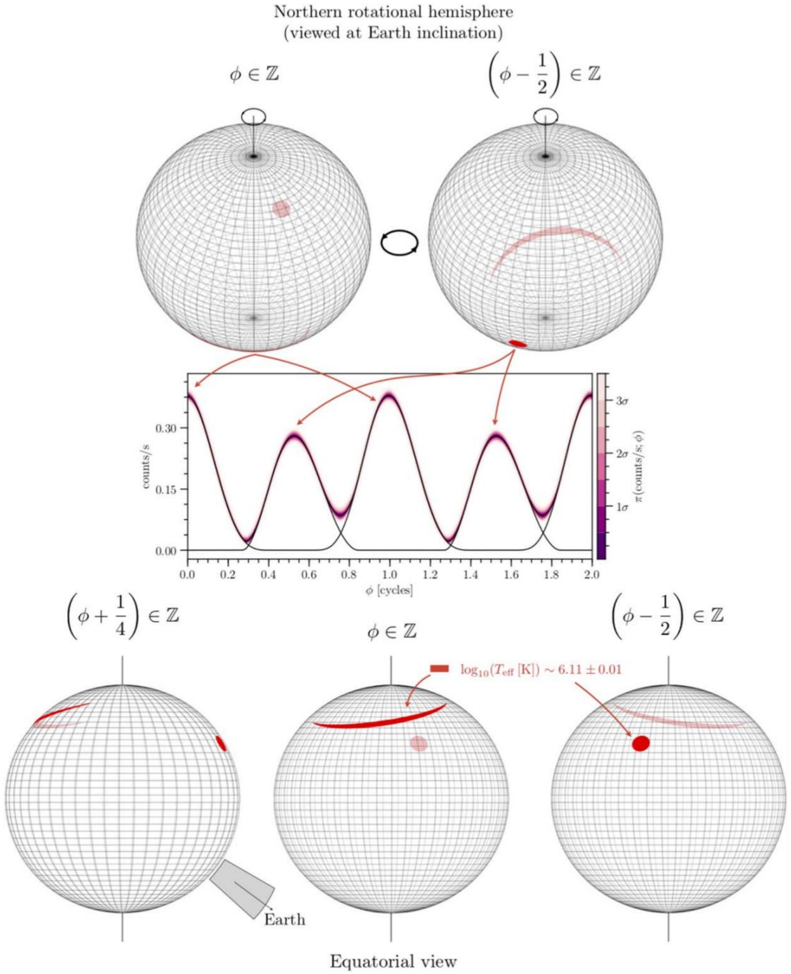

The Astrophysical Journal Letters, 887:L21 (60pp), 2019 December 10

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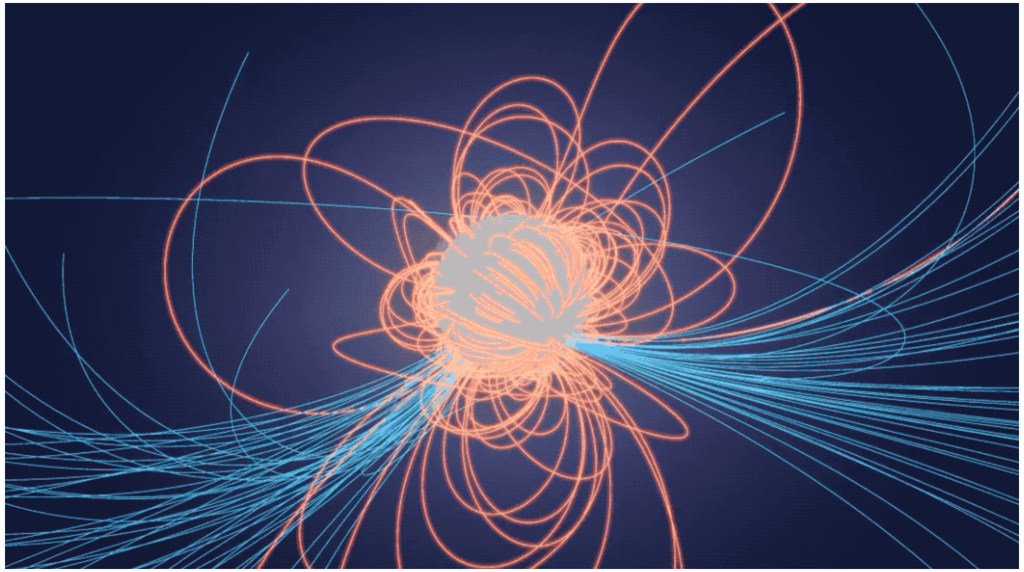
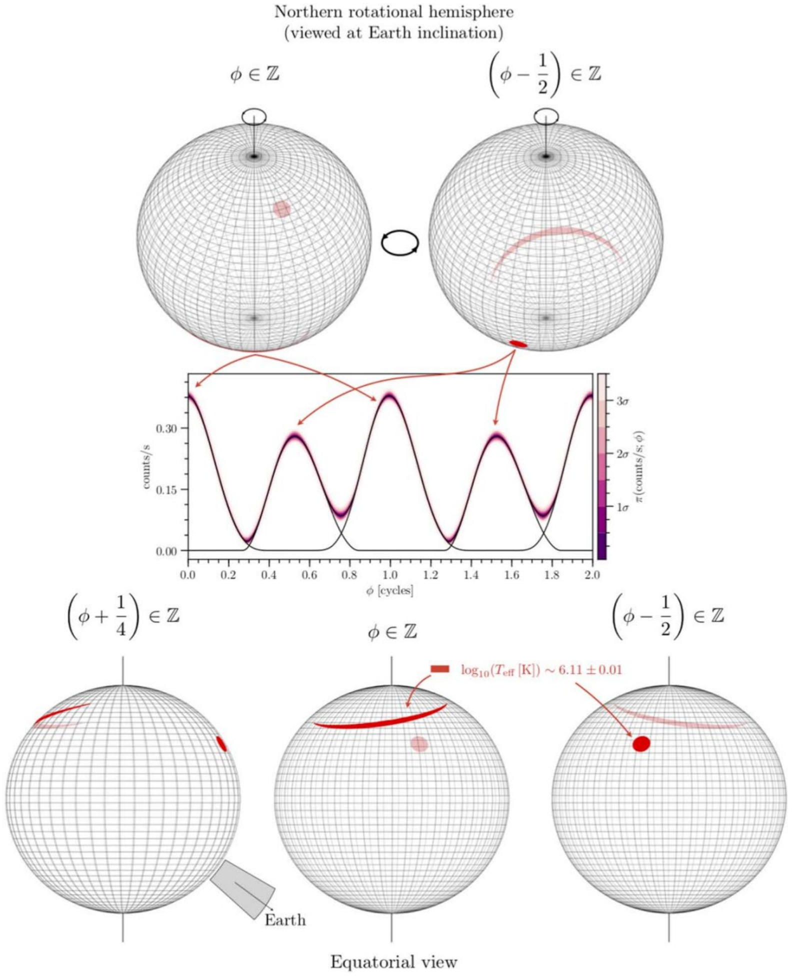


Same set of NS observations.

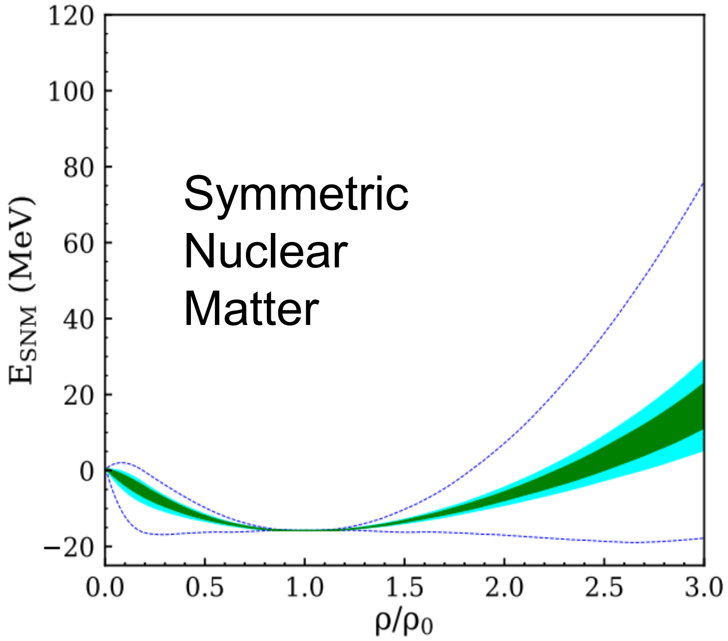
Will consider both, but error bar scaled up by $\sqrt{2}$.



The Astrophysical Journal Letters, 887:L21 (60pp), 2019 December 10

EoS in this analysis

- $E(\rho,\delta) = E_{meta}(\rho,\delta=0) + \delta^2 S(\rho)$
- $E_{meta}(\rho,\delta=0)$ is Equation of State from metamodeling.
  - The first four orders of derivatives with respect to ρ are independent of each other.
  - NOT a simple power law.
  - Can still be expanded in Taylor expansion around $x = \frac{\rho-\rho_0}{3\rho_0}$,
    - $E_{meta}(\rho,\delta=0) \approx E_{sat} + \frac{1}{2}K_{sat}x^2 + \frac{1}{6}Q_{sat}x^3 + \frac{1}{24}Z_{sat}x^4 + \cdots$



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EoS in this analysis

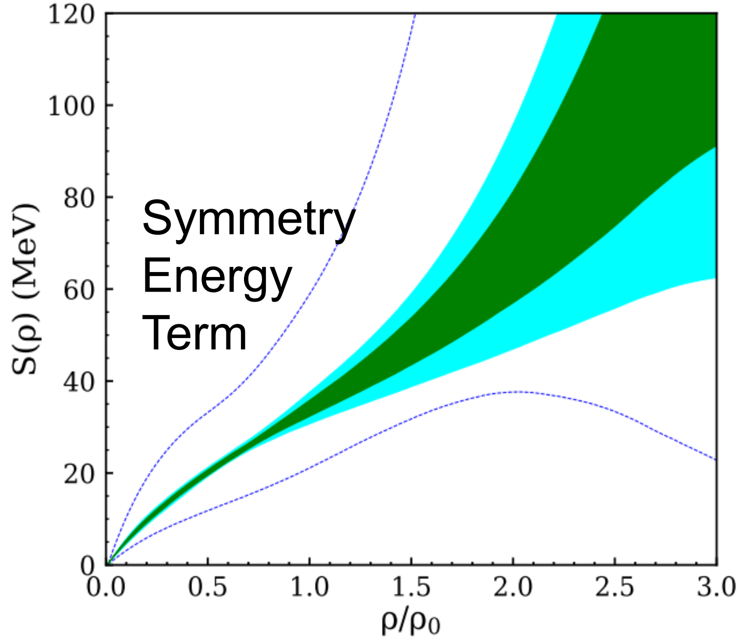
- $E(\rho,\delta) = E_{meta}(\rho,\delta = 0) + \delta^2 S(\rho)$
- $S(\rho)$ is a custom expansion from W.G. Lynch, Physics Letters B 137098 (2022).
  - Formulated to match constraints on symmetry energy term from various experiments.

$$S(\rho) = S_{kin}(\rho) + S_{int}(\rho)$$

$$S_{int}(\rho) = S_{int}(\rho_{01}) + S'_{int}(\rho - \rho_{01}) + \frac{1}{2}S''_{int}(\rho - \rho_{01})^2 + \frac{1}{6}S'''_{int}(\rho - \rho_{01})^3$$

Where $S_{int}(0) \equiv 0$

$$S_{kin}(\rho) = S_{kin}(\rho_0)(\rho/\rho_0)^{2/3}\,\text{MeV}$$



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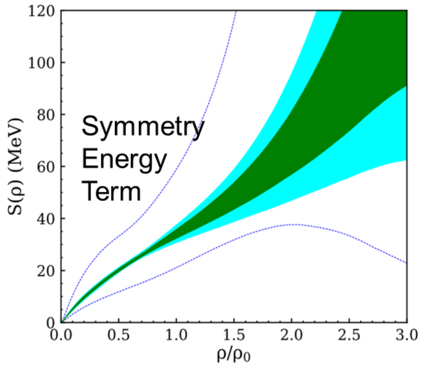
Calculate astronomical predictions from nuclear EoS

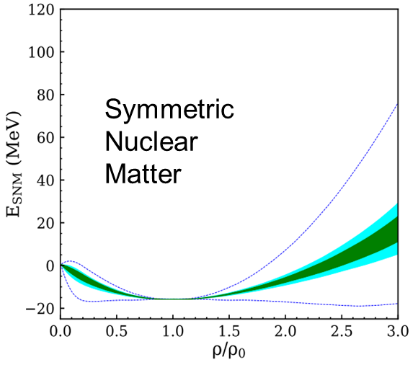
- Tolman-Oppenheimer-Volkoff (TOV) equation

$$\frac{dP(r)}{dr} = -\frac{(\mathcal{E}(r) + P(r))(M(r) + 4\pi r^3 P(r))}{r^2(1 - 2M(r)/r)},$$

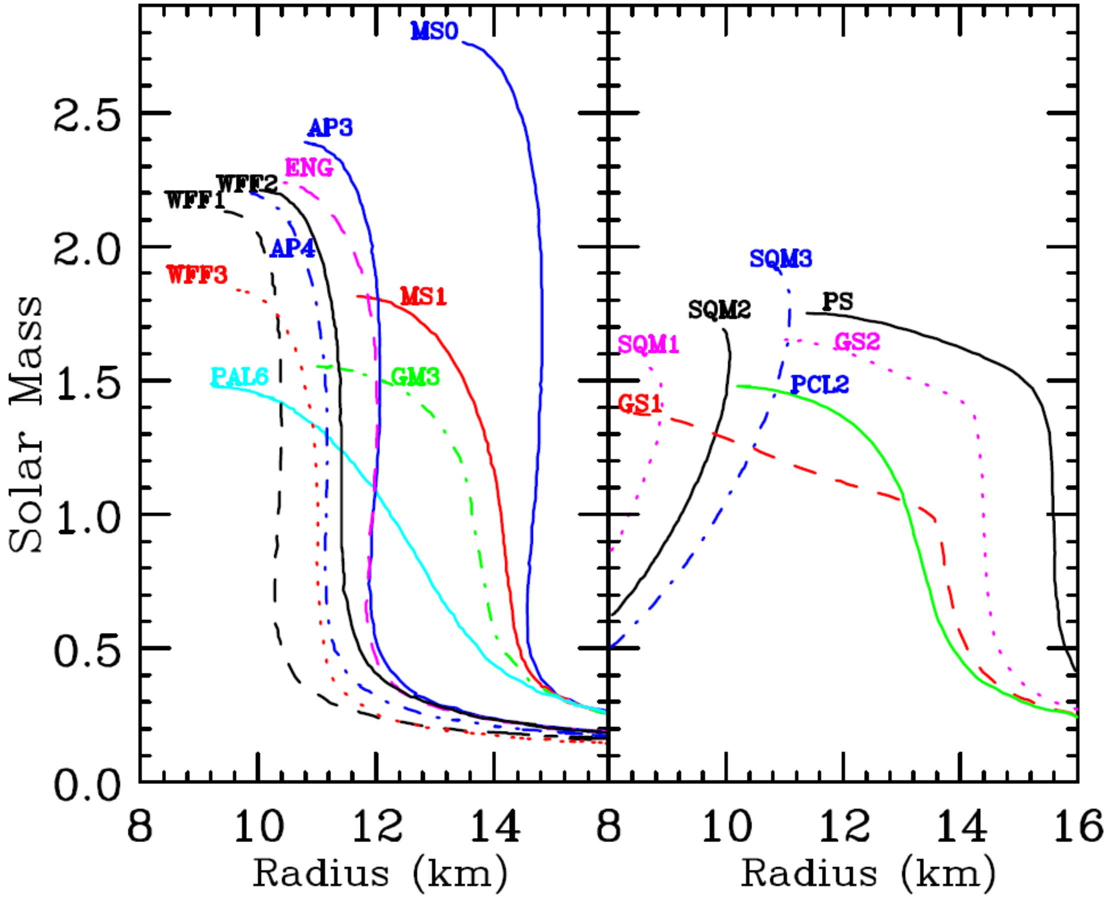
$$\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r).$$

- Convert EoS to NS properties



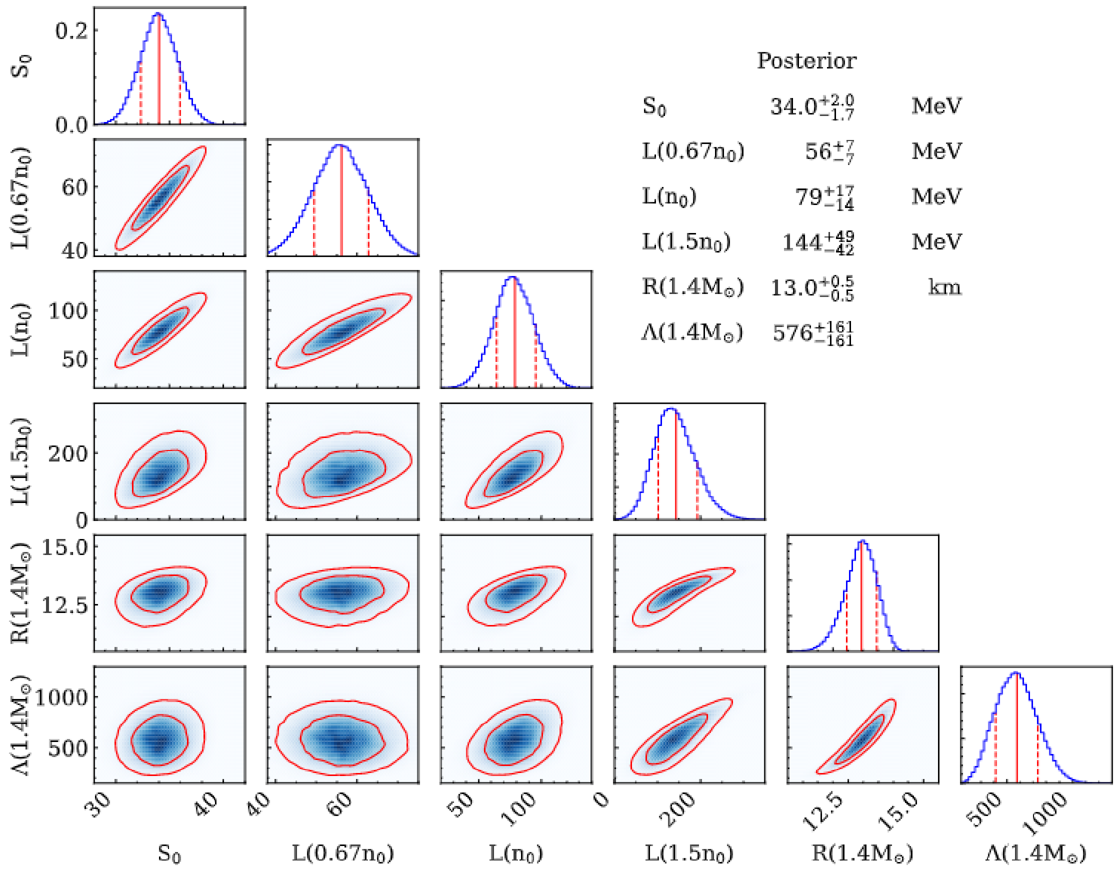


TOV equation



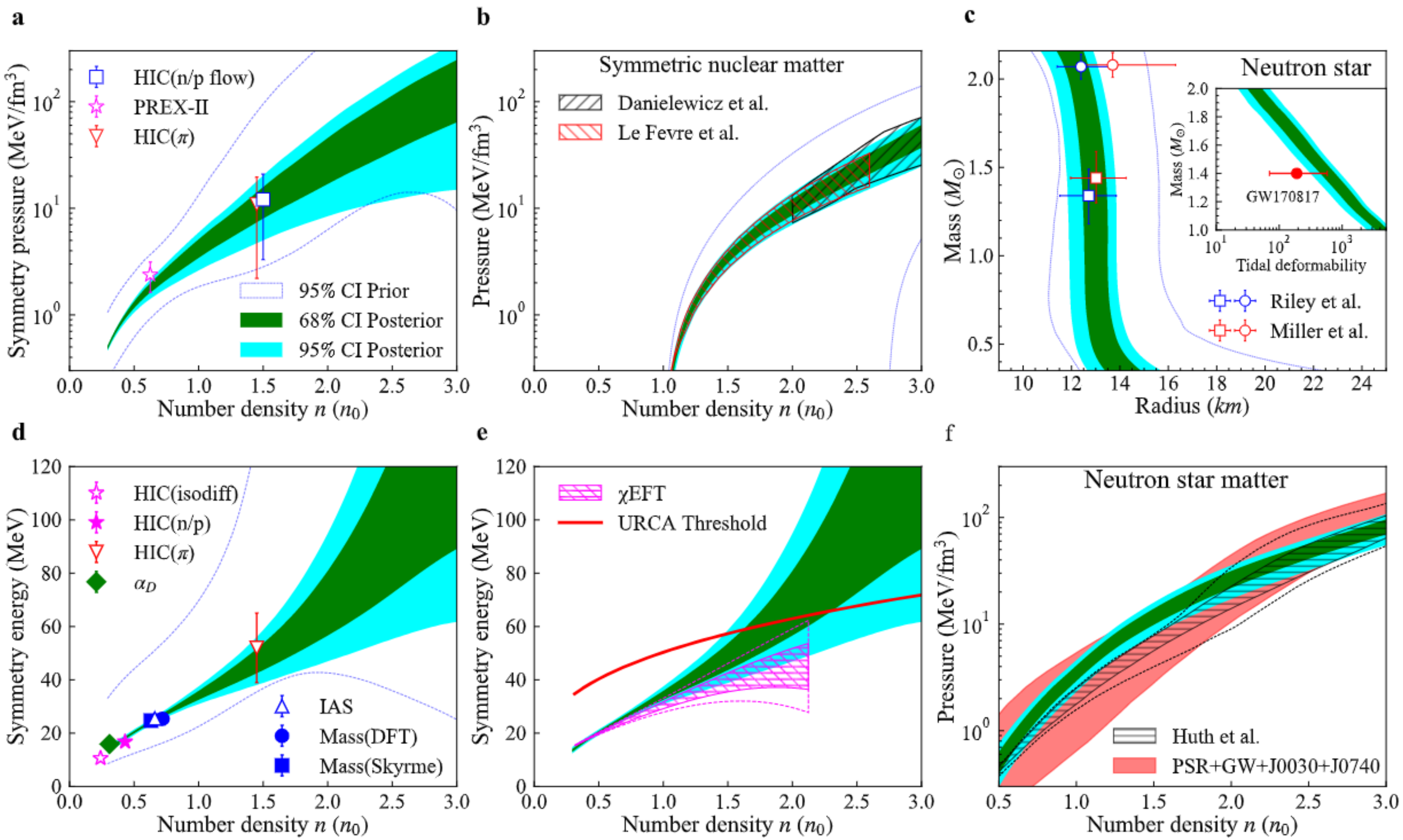
Bayesian analysis – Posterior of selected parameters

Parameters	Priors	Posteriors (Mean and 1-σ standard deviation)
$K_{\rm sat}$ (MeV)	[0, 648]	219^{+28}_{-28}
$Q_{\rm sat}$ (MeV)	[-1100, 2100]	-662^{+304}_{-217}
S_{01} (MeV)	[0, 50]	$24.0^{+0.5}_{-0.4}$
L_{01} (MeV)	[0, 120]	53^{+6}_{-5}
K_{01} (MeV)	[-300, 300]	-51^{+28}_{-25}
$Z_{\rm sat}$ (MeV)		1460^{+940}_{-940}
Q_{01} (MeV)		305^{+132}_{-132}
S_0 (MeV)		$34.0^{+2.0}_{-1.7}$
L (MeV)		79^{+17}_{-14}
$K_{\rm sym}$ (MeV)		21^{+87}_{-87}
$R(1.4M_\odot)$ (km)		$13.0^{+0.5}_{-0.5}$
$\Lambda(1.4M_\odot)$		576^{+161}_{-161}



	Posterior	
S_0	$34.0^{+2.0}_{-1.7}$	MeV
$L(0.67n_0)$	56^{+7}_{-7}	MeV
$L(n_0)$	79^{+17}_{-14}	MeV
$L(1.5n_0)$	144^{+49}_{-42}	MeV
$R(1.4M_\odot)$	$13.0^{+0.5}_{-0.5}$	km
$\Lambda(1.4M_\odot)$	576^{+161}_{-161}	

Constrained EoS



To encourage necessary developments, needed for putting a meaningful constraint on the EOS, the following questions will be considered at the workshop:

– Can we reconcile data from current and previous experiments?

– What other observables could enable the extraction of the EOS?

– Are the nuclear matter EOSs from astrophysics consistent with heavy-ion collision observables in the range rho < 4.0rho_0?

– Can we find a flexible common parametrization of the EOS, applicable to neutron star calculations and different types of heavy-ion collisions simulations?

– What improvements on the constraints on the EOS can we expect from future heavy-ion experiments?

– What development is necessary for transport codes to address the above questions?

Summary and outlook

- Compiled a list of astronomical and terrestrial constraints.
- Provided a systematic way to select EoS from constraints with Bayesian analysis.
- More work are underway to study symmetry energy term at above $1.5\rho_0$.
- More NS will be measured by LIGO and NICER in the future.

SπRIT

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Genie Jhang

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Jon Barney
Justin Estee
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https://groups.nscl.msu.edu/hira/cosmic/SpiritTPC.html

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Nusym2018
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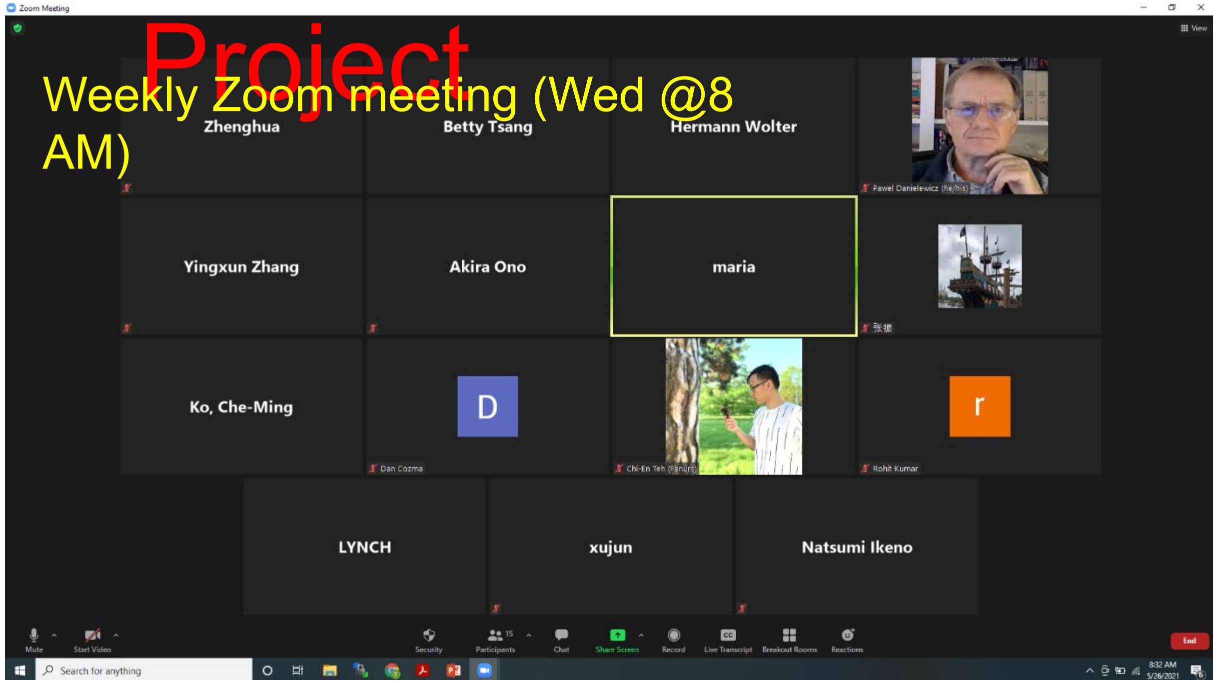
Korea

Transport Models Evaluation Project

- Maria Colonna
- Dan Cozma
- Pawel Danielewicz
- Natsumi Ikeno
- CheMing Ko
- Bill Lynch
- Akira Ono
- Jun Su
- Betty Tsang
- Hermann Wolter
- Jun Xu
- YingXun Zhang
- Zhen Zhang


Weekly Zoom meeting (Wed @8 AM)

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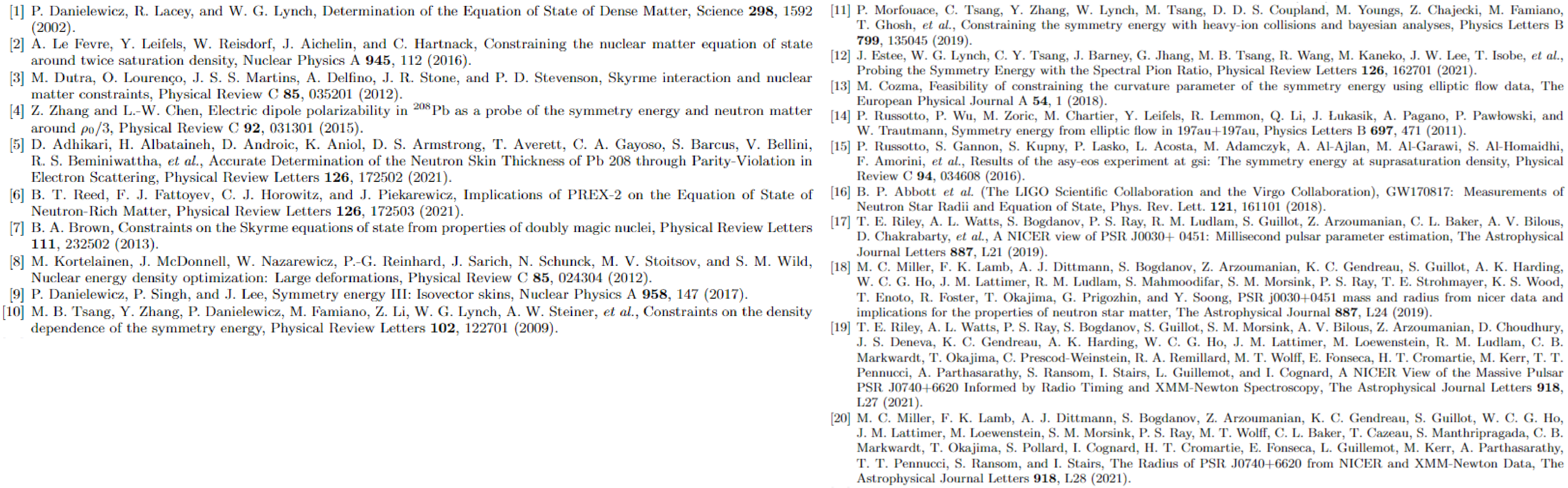

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Back-up slides

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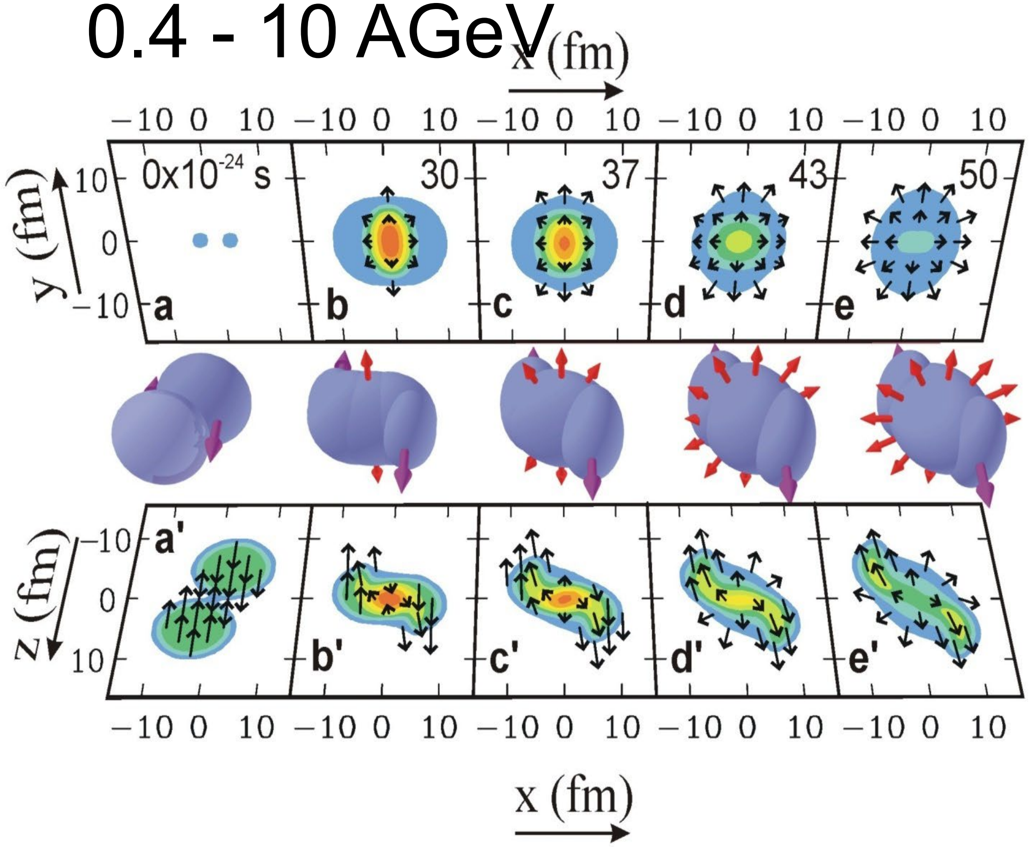
Symmetric matter constraint from the observed nuclear flow

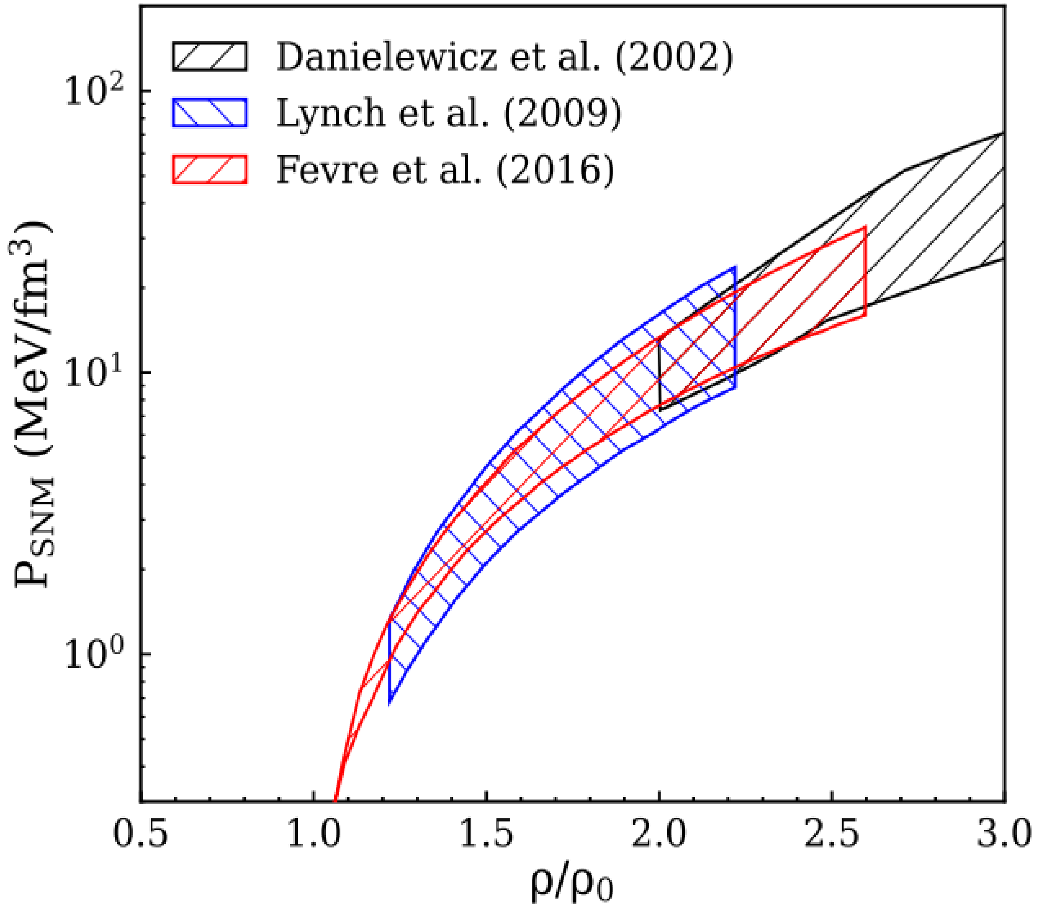
- Elliptical flow from Au + Au @ 0.4 - 10 AGeV

Danielewicz, Lacey, Lynch, Science 298, 1592 (2002)
(Le Fevre et. al, NPA 945, 112 (2016))



pressure contours

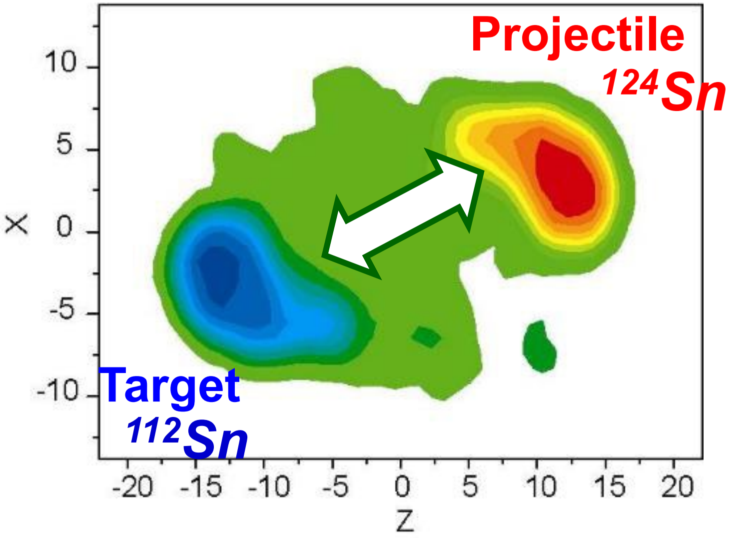
density contours



Symmetry energy term constraint from Isospin diffusion and n/p ratio

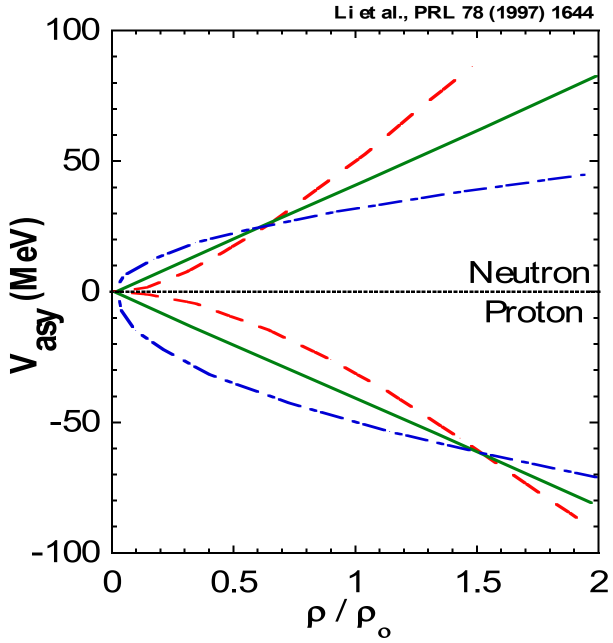
- ^{112}Sn + ^{112}Sn and ^{124}Sn + ^{124}Sn
- 50 AMeV for isospin diffusion, 120 AMeV for n/p ratio

Isospin Diffusion; low ρ



Tsang et al., PRL **92**, 062701 (2004)

$S(0.038\ \mathrm{fm}^{-3}) = 10.3 \pm 1.0$ MeV [10]

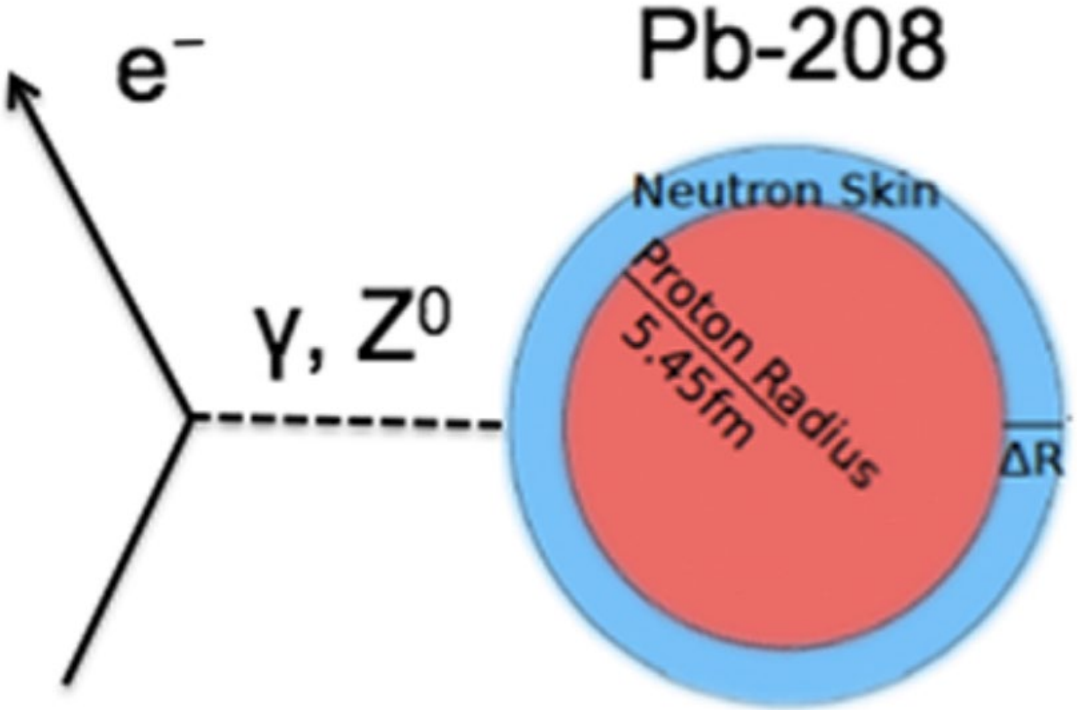


Y(n)/Y(p) ratios
V1(n)/v1(p)
flows

$S(0.069\ \mathrm{fm}^{-3}) = 16.8 \pm 1.2$ MeV [11]

Symmetry energy term constraint from PREX-II

- Lead Radius Experiment (PREX-II)
- Probe neutron radius through parity violating electron scattering.
- Neutron skin ⇔ Pressure of neutron matter.
- Sensitive to symmetry energy term at $\rho = 2/3\rho_0$.
- $\mathrm{L}(0.1\ \mathrm{fm}^{-3}) = 71.5 \pm 22.6$ MeV. [5,6]

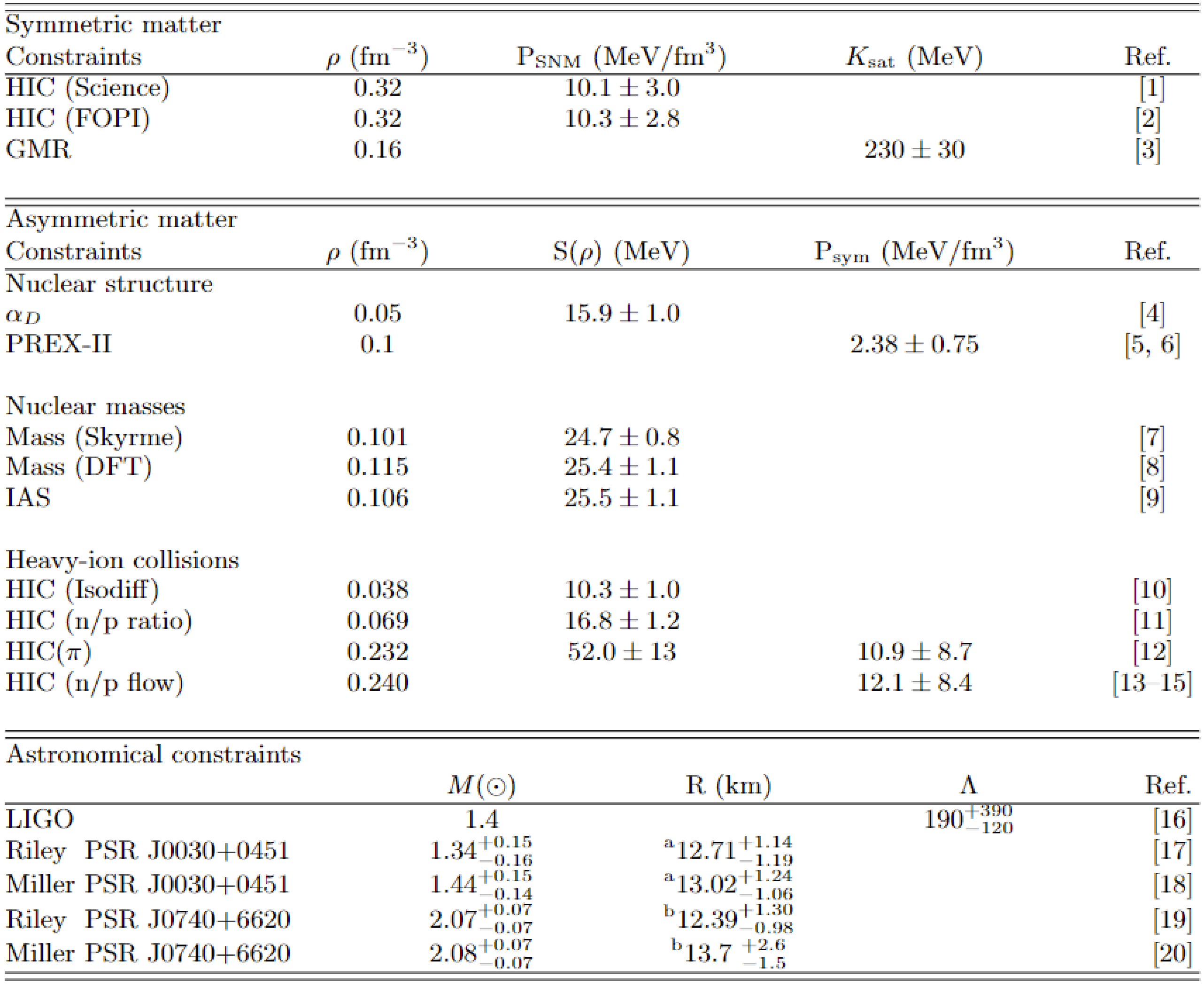


Source:
https://www.physics.umass.edu/news/2012-01-
29-neutron-skin-lead-measured-prex

Symmetry energy term constraints from dipole polarizability and nuclear masses

- Dipole polarizability (α_D): Response of nucleus to the presence of external E-field.
  - measured for ^{208}Pb.
  - Sensitive to density at $\rho = 0.31\rho_0 = 0.05\ fm^{-3}$.
  - $S(0.05\ \mathrm{fm}^{-3}) = 15.9 \pm 1.0$ MeV. [4]
- Nuclear masses: Fit the observed nuclide masses and masses of double magic nuclei with different density functionals.
  - Sensitive to density at $\rho \approx \frac{2}{3}\rho_0$.
  - When fitted with Skyrme, analysis shows $S(0.101\ \mathrm{fm}^{-3}) = 24.7 \pm 0.8$ MeV. [7]
  - When fitted with DFT, analysis shows $S(0.115\ \mathrm{fm}^{-3}) = 25.4 \pm 1.1$ MeV. [8]
- Isobaric analogue state (same mass, different numbers of p and n) analysis shows $S(0.106\ \mathrm{fm}^{-3}) = 25.5 \pm 1.1$ MeV. [9]

Short summary on all constraints



Symmetric matter

Constraints	ρ (fm^{-3})	P_{SNM} (MeV/fm^3)	K_{sat} (MeV)	Ref.
HIC (Science)	0.32	10.1 ± 3.0		[1]
HIC (FOPI)	0.32	10.3 ± 2.8		[2]
GMR	0.16		230 ± 30	[3]

Asymmetric matter

Constraints	ρ (fm^{-3})	S(ρ) (MeV)	P_{sym} (MeV/fm^3)	Ref.
Nuclear structure				
α_D	0.05	15.9 ± 1.0		[4]
PREX-II	0.1		2.38 ± 0.75	[5, 6]
Nuclear masses				
Mass (Skyrme)	0.101	24.7 ± 0.8		[7]
Mass (DFT)	0.115	25.4 ± 1.1		[8]
IAS	0.106	25.5 ± 1.1		[9]
Heavy-ion collisions				
HIC (Isodiff)	0.038	10.3 ± 1.0		[10]
HIC (n/p ratio)	0.069	16.8 ± 1.2		[11]
HIC(π)	0.232	52.0 ± 13	10.9 ± 8.7	[12]
HIC (n/p flow)	0.240		12.1 ± 8.4	[13–15]

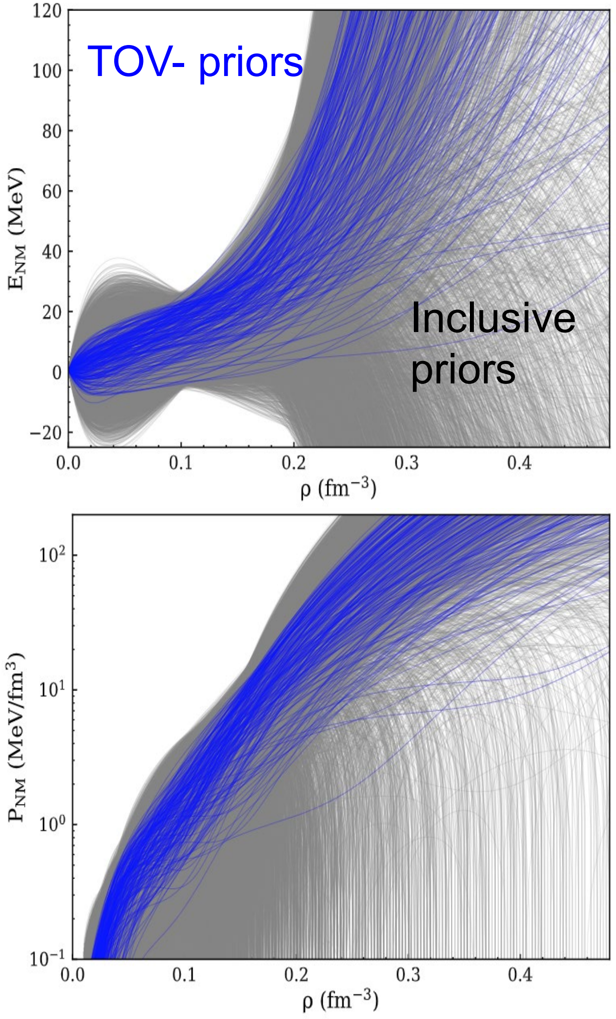
Astronomical constraints

	$M(\odot)$	R (km)	Λ	Ref.
LIGO	1.4		190^{+390}_{-120}	[16]
Riley PSR J0030+0451	$1.34^{+0.15}_{-0.16}$	$^{a}12.71^{+1.14}_{-1.19}$		[17]
Miller PSR J0030+0451	$1.44^{+0.15}_{-0.14}$	$^{a}13.02^{+1.24}_{-1.06}$		[18]
Riley PSR J0740+6620	$2.07^{+0.07}_{-0.07}$	$^{b}12.39^{+1.30}_{-0.98}$		[19]
Miller PSR J0740+6620	$2.08^{+0.07}_{-0.07}$	$^{b}13.7^{+2.6}_{-1.5}$		[20]

Bayesian analysis – Inclusive Prior and TOV Prior

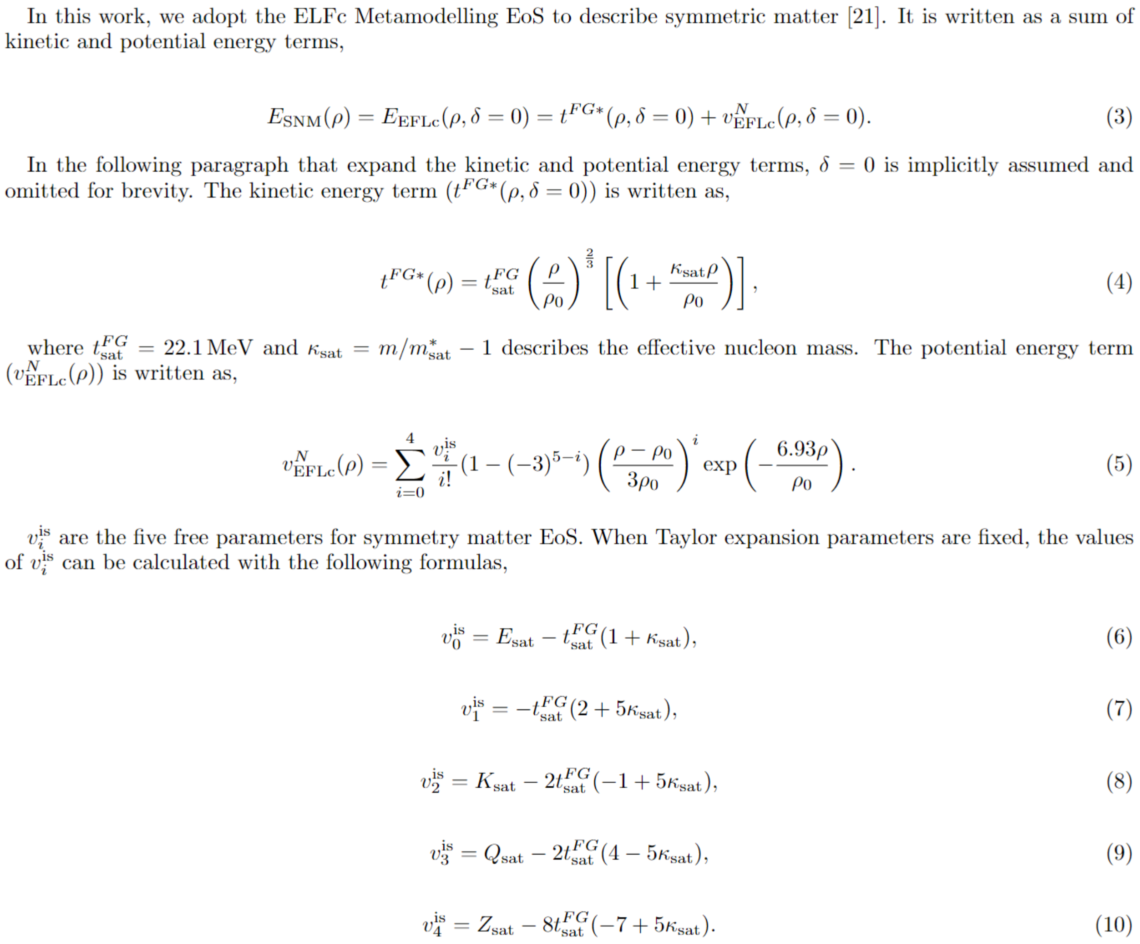
- Inclusive Prior (as wide as possible):

	Prior	
	Min.	Max.
K_{sat}	0	648
Q_{sat}	-1100	2100
$P_{\text{SNM}}(4\rho_0)$	0	300
m_{sat}/m_N	0.50	1.0
S_{int}	0.87	28.9
S'_{int}	-141	379
S''_{int}	-2990	2450



- TOV prior:
  - Inclusive prior that satisfies thermodynamic stability condition.
  - Maximum NS mass > 2.17 solar mass.

Formulation of our EoS



In this work, we adopt the ELFc Metamodelling EoS to describe symmetric matter [21]. It is written as a sum of kinetic and potential energy terms,

$$E_{\rm SNM}(\rho) = E_{\rm EFLc}(\rho, \delta = 0) = t^{FG*}(\rho, \delta = 0) + v^{N}_{\rm EFLc}(\rho, \delta = 0). \tag{3}$$

In the following paragraph that expand the kinetic and potential energy terms, $\delta = 0$ is implicitly assumed and omitted for brevity. The kinetic energy term ($t^{FG*}(\rho, \delta = 0)$) is written as,

$$t^{FG*}(\rho) = t^{FG}_{\rm sat}\left(\frac{\rho}{\rho_0}\right)^{\frac{2}{3}}\left[\left(1 + \frac{\kappa_{\rm sat}\rho}{\rho_0}\right)\right], \tag{4}$$

where $t^{FG}_{\rm sat} = 22.1\,\mathrm{MeV}$ and $\kappa_{\rm sat} = m/m^{*}_{\rm sat} - 1$ describes the effective nucleon mass. The potential energy term ($v^{N}_{\rm EFLc}(\rho)$) is written as,

$$v^{N}_{\rm EFLc}(\rho) = \sum_{i=0}^{4}\frac{v^{\rm is}_i}{i!}(1 - (-3)^{5-i})\left(\frac{\rho - \rho_0}{3\rho_0}\right)^{i}\exp\left(-\frac{6.93\rho}{\rho_0}\right). \tag{5}$$

$v^{\rm is}_i$ are the five free parameters for symmetry matter EoS. When Taylor expansion parameters are fixed, the values of $v^{\rm is}_i$ can be calculated with the following formulas,

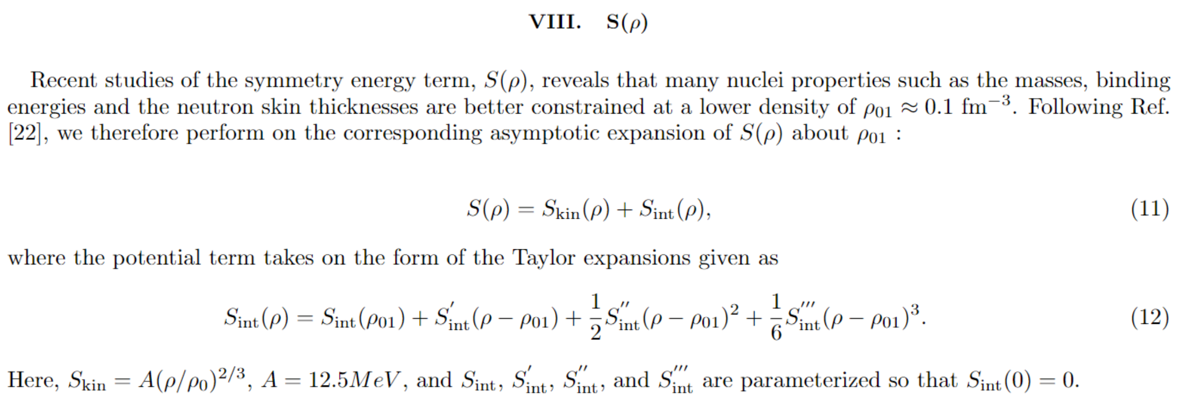
$$v^{\rm is}_0 = E_{\rm sat} - t^{FG}_{\rm sat}(1 + \kappa_{\rm sat}), \tag{6}$$

$$v^{\rm is}_1 = -t^{FG}_{\rm sat}(2 + 5\kappa_{\rm sat}), \tag{7}$$

$$v^{\rm is}_2 = K_{\rm sat} - 2t^{FG}_{\rm sat}(-1 + 5\kappa_{\rm sat}), \tag{8}$$

$$v^{\rm is}_3 = Q_{\rm sat} - 2t^{FG}_{\rm sat}(4 - 5\kappa_{\rm sat}), \tag{9}$$

$$v^{\rm is}_4 = Z_{\rm sat} - 8t^{FG}_{\rm sat}(-7 + 5\kappa_{\rm sat}). \tag{10}$$



VIII. S(ρ)

Recent studies of the symmetry energy term, $S(\rho)$, reveals that many nuclei properties such as the masses, binding energies and the neutron skin thicknesses are better constrained at a lower density of $\rho_{01} \approx 0.1\ \mathrm{fm}^{-3}$. Following Ref. [22], we therefore perform on the corresponding asymptotic expansion of $S(\rho)$ about ρ_{01} :

$$S(\rho) = S_{\rm kin}(\rho) + S_{\rm int}(\rho), \tag{11}$$

where the potential term takes on the form of the Taylor expansions given as

$$S_{\rm int}(\rho) = S_{\rm int}(\rho_{01}) + S^{'}_{\rm int}(\rho - \rho_{01}) + \frac{1}{2}S^{''}_{\rm int}(\rho - \rho_{01})^2 + \frac{1}{6}S^{'''}_{\rm int}(\rho - \rho_{01})^3. \tag{12}$$

Here, $S_{\rm kin} = A(\rho/\rho_0)^{2/3}$, $A = 12.5MeV$, and $S_{\rm int}$, $S^{'}_{\rm int}$, $S^{''}_{\rm int}$, and $S^{'''}_{\rm int}$ are parameterized so that $S_{\rm int}(0) = 0$.

Conversion to NS properties

- To convert each nuclear EoS to neutron star EoS,

1. Calculate the ratios of proton, neutron, electrons and muon by minimizing total energy at different pressures.
2. At low density, connect it to crustal EoS.
3. At high density, if speed of sound exceeds speed of light, meta-modeling EoS will be replaced with EoS of constant speed of sound = c.

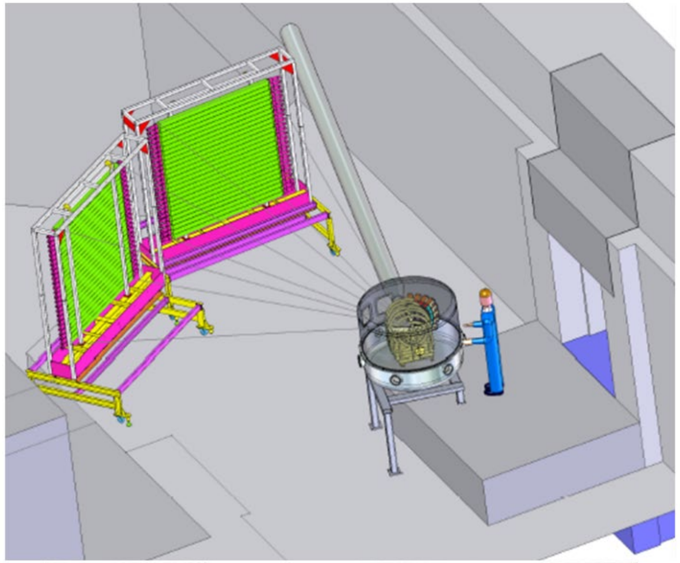
- The converted EoS is put inside Tolman-Oppenheimer-Volkoff (TOV) equation to predict the radius of 1.4 solar mass NS,

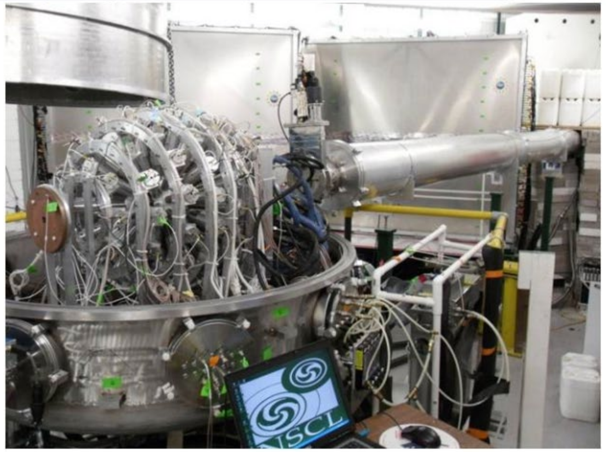
$$\frac{dP(r)}{dr} = -\frac{(\mathcal{E}(r)+P(r))(M(r)+4\pi r^3 P(r))}{r^2(1-2M(r)/r)},$$

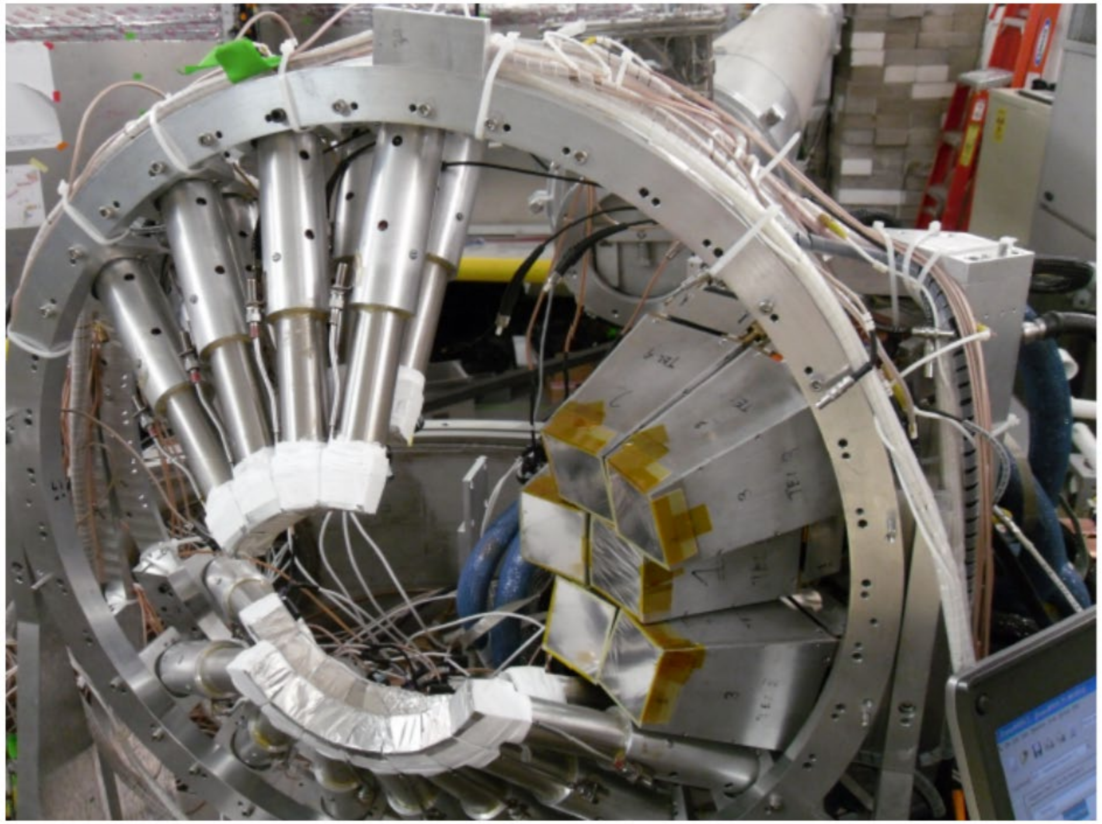
$$\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r).$$

- TOV equation predicts the structure of a static spherical object under general relativity.
- Details are given in C. Y. Tsang , et al. (2020) Phys. Rev. C **102**, 045808
- Can be extended to give Λ, the tidal deformability of NS.

LASSA experiment for n/p ratio







Courtesy Daniel Coupland