A quantum-information theoretic viewpoint of Many-body open quantum systems

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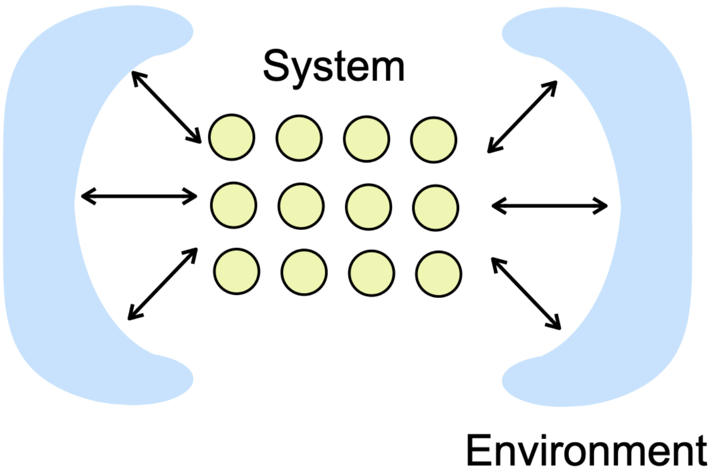
Open systems



Accurate description of experimental systems

Quantum optics e.g. quantum emitters coupled to photonic fields.

Quantum information processing platforms.

Many-body physics

Thermalization, deep-thermalization …

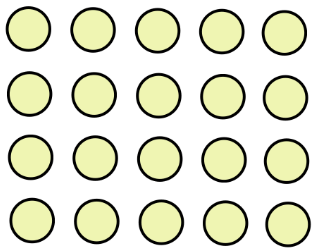
Generalization of classical probabilistic models

Probability distributions → Density matrices.

Classical Langevin equations → Quantum Open system models.

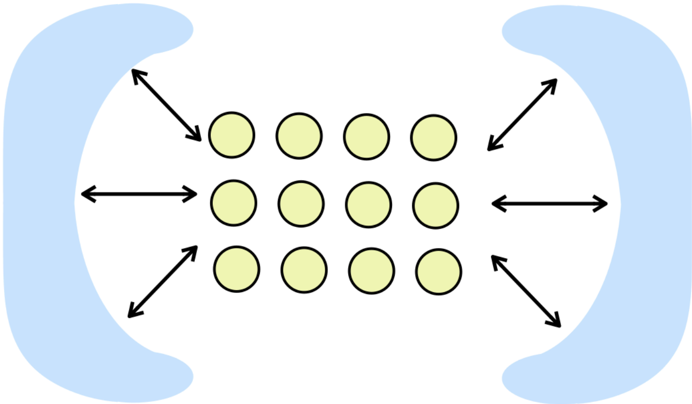
The many-body regime

Large number of interacting subsystems $\Longrightarrow$ exponentially large Hilbert spaces.



Closed Systems

- Well developed numerical tools.
- Well-understood analytical methods.
- Rigorous theory of many-body phases.
- Information-theoretic/Complexity-theoretic understanding.



Open Systems

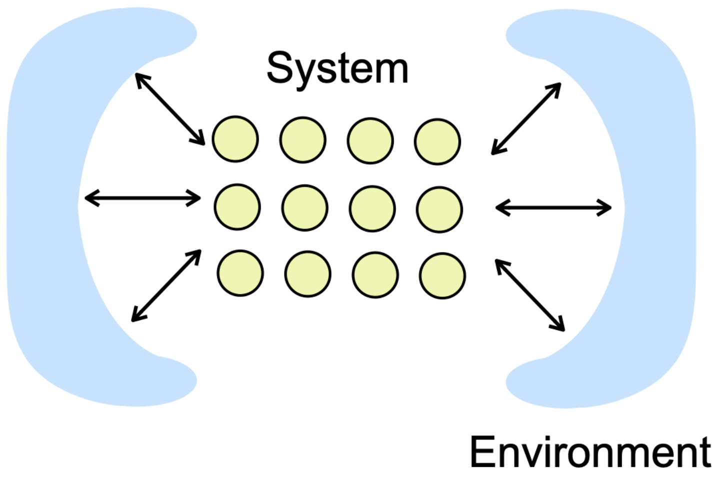
?

Model

$$H_{SE}(t) = \underbrace{H_S}_{} + \sum_{\alpha} \underbrace{X_\alpha}_{} \underbrace{B_\alpha(t)}_{}$$

System Operators

Environment Operators



System dynamics:

$$\rho_S(t) = \mathrm{Tr}_E[U_{SE}(t,0)(\rho_S(0) \otimes \rho_E(0))U_{SE}(0,t)]$$

Environment can be specified via $n-$point correlation functions:

$$K_{\alpha_1,\alpha_2\ldots\alpha_n}(t_1, t_2 \ldots t_n) = \mathrm{Tr}_E[B_{\alpha_1}(t_1)B_{\alpha_2}(t_2)\ldots B_{\alpha_n}(t_n)\rho_E(0)]$$

Models: Stationary and Gaussian environments

Stationarity: $K_{\alpha_1,\alpha_2\ldots\alpha_n}(t_1, t_2\ldots t_n) = K_{\alpha_1,\alpha_2\ldots\alpha_n}(t_1+\tau, t_2+\tau\ldots t_n+\tau)$

Gaussianity: $K_{\alpha_1,\alpha_2\ldots\alpha_n}(t_1, t_2\ldots t_n)$ satisfies Wick's/Isserlis's theorem:

$$K_{\alpha_1,\alpha_2\ldots\alpha_n}(t_1, t_2\ldots t_n) = K_{\alpha_1,\alpha_2}(t_1, t_2)K_{\alpha_3,\alpha_4}(t_3, t_4)\ldots + \text{all other pairings}$$

Open systems with stationary Gaussian environments specified by:

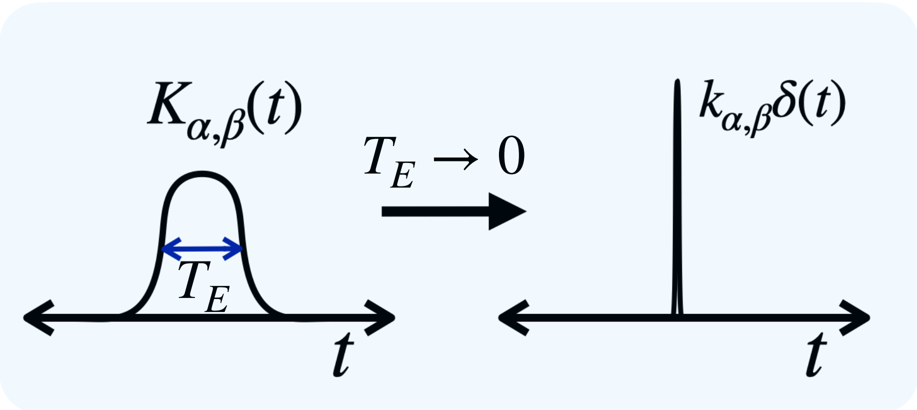
System Hamiltonian H_S, Jump operators X_α and memory kernels $K_{\alpha,\beta}(t)$.

Markovian models

- $K_{\alpha,\beta}(t_1 - t_2) \sim k_{\alpha,\beta}\delta(t_1 - t_2)$, system dynamics described by Lindblad master equation

$$\frac{d}{dt}\rho(t) = -i[H_S, \rho(t)] + \sum_{\alpha,\beta} k_{\alpha,\beta}\left(X_\alpha \rho(t) X_\beta - \frac{1}{2}\{X_\beta X_\alpha, \rho(t)\}\right)$$

- In general, Markov limit and thermodynamic limits **do not** commute.



$$\lim_{N\to\infty} \lim_{T_E\to 0} O_N(T_E) \neq \lim_{T_E\to 0} \lim_{N\to\infty} O_N(T_E)$$

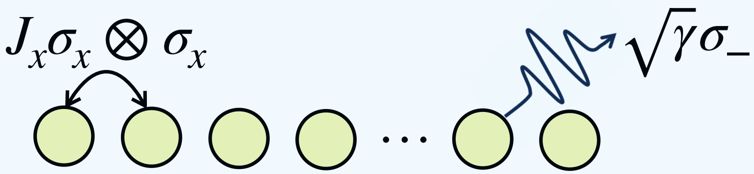
Explicit examples even with Gaussian fermion models

B. Windt, X. Yu, **RT** (2025)

How “complex” are many-body systems?

Specific models

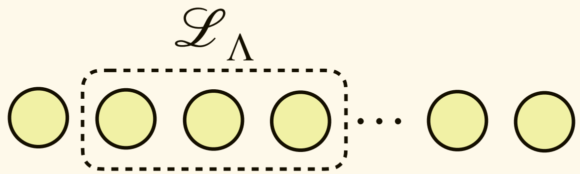
E.g. Fixed point of a master equation with $\sigma_x^{\otimes 2}$ Hamiltonian and σ_- dissipation.



- Correlation functions.
- Entanglement entropies/mutual information.
- Stabilizer entropies/Magic.

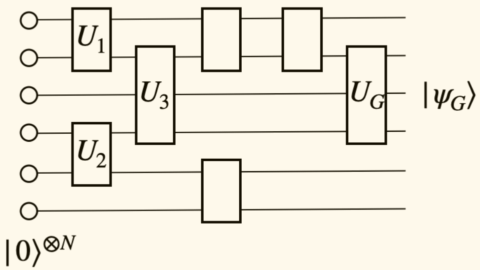
Model families

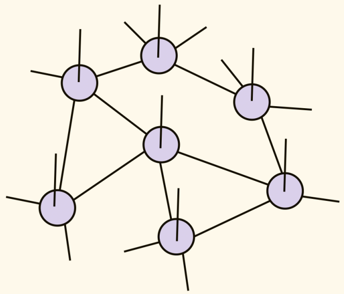
E.g. Geometrically local, Translationally invariant Lindbladians.



- Which quantum states can be prepared using these model families?
- How easy is it to simulate these model families classically or quantumly?

State Preparation





To

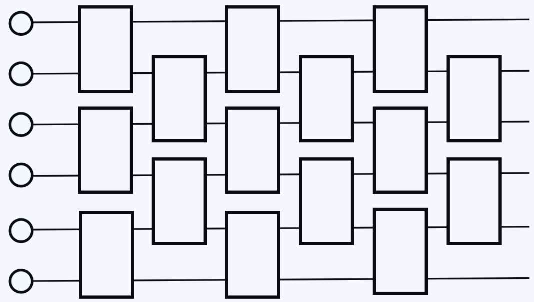
Geometrically local
open system

Measure of the operational capability of a family of open-system models.

Circuit Complexity

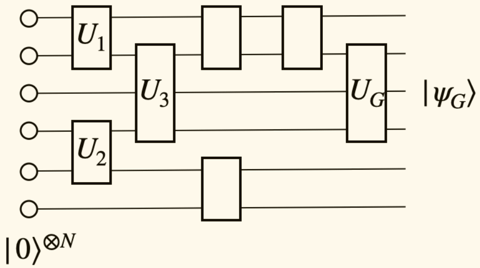
Geometrically local open system dynamics

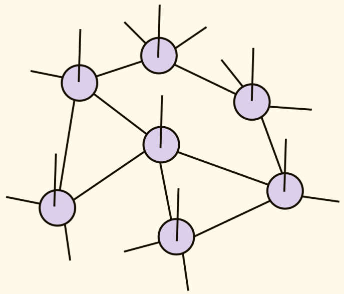
To



Measure of quantum resources needed to simulate open-system models.

State Preparation





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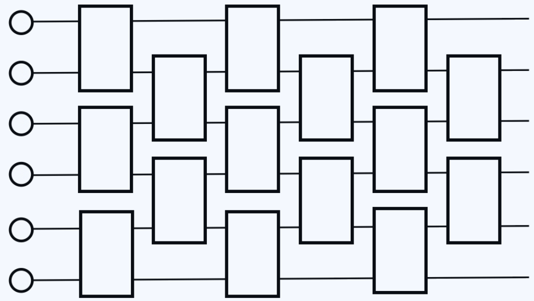
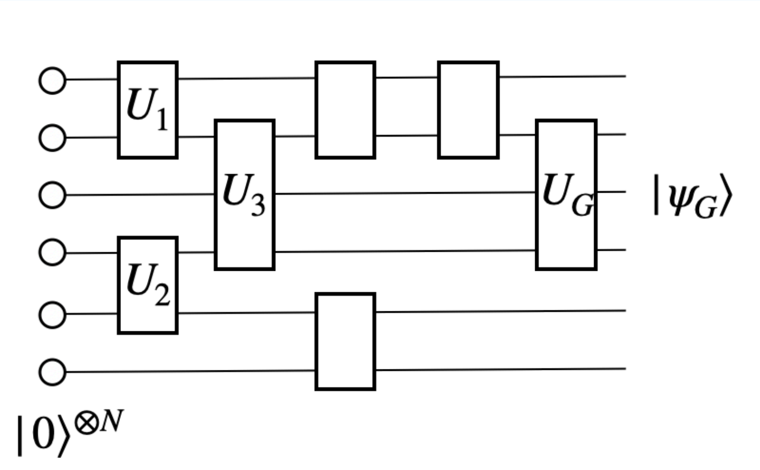


Measure of quantum resources needed to simulate open-system models.

State-preparation with dissipation



Target quantum state:

$$|\psi_G\rangle = \underbrace{U_G U_{G-1} \dots U_1}_{U} |0\rangle^{\otimes N}$$

Lindbladian that prepares $|\psi_T\rangle$ as its unique fixed point?

Yes but with non-local jump operators

$$L_i = U|0\rangle_i\langle 1|U^\dagger$$

No with only $k-$local jump operators

$$|\text{GHZ}\rangle = |0\rangle^{\otimes N} + |1\rangle^{\otimes N}$$

cannot be unique fixed point of a local Lindbladian.

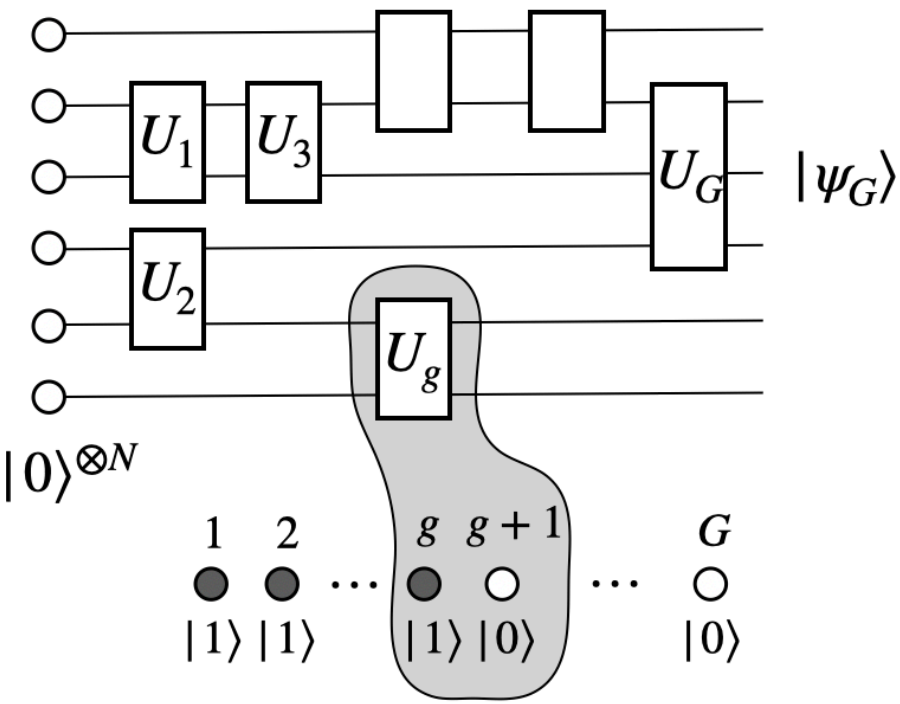
State-preparation with dissipation

5-local lindbladians **can** prepare encodings of the state $|\psi_G\rangle$

$$\rho = \frac{1}{G+1}\Big(|\psi_0\rangle\langle\psi_0| \otimes |00\ldots0\rangle\langle 00\ldots0| + |\psi_1\rangle\langle\psi_1| \otimes |10\ldots0\rangle\langle 10\ldots0| + \ldots |\psi_G\rangle\langle\psi_G| \otimes |11\ldots1\rangle\langle 11\ldots1| \Big).$$

- Post-select the ancillary/clock qubits to get $|\psi_G\rangle$

F. Verstraete, M. Wolf, J. I. Cirac (2009)



Mixing time + post-selection time $\sim O(G^3 \log N)$

D. Baruah, G. Dunnweber, G. Styliaris, **RT** (2025)

State-preparation with dissipation

Restricting the Lindbladians to be geometrically local.

- **In 2D lattices:**

  Mixing time + post-selection $\sim O(N^4 G^3 \log N^2 G)$

  In 1D lattices:

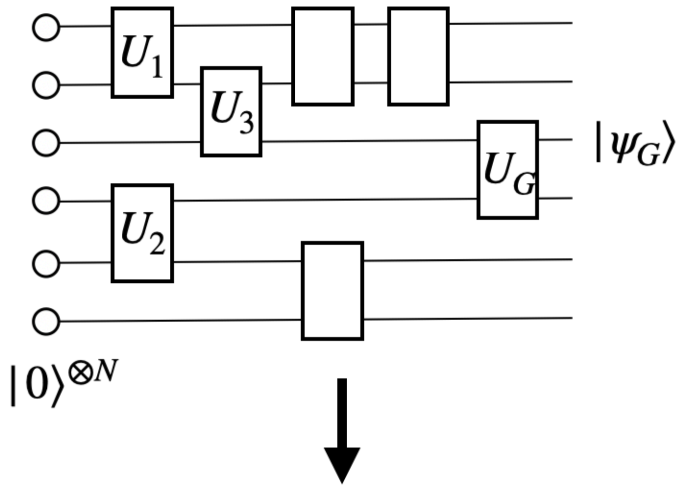
  Mixing time + post-selection $\sim O(N^{12} G^3 \log N^4 G)$

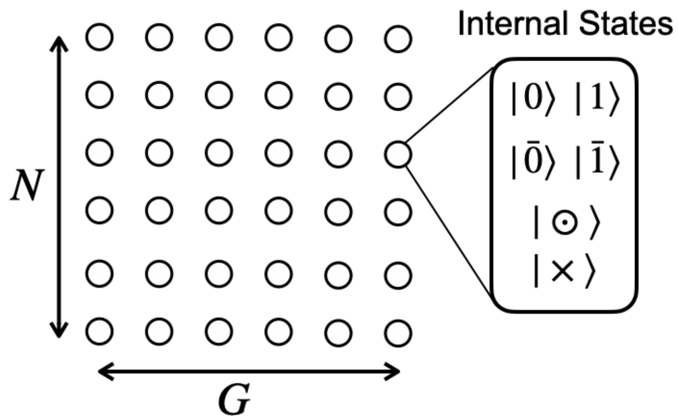
D. Baruah, G. Dunnweber, G. Styliaris, **RT** (2025)

- Translationally invariant (TI) models:

  As powerful as TI circuits on TI product states.

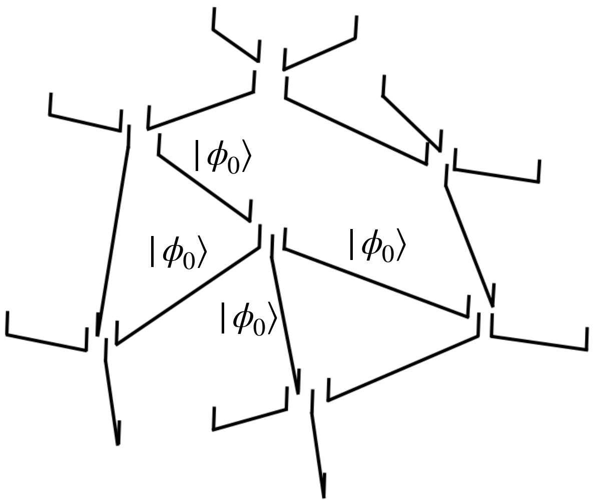
G. Dunnweber, J.I. Cirac, G. Styliaris, **RT** (2025)



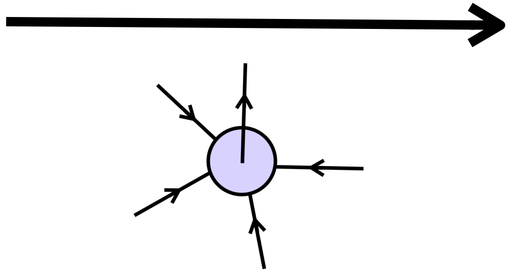


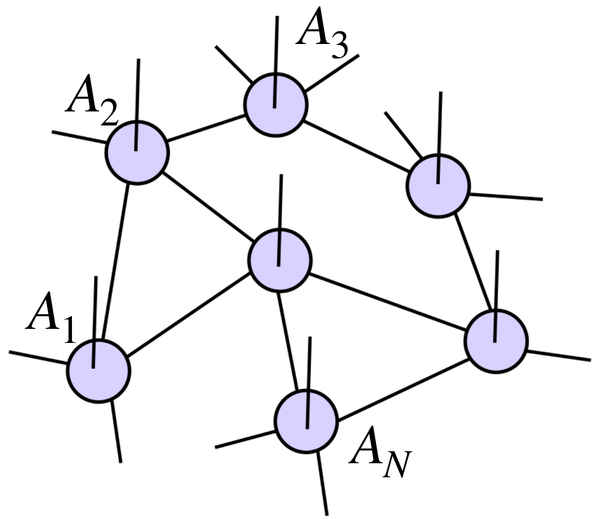
Tensor network states

Maximally entangled states on each edge



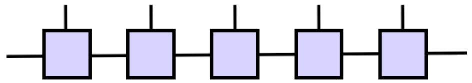
Apply 1-to-1 map on each vertex



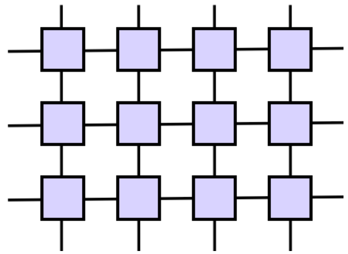


States specified as *injective* tensor networks.

- Approximate ground states of gapped Hamiltonians.
- Satisfy an area-law for entanglement entropy.



Matrix Product States



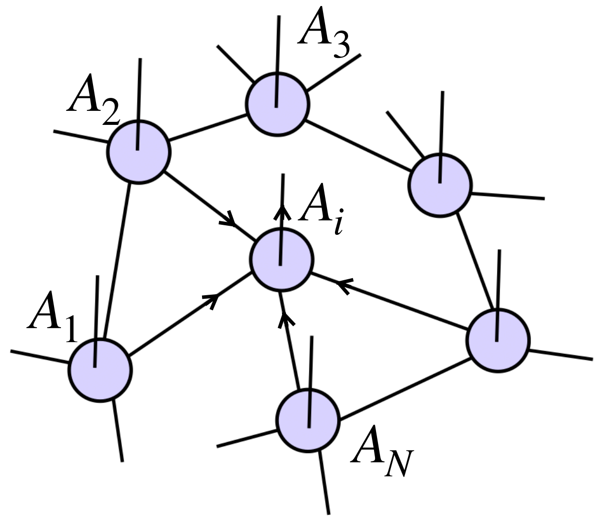
2D projected entangled-pair states

Dissipative preparation of tensor network states

Def. $\delta-$isometric tensor network state

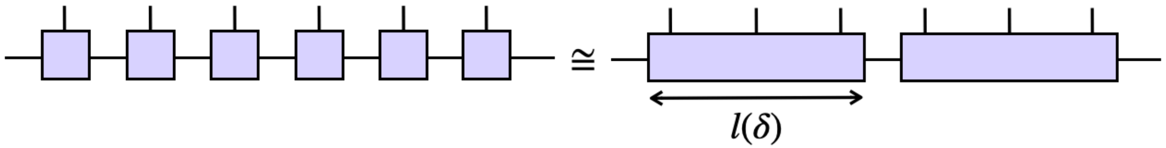
A tensor network state $|\psi\rangle$ specified on a graph $\mathcal{G}$ is δ-isometric if for each tensor A_i

$$\|A_i^\dagger A_i - I\| \leq \delta$$



In 1D

$\delta-$unitarity can be provably satisfied by blocking



In 2D

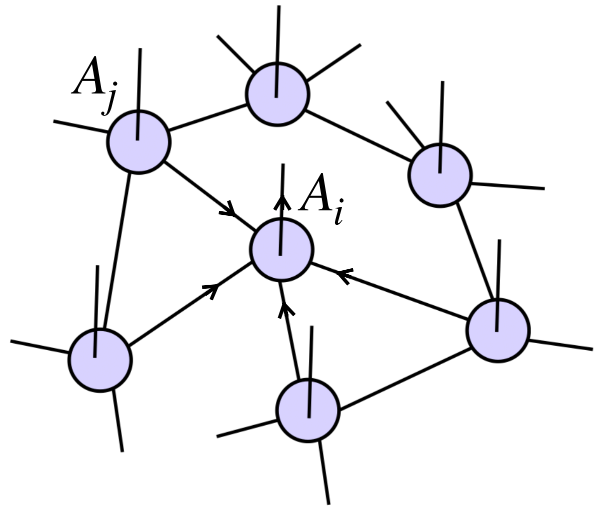
Conjecture: $\delta-$unitarity can be satisfied by blocking (mod low-depth circuit).

Dissipative preparation of tensor network states

Parent Lindbladian:

$$\mathscr{L}(\rho) = \sum_{i,j,\alpha} L_{i,j;\alpha} \rho L^{\dagger}_{i,j;\alpha} - \frac{1}{2}\{L^{\dagger}_{i,j;\alpha} L_{i,j;\alpha}, \rho\}$$

$$L_{i,j;\alpha} = A_i A_j |\phi_0\rangle\langle\phi_\alpha| A_i^{-1} A_j^{-1}$$



Result: Parent Lindbladian for $\delta-$isometric TNS

If $\delta < c(\mathscr{G})$ where $c(\mathscr{G})$ is a constant that depends only on the degree of the graph $\mathscr{G}$, then there is a local Lindbladian which prepares $|\psi\rangle$ as a unique fixed point in time $\sim O(\log(N))$.

D. Baruah, G. Styliaris, J.I. Cirac, **RT** (2025)

Dissipative vs unitary state preparation

- Every 1D injective tensor network state can be dissipatively prepared in $O(\log N)$.

  Previous best dissipative preparation: Time $\sim O(N^{\log(N)})$

  Unitary state preparation: Time $\sim O(\log(N))$ F. Verstraete et al (2009)

  D. Malz, G. Styliaris, Z. Wei et al (2024)

- In 2 and higher dimensions, sufficiently small δ

  Unitary state preparation: Time $\sim O(\log^{d+1}(N))$ Y. Ge et al (2015)

  $\Longrightarrow$ Polynomial speed-up over known unitary methods.

- On non-geometrically local graphs, sufficiently small δ

  Unitary state preparation: Time $\sim O(N^3)$ Y. Ge et al (2015)

  $\Longrightarrow$Exponential speed-up over known unitary methods.

Robustness guarantees

- Preparation of TNS with parent Lindbladian provides natural robustness to initialization errors.

- Experimental noise:

$$\text{Target: } \mathscr{L} = \sum_i \mathscr{L}_i, \quad \text{Experiment: } \mathscr{L} = \sum_i \mathscr{L}_i + \delta \sum_i \mathscr{N}_i$$

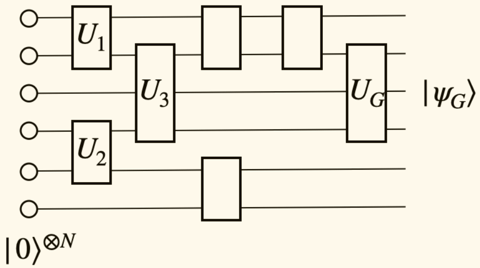
✗ Full state is almost orthogonal: $\langle \psi | \rho_\delta | \psi \rangle \sim \exp(-\delta N)$

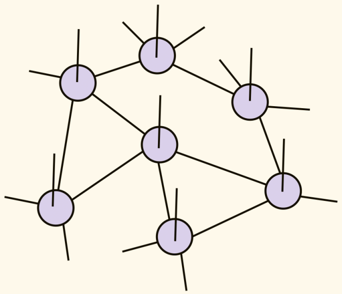
✓ Dissipatively prepared geometrically-local TNS, reduced states are robust:

$$\text{For } |A| \leq O(1), \|\mathrm{Tr}_{A^c}(|\psi\rangle\langle\psi|) - \mathrm{Tr}_{A^c}(\rho_\delta)\| \leq O(\delta)$$

D. Baruah, G. Styliaris, J.I. Cirac, **RT** (2025)

State Preparation





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Circuit Complexity

Geometrically local open system dynamics

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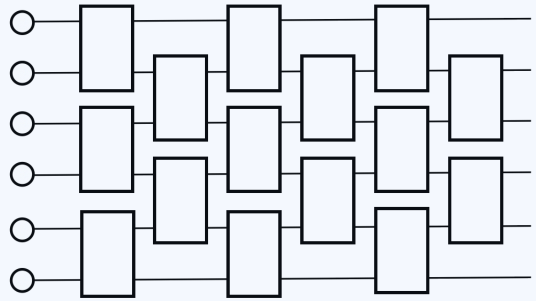
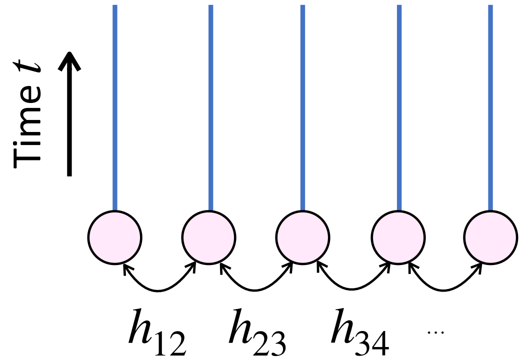
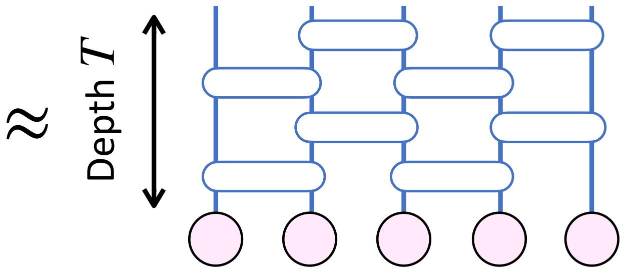


Measure of quantum resources needed to simulate open-system models.

Quantum simulation of lattice models



Closed system $H_S = \sum_i h_{i,i+1}$



Target: $\|U - e^{-iH_S t}\| \leq \varepsilon$

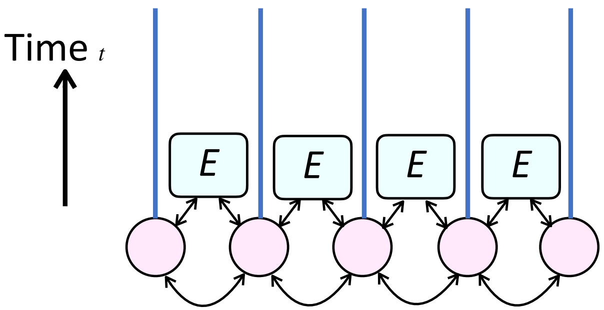
Circuit depth $T \sim O(t)$

Higher order Trotter Product formulae

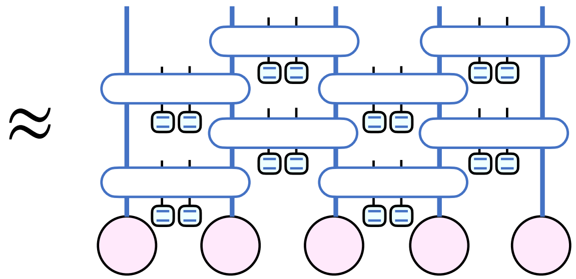
Su, Childs (2019).

Lieb-Robinson bounds + Quantum Signal Processing.

Hastings, Haah et al (2018).



Open system $\mathscr{E}_S(t)$



Target: $\|\mathscr{E} - \mathscr{E}_S(t)\|_\diamond \leq \varepsilon$

Implement $\mathscr{E}$ in $O(t)$ depth and with $O(nt)$ ancillas?

$O(t \times \text{poly}(n))$ gate count attained using Quantum Signal Processing.

X. Li, C. Wang (2022).

Challenge with open-system simulation

Optimal simulation requires forward and backward evolution

Example, trotterization: $H_S = A + B, \quad A = h_{12} + h_{34} + \ldots, B = h_{23} + h_{45} + \ldots$

Second order:

$$e^{-i\varepsilon H_S} \approx \mathcal{S}_2(\varepsilon) = e^{-i\varepsilon A/2} e^{-i\varepsilon B} e^{-i\varepsilon A/2}$$

Fourth order:

$$e^{-i\varepsilon H_S} \approx \mathcal{S}_4(\varepsilon) = \mathcal{S}_2^2(0.4145\varepsilon) e^{-0.3289\varepsilon A} e^{0.6579\varepsilon B} e^{-0.3289\varepsilon A} \mathcal{S}_2^2(0.4145 e^{-0.3289\varepsilon A} \varepsilon)$$

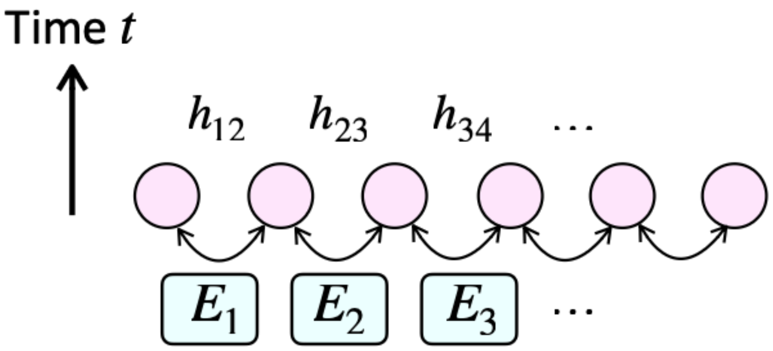
Backward-time evolution

Open systems: Harder to do backward-time evolution.

System evolution $\mathscr{E}_S(t)$: A channel but $\mathscr{E}_S^{-1}(t)$ is not necessarily a channel.

System-enviroment evolution $U_{SE}(t)$: Inverse is a unitary but $H_{SE}(t)$ is an unbounded and time-dependent Hamiltonian.

Quantum simulation of lattice models



$$H_{SE}(t) = \sum_i h_{i,i+1} + \sum_{i,\alpha} X_{i,i+1} B_{i,\alpha}(t)$$

$$K_{i,\alpha,\beta}(t) = \mathrm{Tr}[B_{i,\alpha}(t) B_{i,\beta}(0) \rho_E(0)]$$

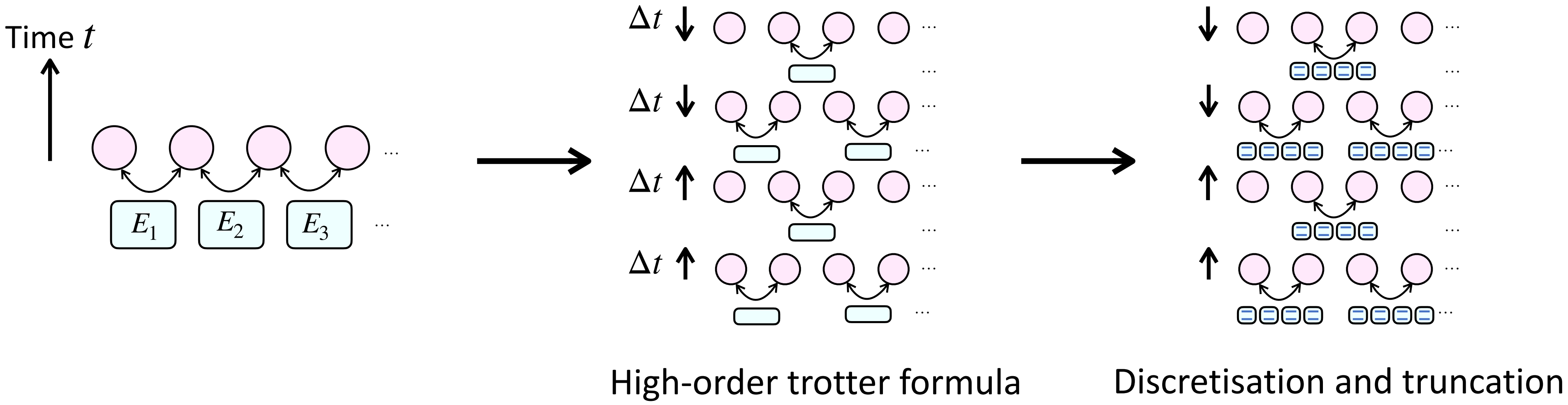
Assumptions: $K_{i,\alpha,\beta}(t)$ decays superpolynomially to 0 as $|t| \to \infty$ and has smooth derivatives.

Result:

- The channel describing system dynamics $\mathscr{E}_S(t)$ can be simulated to an error ε with depth $T = O(t(nt/\varepsilon)^{o(1)})$.
- With circuit of depth $T = O(t(t^{d+1}/\varepsilon)^{o(1)})$ and local observable can be simulated to error ε

X. Yu et al **RT** (2025)

Circuit complexity for open system dynamics



- Develop a high-order trotter formula for $H_{SE}(t)$ with trotter error $O(nt(\Delta t)^p)$.

  Not a direct application of time-dependent trotter formulae.

- Provide a near-optimal discretisation and truncation of the trotterized unitary .

Summary

- Open system with Gaussian environments have a succinct description with system Hamiltonian, jump operators and memory kernels which include both non-Markov and Markov models.

- Geometrically local open systems can be used to prepare quantum states described by both quantum circuits and tensor networks.

- Geometrically local open systems, under smoothness assumption on the system-environment interactions, can be simulated near-optimally with quantum circuits.