Bridging Theory and Experiment
at the Electron-Ion Collider

4 June, 2025

QED nuclear medium effects at EIC energies

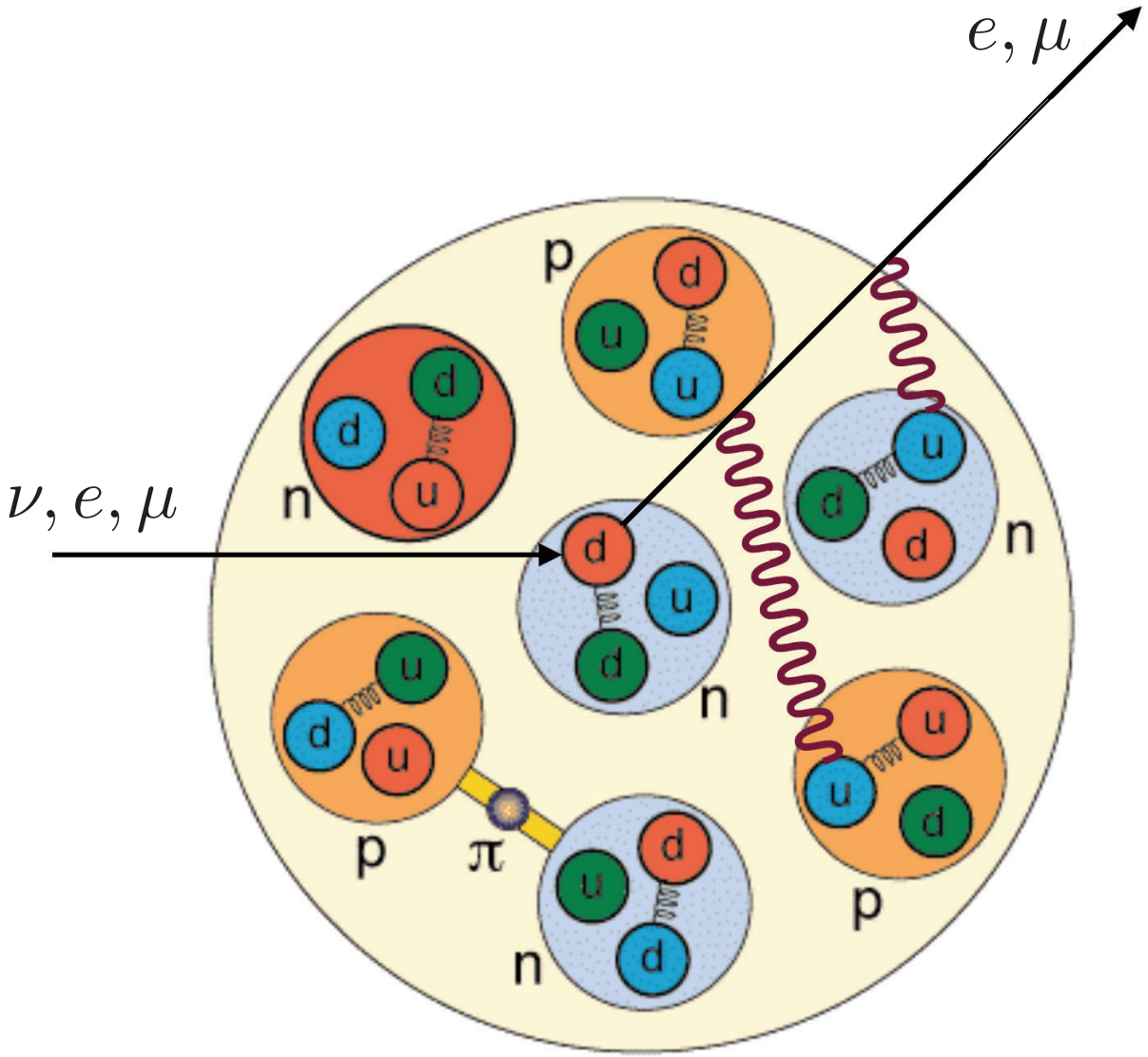
Shohini Bhattacharya, O. T., and Ivan Vitev, accepted in Phys. Rev. D, arXiv: 2502.06943

O. T. and Ivan Vitev, Phys. Lett. B 805, 135466 (2022)
O. T. and Ivan Vitev, Phys. Rev. D 108 9 (2023)
O. T. and Ivan Vitev, Phys. Rev. D 109, 7, (2024)

Oleksandr (Sasha) Tomalak

tomalak7.github.io

QED medium effects



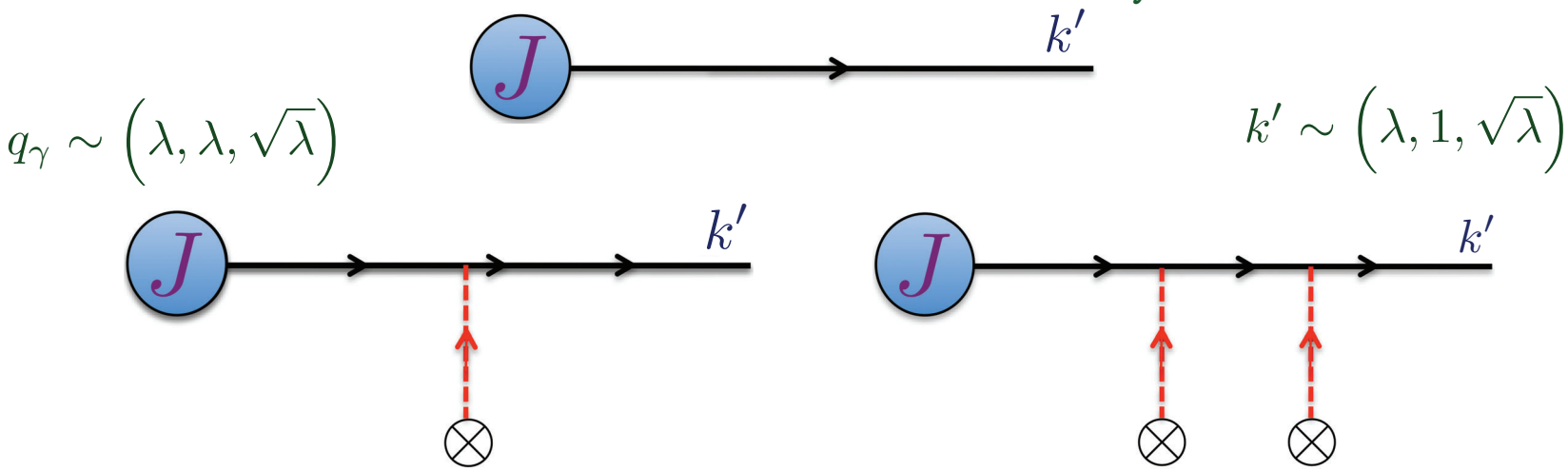
- charged lepton exchanges photons with nuclear medium

$\mathrm{SCET_G}$ formulation

- forward scattering is dominant process

- Glauber photons exchanged with a nuclear charge distribution

QCD: G. Ovanesyan and I. Vitev, JHEP (2011)



- change: integral along final lepton direction over charge and potential

$$\delta\sigma_f \sim \int_{\text{lepton line}}^{\text{final}} \rho(z)\,\mathrm{d}z \int \frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0\left(\vec{k}, \vec{k}' - \vec{q}_\perp\right) - \sigma_0\left(\vec{k}, \vec{k}'\right)\right)$$

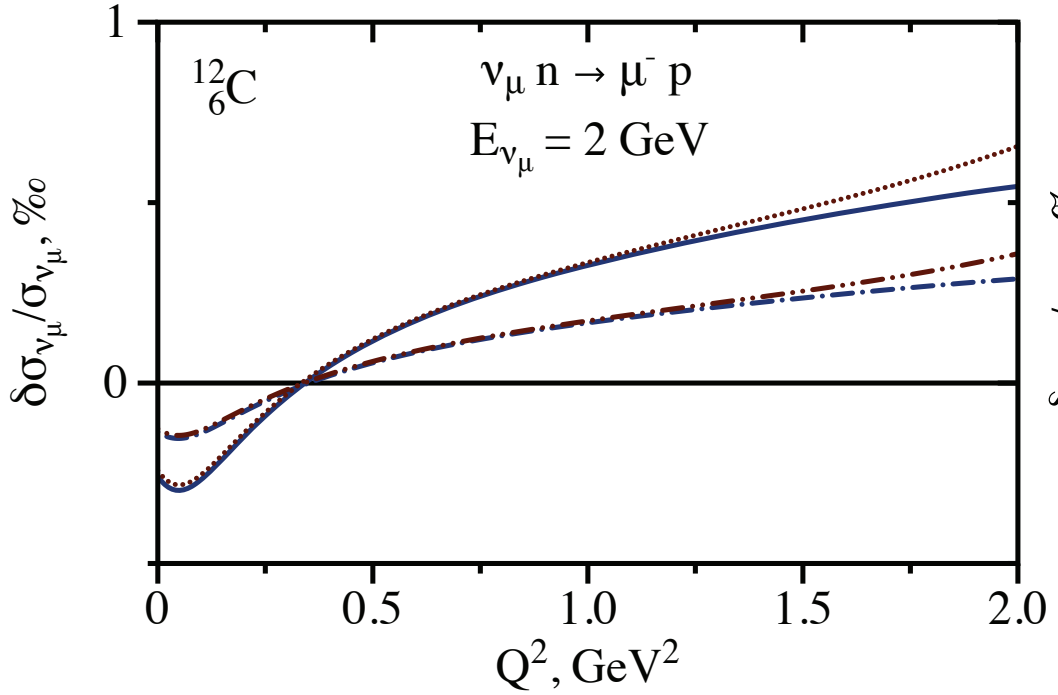
- leading-order cross sections are distorted
- EFT and full QED calculations are performed

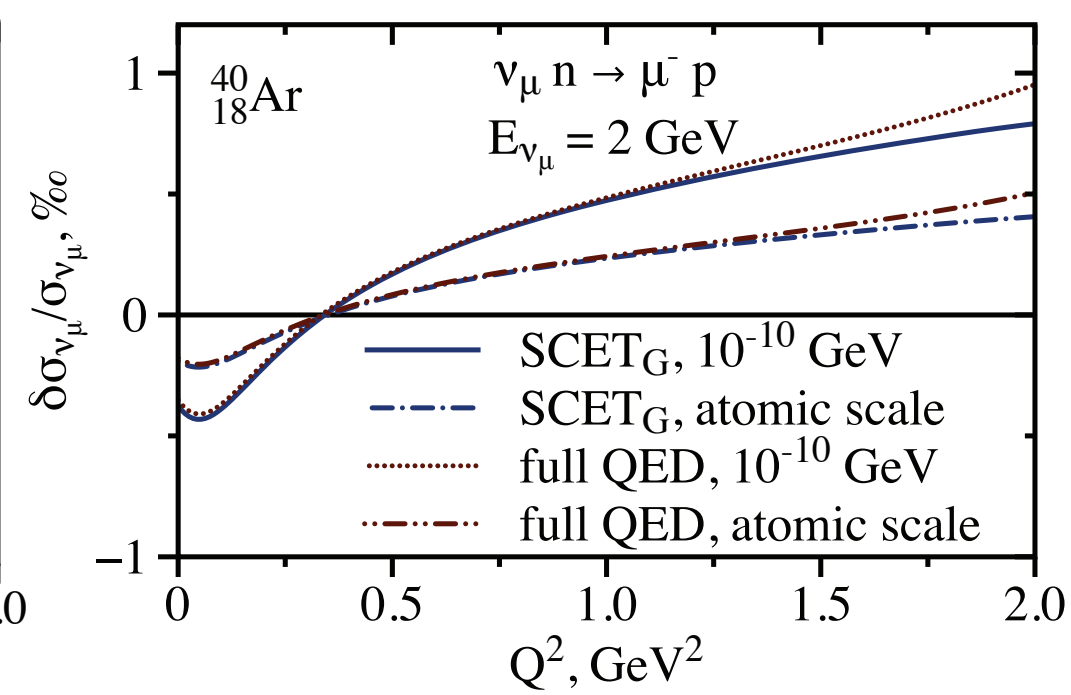
Neutrino scattering

- relative correction per nucleon

IR regularization

$$v\left(q_\perp^2\right) = \frac{e^2}{q_\perp^2 + \lambda^2}$$





flavor-independent at GeV energies

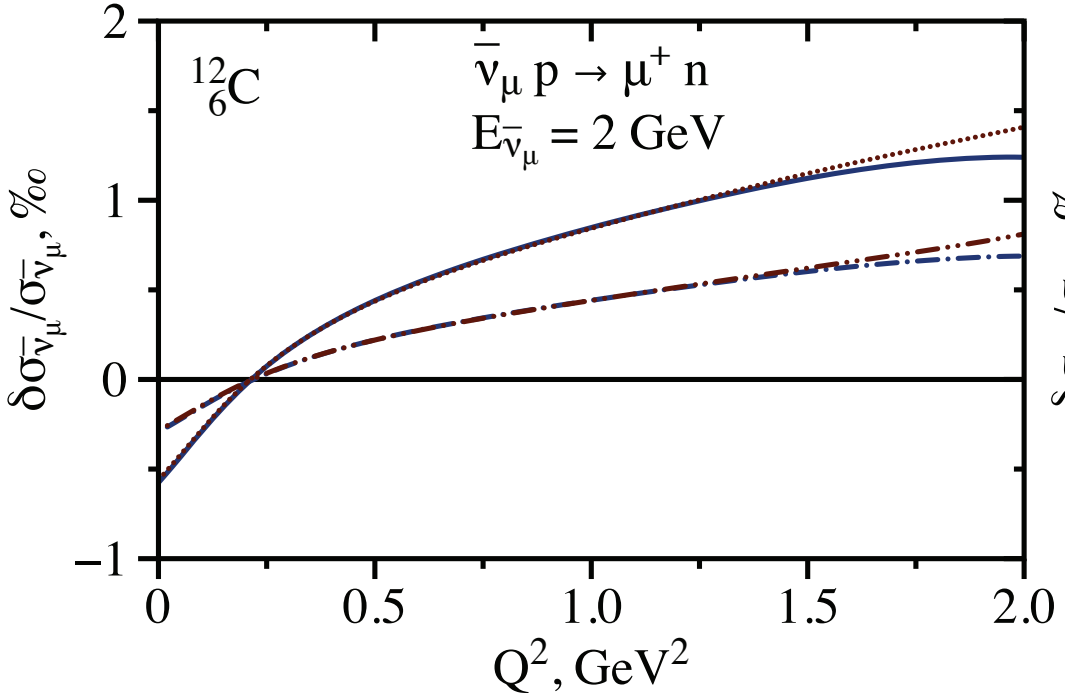
- permille-level distortion of cross sections: O(α^2) correction
- smaller correction to inclusive cross section

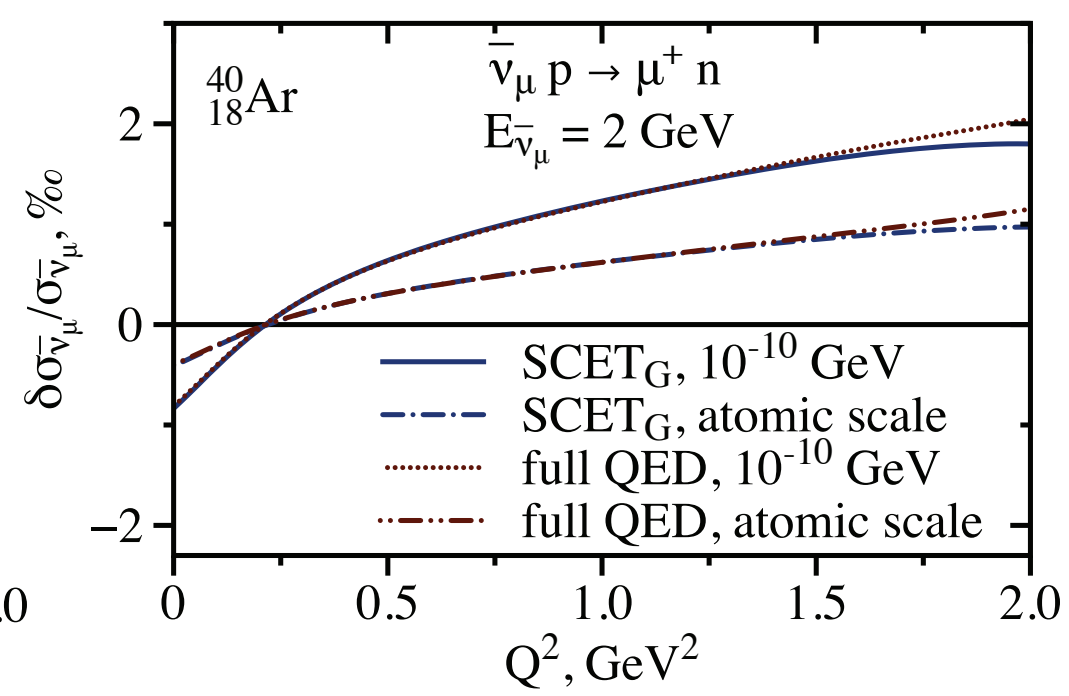
Antineutrino scattering

- relative correction per nucleon

IR regularization

$$v\left(q_\perp^2\right) = \frac{e^2}{q_\perp^2 + \lambda^2}$$





flavor-independent at GeV energies

- permille-level distortion of cross sections: O(α^2) correction
- larger correction than for neutrino scattering

$\mathrm{SCET_G}$ formulation

- forward scattering is dominant process

- Glauber photons exchanged with a nuclear charge distribution

- add initial-state exchanges, no interference with final-state exchanges

- change: integral along initial lepton direction over charge and potential

$$\delta\sigma_i \sim \int_{\text{lepton line}}^{\text{initial}} \rho(z)\,\mathrm{d}z \int \frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0\left(\vec{k}+\vec{q}_\perp,\vec{k}'\right) - \sigma_0\left(\vec{k},\vec{k}'\right)\right)$$

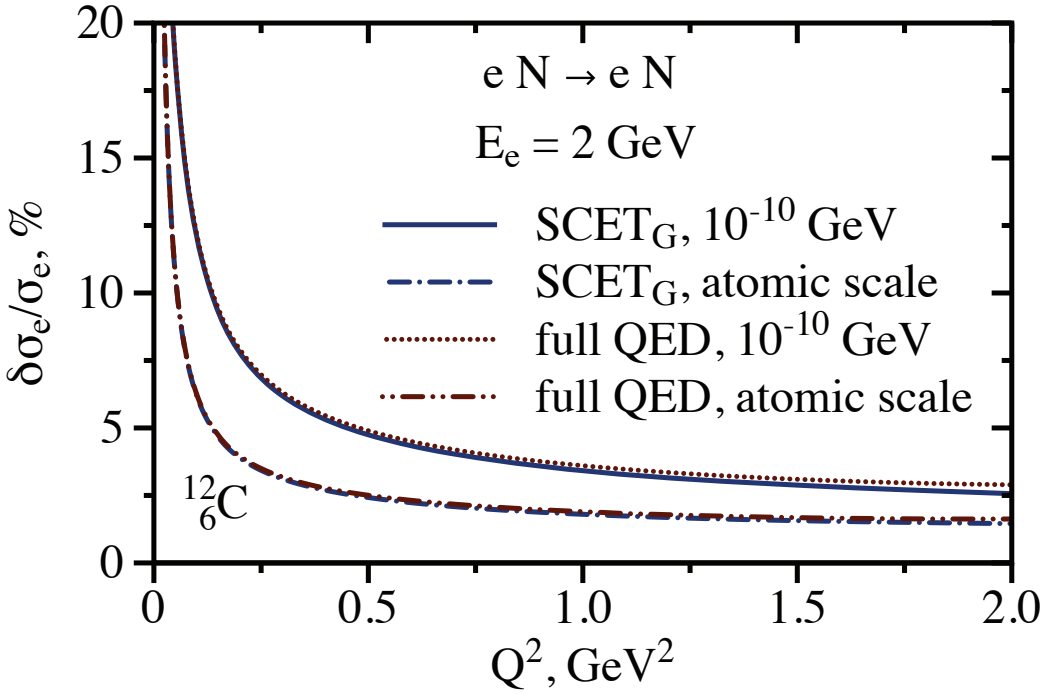
- change: integral along final lepton direction over charge and potential

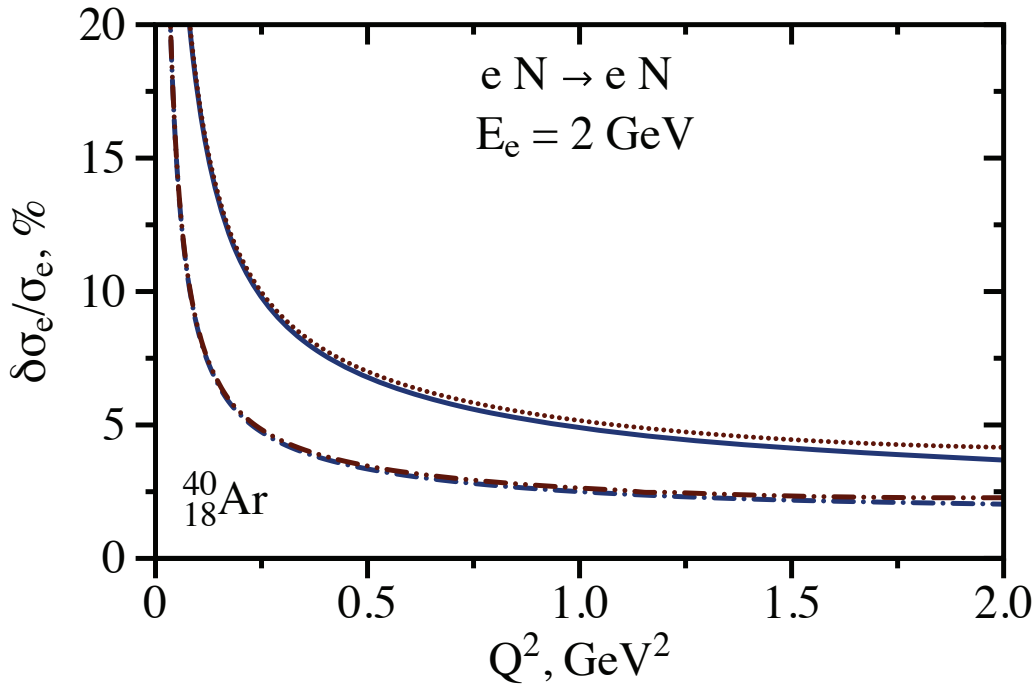
$$\delta\sigma_f \sim \int_{\text{lepton line}}^{\text{final}} \rho(z)\,\mathrm{d}z \int \frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0\left(\vec{k},\vec{k}'-\vec{q}_\perp\right) - \sigma_0\left(\vec{k},\vec{k}'\right)\right)$$

- leading-order cross sections are distorted
- EFT and full QED agree above the lepton mass scale

Electron scattering

- relative correction per nucleus after incoherent sum over nucleons

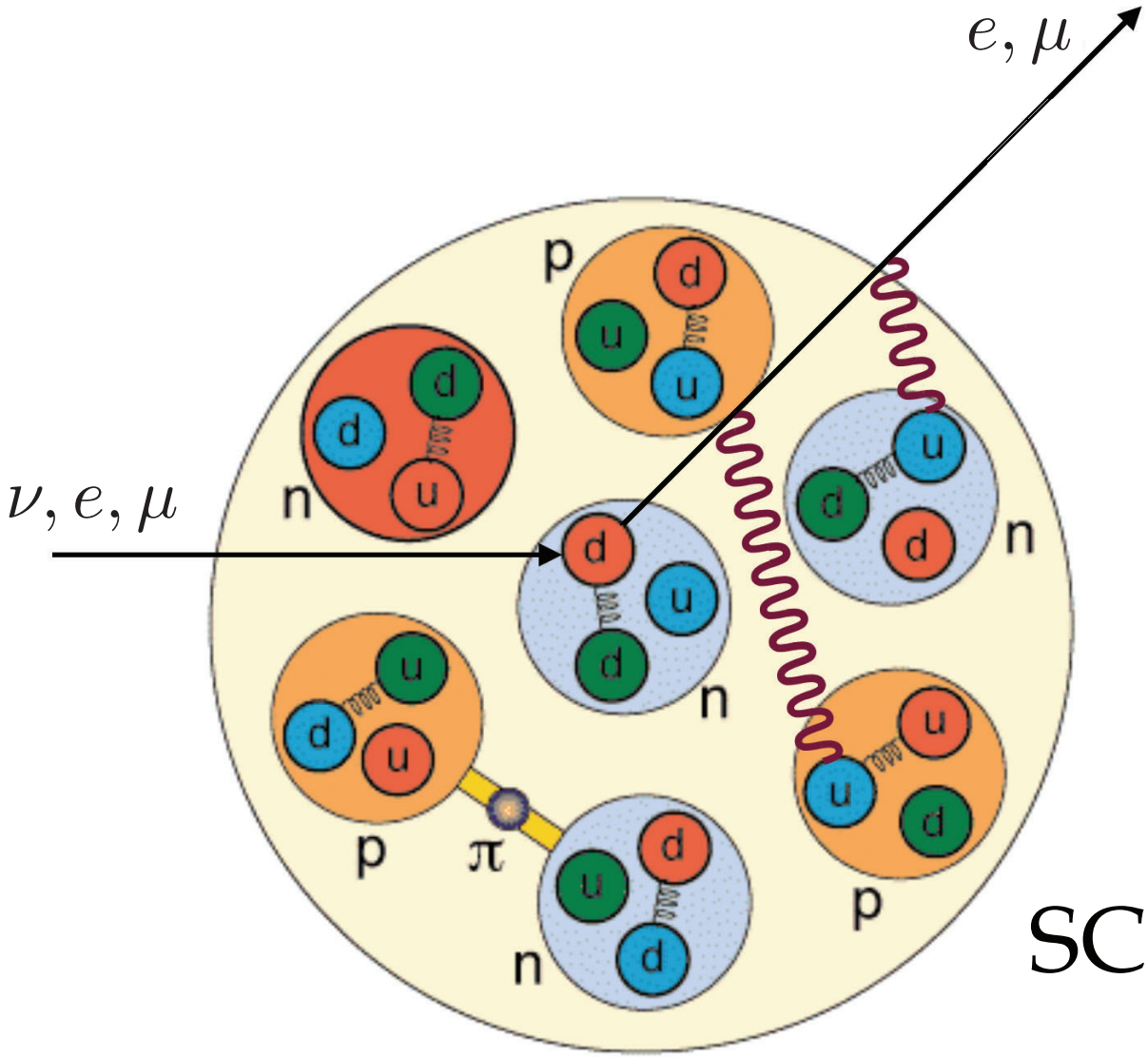




O. T. and Ivan Vitev, Phys. Lett. B 805, 135466 (2022)

- percent-level at low momentum transfers: O(α^2) correction
- critical new effect for electron scattering experiments

QED medium effects

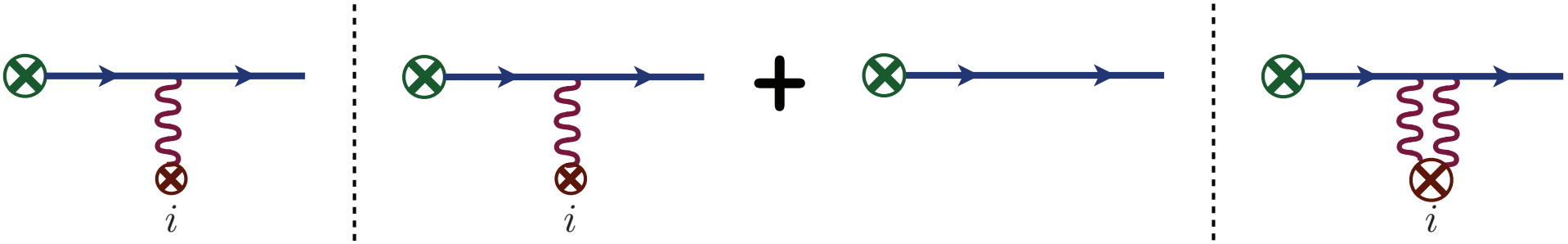


$SCET_G$ only

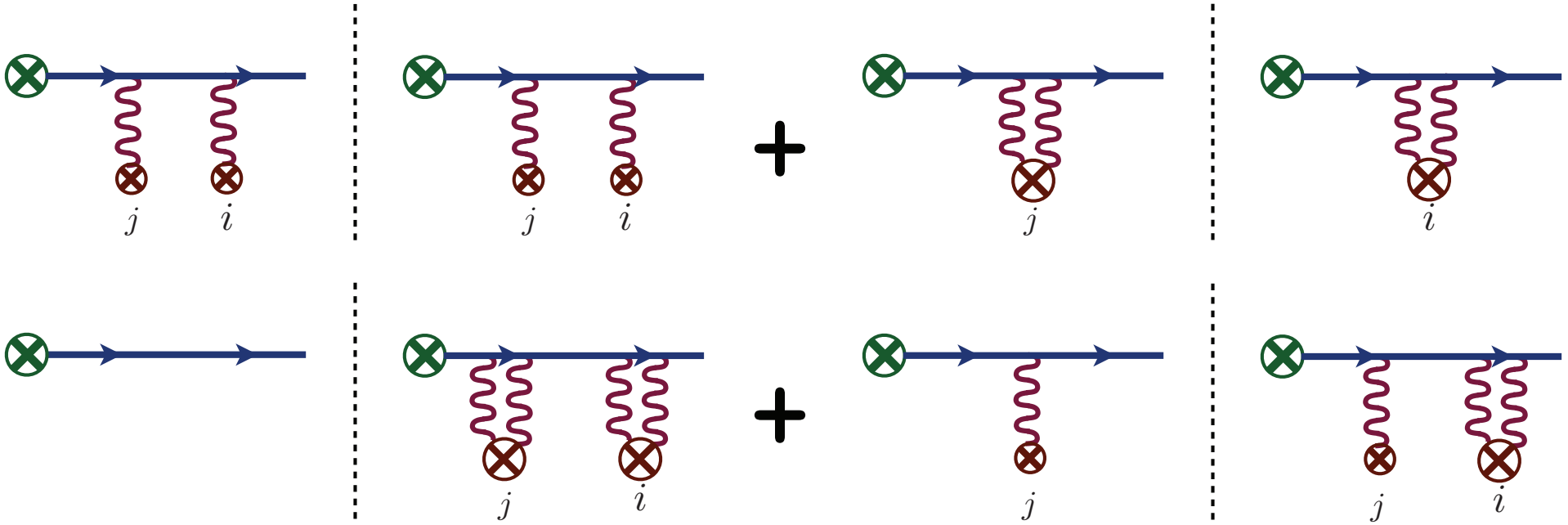
- >10000 interactions along the lepton trajectory resumed

QED medium effects

1st order in opacity

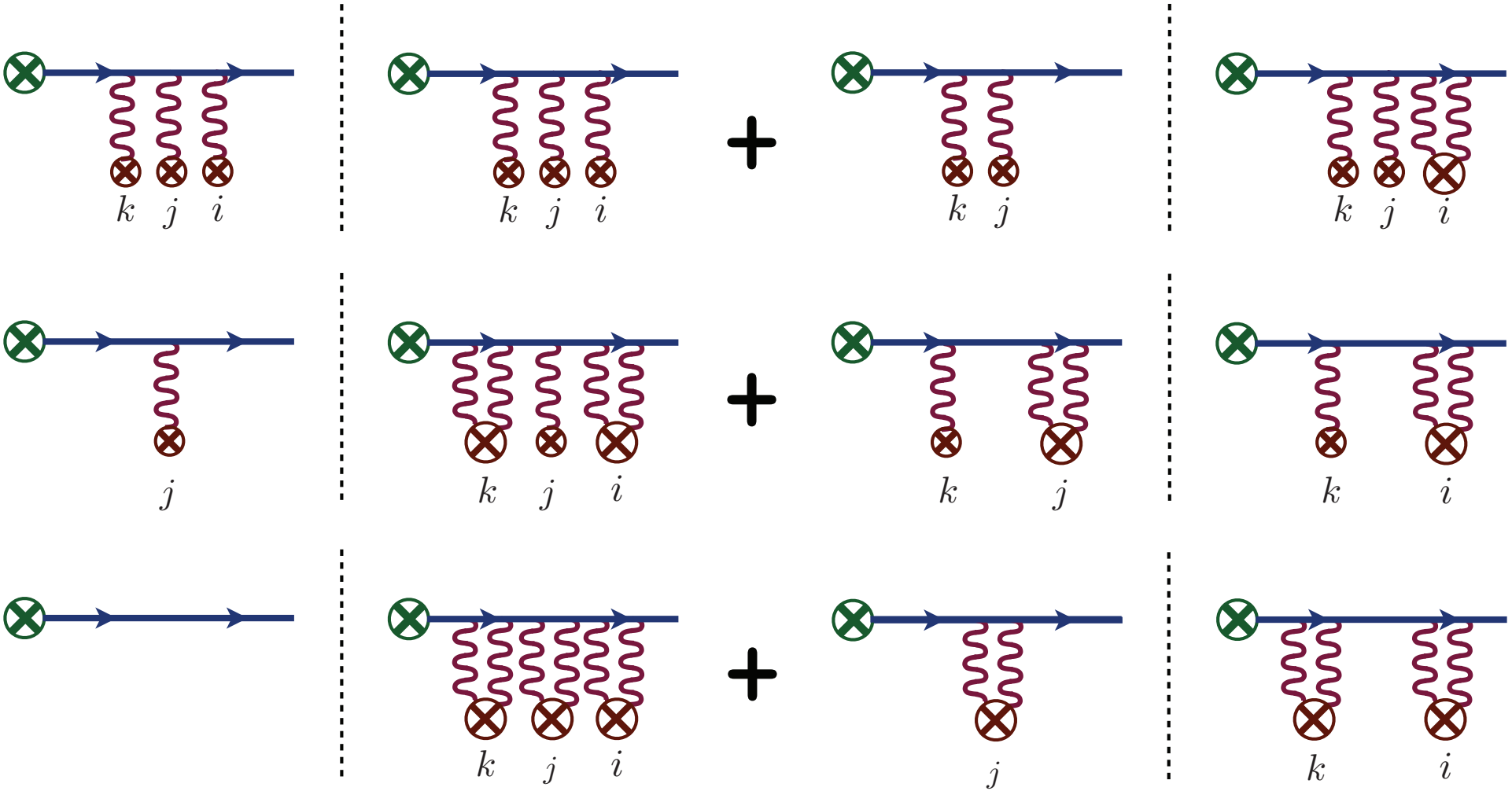


2nd order in opacity



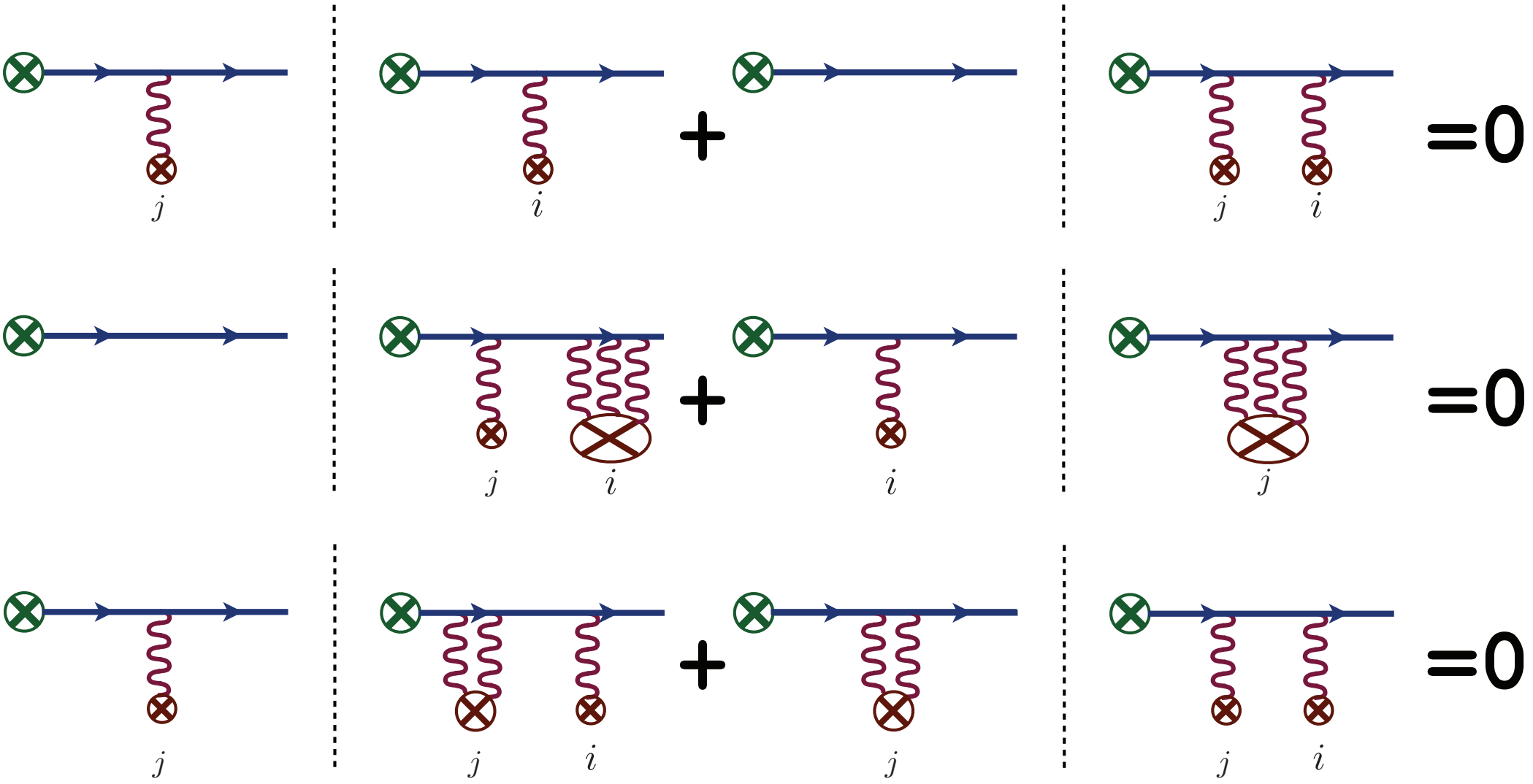
QED medium effects

3rd order in opacity



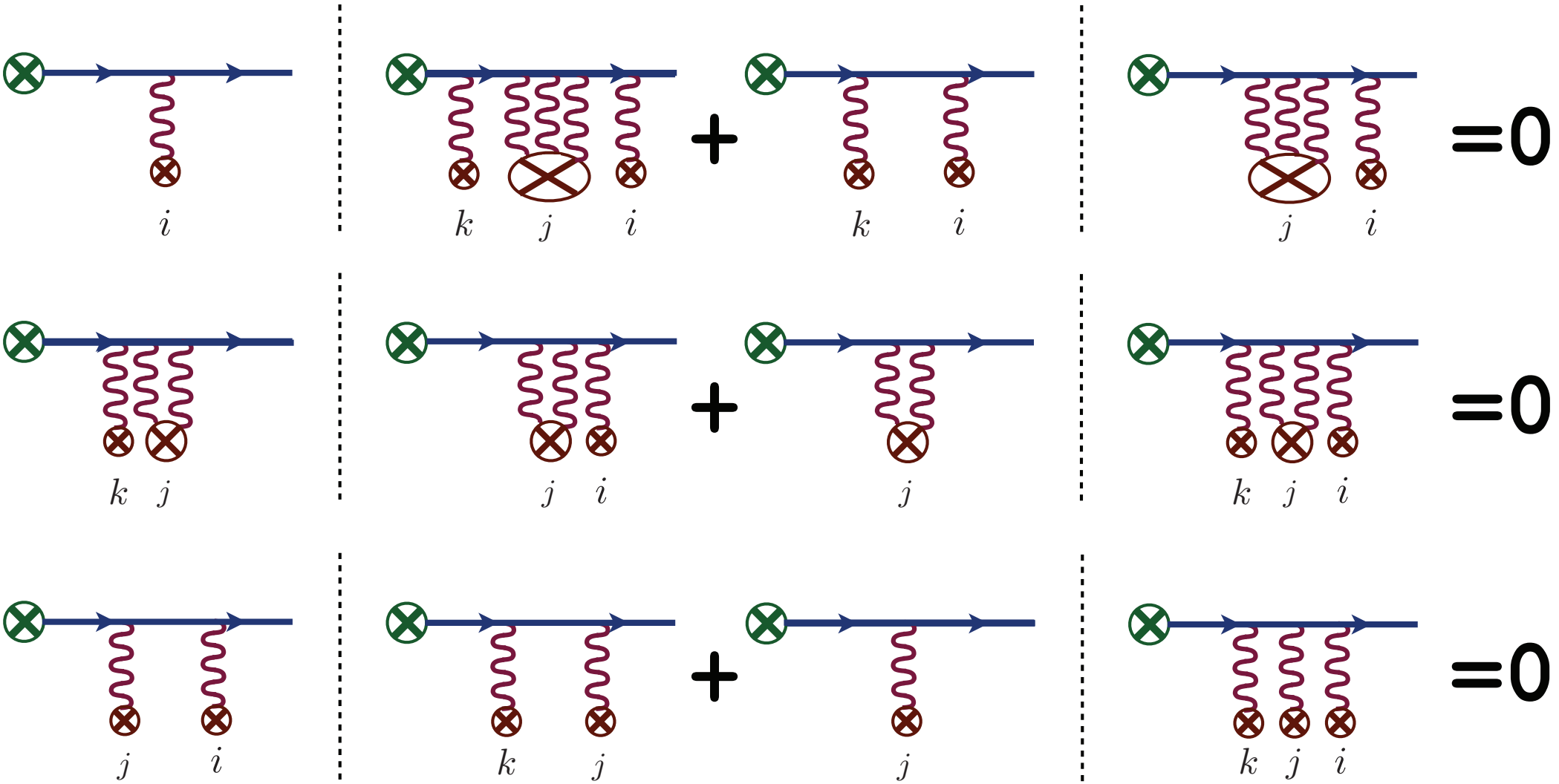
QED medium effects

2nd order in opacity



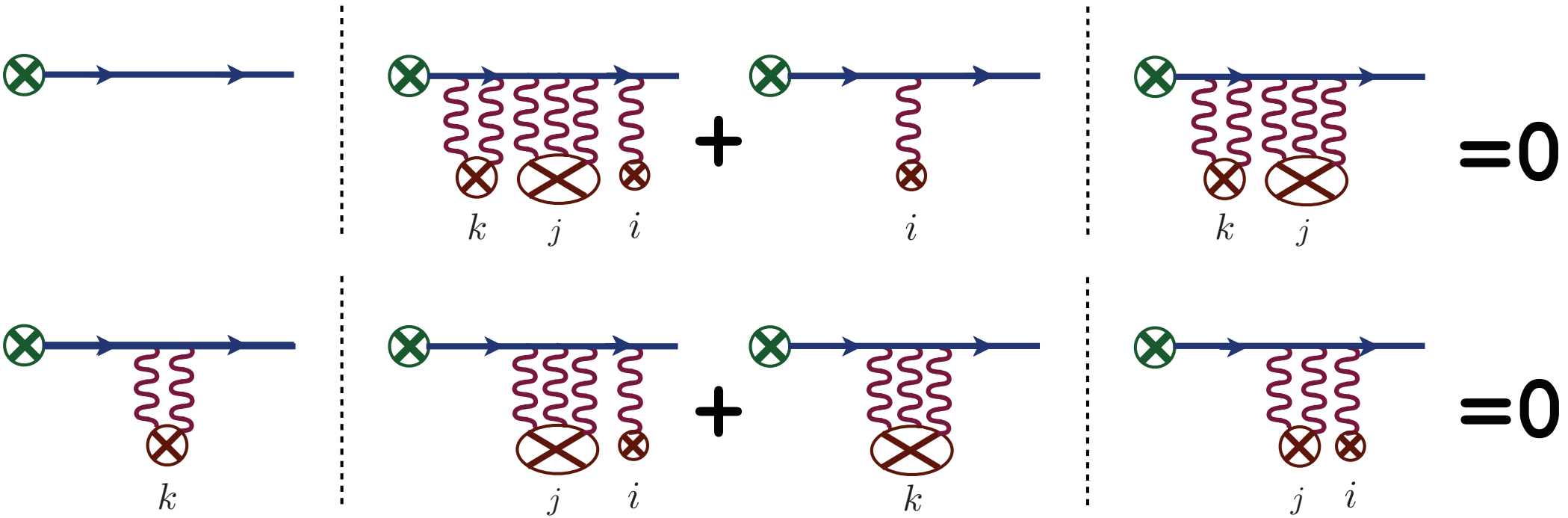
QED medium effects

3rd order in opacity



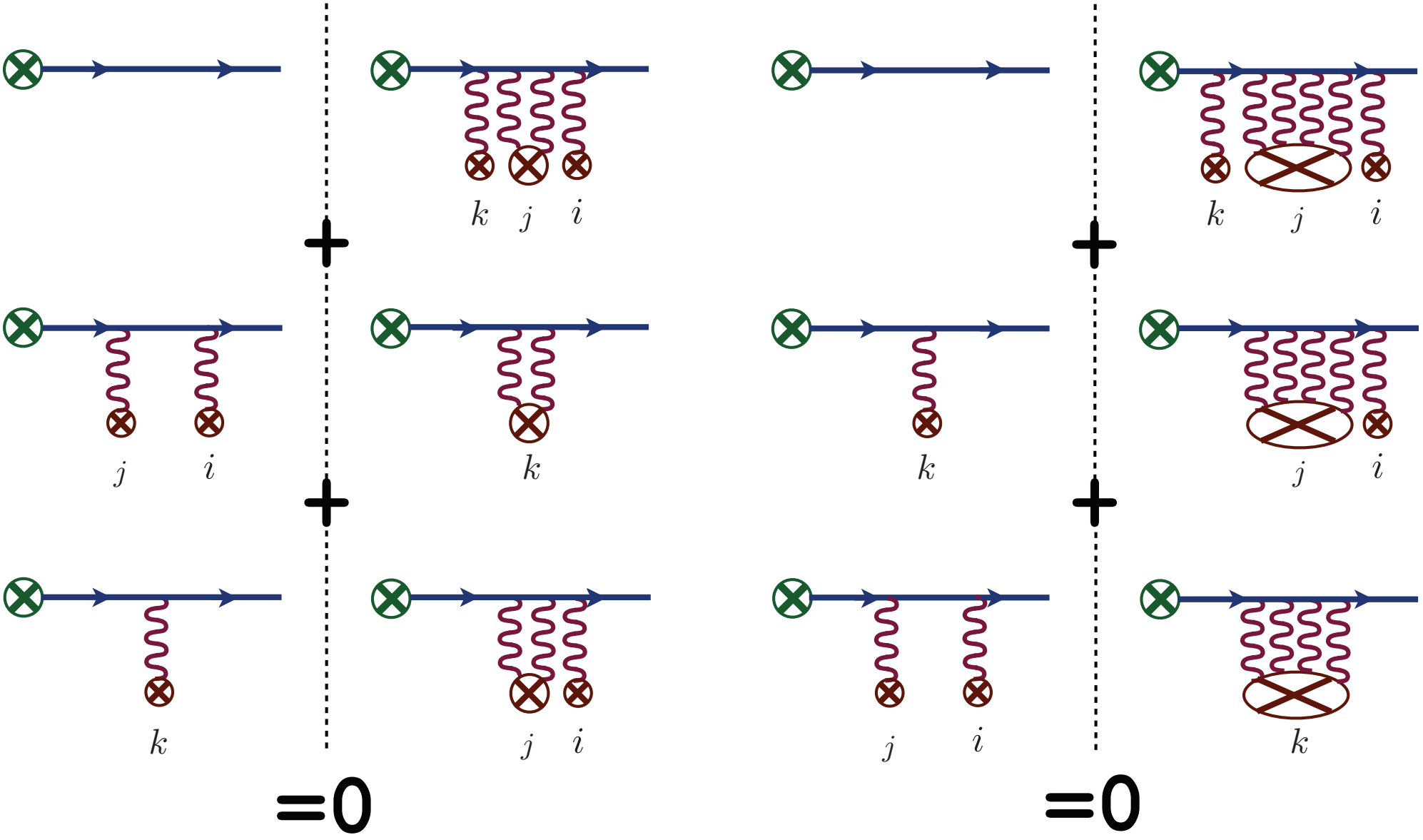
QED medium effects

3rd order in opacity



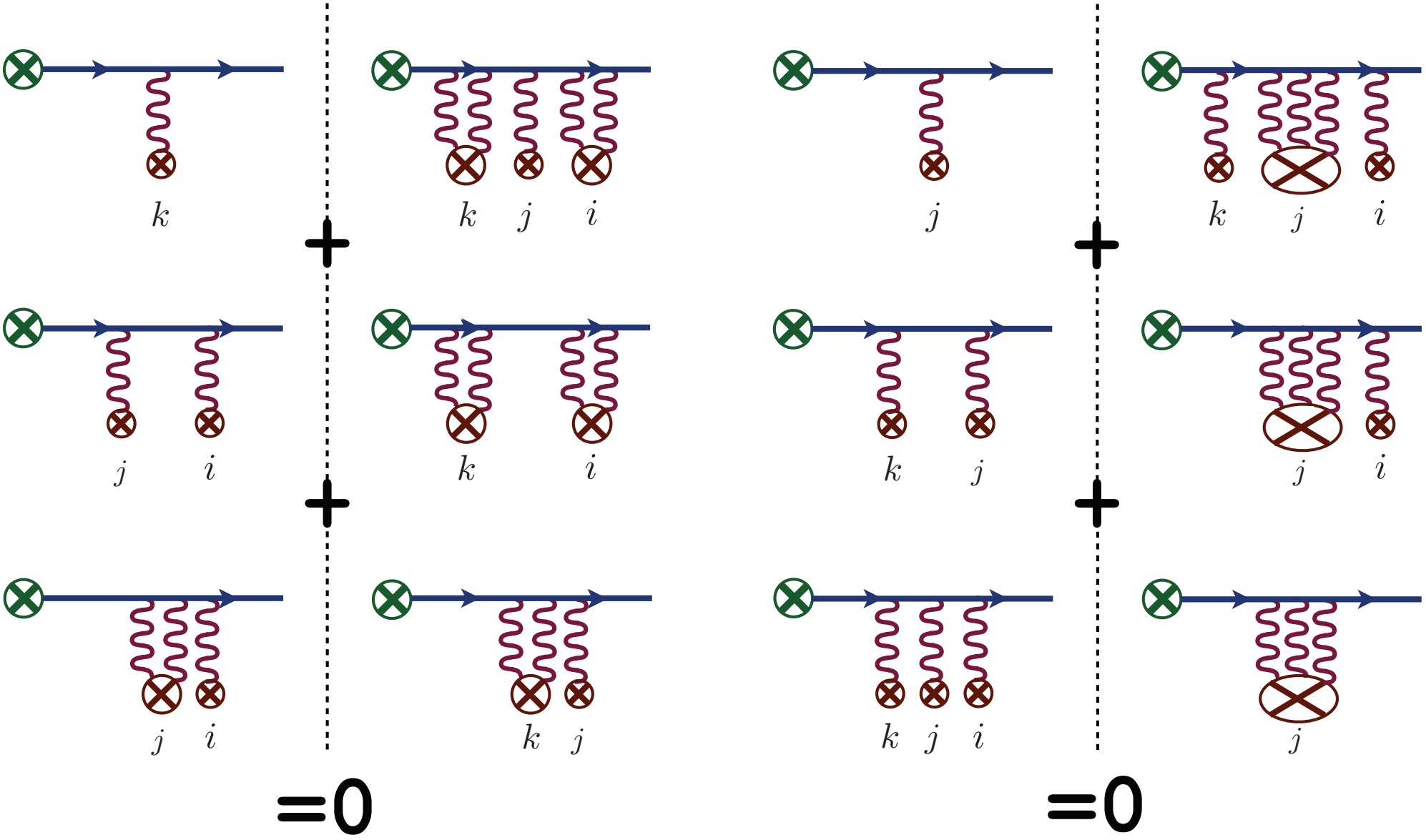
QED medium effects

3rd order in opacity



QED medium effects

3rd order in opacity



Exact expression for nth order

- cross-section modification: $\vec{q}_\perp = \vec{q}_{1,\perp} + ... + \vec{q}_{n,\perp}$

$$\sum_{z^n>...>z^1>0} \frac{N_{z^1}}{S_\perp^{z^1}} ... \frac{N_{z^n}}{S_\perp^{z^n}} \int \frac{\mathrm{d}^2\vec{q}_{1,\perp}}{(2\pi)^2} ... \frac{\mathrm{d}^2\vec{q}_{n,\perp}}{(2\pi)^2} v\left(q_{1,\perp}\right)^2 ... v\left(q_{n,\perp}\right)^2$$

$$\times \left[\mathrm{d}\sigma^{\mathrm{LO}}\left(\vec{p}' + \vec{k}_\gamma - \vec{q}_\perp\right) - \sum_{i=1}^{n} \mathrm{d}\sigma^{\mathrm{LO}}\left(\vec{p}' + \vec{k}_\gamma - \vec{q}_\perp + \vec{q}_{i,\perp}\right) \right.$$

$$+ \sum_{i=2}^{n} \sum_{j=1}^{i-1} \mathrm{d}\sigma^{\mathrm{LO}}\left(\vec{p}' + \vec{k}_\gamma - \vec{q}_\perp + \vec{q}_{i,\perp} + \vec{q}_{j,\perp}\right)$$

$$\left. + ... + (-1)^{n-1} \sum_{i=1}^{n} \mathrm{d}\sigma^{\mathrm{LO}}\left(\vec{p}' + \vec{k}_\gamma - \vec{q}_{i,\perp}\right) + (-1)^n \mathrm{d}\sigma^{\mathrm{LO}}\left(\vec{p}' + \vec{k}_\gamma\right) \right]$$

- all-orders expression: the same as in QCD medium

Resummation for constant density

- constant charge density approximation:

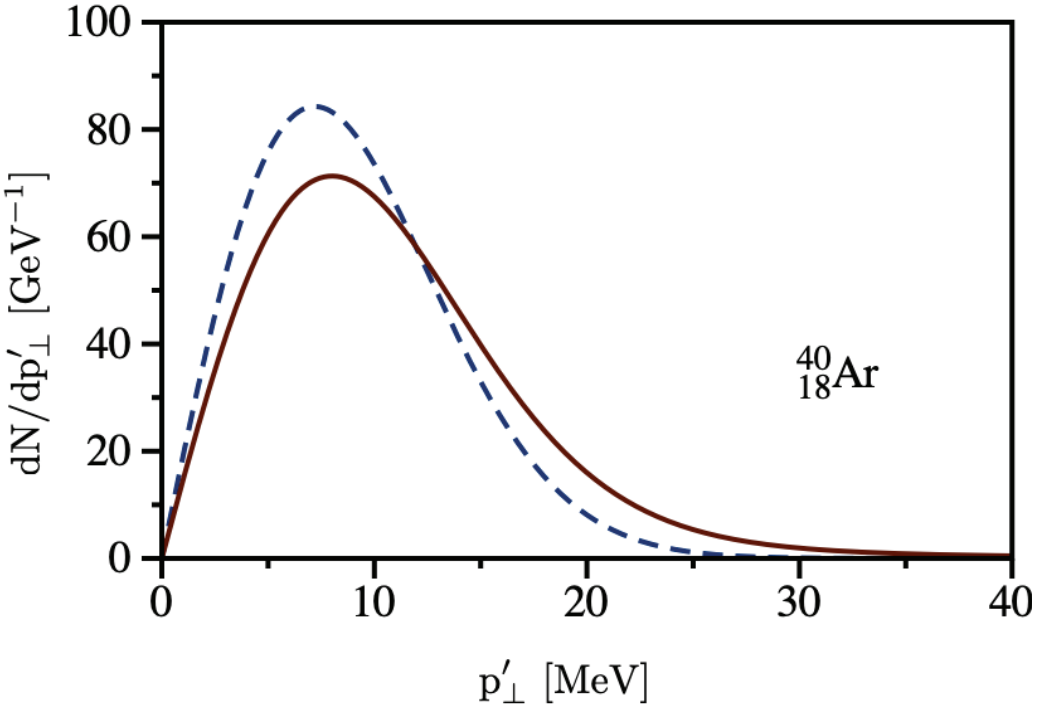
$$\frac{\mathrm{d}N\left(\vec{p}'_{\perp}\right)}{\mathrm{d}^2\vec{p}'_{\perp}} = \sum_{n=0}^{\infty} \frac{\mathrm{d}N^{(n)}\left(\vec{p}'_{\perp}\right)}{\mathrm{d}^2\vec{p}'_{\perp}}$$

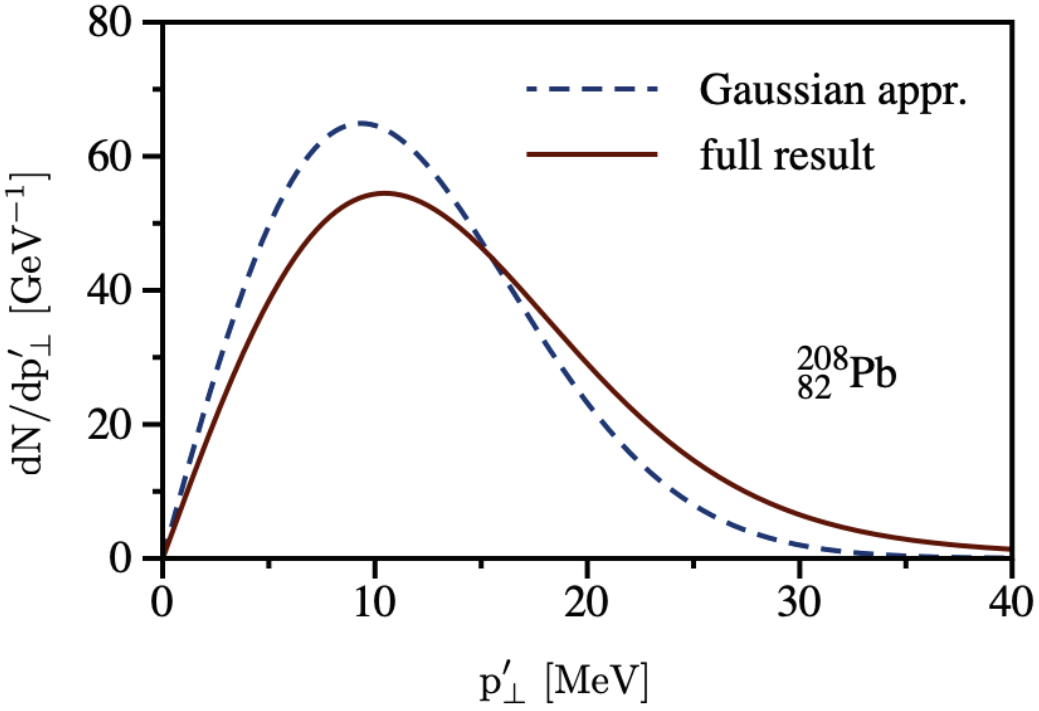
QCD: G. Ovanesyan and I. Vitev, JHEP (2011)

- resummation of all orders as in QCD

Broadening of electron tracks

- multiple re-scattering generates transverse momentum





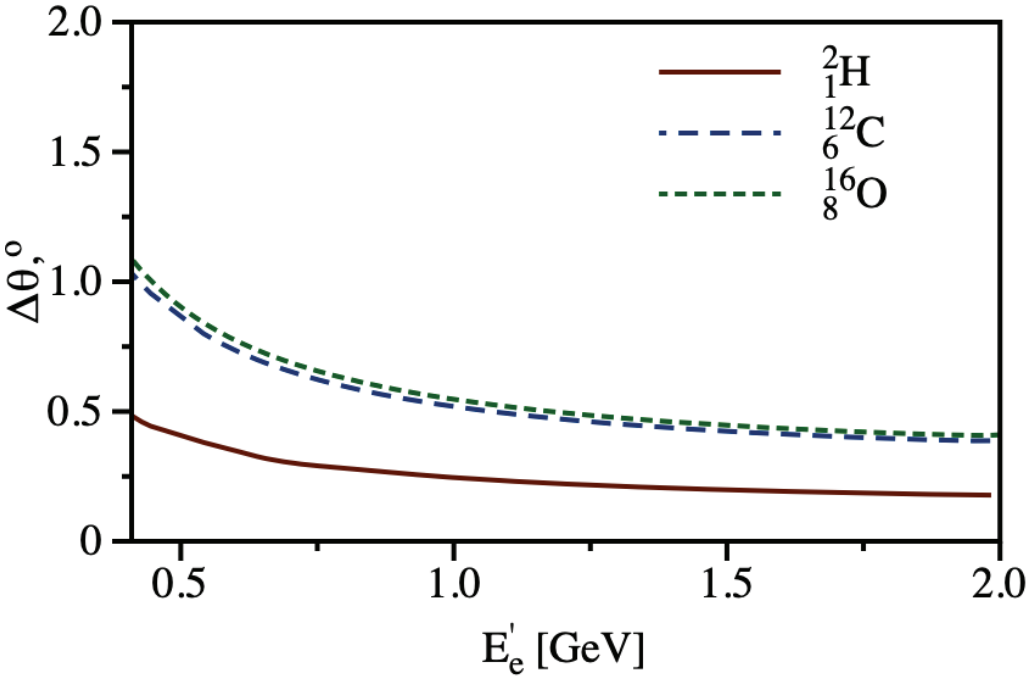
O. T. and Ivan Vitev, Phys. Rev. D 108 9, 9 (2023)

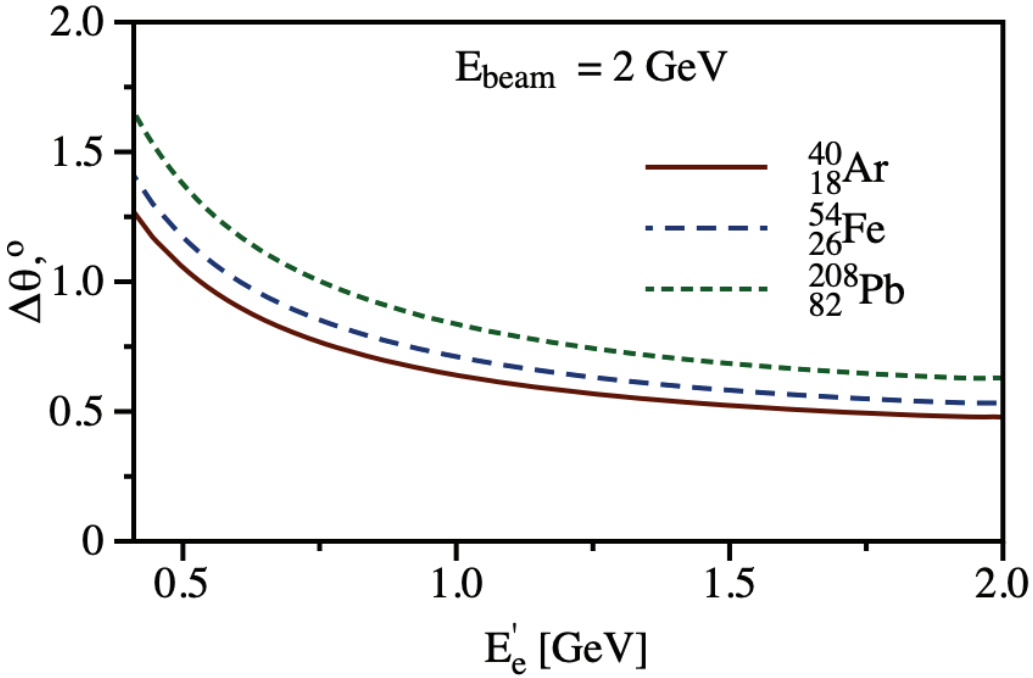
- exact resummation vs Gaussian approximation: nuclear size scale

- Glauber exchange induces 10-30 MeV transverse momentum

Broadening of electron tracks

- r. m. s. deflection angle after multiple rescattering





- nucleus approximated as sphere of constant density

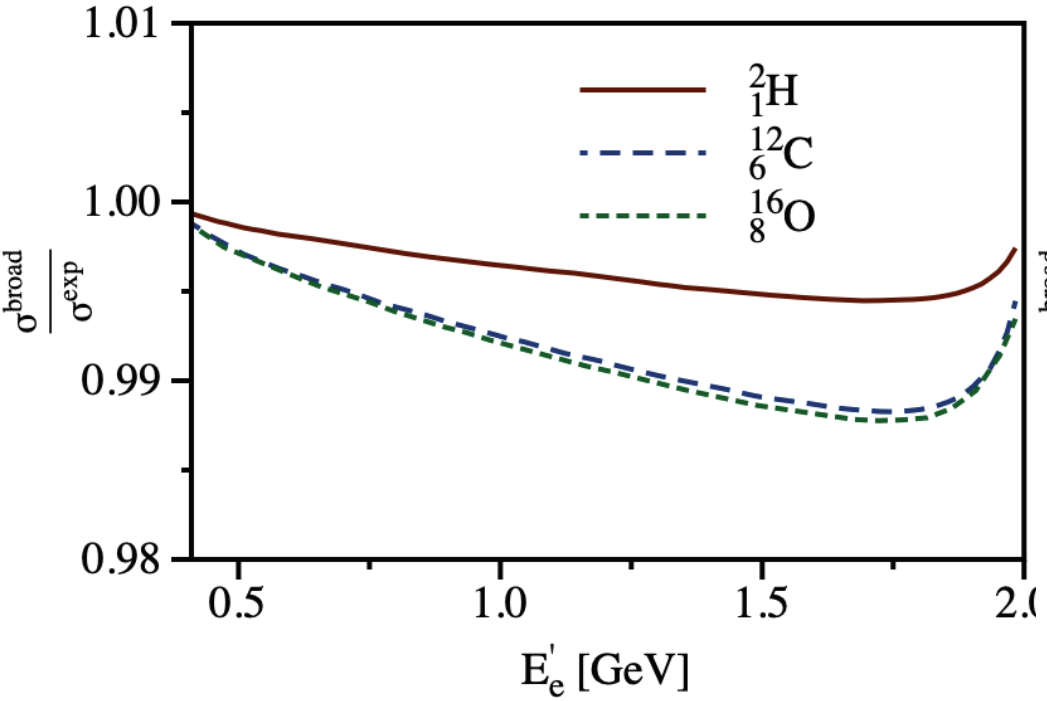
- sizable deflection of electron tracks $\sqrt{<(\Delta\theta)^2>} \sim 1/E$

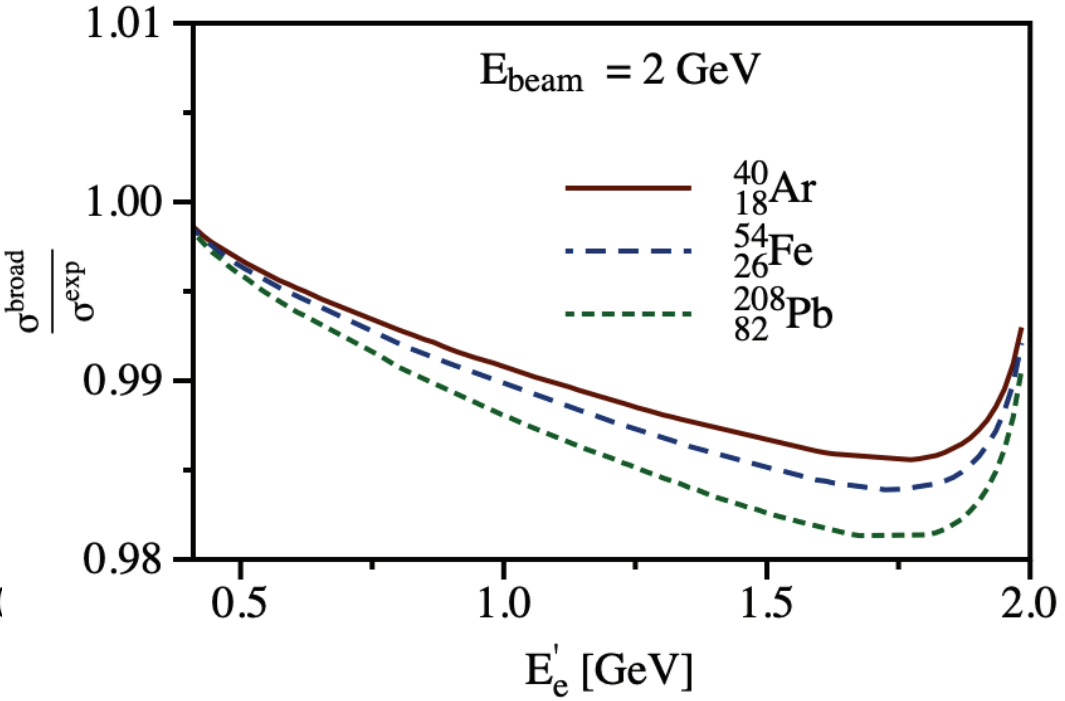
Effect on unpolarized cross section

- initial and final re-scattering is taken into account

JLab energies

- momentum transfer from electron kinematics

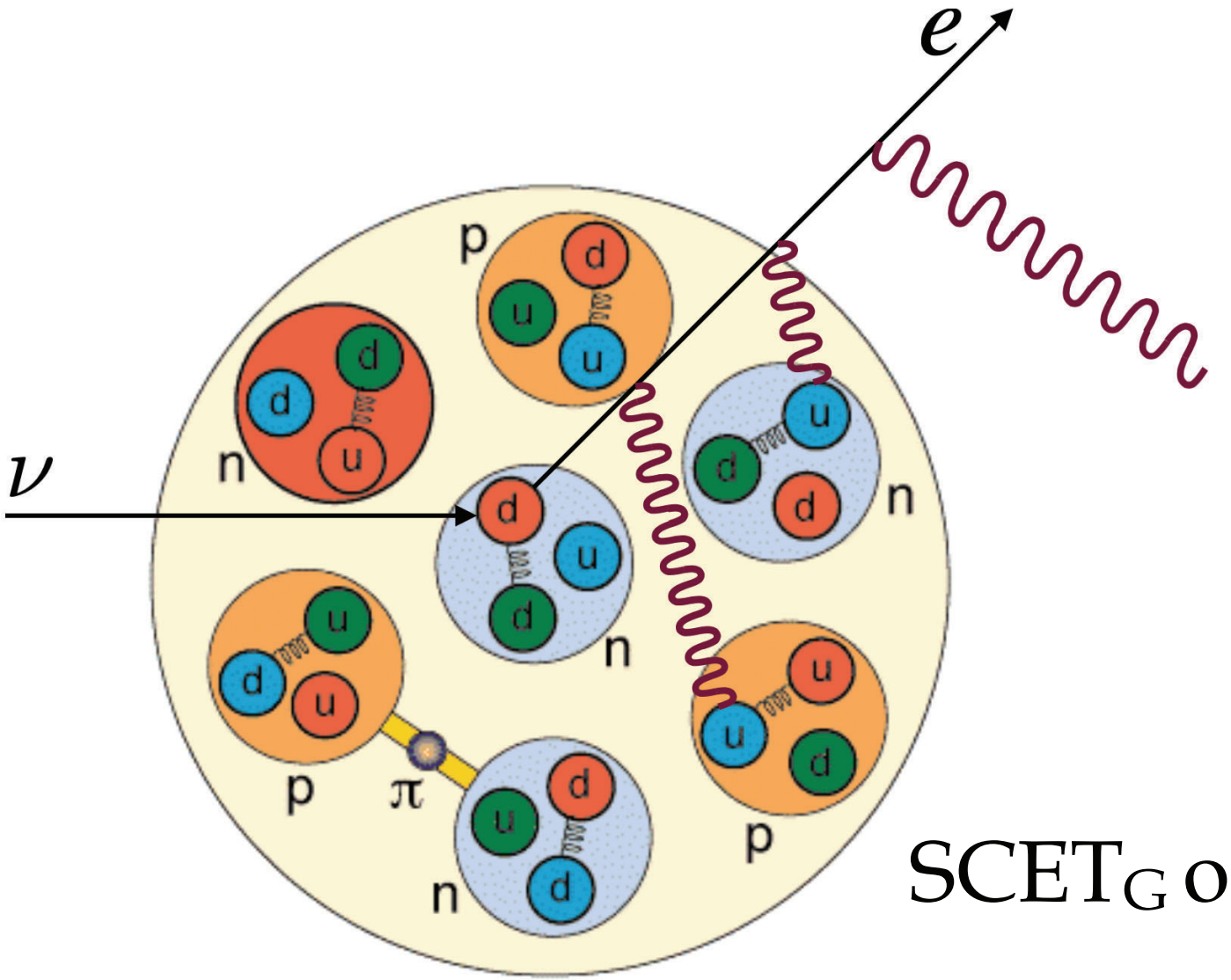




- nucleus approximated as sphere of constant density

- percent-level electron-nucleus cross-section suppression

QED medium-induced radiation



$SCET_G$ only

- >10000 interactions along the lepton trajectory resumed

Exact expression for nth order

- cross-section modification: $\vec{q}_\perp = \vec{q}_{1,\perp} + ... + \vec{q}_{n,\perp}$

$$
\begin{aligned}
& e^2 \left| \frac{p' \cdot \varepsilon^\star}{p' \cdot k_\gamma} - \frac{v \cdot \varepsilon^\star}{v \cdot k_\gamma} \right|^2 \frac{\mathrm{d}^3 \vec{k}_\gamma}{(2\pi)^3 \, 2E_\gamma} \sum_{z^n > ... > z^1 > 0} \frac{N_{z^1}}{S_\perp^{z^1}} ... \frac{N_{z^n}}{S_\perp^{z^n}} \int \frac{\mathrm{d}^2 \vec{q}_{1,\perp}}{(2\pi)^2} ... \frac{\mathrm{d}^2 \vec{q}_{n,\perp}}{(2\pi)^2} v\left(q_{1,\perp}\right)^2 ... v\left(q_{n,\perp}\right)^2 \\
& \times \Big[\mathrm{d}\sigma^{\mathrm{LO}} \left(\vec{p}' + \vec{k}_\gamma - \vec{q}_\perp \right) \left| \Gamma^{(n)} \left(\vec{q}_{1,\perp}, ..., \vec{q}_{n,\perp}, z^1, ..., z^n \right) \right|^2 \\
& - \sum_{i=1}^{n} \mathrm{d}\sigma^{\mathrm{LO}} \left(\vec{p}' + \vec{k}_\gamma - \vec{q}_\perp + \vec{q}_{i,\perp} \right) \left| \Gamma^{(n-1)} \left(\vec{q}_{1,\perp}, ..., \vec{q}_{i-1,\perp}, \vec{q}_{i+1,\perp}, ..., \vec{q}_{n,\perp}, z^1, ..., z^{i-1}, z^{i+1}, ..., z^n \right) \right|^2 \\
& + \sum_{i=2}^{n} \sum_{j=1}^{i-1} \mathrm{d}\sigma^{\mathrm{LO}} \left(\vec{p}' + \vec{k}_\gamma - \vec{q}_\perp + \vec{q}_{i,\perp} + \vec{q}_{j,\perp} \right) \\
& \times \left| \Gamma^{(n-2)} \left(\vec{q}_{1,\perp}, ..., \vec{q}_{j-1,\perp}, \vec{q}_{j+1,\perp}, ..., \vec{q}_{i-1,\perp}, \vec{q}_{i+1,\perp}, ..., \vec{q}_{n,\perp}, z^1, ..., z^{j-1}, z^{j+1}, ..., z^{i-1}, z^{i+1}, ..., z^n \right) \right|^2 \\
& + ... + (-1)^{n-1} \sum_{i=1}^{n} \mathrm{d}\sigma^{\mathrm{LO}} \left(\vec{p}' + \vec{k}_\gamma - \vec{q}_{i,\perp} \right) \left| \Gamma^{(1)} \left(\vec{q}_{i,\perp}, z^i \right) \right|^2 + (-1)^n \, \mathrm{d}\sigma^{\mathrm{LO}} \left(\vec{p}' + \vec{k}_\gamma \right) \Big]
\end{aligned}
$$

Exact expression for nth order

- cross-section modification: $\vec{q}_\perp = \vec{q}_{1,\perp} + ... + \vec{q}_{n,\perp}$

$$e^2 \left| \frac{p' \cdot \varepsilon^\star}{p' \cdot k_\gamma} - \frac{v \cdot \varepsilon^\star}{v \cdot k_\gamma} \right|^2 \frac{\mathrm{d}^3 \vec{k}_\gamma}{(2\pi)^3 \, 2E_\gamma} \sum_{z^n > ... > z^1 > 0} \frac{N_{z^1}}{S_\perp^{z^1}} ... \frac{N_{z^n}}{S_\perp^{z^n}} \int \frac{\mathrm{d}^2 \vec{q}_{1,\perp}}{(2\pi)^2} ... \frac{\mathrm{d}^2 \vec{q}_{n,\perp}}{(2\pi)^2} v\left(q_{1,\perp}\right)^2 ... v\left(q_{n,\perp}\right)^2$$

$$\times \Big[\mathrm{d}\sigma^{\mathrm{LO}} \left(\vec{p}' + \vec{k}_\gamma - \vec{q}_\perp \right) \left| \Gamma^{(n)} \left(\vec{q}_{1,\perp}, ..., \vec{q}_{n,\perp}, z^1, ..., z^n \right) \right|^2$$

$$- \sum_{i=1}^{n} \mathrm{d}\sigma^{\mathrm{LO}} \left(\vec{p}' + \vec{k}_\gamma - \vec{q}_\perp + \vec{q}_{i,\perp} \right) \left| \Gamma^{(n-1)} \left(\vec{q}_{1,\perp}, ..., \vec{q}_{i-1,\perp}, \vec{q}_{i+1,\perp}, ..., \vec{q}_{n,\perp}, z^1, ..., z^{i-1}, z^{i+1}, ..., z^n \right) \right|^2$$

$$+ \sum_{i=2}^{n} \sum_{j=1}^{i-1} \mathrm{d}\sigma^{\mathrm{LO}} \left(\vec{p}' + \vec{k}_\gamma - \vec{q}_\perp + \vec{q}_{i,\perp} + \vec{q}_{j,\perp} \right)$$

$$\times \left| \Gamma^{(n-2)} \left(\vec{q}_{1,\perp}, ..., \vec{q}_{j-1,\perp}, \vec{q}_{j+1,\perp}, ..., \vec{q}_{i-1,\perp}, \vec{q}_{i+1,\perp}, ..., \vec{q}_{n,\perp}, z^1, ..., z^{j-1}, z^{j+1}, ..., z^{i-1}, z^{i+1}, ..., z^n \right) \right|^2$$

$$+ ... + (-1)^{n-1} \sum_{i=1}^{n} \mathrm{d}\sigma^{\mathrm{LO}} \left(\vec{p}' + \vec{k}_\gamma - \vec{q}_{i,\perp} \right) \left| \Gamma^{(1)} \left(\vec{q}_{i,\perp}, z^i \right) \right|^2 + (-1)^n \, \mathrm{d}\sigma^{\mathrm{LO}} \left(\vec{p}' + \vec{k}_\gamma \right) \Big]$$

$$\Gamma^{(n)} \left(\vec{q}_{1,\perp}, ..., \vec{q}_{n,\perp}, z^1, ..., z^n \right) \approx 1$$

- resummation of soft radiation at leading power

$x = \dfrac{E_\gamma}{E_\ell + E_\gamma}$

Exact expression for nth order

- cross-section modification: $\vec{q}_\perp = \vec{q}_{1,\perp} + ... + \vec{q}_{n,\perp}$

$$\frac{\alpha}{\pi}\frac{1+(1-x)^2}{x}\ln\frac{(E_\ell+E_\gamma)\,\Delta\theta}{m_\ell}\sum_{z^n>...>z^1>0}\frac{N_{z^1}}{S_\perp^{z^1}}...\frac{N_{z^n}}{S_\perp^{z^n}}\int\frac{\mathrm{d}^2\vec{q}_{1,\perp}}{(2\pi)^2}...\frac{\mathrm{d}^2\vec{q}_{n,\perp}}{(2\pi)^2}v\left(q_{1,\perp}\right)^2...v\left(q_{n,\perp}\right)^2$$

$$\times\Big[\mathrm{d}\sigma^{\mathrm{LO}}\left(\vec{p}\,' + \vec{k}_\gamma - \vec{q}_\perp\right)\left|\Gamma^{(n)}\left(\vec{q}_{1,\perp},...,\vec{q}_{n,\perp},z^1,...,z^n\right)\right|^2$$

$$-\sum_{i=1}^{n}\mathrm{d}\sigma^{\mathrm{LO}}\left(\vec{p}\,' + \vec{k}_\gamma - \vec{q}_\perp + \vec{q}_{i,\perp}\right)\left|\Gamma^{(n-1)}\left(\vec{q}_{1,\perp},...,\vec{q}_{i-1,\perp},\vec{q}_{i+1,\perp},...,\vec{q}_{n,\perp},z^1,...,z^{i-1},z^{i+1},...,z^n\right)\right|^2$$

$$+\sum_{i=2}^{n}\sum_{j=1}^{i-1}\mathrm{d}\sigma^{\mathrm{LO}}\left(\vec{p}\,' + \vec{k}_\gamma - \vec{q}_\perp + \vec{q}_{i,\perp} + \vec{q}_{j,\perp}\right)$$

$$\times\left|\Gamma^{(n-2)}\left(\vec{q}_{1,\perp},...,\vec{q}_{j-1,\perp},\vec{q}_{j+1,\perp},...,\vec{q}_{i-1,\perp},\vec{q}_{i+1,\perp},...,\vec{q}_{n,\perp},z^1,...,z^{j-1},z^{j+1},...,z^{i-1},z^{i+1},...,z^n\right)\right|^2$$

$$+...+(-1)^{n-1}\sum_{i=1}^{n}\mathrm{d}\sigma^{\mathrm{LO}}\left(\vec{p}\,' + \vec{k}_\gamma - \vec{q}_{i,\perp}\right)\left|\Gamma^{(1)}\left(\vec{q}_{i,\perp},z^i\right)\right|^2 + (-1)^n\,\mathrm{d}\sigma^{\mathrm{LO}}\left(\vec{p}\,' + \vec{k}_\gamma\right)\Big]$$

$$\Gamma^{(n)}\left(\vec{q}_{1,\perp},...,\vec{q}_{n,\perp},z^1,...,z^n\right)\approx 1$$

- resummation of soft radiation at leading power
- approximate resummation of collinear radiation

QED medium-induced radiation

broadening with radiation: p_T spectrum is multiplied with soft (collinear) function in vacuum for observables including soft (collinear) photons

soft (collinear) functions in vacuum

O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret editors suggestion in Phys. Rev. D (2022)

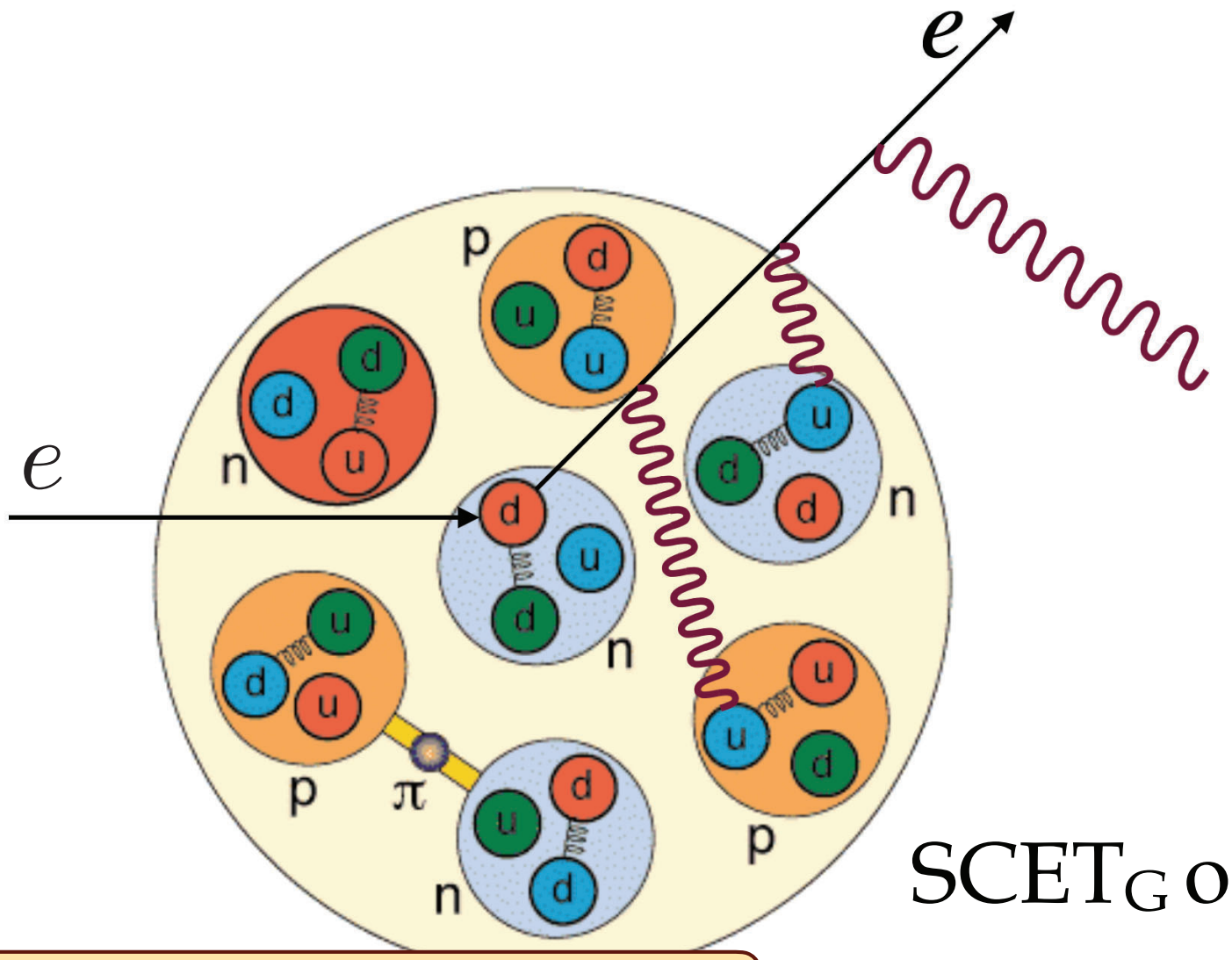
relative cross-section correction at each order in expansion on number of re-scatterings: the same for soft, collinear, and no-radiation cases

vanishing spectrum of soft or collinear medium-induced photons

O. T. and Ivan Vitev, Phys. Rev. D 109, 7, (2024)

- separation of scales: exact resummation of medium effects

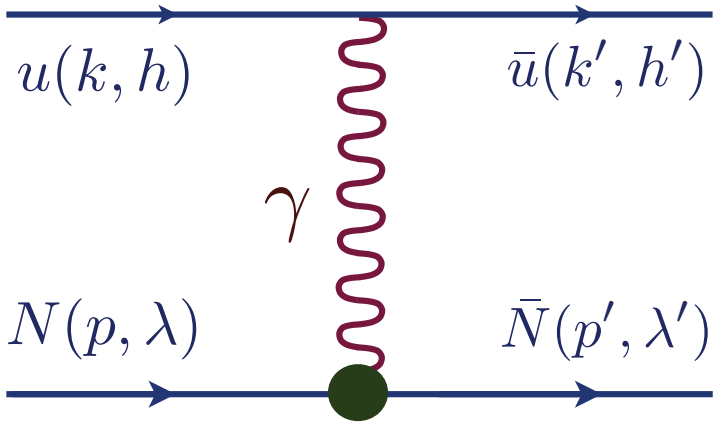
EIC energies



SCET$_G$ only

- elastic electron-nucleon scattering
- neutral-current unpolarized DIS

Elastic process



photon-proton vertex

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P(Q^2)$$

Dirac and Pauli form factors

lepton energy and momentum transfer $\quad Q^2 = -(k-k')^2$

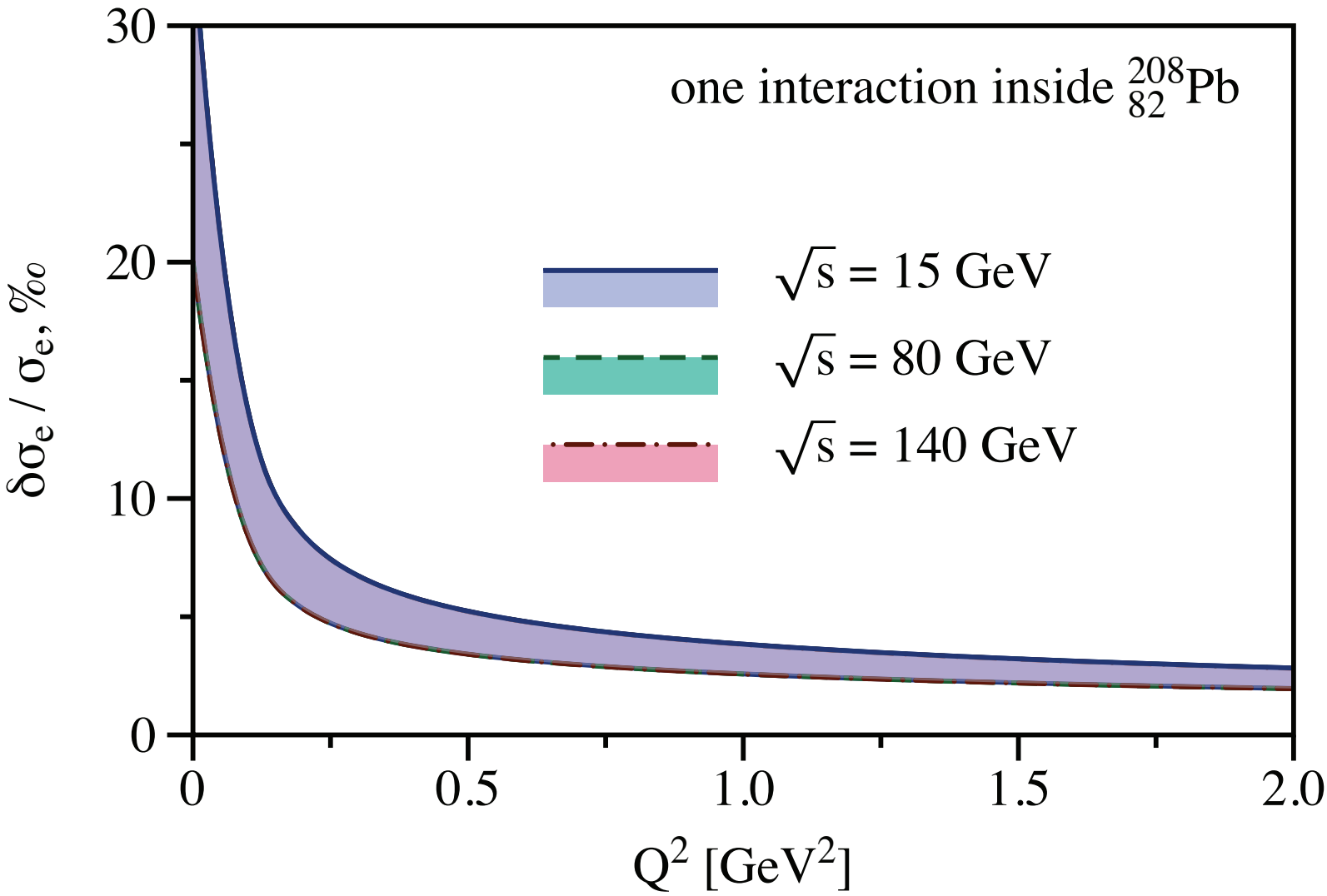
$\tau \quad \varepsilon$ kinematical variables $\qquad \tau = \dfrac{Q^2}{4M^2}$ $\qquad \varepsilon \leftrightarrow \theta_{\rm lab}$

Sachs electric and magnetic form factors

$$G_E = F_D - \tau F_P \qquad G_M = F_D + F_P$$

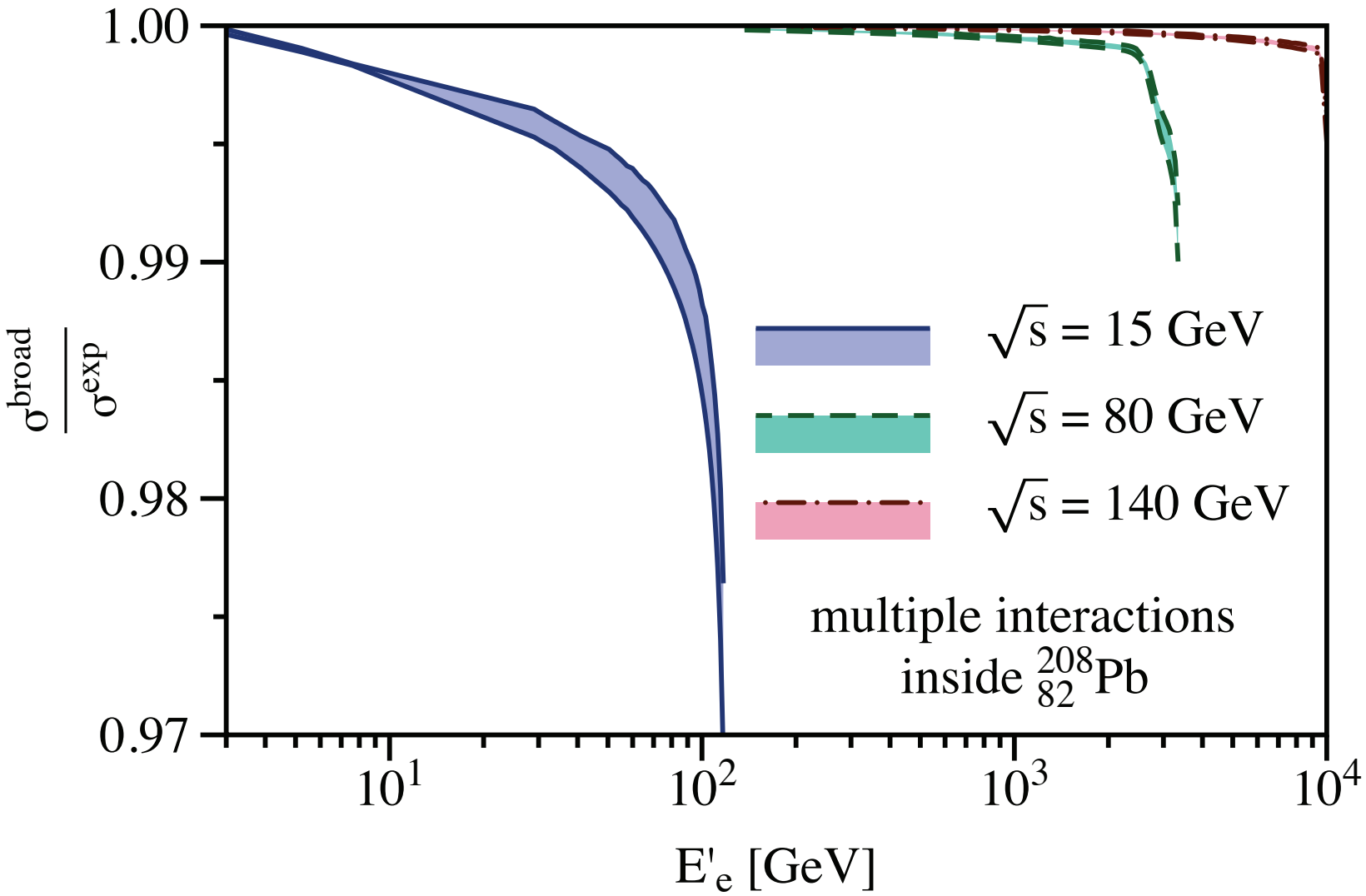
$$\frac{\mathrm{d}\sigma^{\rm unpol}}{\mathrm{d}\Omega} \sim G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

Elastic process. 1 rescattering



- saturated regime at EIC energies
- sizable effects for forward scattering angles

Elastic process. Resumed result



Shohini Bhattacharya, O. T., and Ivan Vitev, accepted in Phys. Rev. D, arXiv: 2502.06943

- sizable effects at low beam energies

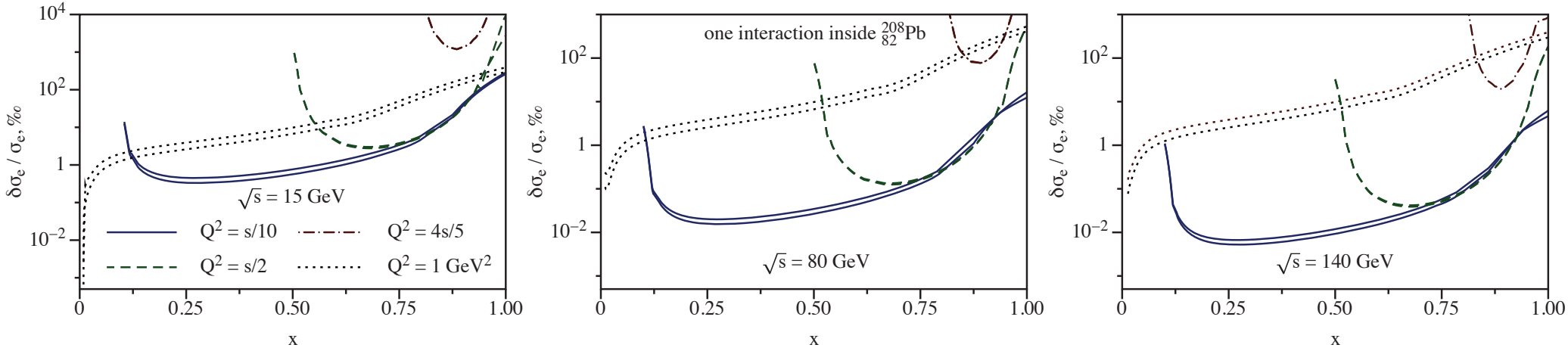
Neutral-current DIS

lepton energy and momentum transfer $Q^2 = -(k - k')^2$

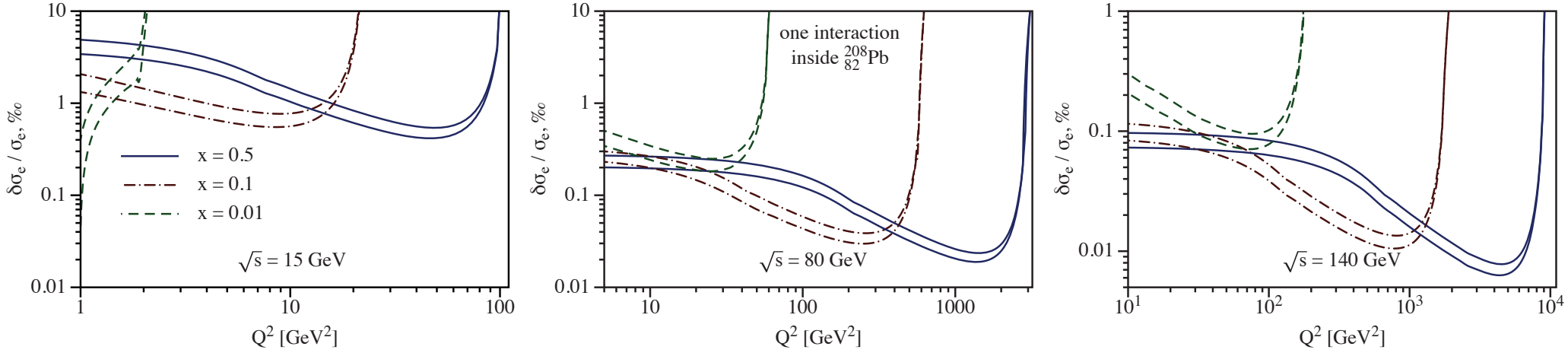
Bjorken variable $x = \dfrac{Q^2}{2M\,(E - E')}$

DIS. 1 rescattering

- fixed Q^2:



- fixed x:



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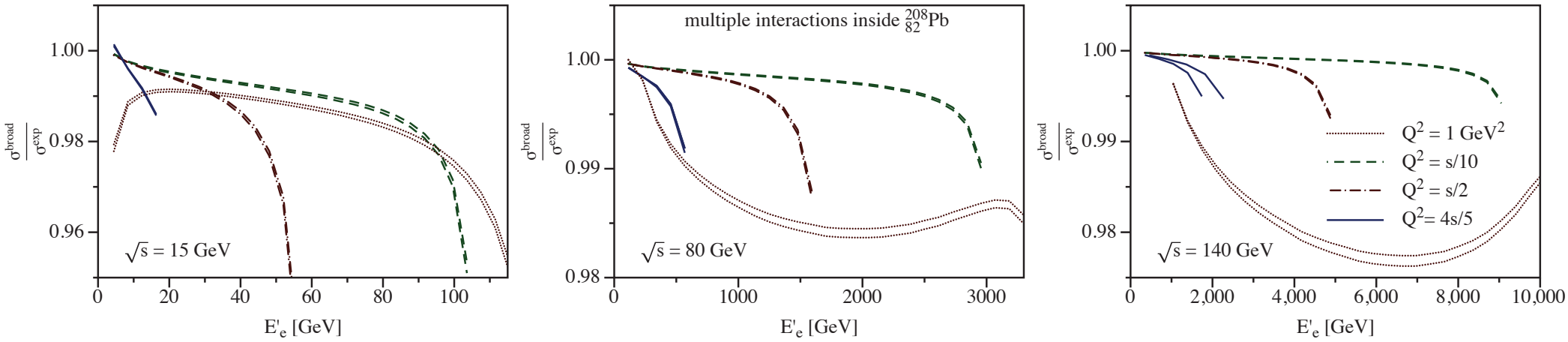
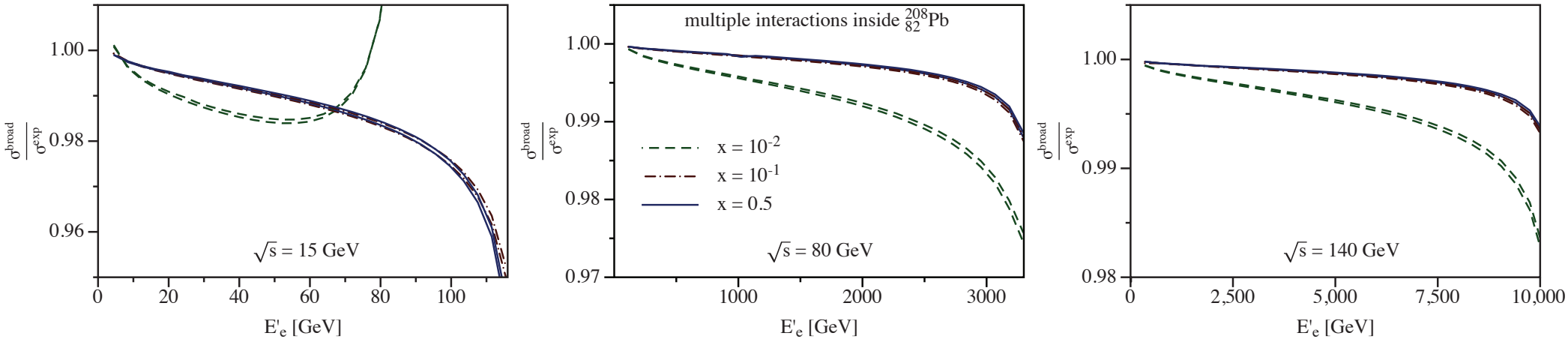
- sizable effects for forward scattering and large x

DIS. Resumed result

- fixed Q^2:



- fixed x:



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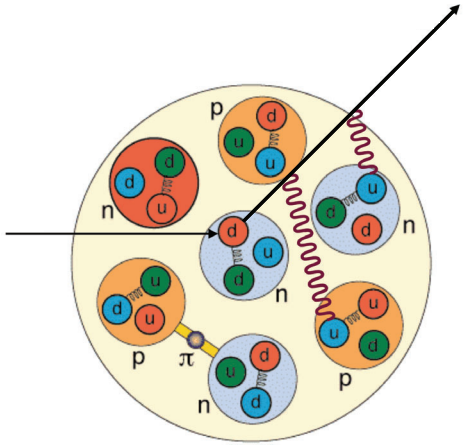
- sizable effects at low beam energies and forward scattering

Conclusions



formulation of

QED nuclear medium

effects

virtual corrections at 1st order in opacity: $SCET_G$ and full QED

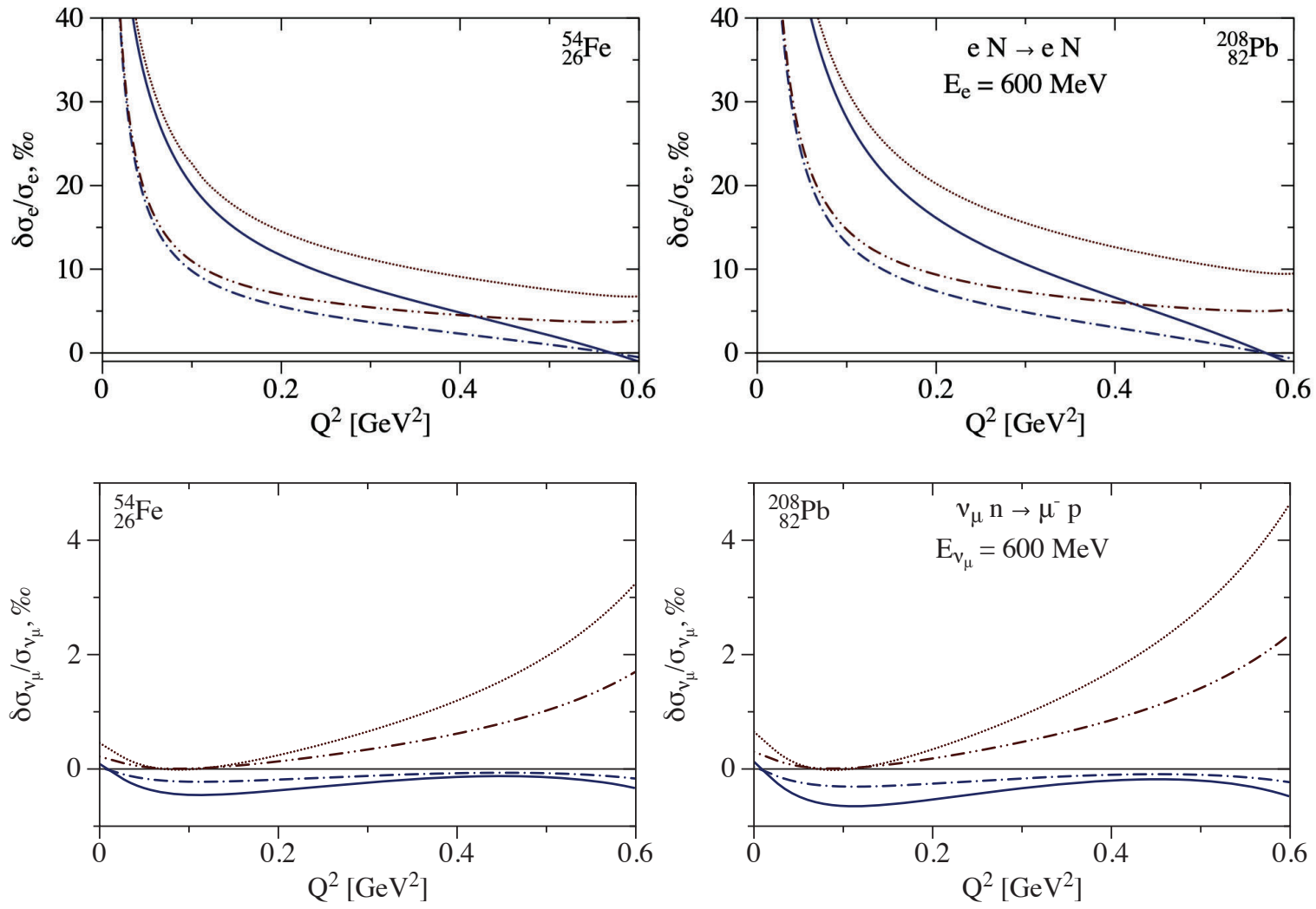
broadening and radiation: $SCET_G$

verified: $SCET_G$ works perfectly at GeV energies and above

Cross sections at 600 MeV beam energy

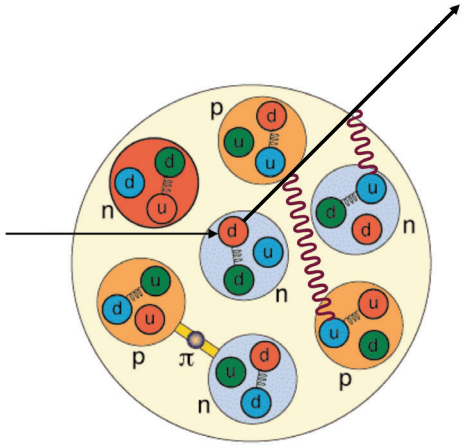
1st order in opacity

EIC is in safe regime !!!



- QED and SCET$_G$ significantly differ at 100th of MeV energy

Conclusions



formulation of

QED nuclear medium

effects

virtual corrections at 1st order in opacity: $SCET_G$ and full QED

broadening and radiation: $SCET_G$

verified: $SCET_G$ works perfectly at GeV energies and above

but not for 100th MeV !!!

found: a) sizable deflection of charged lepton tracks

b) multiple rescattering: %-level corrections and above

c) vanishing nuclear medium-induced photon energy spectra

d) radiation sizably (~10-20 %) modifies broadening

Thanks for your attention !!!