

New Physics Searches at the  
Precision Frontier  
INT-23-1b, Seattle, USA

9 May, 2023

QED radiative corrections to  
charged-current neutrino scattering  
for accelerator neutrinos

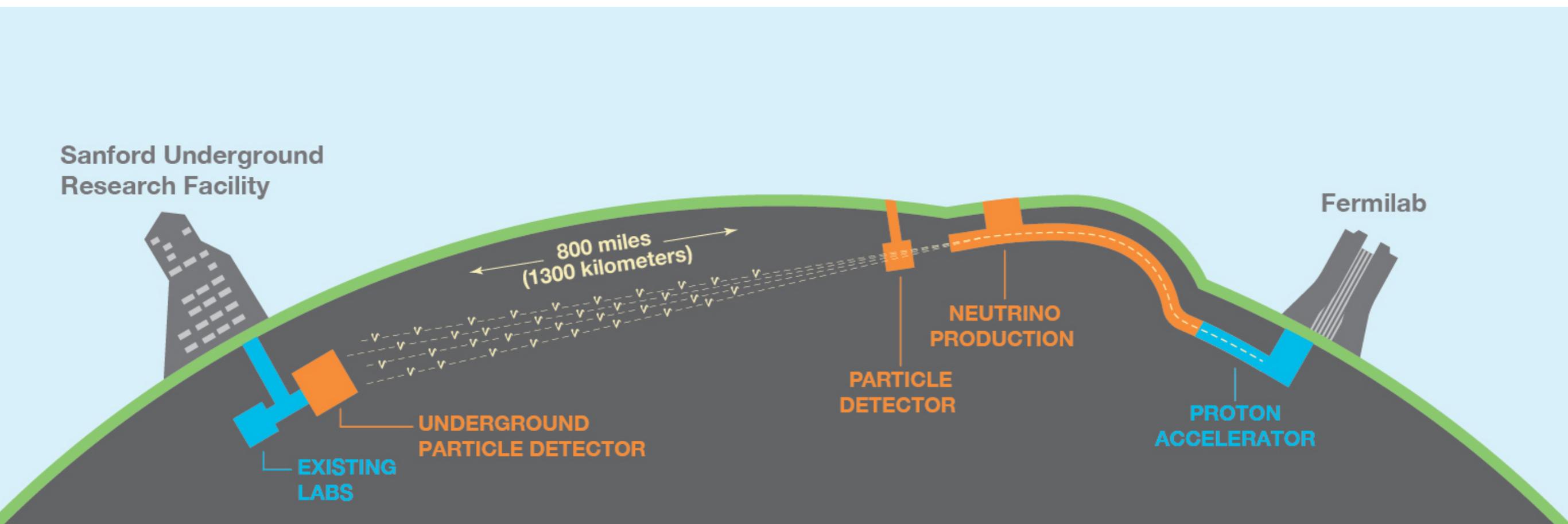


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LA-UR-23-23974

# Neutrino experiments

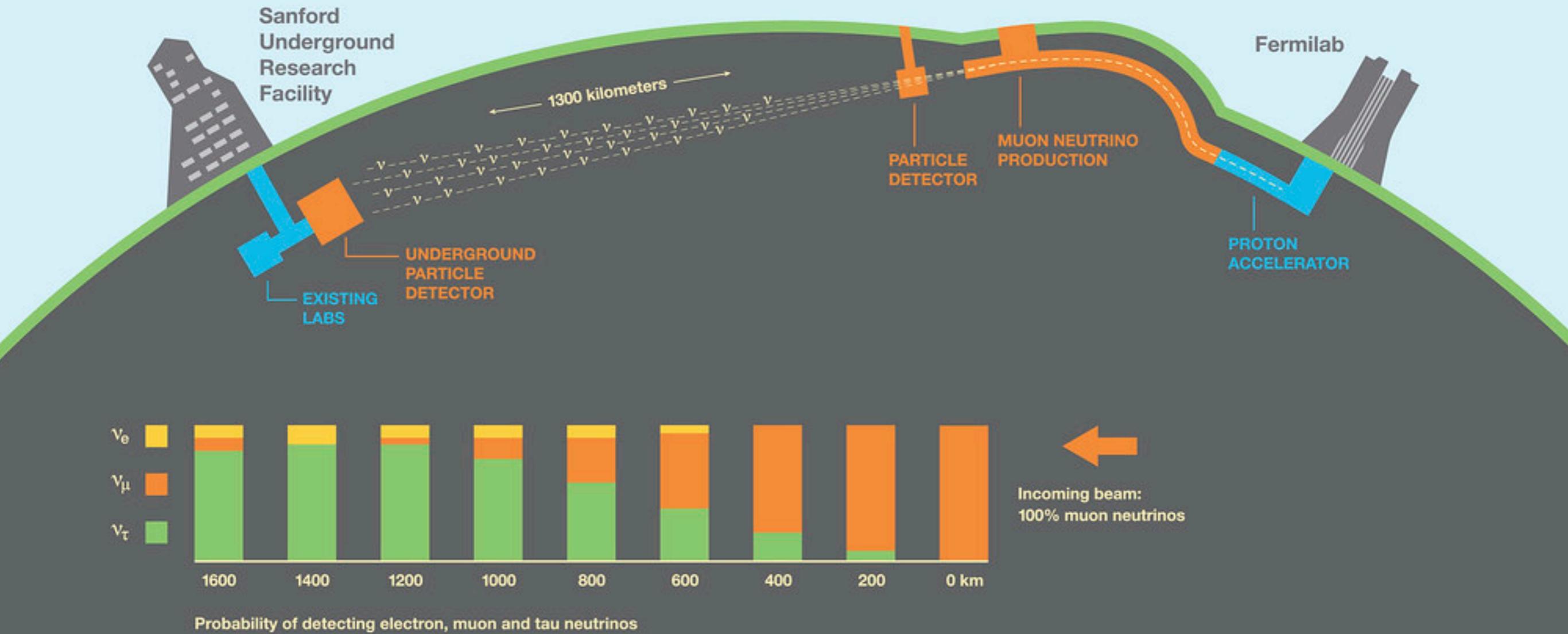
- DUNE and Hyper-K: leading-edge  $\nu$  science experiments



- origin of matter-antimatter asymmetry  $\delta_{CP}$
- mass hierarchy and oscillation parameters PMNS matrix,  $\Delta m_{31}^2$
- Grand Unified Theories proton decay
- dynamics of supernova explosion wait for one;)

# Neutrino experiments

## Deep Underground Neutrino Experiment



$$N_\nu \sim \int dE_\nu \Phi_\nu(E_\nu) \times \sigma(E_\nu) \times R(E_\nu, E_\nu^{\text{rec}})$$

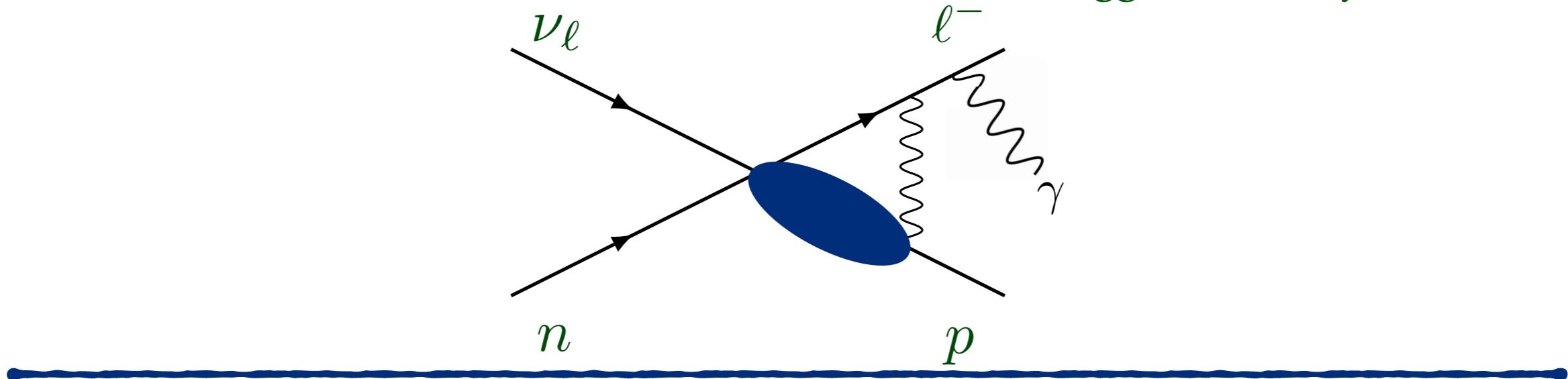
- precise neutrino physics: need in cross sections



O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

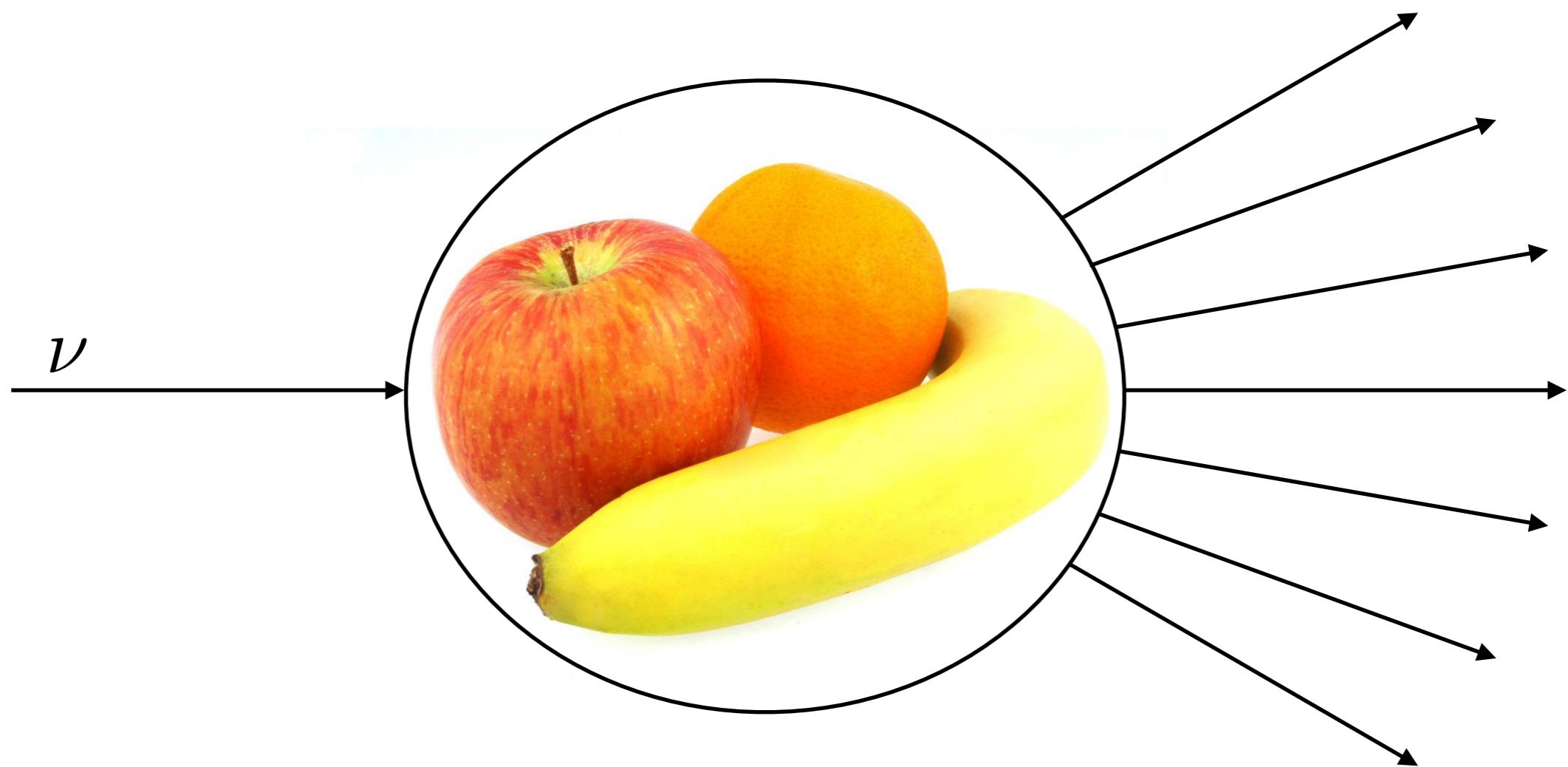
# Radiative corrections in charged-current elastic scattering on free nucleons

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret  
editors suggestion in Phys. Rev. D (2022)

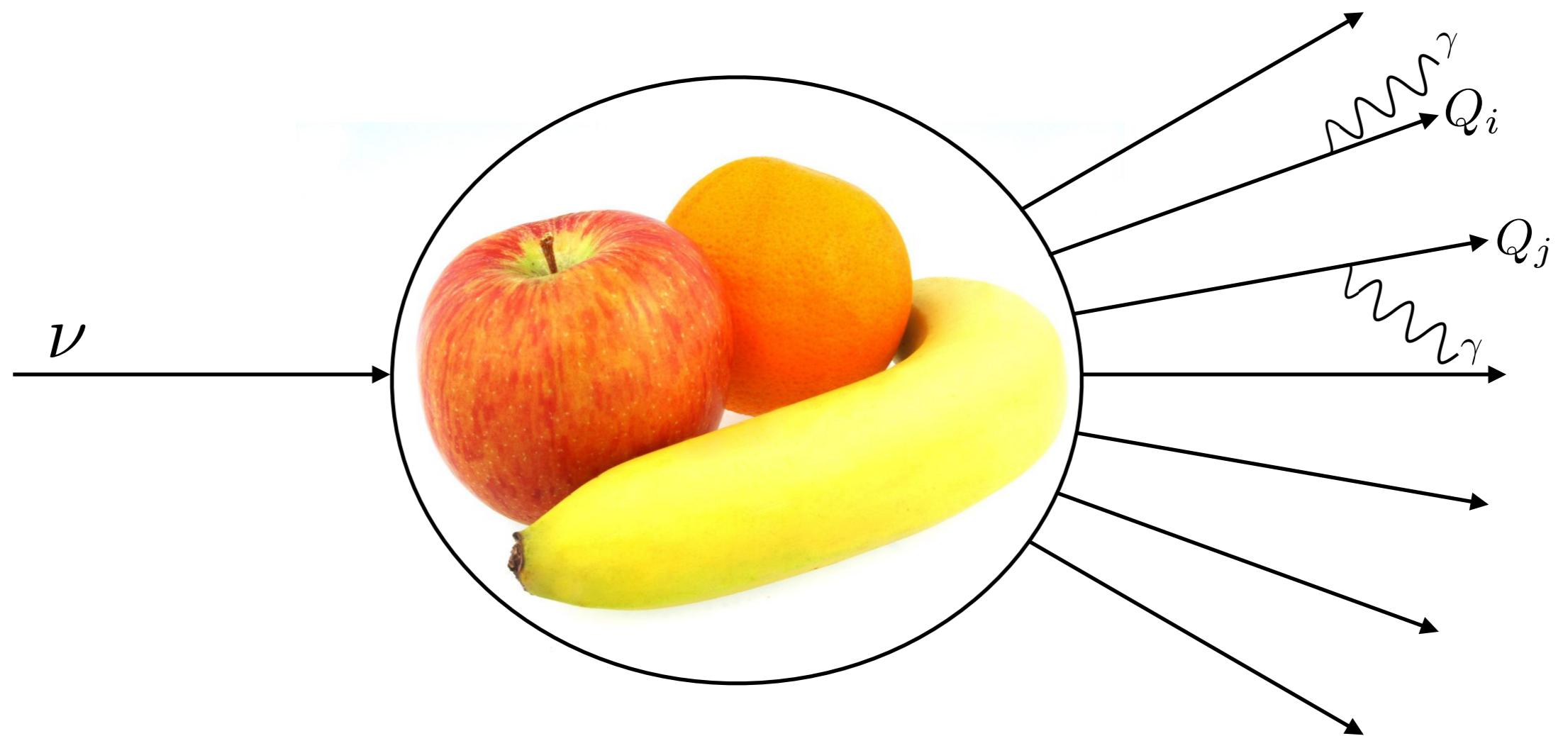


# Neutrino interactions

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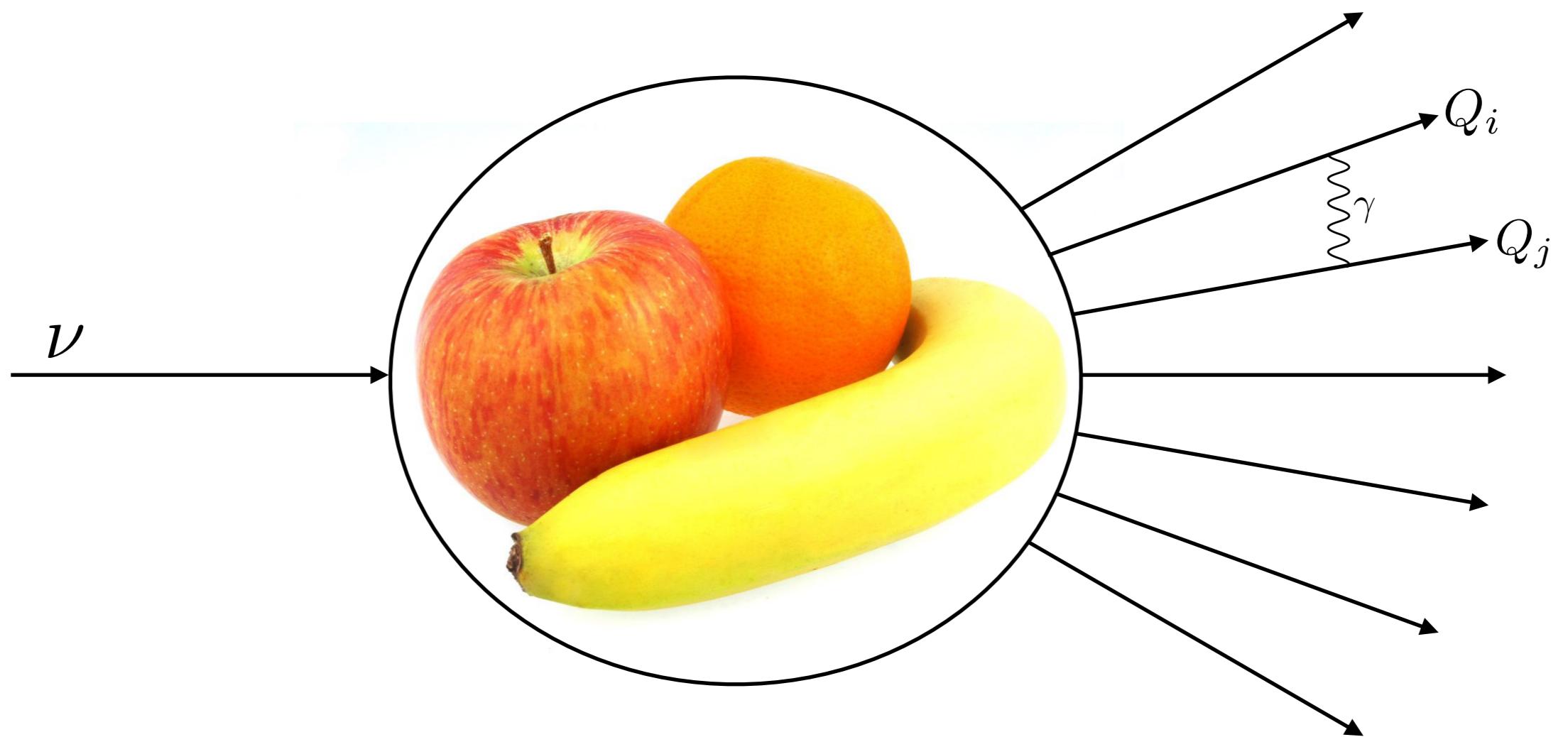


# QED corrections



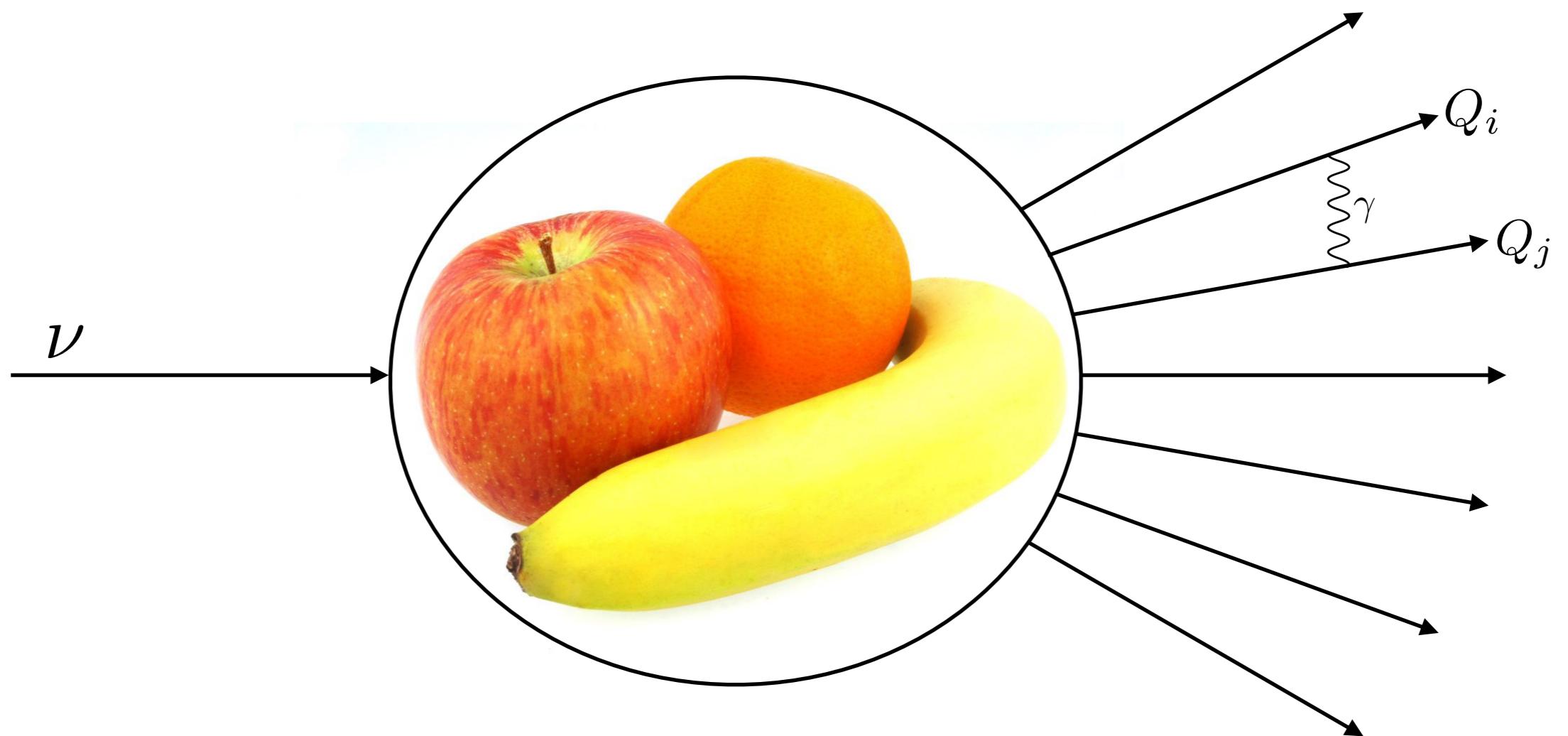
- all charged particles couple to real and virtual photons

# QED corrections



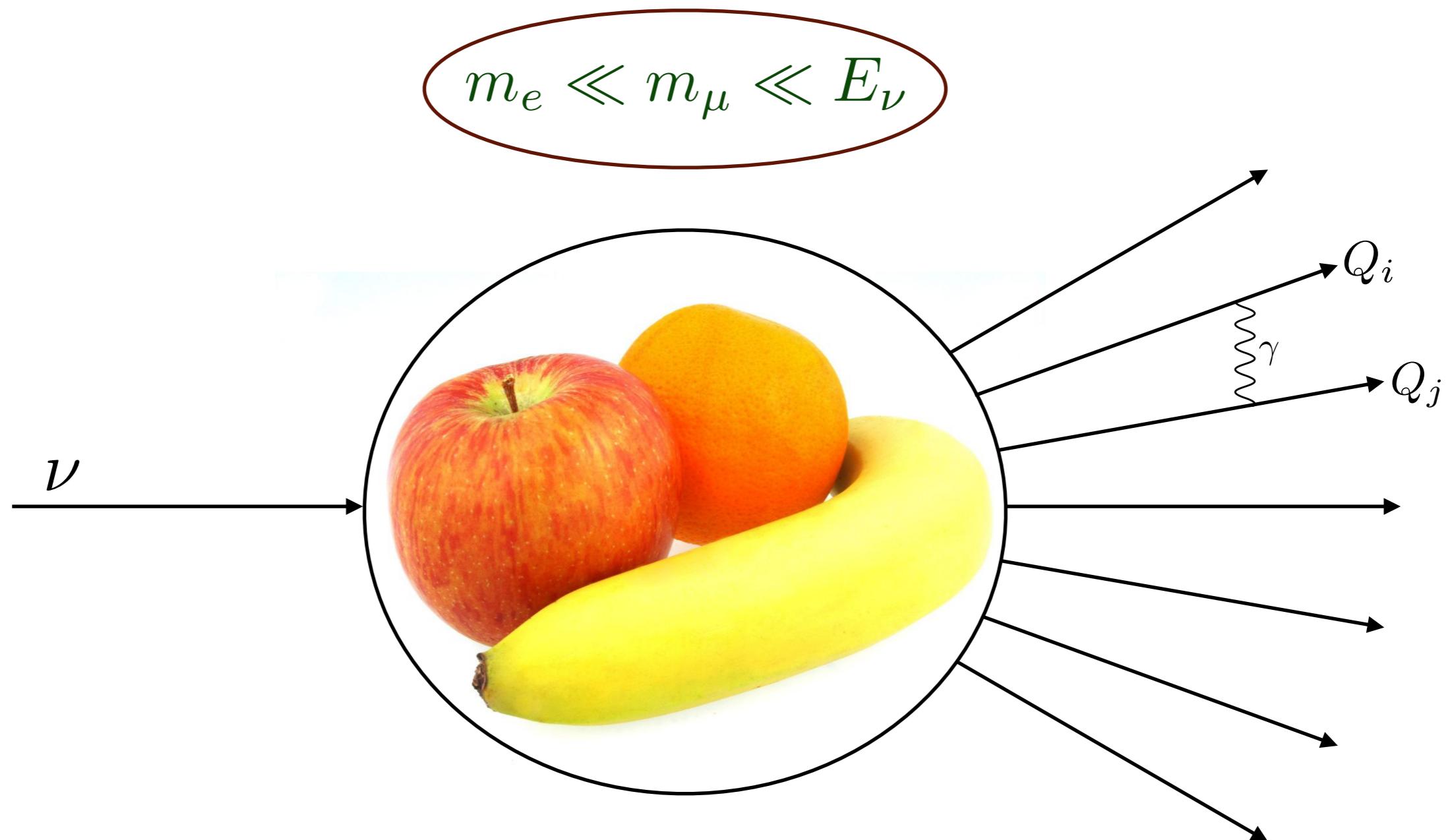
- all charged particles couple to real and virtual photons

# QED corrections



-  $\frac{\alpha}{\pi} \sim 0.2 \%$  suppression by electromagnetic coupling constant

# QED corrections



$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \ln \frac{E_\nu}{m_e} \sim 6 - 10 \text{ or } \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

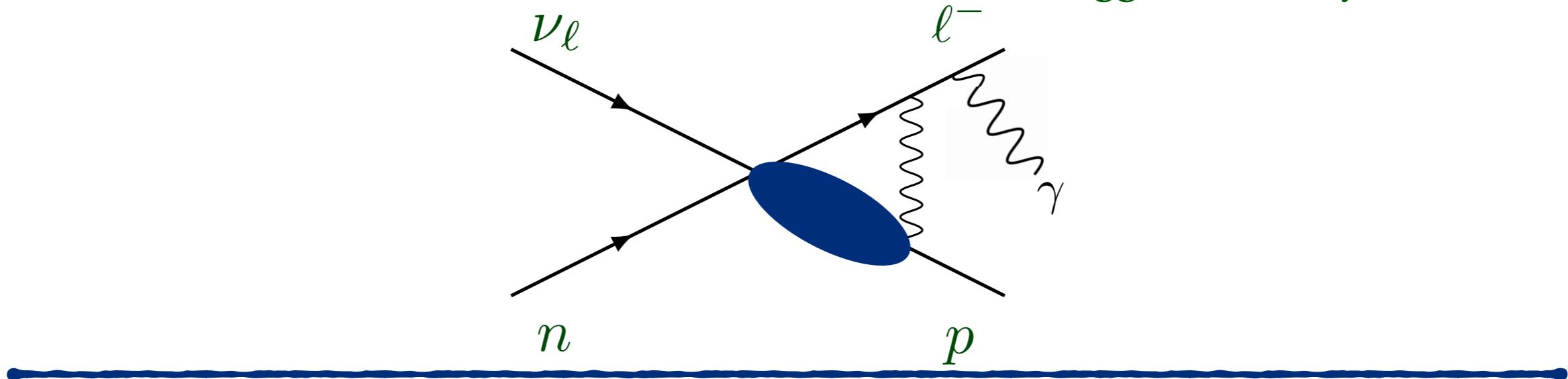
- scale separation introduces large flavor-dependent QED logarithms



O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

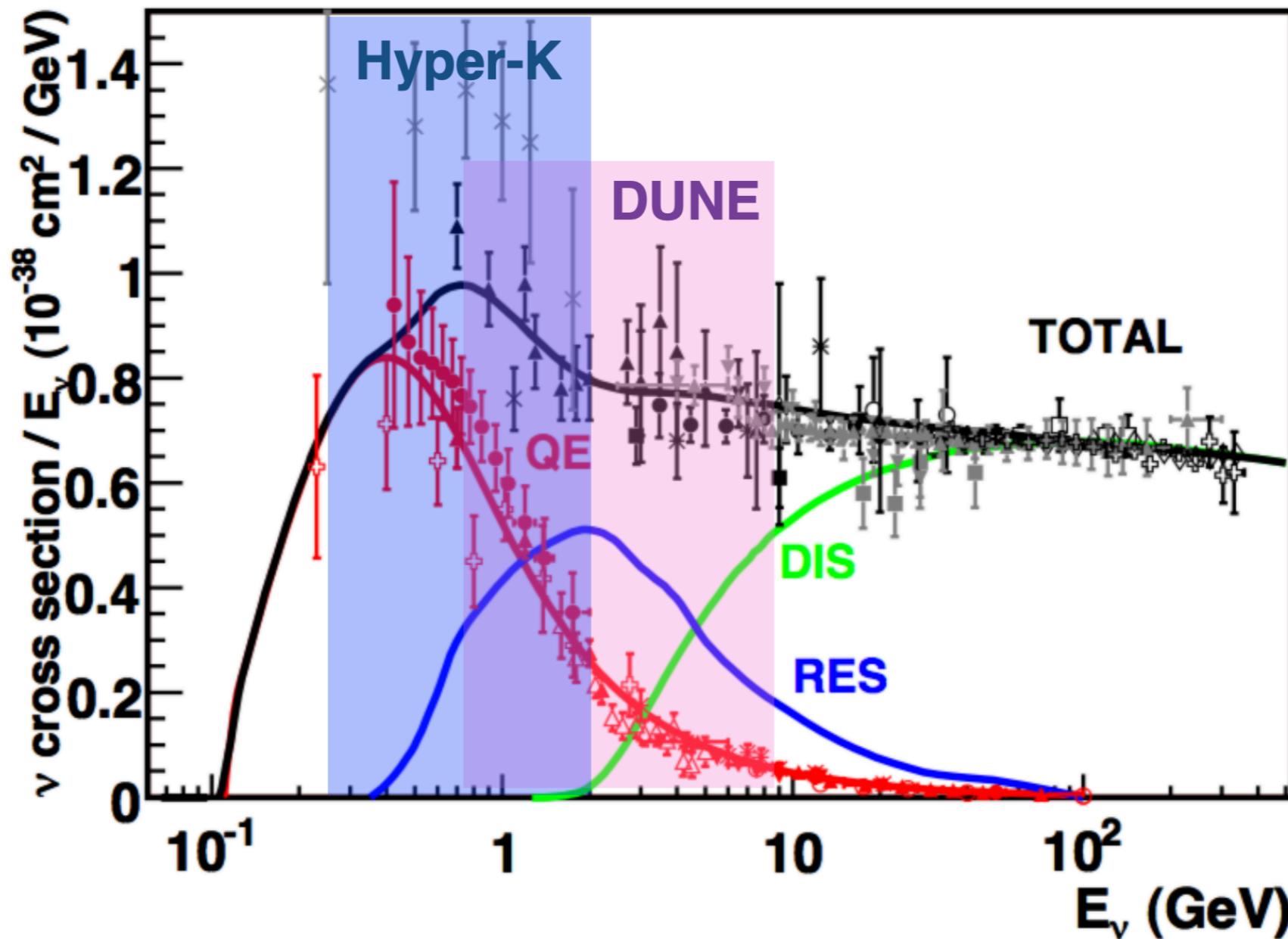
# Radiative corrections in charged-current elastic scattering on free nucleons

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# CCQE. Why should we care?

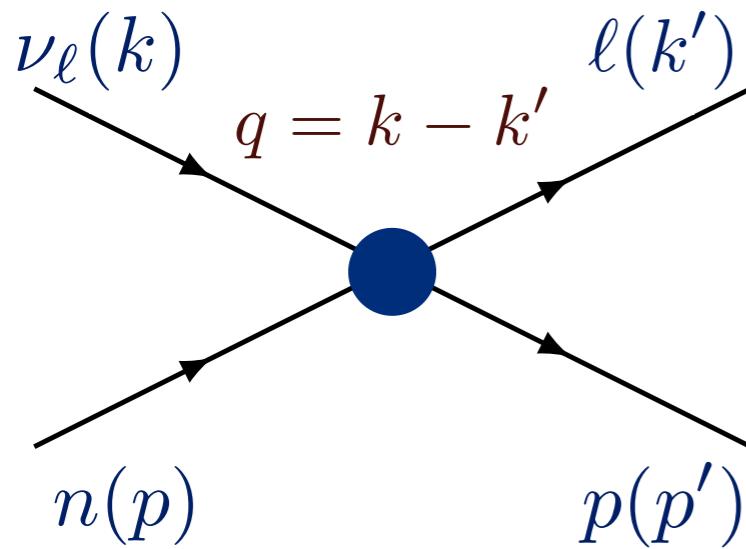
- neutrino-nucleus cross sections and future accelerator-based fluxes



Formaggio  
and Zeller  
(2013)

- basic process: bulk of events at Hyper-K and DUNE
- channel for reconstruction of neutrino energy

# CCQE scattering on free nucleon



neutrino energy

$$E_\nu$$

momentum transfer

$$Q^2 = -q^2$$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu(Q^2) = \langle p | \bar{u} (\gamma^\mu - \gamma^\mu \gamma_5) d | n \rangle$$

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P^V(Q^2) + \gamma^\mu \gamma_5 F_A(Q^2) + \frac{q^\mu}{M} \gamma_5 F_P(Q^2)$$

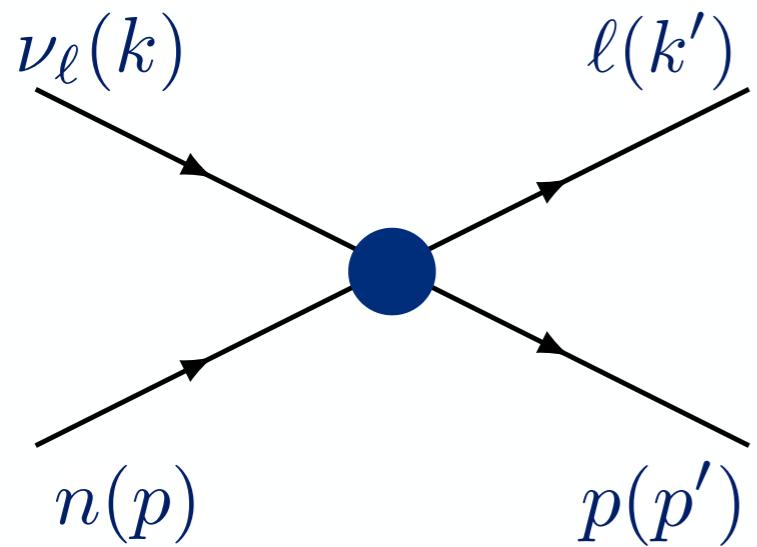
form factors: isovector Dirac and Pauli      axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

tree-level amplitude

$$T = \frac{G_F V_{ud}}{\sqrt{2}} (\bar{\ell}(k') \gamma_\mu (1 - \gamma_5) \nu_\ell(k)) (\bar{p}(p') \Gamma^\mu(Q^2) n(p))$$

# CCQE scattering on free nucleon



$$\nu = E_\nu/M - \tau - r^2$$

$$r = \frac{m_\ell}{2M} \quad \tau = \frac{Q^2}{4M^2}$$

unpolarized cross section

$$\frac{d\sigma}{dQ^2} \sim \frac{M^2}{E_\nu^2} \left( (\tau + r^2) A(Q^2) - \nu B(Q^2) + \frac{\nu^2}{1+\tau} C(Q^2) \right)$$

Llewellyn Smith (1972)

- structure-dependent functions

$$A = \tau \left( G_M^V \right)^2 - \left( G_E^V \right)^2 + (1 + \tau) F_A^2 - \cancel{r^2} \underbrace{\left( \left( G_M^V \right)^2 + F_A^2 - 4\tau F_P^2 + 4F_A F_P \right)}$$

$$B = \pm 4\tau F_A G_M^V$$

$$C = \tau \left( G_M^V \right)^2 + \left( G_E^V \right)^2 + (1 + \tau) F_A^2$$

- pseudoscalar form factor contribution is suppressed by lepton mass
- cross section is sensitive to both vector and axial contributions

# Elastic scattering on free nucleon

- only 3 experiments performed with deuterium bubble chamber  
direct access to form-factor shape

ANL 1982: 1737 events

BNL 1981: 1138 events

FNAL 1983: 362 events

world data: ~3200 events



Fermilab bubble chamber, Richard Drew

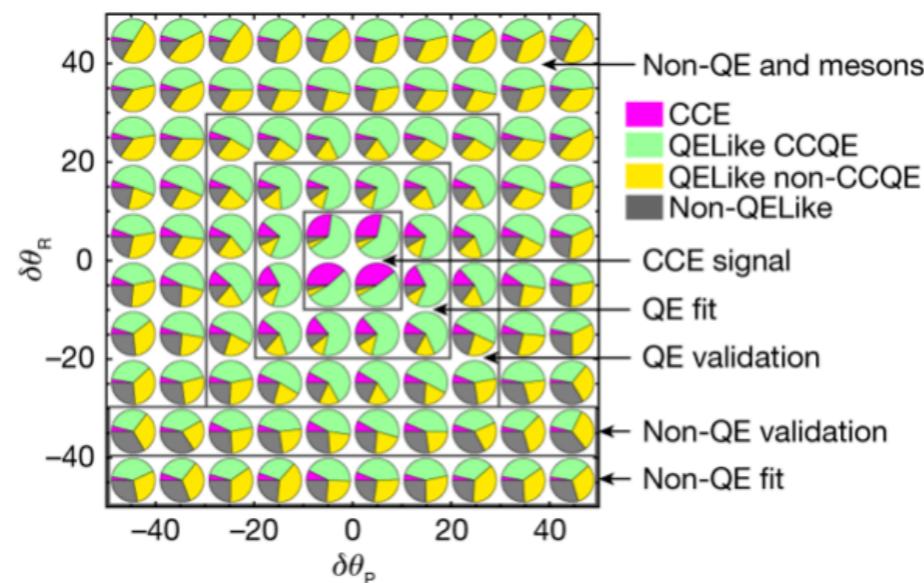
- axial form factor extracted based on electromagnetic structure

# MINERvA result with free protons

- idea of scattering on molecular hydrogen realized !!!

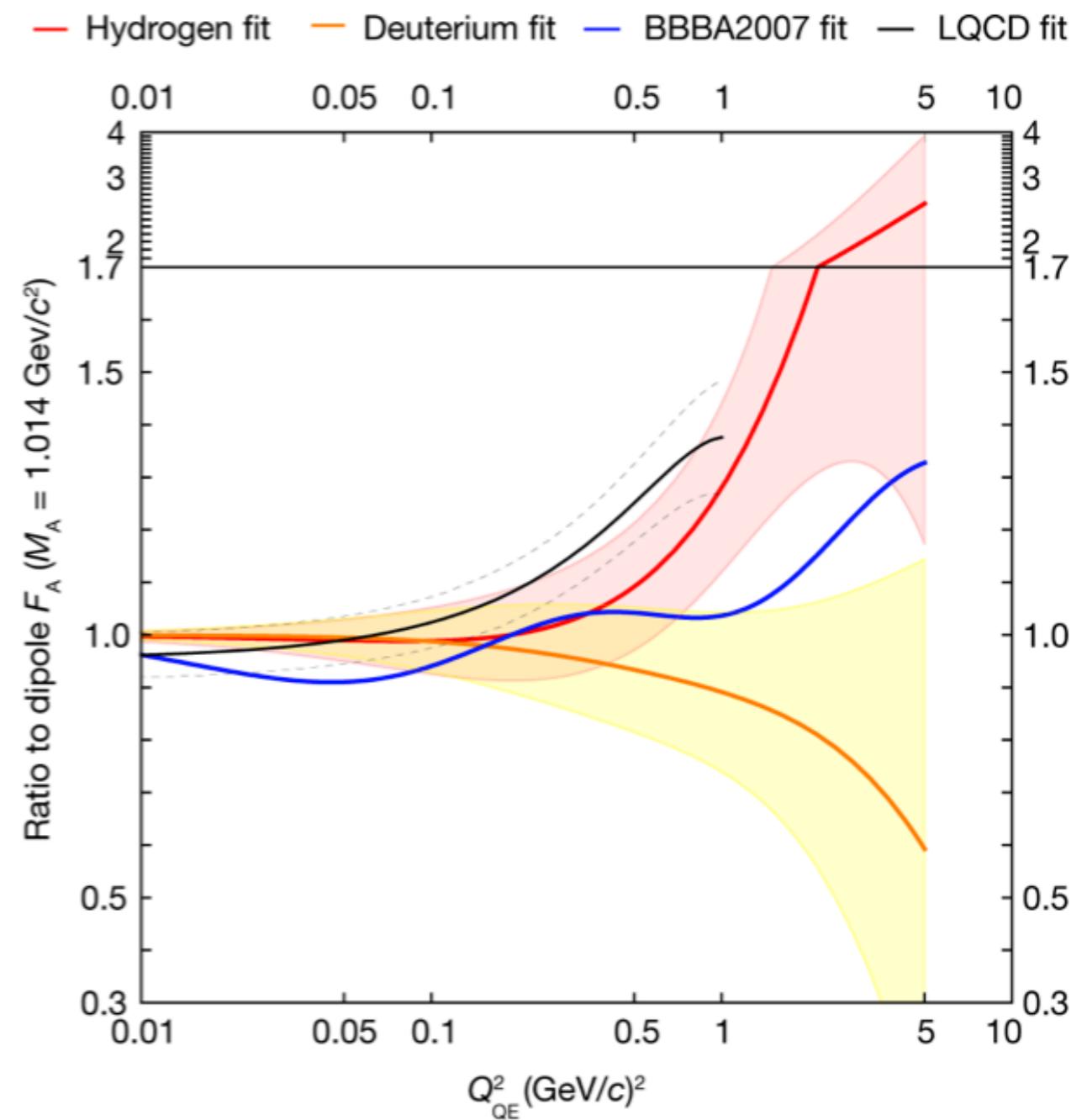


muon kinematic selection



5580 events over  
12500 background

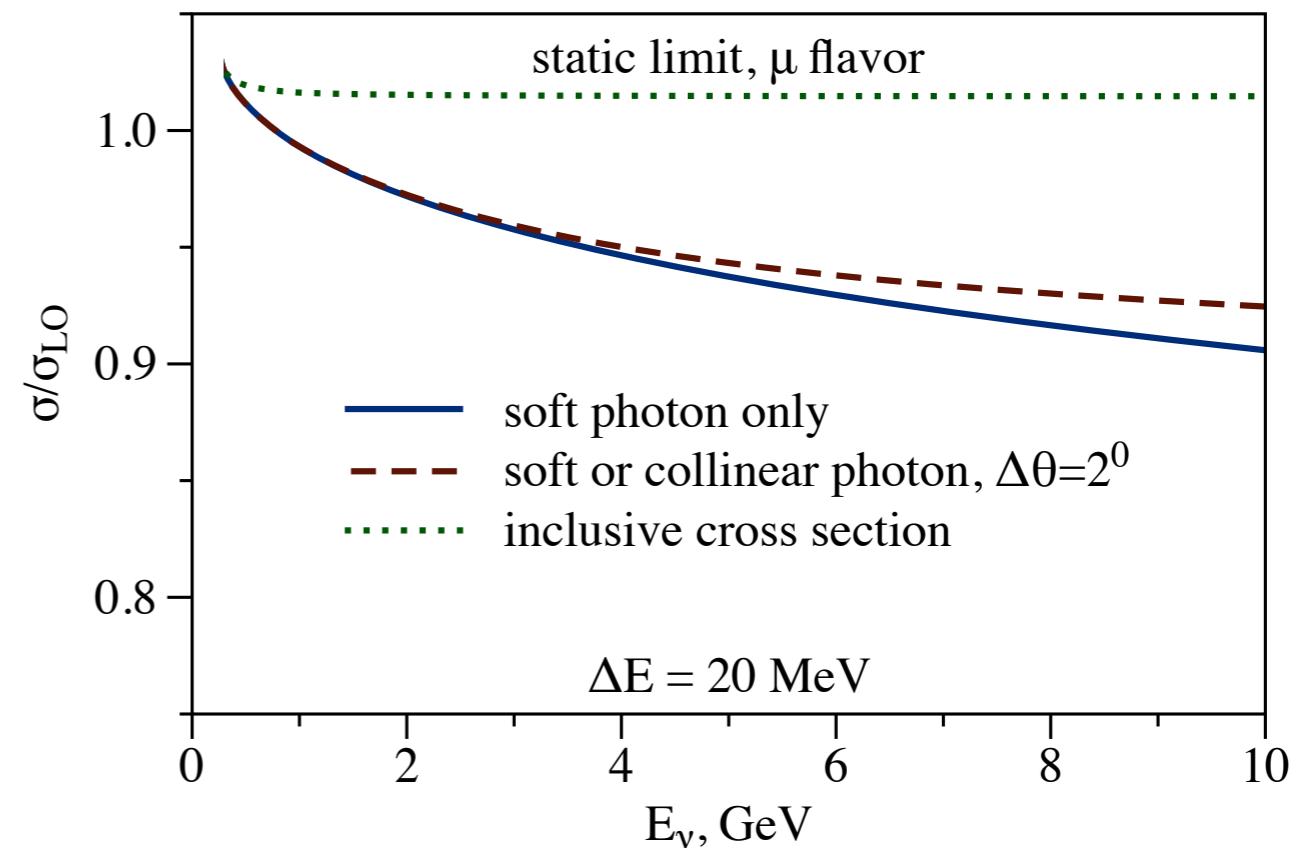
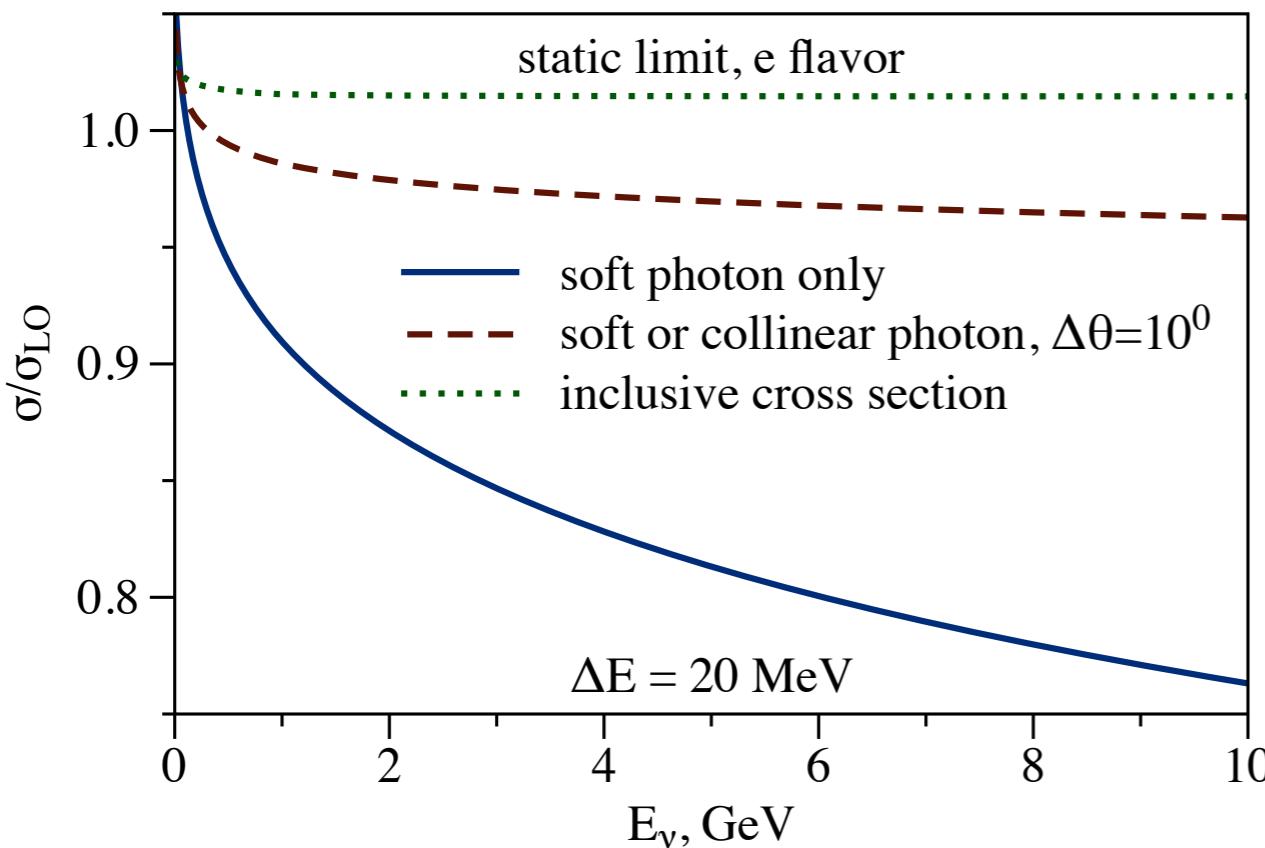
background nuclear events  
constrained by scattering of  $\nu$



- 1st measurement of axial form factor on “free” protons  $\bar{\nu}_\mu p \rightarrow \mu^+ n$

# Static nucleon limit

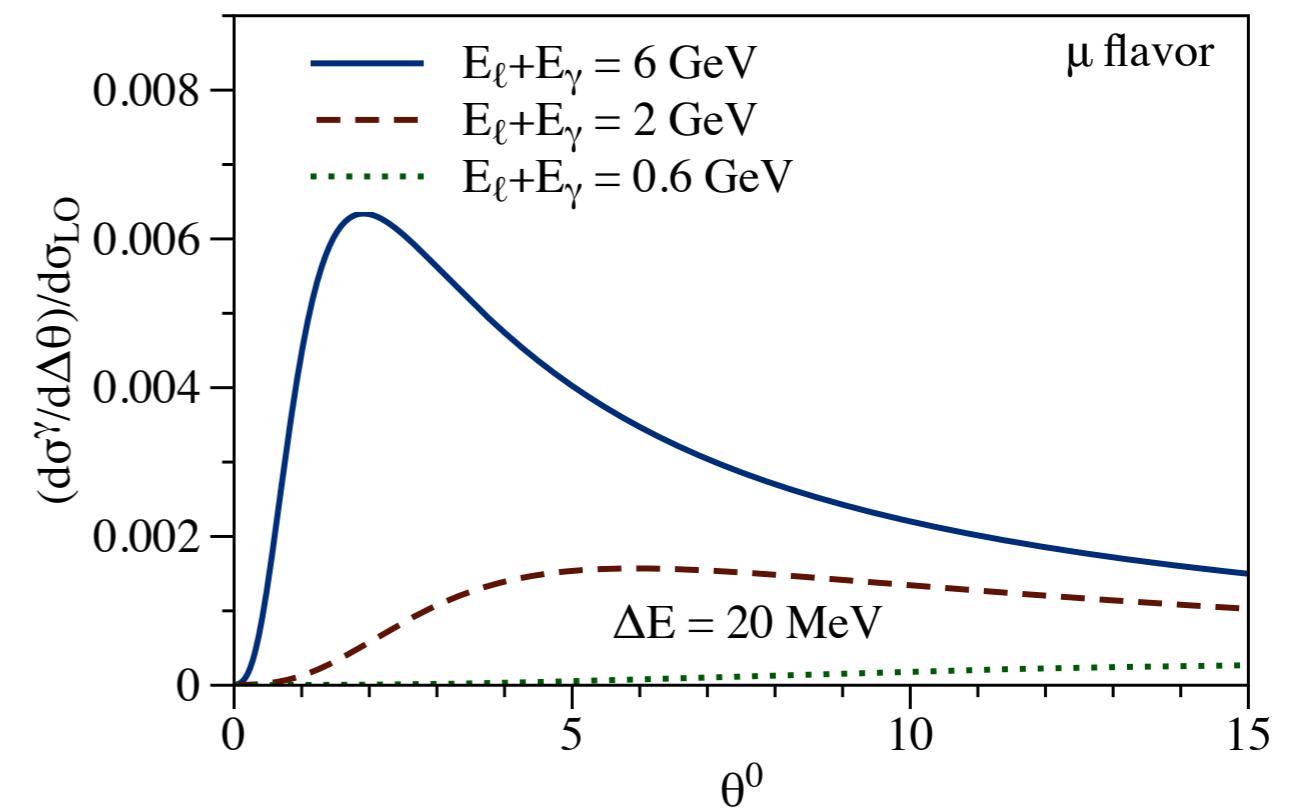
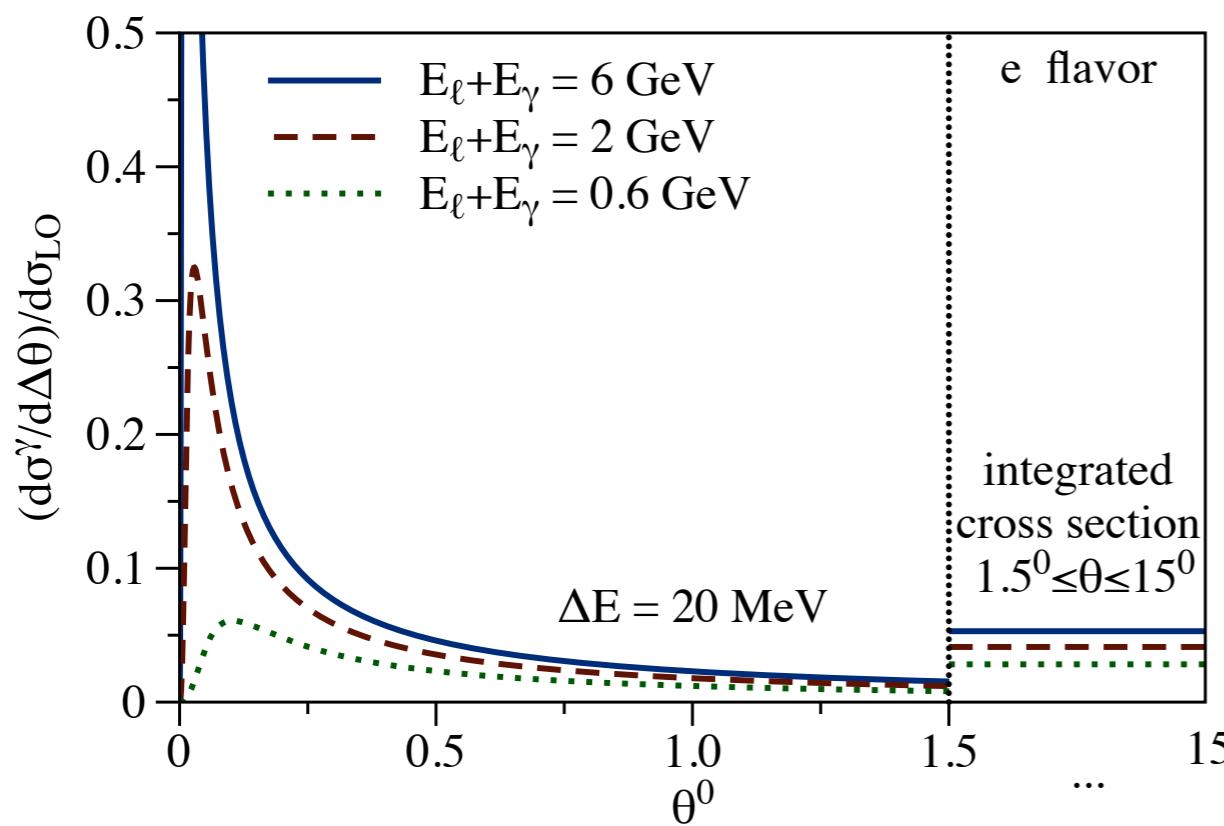
- formal limit of infinitely heavy nucleus  $m_\ell \ll E_\ell \ll M$
- provides correct soft and collinear logarithms
- soft-photon energy  $< 20$  MeV, jet size:  $10^\circ$  for electron and  $2^\circ$  for muon



- flavor-dependent effect, same for  $\nu_\ell n \rightarrow \ell^- p$  vs  $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- collinear observable: cancellation of virtual vs real logs
- inclusive observables ( $+\gamma$ ): few % level, flavor independent

# Electron vs muon jets

- factorization for radiation of collinear photons
- cone angle is defined to lepton direction
- photons of energy  $> 20$  MeV, fixed energy in the cone



- flavor-dependent effect, same for  $\nu_\ell n \rightarrow \ell^- p$  vs  $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- forward-peaked radiation for electron flavor
- negligible radiation for muons with shifted peak position

# Factorization approach

- cross section is given by **factorization formula**

$$d\sigma \sim S\left(\frac{\Delta E}{\mu}\right) J\left(\frac{m_\ell}{\mu}\right) H\left(\frac{M}{\mu}\right)$$



- $M$  - determine **hard function** at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

- $m_\mu$  - soft and collinear functions are evaluated **perturbatively**

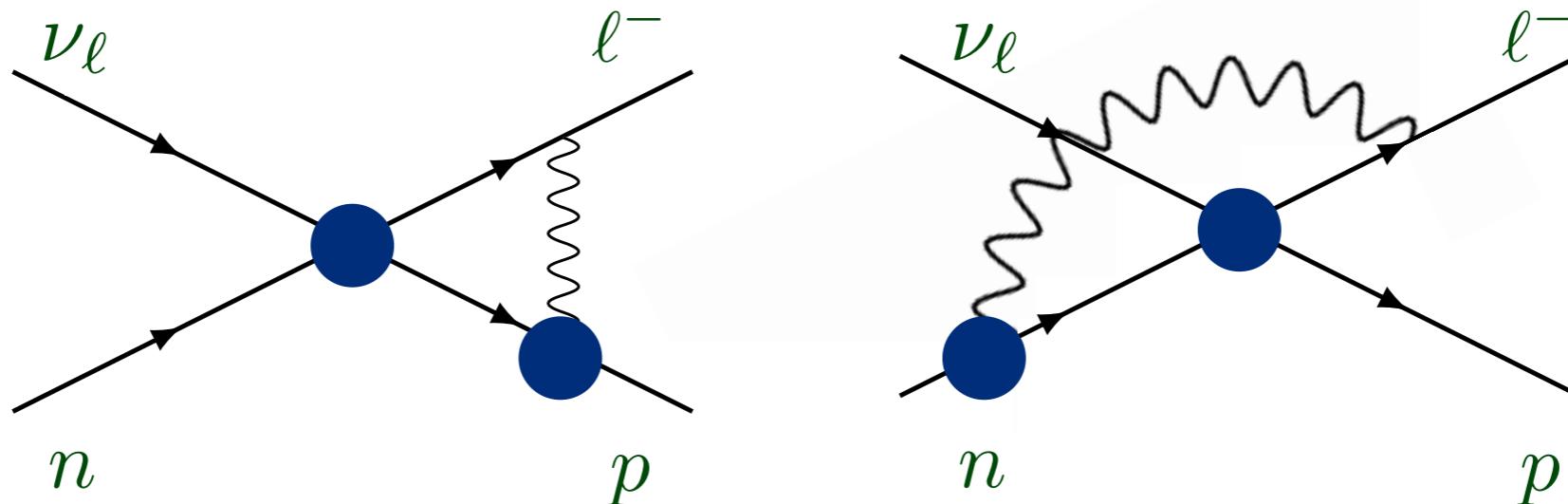
$\Delta E$

SCET power expansion parameter

$$\lambda \sim \frac{m_\mu^2}{E_\nu^2} \sim (\Delta\theta)^2 \sim \frac{\Delta E}{E_\nu}$$

$\ln \lambda$  enhancements

# Hadronic model at GeV scale



- exchange of photon between the charged lepton and nucleons
- assume **onshell form** for each interaction with dipole form factors  
discussed for neutrino-nucleon scattering: Graczyk (2013)
- add **self energy** for charged particles
- reproduce soft and collinear regions of SCET

- best determination of hard function

# Factorization approach

- cross section is given by **factorization formula**

$$d\sigma \sim S\left(\frac{\Delta E}{\mu}\right) J\left(\frac{m_\ell}{\mu}\right) H\left(\frac{M}{\mu}\right)$$

—  $M$

- determine **hard function** at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

—  $m_\mu$

- RGE evolution of the hard function to scales  $\Delta E, m_\ell$

—  $\Delta E$

- soft and collinear functions are evaluated **perturbatively**

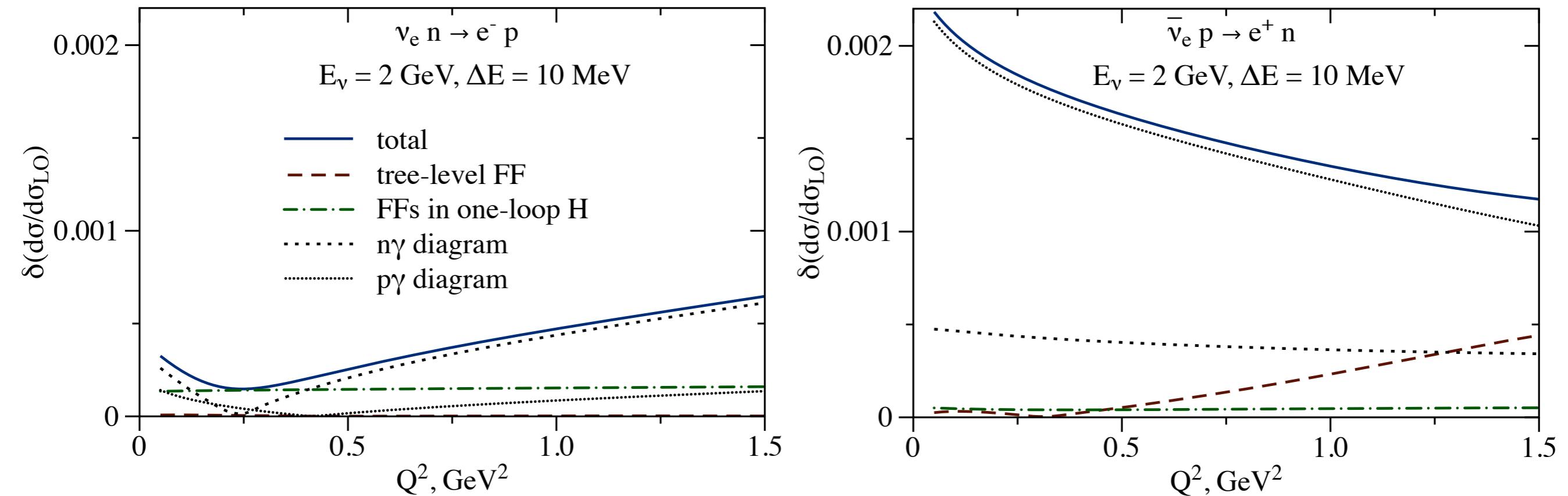
—  $m_e$

- calculate cross section at low energies accounting for **all large logs**  
ep scattering with soft radiation only: Richard J. Hill (2016)

- soft and collinear functions determined **analytically**
- hard function describes physics at GeV energies

# Error budget

- uncertainties from hard function



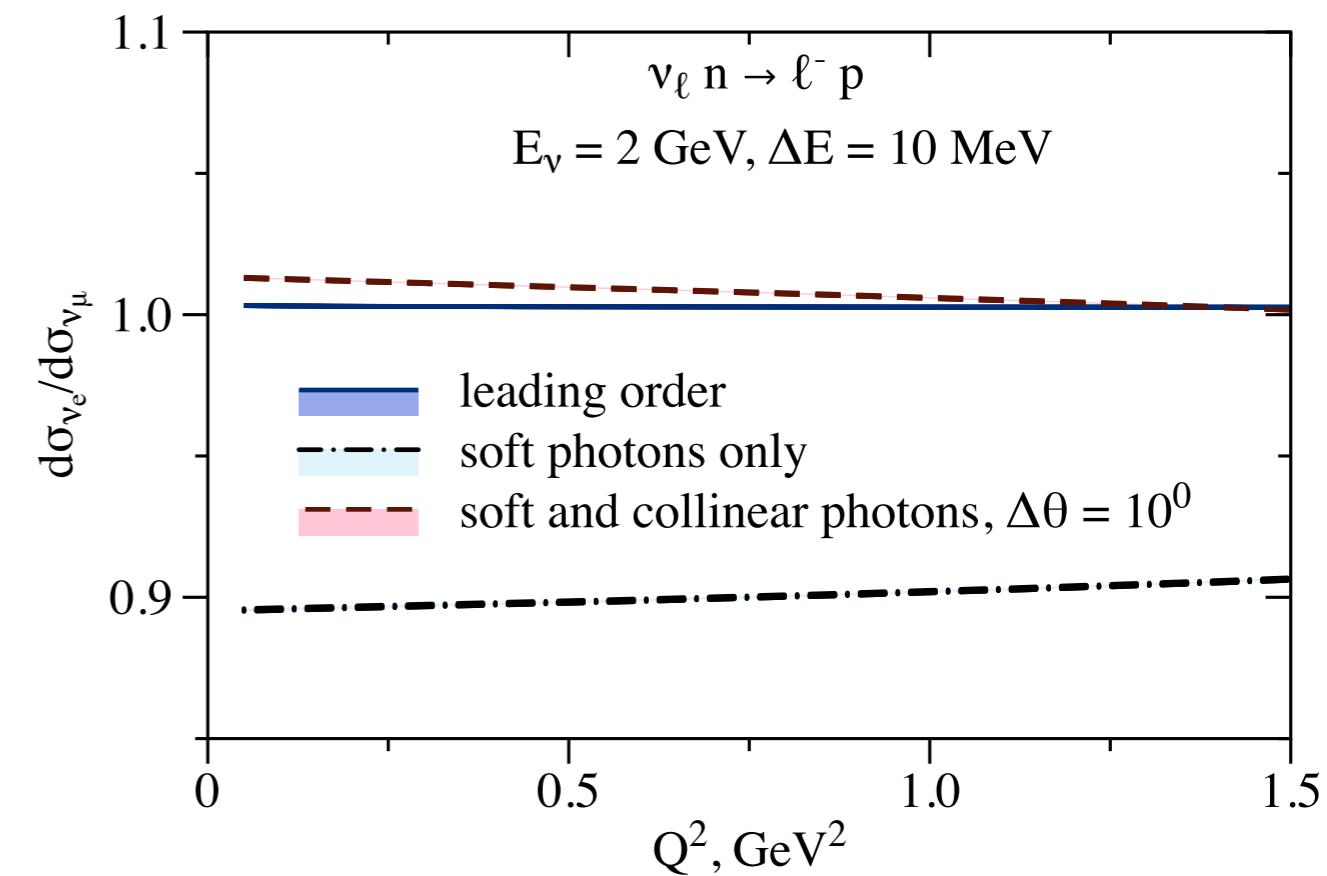
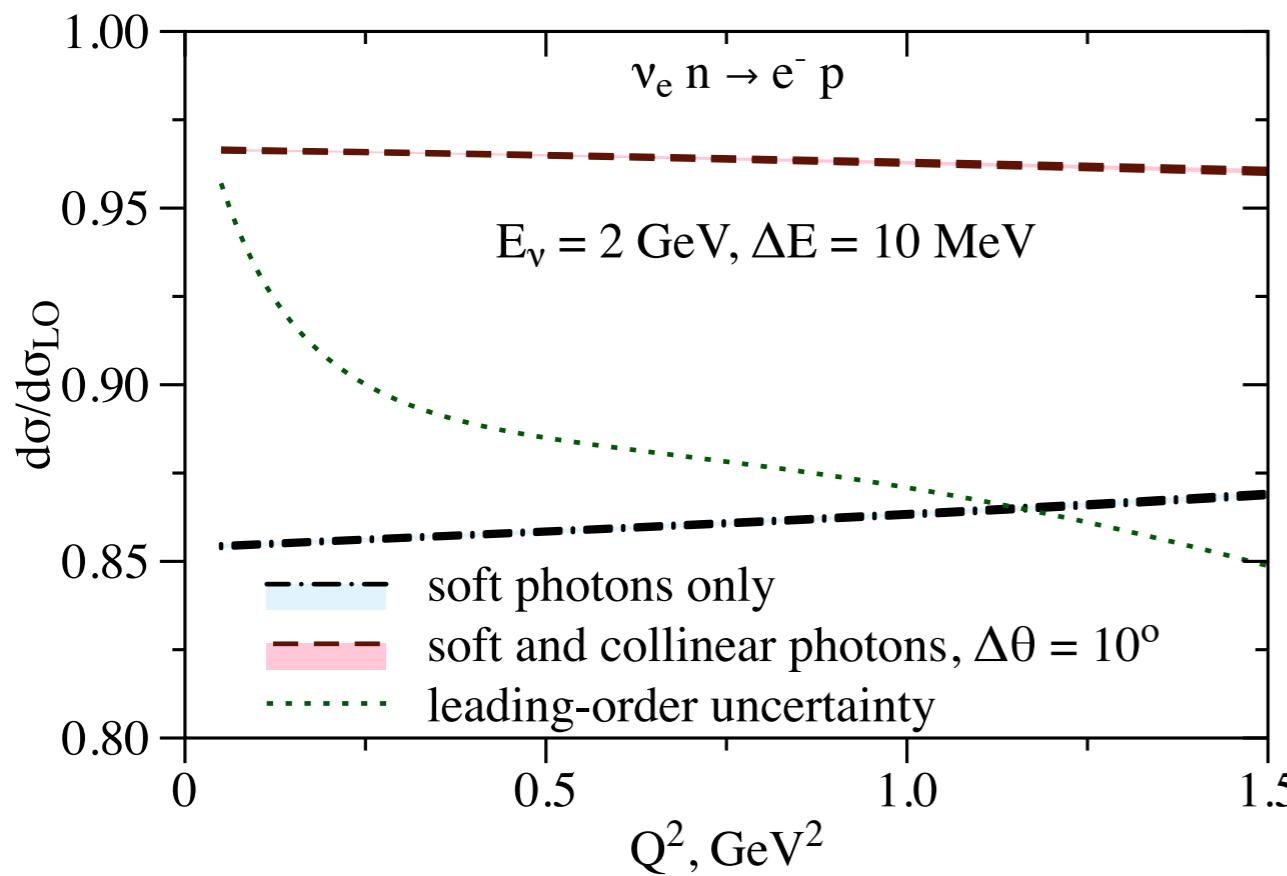
- nucleon form factors
- add perturbative uncertainty by variation of scale

Meyer, Betancourt, Gran and Hill (2016)  
Kaushik Borah, Gabriel Lee, Richard J. Hill and O.T. (2020)

- uncertainty of permille level for the ratio to LO result

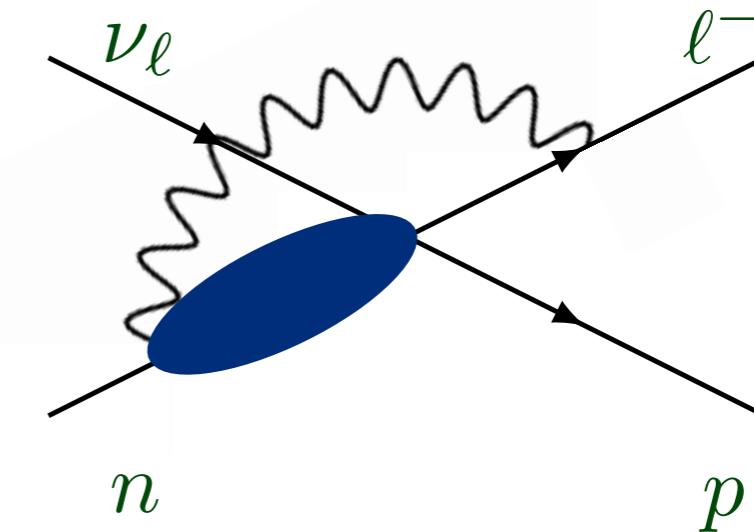
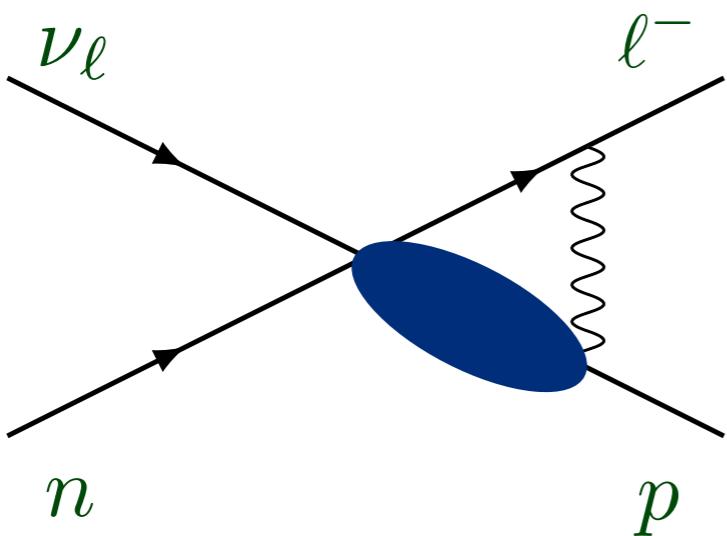
# Exclusive observables

- cancellation of uncertainties from hard function for  $e/\mu$  and ratio to LO

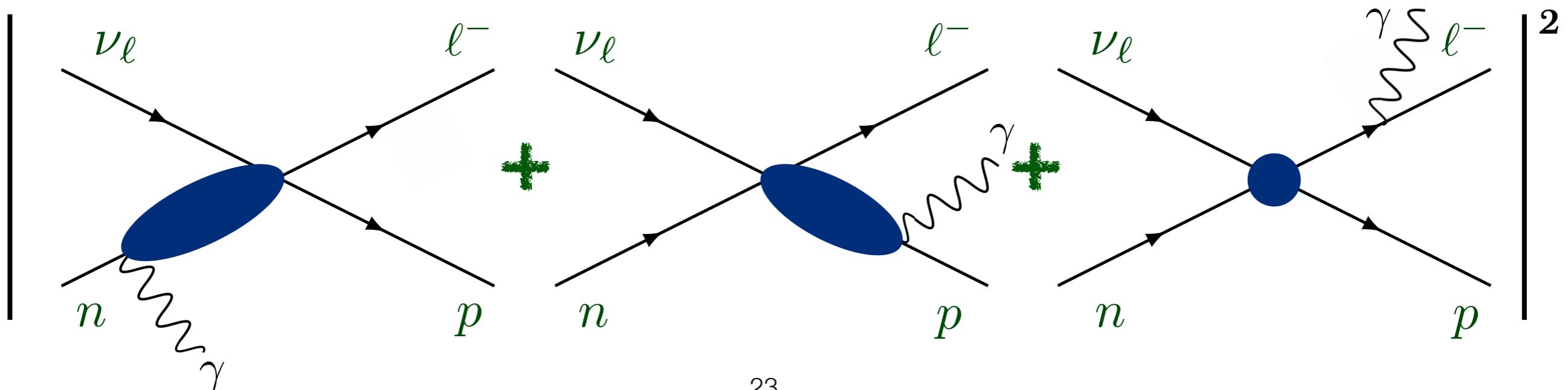


- ratios: cancellation of uncertainty from hard function

# Inclusive observables

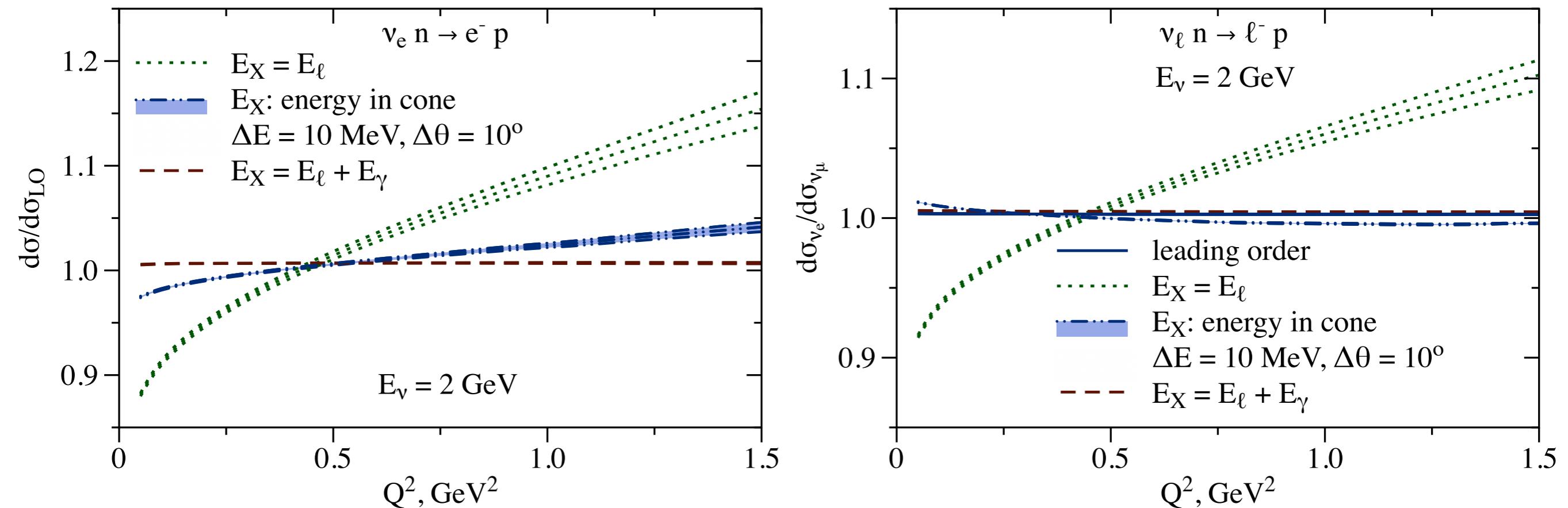


- the same gauge-invariant model for the real radiation
- arbitrary hard photons are part of the observable



# Inclusive observables

- kinematics  $Q^2 = 2M(E_\nu - E_X)$  is reconstructed with 3 different  $E_X$



- dependence on reconstruction of kinematics and cuts
- predict  $\sigma_{\nu_e}$  from  $\sigma_{\nu_\mu}$  measurements with neutrino beam

# Electron/muon ratio

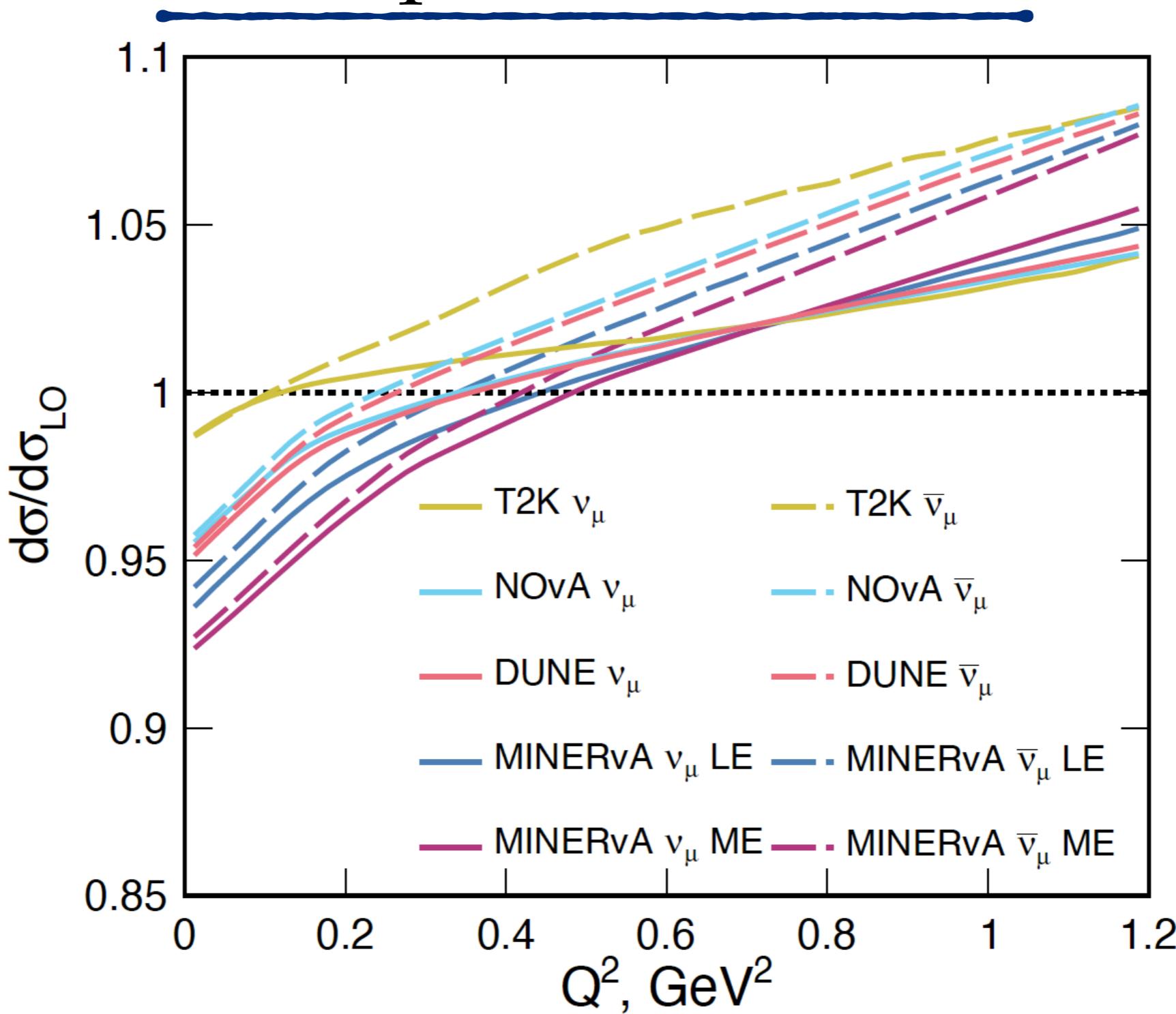
	$E_\nu$ , GeV		$\left(\frac{\sigma_e}{\sigma_\mu} - 1\right)_{\text{LO}}$ , %	$\frac{\sigma_e}{\sigma_\mu} - 1$ , %
T2K/HyperK	0.6	$\nu$	$2.47 \pm 0.06$	$2.84 \pm 0.06 \pm 0.37$
		$\bar{\nu}$	$2.04 \pm 0.08$	$1.84 \pm 0.08 \pm 0.20$
NOvA/DUNE	2.0	$\nu$	$0.322 \pm 0.006$	$0.54 \pm 0.01 \pm 0.22$
		$\bar{\nu}$	$0.394 \pm 0.003$	$0.20 \pm 0.01 \pm 0.19$

TABLE II: Inclusive electron-to-muon cross-section ratios for neutrinos and antineutrinos without kinematic cuts. Uncertainties at leading order are from vector and axial nucleon form factors. For the final result, we include an additional hadronic uncertainty from the one-loop correction to the first uncertainty, and provide a second uncertainty as the magnitude of the radiative correction.

$$\frac{\sigma(m_\ell \rightarrow 0)}{\sigma(m_\ell = 0)} \approx 1 + A m_\ell^2 + \alpha B m_\ell^2 \ln m_\ell$$

- inclusive cross sections and flavor ratios determined by KLN
- nuclear effects: suppressed by expansion parameters squared

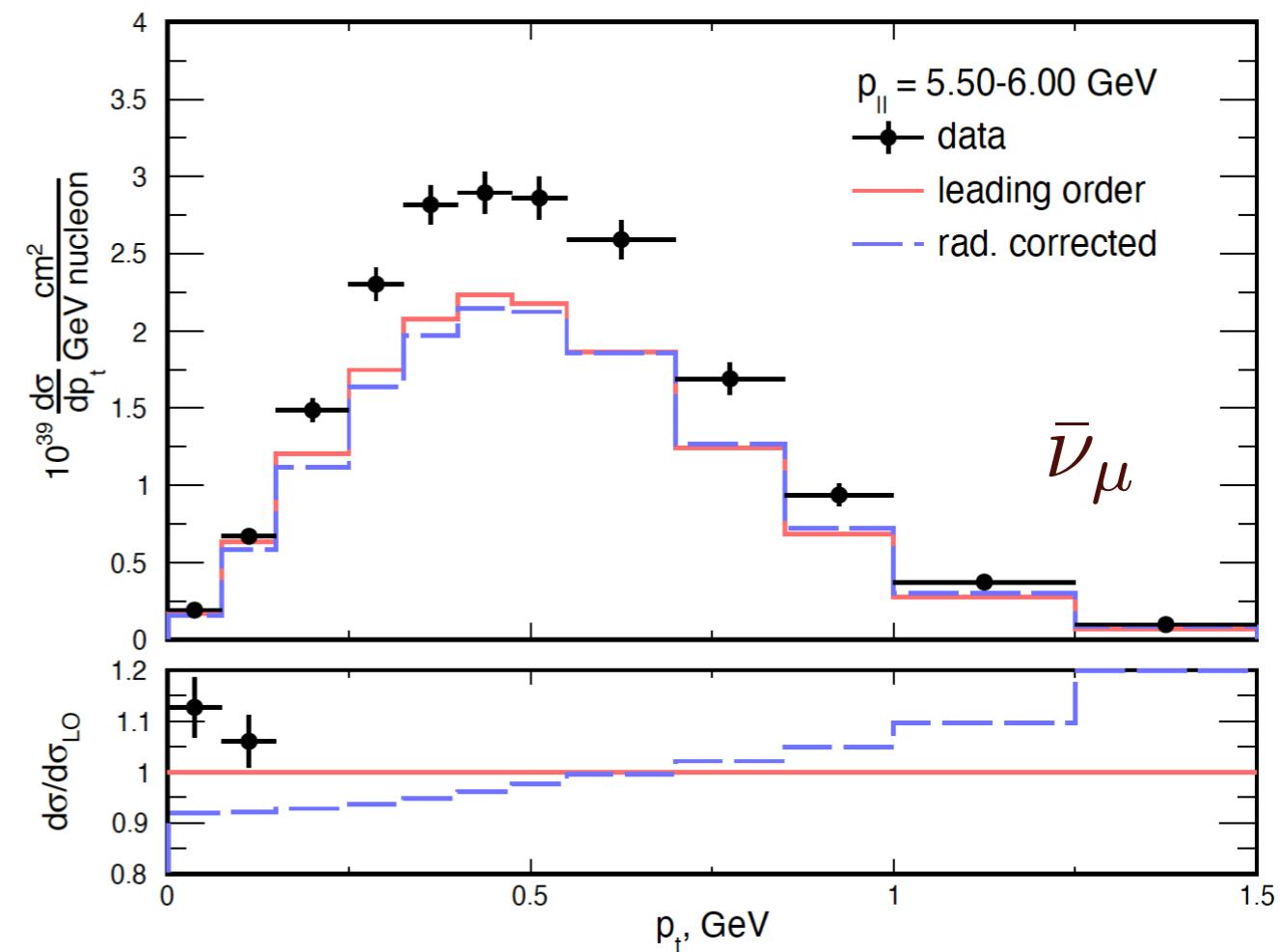
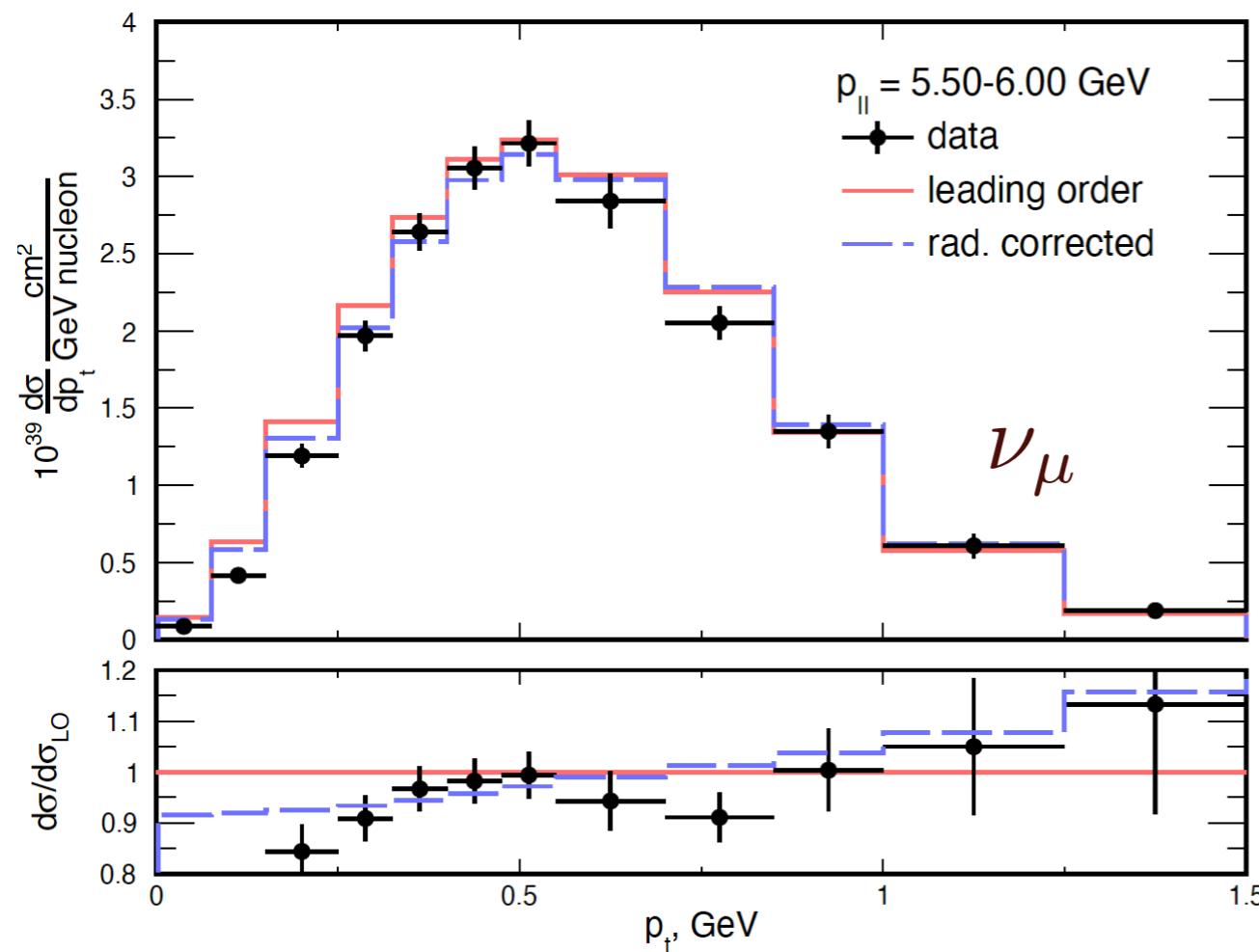
# Comparison to data



- lepton energy spectra with lepton kinematics
- NEUT generator + flux averaging

# Comparison to data

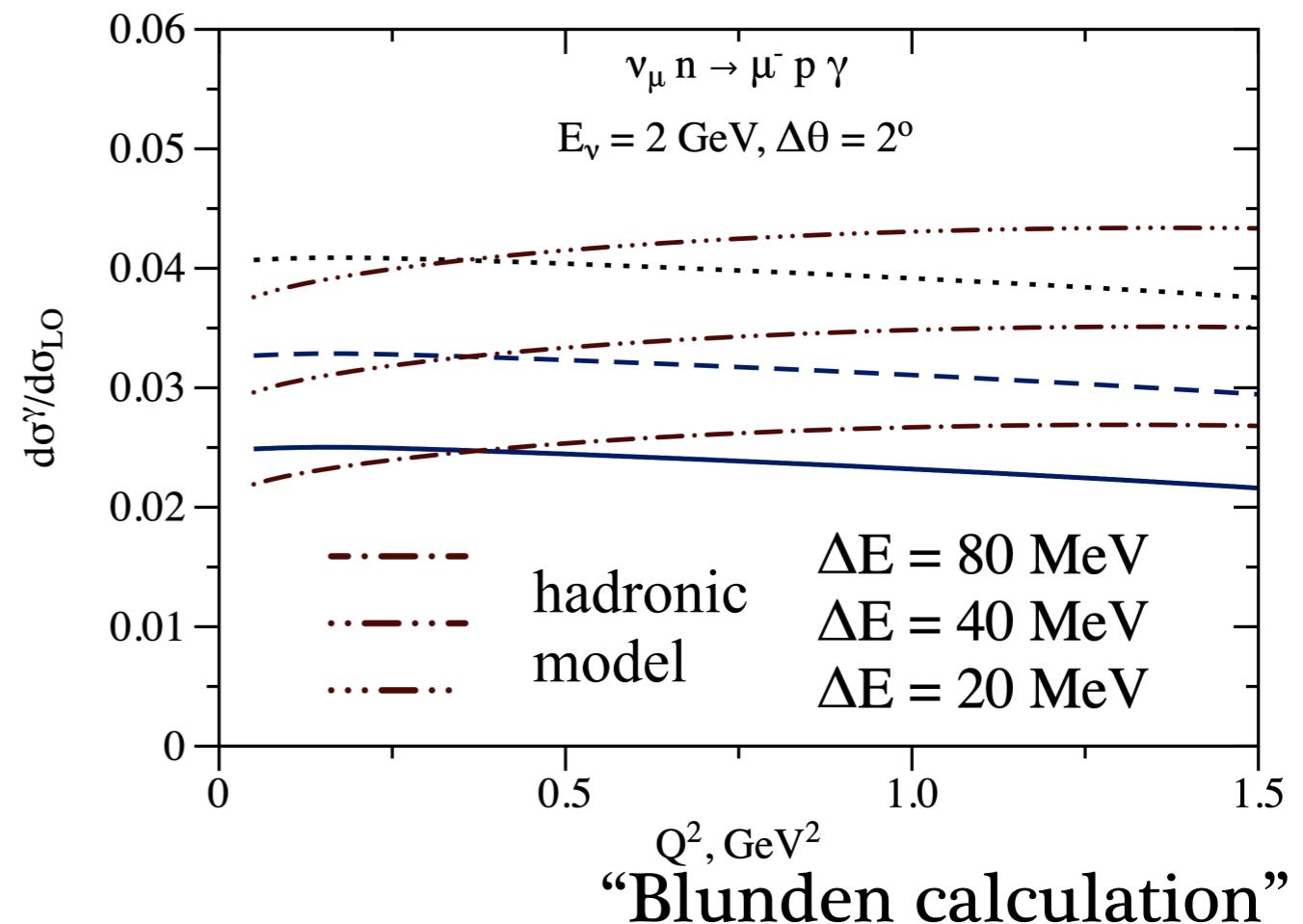
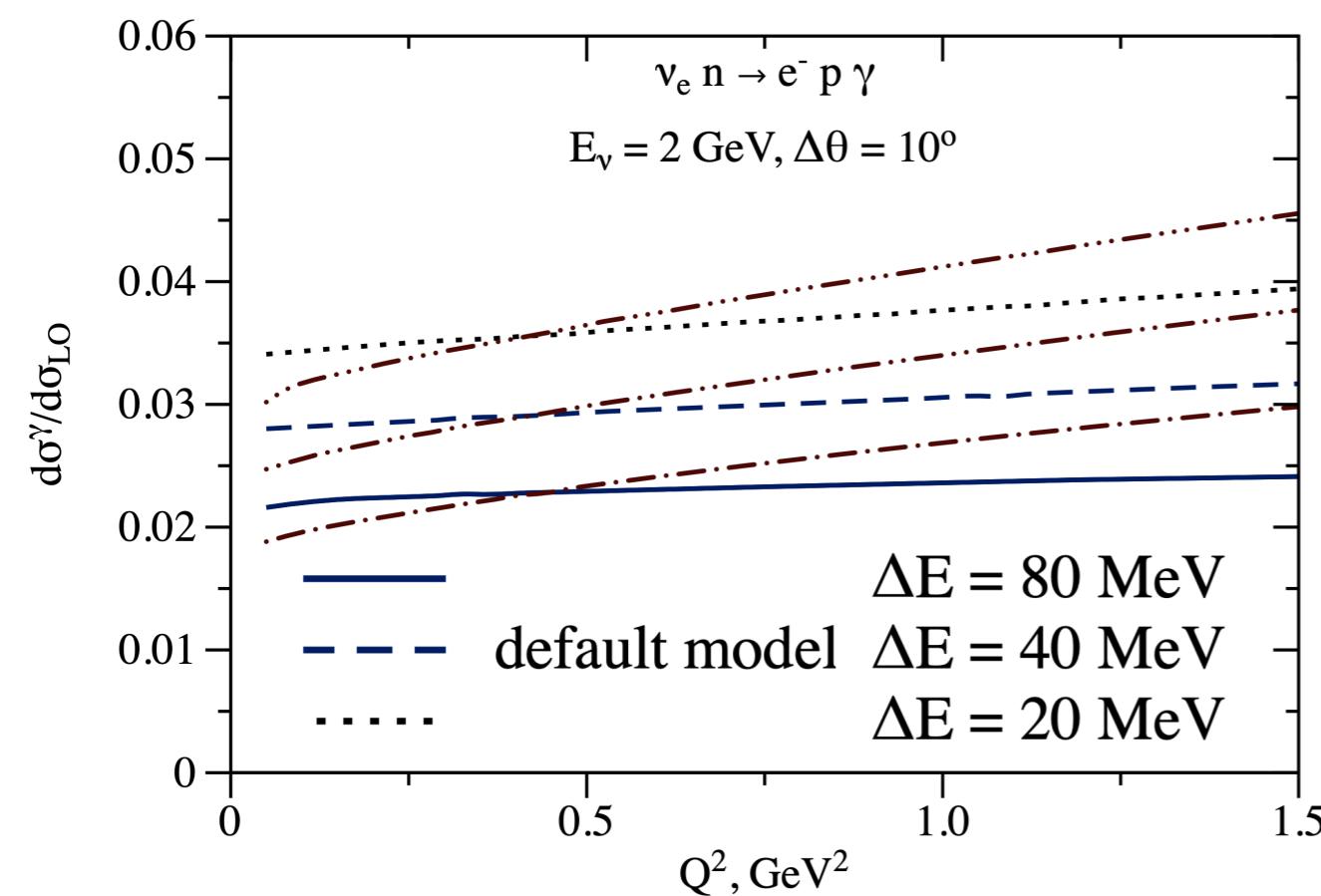
- medium-energy flux data from MINERvA@FERMILAB



- electron flavor: measurements are uncertain
- muon flavor: comparable to experimental precision

# Radiation of hard photons

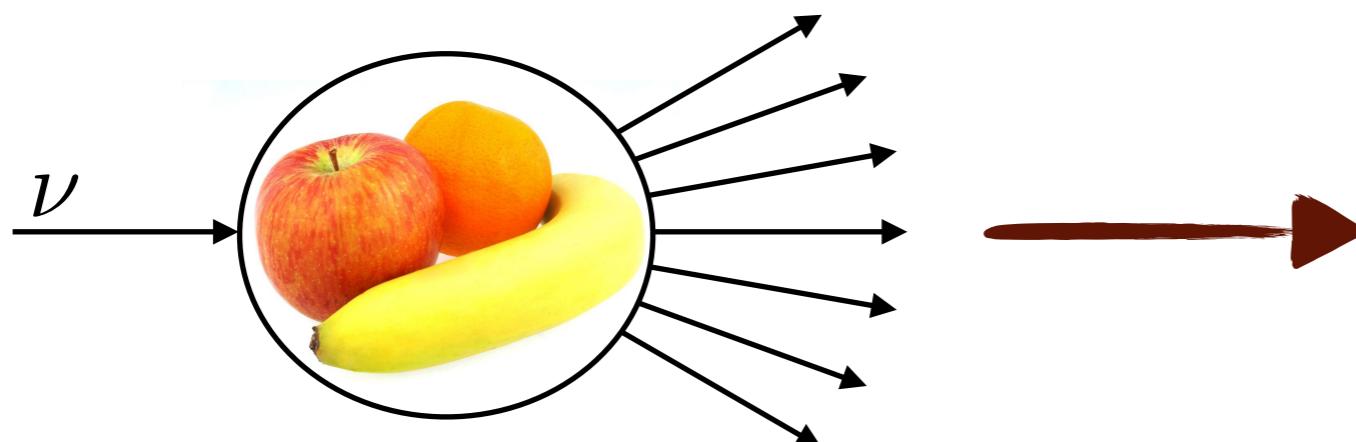
- model-dependent description for radiation of hard photons



- photon energies are above 20, 40, and 80 MeV: default vs “SIFF”  
“hadronic model”

- % -level radiation of non-collinear hard photons
- $10^{-4}$  flavor misidentification rate for NOvA&T2K kinematics

# Conclusions



radiative corrections  
in EFT framework

- radiative corrections to neutrino-nucleon cross sections  
formulated in factorization framework
- charged-current elastic electron vs muon cross-section ratios  
evaluated from theory with sub-percent uncertainty

Thanks for your attention !!!