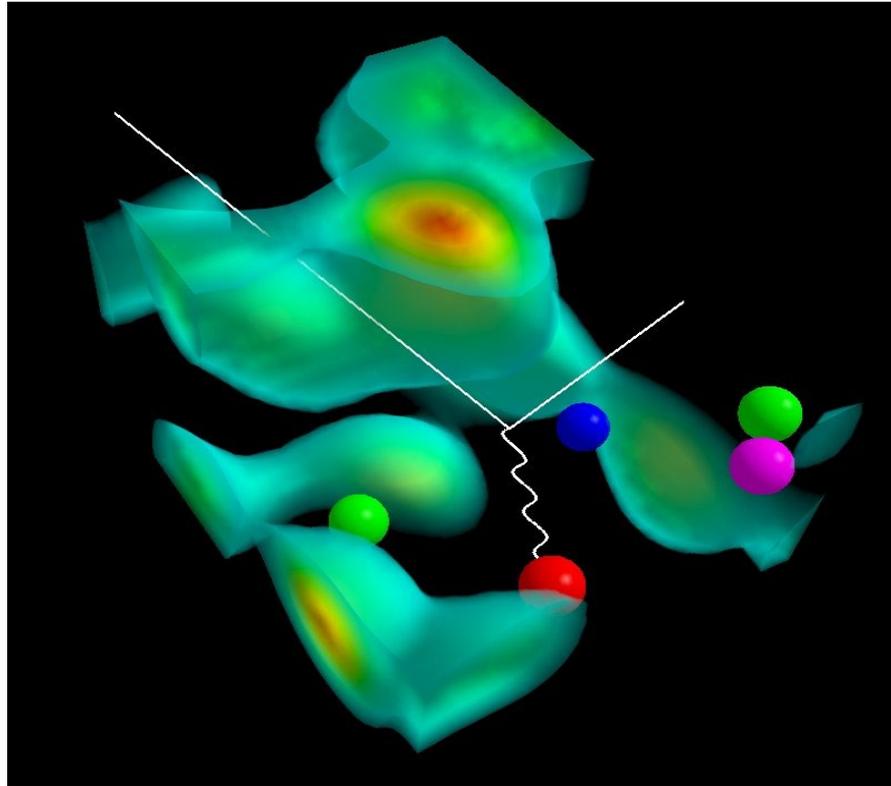


The Structure and Mass of Baryons



Anthony W. Thomas

Origin of the Visible Universe: Unraveling the Proton Mass
INT - 16th June 2022

Outline

I. Excitations of the nucleon are a vital piece of the puzzle

New Insight into the Quark Model

- The $\Lambda(1405)$ IS a $K\bar{N}$ -N bound state
- The Roper IS generated by πN - σN - $\pi\Delta$ rescattering
- The Quark Model is not so bad!

II. Pressure distribution in the proton

Spectroscopy

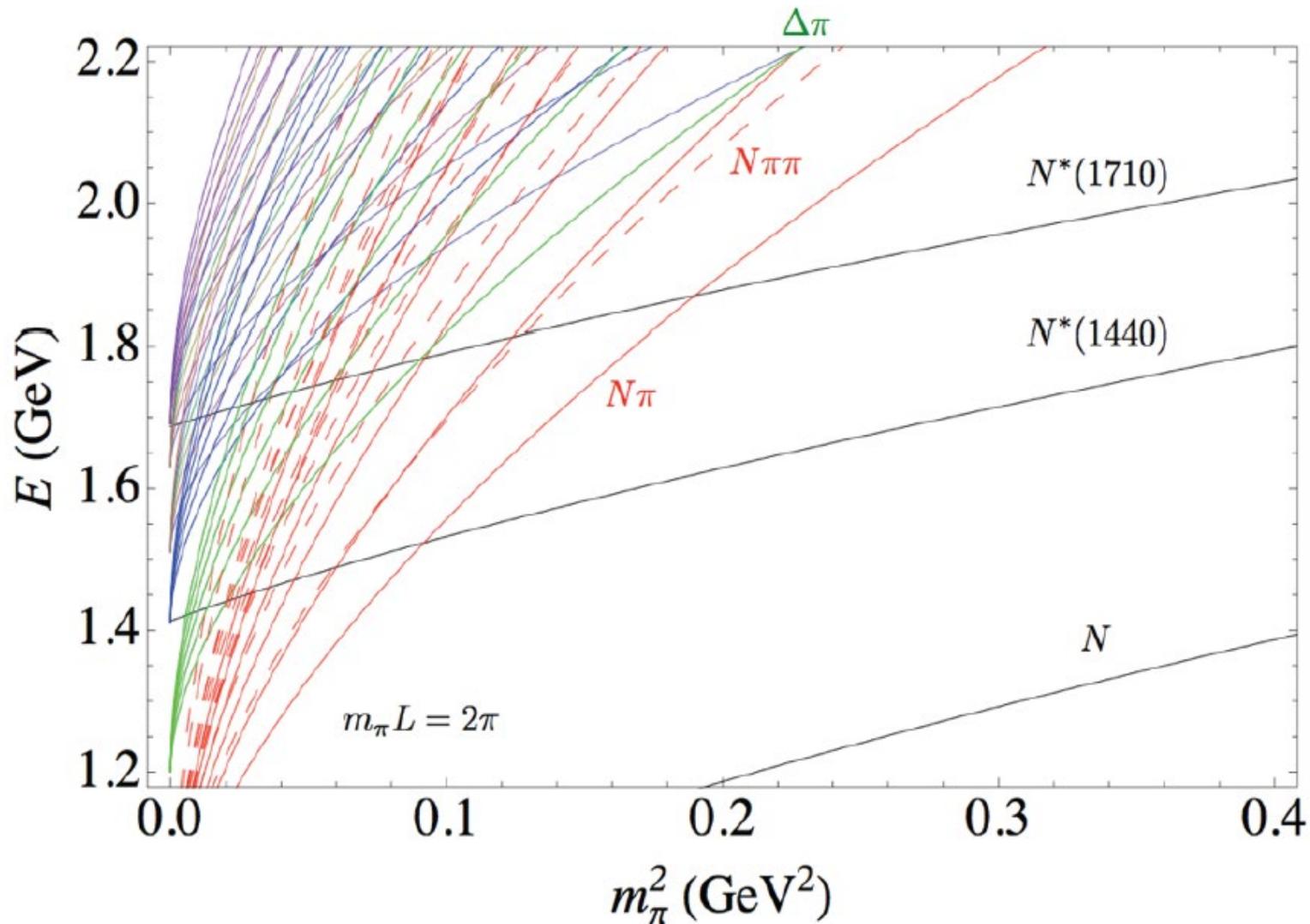
- how do excited states emerge from QCD ?
- what are the fundamental degrees of freedom ?
- Lattice QCD provides extremely valuable information

Resonances are very complicated – and the lattice is not

- **Everything is stable – an eigenstate of the QCD Hamiltonian**
- **Whereas real resonances decay like crazy.....**
- **Lüscher has a method to derive phase shifts at discrete energies when there is one open channel**
- **That approach has been generalized to coupled channels by Hansen and Sharpe (Phys.Rev. D86 (2012) 016007) and Lellouch and Lüscher (Comm.Math.Phys., 219 (2011) 31) BUT it becomes very complicated**

Interesting cases have many open channels
– at least at realistic quark masses

In General: Multiple open channels



and then there is: σN , ωN , ρN etc....

The $\Lambda(1405)$

- We have unambiguous evidence that it is a $K\bar{N}$ bound state! 50 years after speculation by Dalitz *et al.*
- To be fair Dalitz had no quark model then so there was not much else it could be at that time.
- Rather than the Lüscher method we apply **Hamiltonian Effective Field Theory**
 - shown to be equivalent for phase shifts*
 - **BUT also provides information on eigenstates**
- Carry out a Hamiltonian analysis of lattice data
- Examine the **strange magnetic form factor** of $\Lambda(1405)$

* Wu *et al.*, Phys. Rev. C 90 (2014) 5, 055206

First calculation after QCD incorporating chiral symmetry

PHYSICAL REVIEW D

VOLUME 31, NUMBER 5

1 MARCH 1985

S-wave meson-nucleon scattering in an SU(3) cloudy bag model

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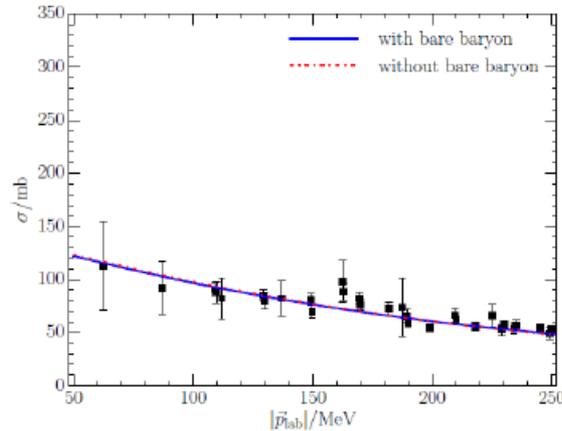
(Received 8 June 1984)

The cloudy bag model (CBM) is extended to incorporate chiral $SU(3) \times SU(3)$ symmetry, in order to describe *S*-wave KN and $\bar{K}N$ scattering. In spite of the large mass of the kaon, the model yields reasonable results once the physical masses of the mesons are used. We use that version of the CBM in which the mesons couple to the quarks with an axial-vector coupling throughout the bag volume. This version also has a meson-quark contact interaction with the same spin-flavor structure as the exchange of the octet of vector mesons. The present model strongly supports the contention that the $\Lambda^*(1405)$ is a $\bar{K}N$ bound state.

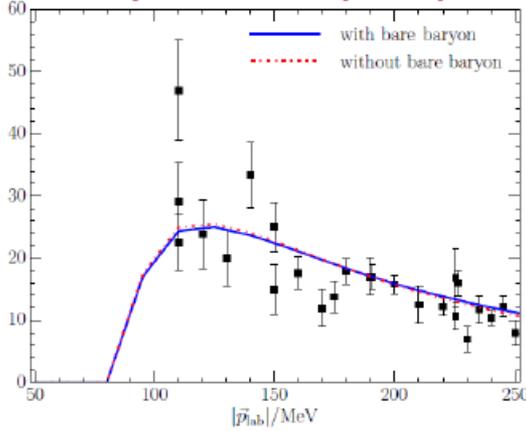
But now we can use QCD itself

Hamiltonian fit to existing data

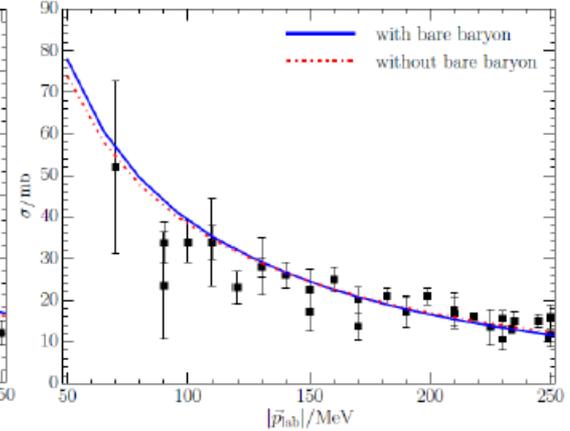
Zhan-wei Liu etc. Phys.Rev. D95 (2017) no.1, 014506



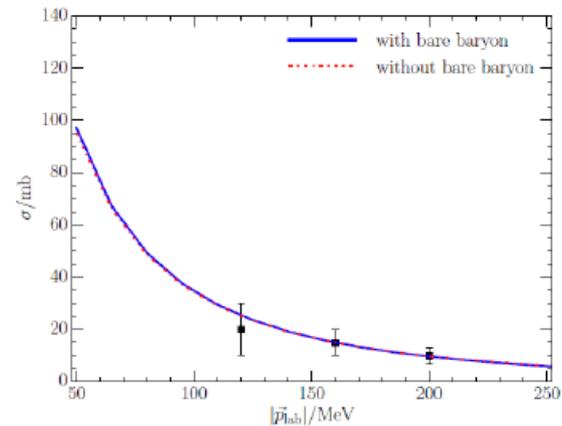
(a) $K^-p \rightarrow K^-p$



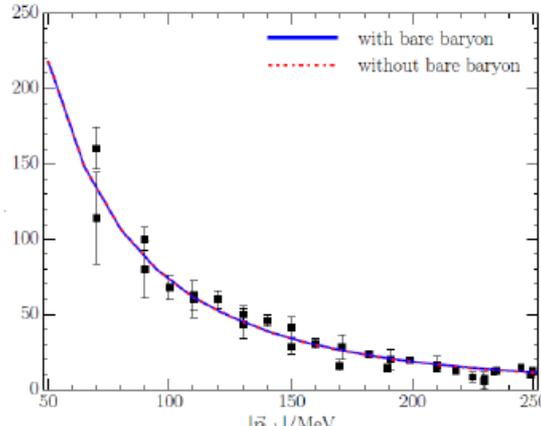
(b) $K^-p \rightarrow \bar{K}^0 n$



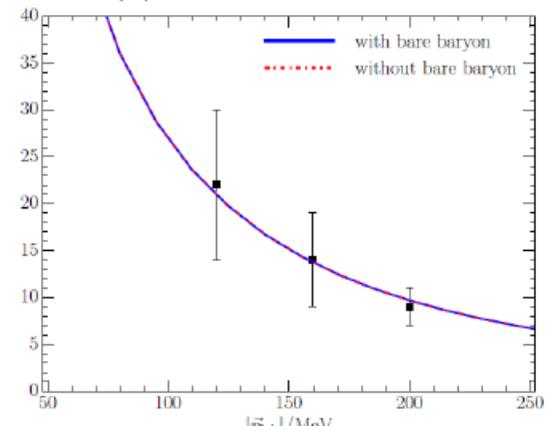
(c) $K^-p \rightarrow \pi^- \Sigma^+$



(d) $K^-p \rightarrow \pi^0 \Sigma^0$



(e) $K^-p \rightarrow \pi^+ \Sigma^-$



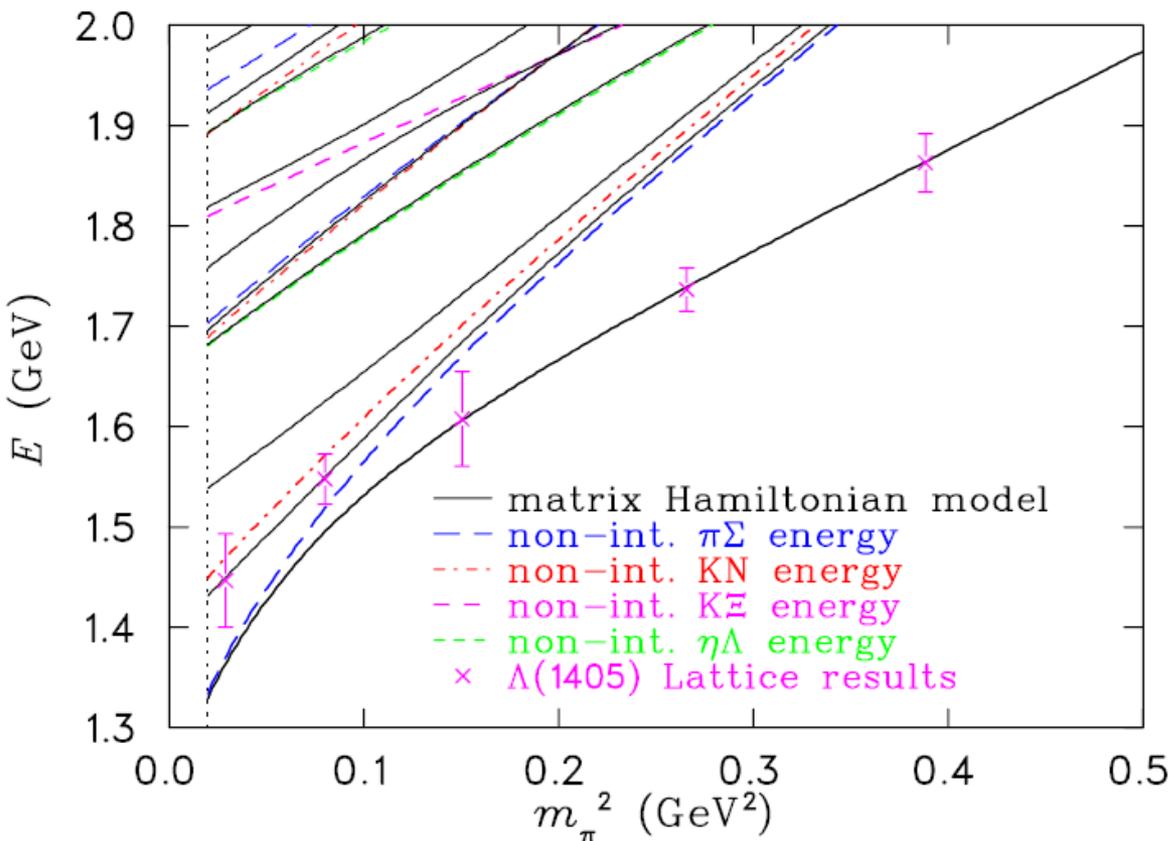
(f) $K^-p \rightarrow \pi^0 \Lambda$

Include $\pi\Sigma$, $K\bar{n}$, $\eta\Lambda$ and $K\Xi$ channels

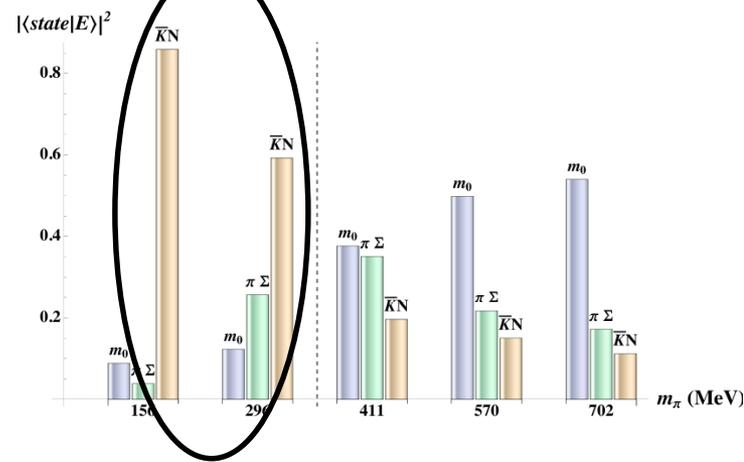
Similar work by Valencia, Bonn, JLab and other groups

Low lying negative parity state : $\Lambda(1405)$

Clear evidence that it is a $\bar{K}N$ bound state



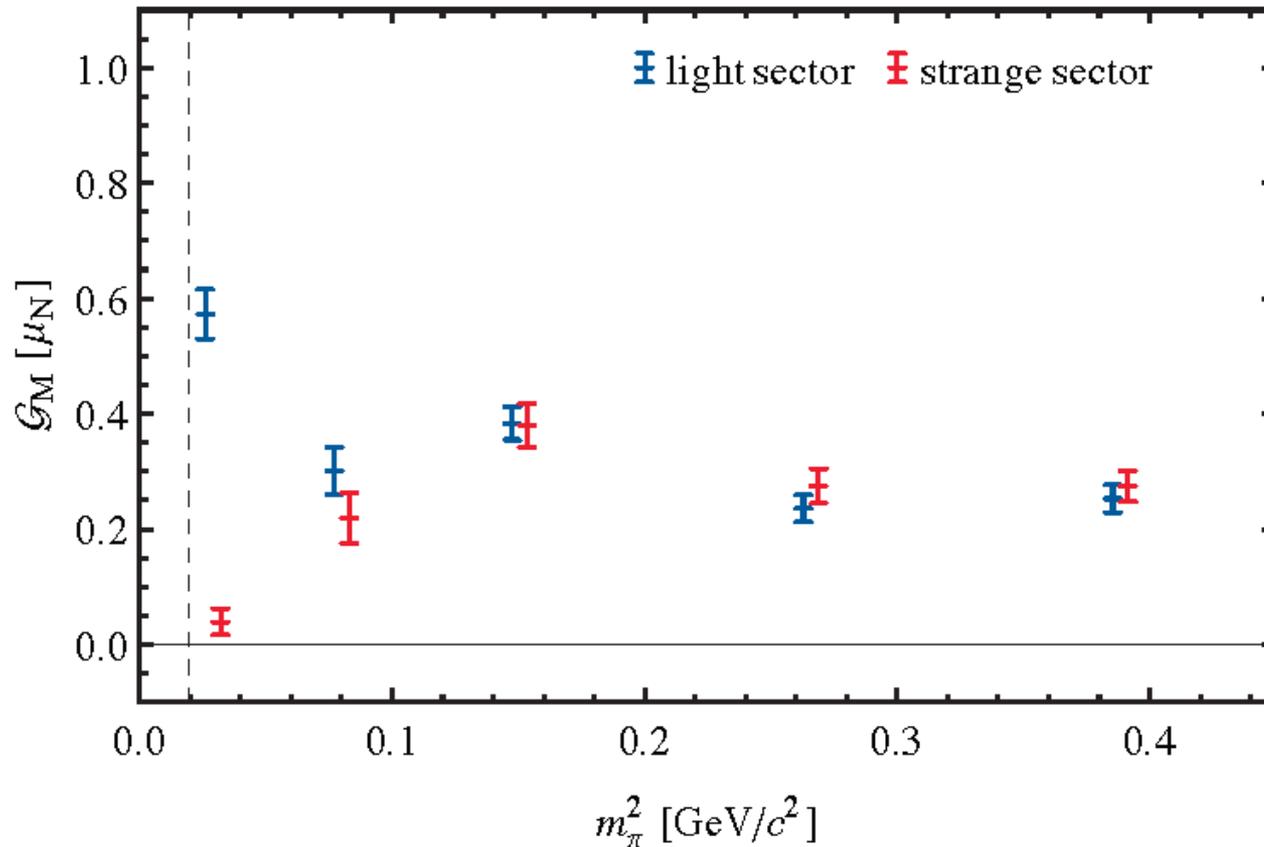
Hamiltonian approach allows one to examine the eigenstates:



Hall, Leinweber, Menadue, Young, AWT
 – Phys. Rev. Lett. 114 (2015) 13

Lattice Magnetic Form Factor Calculations

- Calculation of the individual quark contributions to the magnetic form factor confirms that it is a $K\bar{b}$ -N bound state



Only an $L=0$ $K\bar{b}$ -N state gives vanishing strange moment

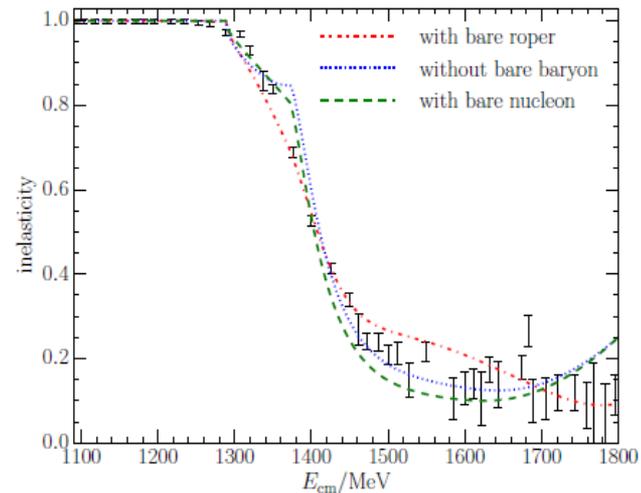
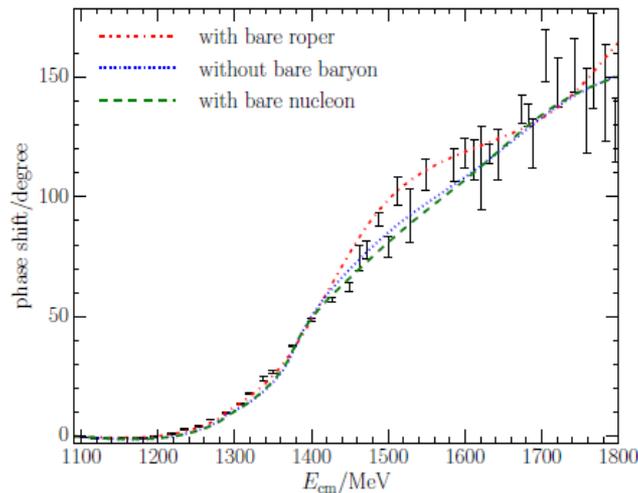
**Note that Lattice QCD allows us
to study hadron structure IN QCD as a
function of quark mass – a powerful tool**

Roper Resonance

Again this has long been a challenge for the quark model, as it is the 1st positive parity excited state and lies below the N(1535), the 1st negative parity state

Bare Roper Case: $m_0 = 2.03$ GeV

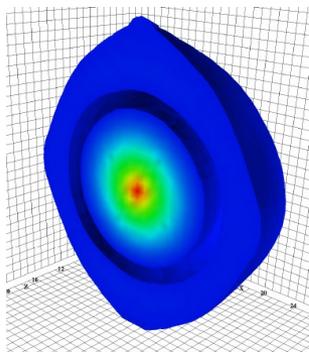
- Consider πN , $\pi\Delta$ and σN channels, dressing a bare state.
- Fit to phase shift and inelasticity



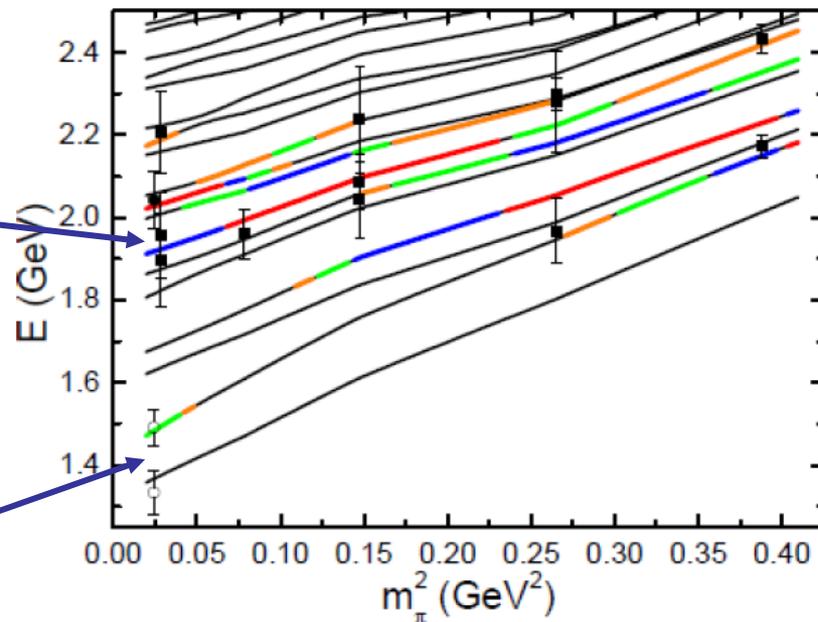
- Fit yields a pole at $1380 - i87$ MeV.
- Compare PDG estimate $1365 \pm 15 - i95 \pm 15$ MeV.

Comparison of HEFT Results with Lattice Energy Levels

- Blue indicates high “bare state” (i.e. 3-quark) content. This matches the lowest state found with a 3-quark interpolating field



- Lattice calculations of Lang et al., Phys. Rev. D 95, 014510 (2017), using baryon-meson interpolating fields, especially $N\sigma$
- Matched by Hamiltonian levels but with little or no 3-quark content



The first scenario with a bare state for P11 around the pole at 2.0 GeV can fit both Lattice data and experimental data well, it indicates that $N^*(1440)$ seems a molecule state, and first radial excitation of nucleon should be around 2.0 GeV.

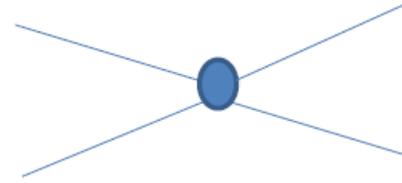
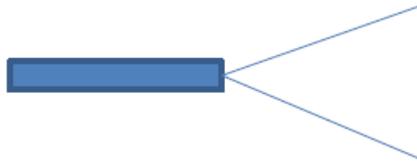
Wu et al., Phys. Rev. D 97 (2018) 9, 094509

**Clear conclusion is that the Roper is
dynamically generated by coupling
to the $N\sigma$ and $\Delta\pi$ channels**

N(1535) is a 3-quark state

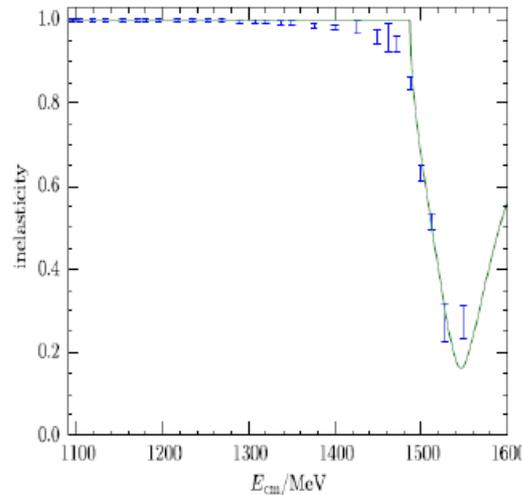
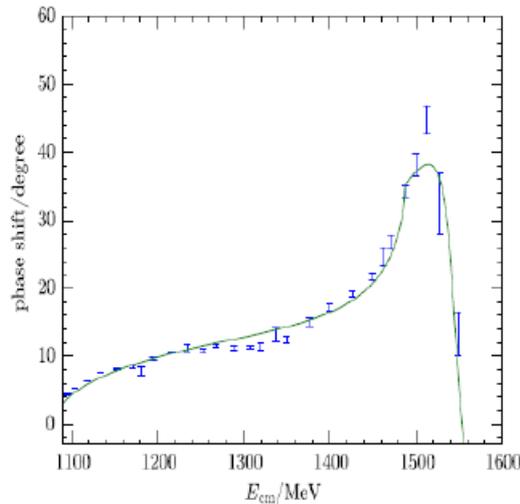
Zhan-wei Liu et al. Phys.Rev.Lett. 116 (2016) no.8, 082004

- 2 Channels: πN and ηN



$$G_{iN}^2(k) = \left(3g_{N_0^*iN}^2 / 4\pi^2 f^2 \right) \omega_i(k) u^2(k)$$

$$\frac{3g_{\pi N}^S \tilde{u}(k) \tilde{u}(k')}{4\pi^2 f^2}$$



$$g_{\pi N}^S = -0.0608 \pm 0.0004$$

$$m_0 = 1601 \pm 14 \text{ MeV}$$

$$g_{N_0^* \pi N} = 0.186 \pm 0.006$$

$$g_{N_0^* \eta N} = 0.185 \pm 0.017,$$

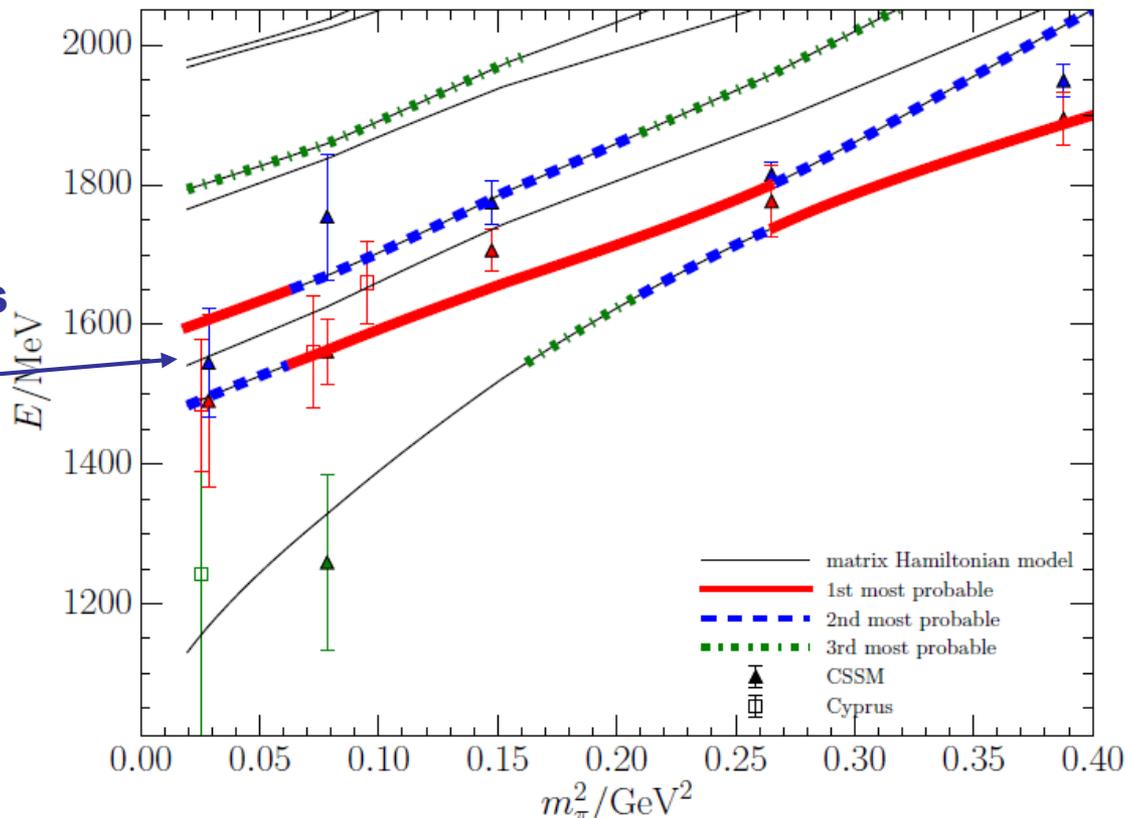
$$\chi_{\text{DOF}}^2 = 6.8$$

$$1531 \pm 29 - i88 \pm 2 \text{ MeV}$$

N(1535) is a 3-quark state

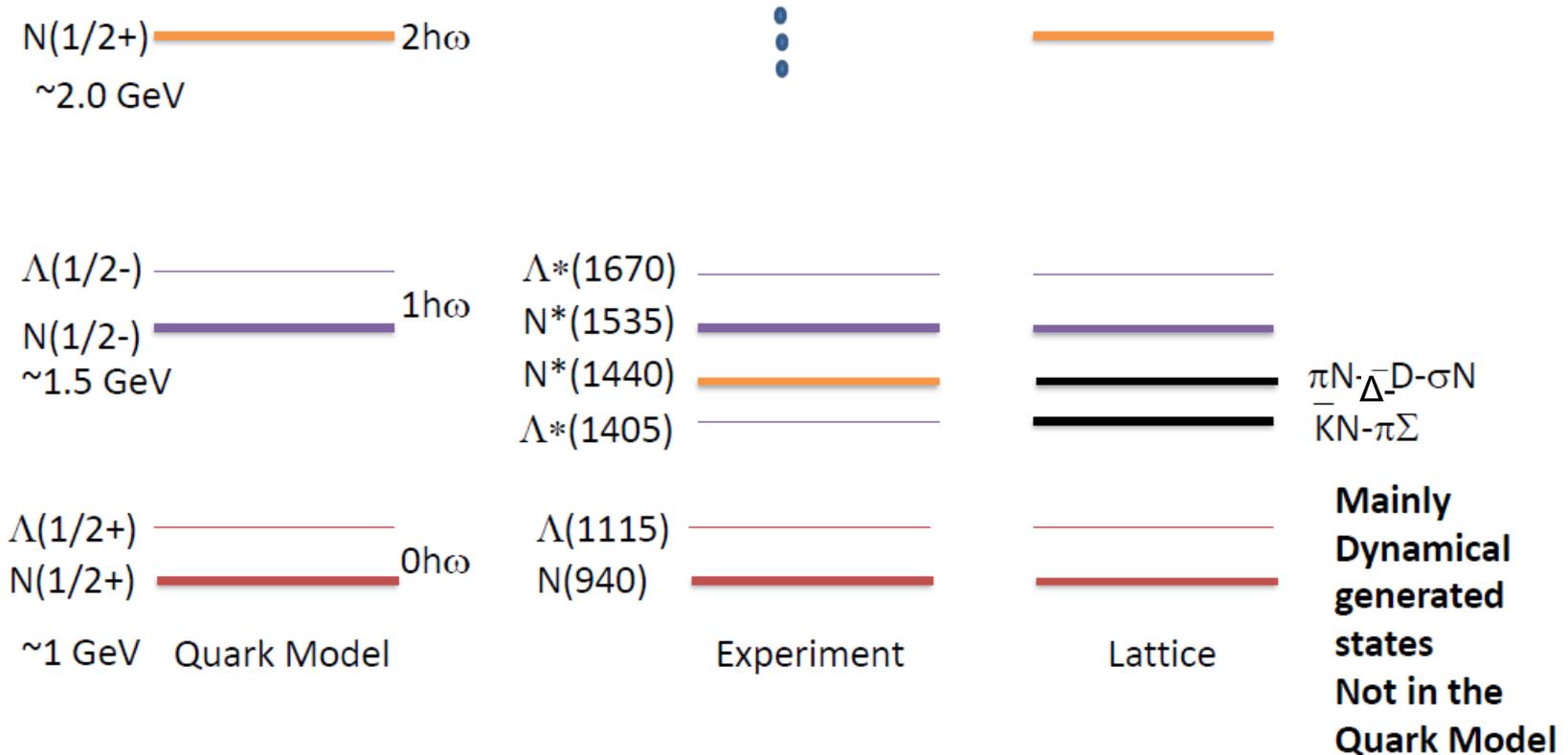
Hamiltonian Model N^* Spectrum: 3 fm

Hamiltonian eigenstates dominated by 3-quark state match the lattice result with 3-quark interpolating field



Liu *et al.*, PRL 116 (2016) 082004

Once the nature of key states becomes clear the quark model makes sense



Pressure and Shear in the Proton

Theoretical developments

There has been a long history of study of the energy-momentum tensor of the proton

Starting with Pagels more than 50 years ago there has been much more work recently by amongst others, Cloët, Ji, Lorcé, Miller, Polyakov.. and their collaborators

Most recently the experimental work at Jefferson Lab produced the first data on the distribution of quark pressure and shear inside the proton

JLab result: Nature May 2018

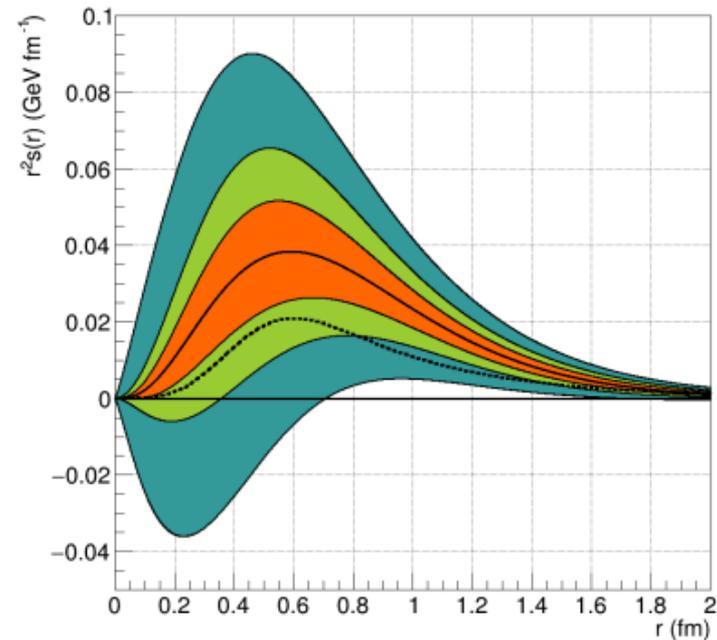
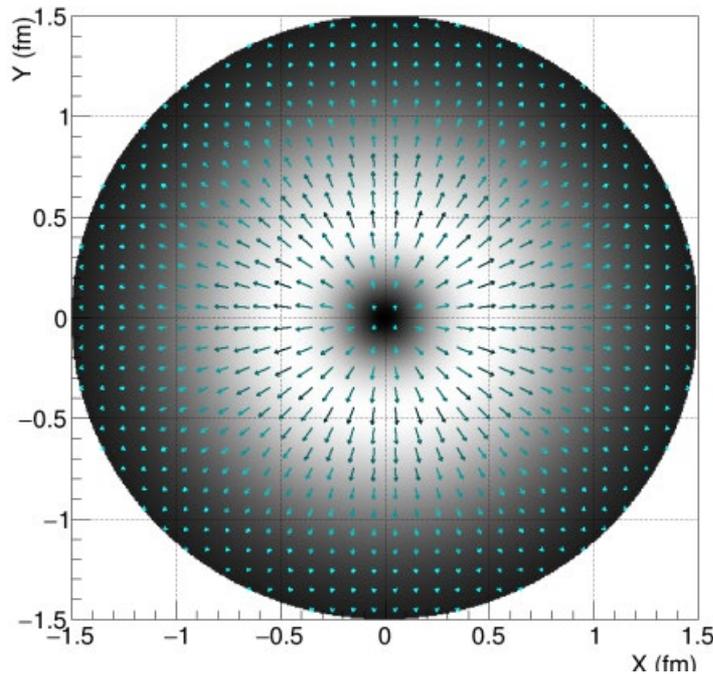
and arXiv: 2104.02031

LETTER

<https://doi.org/10.1038/s41586-018-0060-z>

The pressure distribution inside the proton

V. D. Burkert^{1*}, L. Elouadrhiri¹ & F. X. Girod¹



Lattice studies

Pioneering work on the pion in the PhD thesis at Regensburg in 2007 by Dirk Brömmel

– crucial for our work

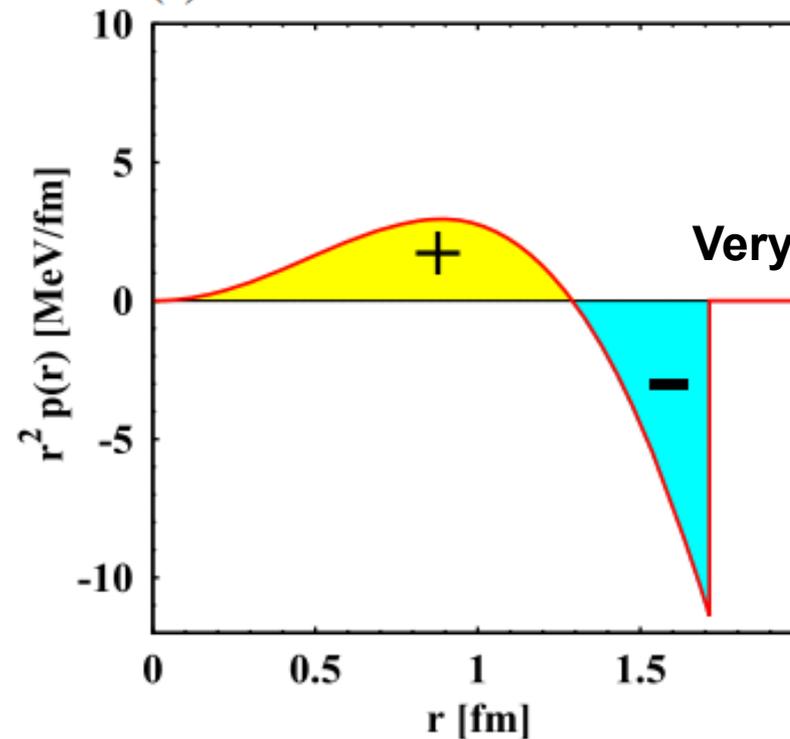
Recent calculations at 450 MeV pion mass by Shanahan and Detmold for the proton (Phys. Rev. Lett. 122 (2019) 7, 072003) and by Pefkou, Hackett and Shanahan for the gluon contributions in a variety of hadrons (Phys. Rev. D 105 (2022) 5, 054509)

... follows earlier work by Yang, Syritsyn, Bali, Hägler, Bratt, Sternbeck and their collaborators

MIT Bag Model study

$$p(r) = \left[\frac{N_q N^2}{12\pi} \alpha_+ \alpha_- \left(j_0 j_1' - j_0' j_1 + \frac{2j_0 j_1}{r} \right) - B \right] \theta_V$$

$$s(r) = \left[\frac{N_q N^2}{4\pi} \alpha_+ \alpha_- \left(j_0 j_1' - j_0' j_1 - \frac{j_0 j_1}{r} \right) \right] \theta_V.$$



Very large bag radius

Neubelt et al., Phys. Rev. D101 (2020) 034013

Cloudy Bag Model

With sufficiently large confinement radius a perturbative treatment of the pion cloud is proven

$$|\tilde{N}\rangle = \sqrt{Z} (|N\rangle + |N\pi\rangle + |\Delta\pi\rangle + \text{higher order terms})$$

In the Breit frame the $|N\pi\rangle$ component yields:

$$\begin{aligned} & \langle N(\vec{\Delta}/2), s | T^{ij}(0) | N(-\vec{\Delta}/2), s \rangle_{\pi N} \\ &= \frac{3g_A^2}{32\pi^3 f_\pi^2} \int d^3\vec{k} \frac{E_{|\vec{k}+\vec{\Delta}/2} u(k) u(|\vec{k} + \vec{\Delta}|) (k^2 + \vec{k} \cdot \vec{\Delta})}{2\omega_k^2 \omega_{|\vec{k}+\vec{\Delta}|}^2} \\ & \langle \pi(\vec{k} + \vec{\Delta}) | T^{ij}(0) | \pi(\vec{k}) \rangle \end{aligned}$$

and we use:

$$\begin{aligned} & \langle \pi(\vec{\Delta}/2) | T_{q,g}^{ij}(0) | \pi(-\vec{\Delta}/2) \rangle \\ &= 2K^i K^j A_{q,g}(t) + 2m_\pi^2 \bar{c}_{q,g}^\pi(t) + \frac{1}{2} (\Delta^i \Delta^j - g^{ij} \Delta^2) D_{q,g}^\pi(t) \end{aligned}$$

CBM continued...

This leads to: $D_{q,g}^{\pi N}(t) = D_{\pi/N}(t) D_{q,g}^{\pi}(t)$

where

$$D_{\pi/N} = \frac{3g_A^2}{32\pi^2 f_\pi^2} \int_0^\infty dk k^2 \frac{u(k)}{k^2 + m_\pi^2}$$

$$\int_{-1}^{+1} dx \frac{u(y) \sqrt{m_N^2 + z(k^2 + k\Delta x)}}{y^2 + m_\pi^2}$$

with a similar result for the $|\Delta\pi\rangle$ component.

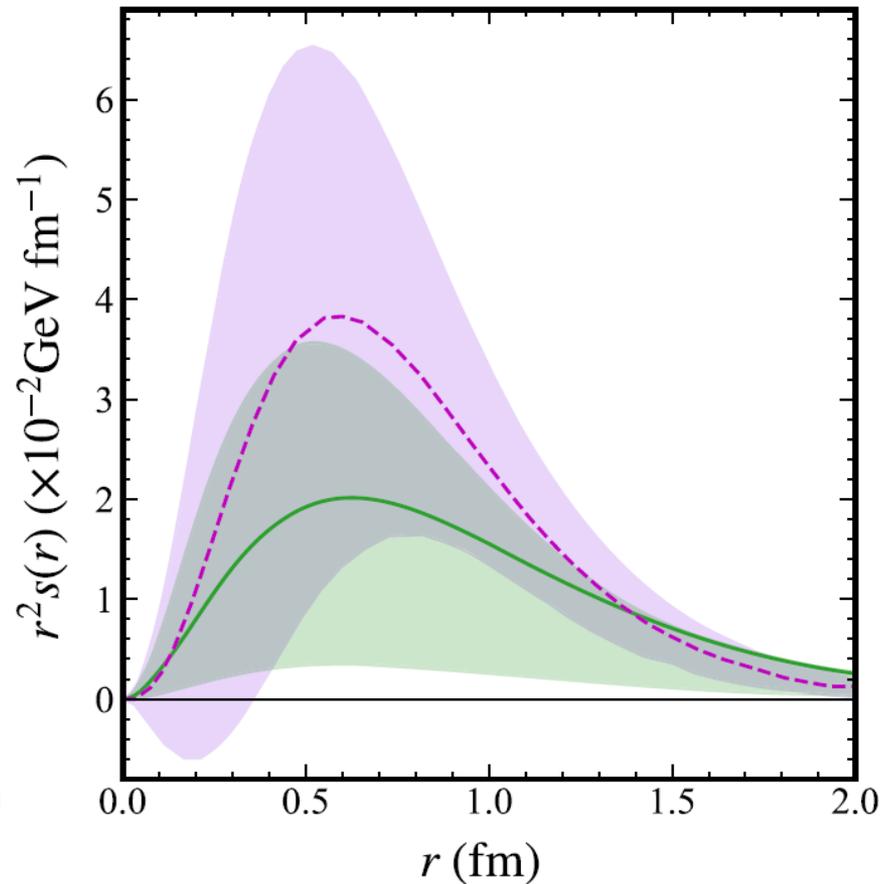
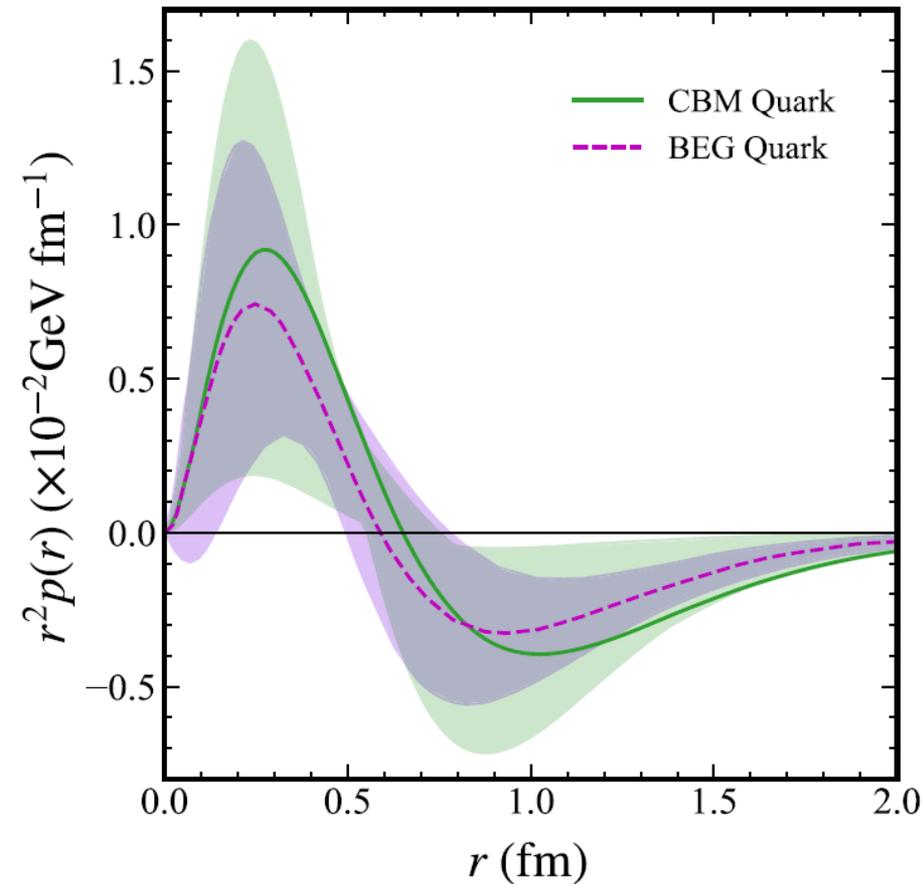
Finally, using $\tilde{D}_{q,g}(r) = \int \frac{d^3\vec{\Delta}}{2E_{\Delta/2}(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} D_{q,g}(-\vec{\Delta}^2)$

we obtain distribution of pressure and shear

$$s(r) = -\frac{1}{2} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}(r)$$

$$p(r) = \frac{1}{3} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}(r)$$

Comparison of Cloudy Bag Model with JLab Data



Comments

Major uncertainty in the present calculation is the old calculation of the pion D-function.

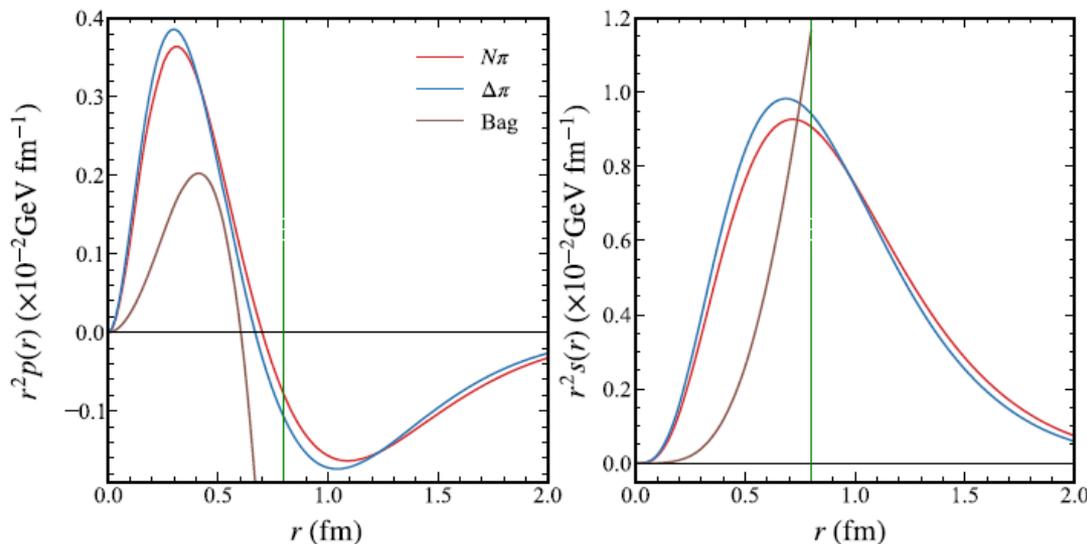
It involves the extrapolation (using formulas given by Brömmel) of a relatively old lattice calculation.

The published letter presents parametrizations of the functions $D_{\pi/N}$ and $D_{\pi/\Delta}$ so that more accurate curves can be generated when modern pion results are presented

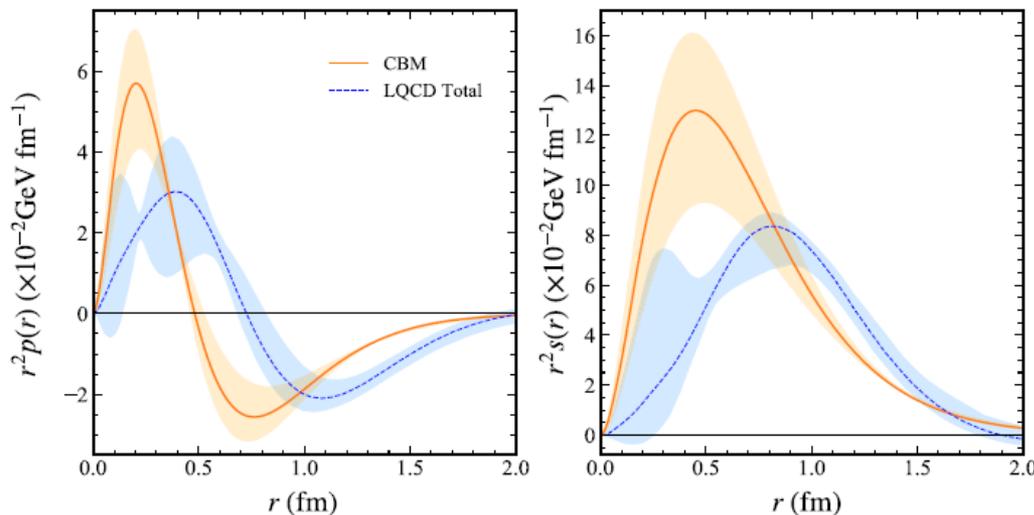
Summary

- **New techniques applied to lattice QCD provide hitherto unimagined insights into hadron structure**
- **The quark model has new life**
- **There has been remarkable experimental progress in the determination of the pressure and shear distribution in the proton, along with new developments in lattice QCD**
- **The pion cloud of the nucleon plays a key role in understanding the experimental data**

Back-up



Separate πN and $\pi\Delta$ contributions at physical pion mass



Results at 450 MeV compared with Pefkou et al.

