Perturbative calculations of few-nucleon observables in χEFT up to N^3LO

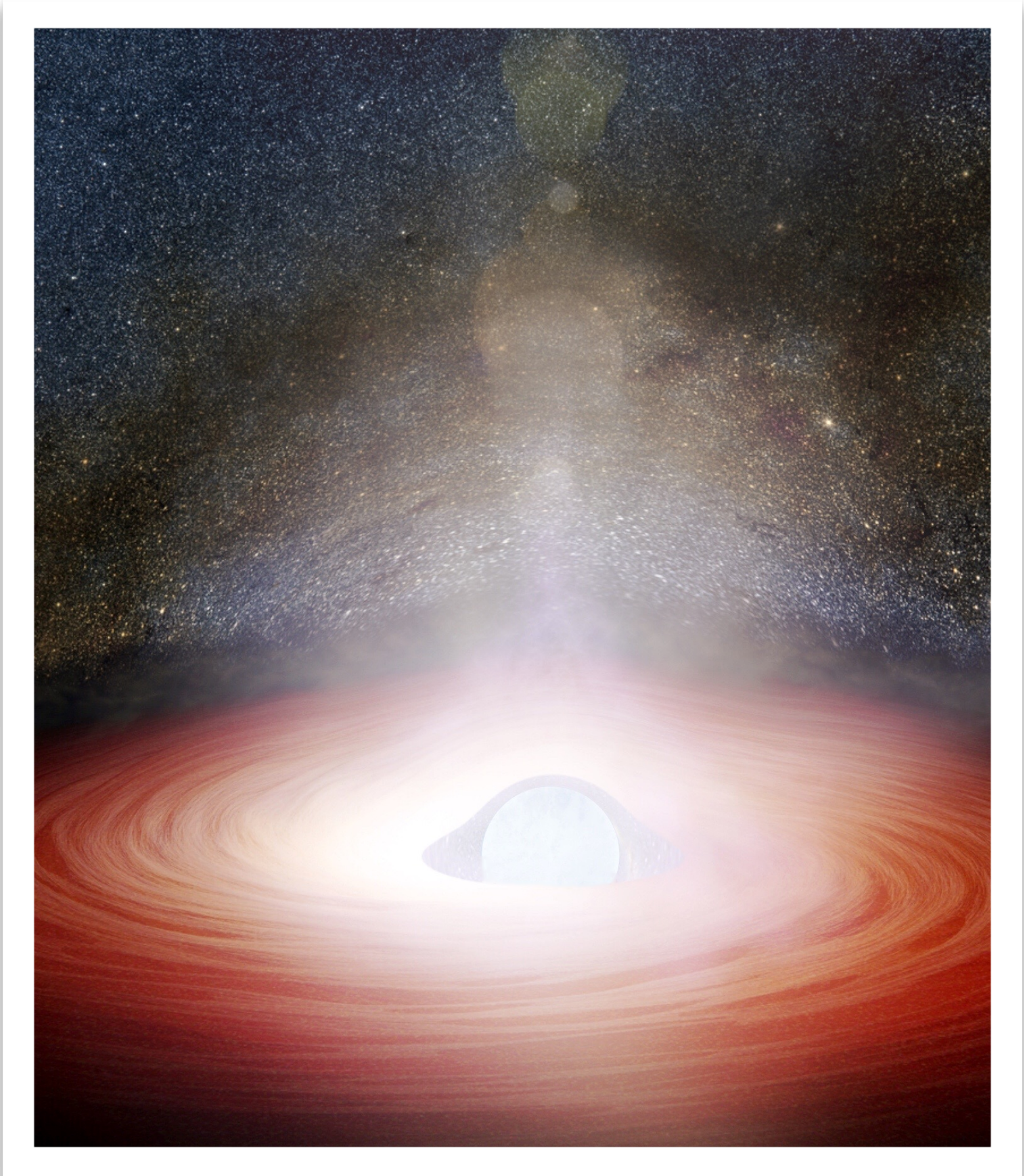
INT Seattle, 21 May 2026

Nuclear Hamiltonians for Advancing Nuclear Physics and Beyond

Swedish Research Council

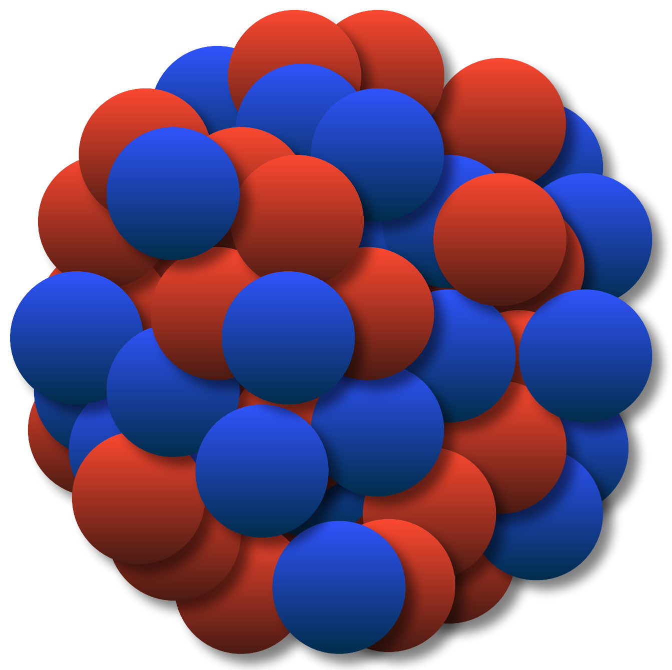
Oliver Thim | Theoretical Subatomic Physics | Chalmers University of Technology

The atomic nucleus

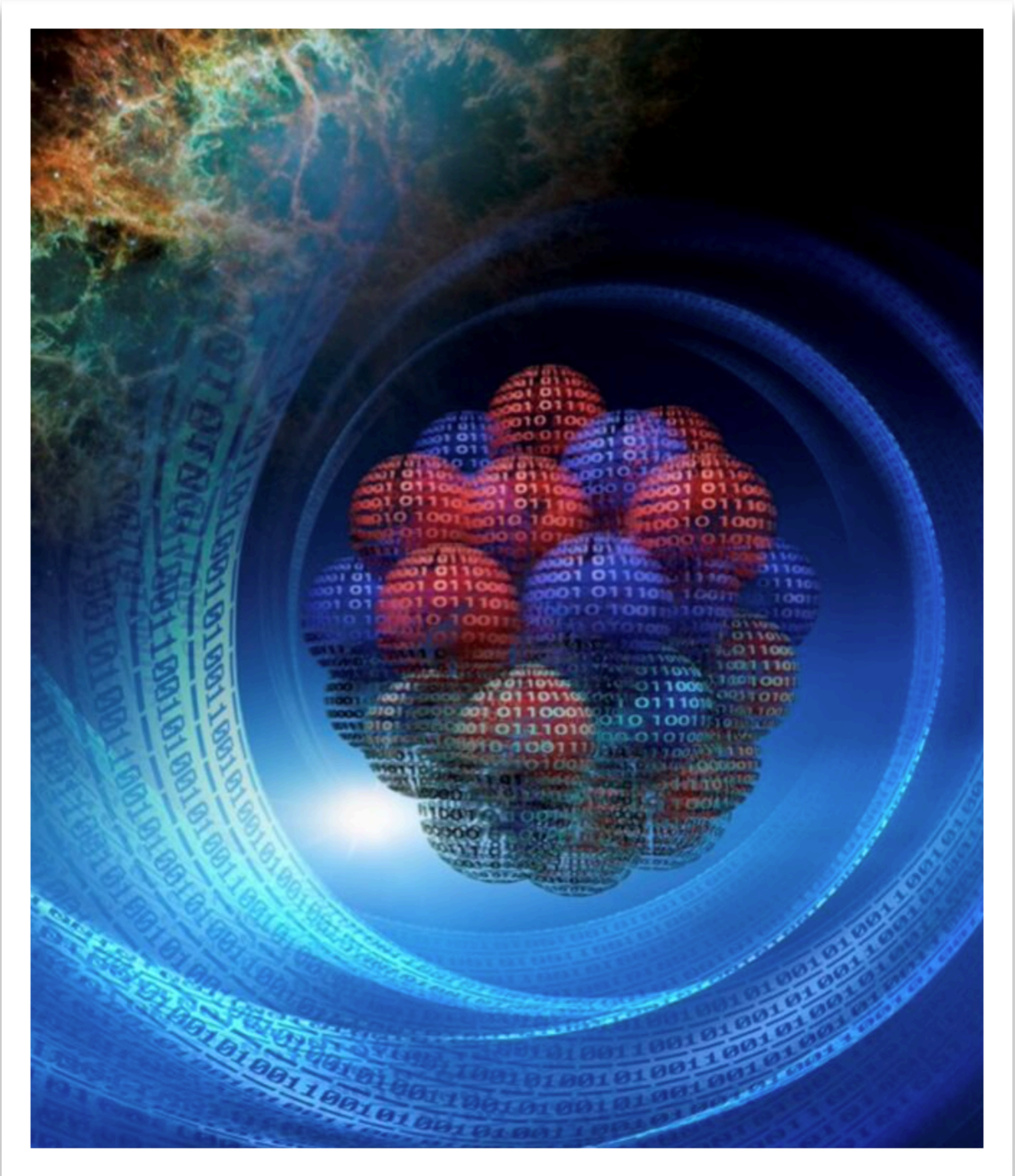


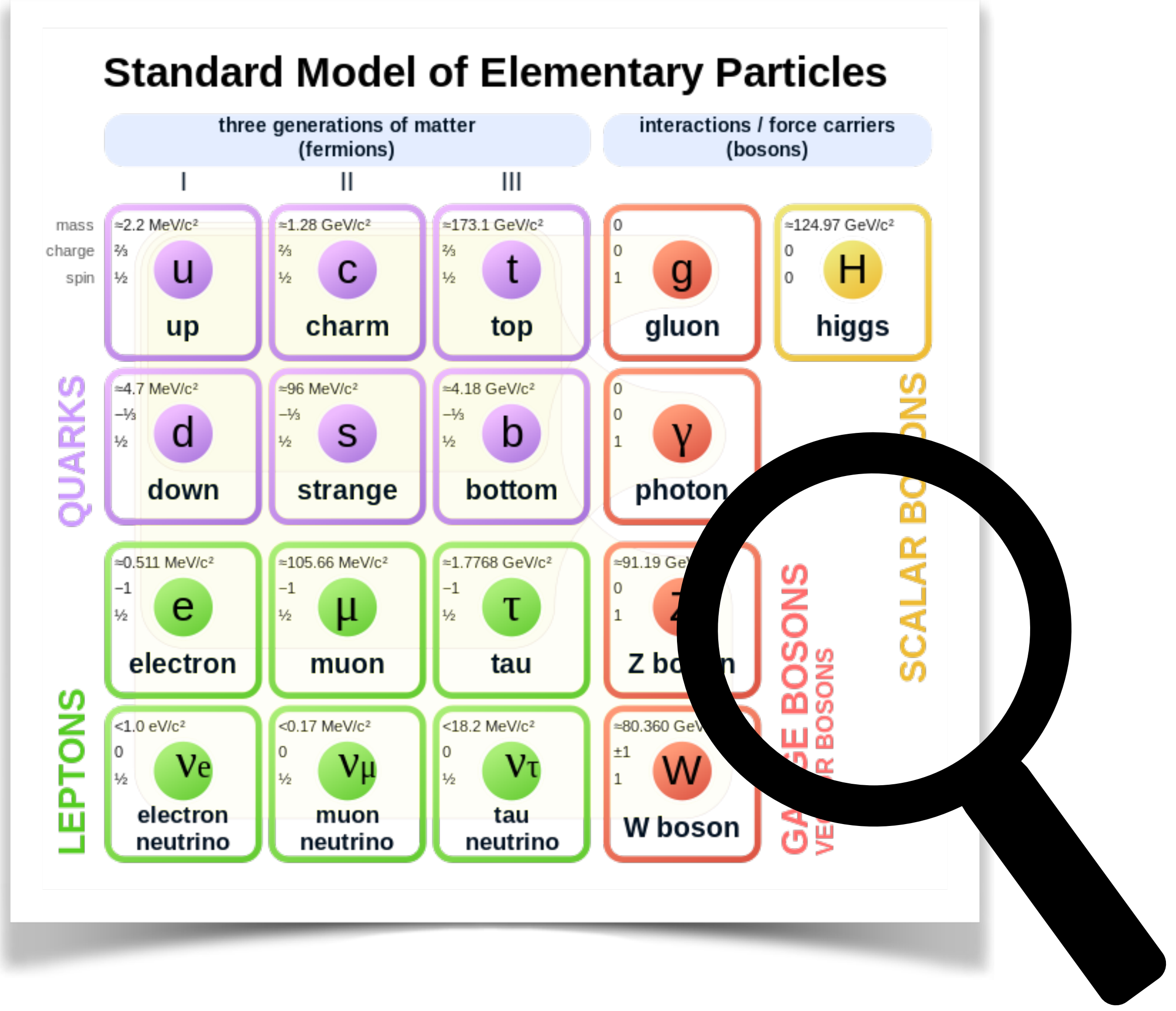
$\sim 10^{4}$ m

$\sim 10^{-15}$ m

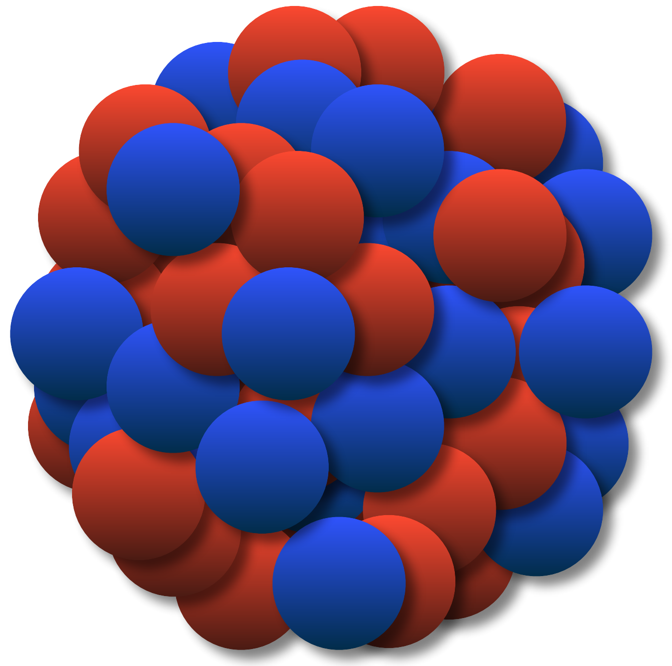


$$H\,|\psi\rangle = E\,|\psi\rangle$$





Ab initio nuclear theory



$$H|\psi\rangle = E|\psi\rangle$$

Goal

Simultaneously describe nuclear phenomena across a wide range of energies and mass numbers, starting from nuclear forces with roots in QCD.

Key challenges

- Construct H to keep the connection to QCD.
- Compute precise predictions for nuclear observables.

An ab initio approach:

Part I

$\mathscr{L}$ $(+\delta\mathscr{L})$

QCD ⇒ χEFT (EFT of N, π)

Part II

$\delta\mathcal{O}_{\text{exp}}$

Data, (NN scattering)

⇓

⇒ V_{NN}, V_{3N},... forces ⇒

Part III

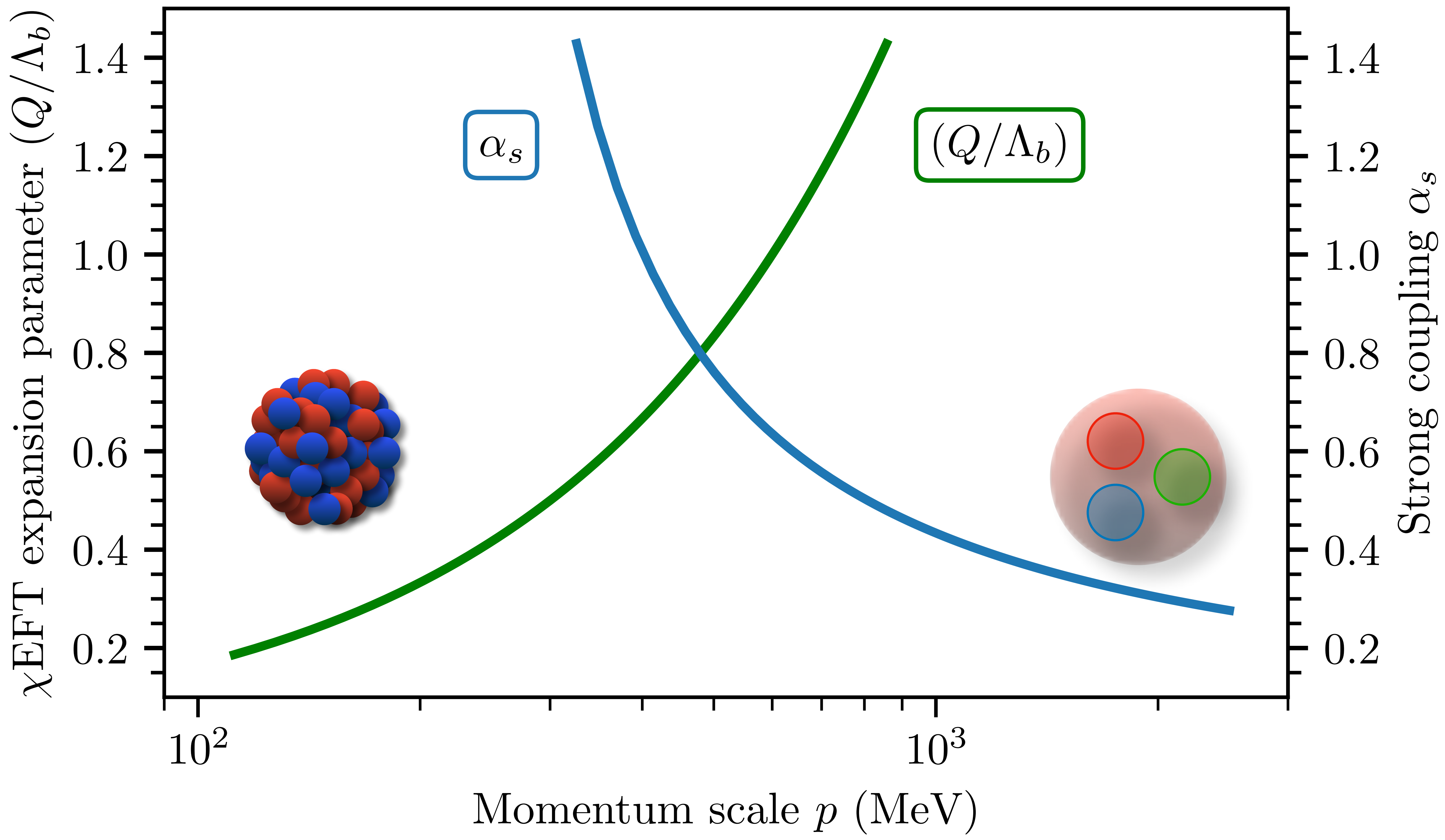
δE_m

⇒ Method for solving SE ⇒ $E_{\text{gs}} + \delta E_{\mathscr{L}} + \delta E_{\mathcal{O}_{\text{exp}}} + \delta E_m$

Part I

The nuclear force from EFT

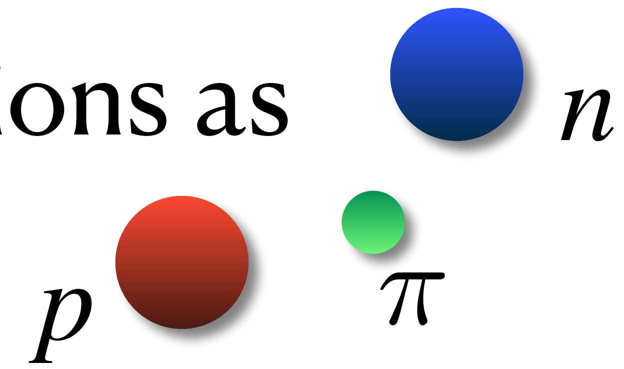
The nuclear force from EFT



χEFT

Weinberg 90's:

S. Weinberg, (1979), (1990), (1991)



- Use protons, neutrons, and pions as degrees of freedom.
- Formulate the most general dynamics consistent with the symmetries of QCD.

$$m_u \approx m_d \approx 0$$

$$SU(2)_L \times SU(2)_R \longrightarrow SU(2)_V$$

$\Longrightarrow$ Goldstone bosons: (π^0, π^+, π^-)

- Perturbative expansion in $\dfrac{m_\pi\ , p}{\Lambda_b} \equiv \dfrac{Q}{\Lambda_b}$.

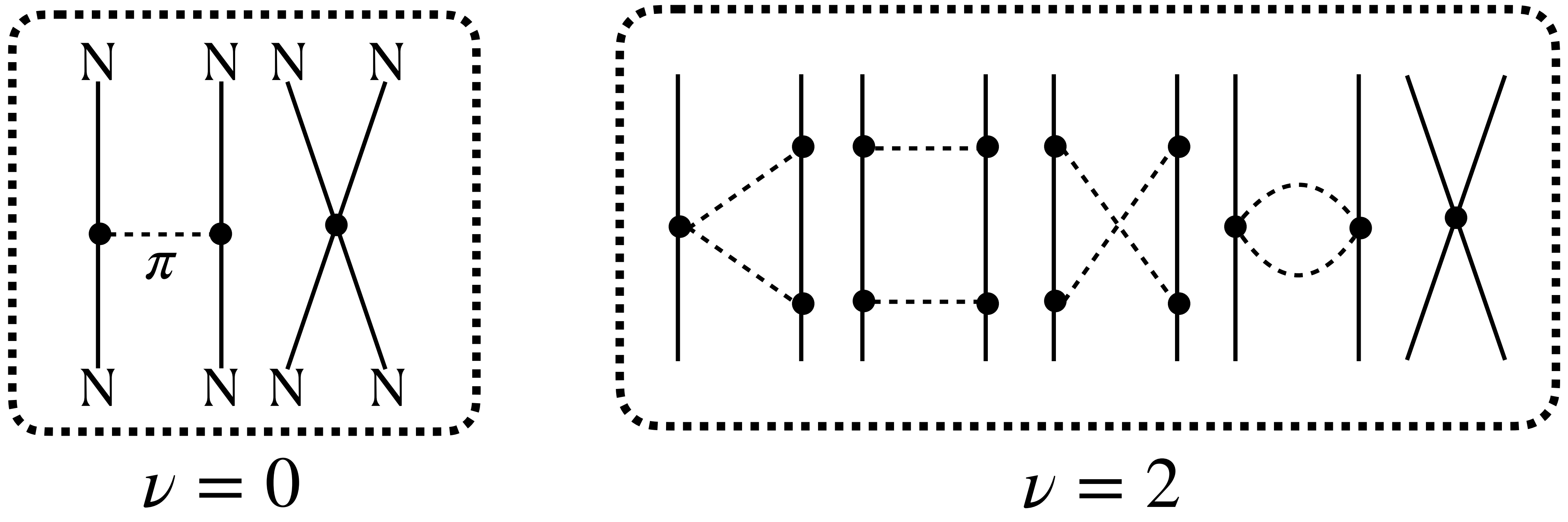
$$\Lambda_b \sim 1\ \text{GeV}$$

Weinberg Power Counting (PC)

- Construct nucleon-nucleon (NN) **potentials**:

E. Epelbaum, H.-W. Hammer, and U.-G. Meissner, Rev. Mod. Phys. **81,** (2009)
R. Machleidt and D. R. Entem, Phys. Rep. **503** (2011)
H.-W Hammer, S. König, and U. van Kolck, Rev. Mod. Phys. **92,** (2020)

$$V = V_{\mathrm{NN}}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\mathrm{NN}}^{(2)}(\boldsymbol{\alpha}^{(2)}) + \ldots$$



- Organize diagrams: $(Q/\Lambda_b)^\nu$

$Q \sim m_\pi,\ \Lambda_b \sim 1$ GeV $\rightarrow H|\psi\rangle = E|\psi\rangle$

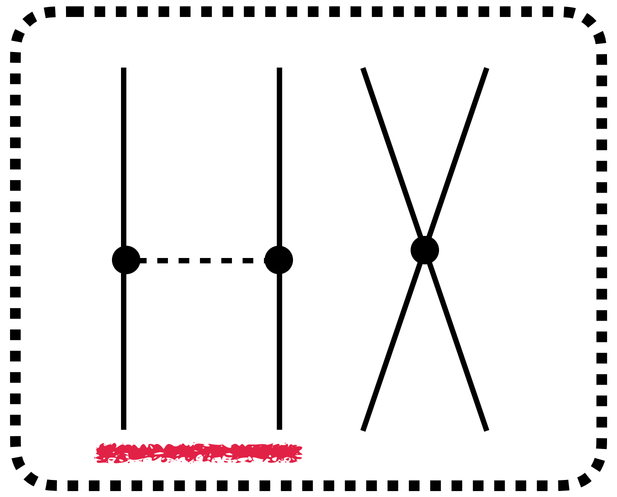
✓ χEFT with WPC: Successful descriptions of two- and three-nucleon forces and interaction currents.

✓ Systematic expansion with quantifiable theoretical error.

$$T_{l'l}^{js}(p', p; p_0) = V_{l'l}^{js}(p', p) + \sum_{l''} \int_0^{\Lambda} dk\ k^2\ V_{l'l''}^{js}(p', k) \frac{m_N}{p_0^2 - k^2 + i\epsilon} T_{l''l}^{js}(k, p; p_0)$$

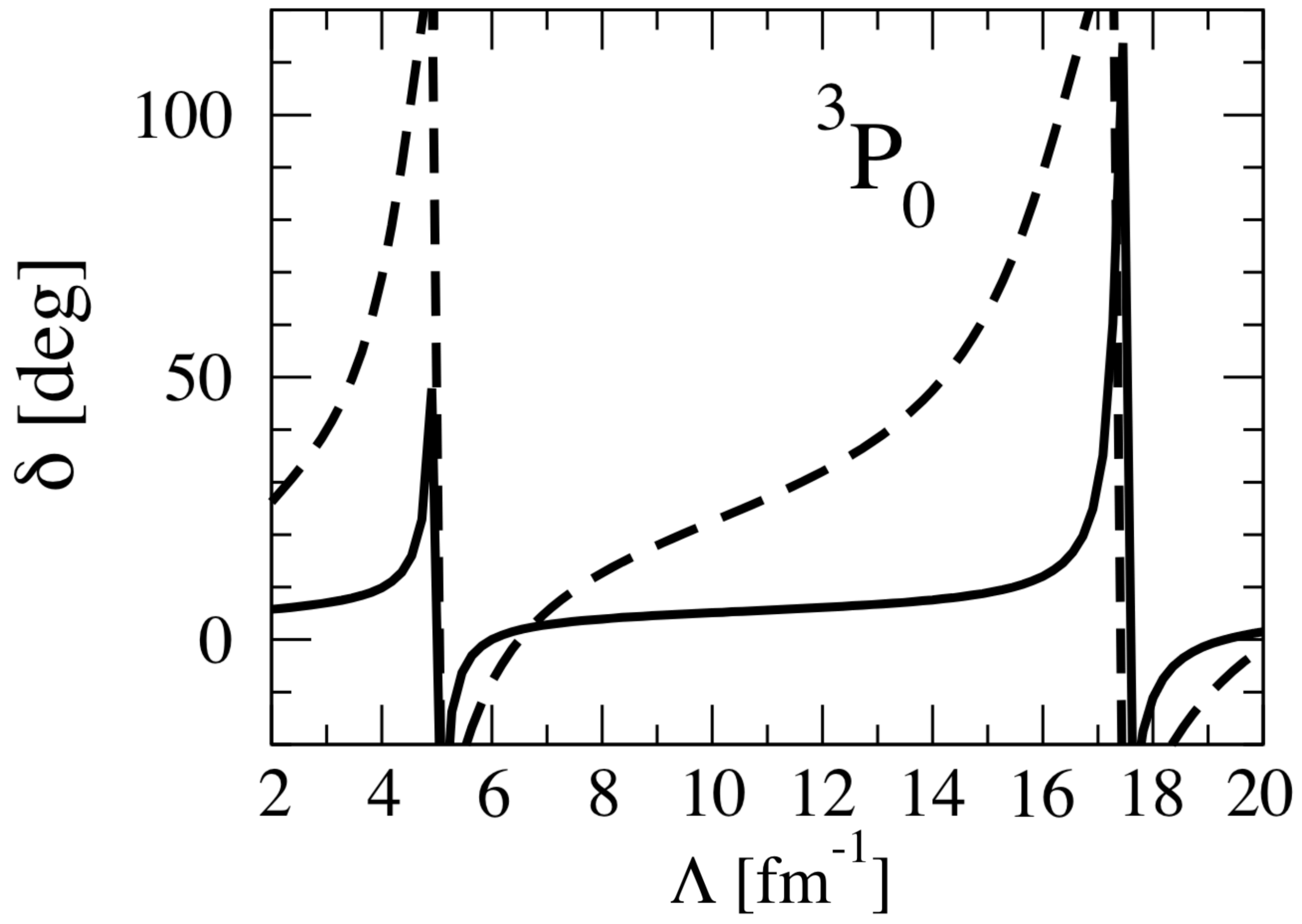
- Predictions of observables **depend on Λ**.

Cutoff dependence in WPC



$\nu = 0$

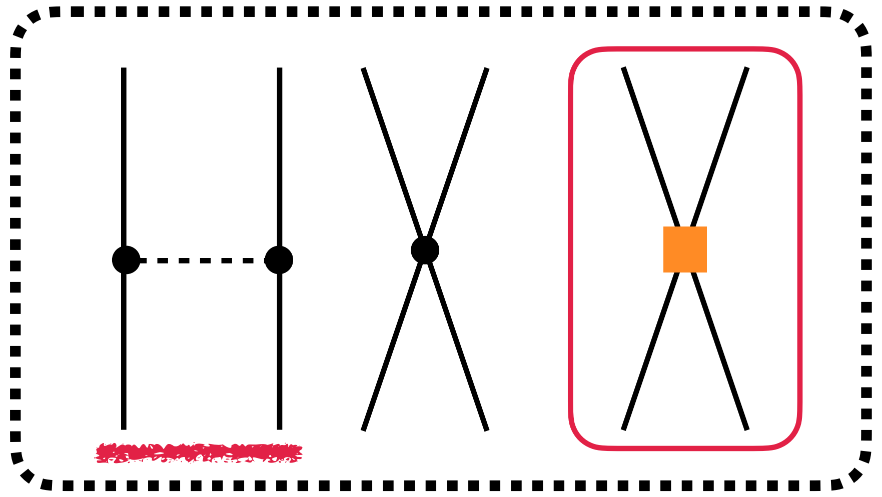
$$T^{js}_{l'l}(p',p;p_0) = V^{js}_{l'l}(p',p) + \sum_{l''}\int_0^{\Lambda} dk\ k^2\ V^{js}_{l'l''}(p',k)\frac{m_N}{p_0^2 - k^2 + i\epsilon}T^{js}_{l''l}(k,p;p_0)$$



A. Nogga *et al.*, Phys. Rev. C **72**, (2005)

Origin: Singular attraction ($\sim -1/r^3$)
due to the tensor force in OPE.

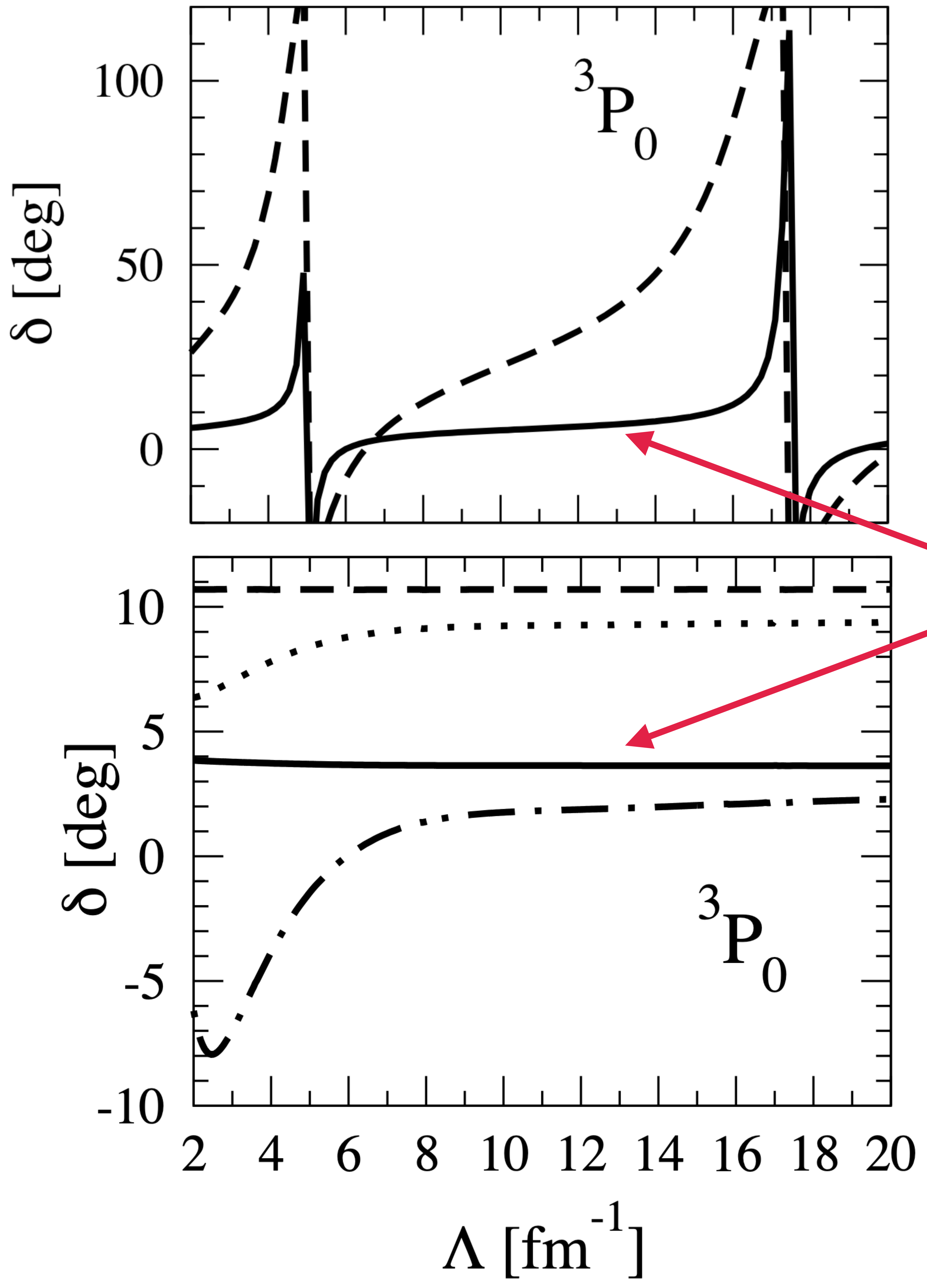
Cutoff dependence in WPC



$\nu = 0$

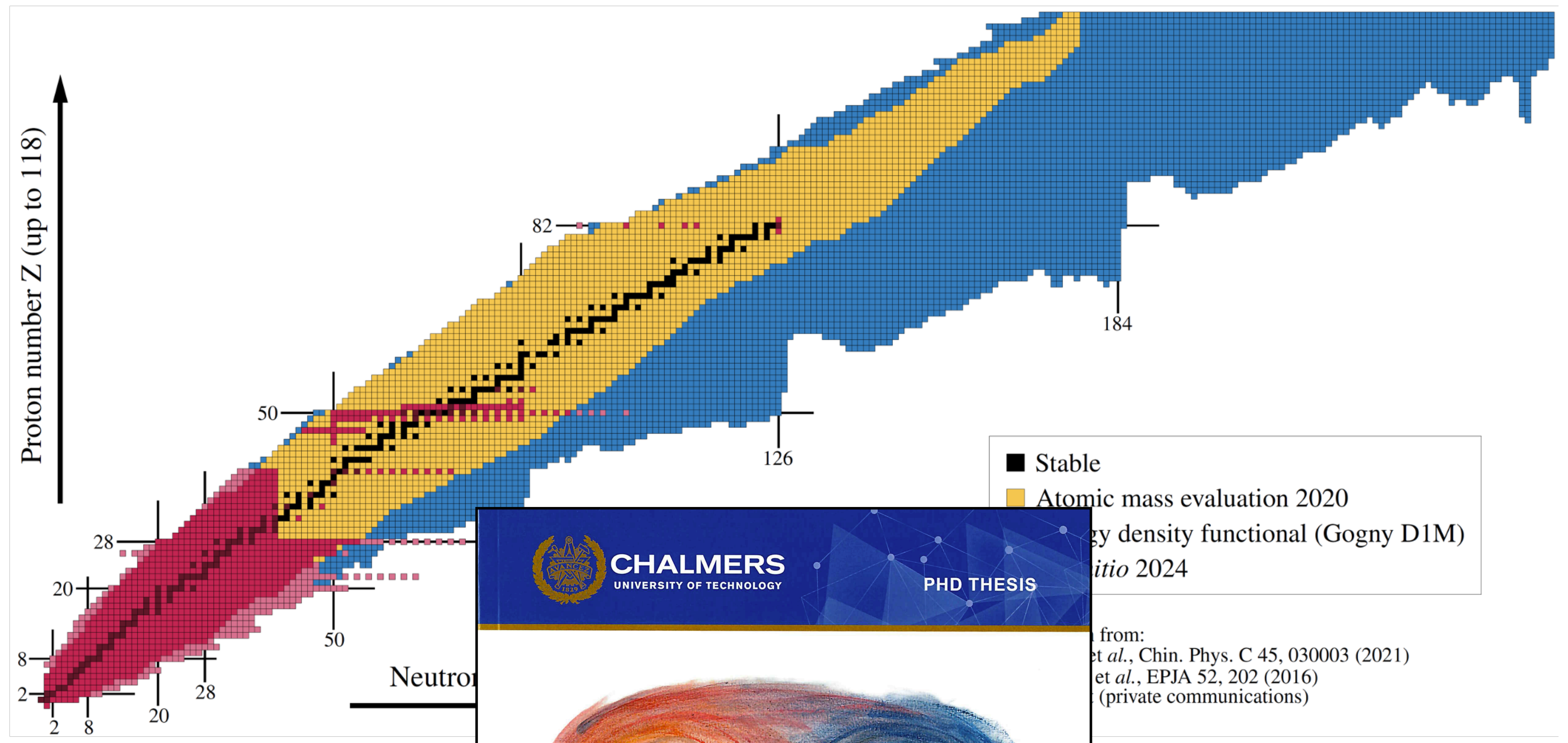
$$T^{js}_{l'l}(p', p; p_0) = V^{js}_{l'l}(p', p) + \sum_{l''} \int_0^{\Lambda} dk\; k^2\; V^{js}_{l'l''}(p', k) \frac{m_N}{p_0^2 - k^2 + i\epsilon} T^{js}_{l''l}(k, p; p_0)$$

Origin: Singular attraction ($\sim -1/r^3$)
due to the tensor force in OPE.



A. Nogga *et al.*, Phys. Rev. C **72**, (2005)

Possible solutions to the Λ dependence



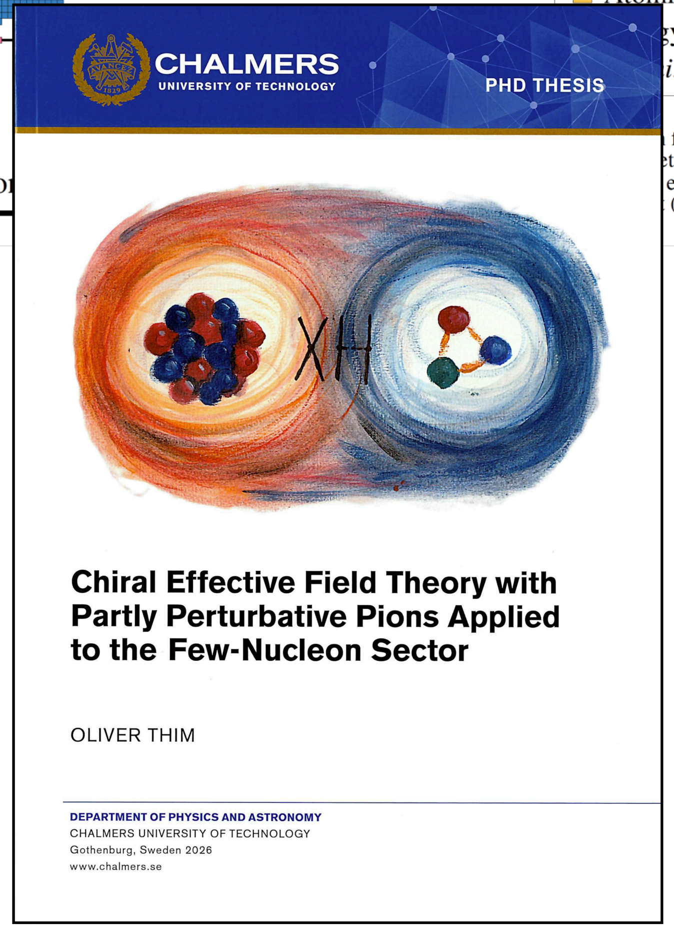
1. Use Weinberg PC and keep the cutoff $\Lambda \lesssim \Lambda_b$.

   D. R. Entem, R. Machleidt, Phys. Rev. C. **68** (2003)
   A. M. Gasparyan, E. Epelbaum, Phys. Rev. C. **107** (2023)
   E. Epelbaum, *et al*. Eur. Phys. J. A **56** (2020)
   P. Reinert, H. Krebs, E. Epelbaum, Eur. Phys. J. A **54** (2018)
   …

2. Treat pions perturbatively (Kaplan, Savage, Wise 1998).

   D. B. Kaplan *et al.,* Nucl. Phys. B **534** (1998)

   Y. P Teng, H.-W. Griesshammer, Eur. Phys. J. A **61** (2025)
   S. Lyo *et al.,* arXiv:2511.12522 (2025)



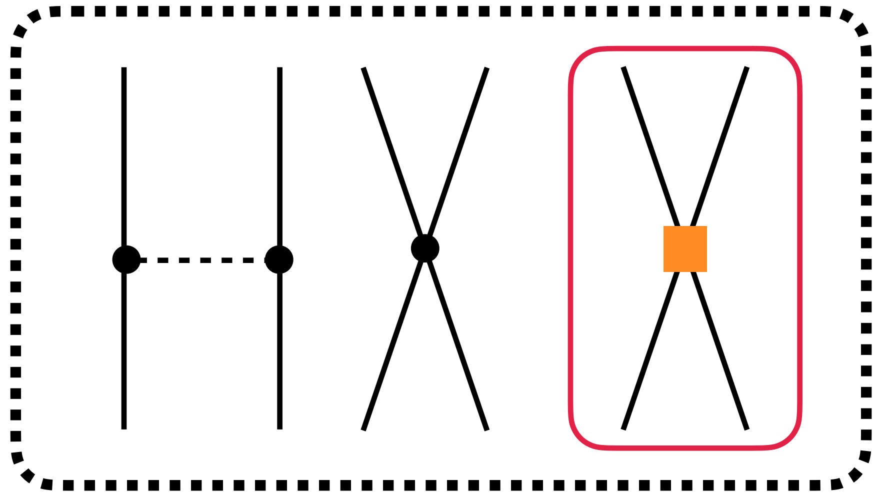
3. Promote counterterms with LECs to lower orders.

   $\Longrightarrow$ **All contributions beyond LO need to be treated in perturbation theory.**

A. Nogga *et al.,* Phys. Rev. C **72**, (2005)

U. van Kolck, Front. in Phys. **8** (2020)

B. Long and U. van Kolck, Ann. Phys. **323**, (2008)

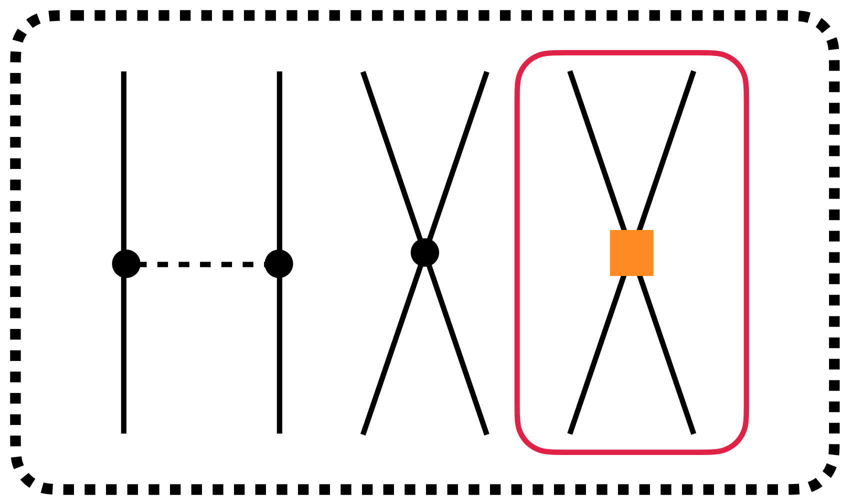


3. Modified Weinberg PC
(Partly perturbative pions)

- What happens if we impose **RG-invariance** and promote counterterms to lower orders to achieve this?

Must be treated perturbatively

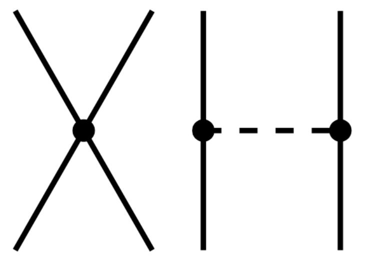
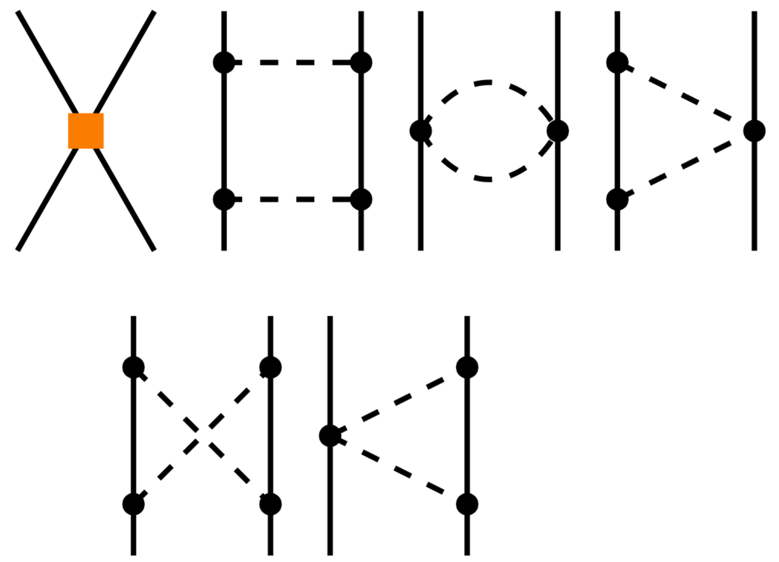
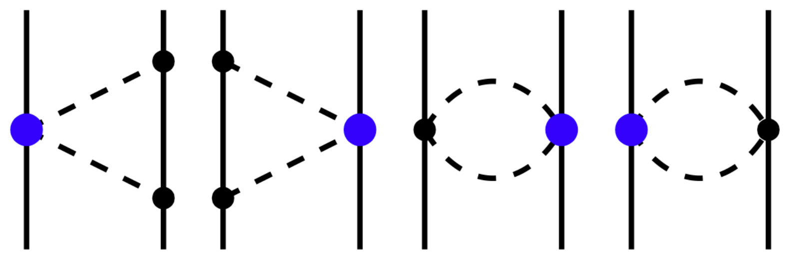
$$V = V_{\rm NN}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\rm NN}^{(1)}(\boldsymbol{\alpha}^{(1)}) + V_{\rm NN}^{(2)}(\boldsymbol{\alpha}^{(2)}) + V_{\rm NN}^{(3)}(\boldsymbol{\alpha}^{(3)}) + \ldots$$



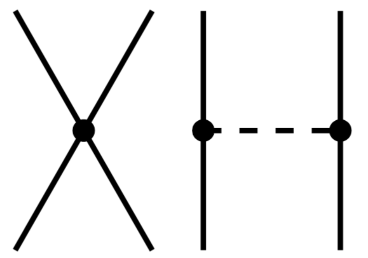
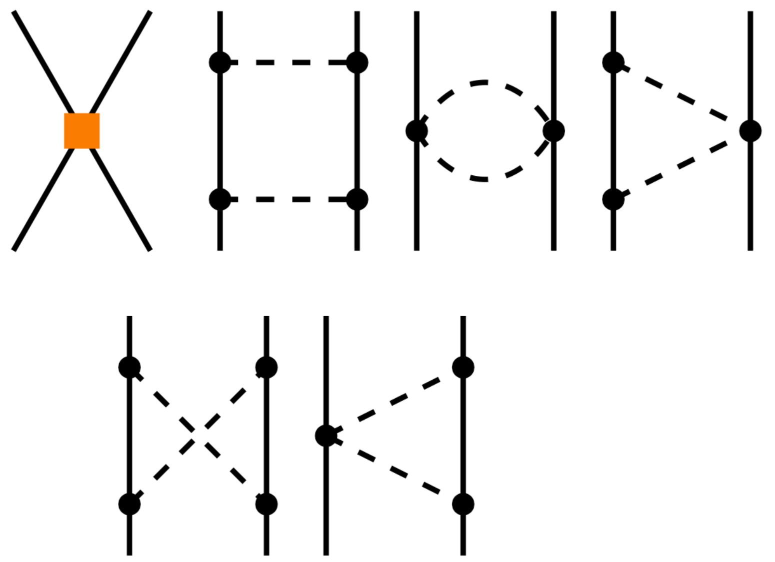
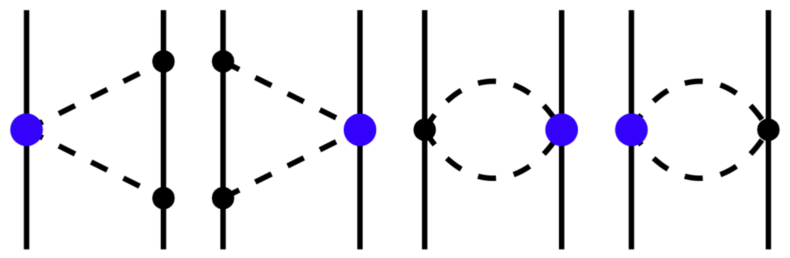
A. Nogga *et al.*, Phys. Rev. C **72**, (2005)
B. Long and U. van Kolck, Ann. Phys. **323**, (2008)
B. Long, C. J. Yang, Phys. Rev. C **84**, (2011), Phys. Rev. C **85**, (2012), Phys. Rev. C **86**, (2012)

A version of the Long and Yang PC

Weinberg PC (NN only)

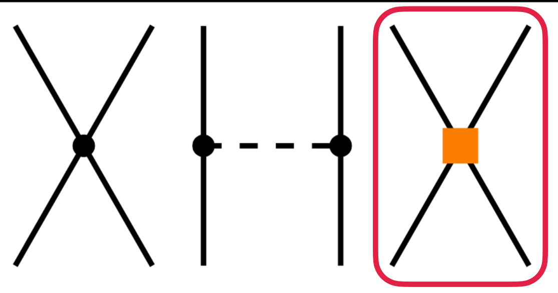
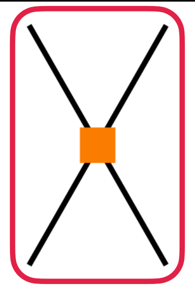
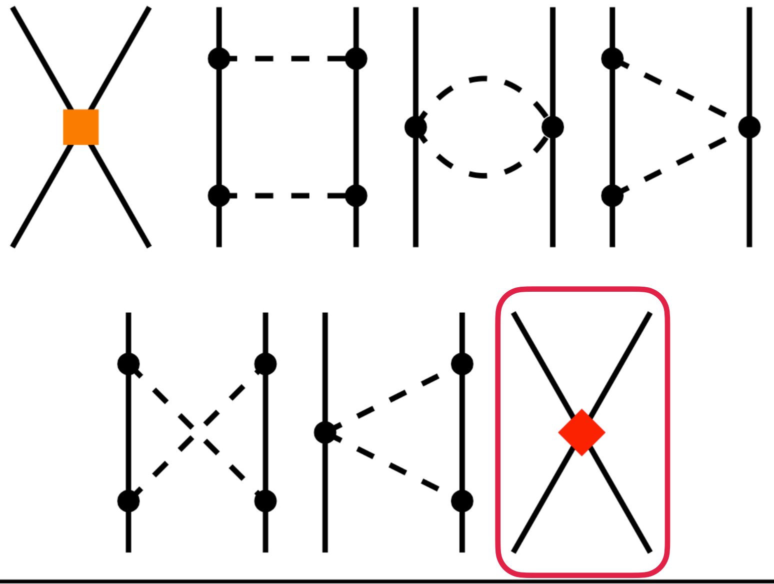
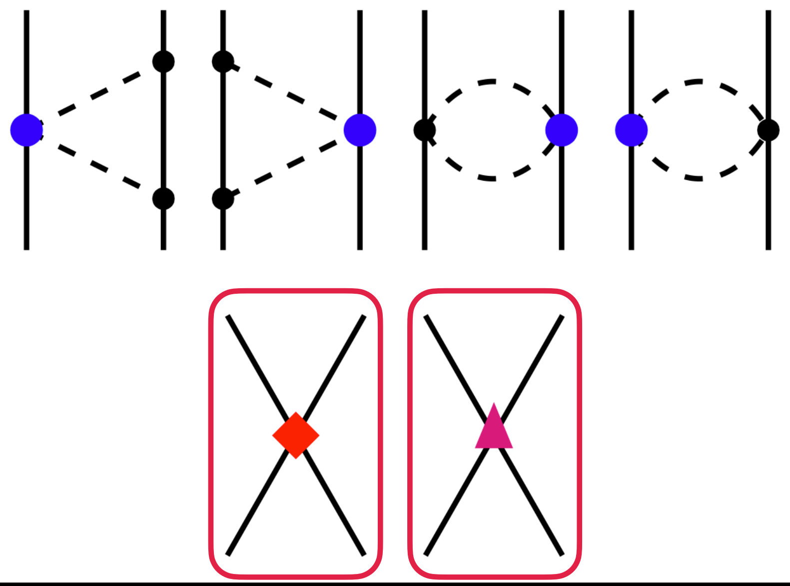
LO $(Q/\Lambda_b)^0$	
NLO $(Q/\Lambda_b)^1$	
N²LO $(Q/\Lambda_b)^2$	
N³LO $(Q/\Lambda_b)^3$	

A version of the Long and Yang PC

Order	Channels: $^1S_0$, $^3P_0$, $^1P_1$, $^3P_1$, $^3S_1{-}^3D_1$, $^3P_2{-}^3F_2$
	Non-perturbative contributions (LO)
LO $(Q/\Lambda_b)^0$	
NLO $(Q/\Lambda_b)^1$	
N^2LO $(Q/\Lambda_b)^2$	
N^3LO $(Q/\Lambda_b)^3$	

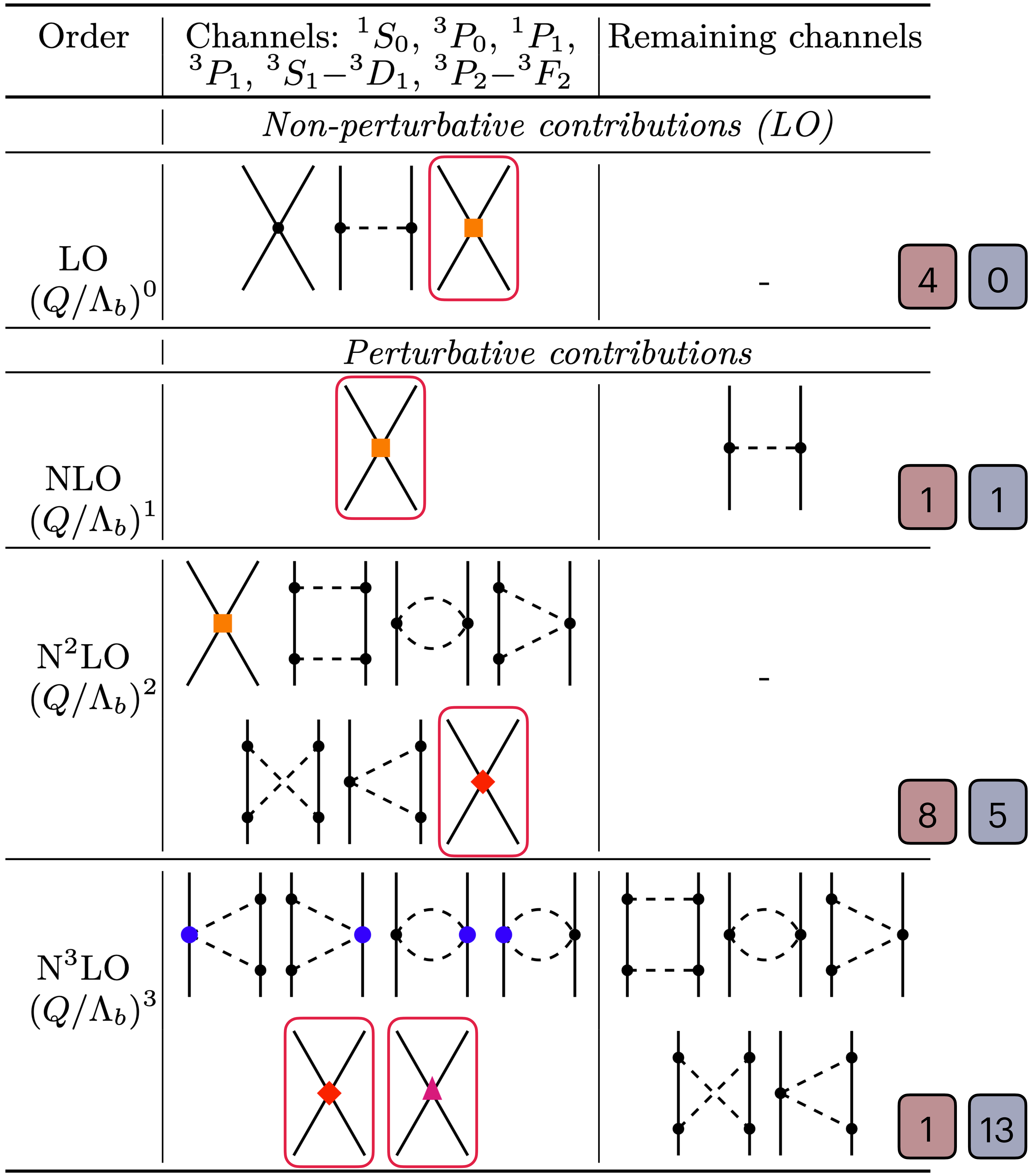
- One-pion exchange at LO only in $\ell \leq 1$.

A version of the Long and Yang PC

Order	Channels: $^1S_0$, $^3P_0$, $^1P_1$, $^3P_1$, $^3S_1-{}^3D_1$, $^3P_2-{}^3F_2$
	Non-perturbative contributions (LO)
LO $(Q/\Lambda_b)^0$	
NLO $(Q/\Lambda_b)^1$	
N^{2}LO $(Q/\Lambda_b)^2$	
N^{3}LO $(Q/\Lambda_b)^3$	

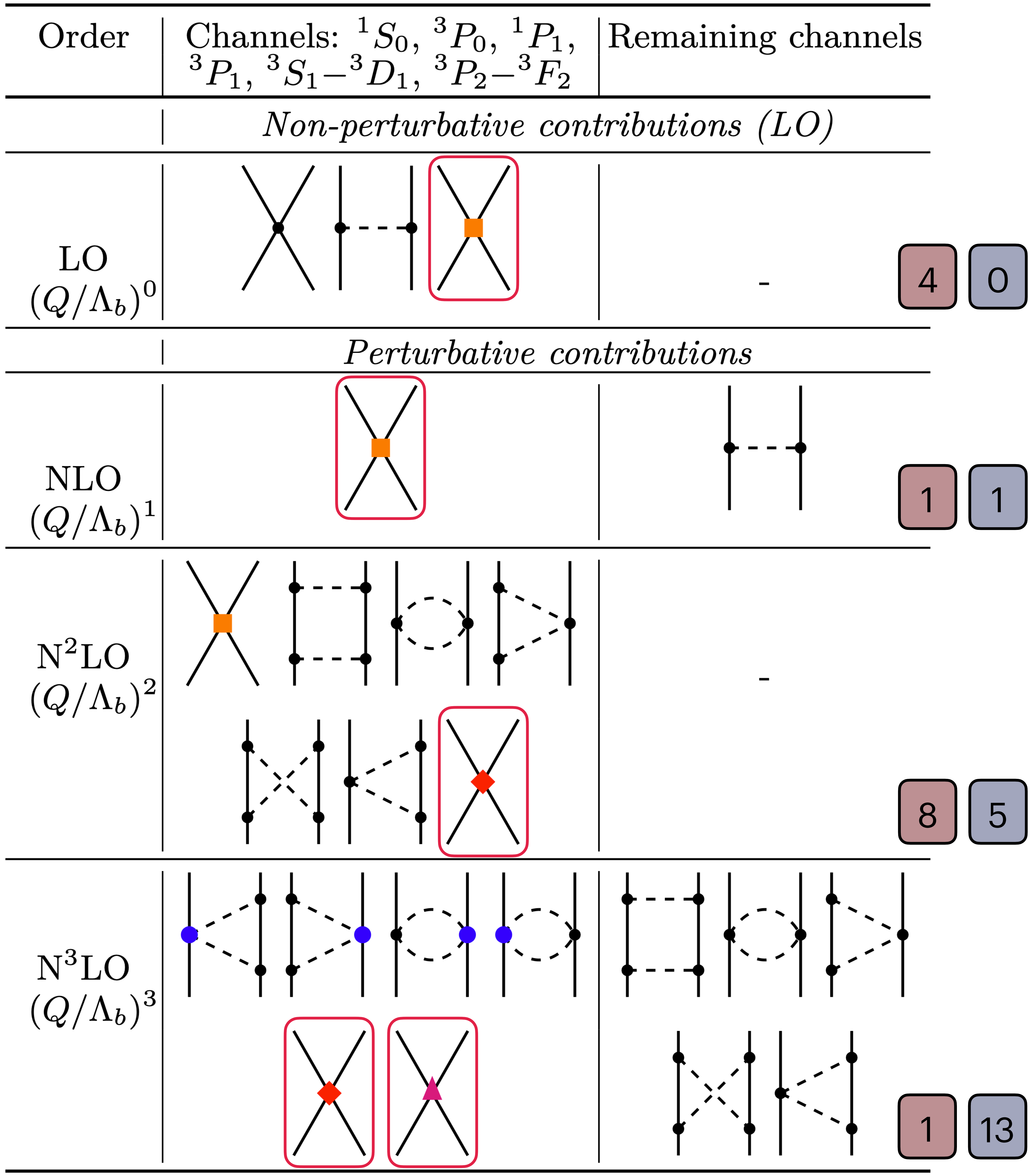
- One-pion exchange at LO only in $\ell \leq 1$.
- Promote counterterms to lower order.

A version of the Long and Yang PC



- One-pion exchange at LO only in $\ell \leq 1$.
- Promote counterterms to lower order.
- Treat all subleading interactions perturbatively.

A version of the Long and Yang PC



- One-pion exchange at LO only in $\ell \leq 1$.
- Promote counterterms to lower order.
- Treat all subleading interactions perturbatively.

Can χEFT with partly perturbative pions describe nuclear observables in $A \geq 2$ systems?

- Studies at LO and NLO for $A > 2$ observables.

Y.-H. Song, R. Lazauskas, and U. van Kolck, Phys. Rev. C **96**, (2017)

C. J. Yang *et al.*, Phys. Rev. C **103**, (2021)

- We have developed the machinery to go up to N^3LO.

Part II

Constructing NN potentials

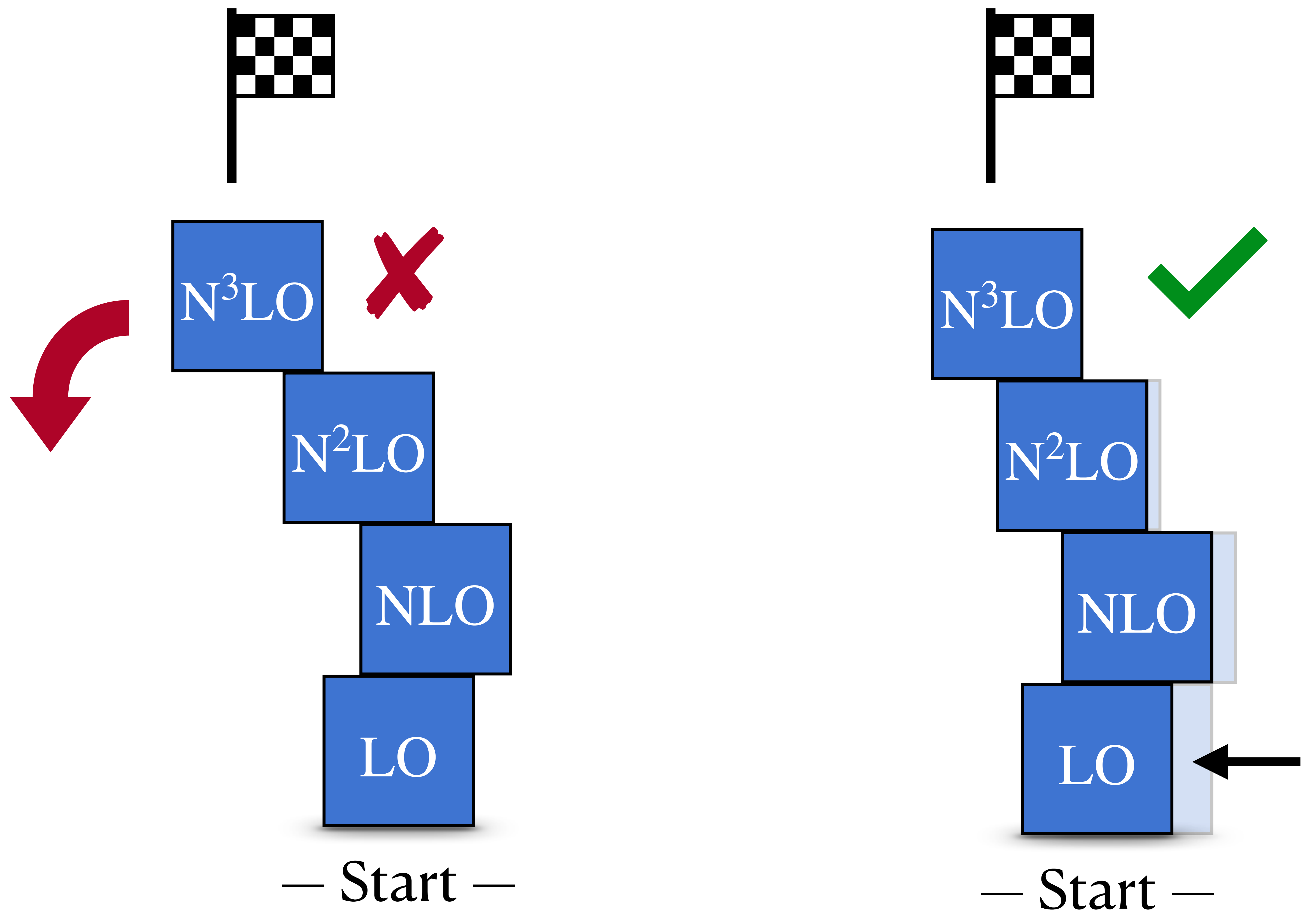
Determining the values of LECs

- Compute NN scattering amplitudes perturbatively:

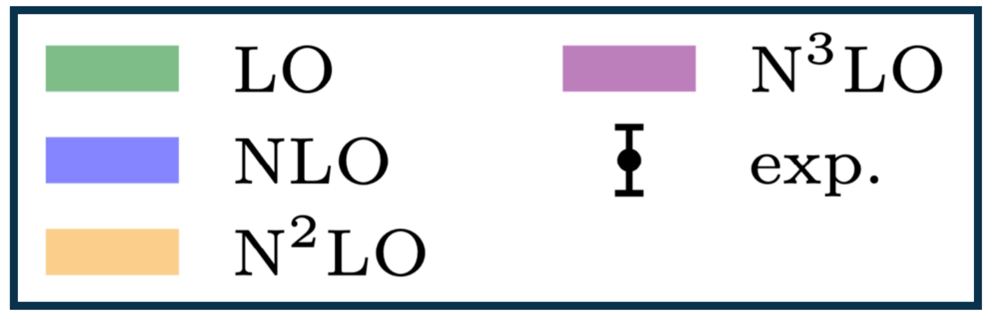
$$\text{LO:} \quad T^{(0)} = V^{(0)} + V^{(0)} G_0^{+} T^{(0)}$$
$$\text{NLO:} \quad T^{(1)} = \Omega_-^{\dagger} V^{(1)} \Omega_+$$
$$\text{N}^2\text{LO:} \quad T^{(2)} = \Omega_-^{\dagger} \left(V^{(2)} + V^{(1)} G_1^{+} V^{(1)} \right) \Omega_+$$

$$T = T^{(0)} + T^{(1)} + T^{(2)} + T^{(3)} + \ldots$$

- Calibrate LECs to empirical phase shifts.
- Conclusion: **The foundation (LO)** is very important!

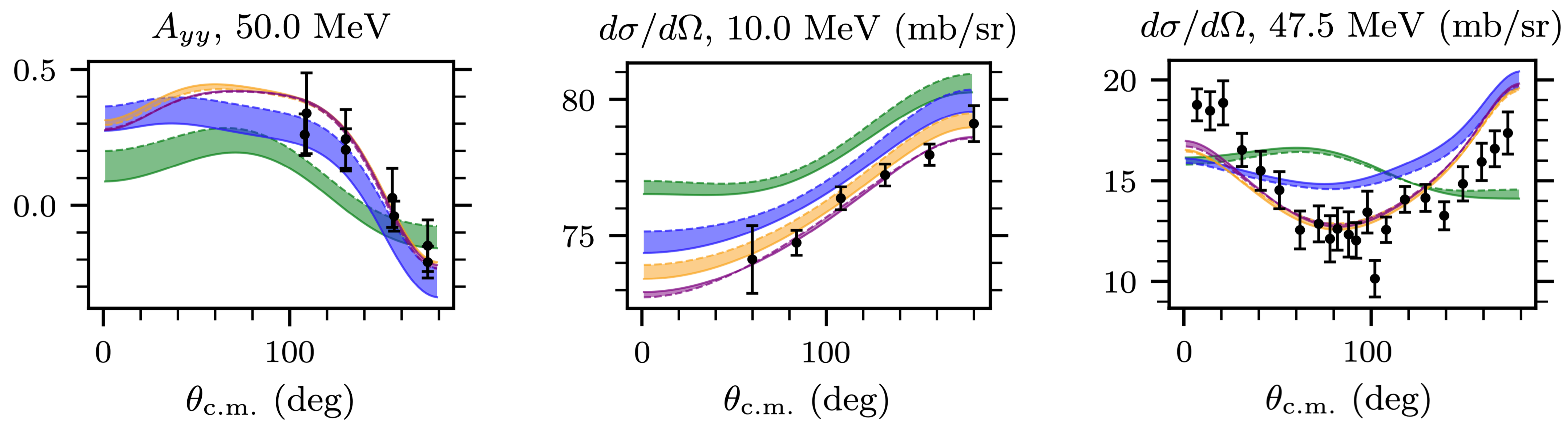


Predicted NN scattering observables



$\Lambda = 500$ MeV, $\Lambda = 2500$ MeV

OT, A. Ekström, and C. Forssén, Phys. Rev. C **109**, (2024)



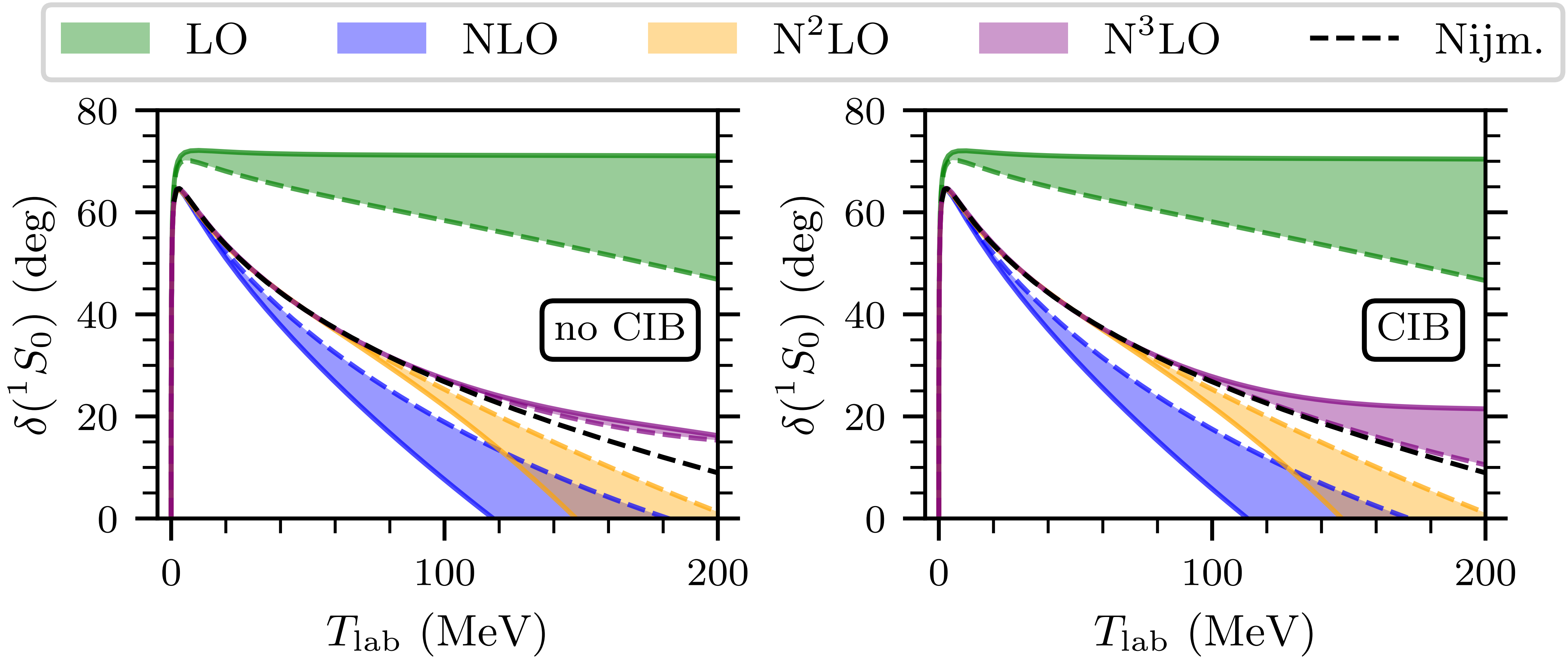
- Clear improvement order-by-order.
- Energy-dependent accuracy.
- Residual cutoff dependence gets smaller with increasing order.

But what about *exceptional* cutoffs?

A. M. Gasparyan and E. Epelbaum, Phys. Rev. C. **107** (2023)

Low-energy theorems: $^1S_0$

Phase shifts in $^1S_0$



Predicted effective range parameters (LETs)

$^1S_0$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
Empirical (Ref. [27])	−23.735(16)	2.68(3)	−0.48(2)	3.9(1)	−19.6(5)
$\Lambda = 500$ MeV, (no CIB)					
LO	*	1.71(0)	−1.77(0)	8.54(0)	−47.0(3)
NLO	*	*	−0.64(0)	4.79(0)	−29.9(2)
N^{2}LO	*	2.72(0)	−0.71(0)	5.05(0)	−29.3(2)
N^{3}LO	*	2.69(0)	−0.66(0)	5.42(0)	−31.0(2)
$\Lambda = 500$ MeV, (CIB)					
LO	*	1.68(0)	−1.55(0)	6.63(0)	−31.64(8)
NLO	*	*	−0.45(0)	3.42(0)	−18.95(8)
N^{2}LO	*	2.70(0)	−0.55(0)	3.77(0)	−18.8(2)
N^{3}LO	*	2.68(0)	−0.50(0)	4.02(0)	−19.8(2)

OT, Few-Body Syst. 65, 69 (2024)

$$F(k) \equiv k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \mathcal{O}\left(k^{10}\right)$$

- Leading **CIB** (pion mass splitting) in the one-pion exchange is **significant** in $^1S_0$.

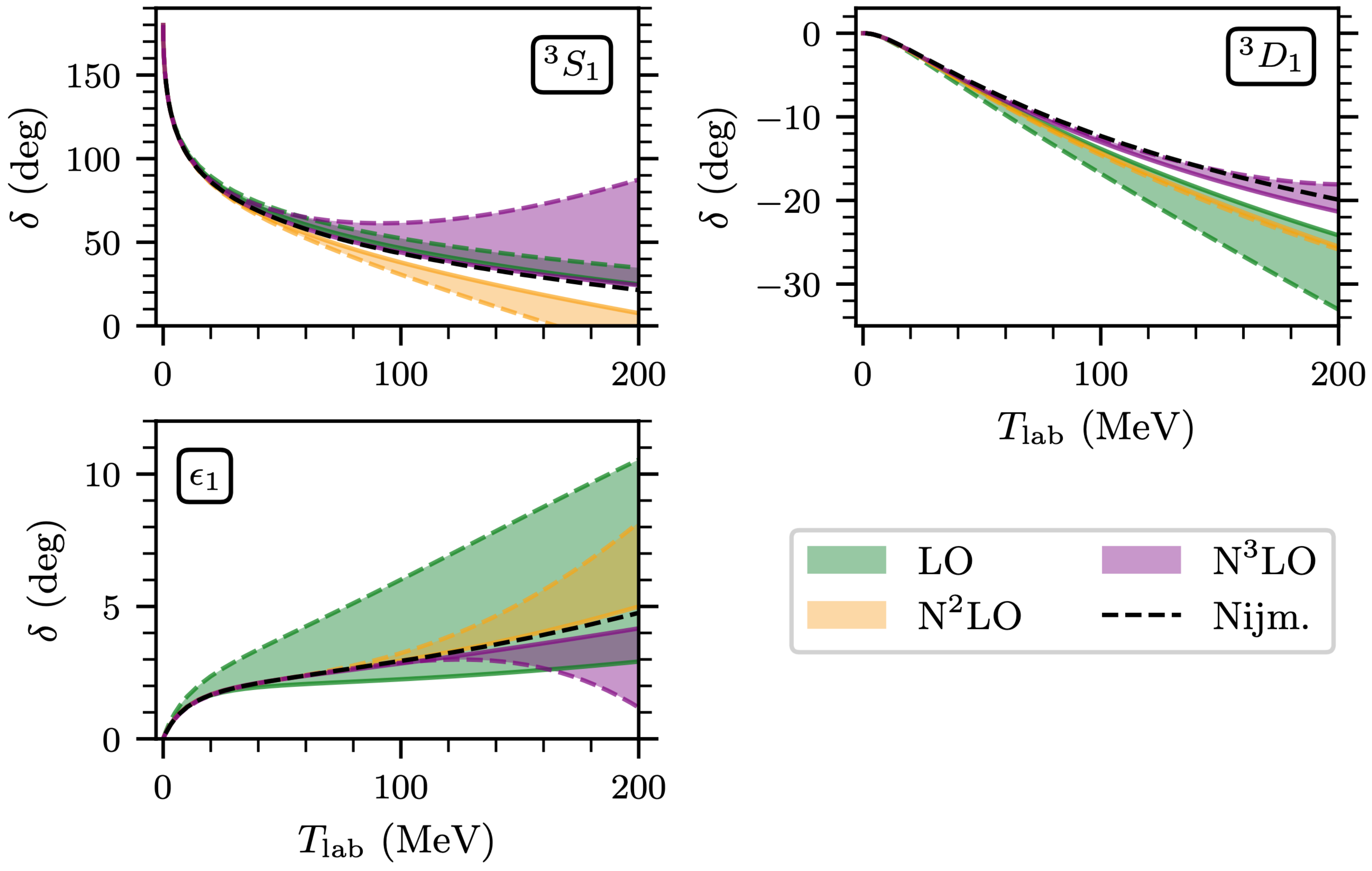
KSW: Perturbative pions

δ ($^1S_0$ channel)	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)
low energy theorem	−3.3	17.8	−108.0
partial wave analysis	−0.48	3.8	−17.0

T.D. Cohen, J.M. Hansen, Phys. Rev. C **59**, (1999)

Low-energy theorems: $^3S_1$

Phase shifts in $^3S_1$— $^3D_1$



Predicted effective range parameters (LETs)

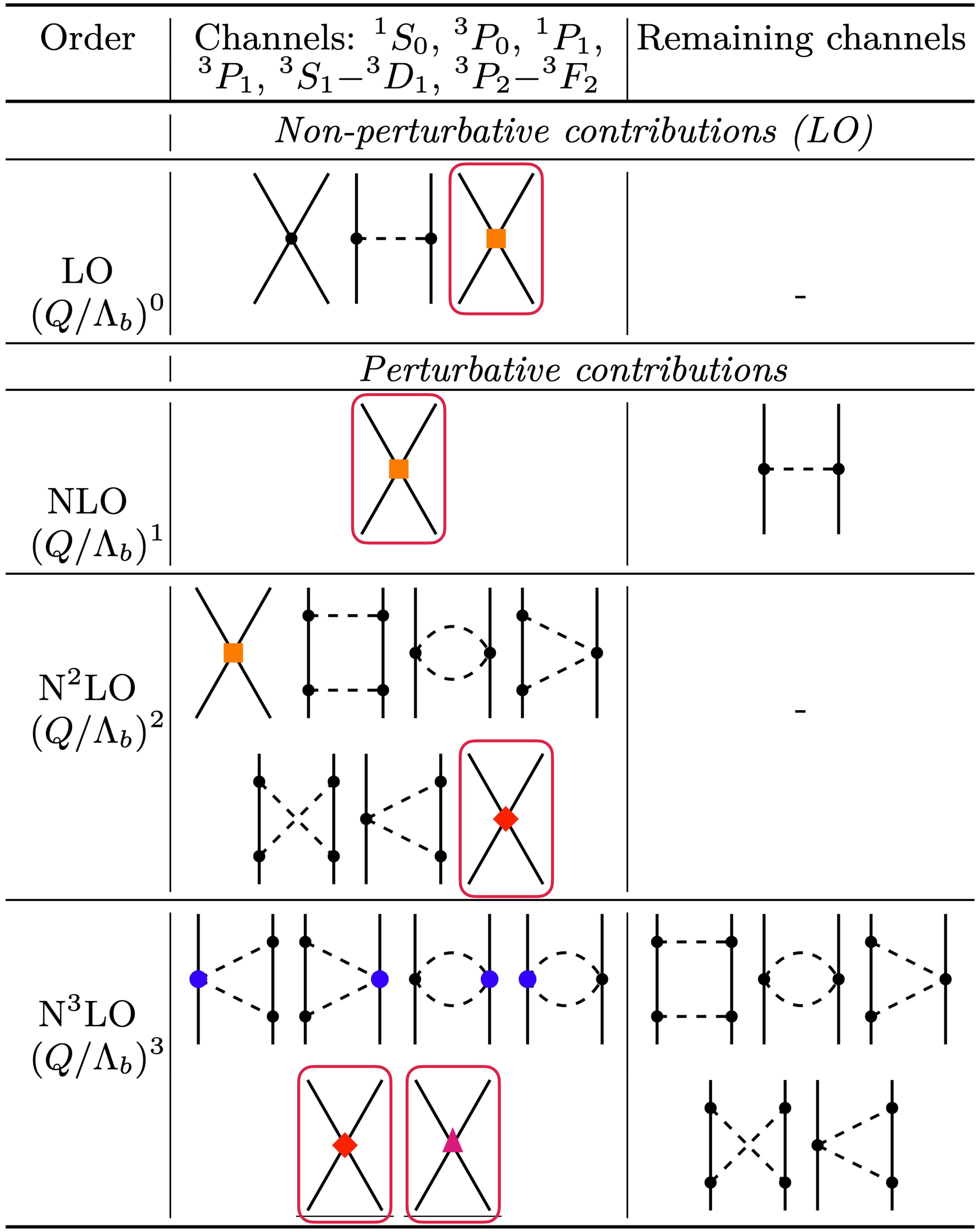
$^3S_1$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
Empirical (Ref. [27])	5.42	1.75	0.045	0.67	−3.94
$\Lambda = 500$ MeV					
LO	∗	1.58(0)	−0.10(0)	0.89(0)	−5.5(2)
N^{2}LO	∗	∗	0.14(0)	0.80(0)	−4.2(2)
N^{3}LO	∗	∗	−0.06(0)	0.46(0)	−3.7(2)
$\Lambda = 2500$ MeV					
LO	∗	1.66(0)	−0.01(0)	0.79(0)	−4.7(2)
N^{2}LO	∗	∗	0.09(0)	0.74(0)	−4.2(7)
N^{3}LO	∗	∗	0.04(0)	0.67(2)	−4.0(9)

- CIB in one-pion exchange is **not significant** in $^3S_1$.
- Cutoff independence for $\Lambda \gtrsim 750$ MeV.

✓ Both phase shift and LETs are accurate, and improved for high cutoffs.

OT, Few-Body Syst. 65, 69 (2024)

In summary



Constructed potentials show consistency in the two-nucleon sector.

Treated perturbatively

$$V = V_{\mathrm{NN}}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\mathrm{NN}}^{(1)}(\boldsymbol{\alpha}^{(1)}) + V_{\mathrm{NN}}^{(2)}(\boldsymbol{\alpha}^{(2)}) + V_{\mathrm{NN}}^{(3)}(\boldsymbol{\alpha}^{(3)}) + \ldots$$

Code for perturbative NN computations available:
https://github.com/othim/nn-mwpc

Part III

Predicting ground-state energies for $A \leq 6$.

Bound-state computations in perturbation theory

$$H|\psi\rangle = E|\psi\rangle$$

$$V = V_{\rm NN}^{(0)}(\boldsymbol{\alpha}^{(0)}) + \overbrace{V_{\rm NN}^{(1)}(\boldsymbol{\alpha}^{(1)}) + V_{\rm NN}^{(2)}(\boldsymbol{\alpha}^{(2)}) + V_{\rm NN}^{(3)}(\boldsymbol{\alpha}^{(3)})}^{\text{Treated perturbatively}} + \ldots$$

- Desired features:
  - Straightforward to implement in existing code frameworks.
  - Small computational overhead.
- Plan: Benchmark in ^{3}H with both the **Rayleigh-Schrödinger (RS)** and the **Finite Difference (FD)** method.
- Question: How well do the perturbative calculations converge?

The Rayleigh-Schrödinger approach

Treated perturbatively

Potential: $V = V_{\mathrm{NN}}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\mathrm{NN}}^{(1)}(\boldsymbol{\alpha}^{(1)}) + V_{\mathrm{NN}}^{(2)}(\boldsymbol{\alpha}^{(2)}) + V_{\mathrm{NN}}^{(3)}(\boldsymbol{\alpha}^{(3)}) + \dots$

LO:

$$H^{(0)} |\Psi_n^{(0)}\rangle = E_n^{(0)} |\Psi_n^{(0)}\rangle$$

NLO:

$$E^{(1)} = \langle\Psi_0^{(0)}|V_{\mathrm{NN}}^{(1)}|\Psi_0^{(0)}\rangle$$

N^2LO:

$$E^{(2)} = \langle\Psi_0^{(0)}|V_{\mathrm{NN}}^{(2)}|\Psi_0^{(0)}\rangle + \sum_{n\neq 0} \frac{|\langle\Psi_0^{(0)}|V_{\mathrm{NN}}^{(1)}|\Psi_n^{(0)}\rangle|^2}{E_0^{(0)} - E_n^{(0)}}$$

⋮

– Need full LO spectrum beyond NLO!

\+ Easy to implement for ^{3}H in the NCSM.

OT, A. Ekström, and C. Forssén, Phys. Rev. C **112** (2025)

A numerical finite difference approach

1. Define a Hamiltonian:

$$H(\boldsymbol{x}) = H^{(0)} + \sum_{\nu=1}^{3} x_\nu V^{(\nu)}$$

2. Solve the Schrödinger equation for multiple values of $\boldsymbol{x}$:

$$H(\boldsymbol{x})|\Psi_n(\boldsymbol{x})\rangle = E_n(\boldsymbol{x})|\Psi_n(\boldsymbol{x})\rangle$$

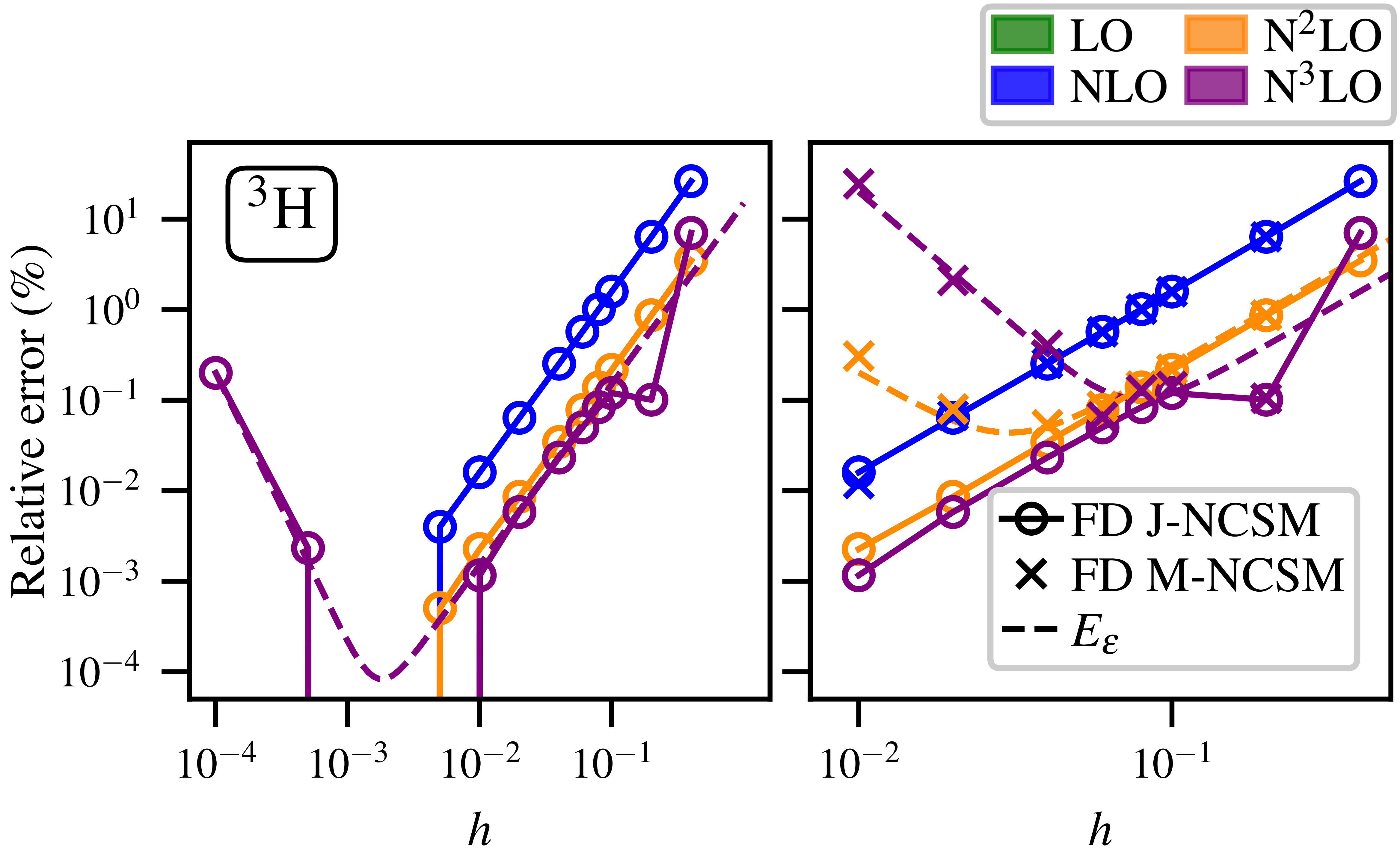
3. Compute derivatives numerically:

$$E_0^{(1)} = \partial_1 E_0(\boldsymbol{x})$$

$$E_0^{(2)} = \partial_2 E_0(\boldsymbol{x}) + \frac{1}{2}\partial_1^2 E_0(\boldsymbol{x})$$

$$E_0^{(3)} = \partial_3 E_0(\boldsymbol{x}) + \partial_1\partial_2 E_0(\boldsymbol{x}) + \frac{1}{6}\partial_1^3 E_0(\boldsymbol{x})$$

Compare to the exact Rayleigh-Schrödinger computations in ^{3}H.



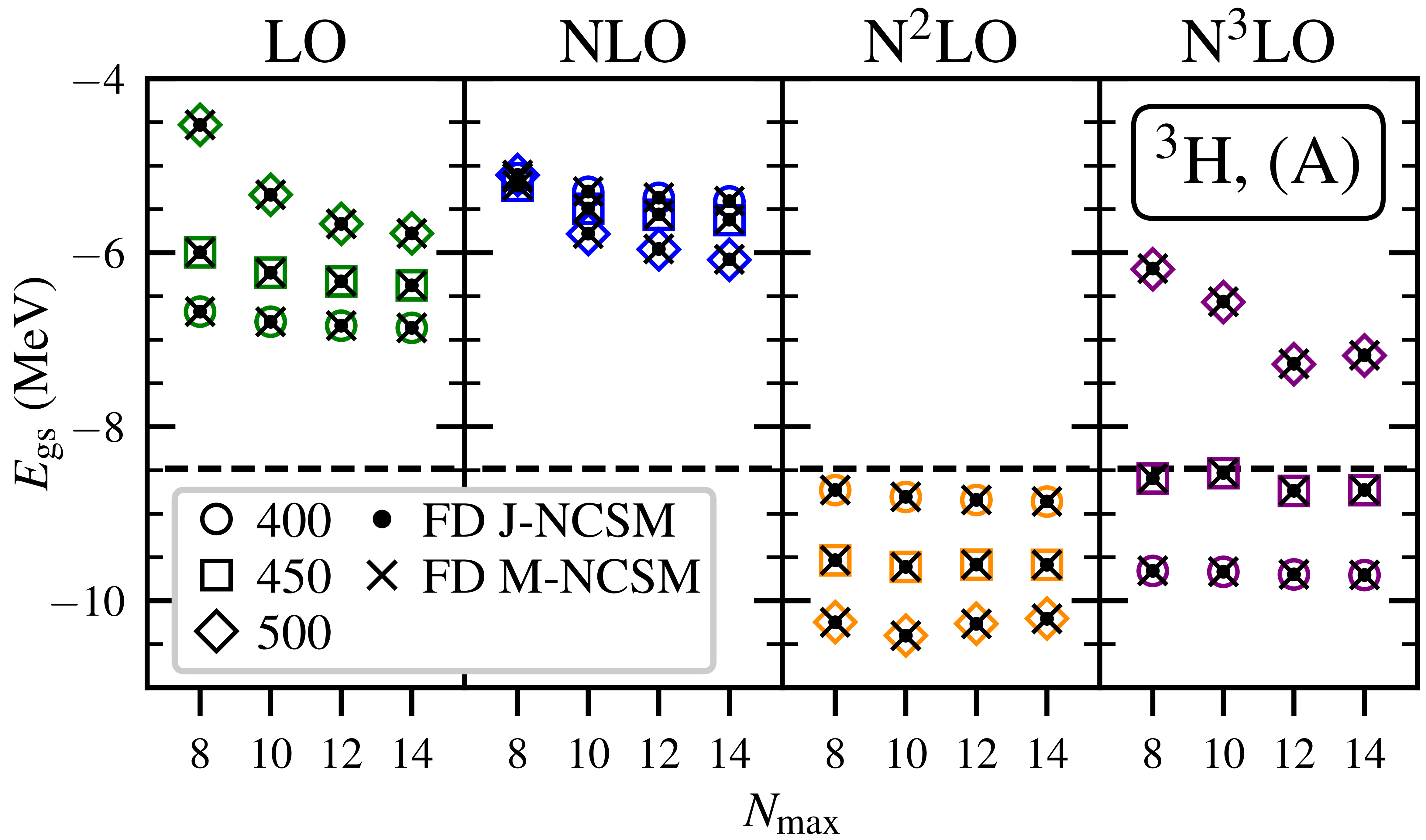
$$E_\epsilon(h) = \epsilon h^{-n} + K h^p$$

Validation of the FD method up to N^3LO

- Validate the **FD** method against exact **RS** computations in ^{3}H.
- Can perform computation up to $\Lambda = 1200$ MeV at N^3LO.
- See big effects from exceptional cutoffs beyond $\Lambda \gtrsim 600$ MeV.

OT, A. Ekström, and C. Forssén, Phys. Rev. C **112** (2025)

- ... but need to be at low cutoffs to converge many-body computations.



- Sub-percent errors.
- Need 13 exact solutions at h^2.

Code available:

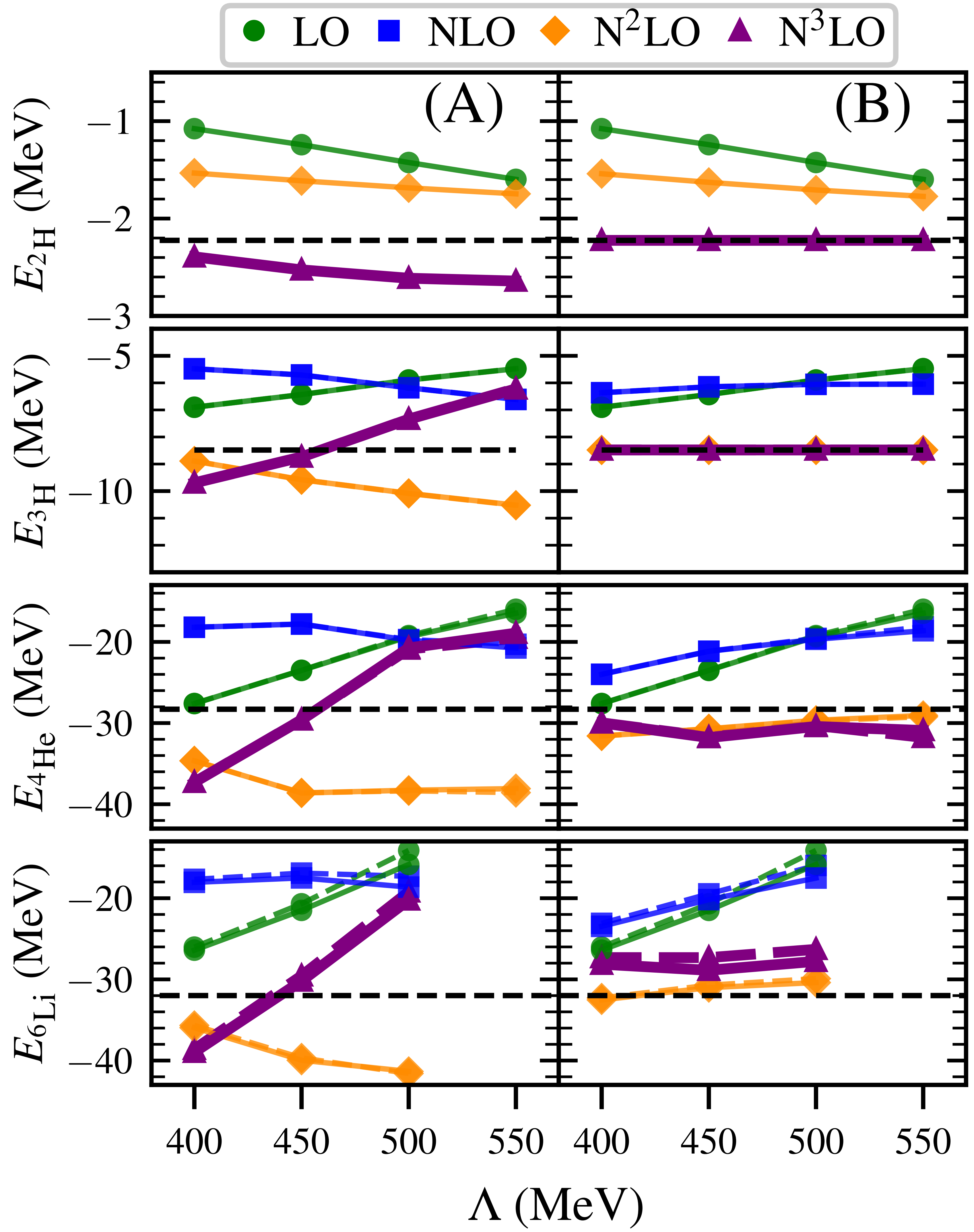
https://github.com/othim/py-ncsm

Calibrating LECs using few-body observables

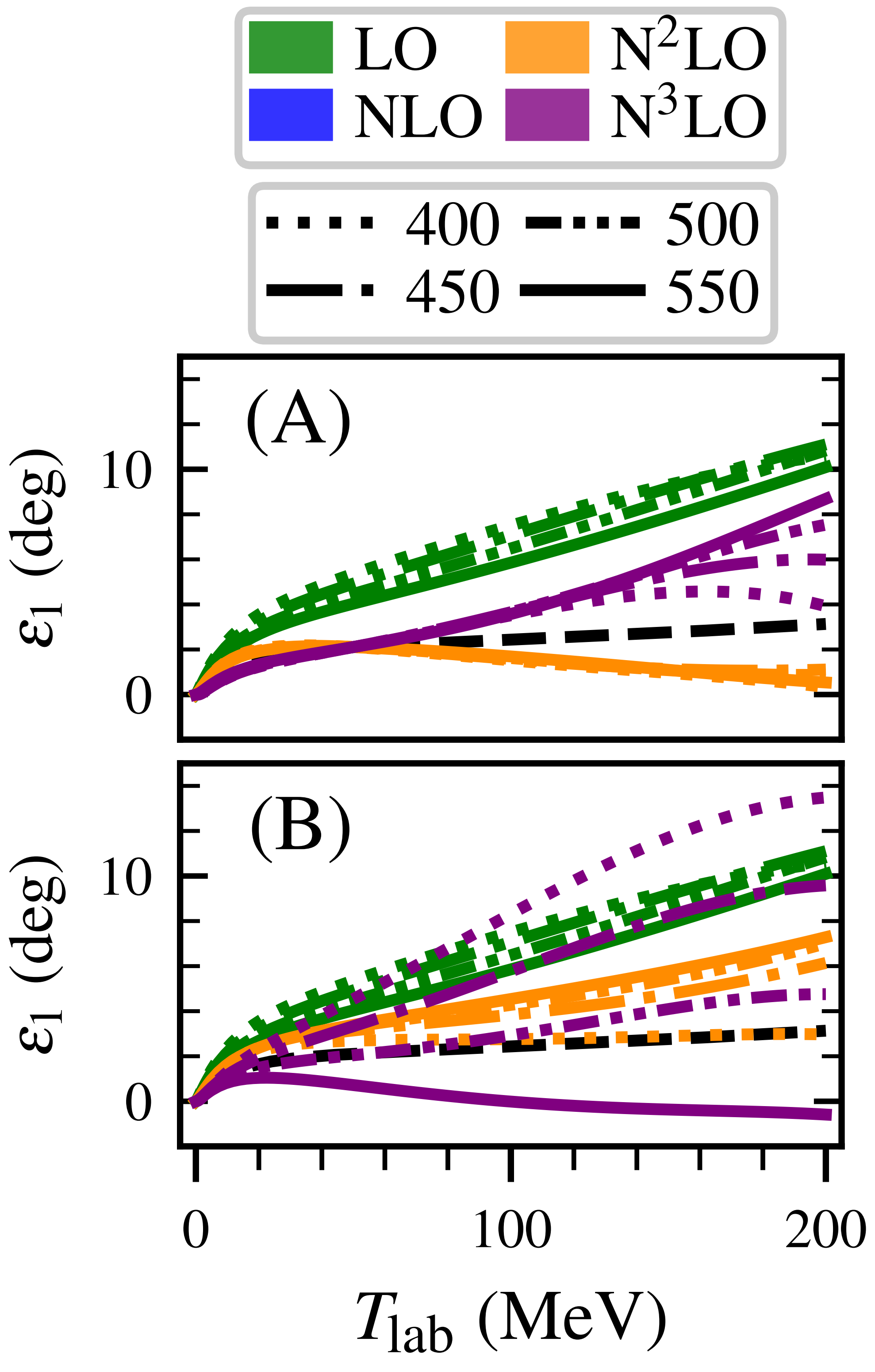
- Interaction **(A)**: Calibrated solely using NN **phase shifts**.
- Interaction **(B)**: Additionally use **few-body input**: ^{3}H @ N^{2,3}LO and ^{2}H @ N^{3}LO.

$\Longrightarrow$

- Eliminates cutoff dependence in ^{4}He and ^{6}Li.
- Induces cutoff dependence in ε_1 (largely influenced by the calibration scheme).



OT, A. Ekström, and C. Forssén, arXiv:2604.14985 (2026)



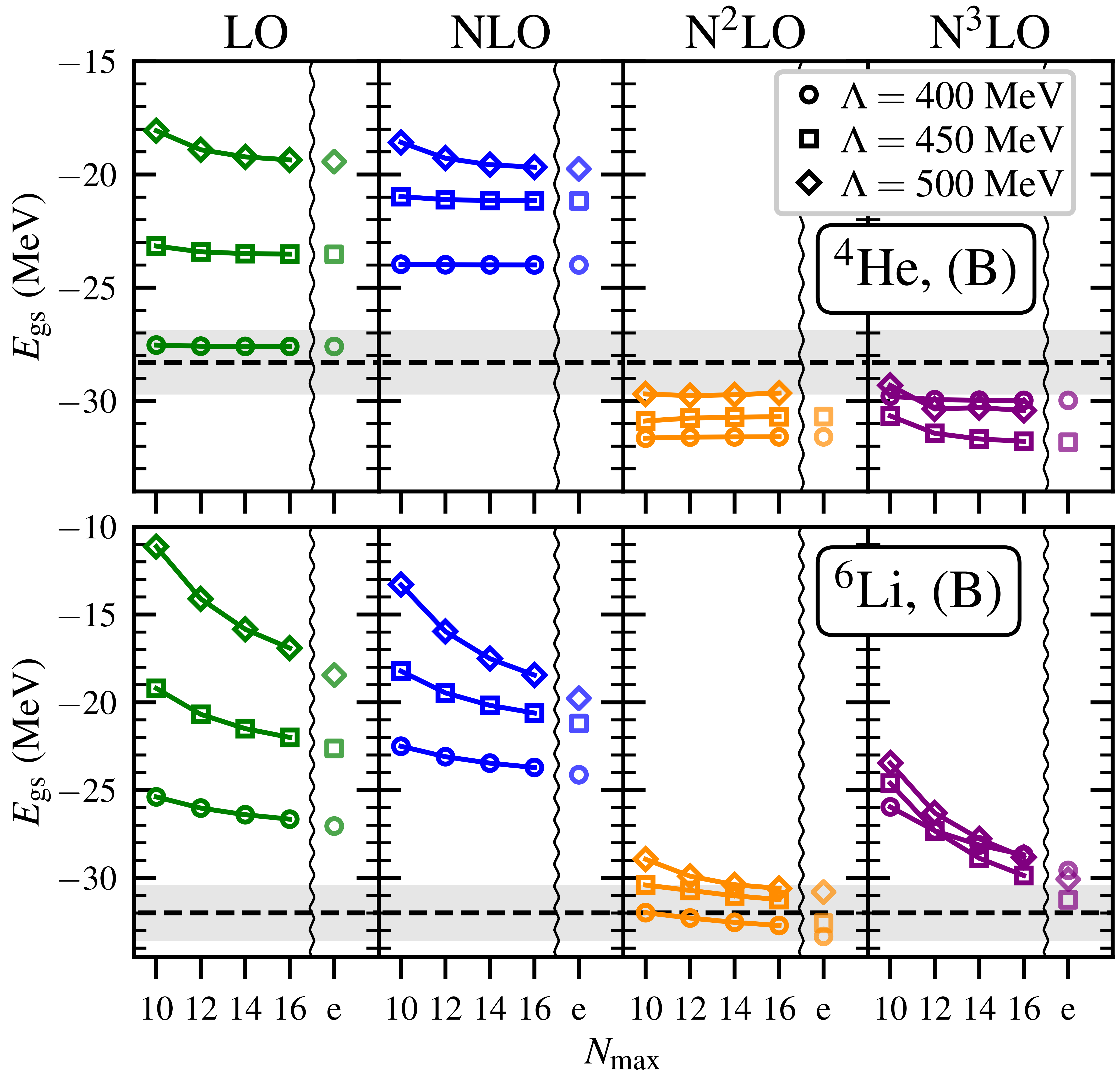
Predictions in ^{4}He and ^{6}Li

- Predictions at $\mathrm{N}^{2,3}$LO close to or **within expected EFT** truncation error.

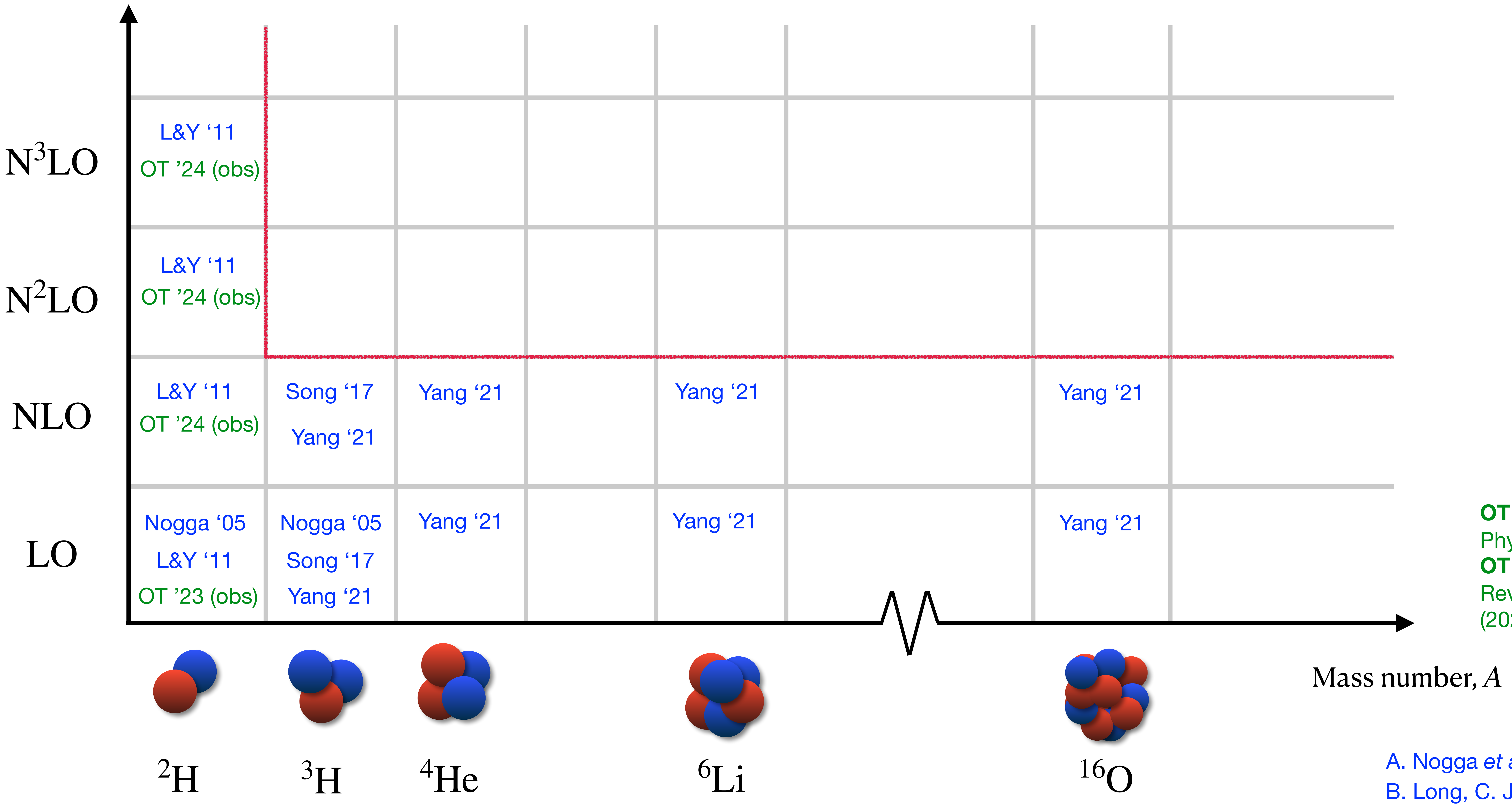
$$\sim \left(\frac{Q}{\Lambda_b}\right)^{\nu+1}$$

- Only **marginally slower** convergence in $N_{\max}$ at higher orders.

Predicted ground-state energies



Partly perturbative pions: progress

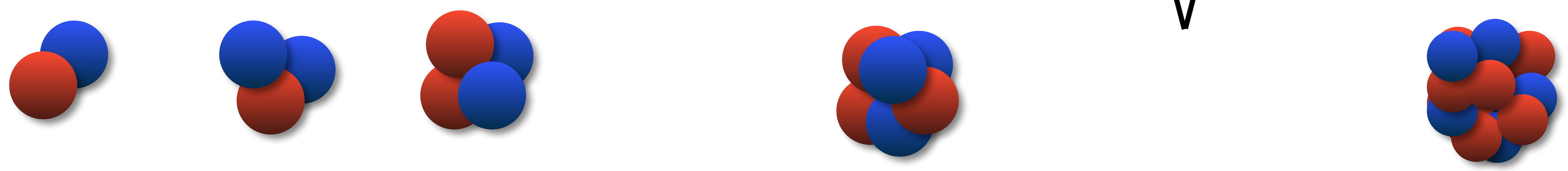


OT, E. May, A. Ekström, and C. Forssén, Phys. Rev. C **108**, (2023)
OT, A. Ekström, and C. Forssén, Phys. Rev. C **109**, (2024), Phys. Rev. C **112** (2025), arXiv:2604.14985 (2026)

A. Nogga *et al.*, Phys. Rev. C **72**, (2005)
B. Long, C. J. Yang, Phys. Rev. C **84**, (2011), Phys. Rev. C **85**, (2012), Phys. Rev. C **86**, (2012)
Y.-H. Song, R. Lazauskas, and U. van Kolck, Phys. Rev. C **96**, (2017)
C. J. Yang *et al.*, Phys. Rev. C **103**, (2021)

Partly perturbative pions: progress

	^{2}H	^{3}H	^{4}He	^{6}Li	^{16}O
N^{3}LO	L&Y '11 OT '24 (obs)	OT '26	OT '26	OT '26	
N^{2}LO	L&Y '11 OT '24 (obs)	OT '25	OT '26	OT '26	
NLO	L&Y '11 OT '24 (obs)	Song '17 Yang '21	Yang '21	Yang '21	Yang '21
LO	Nogga '05 L&Y '11 OT '23 (obs)	Nogga '05 Song '17 Yang '21	Yang '21	Yang '21	Yang '21



Mass number, A

OT, E. May, A. Ekström, and C. Forssén, Phys. Rev. C **108**, (2023)
OT, A. Ekström, and C. Forssén, Phys. Rev. C **109**, (2024), Phys. Rev. C **112** (2025), arXiv:2604.14985 (2026)

A. Nogga *et al.*, Phys. Rev. C **72**, (2005)
B. Long, C. J. Yang, Phys. Rev. C **84**, (2011), Phys. Rev. C **85**, (2012), Phys. Rev. C **86**, (2012)
Y.-H. Song, R. Lazauskas, and U. van Kolck, Phys. Rev. C **96**, (2017)
C. J. Yang *et al.*, Phys. Rev. C **103**, (2021)

Exceptional cutoffs

Origin of exceptional cutoffs

- Compute NN scattering amplitudes perturbatively in ${}^3S_1 - {}^3D_1$:

$$\text{LO:}\quad T^{(0)} = V^{(0)} + V^{(0)} G_0^{+} T^{(0)}$$

$$\text{NLO:}\quad \sout{T^{(1)} = \Omega_-^\dagger V^{(1)} \Omega_+}$$

$$\text{N}^2\text{LO:}\quad T^{(2)} = \Omega_-^\dagger \left(V^{(2)} + \sout{V^{(1)} G_1^{+} V^{(1)}} \right) \Omega_+$$

$$V^{(2)} = V^{(2)}_{2\pi} + V^{(2)}_{\rm ct}$$

$$V^{(2)}_{\rm ct} = \frac{1}{(2\pi)^3} \begin{pmatrix} C^{(1)}_{{}^3S_1} + D^{(0)}_{{}^3S_1}(p'^2 + p^2) & D^{(0)}_{SD} p^2 \\ D^{(0)}_{SD} p'^2 & 0 \end{pmatrix}$$

Compute phase shifts perturbatively:

$$\delta^{(2)}(k) = -\frac{\rho(k) T^{(2)}(k,k;k)}{2i \exp\left(2i\delta^{(0)}(k)\right)}$$

Renormalization conditions:

$$\begin{matrix} {}^3S_1: \\ {}^3S_1: \\ \epsilon_1: \end{matrix} \begin{pmatrix} \delta_0^{(0)}(k_1) + \delta_0^{(2)}(k_1) \\ \delta_0^{(0)}(k_2) + \delta_0^{(2)}(k_2) \\ \epsilon^{(0)}(k_2) + \epsilon^{(2)}(k_2) \end{pmatrix} = \begin{pmatrix} \delta_{0,\rm exp}(k_1) \\ \delta_{0,\rm exp}(k_2) \\ \epsilon_{\rm exp}(k_2) \end{pmatrix}$$

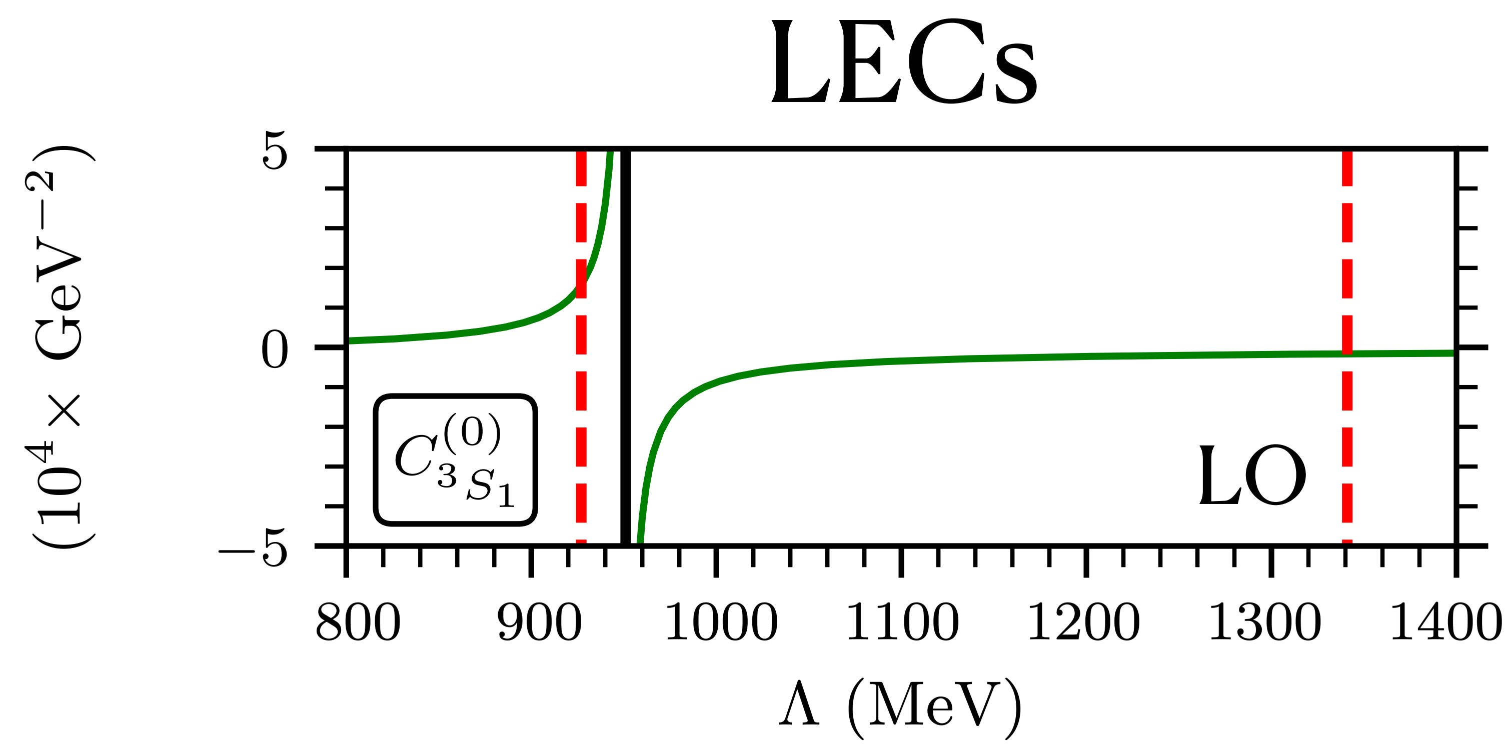
$$\left(\vec{\alpha}^{(2)}\right)^T \equiv \left(C^{(1)}_{{}^3S_1}, D^{(0)}_{{}^3S_1}, D^{(0)}_{SD}\right)$$

$$A_\Lambda \vec{\alpha}^{(2)}(\Lambda) = \vec{\delta}_\Lambda$$

Linear equation for the LECs!

A. M. Gasparyan and E. Epelbaum, Phys. Rev. C. **107** (2023)

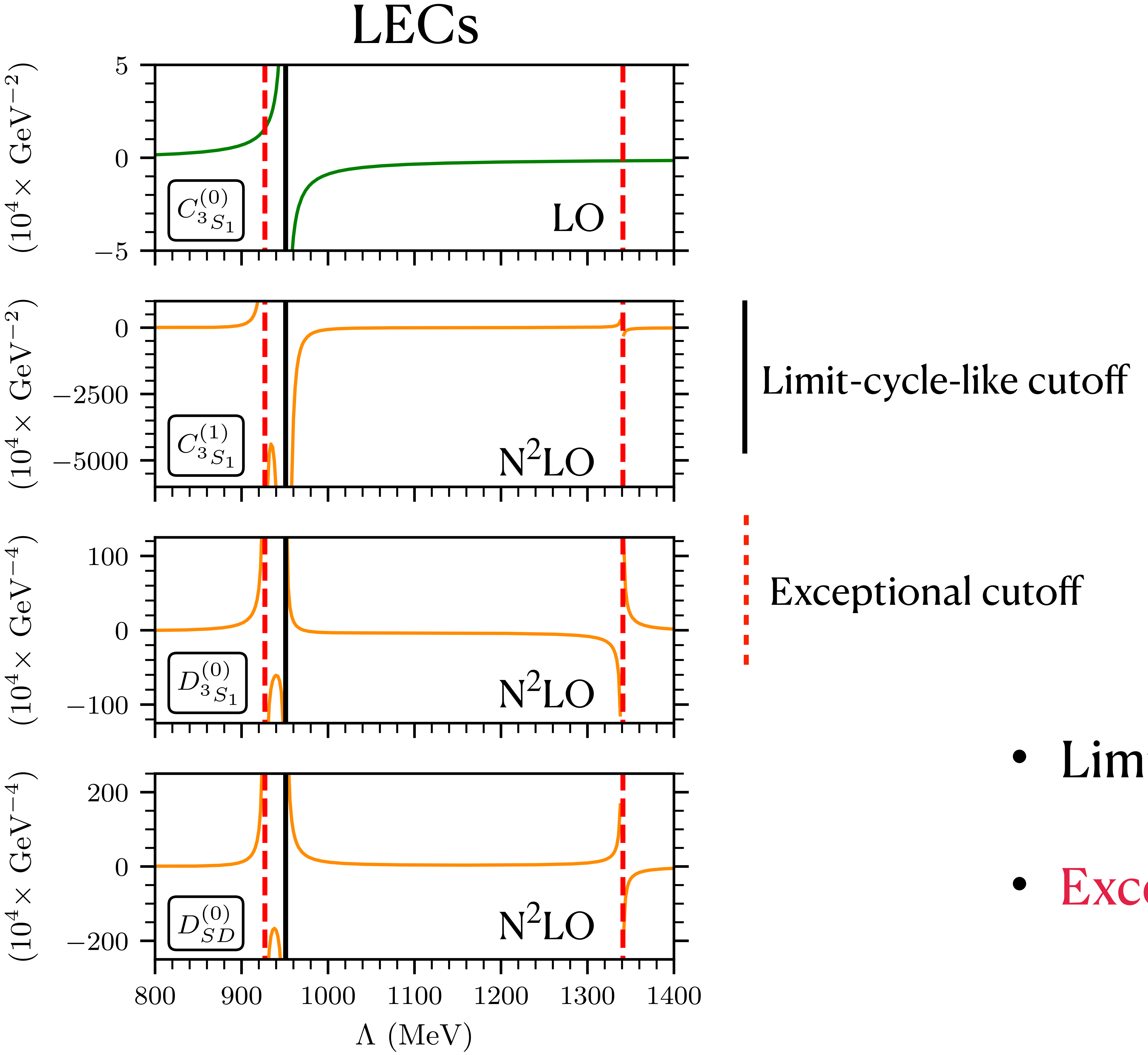
Exceptional cutoffs in $^3S_1 - {}^3D_1$



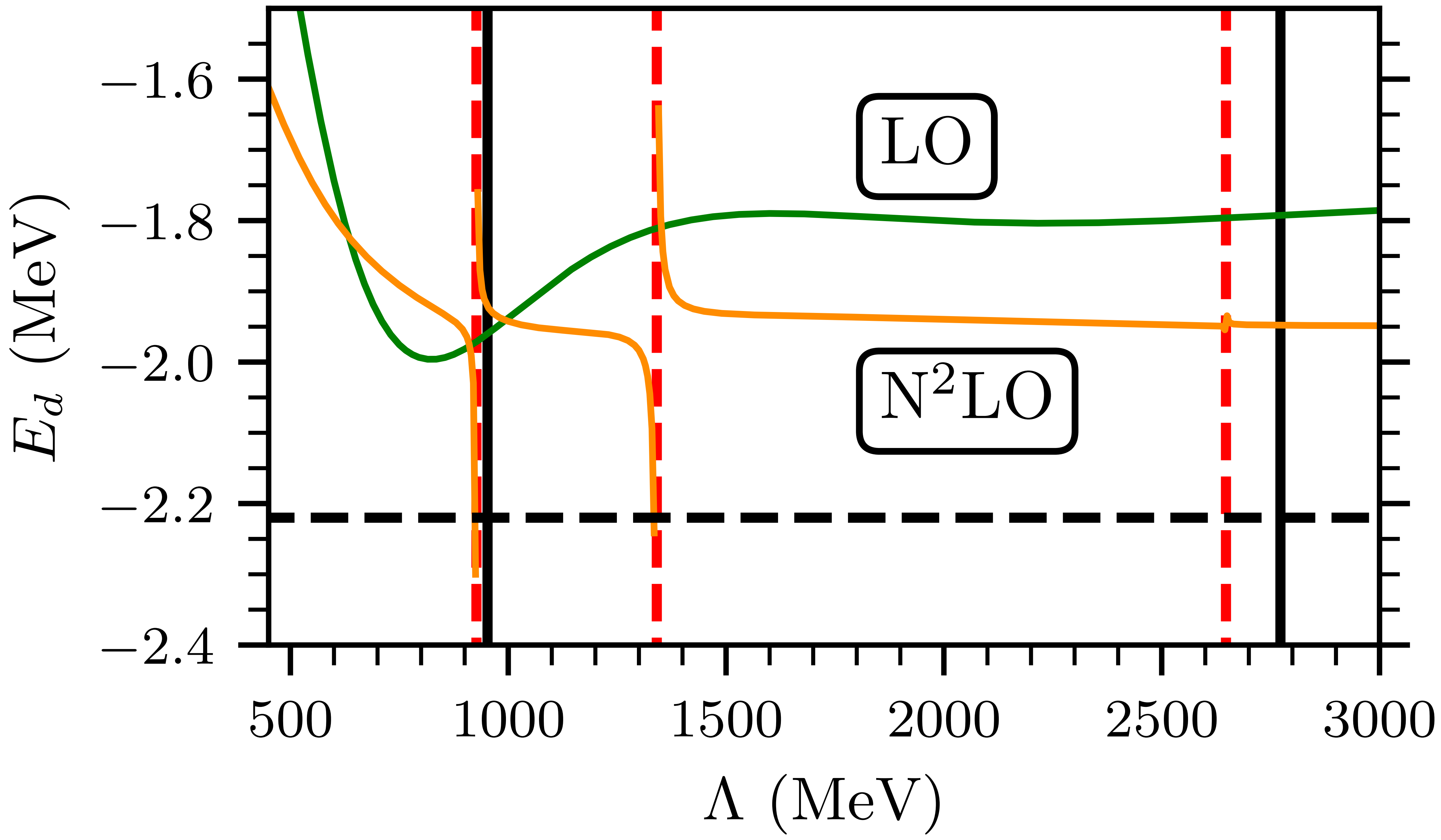
Limit-cycle-like cutoff

Exceptional cutoff

Exceptional cutoffs in $^3S_1 - {}^3D_1$



Deuteron prediction:



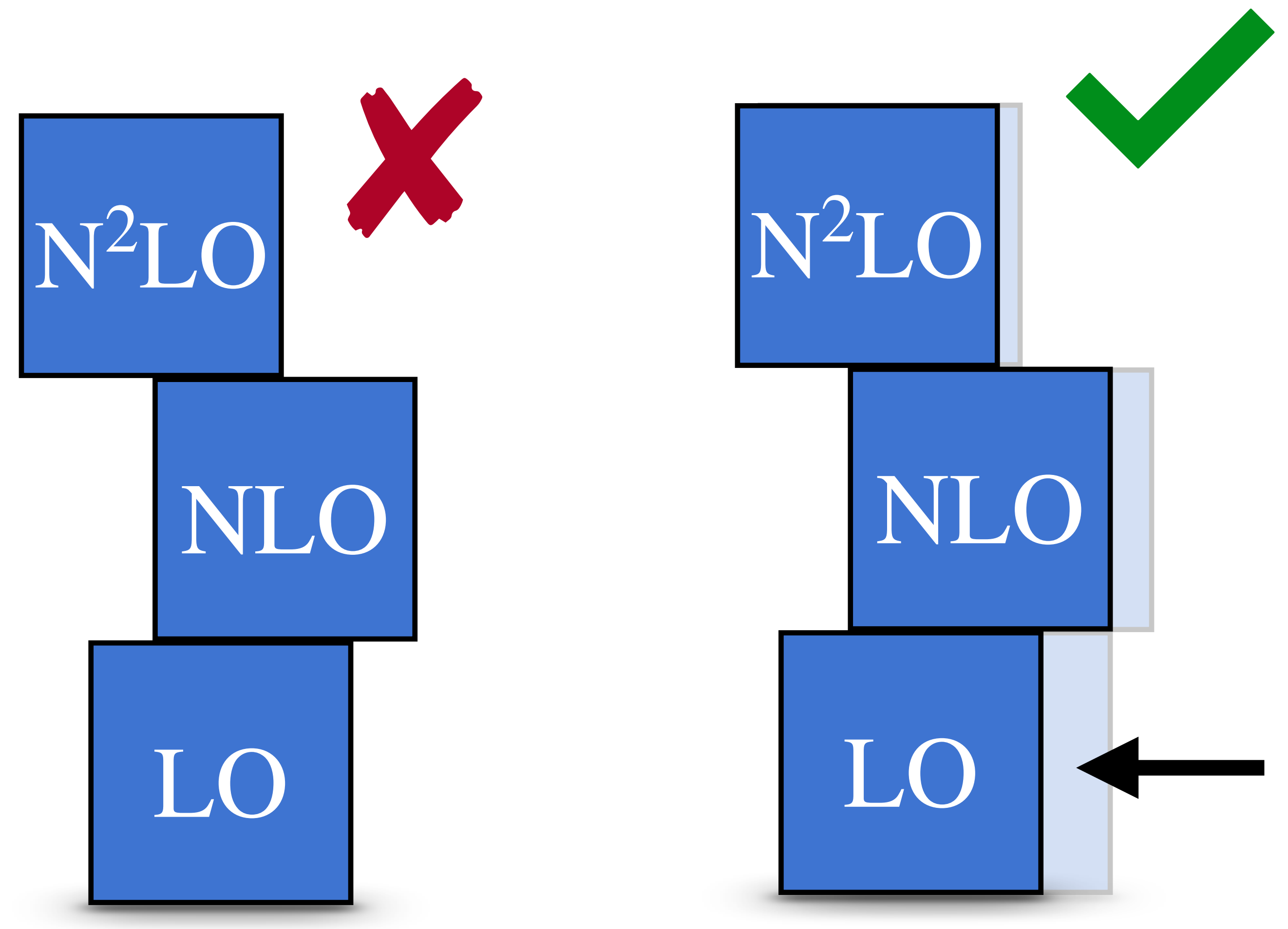
- Limit-cycle-like cutoffs — no divergence in predictions.
- Exceptional cutoffs — diverging predictions.

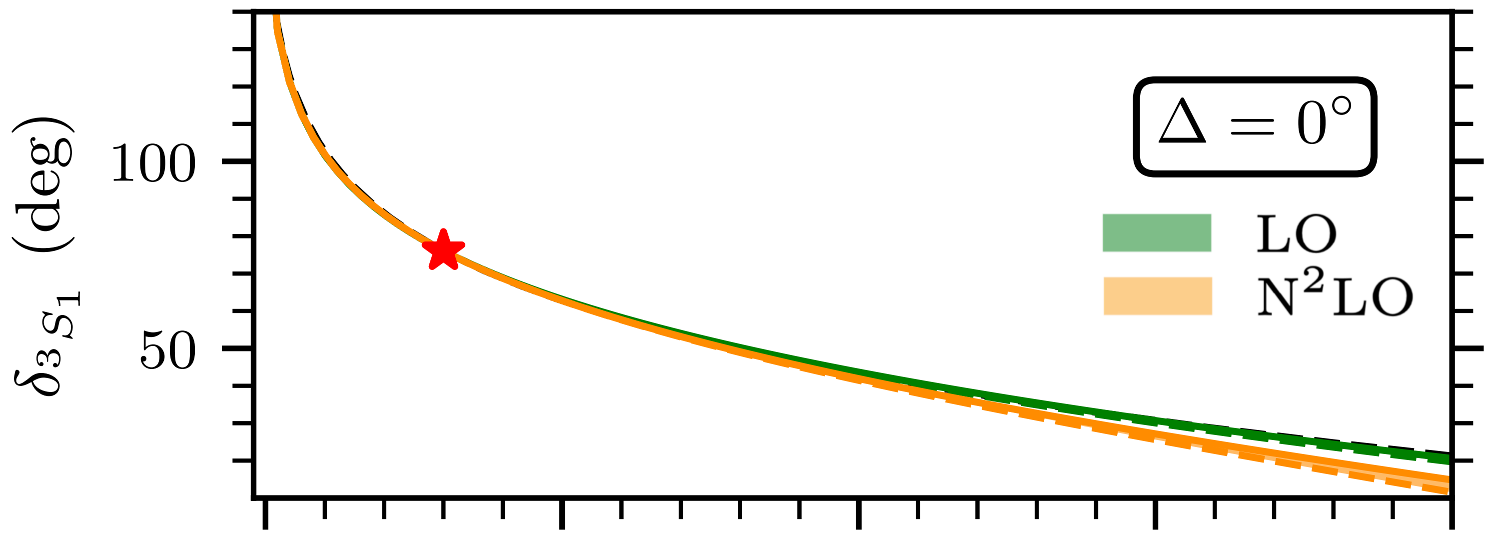
OT, A. Ekström, and C. Forssén, Phys. Rev. C **112** (2025)

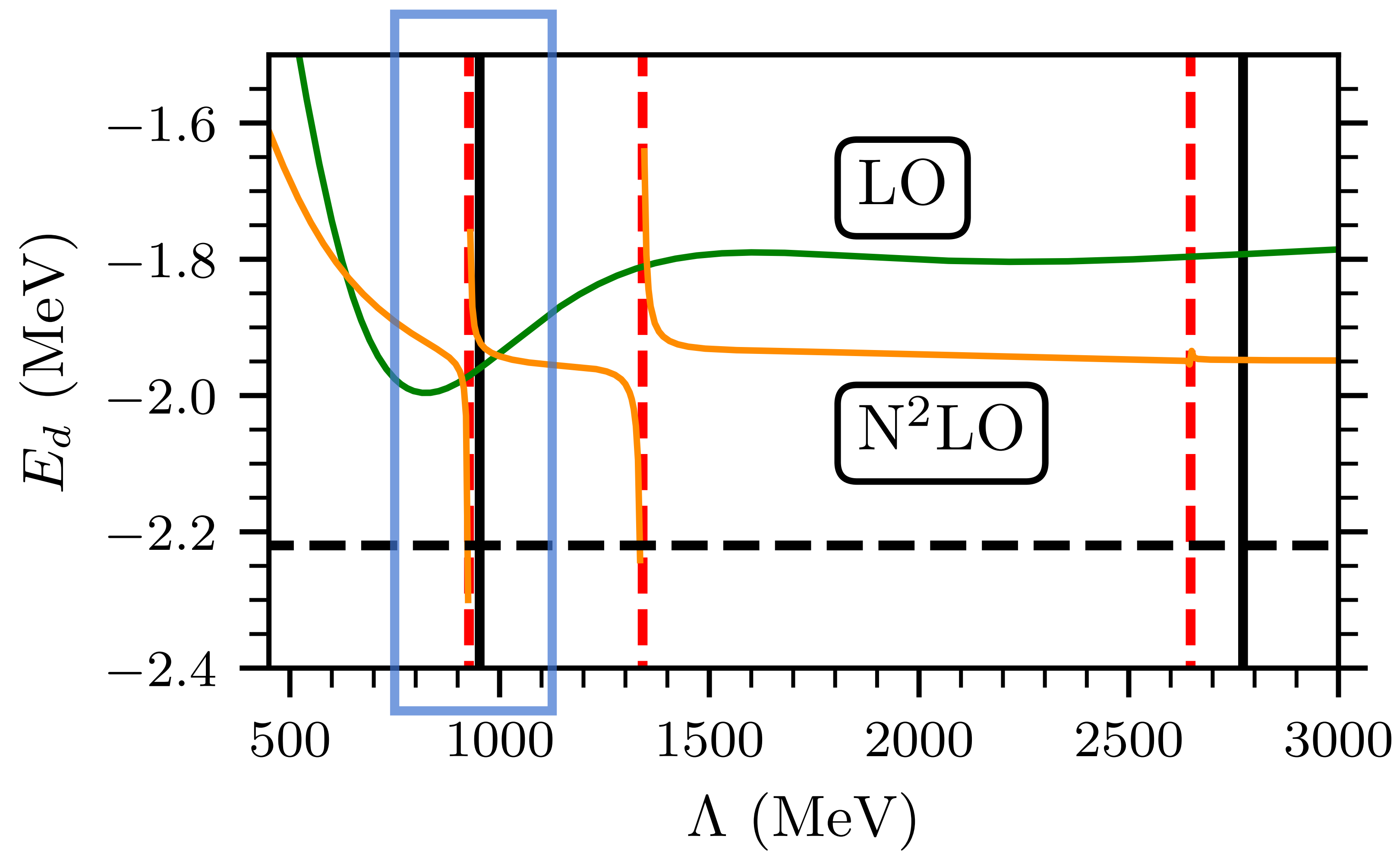
Deuteron at N^{2}LO

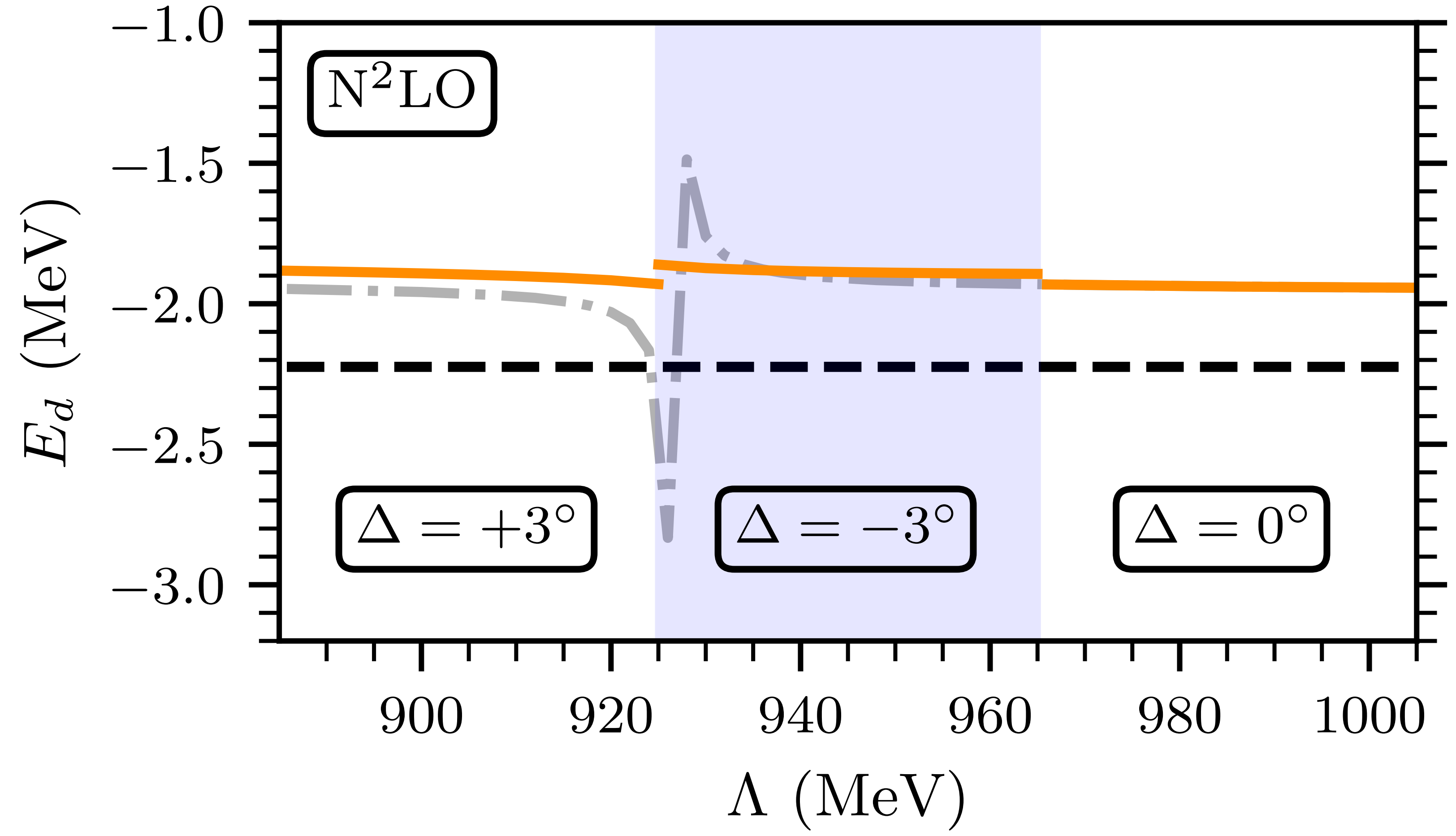
Suggested solution: **Modify LO** within the EFT error.

R. Peng, B. Long, and F.-R. Xu, Phys. Rev. C **110** (2024)



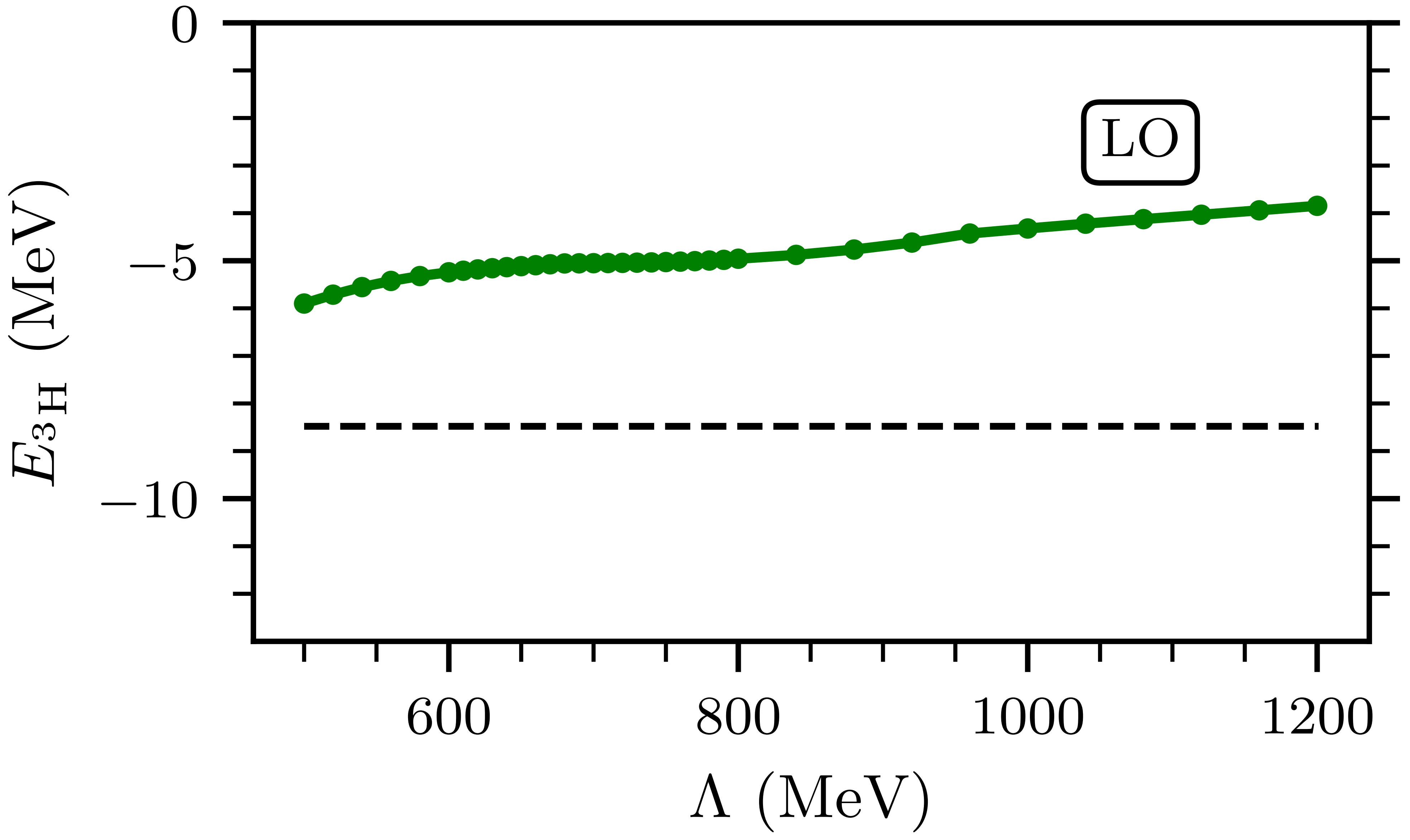




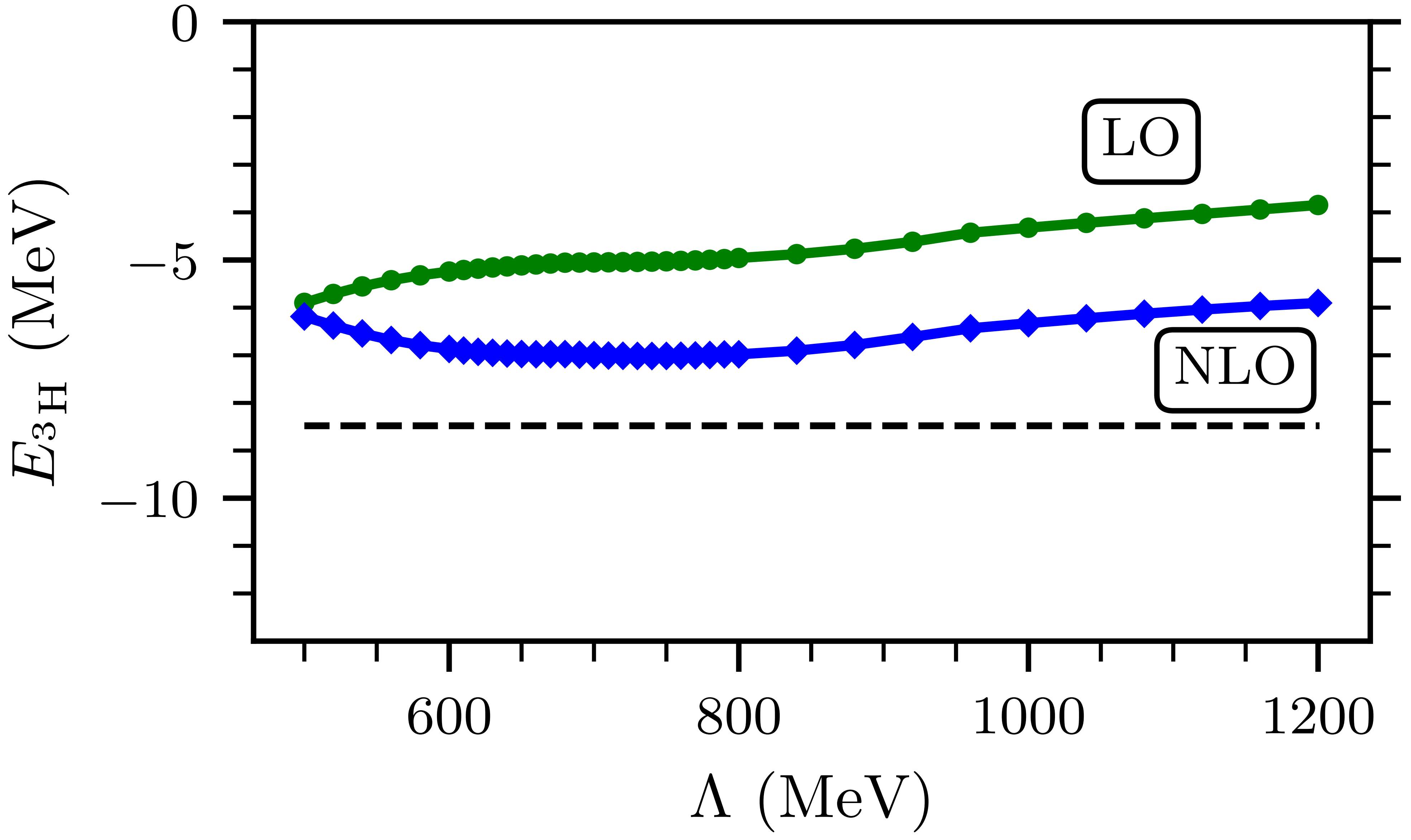


- Divergence mitigated — within EFT error.

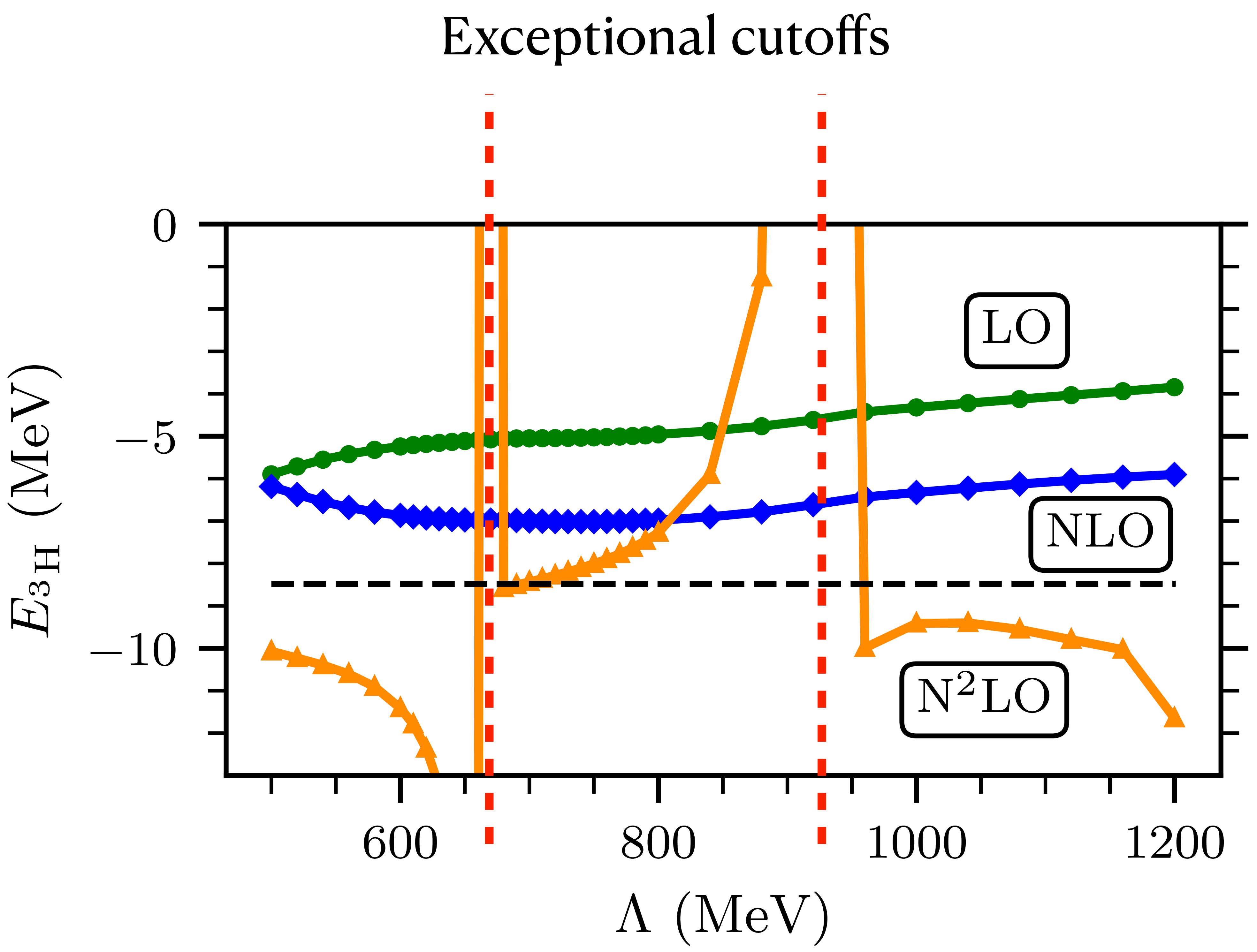
Cutoff dependence of ^{3}H ground state energy

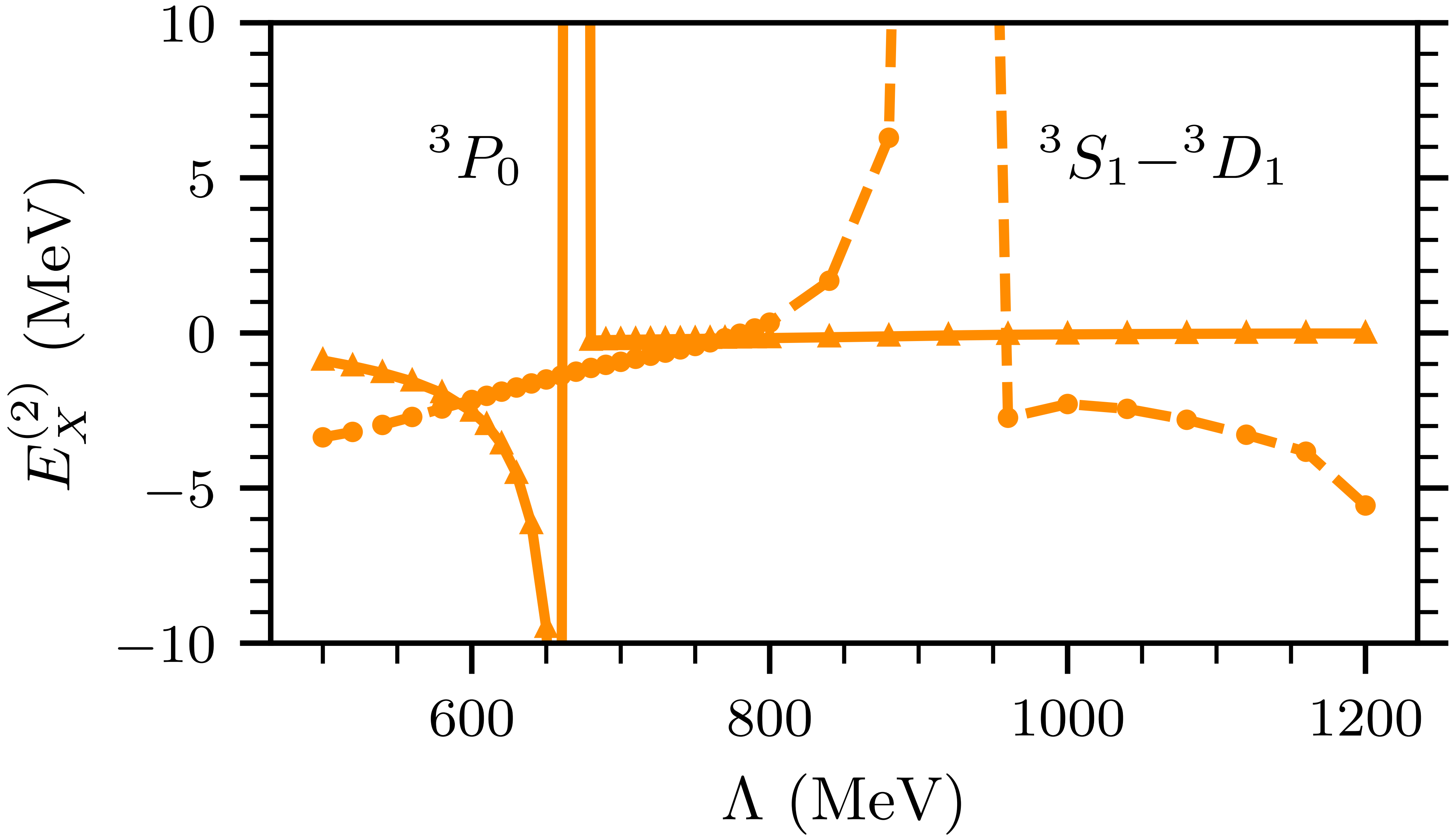


Cutoff dependence of ^{3}H ground state energy



Cutoff dependence of ^{3}H ground state energy

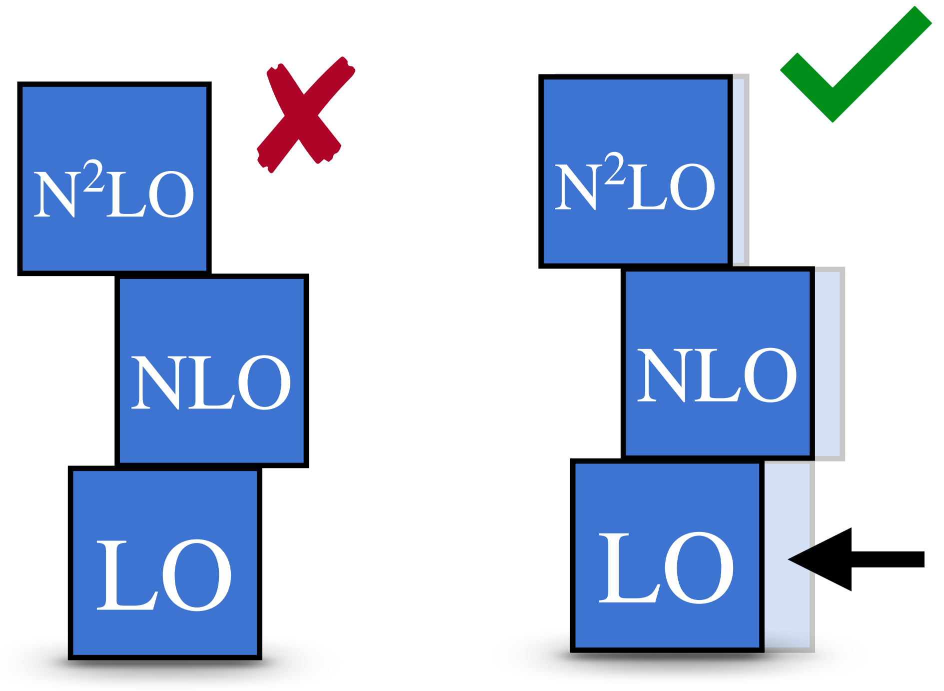


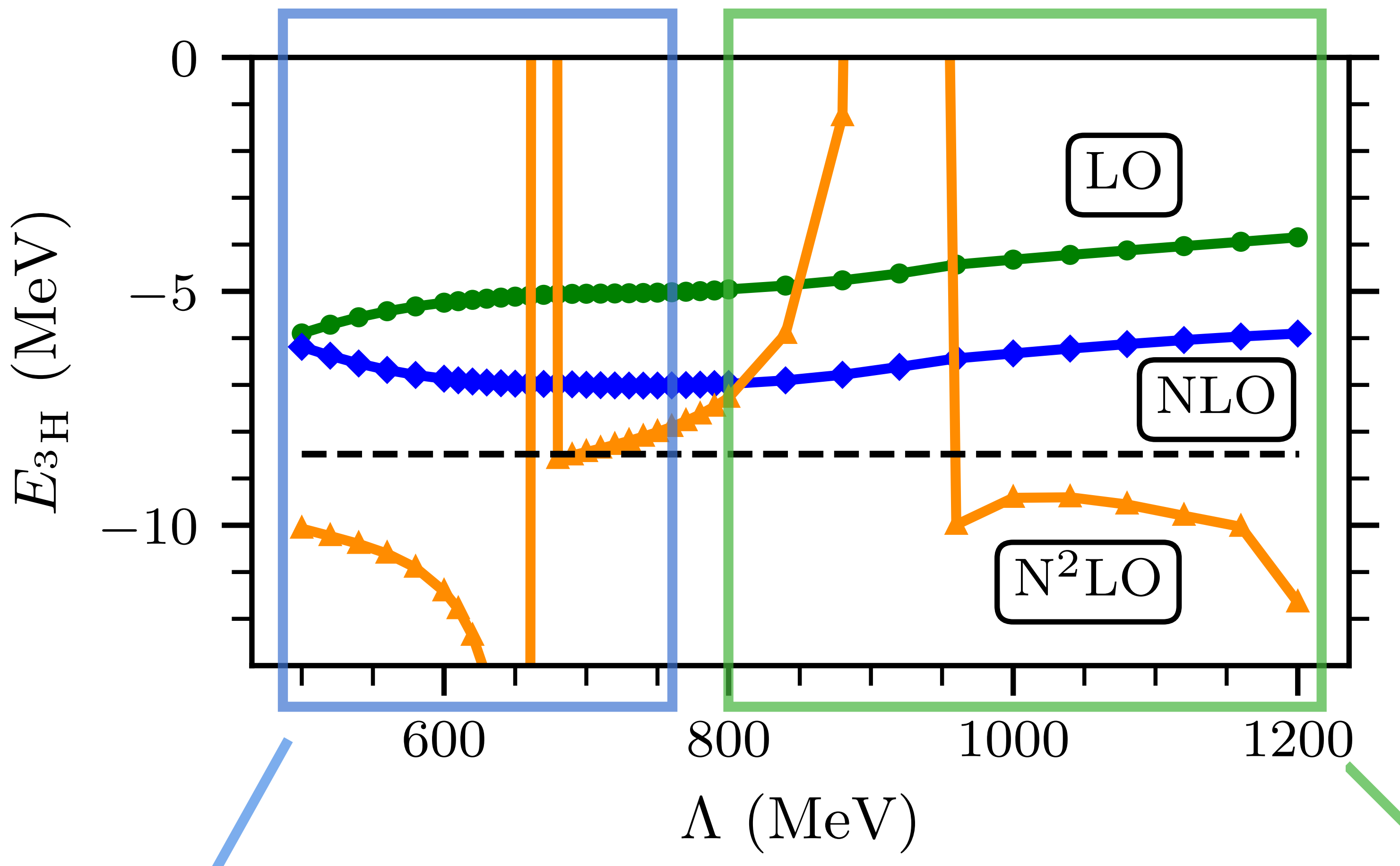


$$E_X^{(2)} \equiv \langle \Psi_0^{(0)} | \sum_{i<j=1}^{3} P_X V_{ij}^{(2)} P_X | \Psi_0^{(0)} \rangle + \sum_{m \neq 0} \frac{|\langle \Psi_0^{(0)} | \sum_{i<j=1}^{3} P_X V_{ij}^{(1)} P_X | \Psi_m^{(0)} \rangle|^2}{E_0^{(0)} - E_m^{(0)}}$$

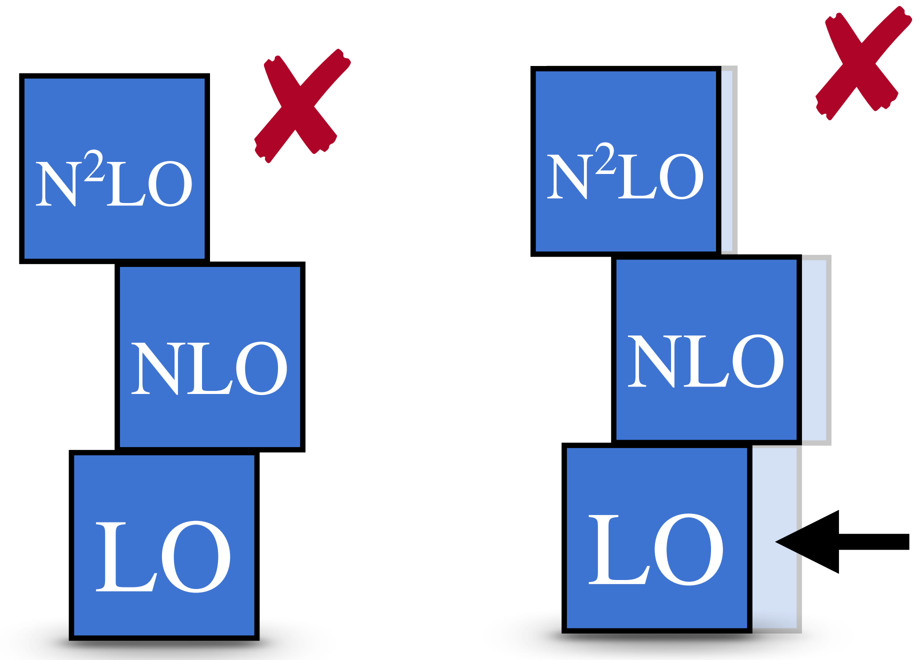
Apply the same principle as in the deuteron

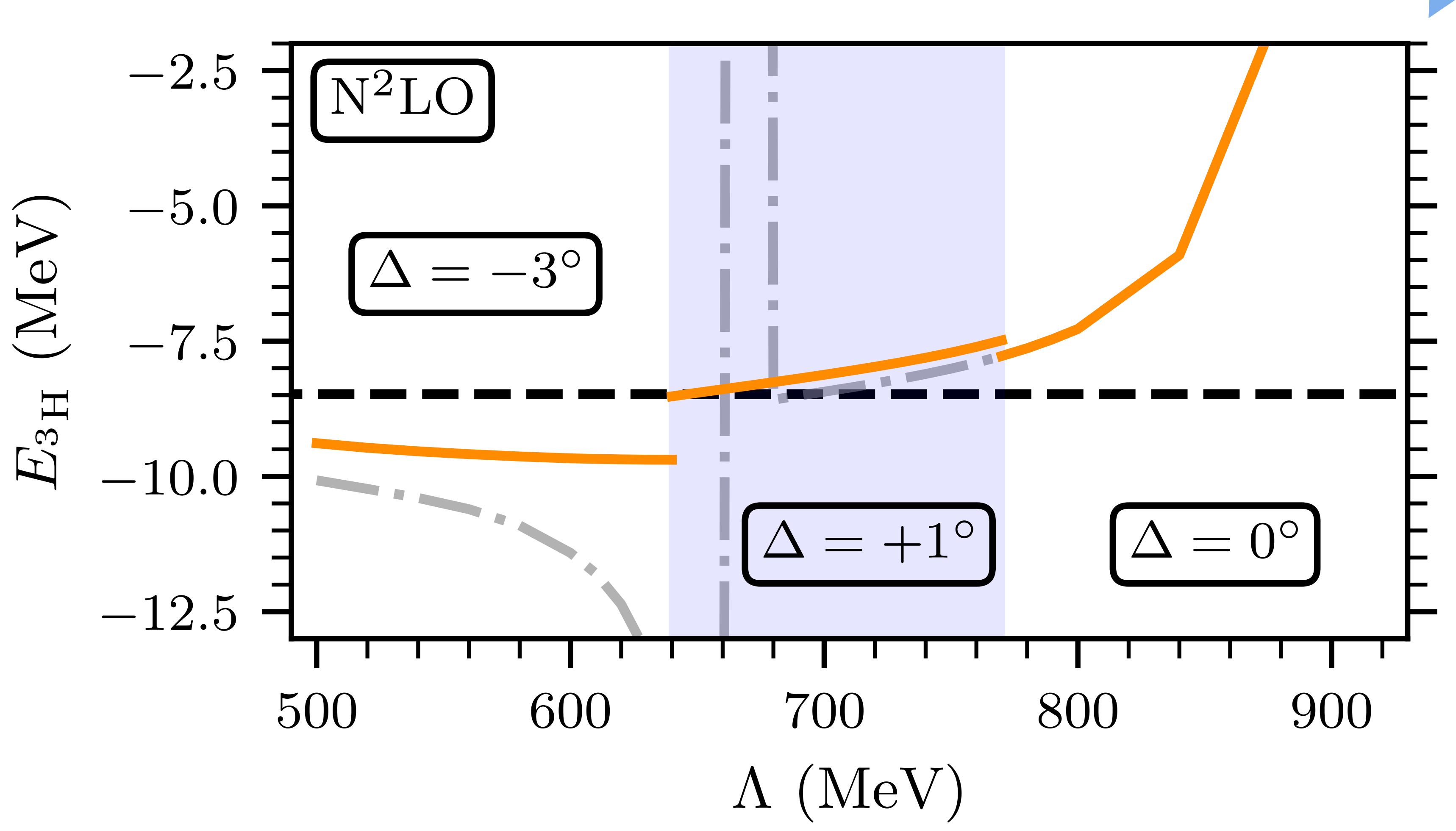
In $^3P_0$

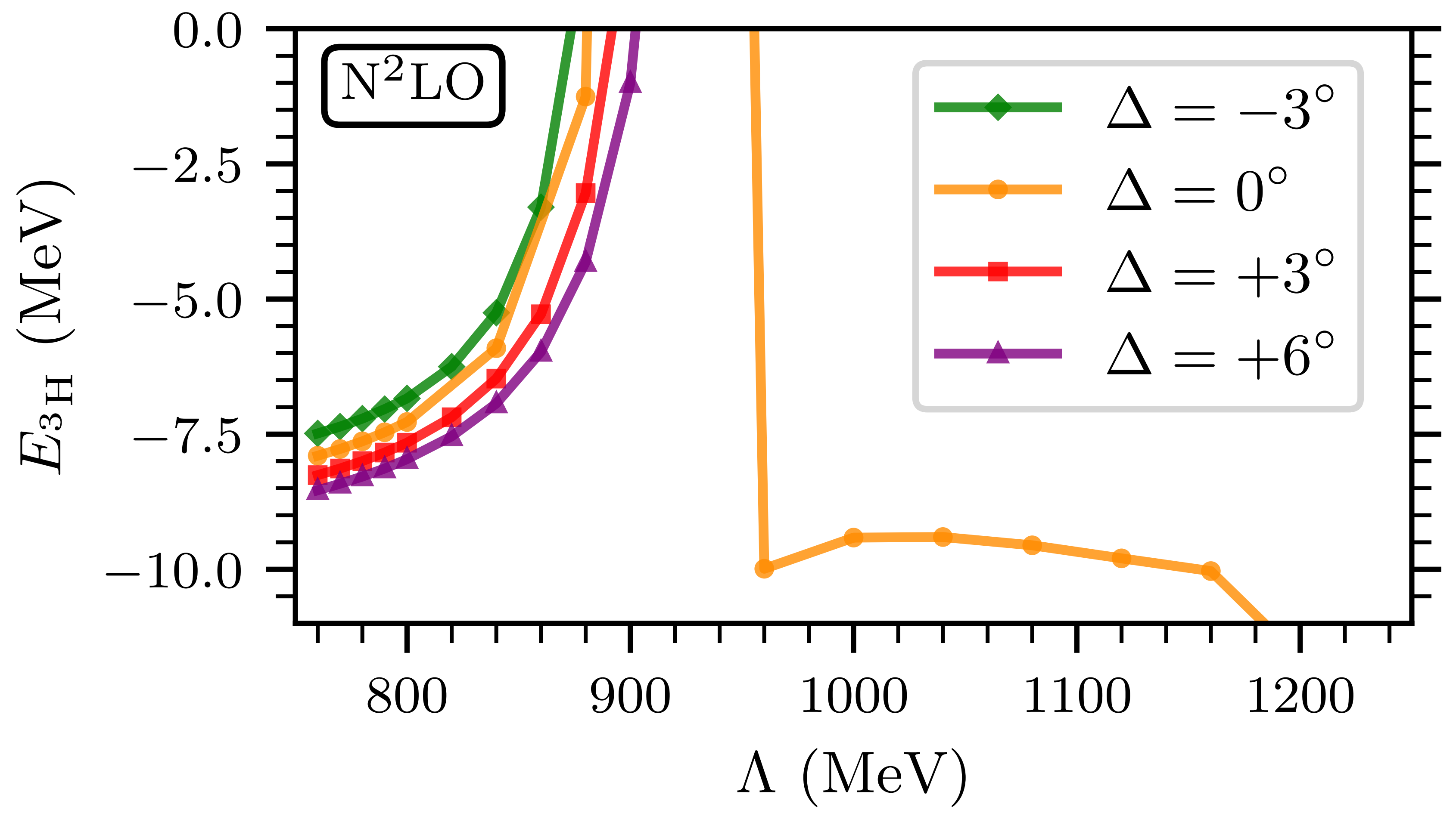




In $^3S_1 - {}^3D_1$







Is this the end?

- Origin of exceptional cutoffs:

A. M. Gasparyan and E. Epelbaum, Phys. Rev. C. **107** (2023)

$$A_\Lambda \vec{\alpha}^{(2)}(\Lambda) = \vec{\delta}_\Lambda$$

Linear equation for the LECs!

- A_Λ — related to derivatives of the LO NN wave function at $r = 0$.

- Appearance of exceptional cutoffs related to a **non-local** LO interaction, which induces a non-trivial **energy dependence**.

M. Pavon Valderrama, Regulator constraints for the perturbative renormalizability of attractive triplets, arXiv:2509.23855 (2025)

  - Indicates that the problem is with the regulation in the PC.
  - Interesting to investigate other regulations.

Summary

Part I / II

- Chiral symmetry constrains the NN interaction.
- Choosing a Power Counting + experimental data $\Longrightarrow V_{\mathrm{NN}}$ (`nn-mwpc`).
- Showed that the long-range potential fulfills Low Energy Theorems.

Part III

- Perturbative computations in ^{3}H up to N^{3}LO: `py-ncsm`!
- Demonstrate reliable computations with **partly perturbative pions** up to ^{6}Li at N^{3}LO using the NCSM.
- Exceptional cutoffs cause divergences in the ^{3}H computations.

Directions to explore further

- Include **three-nucleon** forces and **isospin breaking**.
- Further study the **exceptional cutoffs**.
- Include **uncertainty quantification**.
- Investigate other **perturbative** schemes.

Thank you!