

Constraining nuclear interactions with multi-messenger observations

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9/23/2025, INT program 25-2b: From Colliders to the Cosmos: Exploring the Extremes of Matter
with Experiment and Astrophysical Observation

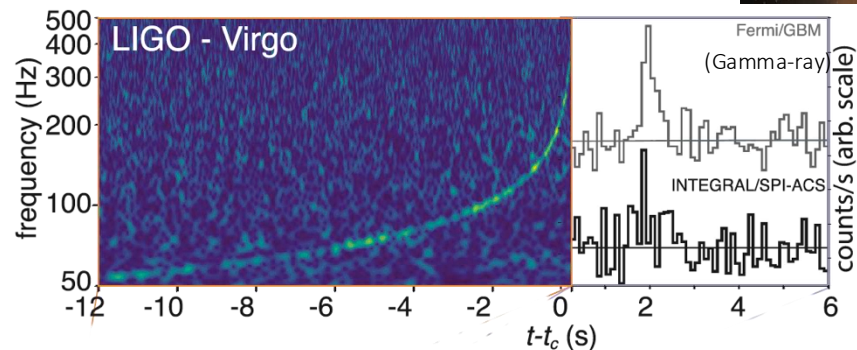
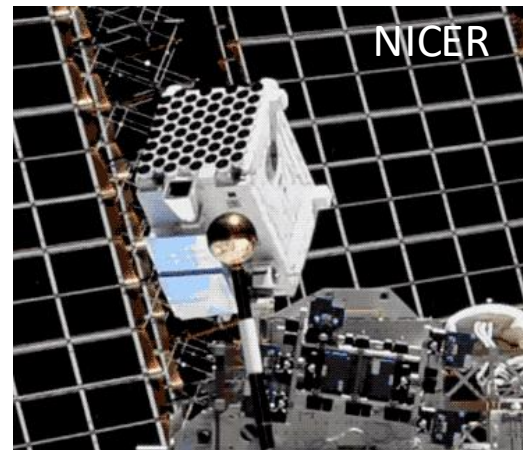
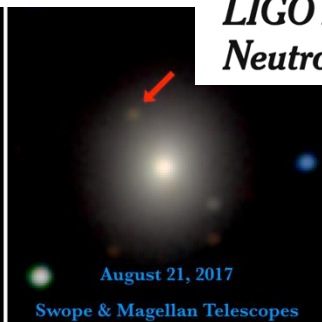
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Neutron Stars

First neutron-star merger
observed on Aug 17, 2017 :

The New York Times

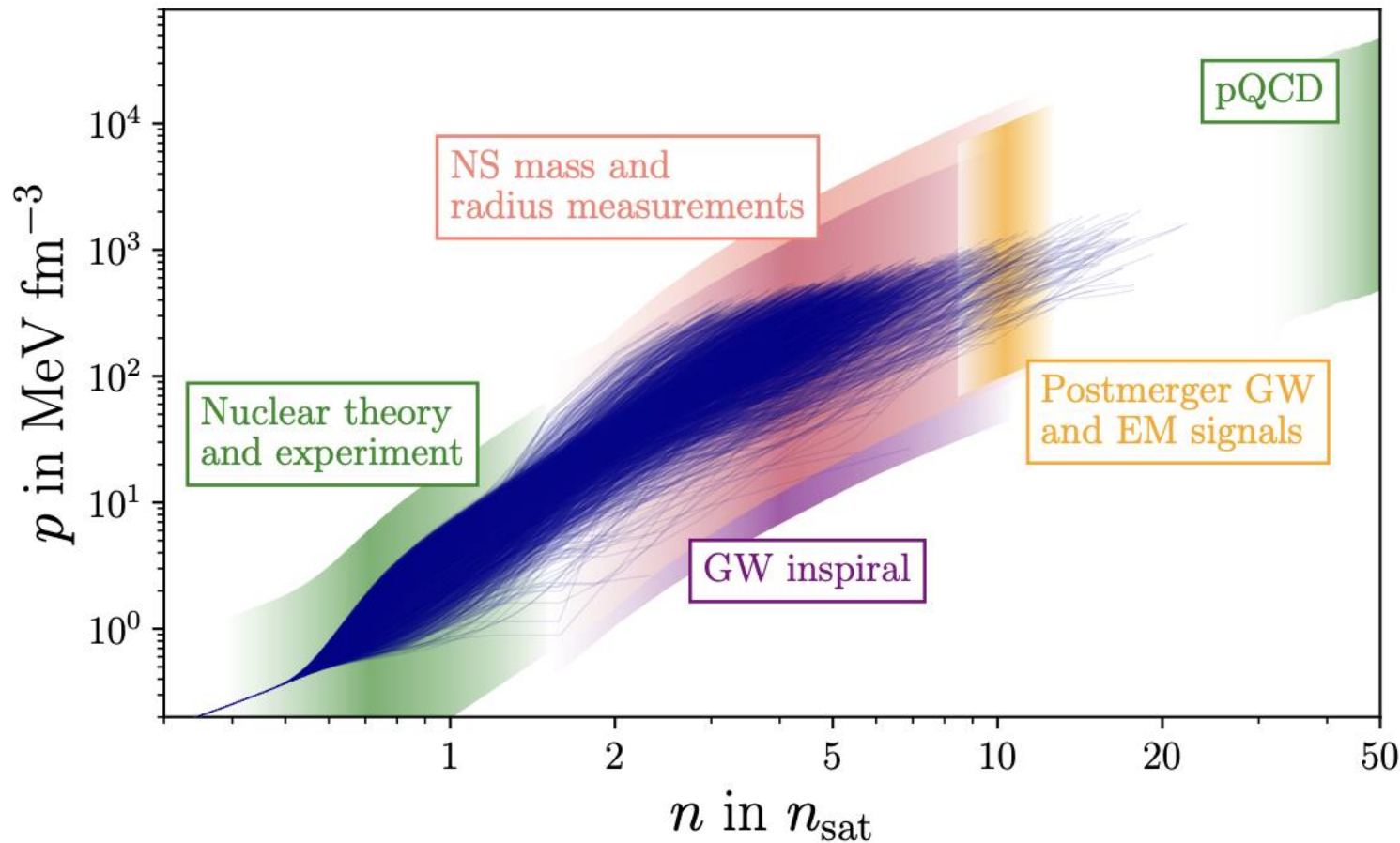
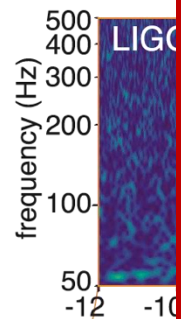
*LIGO Detects Fierce Collision of
Neutron Stars for the First Time*



LIGO/VIRGO collaboration, ApJL 848, L12 (2017)

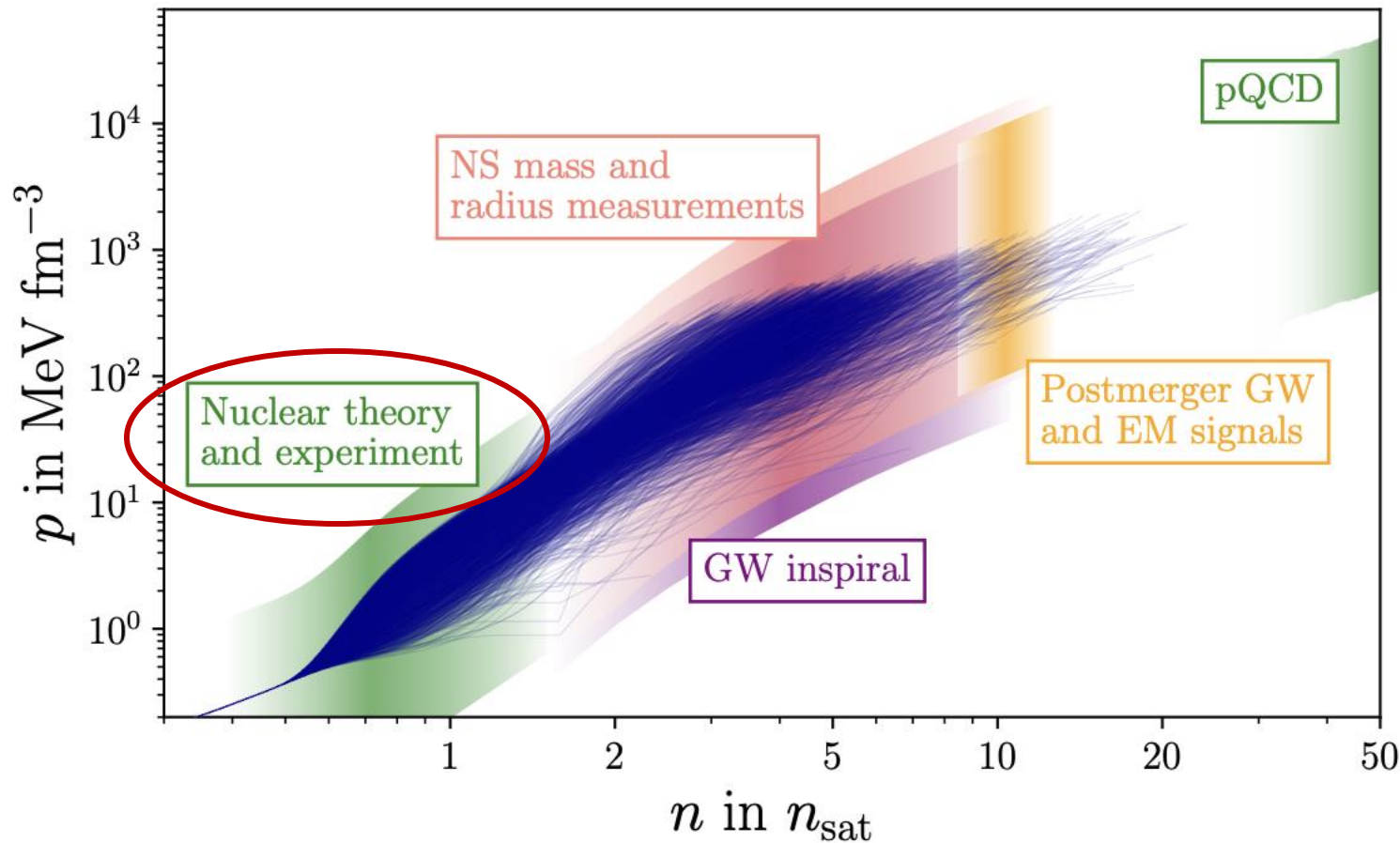
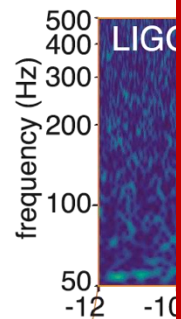
Neu

First neu
observed



Neu

First neu
observed



Neutron Stars

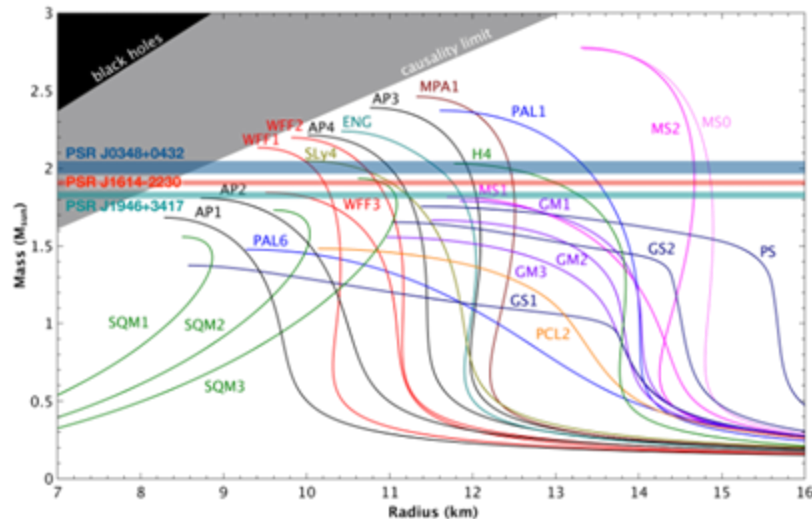
Neutron stars are massive stellar remnants (up to $2 M_{\text{sol}}$) but with radii of only ~ 12 km.

Very large densities, of the order of several times the nuclear saturation density.

Neutron-rich matter in their outer cores.

Stabilized by strong interactions (nuclear forces).

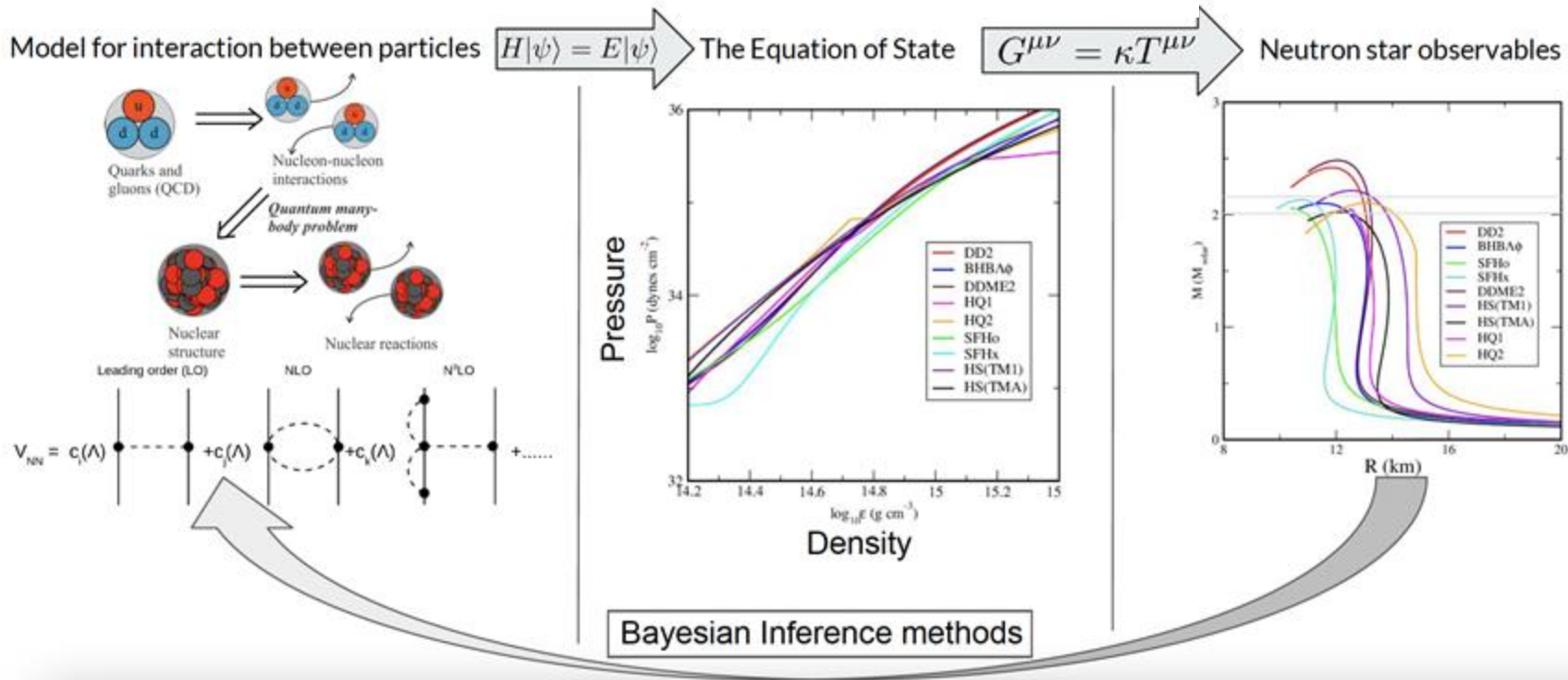
The equation of state (EOS) describes how nuclear forces in the nuclear many-body system determine the properties of the stars.



**Want to (a) determine the EOS with robust uncertainties
and (b) learn about nuclear interactions.**



Neutron Stars



Want to (a) determine the EOS with robust uncertainties and (b) learn about nuclear interactions.



Equation of state

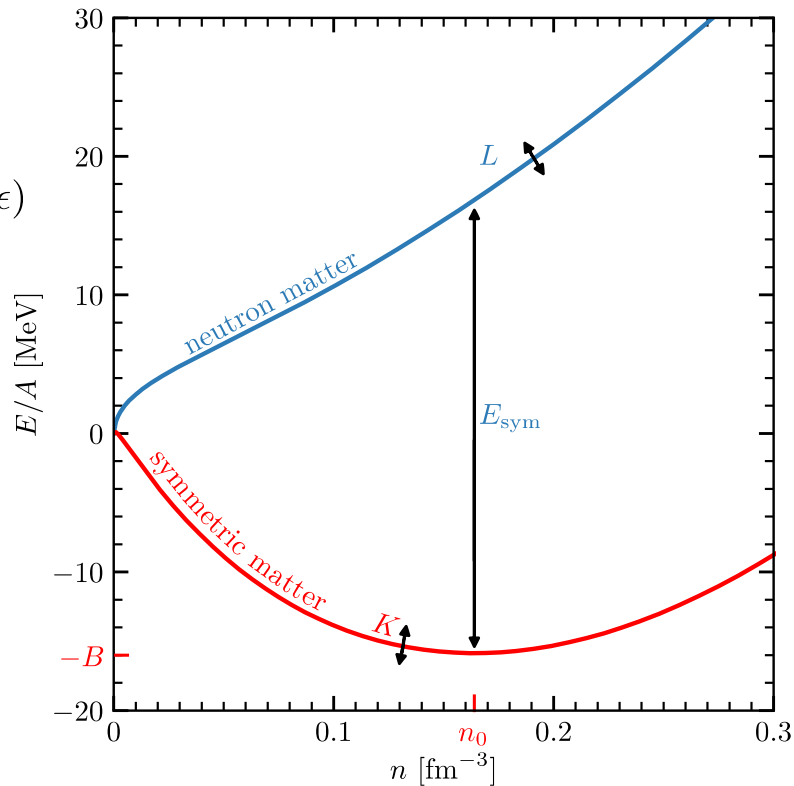
Neutron-star structure depends on the Equation of state $p = p(\epsilon)$

- Baryon density: $n = \frac{A}{V}$
- Energy density: $\epsilon = \frac{E}{V} = n \cdot \frac{E}{A}$
- Pressure: $p = -\frac{\partial E}{\partial V} = -\frac{\partial E/A}{\partial V/A} = n^2 \frac{\partial E/A}{\partial n}$

We can describe a fluid of neutrons and protons by

$$\frac{E}{A}(n, x)$$

where x is the proton fraction, $x = n_p/n$.



- $x = 0.5$: Symmetric nuclear matter: Connection to **laboratory experiments**
- $x = 0.0$: Pure neutron matter: Connection to **astrophysical observations**.
- Difference: Symmetry energy: Connection to **heavy-ion collisions, neutron skins, ...**

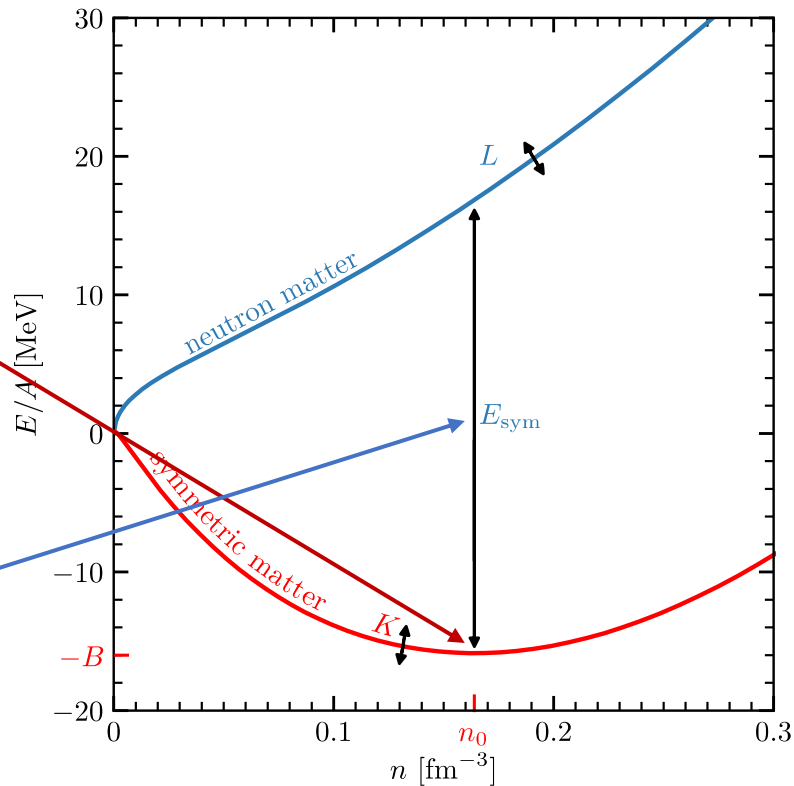


Equation of state

The energy per particle can be Taylor expanded:

$$e_{\text{sat}}(n) = E_{\text{sat}} + K_{\text{sat}} \frac{x^2}{2} + Q_{\text{sat}} \frac{x^3}{3!} + Z_{\text{sat}} \frac{x^4}{4!} + \dots, \quad x := \frac{n - n_{\text{sat}}}{3n_{\text{sat}}}$$

$$e_{\text{sym}}(n) = E_{\text{sym}} + L_{\text{sym}}x + K_{\text{sym}} \frac{x^2}{2} + Q_{\text{sym}} \frac{x^3}{3!} + Z_{\text{sym}} \frac{x^4}{4!} + \dots,$$

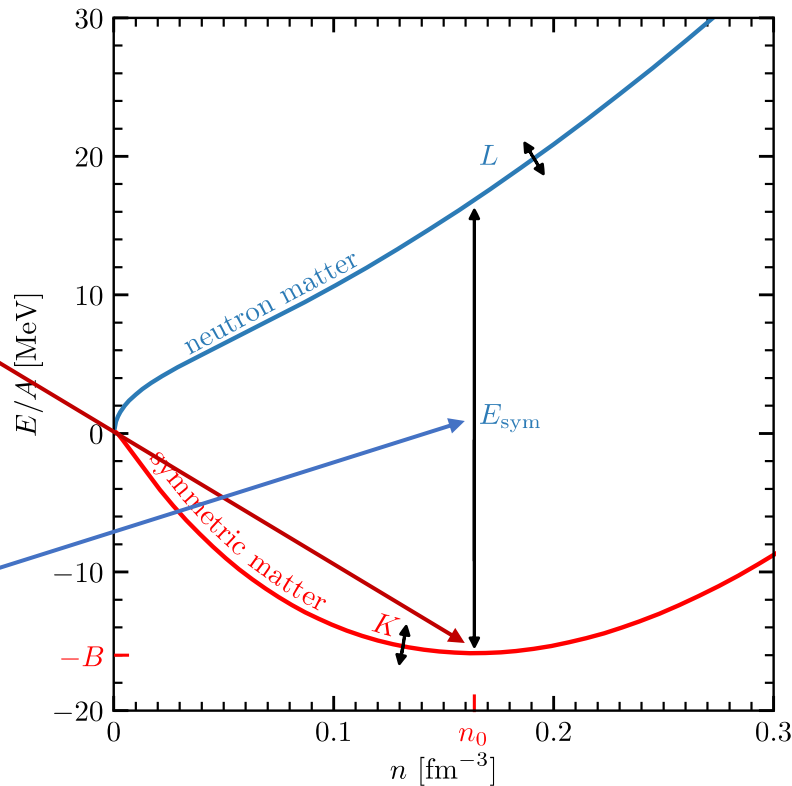


Equation of state

The energy per particle can be Taylor expanded:

$$e_{\text{sat}}(n) = \underbrace{E_{\text{sat}}}_{\text{binding energy per nucleon}} + \underbrace{K_{\text{sat}}}_{\text{incompressibility}} \frac{x^2}{2} + \underbrace{Q_{\text{sat}}}_{\text{symmetry energy}} \frac{x^3}{3!} + \underbrace{Z_{\text{sat}}}_{\text{curvature}} \frac{x^4}{4!} + \dots, \quad x := \frac{n - n_{\text{sat}}}{3n_{\text{sat}}}$$

$$e_{\text{sym}}(n) = \underbrace{E_{\text{sym}}}_{\text{symmetry energy}} + \underbrace{L_{\text{sym}}}_{\text{symmetry parameter}} x + \underbrace{K_{\text{sym}}}_{\text{symmetry incompressibility}} \frac{x^2}{2} + \underbrace{Q_{\text{sym}}}_{\text{symmetry curvature}} \frac{x^3}{3!} + \underbrace{Z_{\text{sym}}}_{\text{symmetry curvature}} \frac{x^4}{4!} + \dots,$$



- These are the **empirical parameters** that describe the properties of dense matter consisting of nucleons.

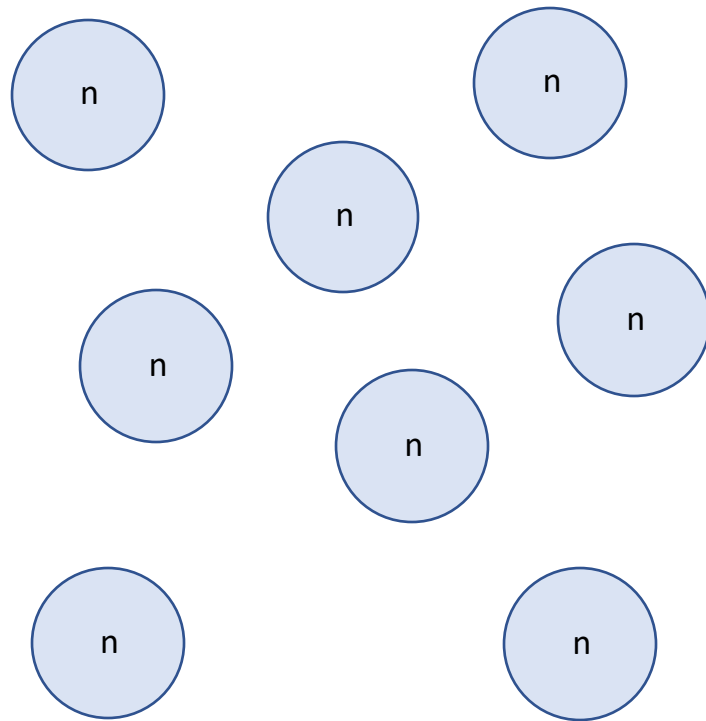


We can model the low-density EOS in terms of these parameters: **the meta-model.**

Microscopic Nuclear Physics

Many different approaches to calculate $\frac{E}{A}(n, x)$ but I will focus on **microscopic calculations** where we solve

$$\mathcal{H}|\psi\rangle = E|\psi\rangle$$



see also Carbone, Drischler, Gandolfi, Hagen, Hebeler, Holt, Lovato, Novario, Piarulli, Schwenk, ...



Microscopic Nuclear Physics

Many different approaches to calculate $\frac{E}{A}(n, x)$ but I will focus on **microscopic calculations** where we solve

$$\boxed{\mathcal{H}}|\psi\rangle = \boxed{E}|\psi\rangle$$

We need:

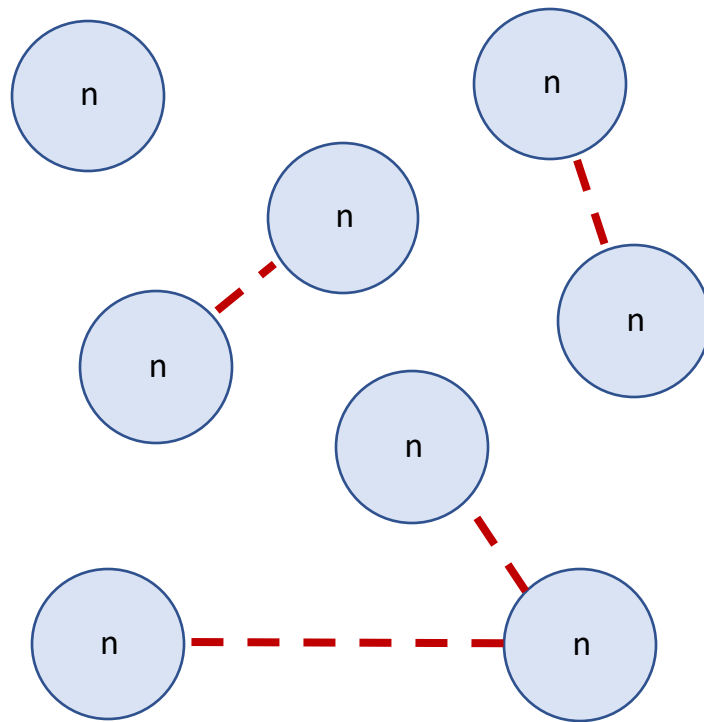
- ❑ A theory for the strong interactions among nucleons

Chiral Effective Field Theory

$$\mathcal{H} = \sum_i \mathcal{T}_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

- ❑ A computational method to solve the many-body Schrödinger equation:

e.g., many-body perturbation theory, quantum Monte Carlo, coupled cluster, self-consistent Green's function, ...



see also Carbone, Drischler, Gandolfi, Hagen, Hebeler, Holt, Lovato, Novario, Piarulli, Schwenk,, ...

Chiral Effective Field Theory

We want to develop an approach that addresses the shortcomings of typical phenomenological approaches:

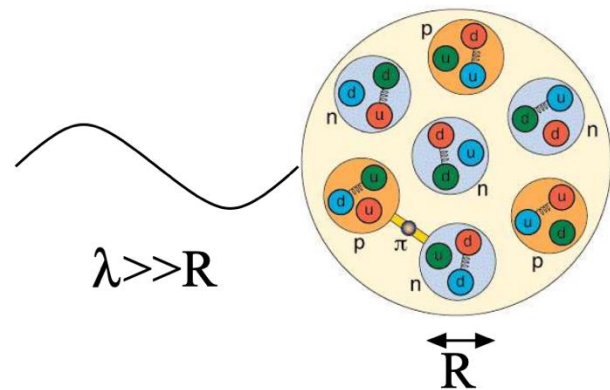
1. We want a description of nuclear interactions in terms of as few parameters as possible that can be adjusted to data, that enables us to make robust predictions.
2. We want a systematically improvable framework to nuclear interactions.
3. This needs to include two-nucleon and many-nucleon forces.
4. Finally, we want to be able to quantify uncertainties of our calculations.



Chiral Effective Field Theory

1. Typical momenta in atomic nuclei $\vec{p}$ are of the order of the pion mass, $m_\pi \sim 140$ MeV. This corresponds to wavelength of the order of 1fm, much larger than necessary to resolve the quark substructure.

Nucleons are relevant **effective** degrees of freedom.

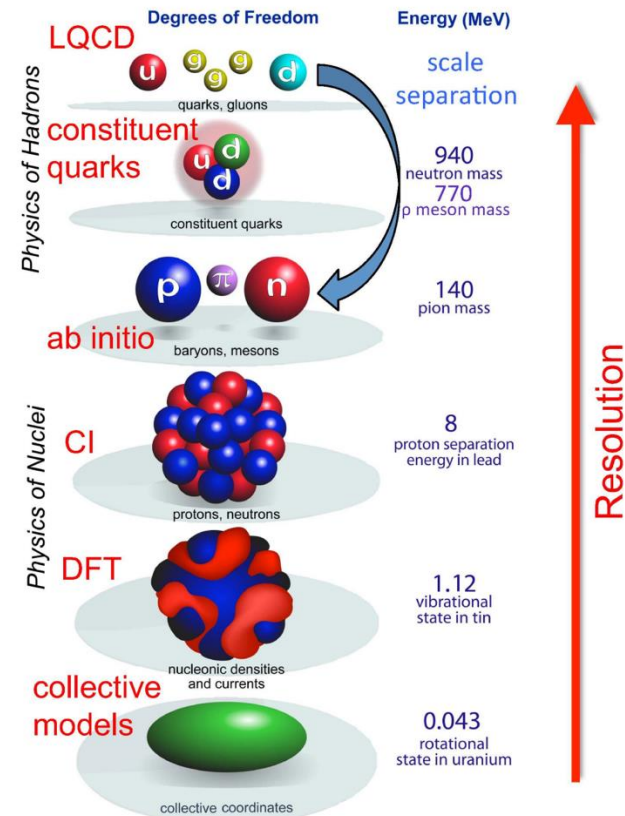


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2. At such momenta, we resolve nucleons and pion exchanges, but heavier degrees of freedom are well separated: **Separation of scales**.

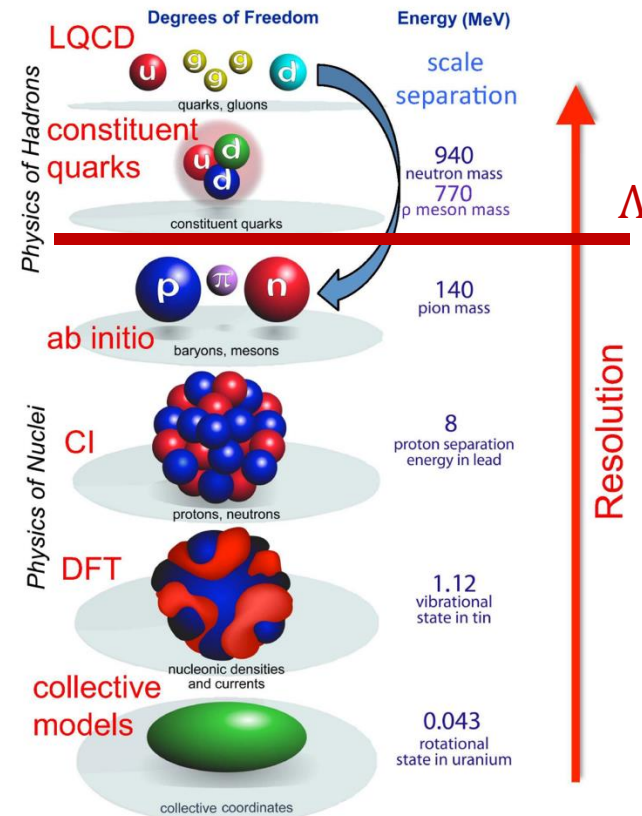


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2. At such momenta, we resolve nucleons and pion exchanges, but heavier degrees of freedom are well separated: **Separation of scales**.
3. The scale controlling the separation is called the **breakdown scale** Λ . Expand interaction in $\vec{p}/\Lambda$

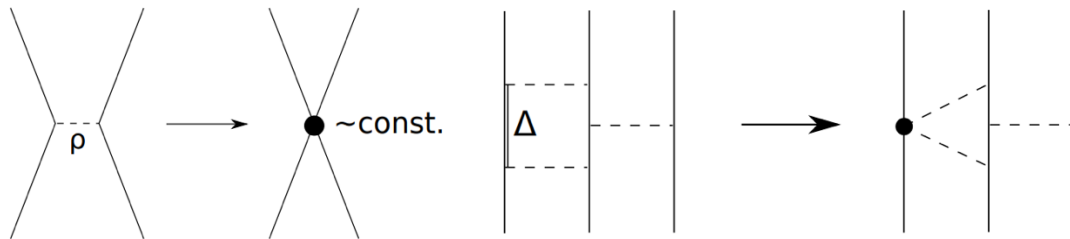


$$\Lambda_B \approx 500 \text{ MeV}$$

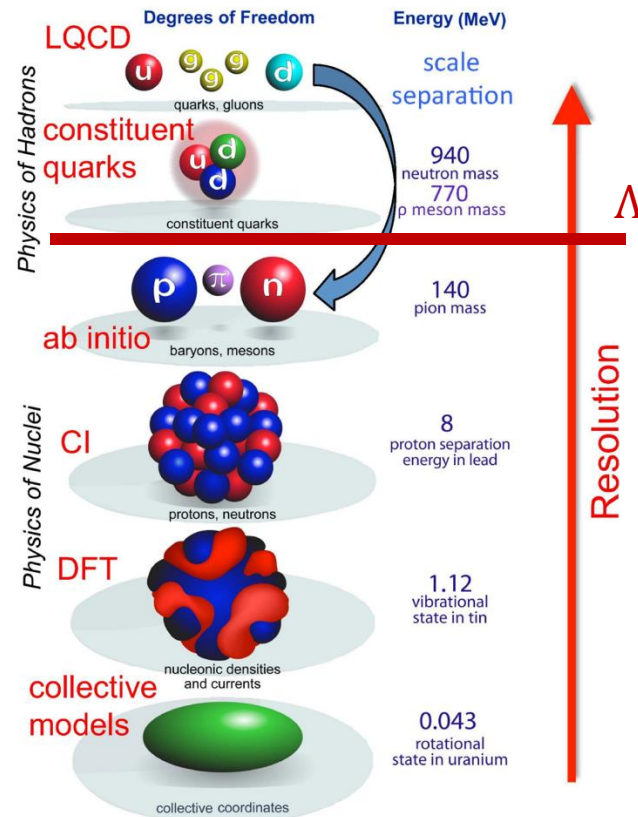
Chiral Effective Field Theory

4. Short-distance physics (above breakdown) is not resolved. It is captured in **low-energy couplings** (LECs) using renormalization. These couplings are natural [of $O(1)$].

Compare to multipole expansion.



4. **Long-range physics**, mediated by pion, is included **explicitly**.



Chiral Effective Field Theory

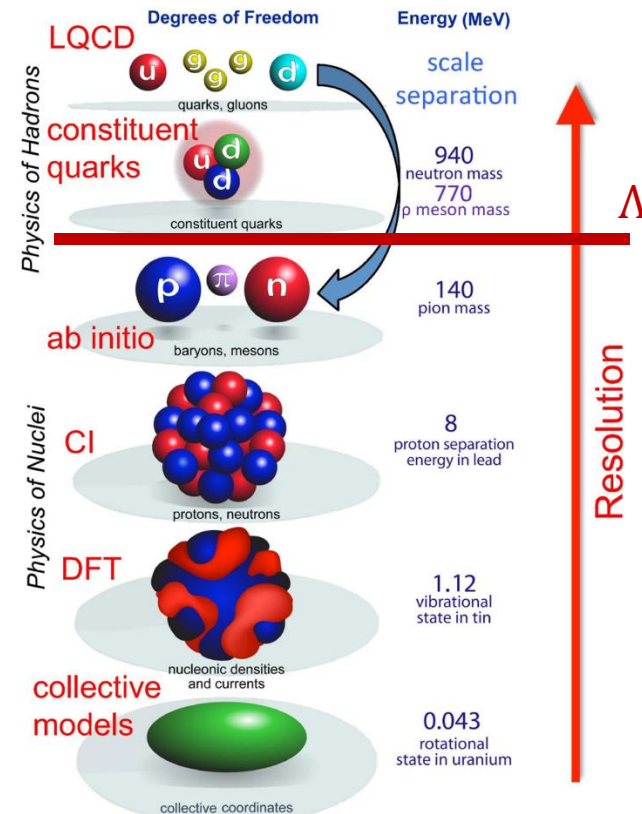
6. This leads to a **systematic expansion** of the underlying Lagrangian (enables error estimates, see later), which breaks down at high momenta:

$$\mathcal{L} = \sum_v \left(\frac{q}{\Lambda_B} \right)^v \mathcal{F}_v(q, g_i)$$

7. The Lagrangian contains all symmetries of QCD, including **chiral symmetry**:

$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \dots$$

8. All unknown parameters, the LECs, have to be determined. They are usually adjusted to reproduce experimental data as “nature knows about everything”.



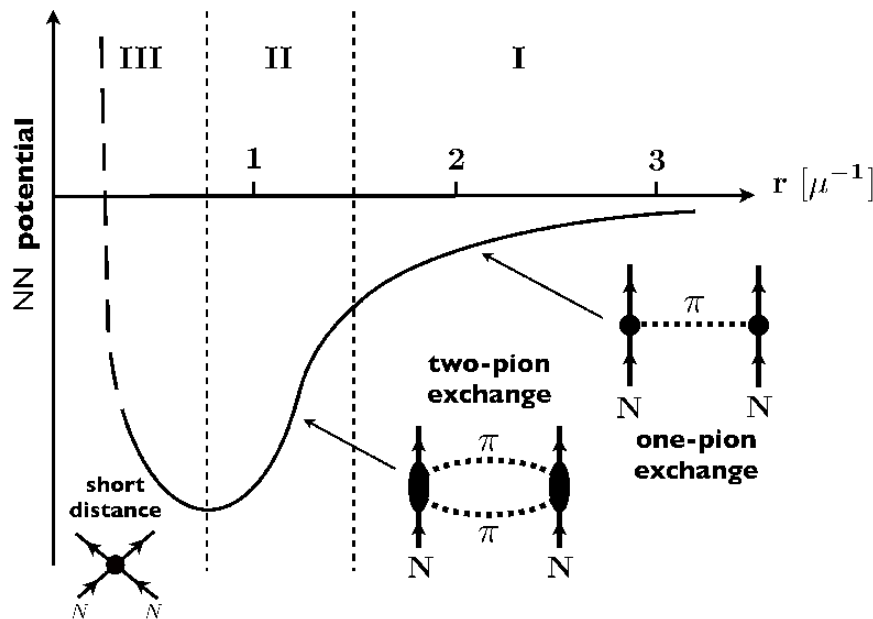
Chiral Effective Field Theory

To understand which diagrams appear at which order, we need a **power counting scheme**. The most used power counting scheme is **Weinberg Power Counting**:

- Use the effective Lagrangian to calculate an effective potential.
- Expand the effective potential in p/Λ and m_π/Λ .
- Employ this potential in few- and many-body solvers/
- Employ a cutoff to remove UV divergences.



Chiral Effective Field Theory

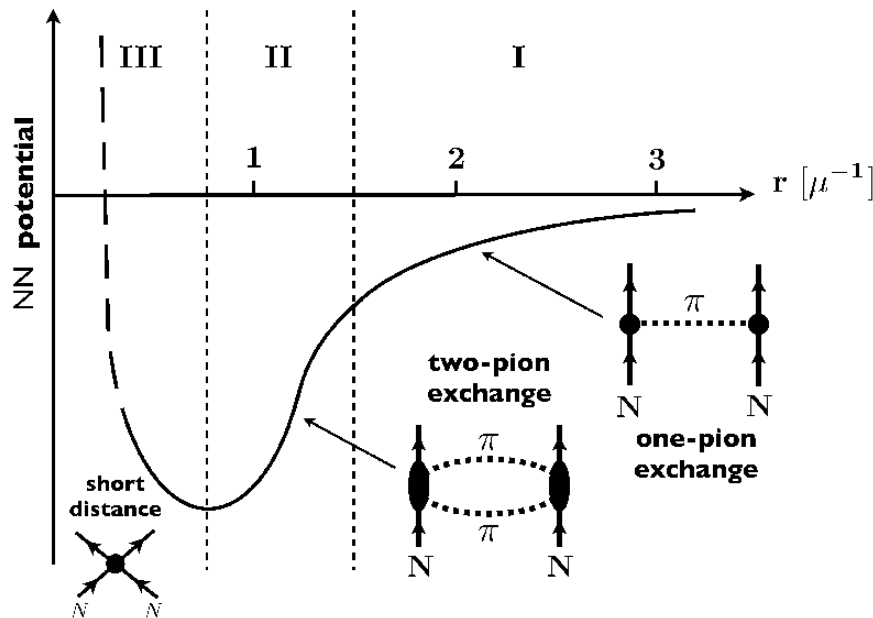


Holt et al., PPNP 73 (2013)

Schematic picture of NN interaction,
potential not observable!



Chiral Effective Field Theory



Holt et al., PPNP 73 (2013)

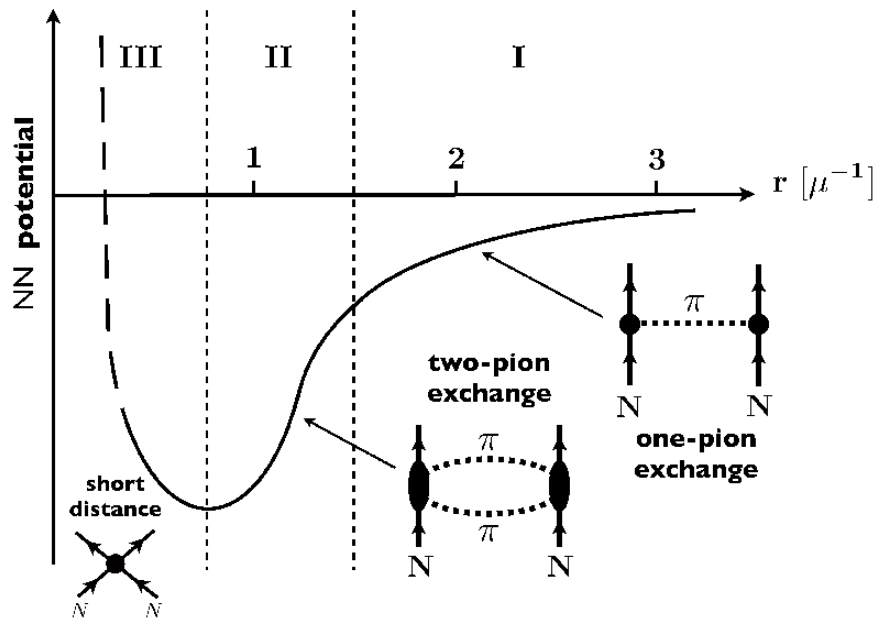
Schematic picture of NN interaction,
potential not observable!

	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$ (2 LECs)	X H	—	—

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...



Chiral Effective Field Theory



Holt et al., PPNP 73 (2013)

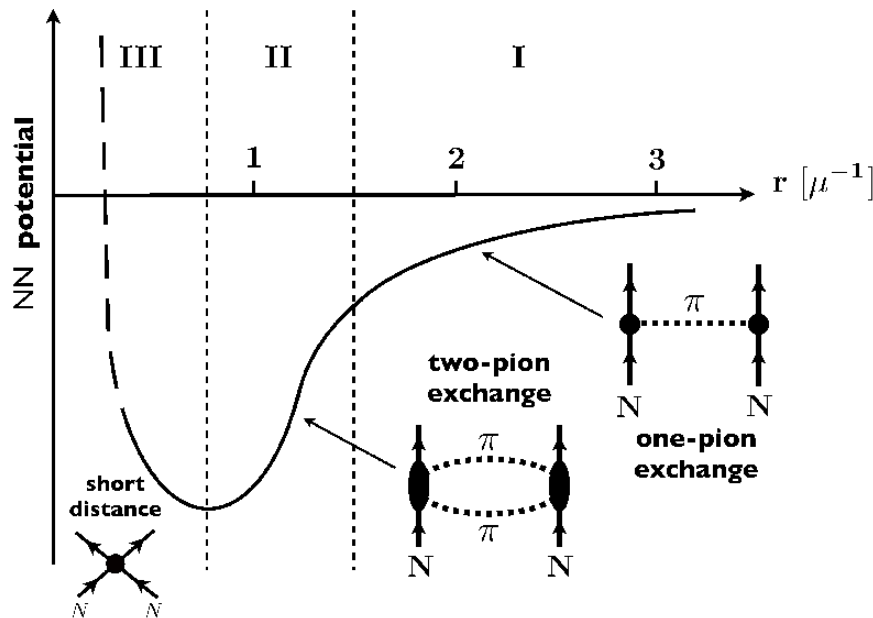
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	NN	3N	4N
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NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$ (7 LECs)			

Weinberg, van Kolck, Kaplan, Savage, Wise,
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Chiral Effective Field Theory



Holt et al., PNP 73 (2013)

	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$ (2 LECs)		—	—
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$ (7 LECs)		—	—
N ² LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$ (2 LECs: 3N)			—
N ³ LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$ (12 LECs)			

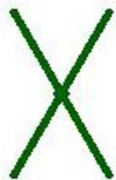
Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...



Hierarchy of nuclear forces: $V_{2N} \gg V_{3N} \gg V_{4N} \gg \dots$

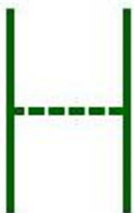
Leading-order (LO) interaction

Contact interactions at LO are momentum-independent (they can only depend on (iso)spin):



$$V_{\text{cont}}^{\text{LO}} = \alpha_1 \mathbb{1} + \alpha_2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \alpha_3 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + \alpha_4 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

We already know the one-pion exchange:



$$V_{\text{OPE}}^{(0)}(\mathbf{q}) = -\frac{g_A^2}{4f_\pi^2} \frac{\boldsymbol{\sigma}_i \cdot \mathbf{q} \boldsymbol{\sigma}_j \cdot \mathbf{q}}{q^2 + m_\pi^2} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$ (2 LECs)			
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$ (7 LECs)			
N ² LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$ (2 LECs: 3N)			
N ³ LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$ (12 LECs)			

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Chiral Effective Field Theory

Systematic expansion of nuclear forces in momentum Q over breakdown scale Λ_b :

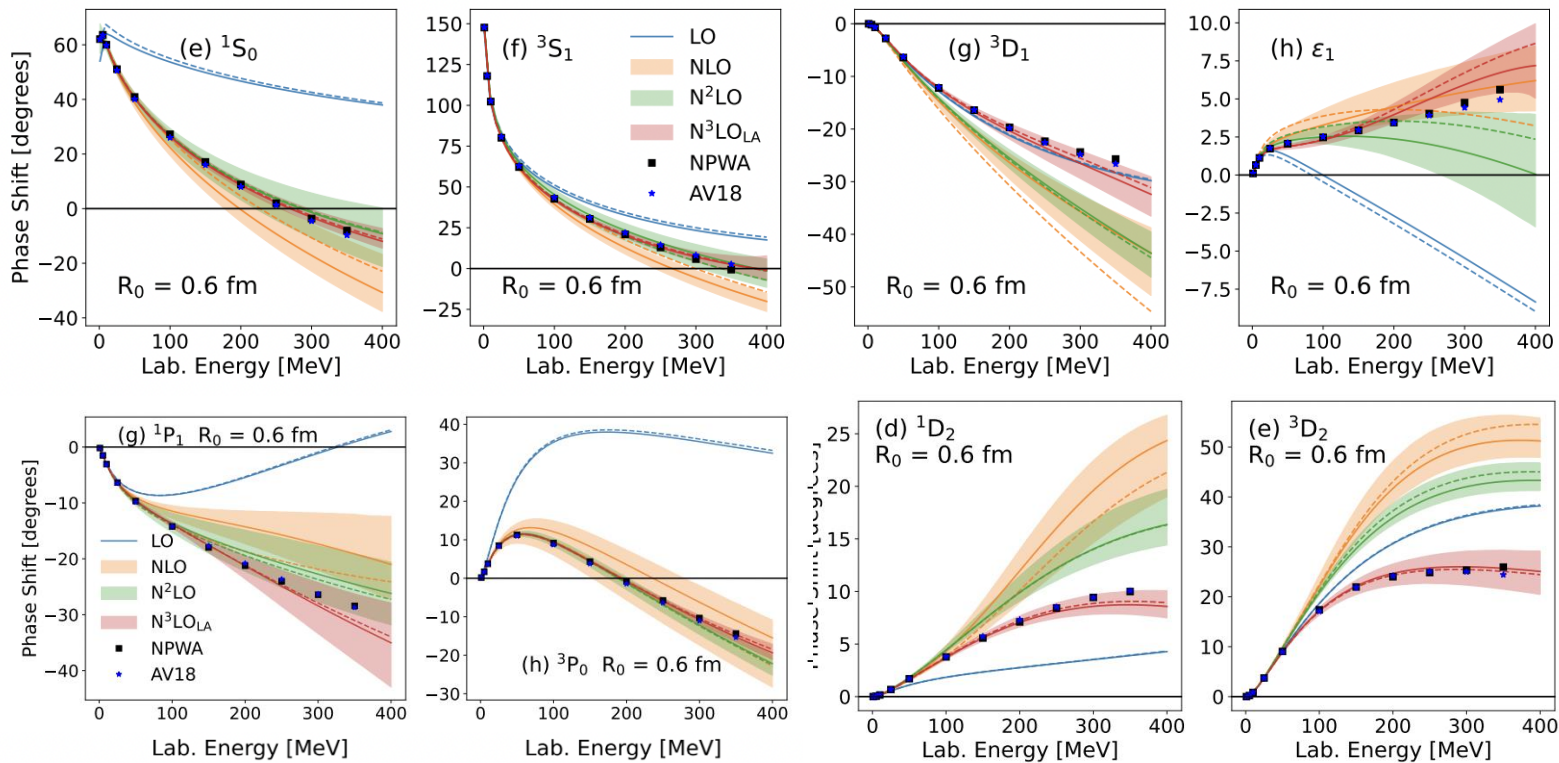
- Based on symmetries of QCD
- Pions and nucleons as explicit degrees of freedom
- Power counting scheme results in systematic expansion, **enables uncertainty estimates!**
- Natural hierarchy of nuclear forces
- **Consistent interactions:** Same couplings for two-nucleon and many-body sector
- Fitting: NN forces in NN system (NN scattering), 3N forces in 3N/4N system (Binding energies, radii)

	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$ (2 LECs)			
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$ (7 LECs)			
N ² LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$ (2 LECs: 3N)			
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Weinberg, van Kolck, Kaplan, Savage, Wise,
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Chiral Effective Field Theory

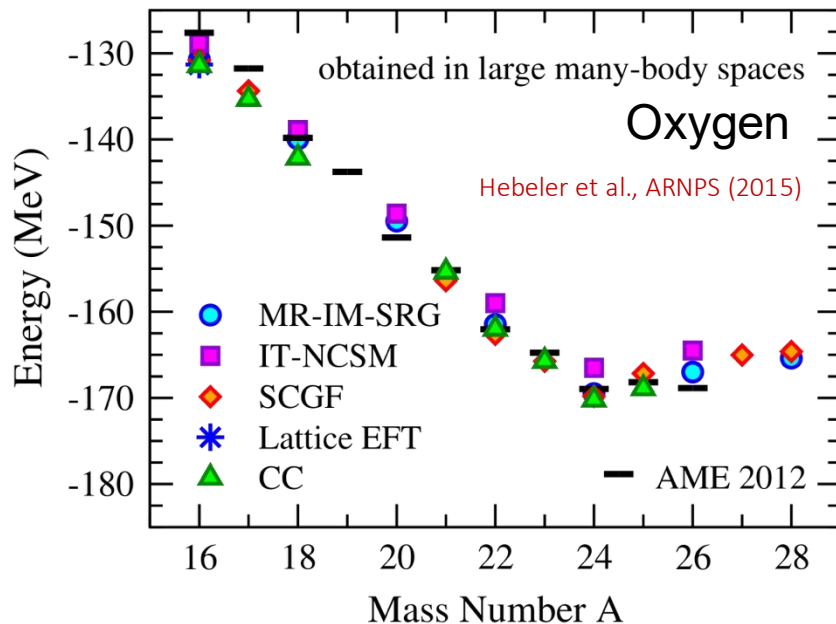


Can work to desired accuracy with **error estimates!**

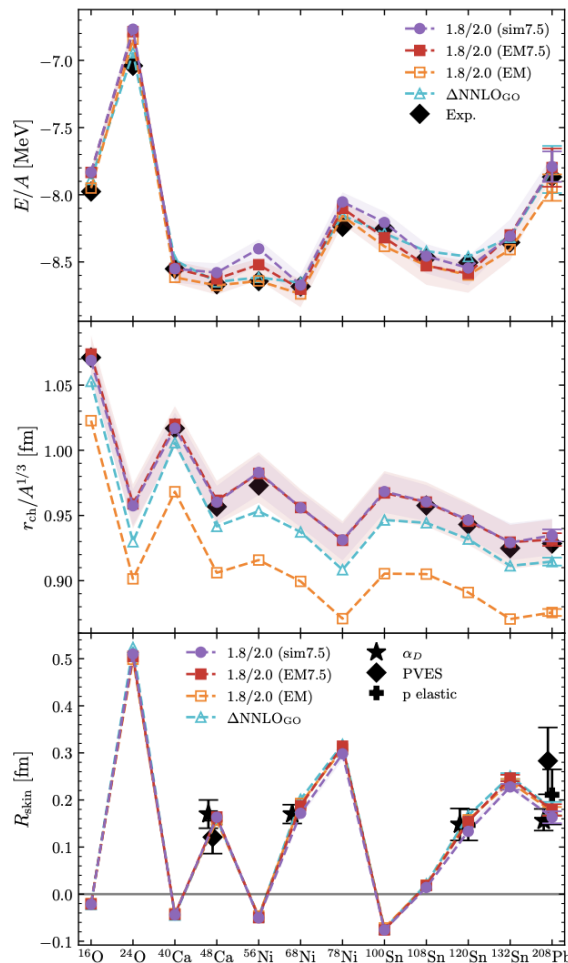
Somasundaram, IT, et al., PRC (2024)

Chiral Effective Field Theory

Results for chiral EFT calculations of nuclei:

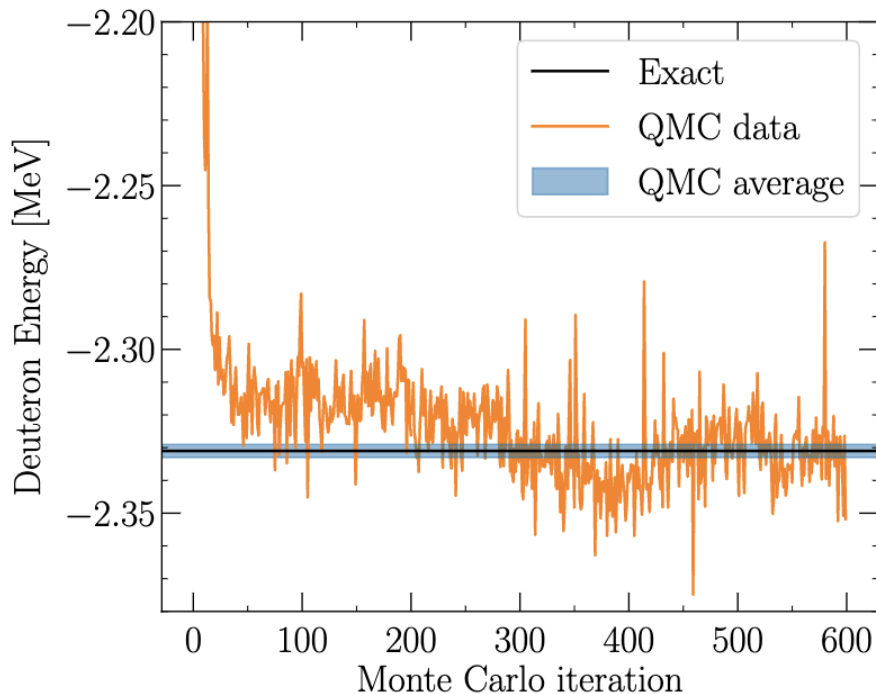


Excellent description of properties of nuclei up to the medium-mass region (fits to light nuclei).



Ab initio approaches – Quantum Monte Carlo

We use quantum Monte Carlo codes to make predictions of the EOS.



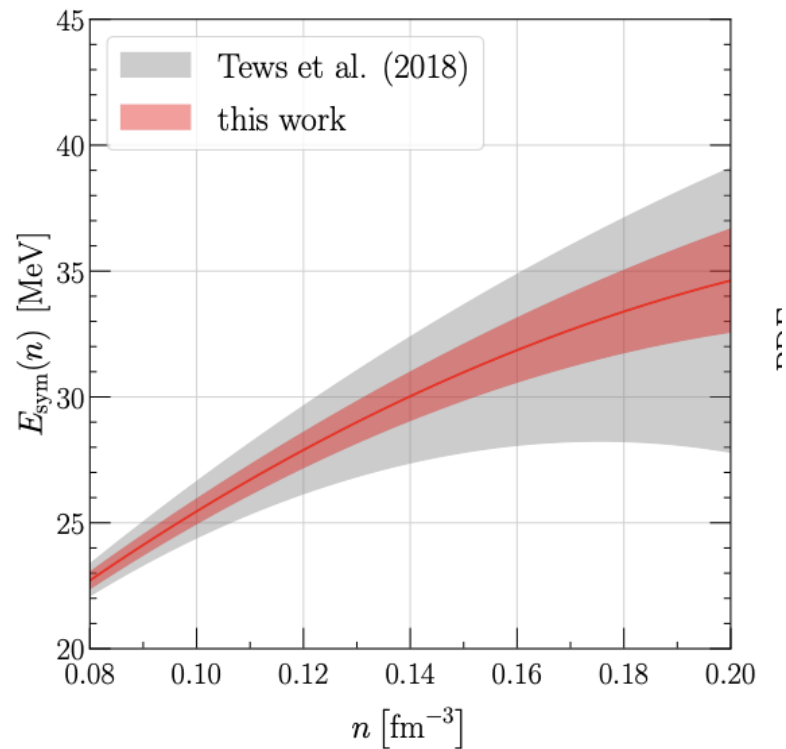
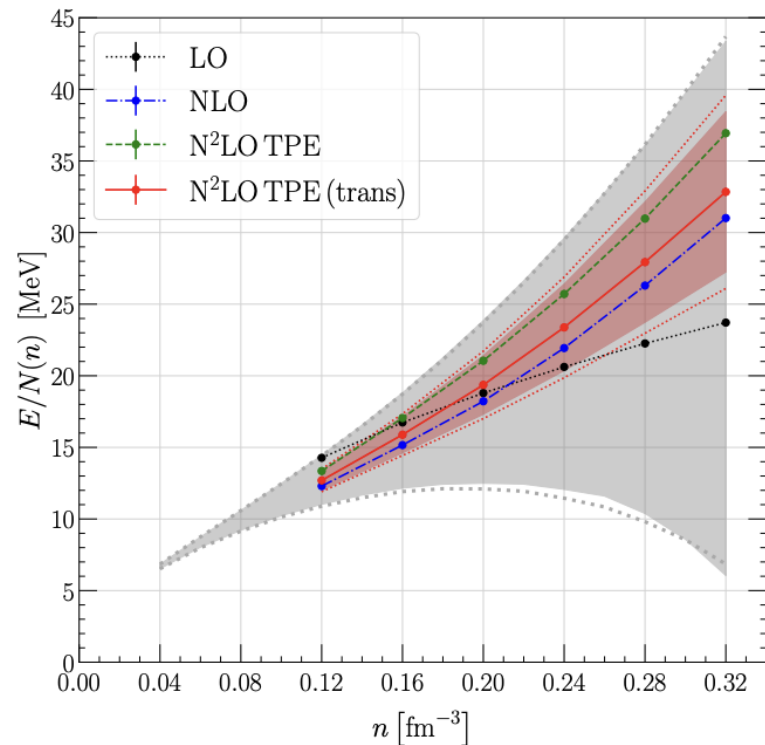
Stochastic propagation of trial wave function in imaginary time to project out the ground state of a system.

Very precise (stat. unc. 1%)

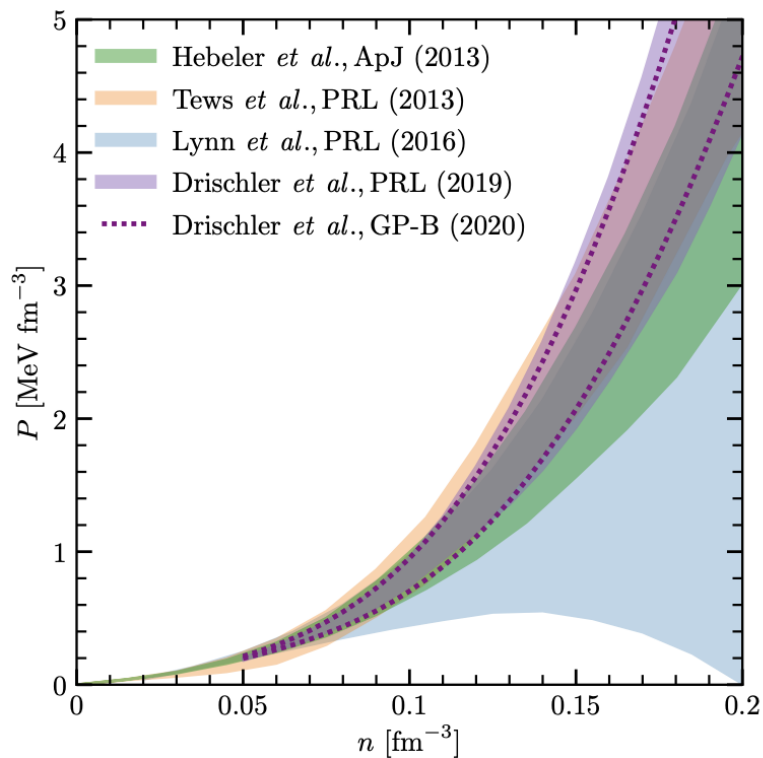
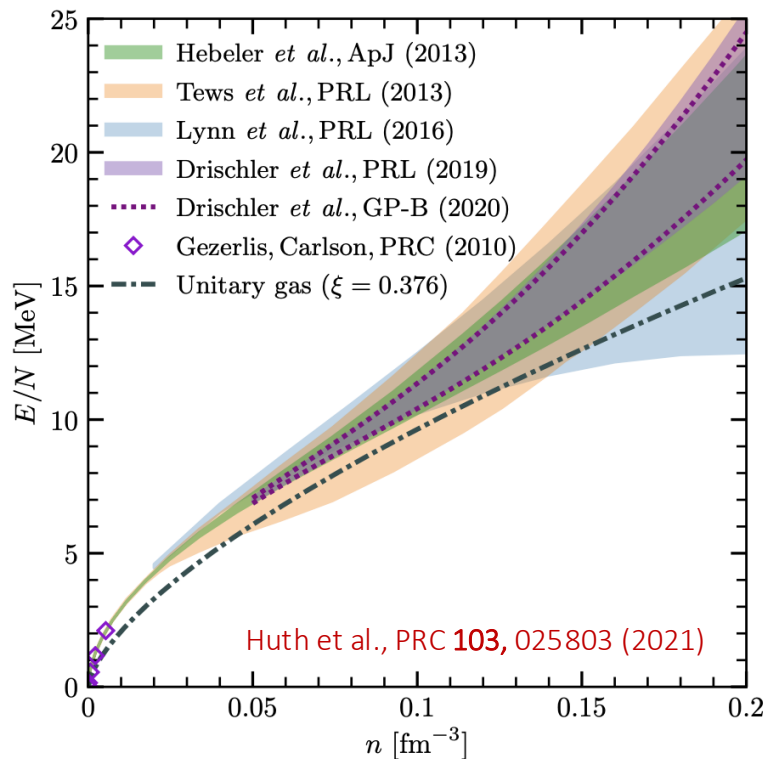
Full EOS calculation for one Hamiltonian costs $O(1 \text{ million})$ CPU-hours using AFDMC code.



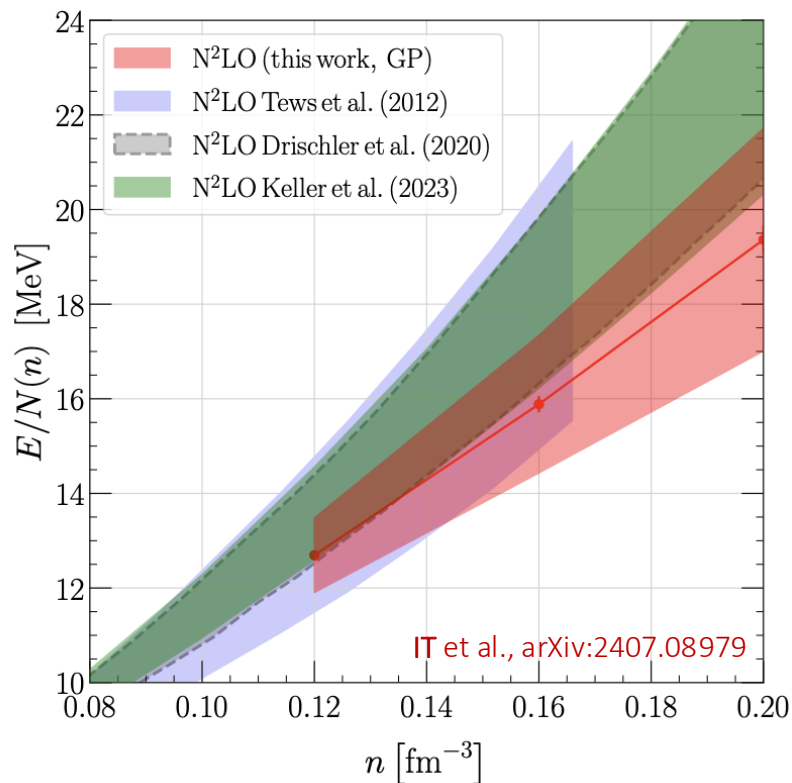
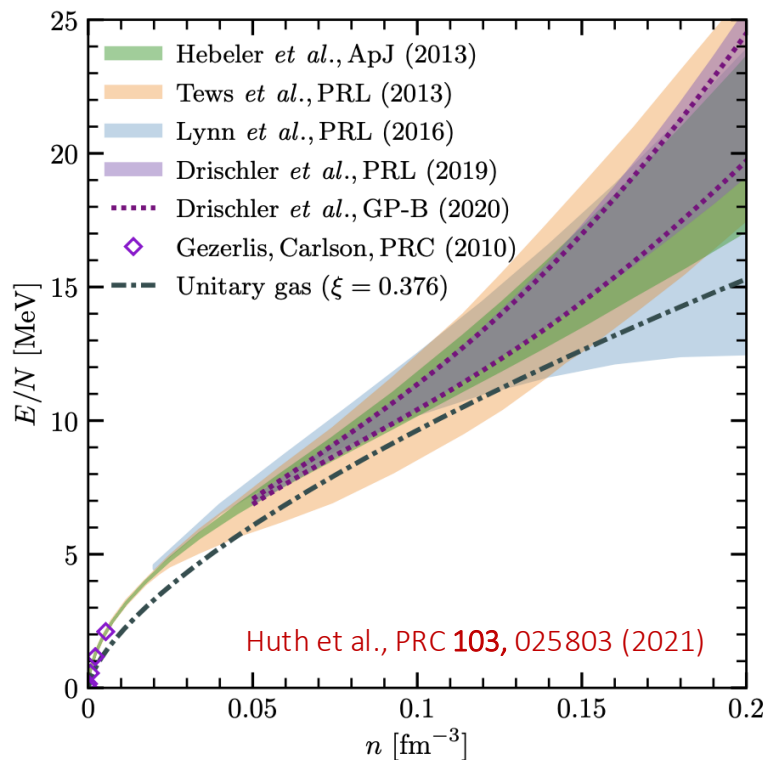
Ab initio approaches – Quantum Monte Carlo



Ab initio approaches – Comparison of different methods



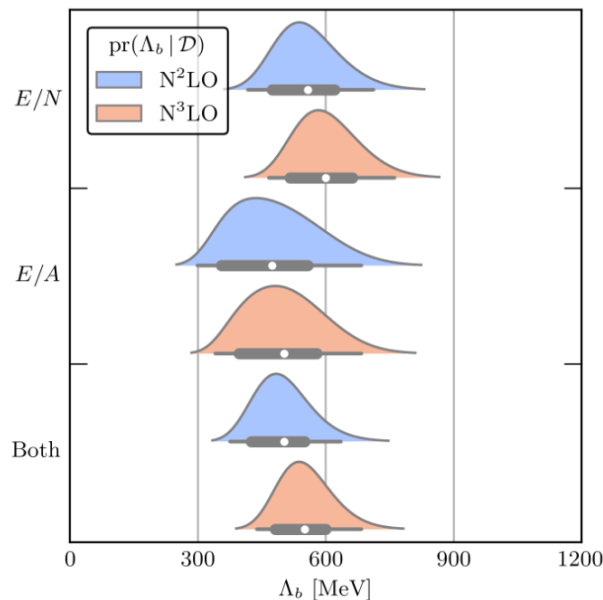
Ab initio approaches – Comparison of different methods



Chiral Effective Field Theory

BUT: There are still many open questions and problems!

- What is the **breakdown scale**? Does it change in the many-body system?



Drischler et al.,
PRC (2020)

	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$ (2 LECs)			
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$ (7 LECs)			
N ² LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$ (2 LECs: 3N)			
N ³ LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$ (15 LECs)			

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...



Chiral Effective Field Theory

BUT: There are still many open questions and problems!

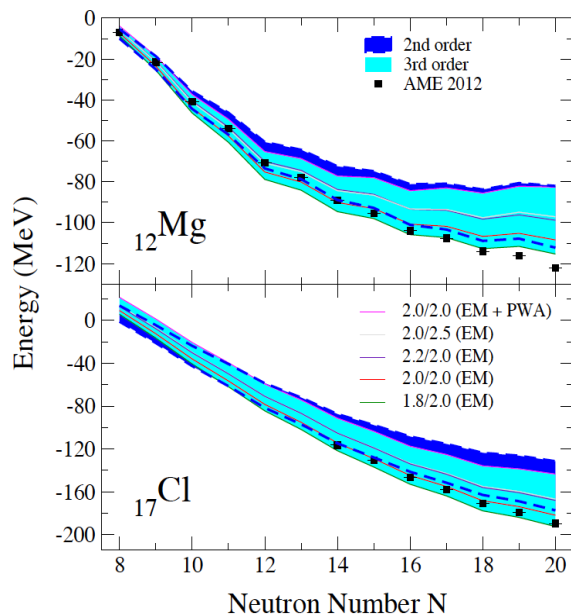
- What is the **breakdown scale**? Does it change in the many-body system?
- How do results depend on the **regularization scheme** (explicit form of the interaction) **and scale** (cutoff necessary in many-body methods)?
- Does this series **converge** in the many-body system?
- How to best determine all **unknown coefficients**?

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LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$ (2 LECs)			
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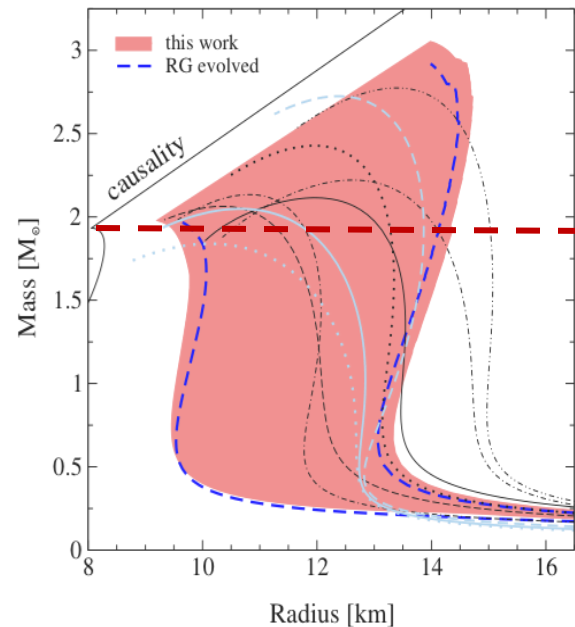
Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...



Uncertainty



Simonis et al., PRC (2016)



Krueger, IT et al., PRC (2013)

Present theoretical predictions for nuclear systems are limited by:

- our incomplete understanding of **nuclear interactions**,
- and our ability to **reliably calculate** these strongly interacting systems.



We need to be able to quantify those. **What is the best approach?**

Uncertainty

We can estimate the uncertainty from the order-by-order convergence of the EFT:

$$\Delta X = X - X_0 \sum_{k=0}^{k_{\max}} c_k Q^k = X_0 \sum_{k=k_{\max}+1}^{\infty} c_k Q^k \quad Q = \frac{\max(p, m_{\pi})}{\Lambda_b}$$

The uncertainty stems from the sum of the truncated terms (**truncation uncertainty**).

Usually, one can use the first truncated term as an estimator for this uncertainty:

$$\Delta X = X_0 c_{k_{\max}+1} Q^{k_{\max}+1}$$

We know X_0 and Q and would only need to estimate the expansion coefficient c .

Suggestion:

Use calculations at previous orders to estimate c at higher orders **a posteriori**, assuming a systematic behavior.



Uncertainty

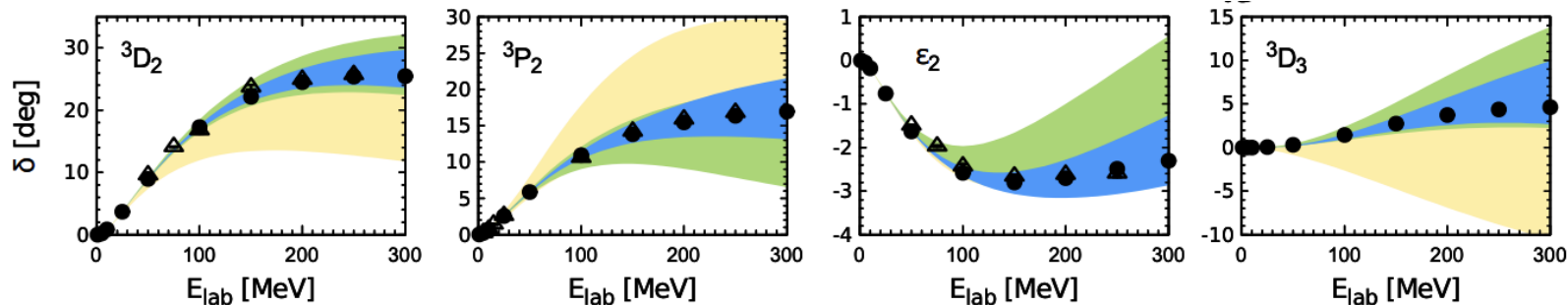
Suggestion by Epelbaum, Krebs, and Meißner (EPJ A 2015): EKM uncertainties.

$$c_{k_{\max}+1} = \max \{c_0, \dots, c_{k_{\max}}\}$$

Then, the uncertainty at e.g. N²LO can be written as

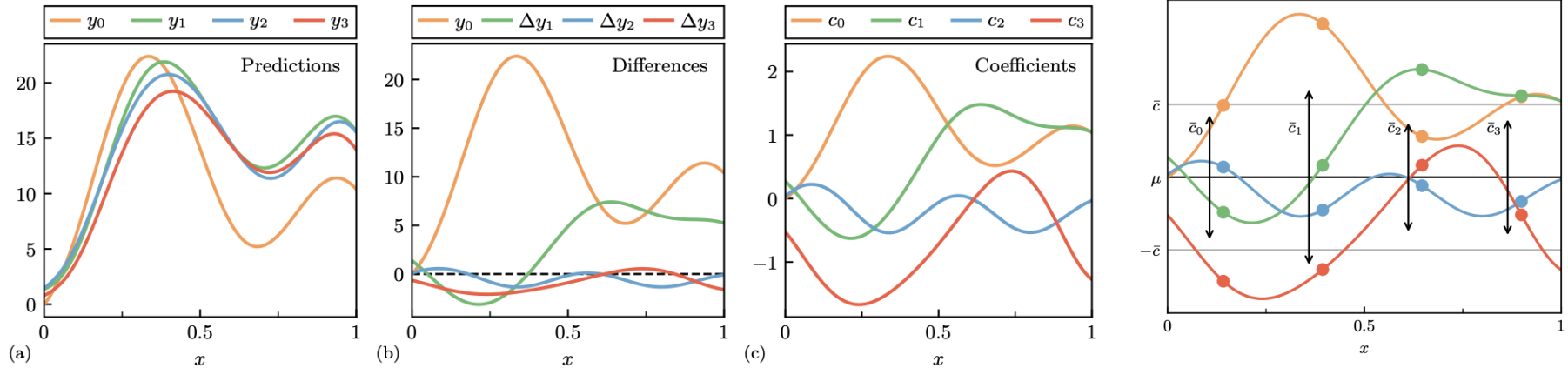
$$\Delta X^{\text{N}^2\text{LO}} = \max \left(Q^4 |X^{\text{LO}} - X^{\text{free}}|, Q^2 |X^{\text{NLO}} - X^{\text{LO}}|, \right. \\ \left. Q |X^{\text{N}^2\text{LO}} - X^{\text{NLO}}| \right)$$

Uncertainty might be overestimated.



Uncertainty

Suggestion by BUQEYE collaboration:



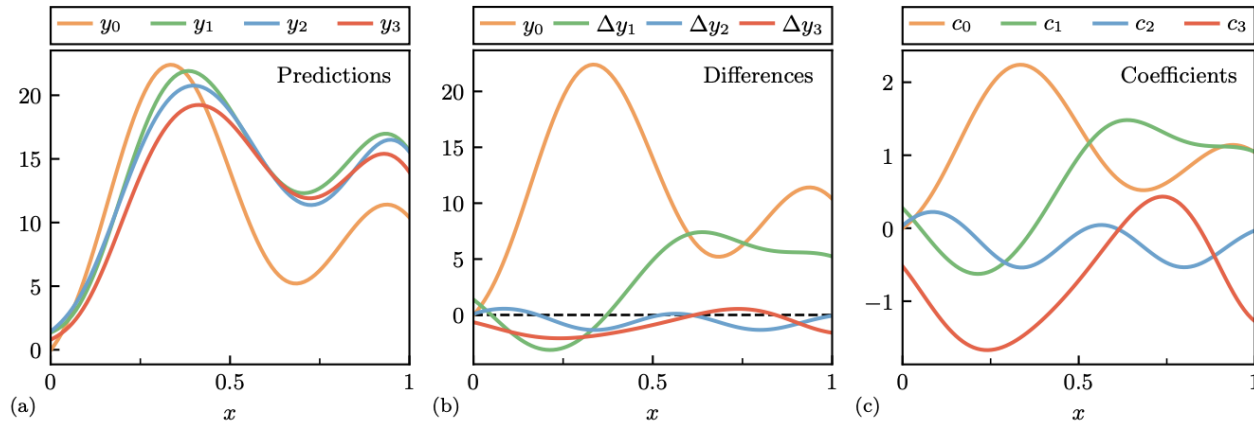
Melendez et al., Phys. Rev. C 100, 044001 (2019)

- Calculate the coefficients, assume they are described by a Gaussian distribution.
- Train a Gaussian Process (GP) to sample coefficients when estimating truncation uncertainty



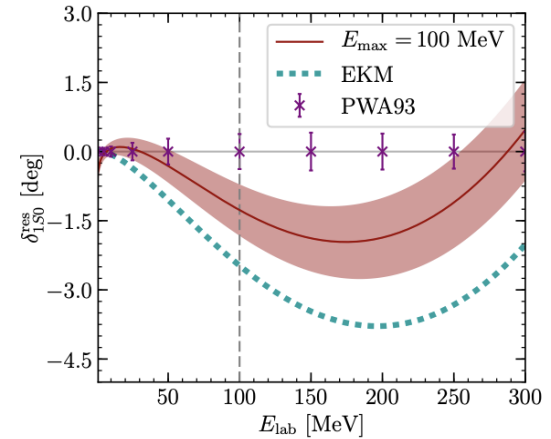
Uncertainty

Suggestion by BUQEYE collaboration:

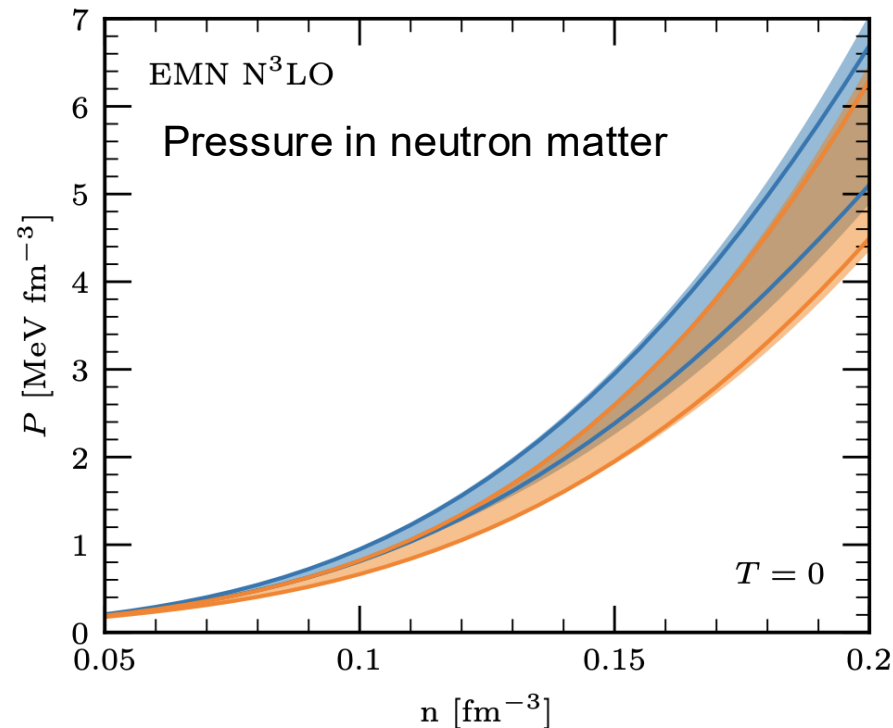


Melendez et al., Phys. Rev. C 100, 044001 (2019)

- Calculate the coefficients, assume they are described by a Gaussian distribution.
- Train a Gaussian Process (GP) to sample coefficients when estimating truncation uncertainty



Uncertainty



Keller et al., PRC (2021)

Estimated from order-by-order calculation:

- Using simple EKM (**bands**):

Epelbaum, Krebs, Meißner, EPJ A (2015)

$$\Delta X^{N^2\text{LO}} = \max \left(Q^4 |X^{\text{LO}} - X^{\text{free}}|, Q^2 |X^{\text{NLO}} - X^{\text{LO}}|, \right. \\ \left. Q |X^{N^2\text{LO}} - X^{\text{NLO}}| \right)$$

$$Q = \frac{\max(p, m_\pi)}{\Lambda_b}$$

- Using Gaussian processes (**lines**).

Drischler et al., PRL and PRC (2020)

Both approaches agree!

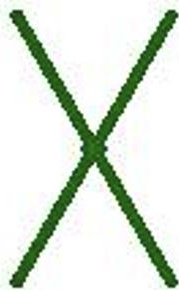
Alternative: Directly map LEC uncertainties to observables, e.g., nuclear matter.

See also Ekstroem, Hagen et al.



Fitting EFT interactions

The EFT potential is generally $V(\mathbf{q}, \mathbf{k}) = V_{\text{cont}}(\mathbf{q}, \mathbf{k}) + V_{\pi}(\mathbf{q}, \mathbf{k})$



$$V_{\text{cont}}^{\text{LO}}(\mathbf{q}, \mathbf{k}) = V_{\text{cont}}^{\text{LO}} = \alpha_1 1 + \alpha_2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \alpha_3 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + \alpha_4 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$\begin{aligned} V_{\text{cont}}^{\text{NLO}}(\mathbf{q}, \mathbf{k}) = & \gamma_1 q^2 + \gamma_2 q^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \gamma_3 q^2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + \gamma_4 q^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + \gamma_5 k^2 + \gamma_6 k^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \\ & + \gamma_7 k^2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + \gamma_8 k^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + \gamma_9 (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)(\mathbf{q} \times \mathbf{k}) + \gamma_{10} (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)(\mathbf{q} \times \mathbf{k}) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ & + \gamma_{11} (\boldsymbol{\sigma}_i \cdot \mathbf{q})(\boldsymbol{\sigma}_j \cdot \mathbf{q}) + \gamma_{12} (\boldsymbol{\sigma}_i \cdot \mathbf{q})(\boldsymbol{\sigma}_j \cdot \mathbf{q}) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + \gamma_{13} (\boldsymbol{\sigma}_i \cdot \mathbf{k})(\boldsymbol{\sigma}_j \cdot \mathbf{k}) \\ & + \gamma_{14} (\boldsymbol{\sigma}_i \cdot \mathbf{k})(\boldsymbol{\sigma}_j \cdot \mathbf{k}) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j. \end{aligned} \quad (24)$$

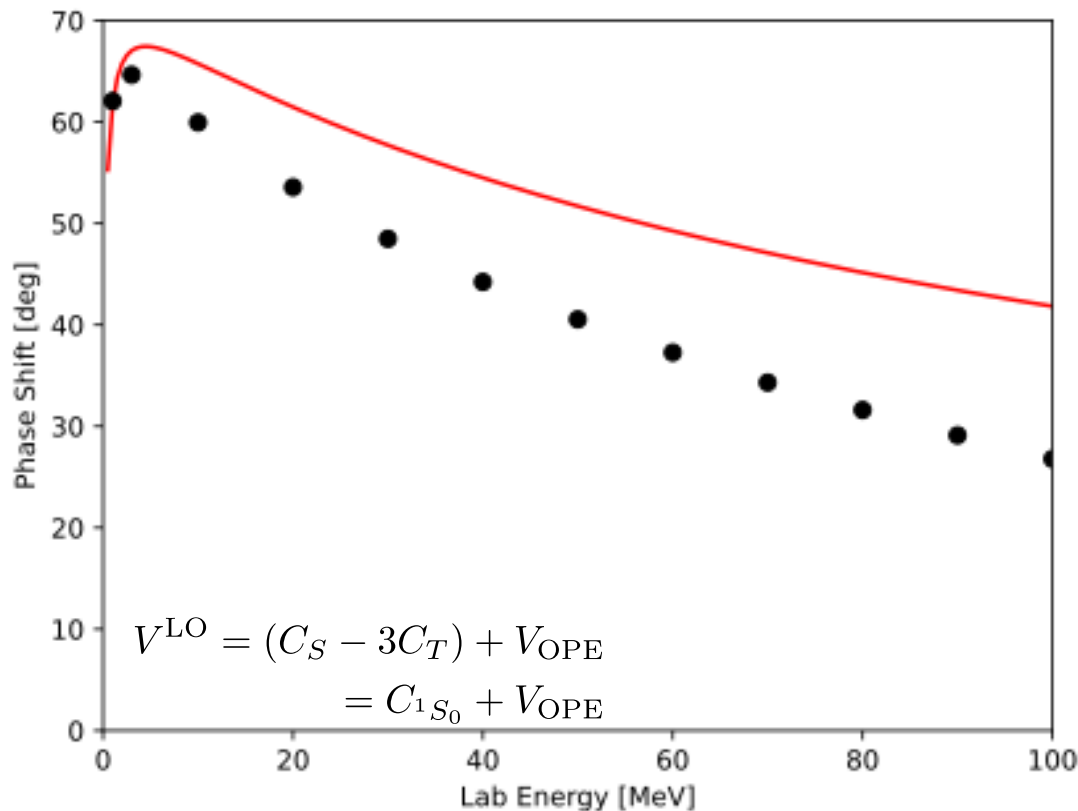


$$V_{\text{OPE}}^{(0)}(\mathbf{q}) = -\frac{g_A^2}{4f_{\pi}^2} \frac{\boldsymbol{\sigma}_i \cdot \mathbf{q} \boldsymbol{\sigma}_j \cdot \mathbf{q}}{q^2 + m_{\pi}^2} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

The Hamiltonian depends on operator LECs, describing the strength of operators in the short-range part and in the pion part.

Fitting EFT interactions

Sketch!



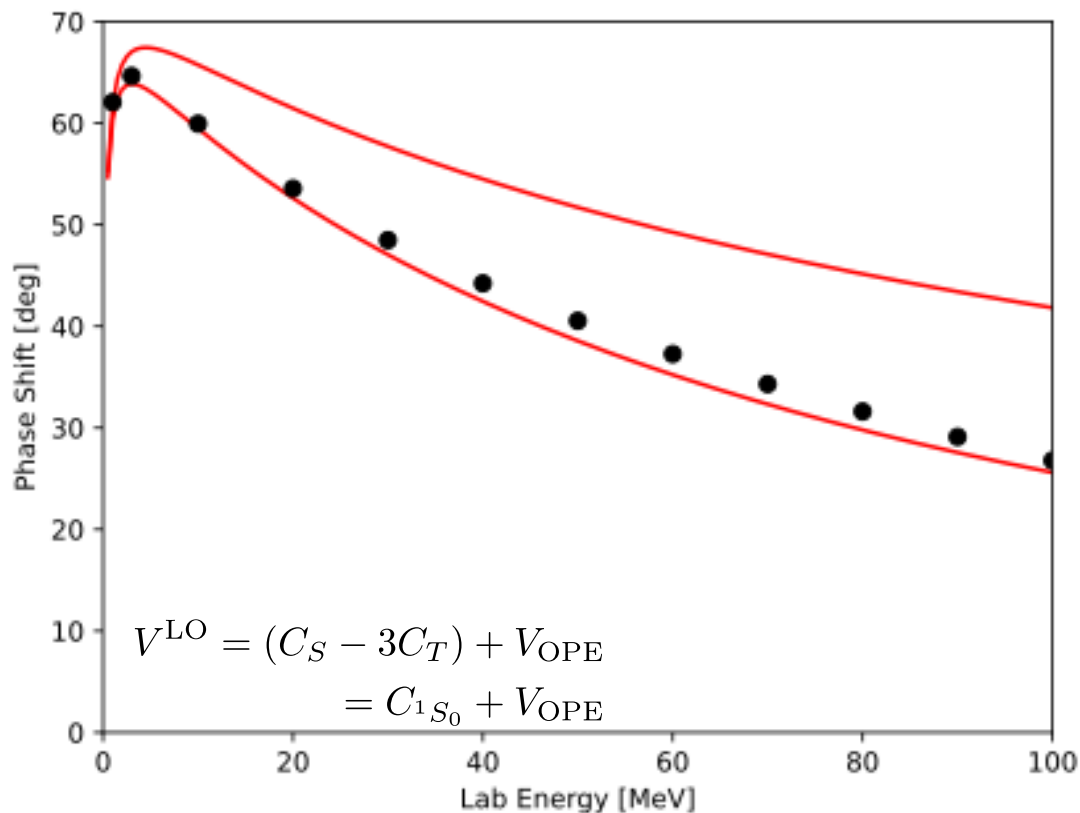
1S_0 phase nucleon-nucleon phase shift

Adjust spectral LEC to phase shifts



Fitting EFT interactions

Sketch!



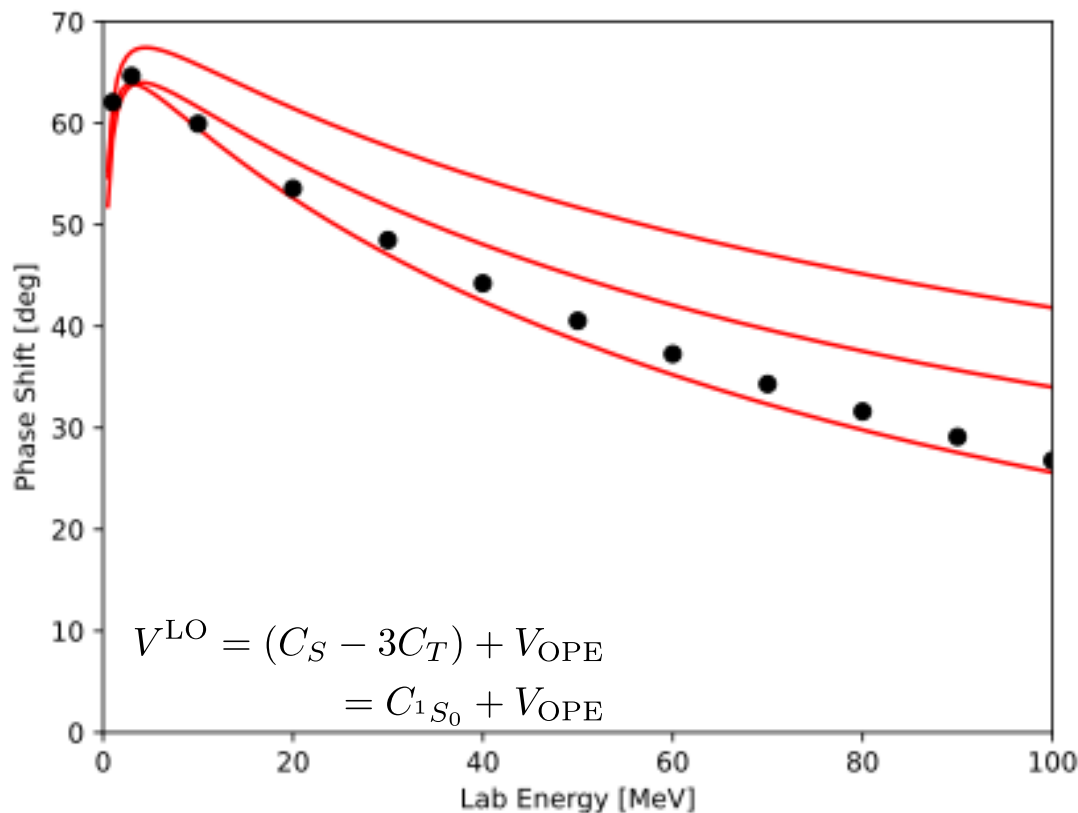
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Fitting EFT interactions

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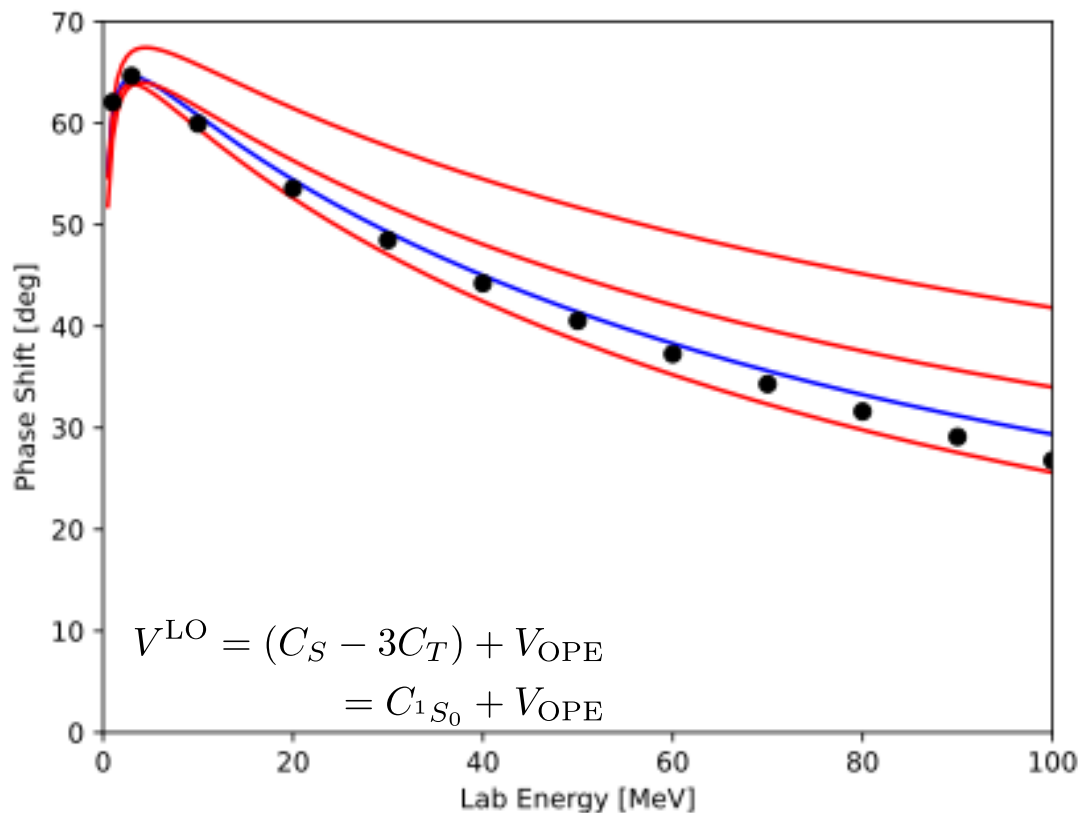
1S_0 phase nucleon-nucleon phase shift

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Fitting EFT interactions

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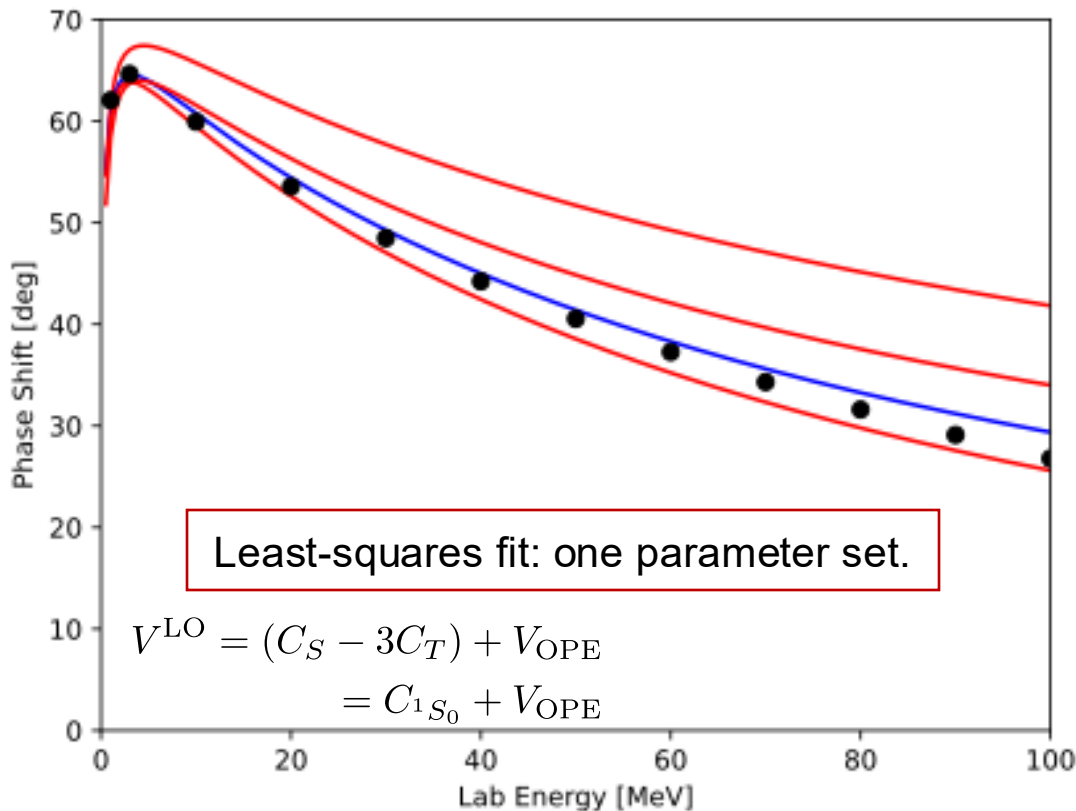
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Fitting EFT interactions

Sketch!



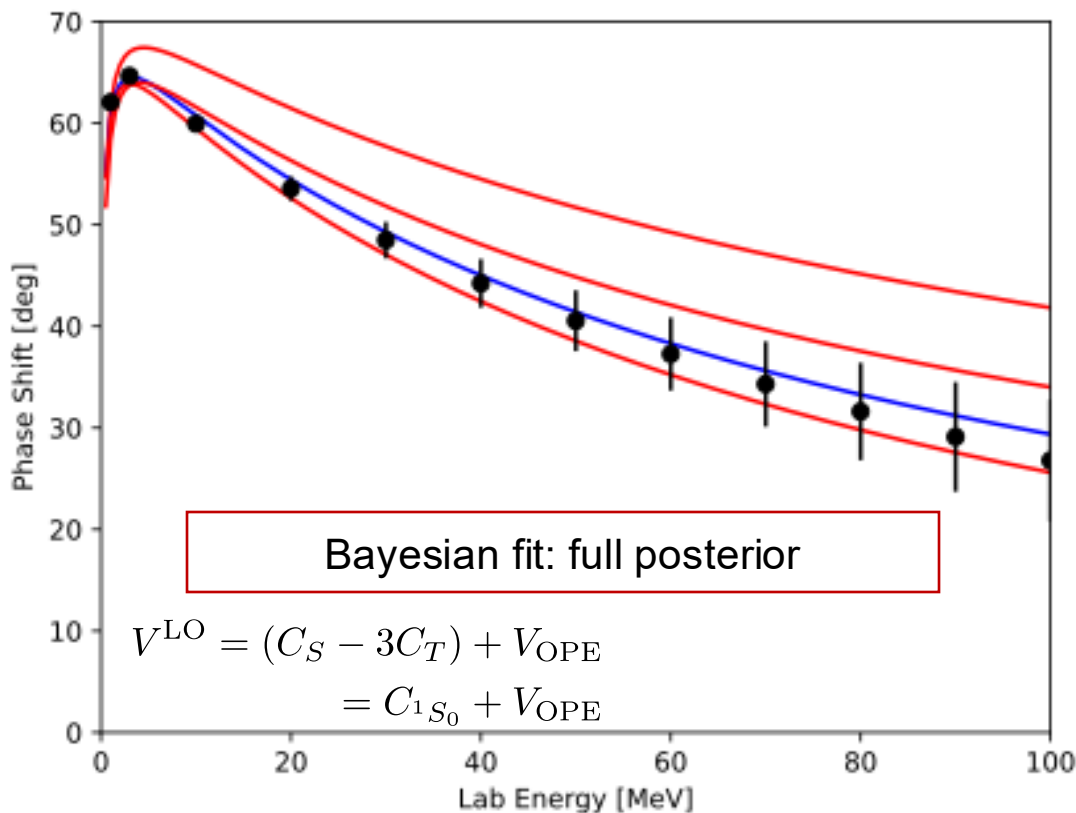
1S_0 phase nucleon-nucleon phase shift

Adjust spectral LEC to phase shifts



Fitting EFT interactions

Sketch!



1S_0 phase nucleon-nucleon phase shift

Adjust spectral LEC to phase shifts



Fitting EFT interactions

The NN interaction is calibrated to scattering data using the method of Bayesian inference. This allows us to incorporate EFT truncation uncertainties in the fit.

$$p(\theta|d, H) = \frac{p(d, H|\theta)p(\theta|H)}{p(d|H)}$$

The Posterior The Likelihood The prior

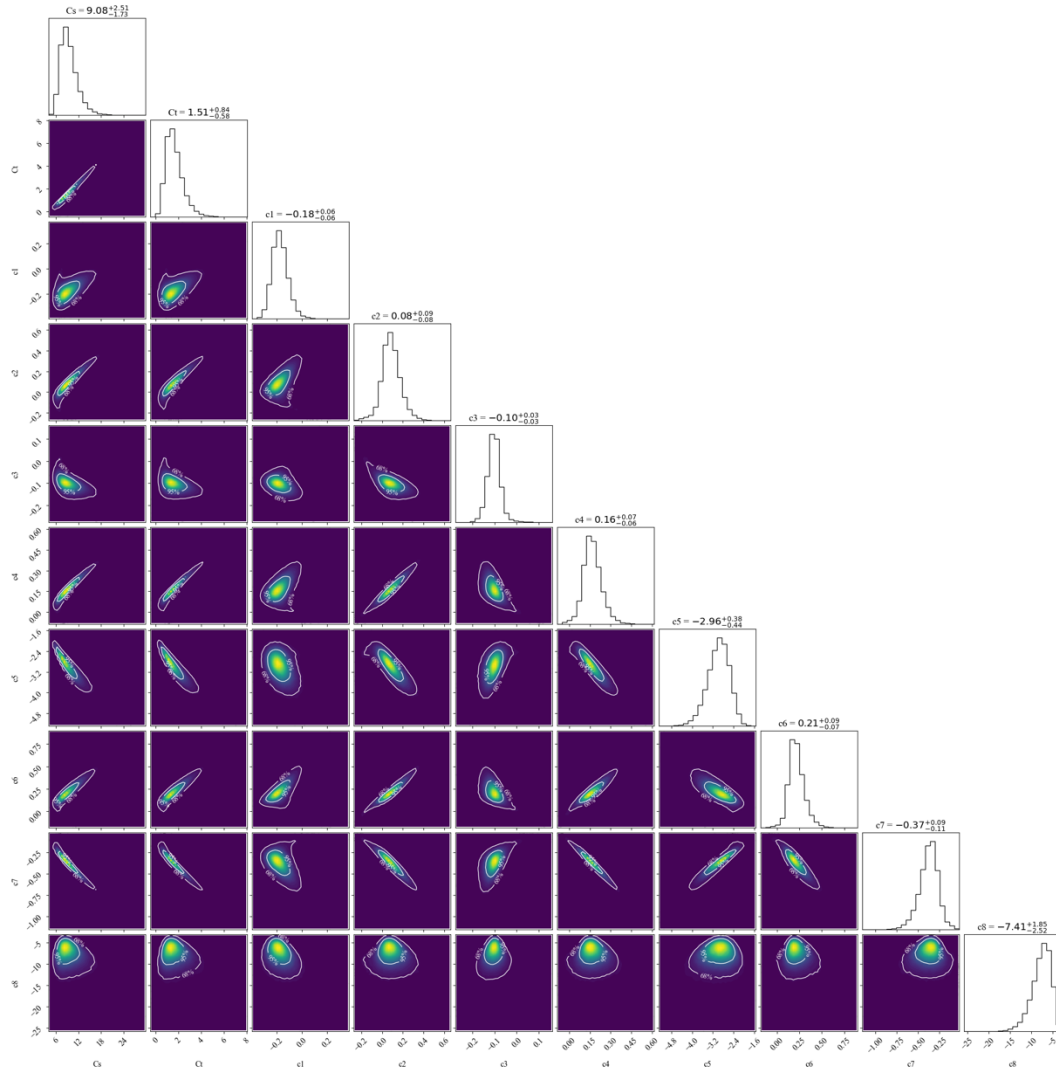
The likelihood accounts for the data, experimental, and theoretical uncertainty:

$$\mathcal{L} \propto \prod_i \exp \left\{ -\frac{1}{2} \left(\frac{X_i^{\text{exp}} - X_i^{\text{theo}}}{\sigma_i} \right)^2 \right\},$$
$$\sigma^2 = \sigma_{\text{exp}}^2 + \sigma_{\text{theo}}^2,$$



Fitting EFT interactions

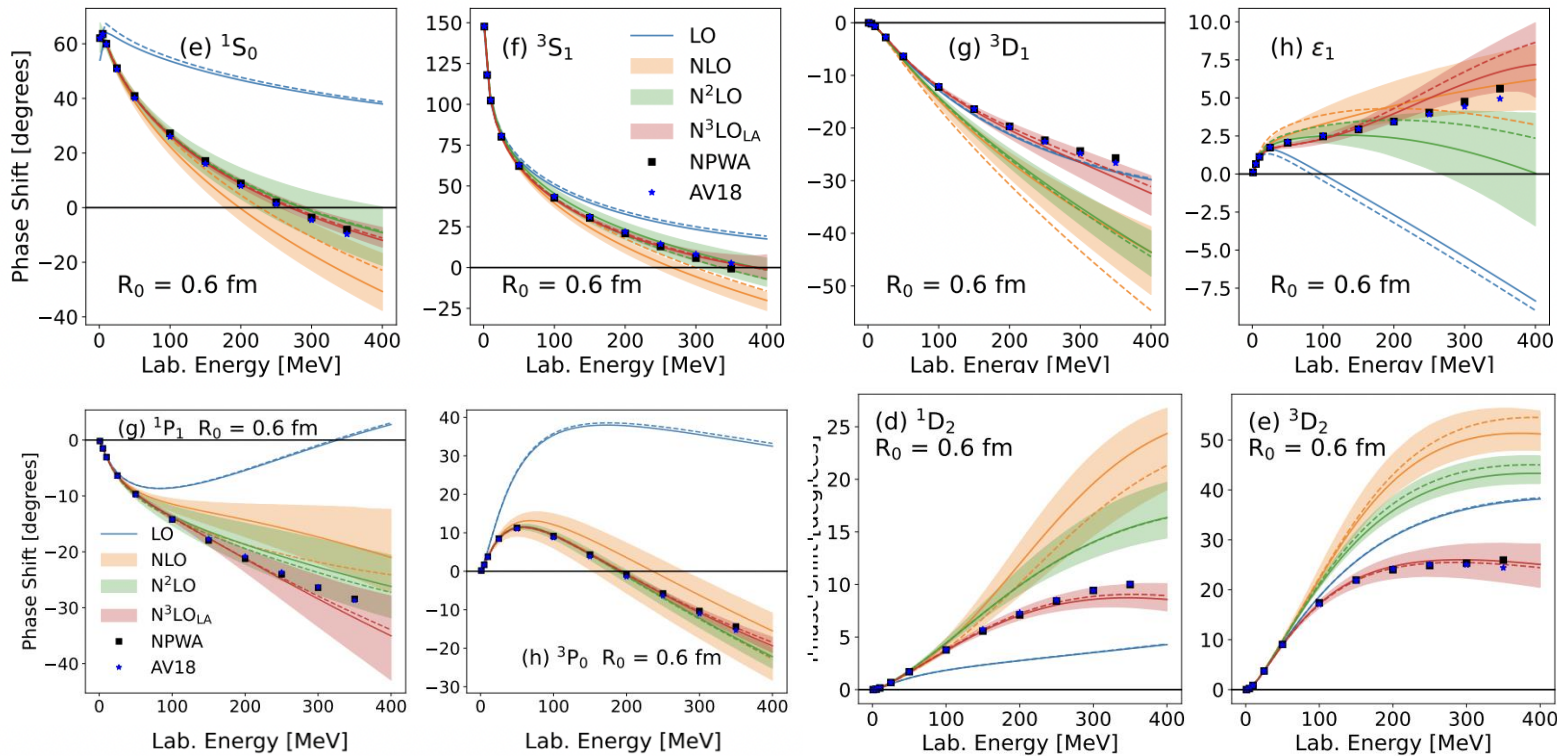
The uncertainty of nuclear interactions and data is mapped into the uncertainty of model parameters (LECs)



More details:
Somasundaram et al., PRC 2023



Fitting EFT interactions



Can work to desired accuracy with **error estimates!**

Somasundaram, IT, et al., PRC (2024)

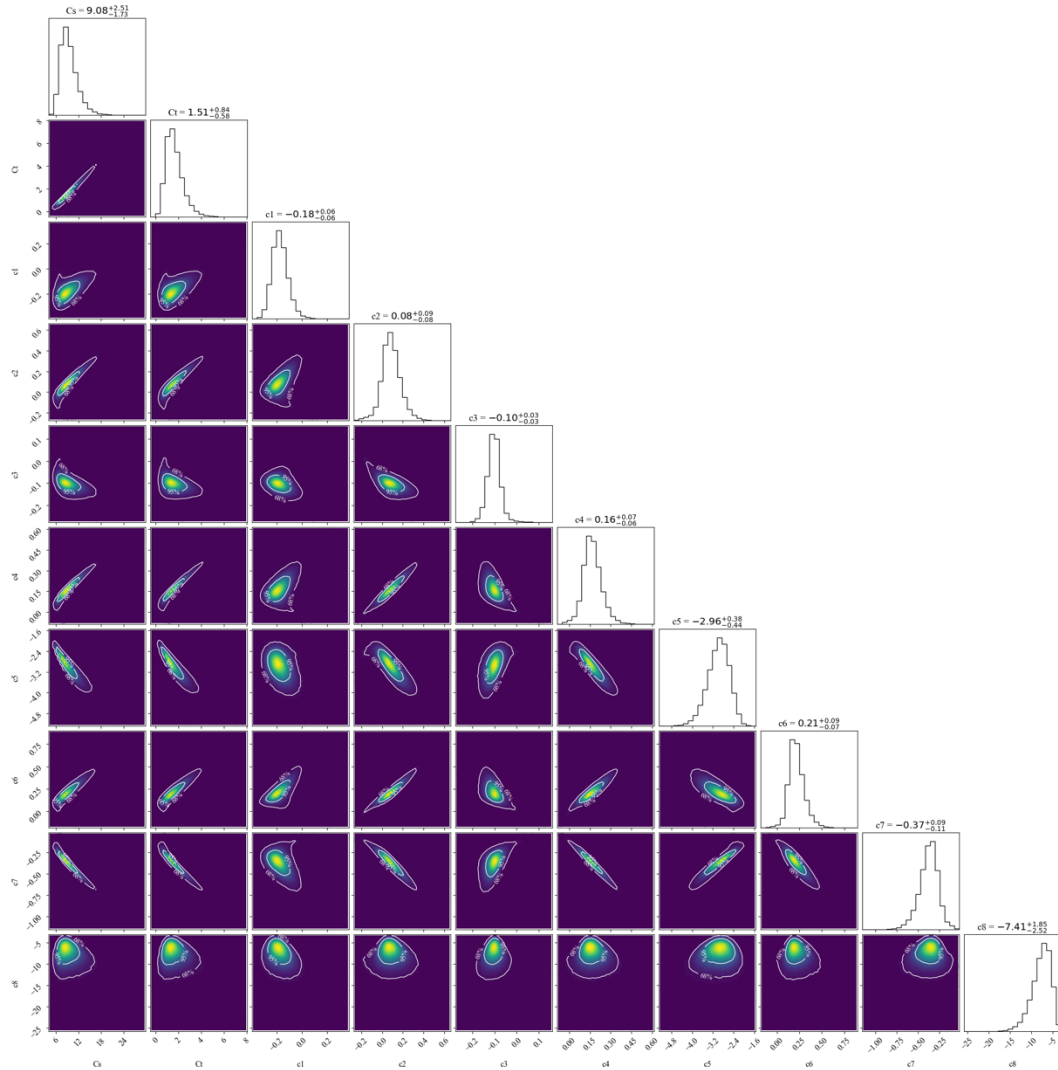
Fitting EFT interactions

The uncertainty of nuclear interactions and data is mapped into the uncertainty of model parameters (LECs)

Alternative: directly map LEC uncertainties to observables, e.g., nuclear matter.

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Fitting EFT interactions

The uncertainty of nuclear interactions and data is mapped into the uncertainty of model parameters

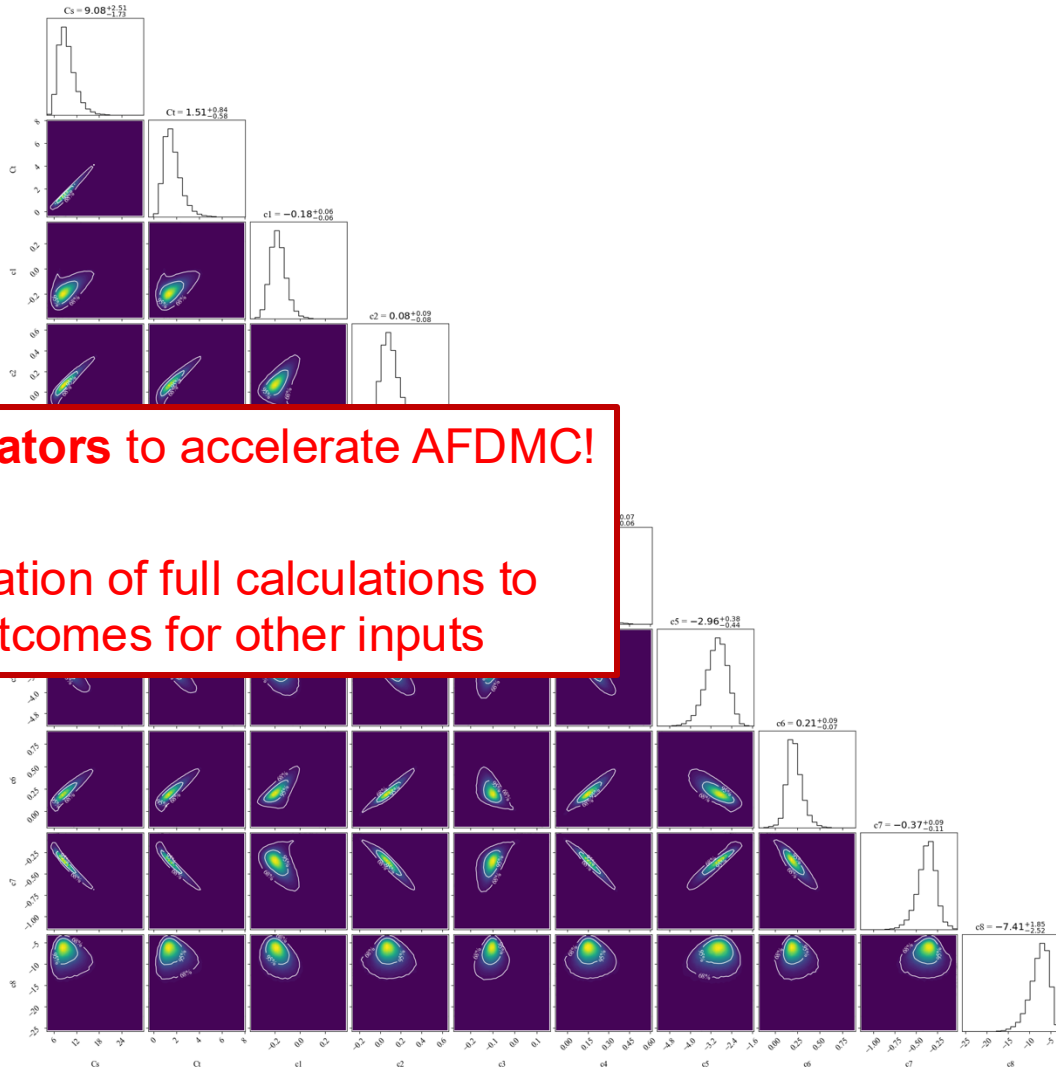
Alternative: direct uncertainties to c_i
e.g., nuclear matter.

We need **emulators** to accelerate AFDMC!

Uses information of full calculations to predict outcomes for other inputs

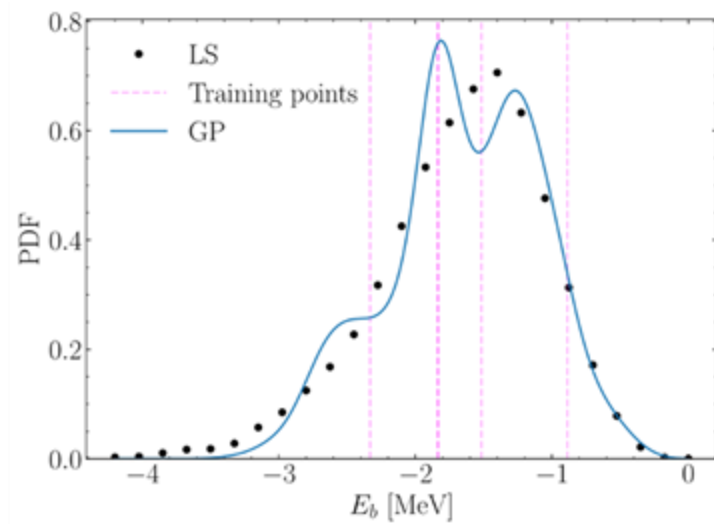
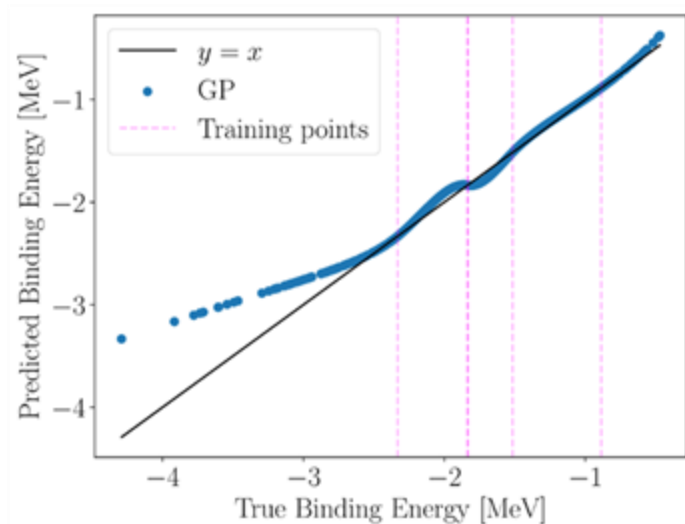
More details:

Somasundaram et al., PRC 2023



Emulators - Case of the deuteron

We have employed several emulators to AFDMC calculations of the deuteron:
Gaussian Process (GP)

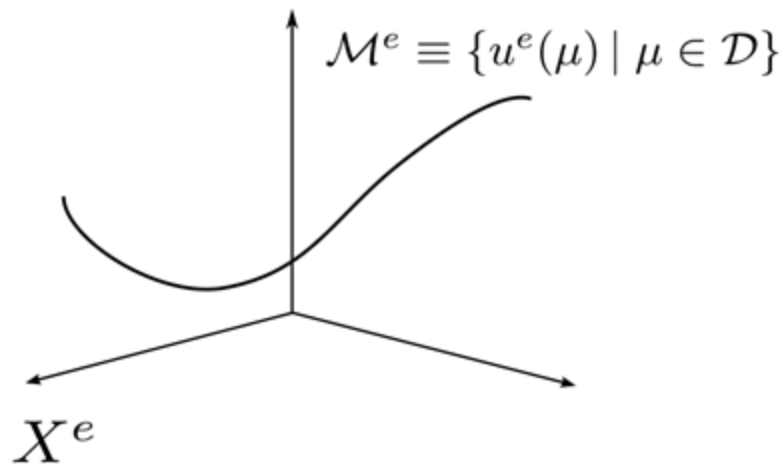


LS: Exact solution with Lippman-Schwinger equations for different sets of couplings..



Reduced-basis model (RBM)

Solution for Hamiltonian $H(\mu)$ lives in lower-dimensional manifold M of full Hilbert space

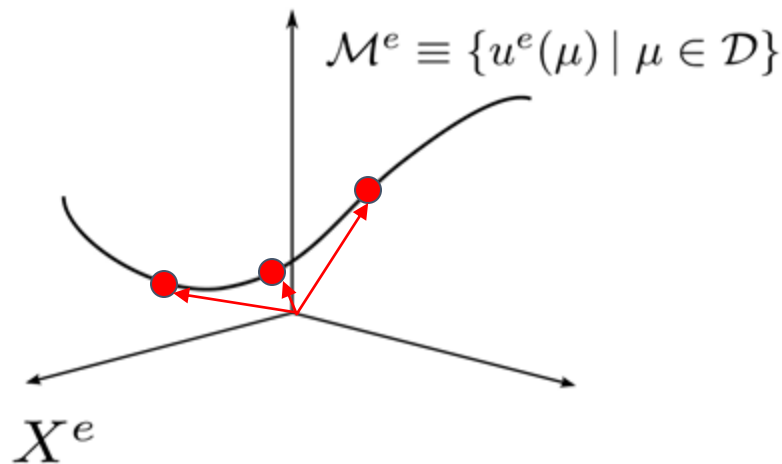


Reduced-basis model (RBM)

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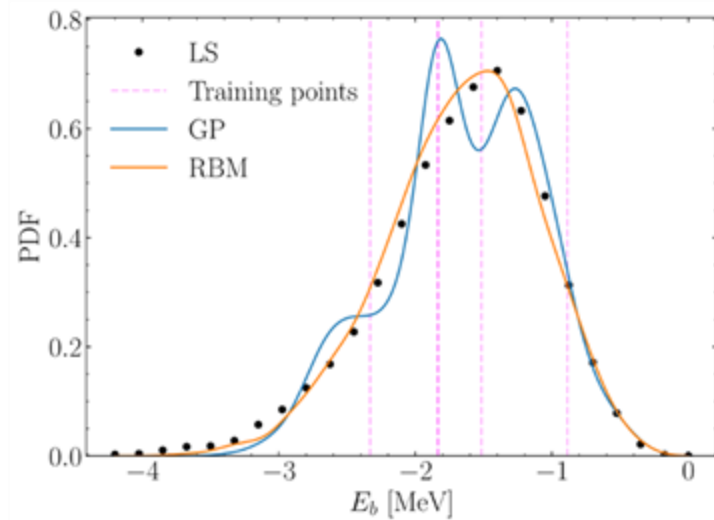
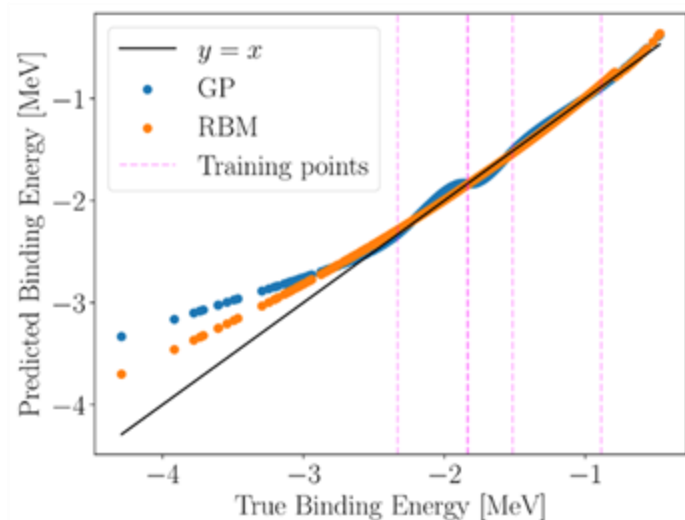
Calculation of N high-fidelity solutions for different μ provides basis for M

Project $H(\mu)$ into this subspace where diagonalization is simple



Emulators - Case of the deuteron

We have employed several emulators to AFDMC calculations of the deuteron: Gaussian Process (GP), Reduced-Basis Method (RBM)



LS: Exact solution with Lippman-Schwinger equations for different sets of couplings..



Parametric Matrix Model (PMM)

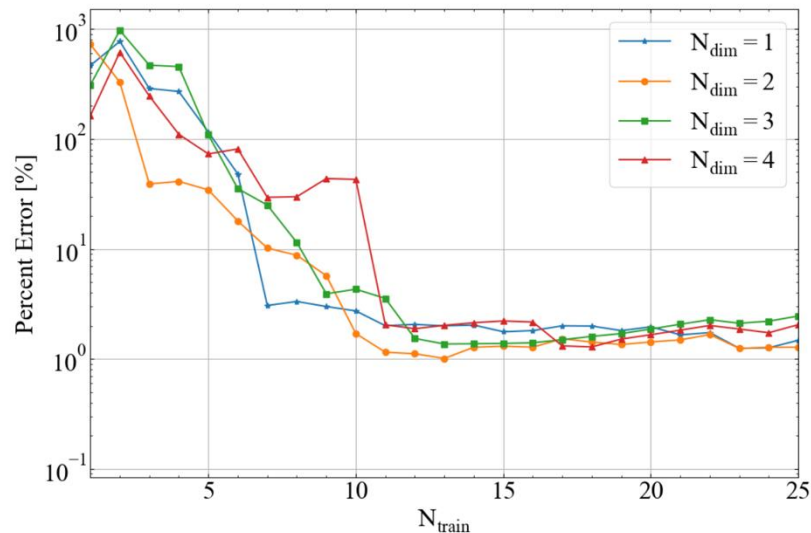
The projected Hamiltonian can be written as:

$$H(\mu) = D + c_1 M_1 + c_2 M_2 + \dots$$

Here, D is a diagonal matrix, M_i are symmetric matrices, and c_i are parameters of the Hamiltonian (low-energy couplings, LECs).

The **PMM** fits the matrix elements to high-fidelity solutions: lowest eigenvalue of $H(\mu)$ is fit to AFDMC energies.

The dimension of $H(\mu)$ is a choice, we will explore different choices.



Armstrong et al. arXiv 2502.03680

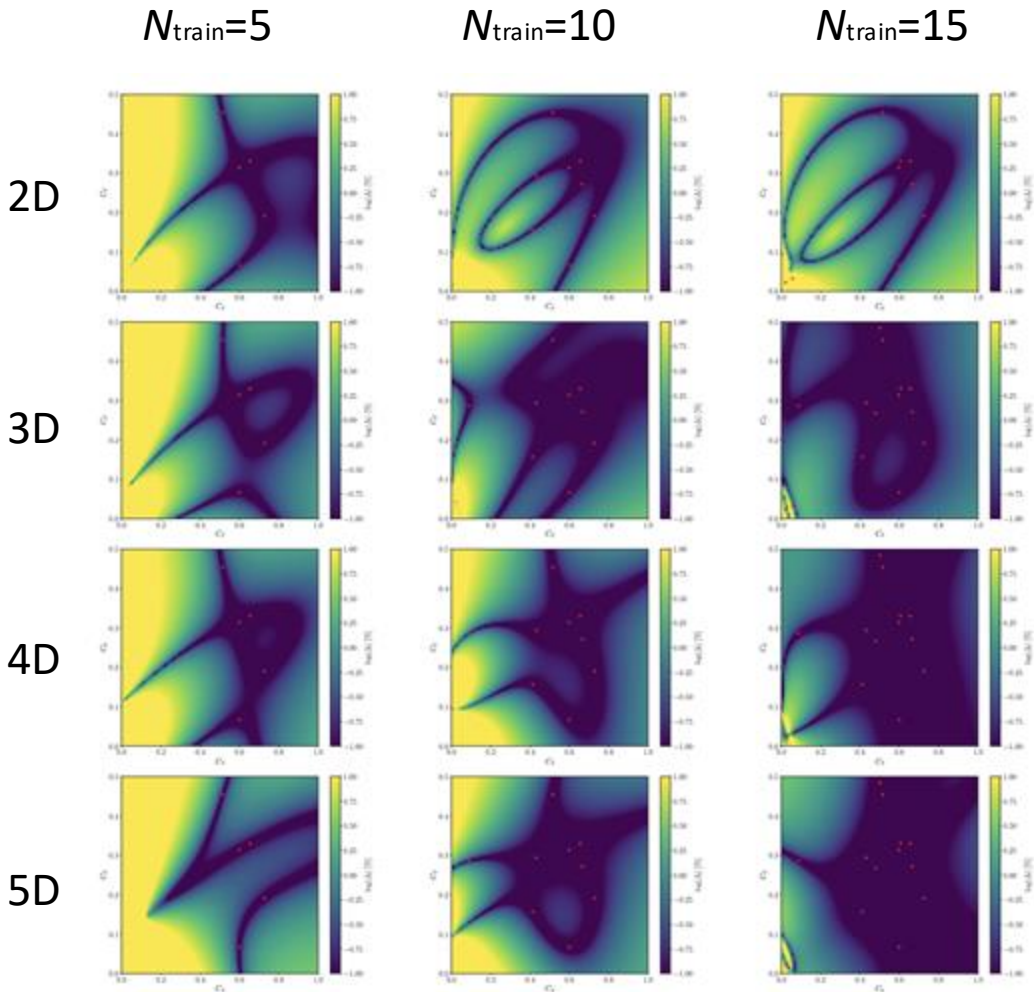


PMM - Toy problem

Randomly set up a 10-D
“Hamiltonian” with 2
parameters.

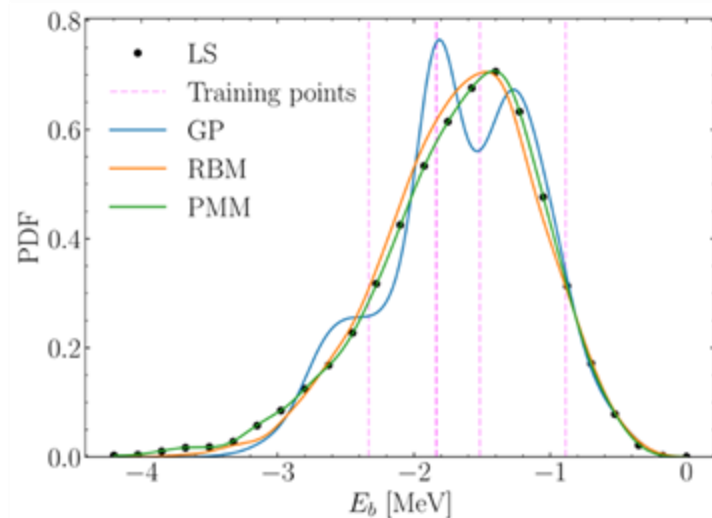
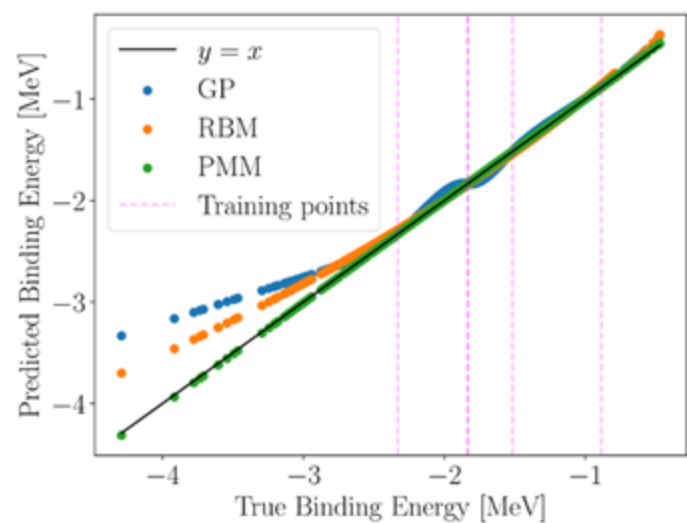
Test reproduction with PMM of
various dimensions and number
of training data.

Overall good description of the
original model with sufficient
dimension/data.



Emulators - Case of the deuteron

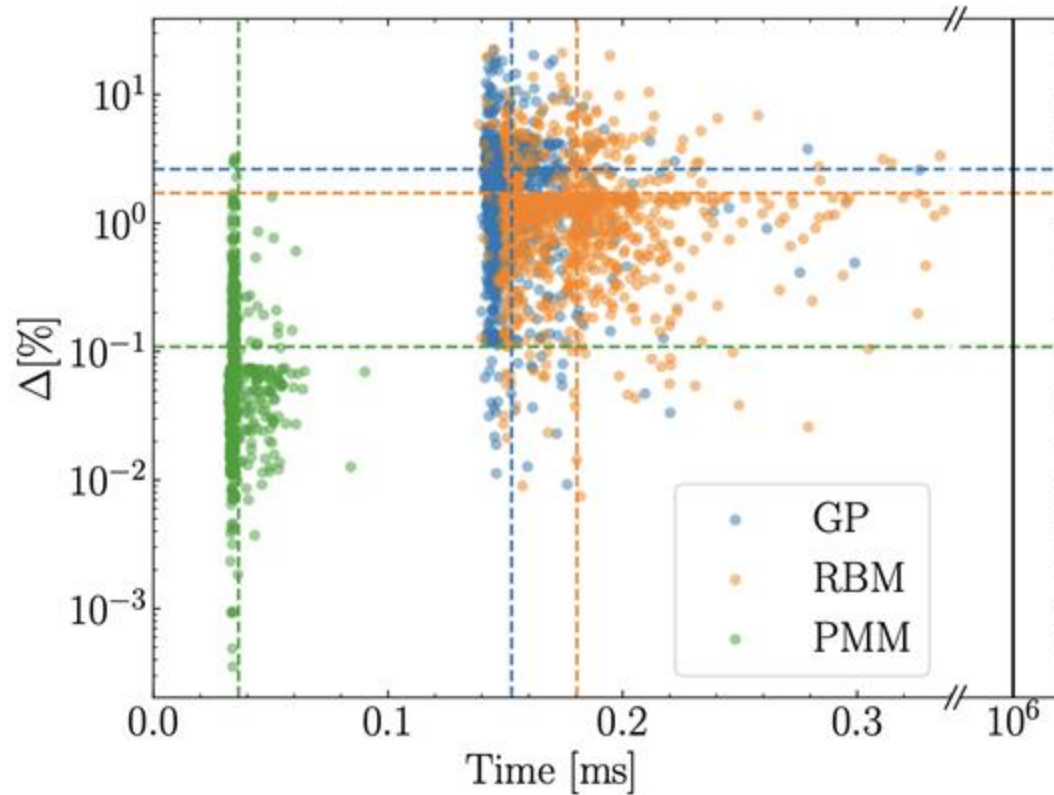
We have employed several emulators to AFDMC calculations of the deuteron: Gaussian Process (GP), Reduced-Basis Method (RBM), **Parametric Matrix Model (PMM)**



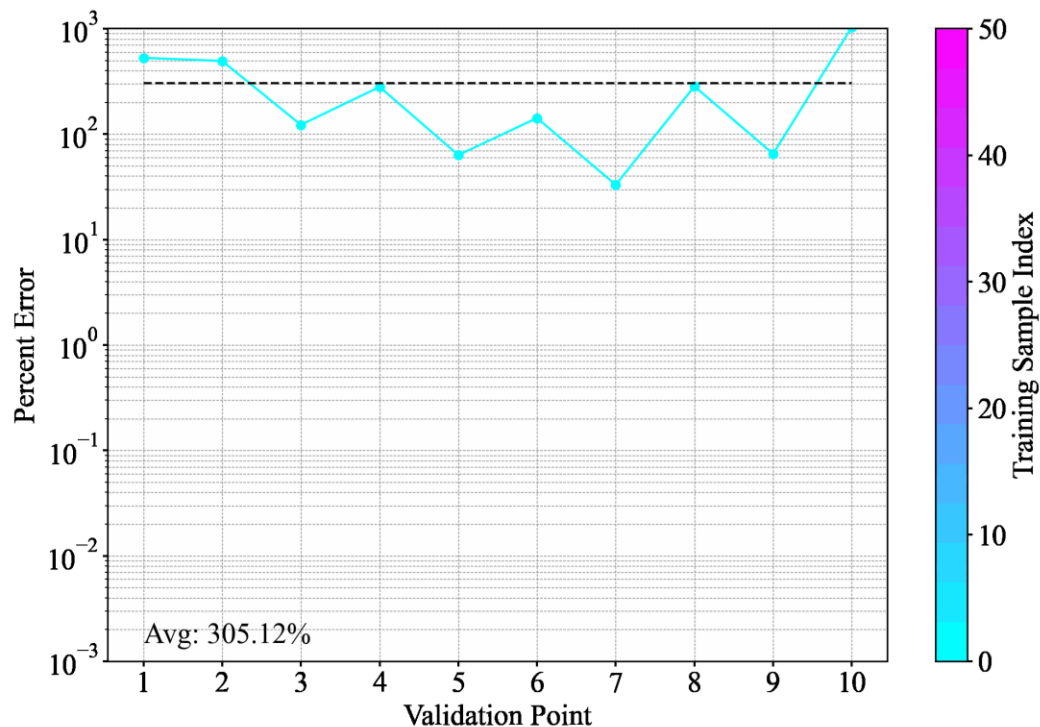
LS: Exact solution with Lippman-Schwinger equations for different sets of couplings..



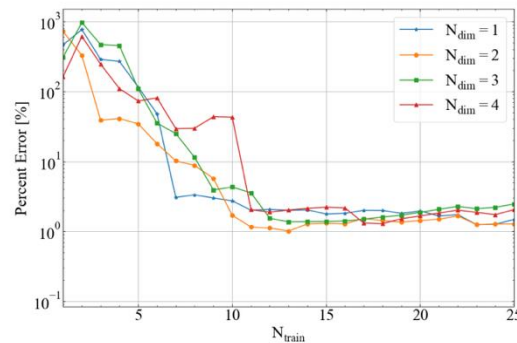
Emulators - Case of the deuteron



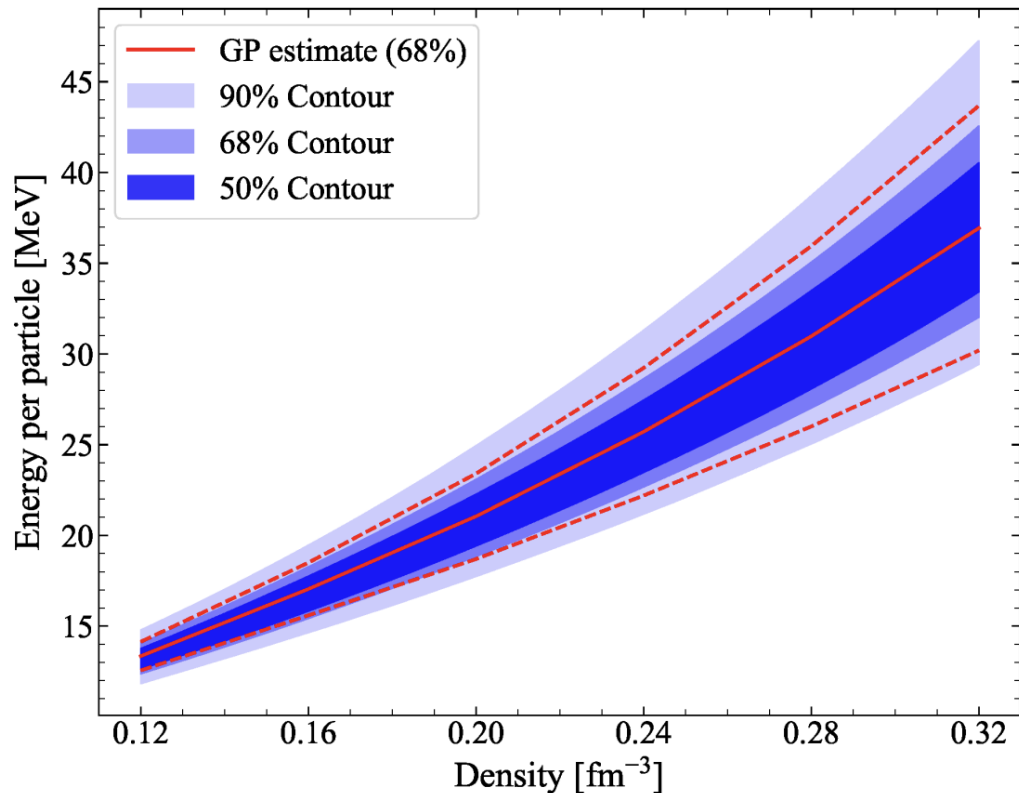
Result for the equation of state



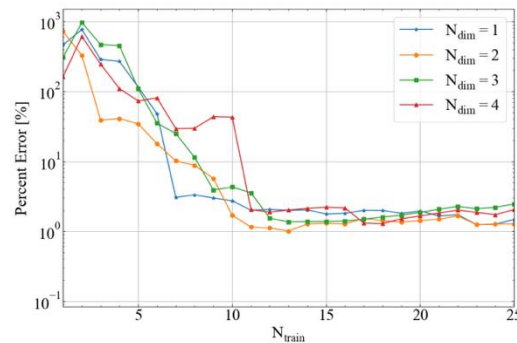
- PMM for AFDMC at $N^2\text{LO}$
- Training for 30 LEC combinations at 5 densities (~ 10 Mio CPU-h)
- Propagate 200,000 interactions to EOS in 1.5s on laptop!



Result for the equation of state

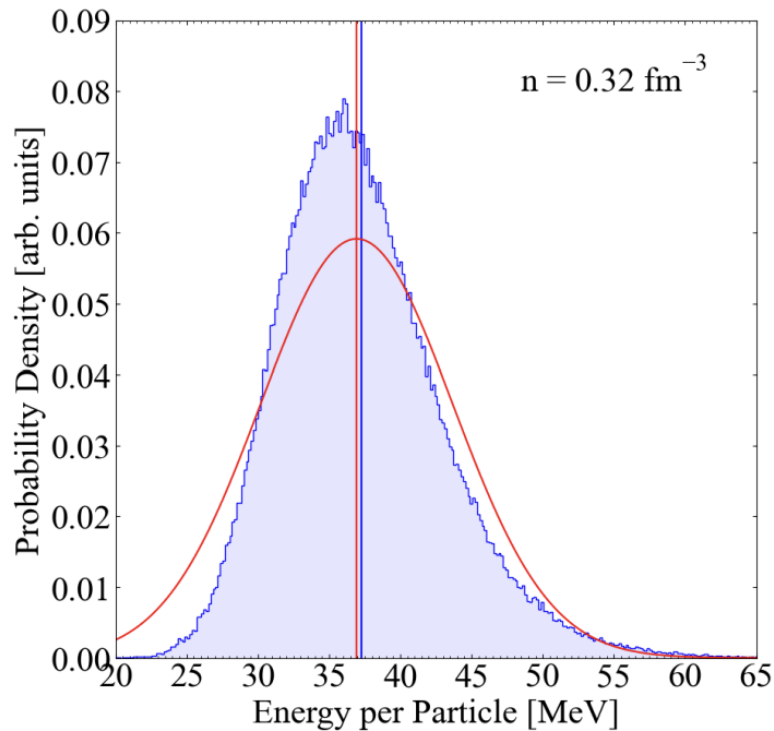
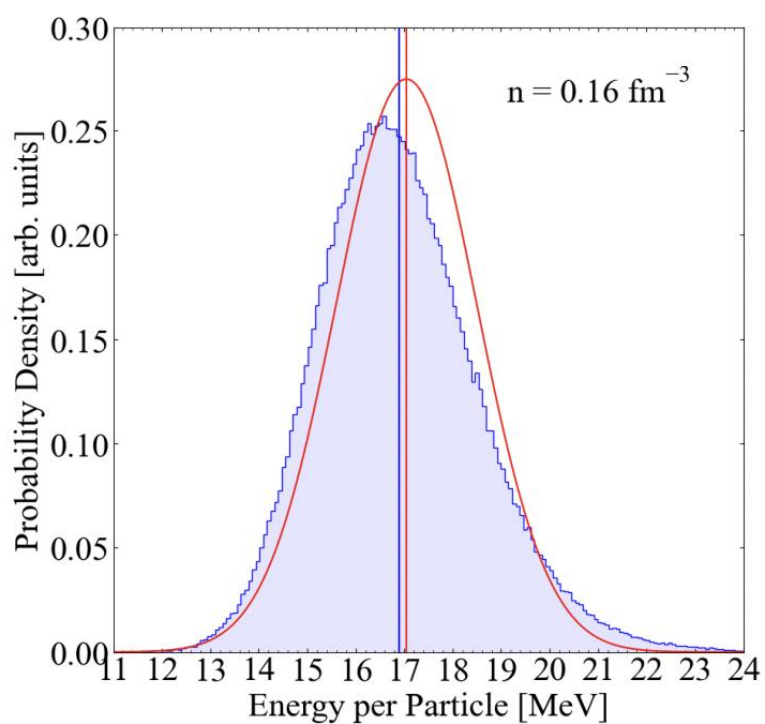


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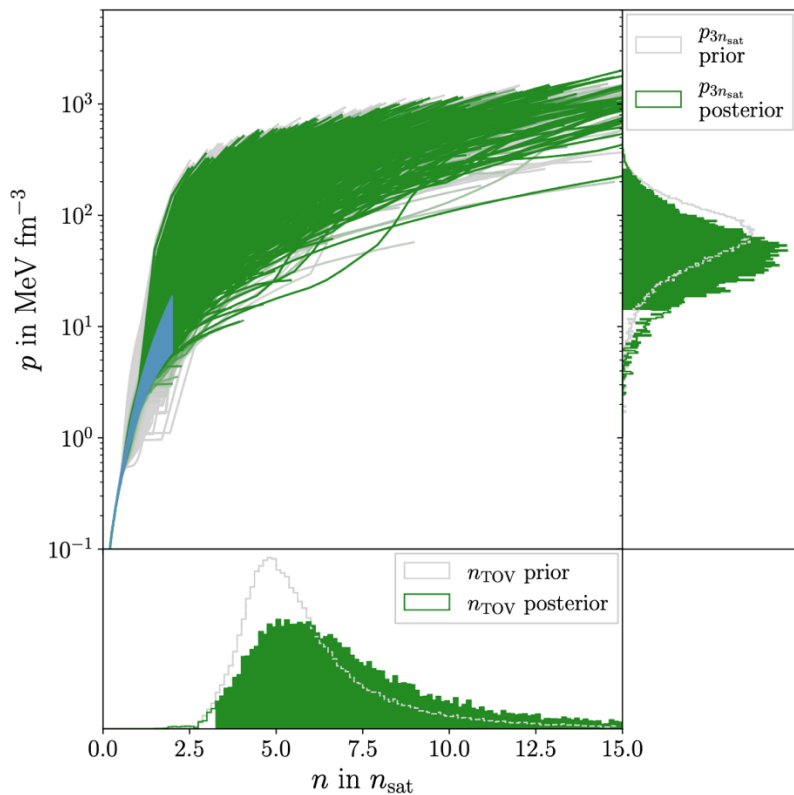


Armstrong et al. arXiv 2502.03680

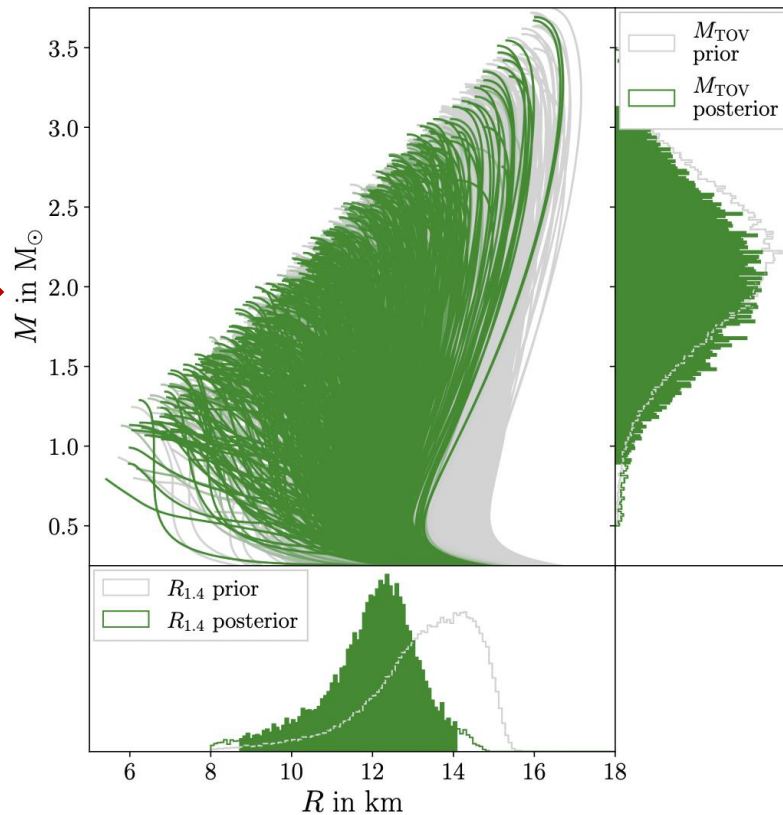
Result for the equation of state



Chiral EFT and neutron stars



TOV eqs.

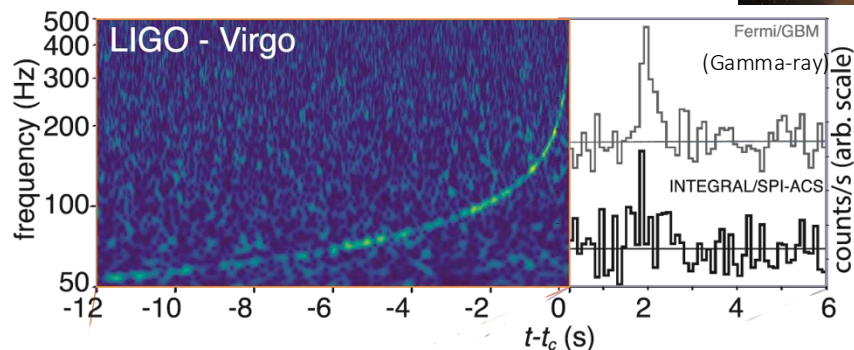
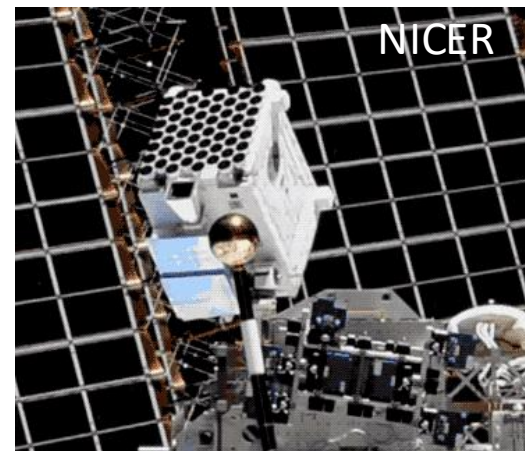
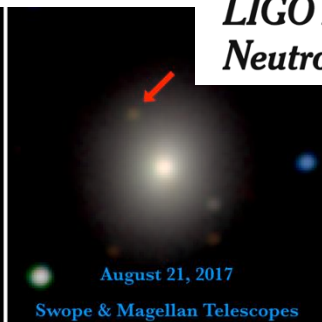


NS (multi-messenger) observations

First neutron-star merger
observed on Aug 17, 2017 :

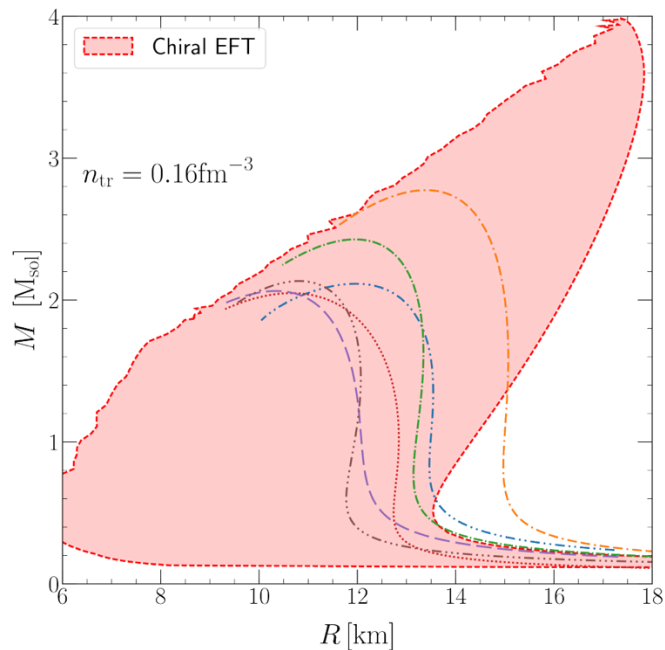
The New York Times

*LIGO Detects Fierce Collision of
Neutron Stars for the First Time*

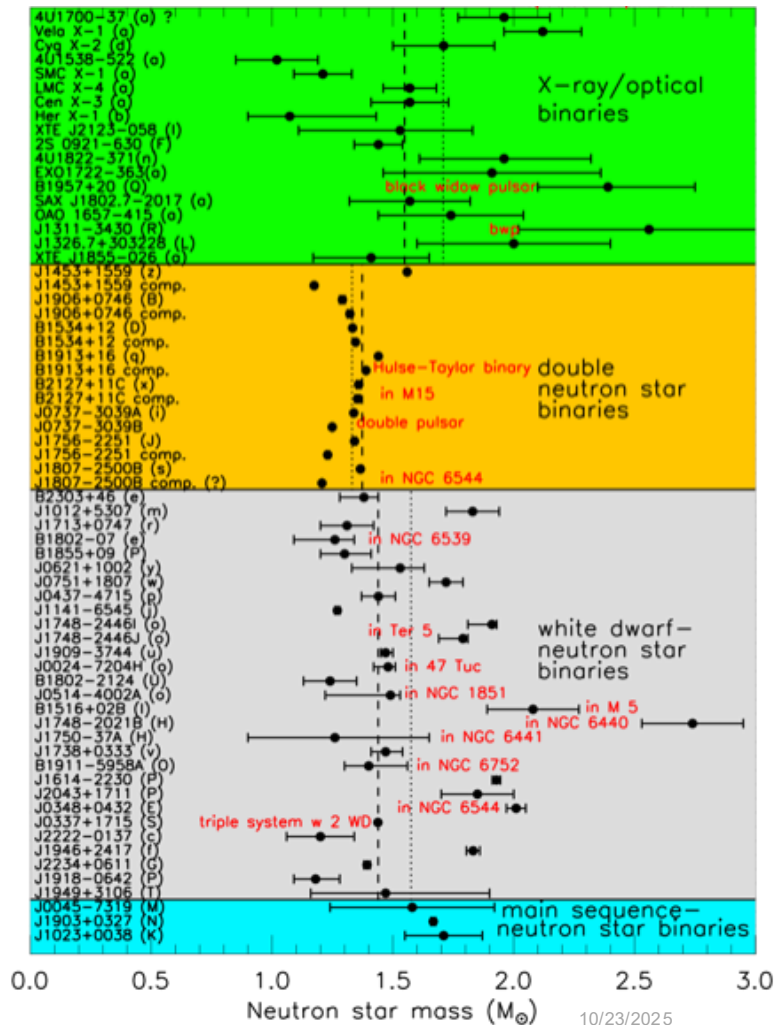


LIGO/VIRGO collaboration, ApJL 848, L12 (2017)

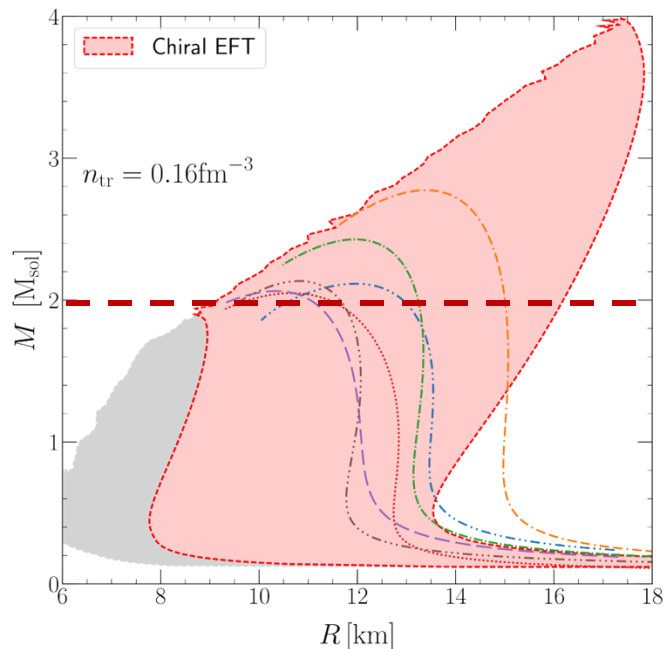
Neutron-star masses



Heaviest observed neutron-stars provide constraints, because all EOS have to be able to reproduce observation.



Neutron-star masses



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A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest¹, T. Pennucci², S. M. Ransom¹, M. S. E. Roberts³ & J. W. T. Hessels^{4,5}

(2010)

A Massive Pulsar in a Compact Relativistic Binary

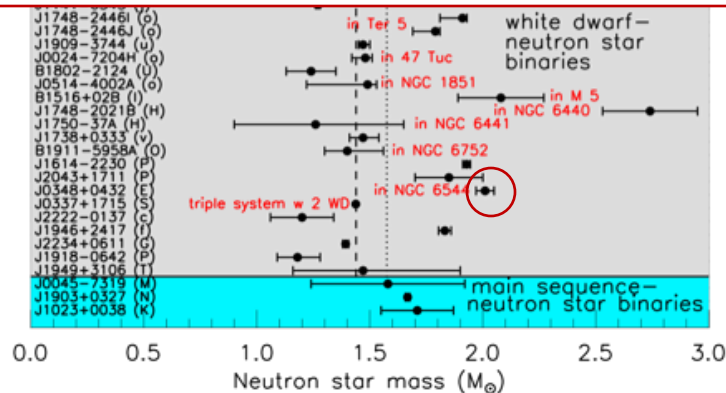
(2013)

John Antoniadis,^{*} Paulo C. C. Freire, Norbert Wex, Thomas M. Tauris, Ryan S. Lynch, Marten H. van Kerkwijk, Michael Kramer, Cees Bassa, Vik S. Dhillon, Thomas Driebe, Jason W. T. Hessels, Victoria M. Kaspi, Vladislav I. Kondratiev, Norbert Langer, Thomas R. Marsh, Maura A. McLaughlin, Timothy T. Pennucci, Scott M. Ransom, Ingrid H. Stairs, Joeri van Leeuwen, Joris P. W. Verbiest, David G. Whelan

Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar

(2019)

H. T. Cromartie^{1*}, E. Fonseca², S. M. Ransom³, P. B. Demorest⁴, Z. Arzoumanian⁵, H. Blumer^{6,7}, P. R. Brook^{6,7}, M. E. DeCesar⁸, D. Dolch⁹, J. A. Ellis¹⁰, R. D. Ferdman¹¹, E. C. Ferrara^{12,13}, N. Garver-Daniels^{6,7}, P. A. Gentile^{6,7}, M. L. Jones^{6,7}, M. T. Lam^{6,7}, D. R. Lorimer^{6,7}, R. S. Lynch¹⁴, M. A. McLaughlin^{6,7}, C. Ng^{15,16}, D. J. Nice⁹, T. T. Pennucci¹⁷, R. Spiewak¹⁸, I. H. Stairs¹⁵, K. Stovall⁴, J. K. Swiggum¹⁹ and W. W. Zhu²⁰

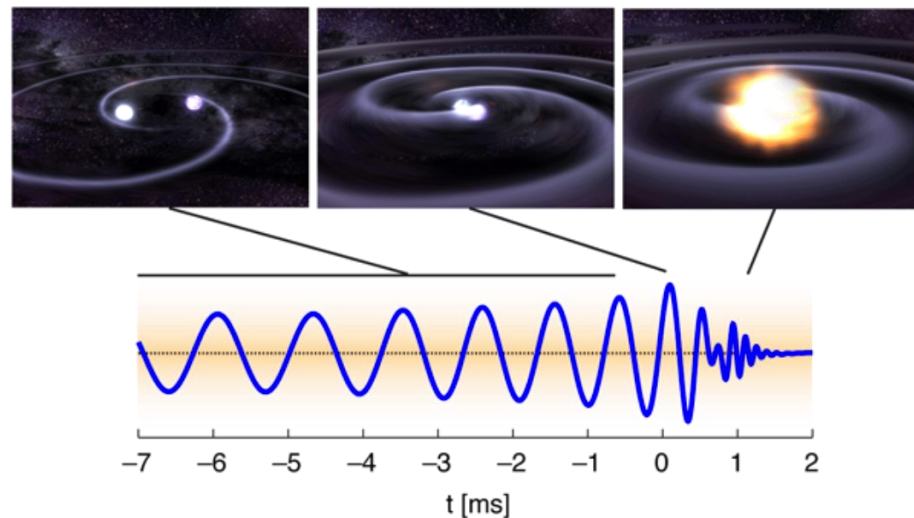
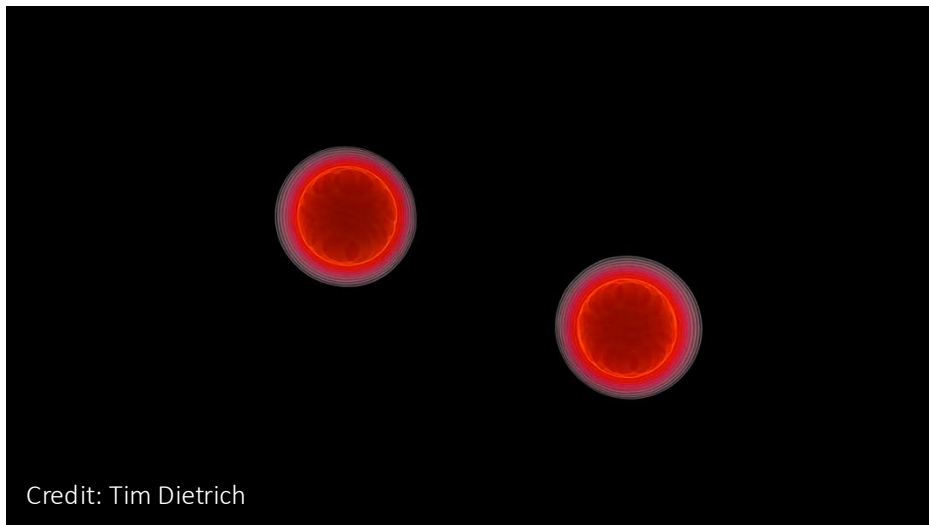


Neutron-star mergers

Gravitational waves from neutron-star merger offer possibility to “measure” the neutron-star radius!

LIGO/VIRGO:

- During merger, neutron stars deform under gravitational field of partner.
- This deformation is measured as “tidal deformability” from gravitational waveform during inspiral phase of neutron-star merger, and probes radius.



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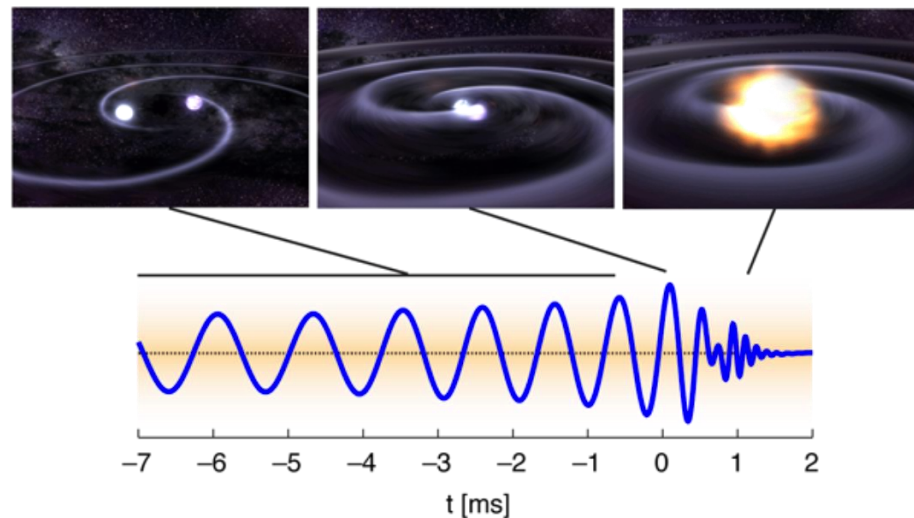
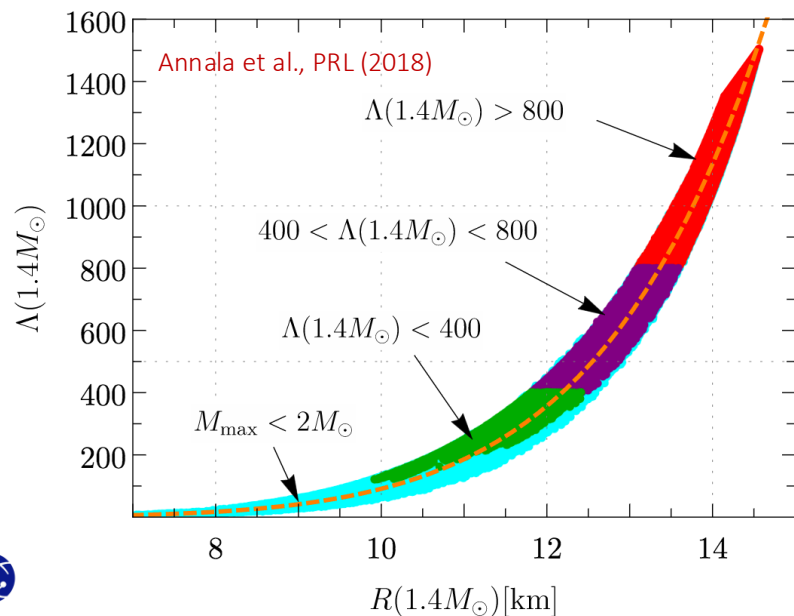


Illustration: (Top) NASA; (Bottom), Alan Stonebreaker

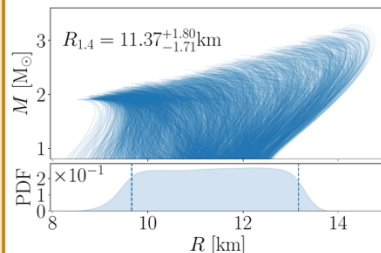
10/23/2025

67

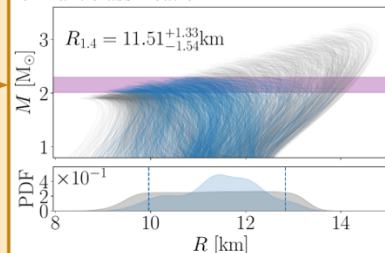
Nuclear-physics Multi-Messenger Astrophysics

Prior construction

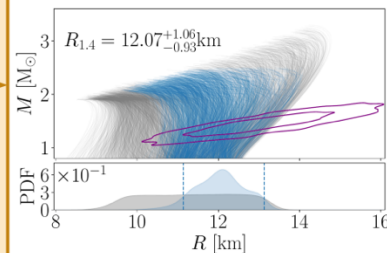
(A) Chiral effective field theory:
EOS derived with the chiral EFT framework



(B) Maximum Mass Constraints:
PSR J0740+6620/ PSR J0348+4032/ PSR J1614-2230 and GW170817/AT2017gfo remnant classification



(C) NICER:
PSR J0030+0451



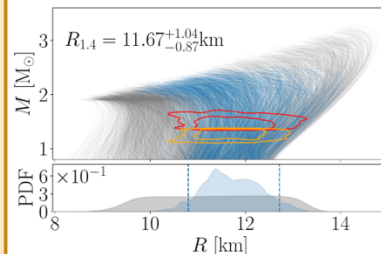
NMMA framework:

Pang et al., Nat. Comm. (2023)

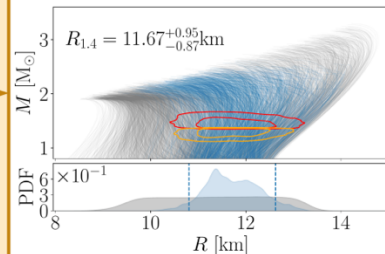
- EOS consistent with theory
- Masses and NICER via published posteriors
- Simultaneous full GW and KN analyses
- Available online.

Parameter estimation

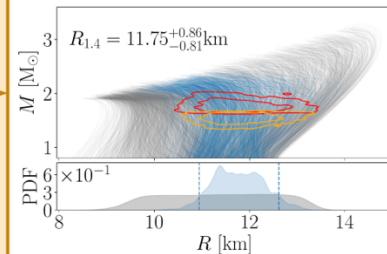
(D) GW170817:
reanalysis with IMRPhenomPv2_NRTidalv2



(E) AT2017gfo:
analysis of the observed lightcurves



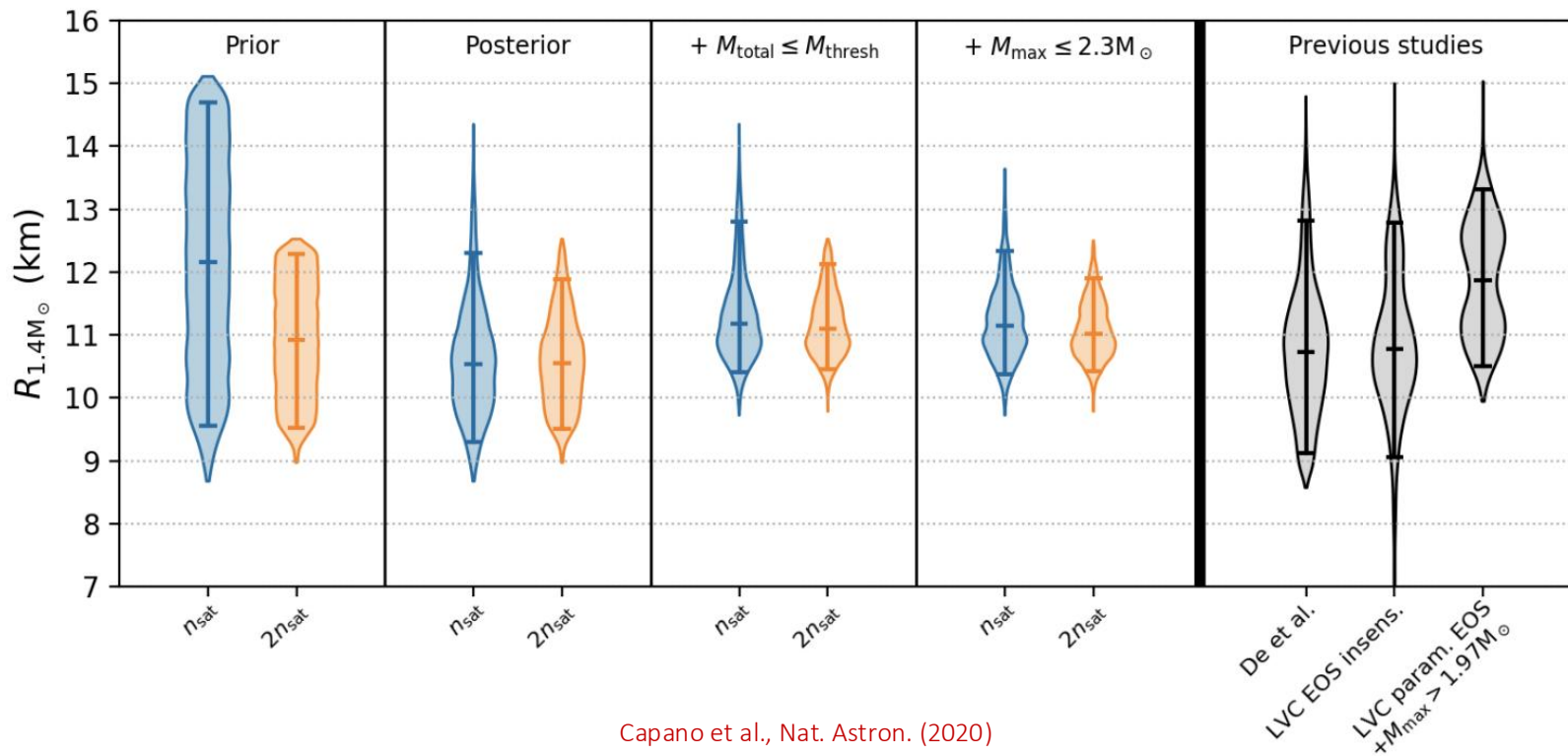
(F) GW190425:
reanalysis with IMRPhenomPv2_NRTidalv2



Dietrich, Coughlin, Pang, Bulla, Heinzl, Issa, IT, Antier, Science (2020)



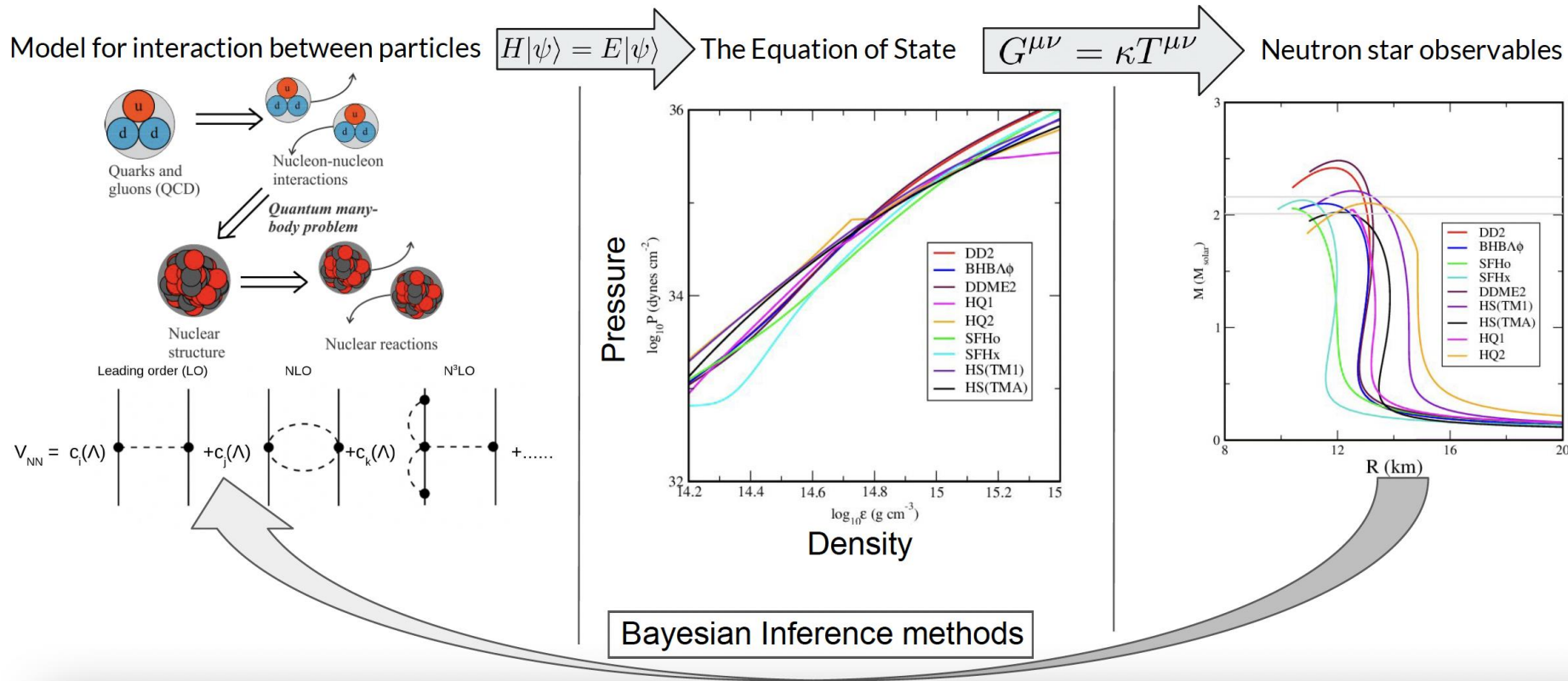
Nuclear-physics Multi-Messenger Astrophysics



Capano et al., Nat. Astron. (2020)

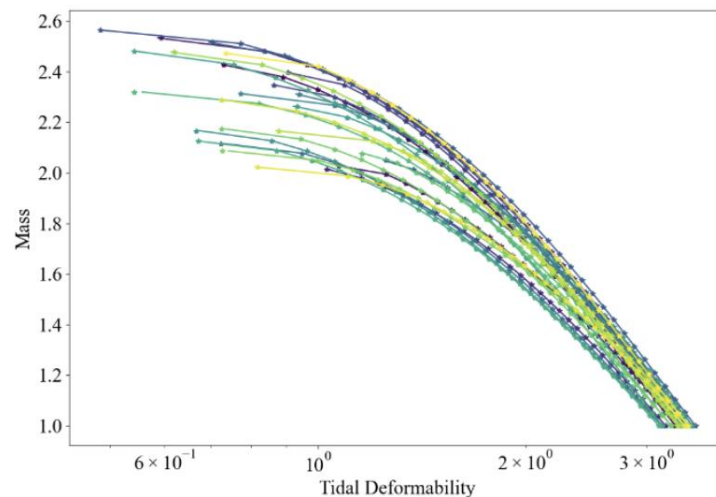
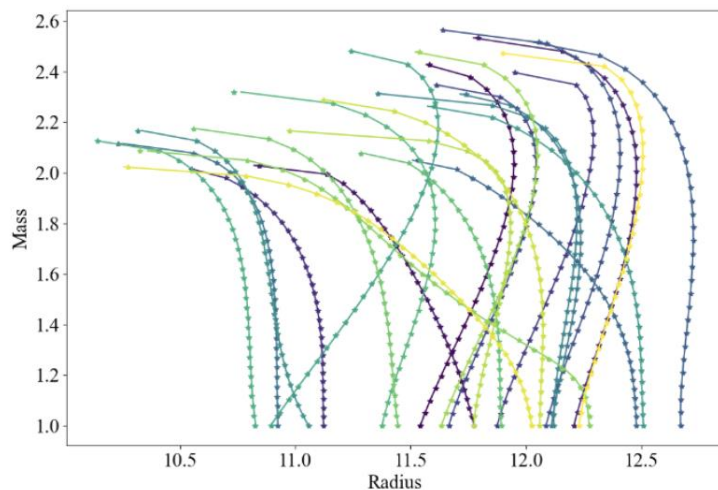


Inferring 3N forces from Observations



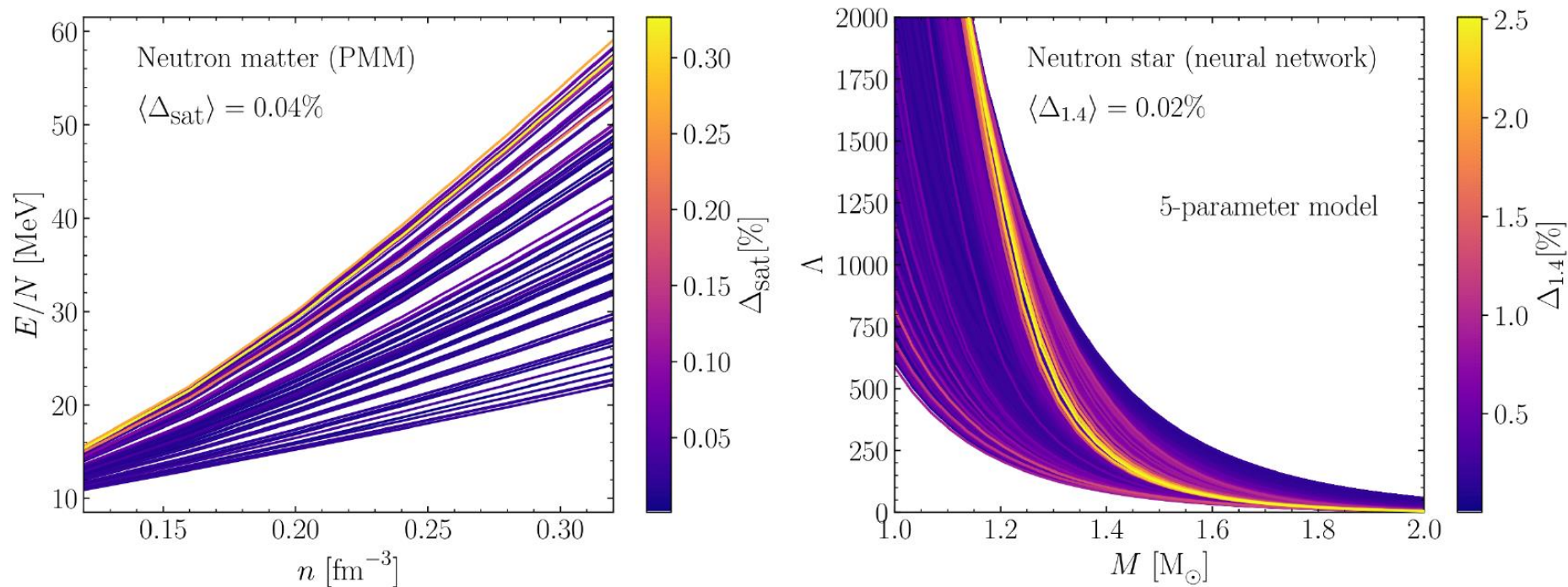
TOV emulators

- Build on Multilayer Perceptron Neural Network
- Trained on $\sim 200k$ solutions to TOV equations for M , R , tidal deformability
- Speed up of factor 200 with average uncertainties of 0.02%

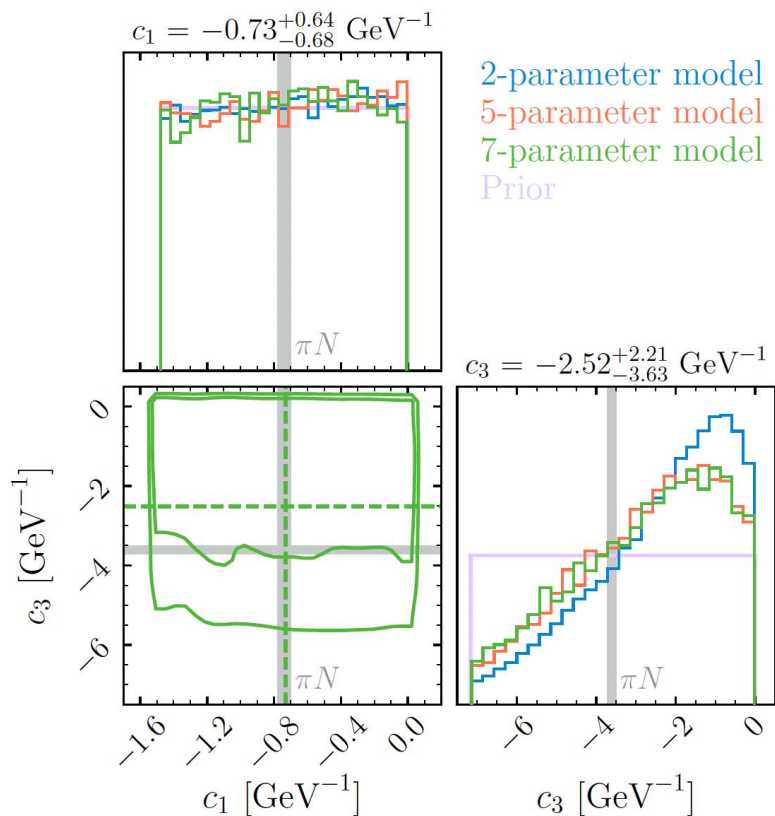


Inferring 3N forces from Observations

Emulators for this task:



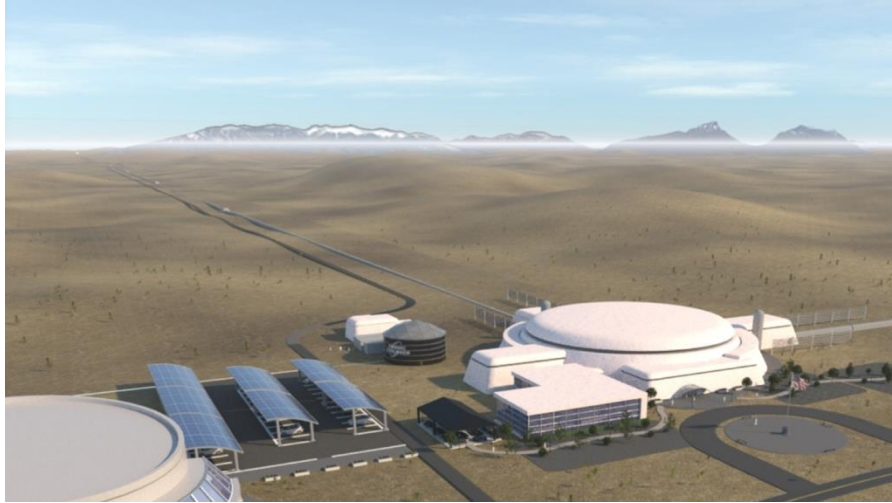
Inferring 3N forces from Observations



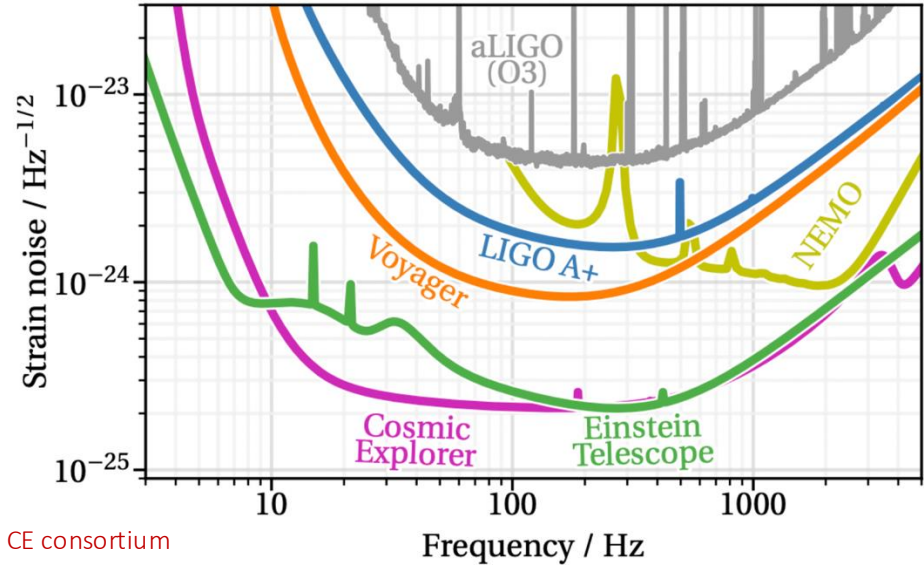
- Use pipeline to measure LECs in leading three-nucleon forces
- Employ three different EOS models with different complexity (number of parameters)
- Current observations provide information in LECs but not competitive to laboratory experiments (gray)



The future: Cosmic Explorer (CE)

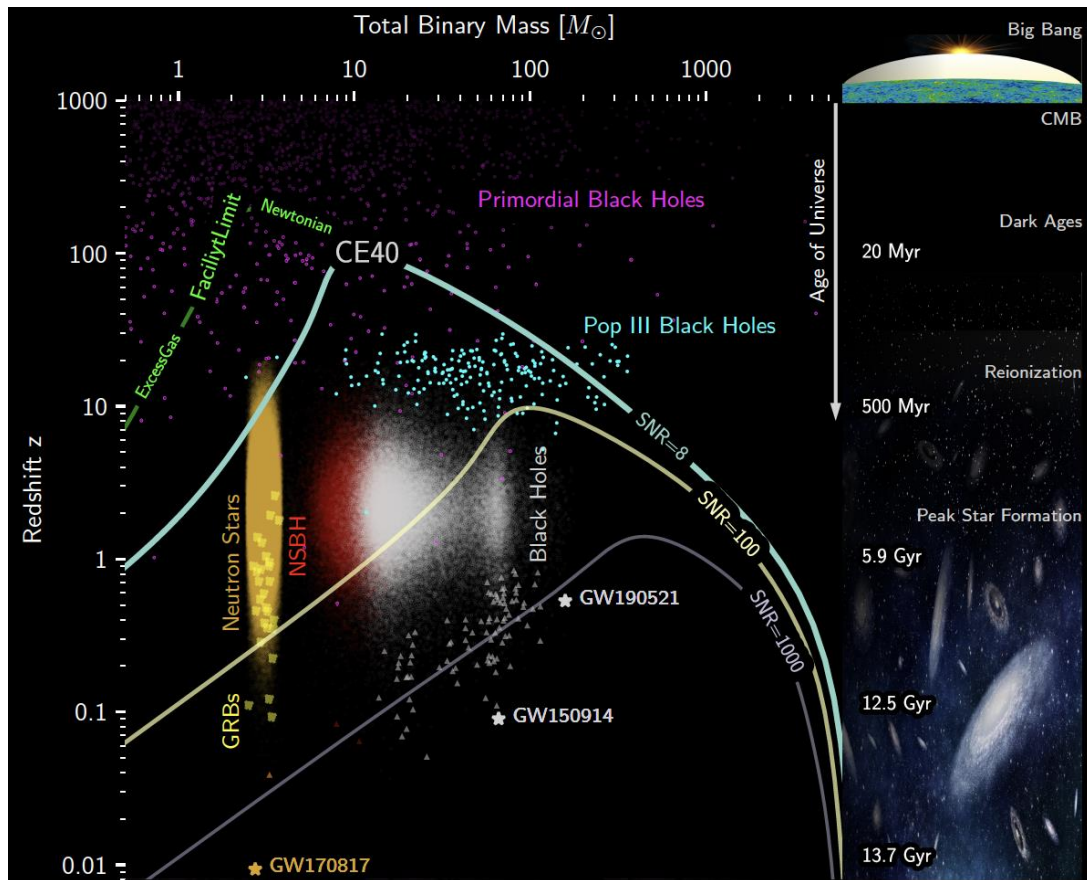


Source: CE consortium



- 3rd- generation Gravitational-Wave Detectors will increase sensitivity by at least factor of 10
- US-proposal: Cosmic Explorer
- EU-proposal: Einstein Telescope

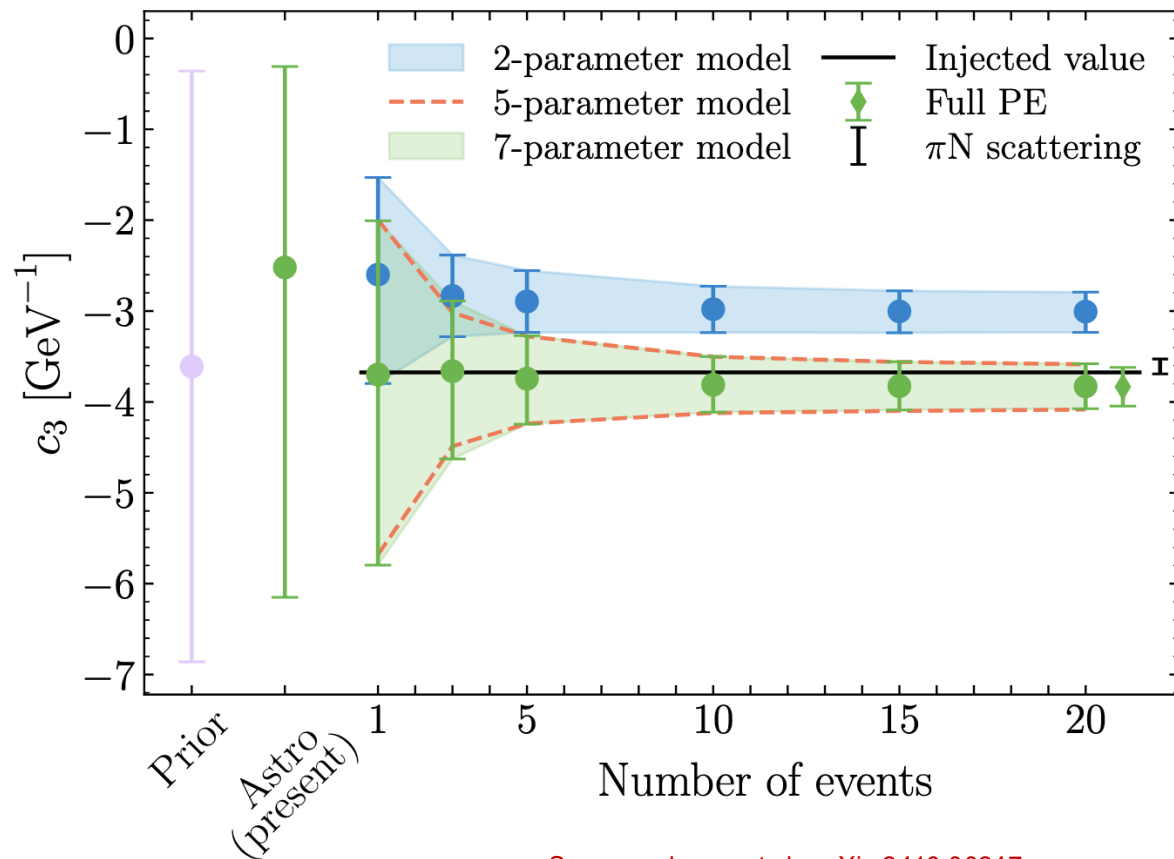
The future: Cosmic Explorer (CE)



- CE will detect the majority of neutron-star mergers in the universe!
- GW170817 would have been observed with an SNR 100 times higher.
- Will measure neutron-star radius to within a few percent!



Inferring 3N forces from Observations

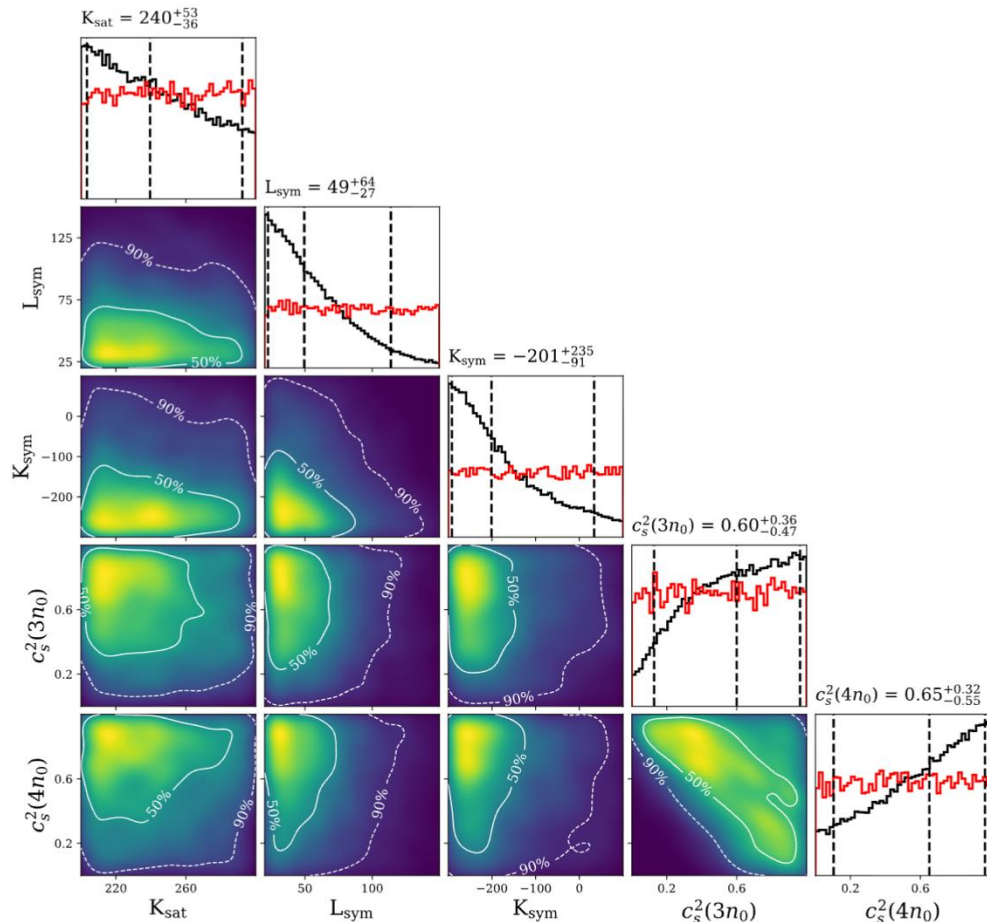


With Cosmic Explorer, astrophysical constraints in LECs are possible!

They are comparable to laboratory measurements.

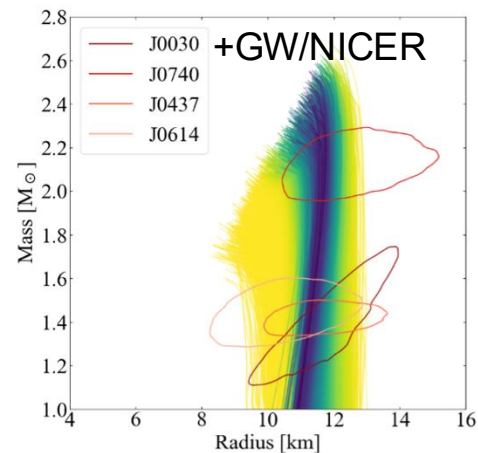
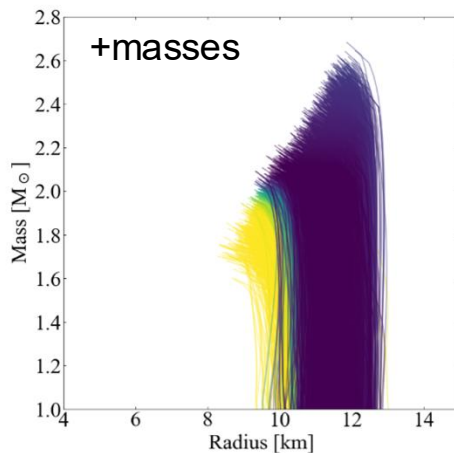
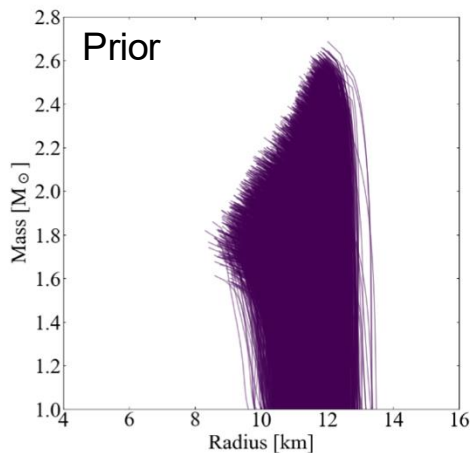
Direct EOS Inference

- Use emulators to directly sample EOS parameters in GW data analysis
- Emulators implemented in PyCBC inference software for EOS models with different number parameters
- Allows for direct sampling, which might be beneficial for large parameters spaces.



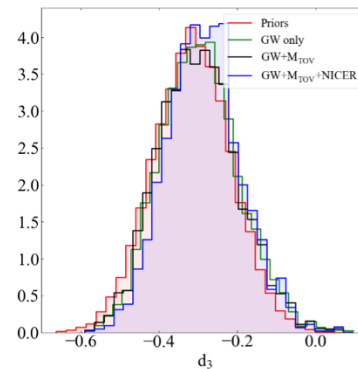
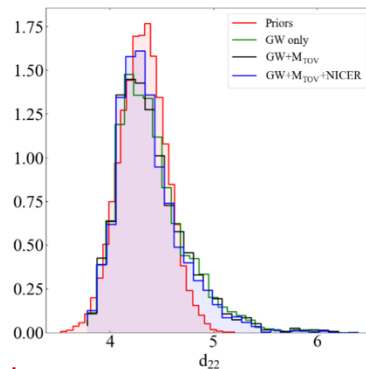
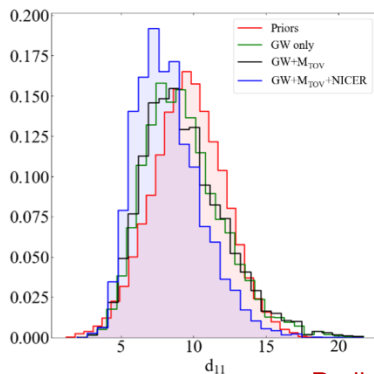
Direct EOS Inference

- Sample LECs of EFT Hamiltonian.
- Not possible without the use of emulators.

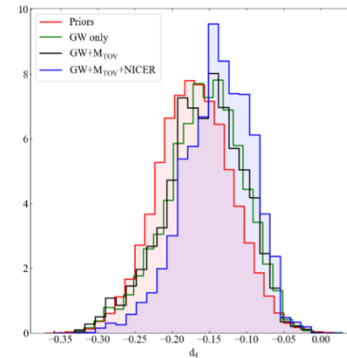
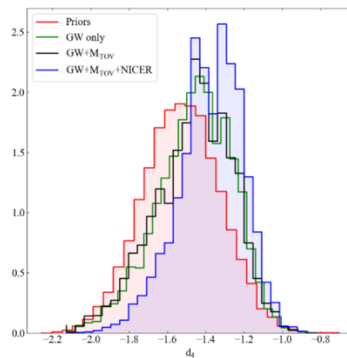
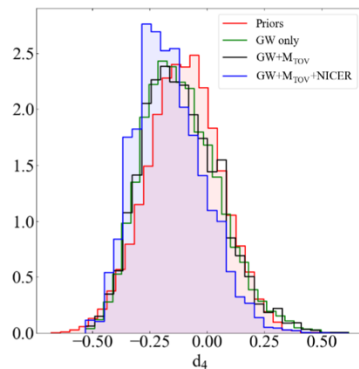


Direct EOS Inference

- Sample LECs of EFT Hamiltonian.
- Not possible without the use of emulators.



Preliminary!



Summary

- Systematic theoretical constraints on the EOS and other neutron-star properties possible from chiral effective field theory.
- Chiral EFT is a low-momentum approach to nuclear forces; it breaks down at around two times the nuclear saturation density (?).
- Importantly, chiral EFT enables systematic theoretical uncertainty estimates.
- We can use astrophysical information on neutron stars to test properties of nuclear interactions and constrain coupling constants.
- Next-generation observatories will be key for this task.

Thanks for your attention!



Challenges/Opportunities

- In which density range do theories of nuclear interactions (such as chiral effective field theory) remain reliable?
- How can we fundamentally improve these theories (power counting, degrees of freedom...)?
- What is the most robust way of quantifying theoretical uncertainties of the calculations?
- We can learn about nuclear interactions at neutron-rich extremes and extreme densities using astrophysics.
- We could study the impact of additional degrees of freedom, such as hyperons.

Thanks for your attention!



Thanks

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Thank you for your
attention!

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