Extended transition operator for ββ decay via Rayleigh–Schrödinger perturbation theory — *Solution to the uncertainty problem*

J. Terasaki, *Inst. of Exp. and Appl. Phys., Czech Tech. Univ. in Prague*

In collaboration with Dr. O. Civitarese (Univ. La Plata)

J. T. and O. Civitarese, PRC **112**, 024304 (2025);
112, L051302 (2025);
arXiv:2509.16605

Jul. 7, 2026
Seattle, USA

Abbreviations

NME: nuclear matrix element

g_A: axial-vector current coupling

g_V: vector current coupling

0νββ: neutrinoless double-β

2νββ: two-neutrino double-β

QRPA: quasiparticle random-phase approximation

RS: Rayleigh-Schrödinger

VC: vertex correction

2BC: two-body current
RS2BC and χ2BC of chiral effective field theory

Talk plan

1. Definition of the uncertainty problem

2. My early stage

3. Our method to solve the problem

Collaboration with Dr. Civitarese (Univ. La Plata)

i) NME with perturbed transition operator

ii) Estimation of effective g_A for 0νββ NME

4. Application of our method to 0νββ NME

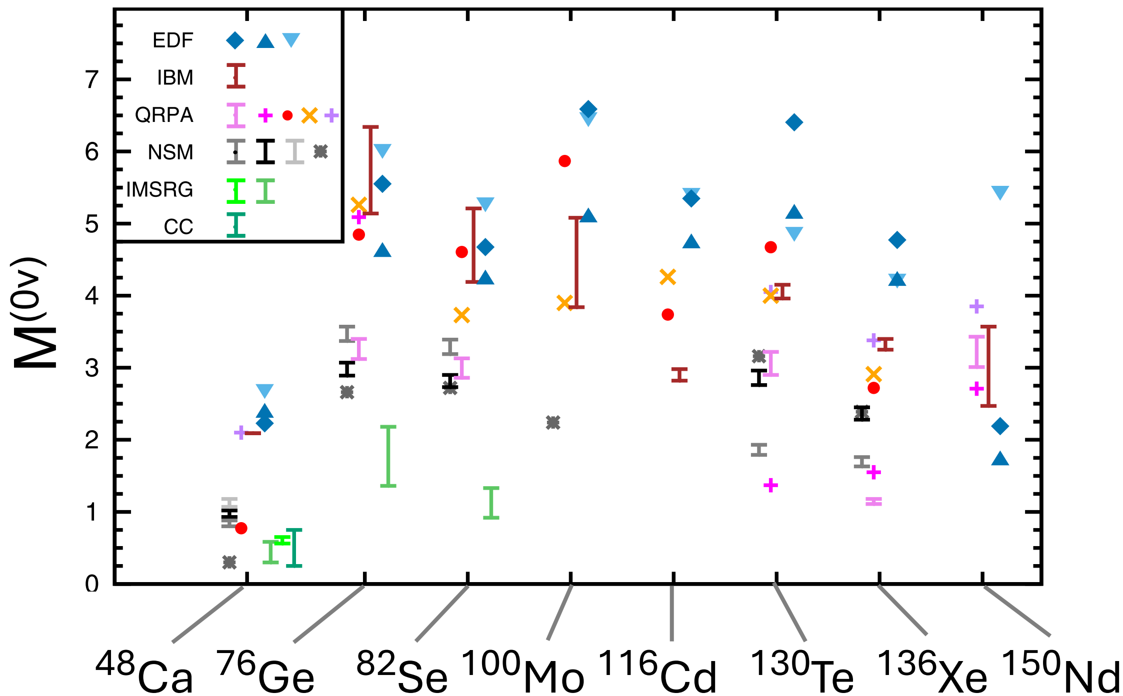
i) ^{136}Xe - ^{136}Ba

ii) ^{76}Ge - ^{76}Se

5. Summary

Compilation of calculated $M^{(0\nu)}$

M. Agostini et al., Rev. Mod. Phys., **95,** 025002 (2023)



The uncertainty of the distribution has not changed since a few decades ago.
The number of calculations increased.

I improved how to apply the QRPA to the ββ NME in

- the overlap of two sets of the intermediate states,

  One ← from the initial state
  The other ← from the final state
  ] Feature of the QRPA approach

  mathematically rigorous, J. T. PRC **87**, 024316 (2013).
- a new method to determine the strength of the isoscalar pairing interaction. It is far from the breaking point of the QRPA. J. T. PRC **102**, 044303 (2020).

I checked

- GT strength function. Compared with exp. data.
- binding energy. Skyrme SkM* intn. is excellent.
- β decay spectrum and strength of ^{138}Xe; acceptable.
- Ikeda sum rule → single-particle space large enough.
- Comparison of spectrum of ^{136}Cs obtained from ^{136}Xe g.s. and that from ^{136}Ba

and others. J. T. and Y. Iwata, PRC **100**, 034325 (2019).

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- a new method to determine the strength of the

$\overbrace{\text{QRPA + SkM*}}^{\text{+ that pairing intn.}}$ is a good approximation for ^{136}Xe and ^{136}Ba.

- GT strength function. Compared with exp. data.
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and others. J. T. and Y. Iwata, PRC **100**, 034325 (2019).

Experimental half-life of ^{136}Xe for 2νββ decay

$$2.18 \times 10^{21}\ \mathrm{yr}$$

A.S. Barabash, Proc. MEDEX'19 (AIP pub., Melville, NY, 2019) p.020002

My cal. (SkM*) value with $g_A = 1.267$ (bare value) is

$$0.03 \times 10^{21}\ \mathrm{yr}$$

Effective g_A to reproduce the exp. half-life is

$$0.422$$

J. T. and O. Civitarese, PRC **112**, 024304 (2025);
112, L051302 (2025)

My assumption:

If the calculation uses no approximation, the exp. half-life can be reproduced with the bare g_A.

Cf. the electric transitions do not need an effective charge if the single-particle space is large enough.

J. T. et al., PRC **78**, 044311 (2008)

The QRPA is a good approximation for nuclear wave function of ^{136}Xe. My interaction strength is not close to the breaking point of the QRPA. My overlap calculation has no problem. The single-particle space is large enough.

But my calculation does not reproduce the exp. half-life without an effective g_A much smaller than the bare value.

What is missing?

The only possibility is that the transition operator is affected by the nucleon-nucleon interaction.

The decay takes time, during which interaction takes place between the nucleons.

Studies on the perturbed transition operator for β decay

A. Arima et al., *in* Adv. in Nucl. Phys. (1987) vol. 18, p. 1

I. S. Towner, Phys. Rep. **155**, 263 (1987)

P. Gysbers et al., Nat. Phys. **15**, 428 (2019) and others

NME with perturbed transition operator

Basic idea

We apply the Rayleigh-Schrödinger perturbation theory

Because the leading-order (usual) NME is derived by that perturbation formula.

J. T. and O. Civitarese, PRC **112**, 024304 (2025)

How?

1. $|\mathcal{J}\rangle = \left|I^{(0)}\right\rangle + \left|I^{(1)}\right\rangle + \left|I^{(2)}\right\rangle$

   : perturbed initial state of the system up to the 2nd-order due to an interaction $\mathcal{V}$ defined later.

   $\left|\tilde{\mathcal{F}}\right\rangle = \left|\tilde{F}^{(0)}\right\rangle + \left|\tilde{F}^{(1)}\right\rangle + \left|\tilde{F}^{(2)}\right\rangle$

: perturbed final state, prepared analogously.

Introduce the weak interaction H_W

Take the 2nd-order term of $\langle\tilde{\mathcal{F}}|H_W|\mathcal{J}\rangle$;

$$\left\langle\tilde{F}^{(0)}\middle|H_W\middle|I^{(2)}\right\rangle + \left\langle\tilde{F}^{(1)}\middle|H_W\middle|I^{(1)}\right\rangle + \left\langle\tilde{F}^{(2)}\middle|H_W\middle|I^{(0)}\right\rangle$$

2. Set $\mathcal{V} = H_W + :V:$, $:V:$ being the residual NN interaction. Take the terms with two H_W and one $:V:$.

Why do we do this?

If the NME is extended in terms of the nuclear structure, the 2nd-order perturbation is necessary. The intermediate states of ββ decay are virtual states. So are the intermediate states of RS perturbation.

3. Remove the double-counting terms.

H_W of the transition operator and that of the perturbation interaction are not distinguished physically.

4. We derive the correction terms due to the perturbation of the transition operator.

We pick up the terms referring to the diagrams. See next slide.

5. For 0νββ NME, the electron matrix element is moved to the phase space factor.

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Why do we do this?

If the NME is extended in terms of the nuclear structure, the 2nd-order perturbation is necessary. The intermediate states of ββ decay are virtual states. So are the intermediate states of RS perturbation.

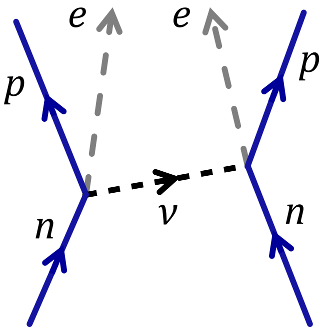
$$\langle \tilde{F}^{(0)} | H_W | I^{(2)} \rangle + \langle \tilde{F}^{(1)} | H_W | I^{(1)} \rangle + \langle \tilde{F}^{(2)} | H_W | I^{(0)} \rangle$$

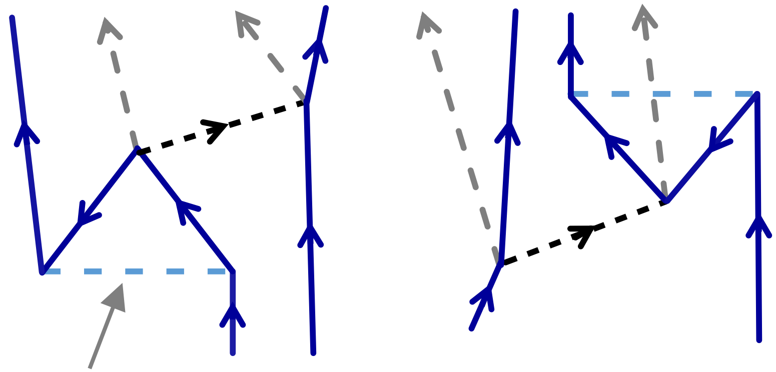
4. We derive the correction terms due to the perturbation of the transition operator.

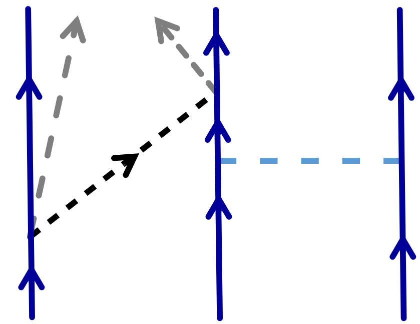
We pick up the terms referring to the diagrams. See next slide.

5. For 0νββ NME, the electron matrix element is moved to the phase space factor.

Diagrams for 0νββ NME







a

$:V:$

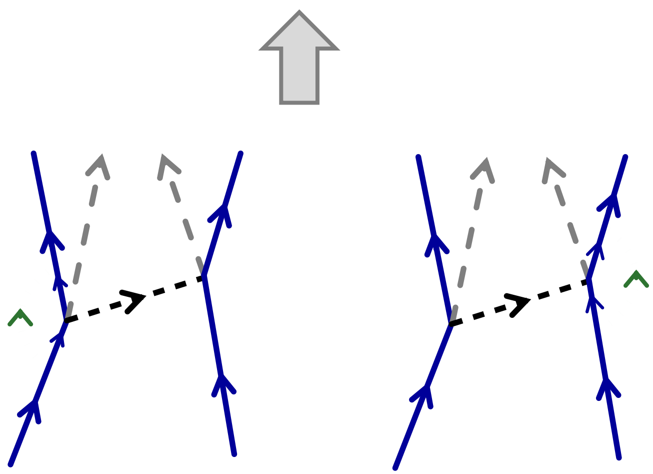
b

c

Leading -order

Vertex correction

RS two-body current



Pion

Perturbed NMEs

$^{136}Xe \rightarrow {}^{136}Ba$, QRPA, Skyrme SkM* + Coulomb + pairing intn.

	0νββ NME		2νββ NME	
	GT	Fermi	GT	Fermi
Leading	3.095	–0.467	0.102	–0.002
VC	1.332	–0.984	0.055	–0.033
RS2BC	–2.731	1.758	–0.192	0.030
Perturbed (sum)	1.696	0.307	–0.035	–0.005

- The summed correction term (VC+RS2BC) of the NME is comparable and anti-coherent to the leading term except for the 2νββ Fermi NME, for which the summed correction is negligible.

Calculated half-life of ^{136}Xe for 2νββ decay

GT and F NME	g_A	Half-life (10^{21} y)
Leading	Bare g_A, 1.267	0.03
Perturbed	Bare g_A, 1.267	0.26
Experimental		2.18

The experimental value from A.S. Barabash, MEDEX'19 Proc. (AIP pub.)

Effective g_A to reproduce exp. 2νββ half-life

Used NME components	$g_A^{\mathrm{eff}}(\exp)$	
	SkM*	SGII
Leading order	0.422	0.563
Perturbed	0.806	0.833

We obtained perturbed GT and Fermi NME components for 0νββ and 2νββ decays.

Our next step

To obtain effective g_A to reproduce the perturbed half-life with the bare g_A for both 0νββ and 2νββ decays.

Are the two effective g_A close or different?

Milestone to solve the uncertainty problem of 0νββ NME

Estimation of effective g_A for 0νββ NME

$$\frac{1}{T_{1/2}^{2\nu}} = G_{2\nu}\left|g_A^2 M_{2\nu}\right|^2$$

Effective g_A used with the leading $M_{2\nu}^{\mathrm{GT}(0)}$ and $M_{2\nu}^{\mathrm{F}(0)}$ to reproduce the experimental $T_{1/2}^{2\nu(\mathrm{exp})}$ is defined by

$$T_{1/2}^{2\nu}(M_{2\nu}^{\mathrm{GT}(0)}, M_{2\nu}^{\mathrm{F}(0)},\ g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{exp})) = T_{1/2}^{2\nu(\mathrm{exp})}.$$

leading

This is the usual effective g_A.

NEW

Effective g_A used with the leading $M_{2\nu}^{\mathrm{GT}(0)}$ and $M_{2\nu}^{\mathrm{F}(0)}$ to reproduce the half-life obtained from the perturbed NME components and the bare g_A

$$T_{1/2}^{2\nu}(M_{2\nu}^{\mathrm{GT}(0)}, M_{2\nu}^{\mathrm{F}(0)},\ g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})) = T_{1/2}^{2\nu}(M_{2\nu}^{\mathrm{GT}},\ M_{2\nu}^{\mathrm{F}},\ g_{\mathrm{A}}^{\mathrm{bare}}).$$

perturbed

Analogously, for the 0νββ decay,

$$T_{1/2}^{0\nu}(M_{0\nu}^{\mathrm{GT}(0)}, M_{0\nu}^{\mathrm{F}(0)},\ g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})) = T_{1/2}^{0\nu}(M_{0\nu}^{\mathrm{GT}},\ M_{0\nu}^{\mathrm{F}},\ g_{\mathrm{A}}^{\mathrm{bare}}).$$

0vββ effective g_A

$$\left\{g_{A,0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})\right\}^2 \approx \left(g_A^{\mathrm{bare}}\right)^2 \left(1 + \frac{M_{0\nu}^{\mathrm{GT(vc)}} + M_{0\nu}^{\mathrm{GT(2bc)}}}{M_{0\nu}^{\mathrm{GT(0)}}}\right)$$

- $M_{0\nu}^{\mathrm{GT(vc)}} + M_{0\nu}^{\mathrm{GT(2bc)}}$ includes the neutrino potential and the residual interaction.
- $M_{0\nu}^{\mathrm{GT(0)}}$ includes the neutrino potential.

The effect of the neutrino potential on $g_{A,0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})$ is weakened.

2vββ effective g_A

$$\left\{g_{A,2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})\right\}^2 \approx \left(g_A^{\mathrm{bare}}\right)^2 \left(1 + \frac{M_{2\nu}^{\mathrm{GT(vc)}} + M_{2\nu}^{\mathrm{GT(2bc)}}}{M_{2\nu}^{\mathrm{GT(0)}}}\right)$$

$M_{2\nu}^{\mathrm{GT(vc)}}$, $M_{2\nu}^{\mathrm{GT(2bc)}}$, and $M_{2\nu}^{\mathrm{GT(0)}}$ independent of the neutrino potential.

Effective g_A numerically obtained

$^{136}Xe \rightarrow {}^{136}Ba$, QRPA, Skyrme (SkM* and SGII)
Coulomb and contact pairing intn.

g_A specification	$g_{\mathrm{A}}^{\mathrm{eff}}$	
	SkM*	SGII
$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld;pt})$	0.796	0.454
$g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld;pt})$	0.696	0.847

Most remarkable result

$$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld;pt}) = 1.14\, g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld;pt}), \qquad (\mathrm{SkM}^*)$$

The effective g_A for the 0νββ and 2νββ are close.

Usual discussion on $g_{A,0\nu}^{\mathrm{eff}}$ and $g_{A,2\nu}^{\mathrm{eff}}$

0νββ decay has a virtual neutrino with momentum

$$0 \le q \le \infty$$

⇨ Conjecture that $g_{A,0\nu}^{\mathrm{eff}}$ and $g_{A,2\nu}^{\mathrm{eff}}$ are quite different.

Argument 1

Major q for 0νββ NME is ≈ 100 MeV.

The 0νββ NME density in the r space conjugate to q has an outstandingly large peak at r = 1 fm.

Nucleon form factors are often included in the 0νββ NME density in the q space, *e.g.*,

$$F(q^2) = \frac{1}{(1 - q^2/M_A^2)^2}, \qquad M_A \approx 1\,\mathrm{GeV}$$

J.D. Vergados, PRC **24**, 640 (1981)

⇨ Contribution of $F(q^2)$ to $g_{A,0\nu}^{\mathrm{eff}}$ is small.

Argument 2

Chiral two-body current (χ2BC) correction to the 0νββ NME

J. Menéndez et al., PRL **107**, 062501 (2011)

Another analysis of χ2BC

J. Engel et al., PRC **89**, 064308 (2014)

They calculated the 0νββ and 2νββ NMEs with and without the χ2BC for ^{76}Ge.

⇨ $g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld;pt}) = 1.10 g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld;pt})$

Argument 3

SGII does not results in similar $g_{\mathrm{A},0\nu}^{\mathrm{eff}}$ and $g_{\mathrm{A},2\nu}^{\mathrm{eff}}$.

$$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld;pt}) = 0.53 g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld;pt})$$

Which is more reliable, SkM* or SGII?

⇨ That is SkM*. SGII overestimates the binding energy.

Argument 1

Chiral two-

J. Menénde

Another an

J. Engel et a

They calcu

the χ2BC f

⇨ $g^{\text{eff}}_{\text{A},0\nu}$(l

	Binding E. /nucleon (MeV)	Discrepancy from exp. value (MeV)
Exp.	8.396	
SkM*	8.415	0.019
SGII	8.603	0.207

Exp. value is from `www.nndc.bnl.gov` (2025)
SkM* and SGII values are from the Hartree-Fock-Bogoliubov calculation of the ground state of ^{136}Xe.

Argument 3

SGII does not results in similar $g^{\text{eff}}_{\text{A},0\nu}$ and $g^{\text{eff}}_{\text{A},2\nu}$.

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⇨ $g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld;pt}) = 1.10 g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld;pt})$

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Which is more reliable, SkM* or SGII?

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Argument 4

An analytical discussion is possible to explain why $g_{\mathrm{A},0\nu}^{\mathrm{eff}}$ and $g_{\mathrm{A},2\nu}^{\mathrm{eff}}$ are close. The effective g_A is the ratio of perturbed and leading NME components. Both the numerator and the denominator include the neutrino potential. J. T. and O. C. PRC **112**, 024304 (2025)

Idea to proceed

Half-life reproduced by the effective g_A

If the perturbed reference is replaced by the unperturbed one,

$$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{ld}) = g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{ld}) = g_A^{\mathrm{bare}}$$

Referring to the half-life with 1st-order perturbation,

$$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt}) \approx g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})$$

The ratio of $g_{\mathrm{A},0\nu}^{\mathrm{eff}}$ and $g_{\mathrm{A},2\nu}^{\mathrm{eff}}$ is close to the convergence at the 1st order. Thus, we speculate

$$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt}^{\infty}) \approx g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{exp})$$

We apply the phenomenological 2vββ g_A for the 0vββ NME.

The advantage: the exp. value nonperturbatively contains the many-body effects.

I will show two kinds of NME$\times g_{\mathrm{A}}^2$ of several groups.

- Previous (usual) one

$$\left(g_A^{\mathrm{bare}}\right)^2 M_{0\nu}^{(0)} = \left(g_A^{\mathrm{bare}}\right)^2 M_{0\nu}^{\mathrm{GT}(0)} - g_V^2 M_{0\nu}^{\mathrm{F}(0)}$$

- Modified NME with $\left(g_{A,0\nu}^{\mathrm{eff}}\right)^2$

$$g_{A,0\nu}^{\mathrm{eff}}(\mathrm{ld;est}) = g_{A,2\nu}^{\mathrm{eff}}(\mathrm{ld;exp}) \frac{g_{A,0\nu}^{\mathrm{eff}}(\mathrm{ld;pt})}{g_{A,2\nu}^{\mathrm{eff}}(\mathrm{ld;pt})},$$

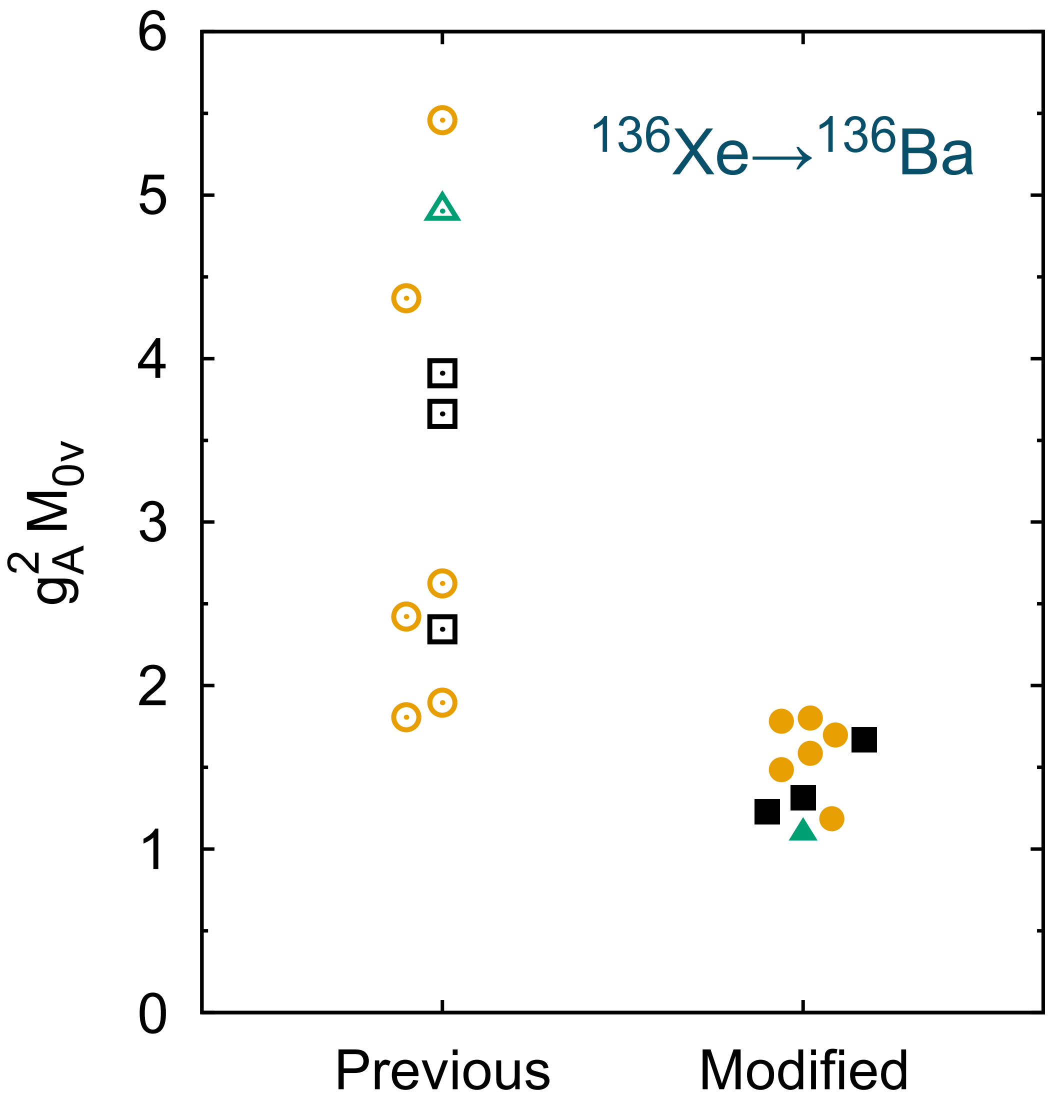
$$\left[g_{A,0\nu}^{\mathrm{eff}}(\mathrm{ld;est})\right]^2 M_{0\nu}^{\mathrm{eff}} = \left[g_{A,0\nu}^{\mathrm{eff}}(\mathrm{ld;est})\right]^2 M_{0\nu}^{\mathrm{GT}(0)} - g_V^2 M_{0\nu}^{\mathrm{F}(0)}$$

This $g_{A,2\nu}^{\mathrm{eff}}(\mathrm{ld;exp})$ can also be obtained for the results of other groups.

g_A^{bare}: 1.25–1.27, depending on the group

We use our correction factor (1.14) for other groups because that of other groups not available.

Application of our method to 0νββ NME of other groups



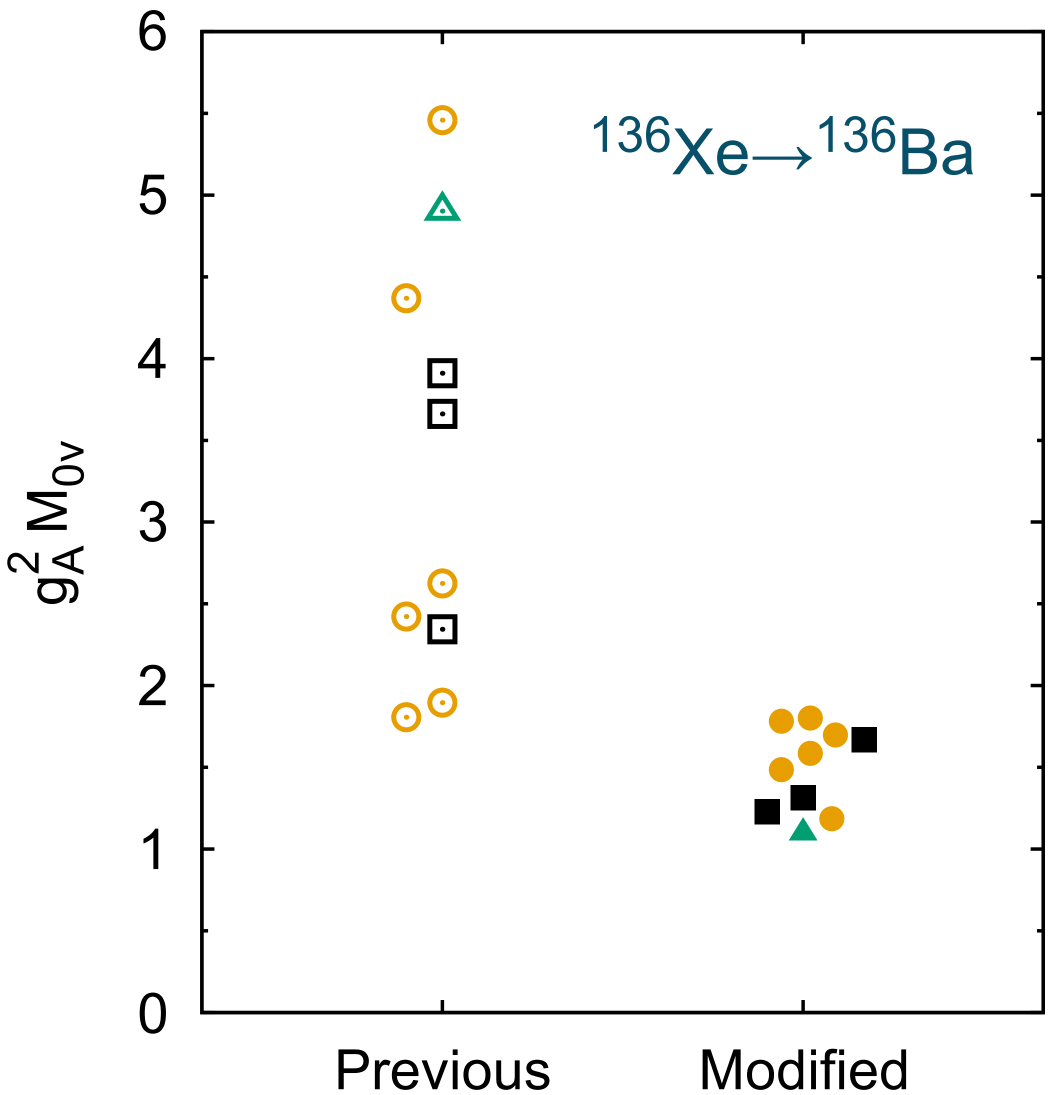
Square: Shell model
Circle: QRPA
Triangle: IBM

Condition on the samples:

Both the 0νββ and 2νββ NMEs are obtained on the same footing.

J. T. and O. Civitarese, arXiv: 2509.16605

Application of our method to 0νββ NME of other groups



A few QRPA groups use g_A = 1.27 and 1.0, and adjust the isoscalar pairing intn. to the exp. $T_{1/2}(2\nu\beta\beta)$.

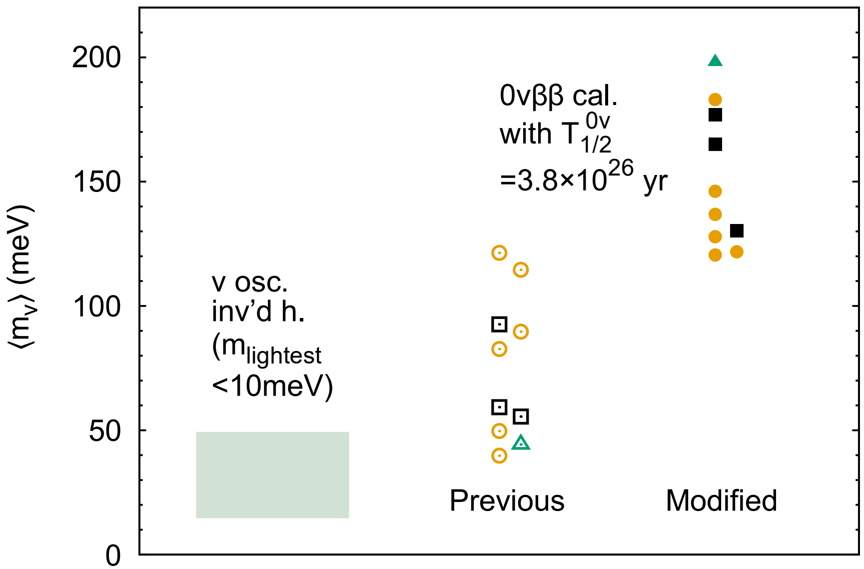
Cal. (1.27) → Previous,

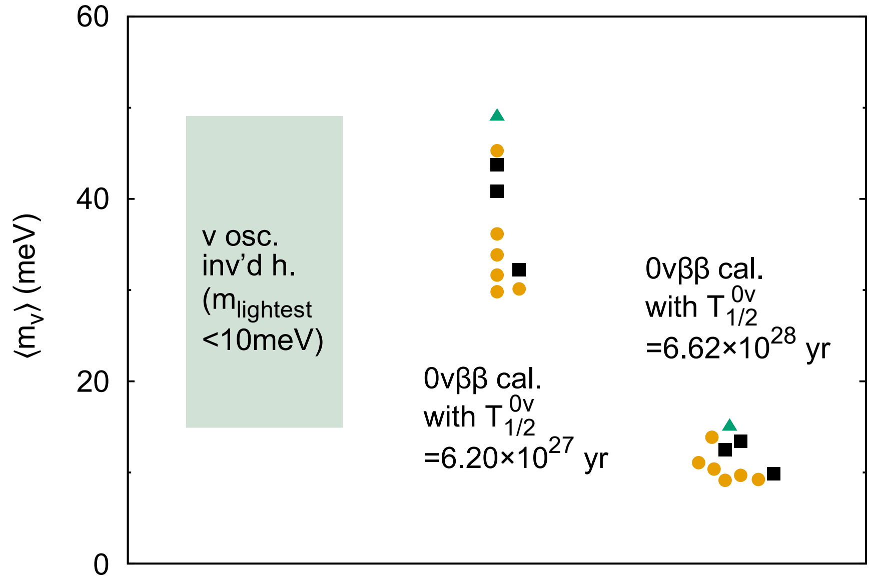
Cal. (1.0) → Modified.

g_A = 1.0 is less close to the QRPA breaking point.

J. T. and O. Civitarese, arXiv: 2509.16605

Together with neutrino oscillation





The upper and lower edges of the rectangular show the region of the effective neutrino mass $\langle m_\nu \rangle$ allowed by the neutrino oscillation data with the assumptions:

- the inverted neutrino eigen mass ordering,
- the lightest neutrino eigen mass < 10 meV

The experimental lower limit of the half-life of ^{136}Xe for the $0\nu\beta\beta$ decay is 3.8×10^{26} yr, S. Abe et al. (KamLAND-Zen) PRL **135**, 262501 (2025)

Perturbed NMEs

^{76}Ge $\rightarrow$ ^{76}Se, QRPA, Skyrme SkM* + Coulomb + pairing intn.

	0νββ NME		2νββ NME	
	GT	Fermi	GT	Fermi
Leading	2.278	–0.368	0.0759	0.00016
VC	0.565	–0.373	–0.0101	0.00403
RS2BC	–0.566	0.370	0.0187	–0.00940
Perturbed (sum)	2.277	–0.373	0.0845	–0.00519

- The summed correction term (VC+RS2BC) vanishes for the 0νββ decay. That of the 2νββ GT is one-tenth the leading term. The 2νββ Fermi term is < one-tenth the GT term.

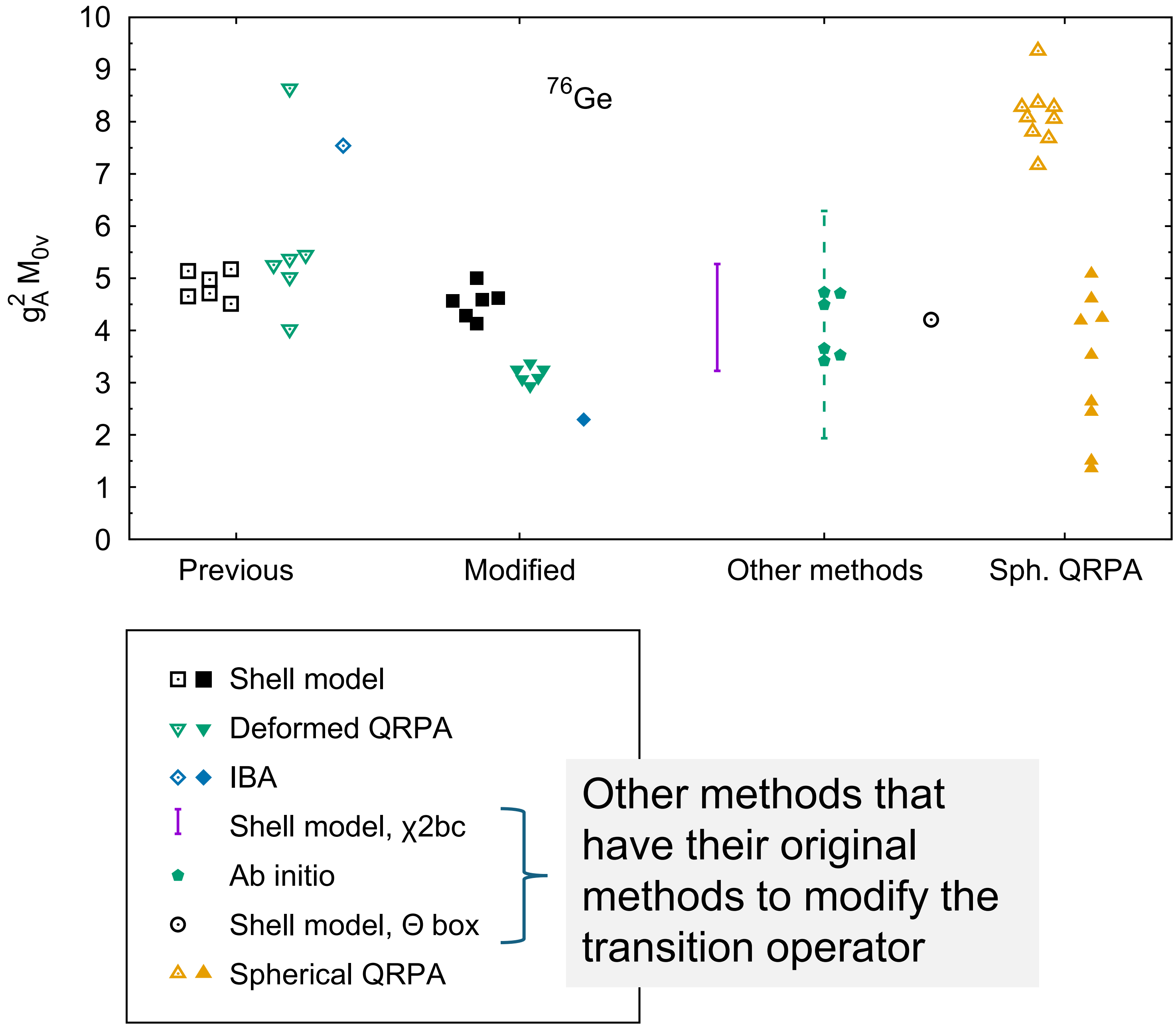
Calculated half-life of ^{76}Ge for $2\nu\beta\beta$ decay

GT and F NME	g_A	Half-life (10^{21} y)
Leading	Bare g_A, 1.267	1.40
Perturbed	Bare g_A, 1.267	1.05
Experimental		1.88

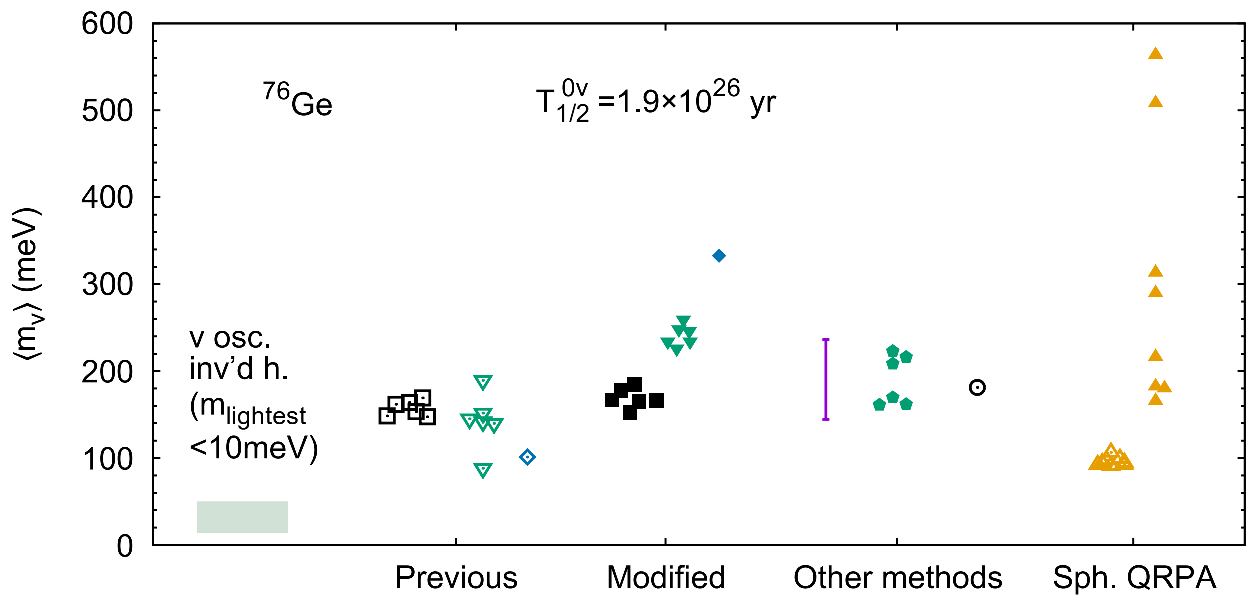
The experimental value from A.S. Barabash, MEDEX'19 Proc. (AIP pub.)

Effective g_A to reproduce exp. $2\nu\beta\beta$ half-life

Used NME components	$g_A^{\mathrm{eff}}(\mathrm{exp})$
	SkM*
Leading order	1.177
Perturbed	1.087

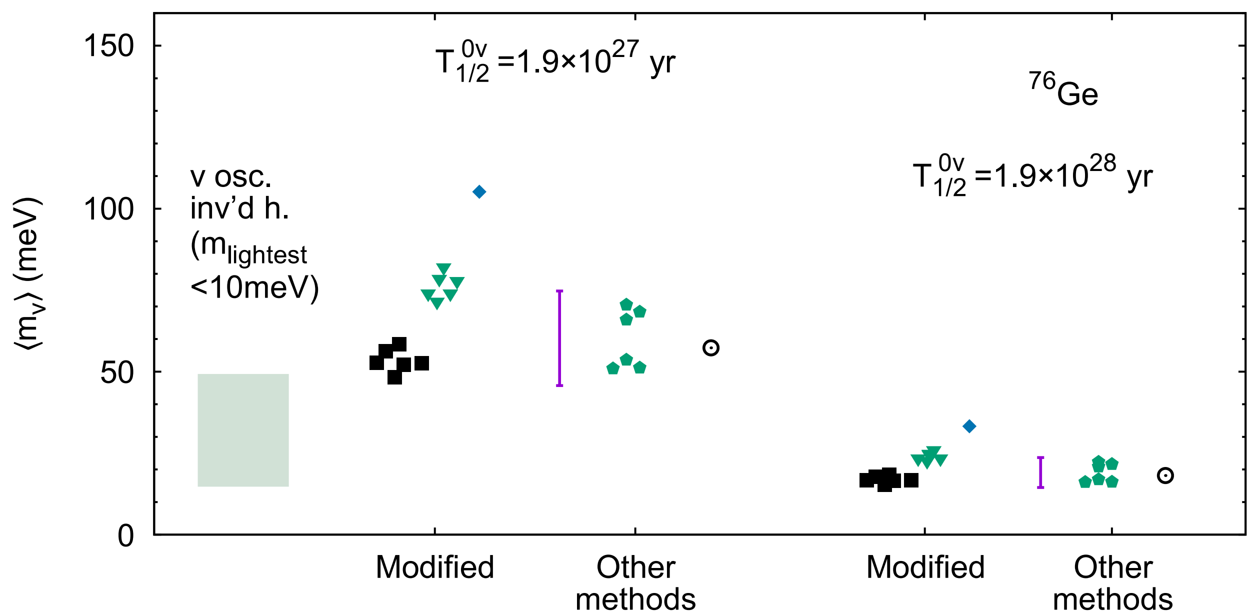


Other methods that have their original methods to modify the transition operator



Current lower limit of exp. $T_{1/2}^{0\nu}$ $=1.9\times10^{26}$ yr

H. Acharya et al. (LEGEND), PRL **136**, 022701 (2026)



If the ordering is the inverted one, and the lightest eigen m_ν is small enough, $T_{1/2}^{0\nu}$ of 10^{27}–10^{28} yr is crucial for finding the 0νββ decay.

Deformation of ^{76}Ge and ^{76}Se

Properties established by spectroscopic studies

- The ground state of ^{76}Ge has a static quadrupole deformation

$$\beta \approx 0.2$$

T. Nikšić, P. Marević, and D. Vretenar, PRC **89**, 044325 (2014).
K. Nomura, PRC **105**, 044301 (2022).

In my HFB calculation with SkM*, $\beta = 0.18$.

A collective rotational band with strong intra-band transitions is observed (`nndc.bnl.gov`).

- The potential energy surface with respect to γ is substantially shallow (all relevant papers).

Properties less established

- Static deformation of the ground state of ^{76}Se

  In the HFB calculation with SkM*, it is spherical.

For ^{76}Ge-^{76}Se

Deformed QRPA → Dispersion of $0\nu\beta\beta$ NME reduced by calibration using $2\nu\beta\beta$ decay → $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs are correlated.

Spherical QRPA → Dispersion of $0\nu\beta\beta$ NME broadened by calibration using $2\nu\beta\beta$ decay → $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs are uncorrelated

^{76}Ge is deformed.

For ^{76}Ge-^{76}Se

Deformed QRPA → Dispersion of $0\nu\beta\beta$ NME reduced by calibration using $2\nu\beta\beta$ decay → $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs are correlated.

This is correct.

Spherical QRPA → Dispersion of $0\nu\beta\beta$ NME broadened by calibration using $2\nu\beta\beta$ decay → $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs are uncorrelated

^{76}Ge is deformed.

Summary

- I improved the QRPA approach to the ββ decay.
  I also investigated the validity of the QRPA to ^{136}Xe.
  ⇨ QRPA is a good approx. for ^{136}Xe.

- A small effective g_A is necessary for 2vββ decay.
  According to the assumption that the exact calculation does not need an effective g_A, something is missing in my calculation. That is not in the QRPA.

- We introduced the transition operator perturbed by the NN interaction by our own method.

- $g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt}) = 1.14\ g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt}),\ \ (\mathrm{SkM}^*)$

- We introduced the idea to use $g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{exp})$ for 0vββ NME.

^{136}Xe-^{136}Ba

- We applied this idea with a minor modification to the results of other groups, and the dramatic reduction of dispersion of the $g_A{}^2$ NME was obtained.
- The predicted $T_{1/2}^{0\nu}$ of ^{136}Xe (10^{27}–10^{28} yr) is longer than many of the previous ones.

^{76}Ge-^{76}Se

- The dispersion of the 0vββ NME is reduced significantly if the spherical QRPA calculations are excluded.
- For the inverted ordering and a small enough lightest m_ν, $T_{1/2}^{0\nu}$ of 10^{27}–10^{28} yr is predicted.
- From the discussion based on deformation, the 0vββ and 2vββ NME are correlated.

Referred calculations

Method		Calculation
QRPA	1	Ours, SkM*
	2	M. Mustnen and J. Engel (2013), SkM*
	3	M. Mustnen and J. Engel (2013), Mod. SkM*
	4	Šimkovic et al. (2018)
	5	D.-L. Fang et al. (2018), AV18, w/o src
	6	D.-L. Fang et al. (2018), CD-Bonn, w/o src
Shell Model	1	J. Menéndez (2018), Argonne src
	2	J. Menéndez (2018), CD-Bonn src
	3	M. Horoi and B. A. Brown (2013), largest space
IBM		J. Barea et al. (2013)

src: short-range correlation.

Method		Calculation
Other operator modification method	1	J. Menéndez et al. (2011), SM+χ2BC
	2	A. Belley et al. (2024), ab initio
	3	L. Coraggio et al. (2020), SM+Θ box

For papers, see J. T. and O. Civitarese, arXiv:2509.16605

Thank you for
your attention.