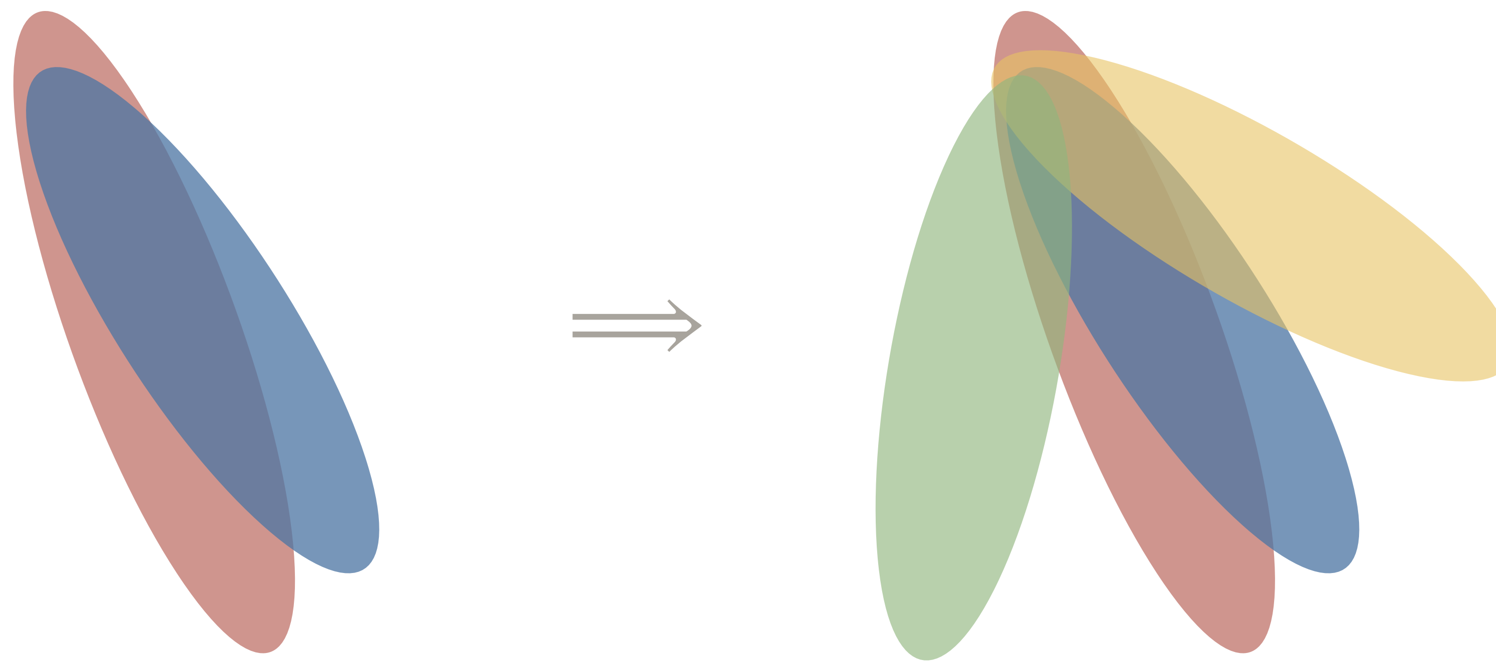
Revisiting α_s extractions from Soft Collinear Effective Theory



Jim Talbert

with G. Bell, C. Lee, Y. Makris, H. Prager, B. Yan

European Commission

MARIE CURIE ACTIONS

03 May 2023 || Institute for Nuclear Theory || U. Washington

The PDG table on α_s

hep-ph/0803.0342 (BS)
hep-ph/1006.3080 (AFHMS)
hep-ph/1501.04111 (HKMS)

To be included in the PDG average, a fit must:

- be published in a peer-reviewed journal…
- include $O(\alpha_s^3)$ fixed-order perturbative results…
- include `reliable' error estimates, including NP effects…

2022 PDG world average:
.1179 +- .0009

Thrust at N³LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$

Riccardo Abbate,[1] Michael Fickinger,[2] André H. Hoang,[3] Vicent Mateu,[3] and Iain W. Stewart[1]

hep-ph/1006.3080

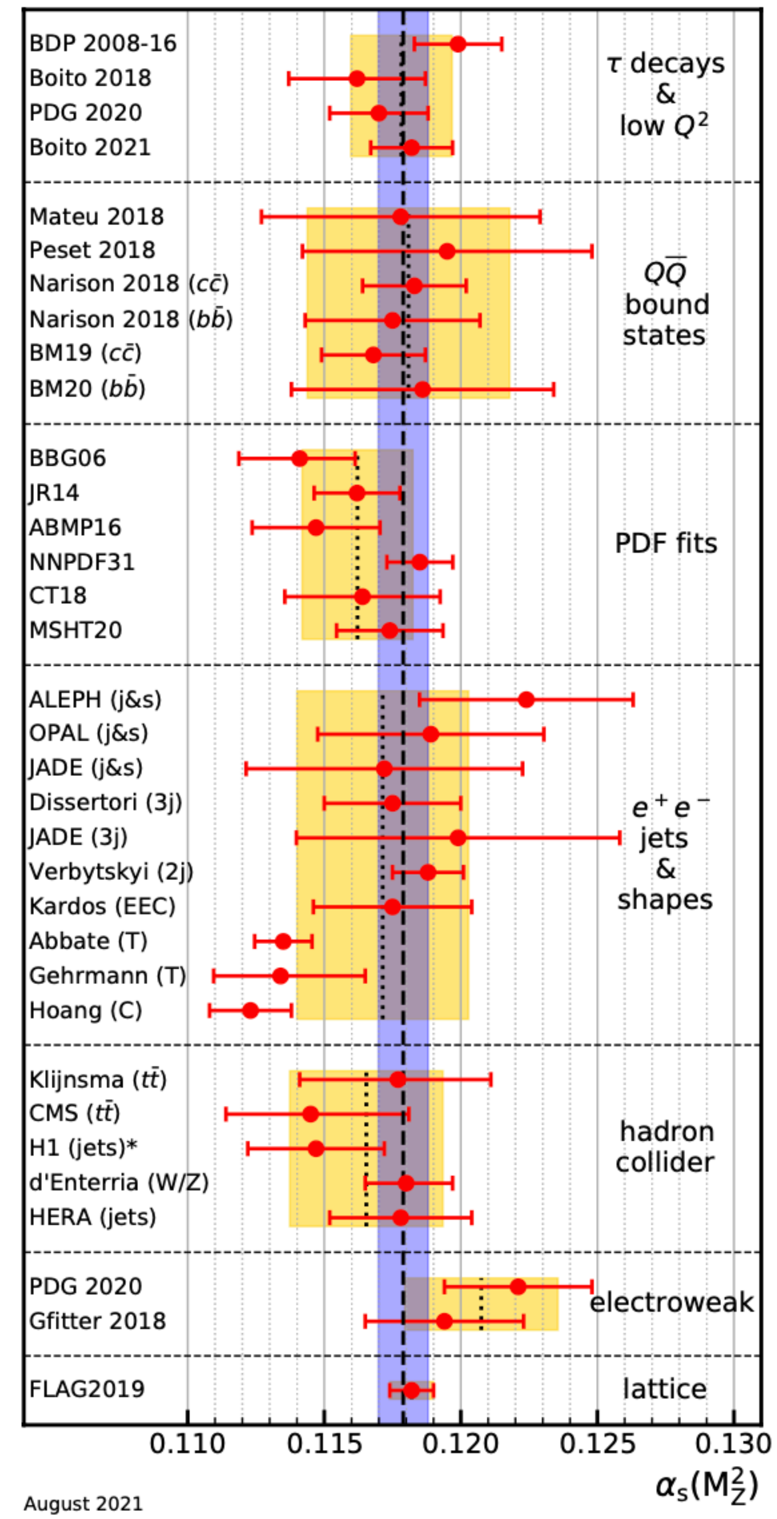
$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\rm exp} \pm (0.0005)_{\rm hadr} \pm (0.0009)_{\rm pert}$$

A Precise Determination of α_s from the C-parameter Distribution

André H. Hoang,[1,2] Daniel W. Kolodrubetz,[3] Vicent Mateu,[1] and Iain W. Stewart[3]

hep-ph/1501.04111

$$\alpha_s(m_Z) = 0.1123 \pm 0.0002_{\rm exp} \pm 0.0007_{\rm hadr} \pm 0.0014_{\rm pert}$$



- **2015 C-parameter result ~4σ away from world average…**

Outline

Event shape distributions and α_s in SCET

Thrust fits revisited

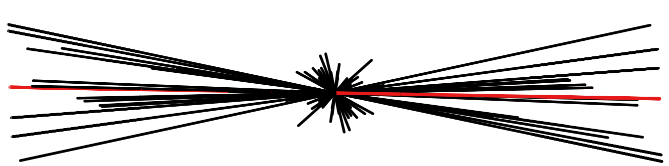
Towards generalized angularity fits

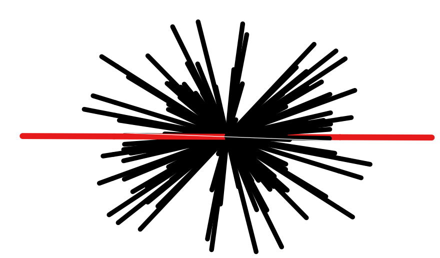
Warning! This is not a formalism talk. This is barely a pheno talk....

This is really a talk on fitting.

Event shapes @ e+e- colliders

- The strong interaction provides the dominant contribution to collider processes involving colored objects.
- As QCD exhibits **asymptotic freedom**, high-energy colliders probe the interaction at weaker coupling, thereby permitting a perturbative description in QFT.
- However, as an event evolves, **many scales are probed**, including the non-perturbative. As a result, QCD spans a rich array of physical phenomena!
- QCD is 'simpler' in electron-positron collisions, as one avoids the messy **initial state physics** of bound protons (PDFs, ISR, multi-pardon scattering, etc.). This facilitates precision QCD studies, e.g. extractions of the strong coupling constant.
- Today we will focus on **event shapes**, which are geometric observables characterizing the 'shape' of final-state momentum distributions of hadronic objects. The dominant channel is **e+e- -> JJ**, as soft and collinear enhancements via gluon emission primarily lead to dijet events.

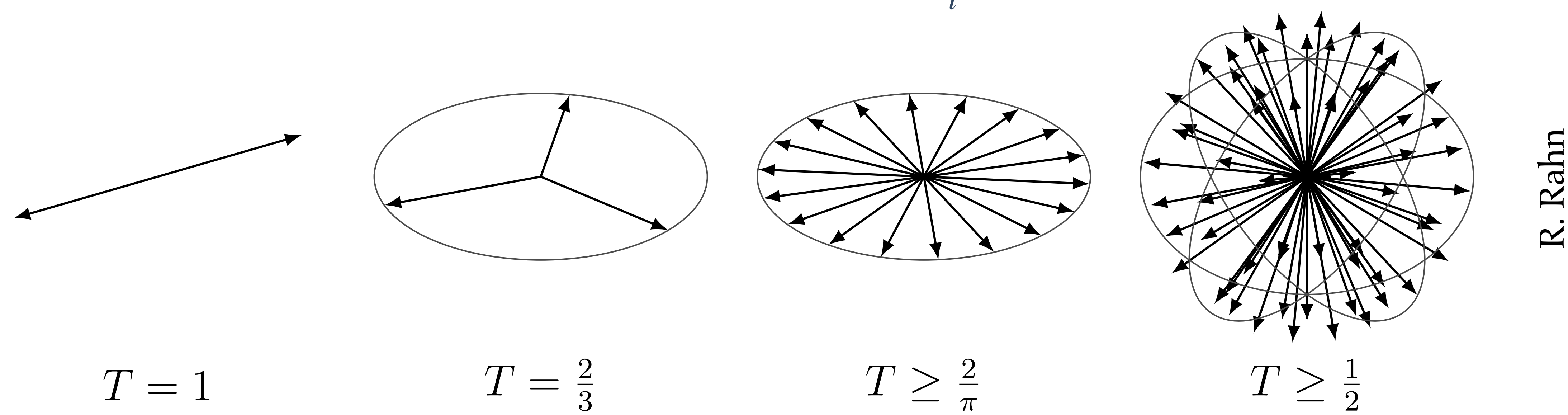




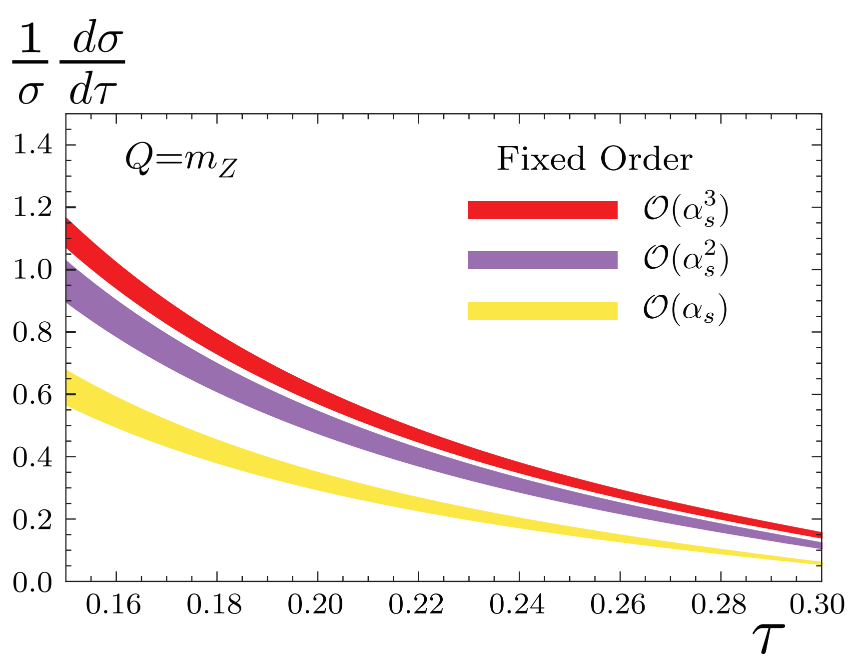
- So, a given event shape would assign a number to these geometric configurations, depending on what is actually getting measured…

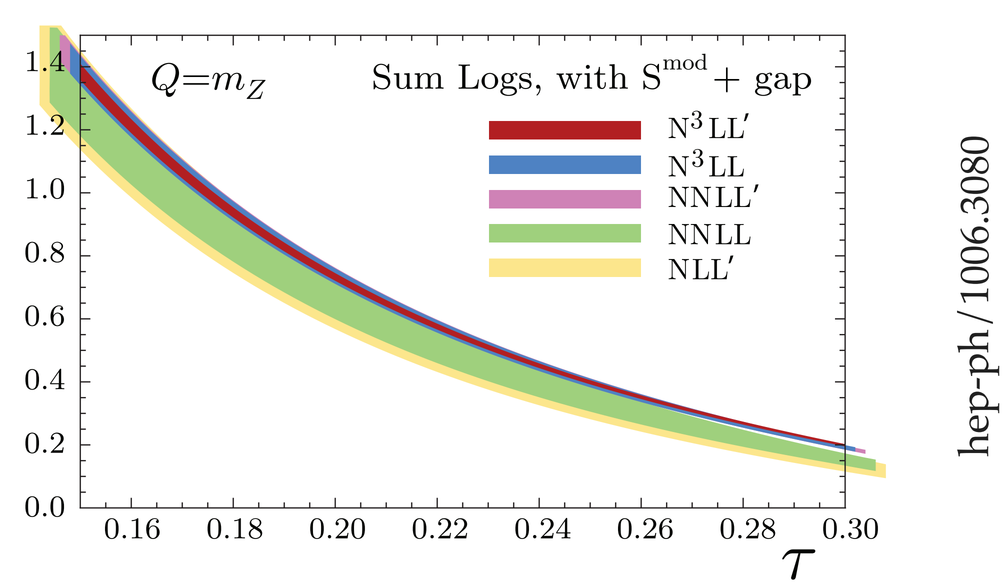
Event shape distributions: thrust

- The classic example is *Thrust:* $\tau = 1 - T = \frac{1}{Q}\sum_i |p_\perp^i|\, e^{-|\eta^i|}$ [Farhi, PRL 39 (1977)]



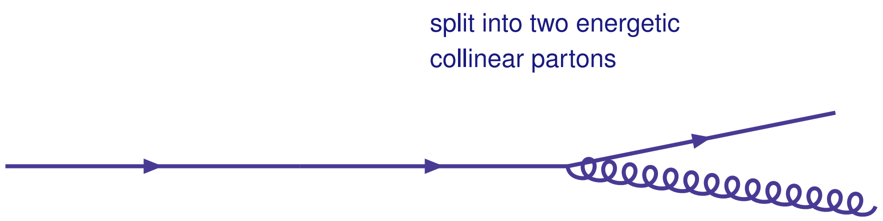
- The fixed-order distribution can readily be computed in QCD, while the current state of the art is a N^{3}LL′ + $O(\alpha_s^3)$ resummation performed with EFT (SCET) techniques:



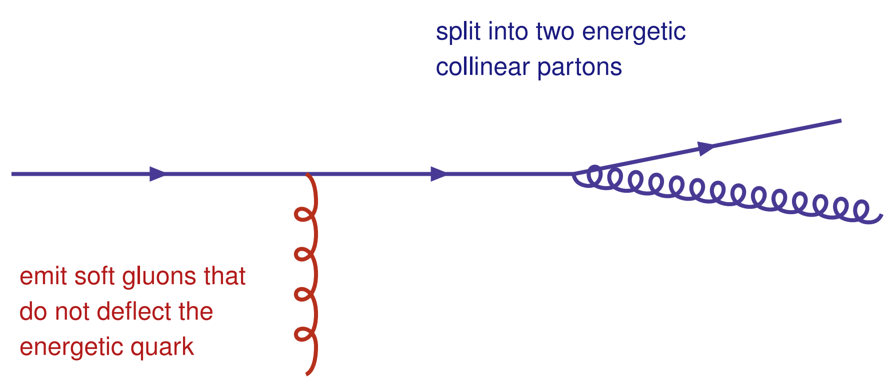


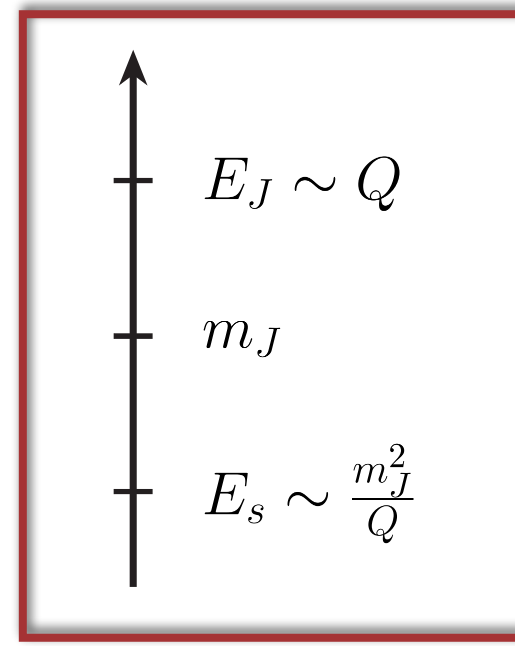
Introducing SCET: intuition

- Event shapes can be predicted with SCET, an effective theory describing collinear and soft degrees of freedom (light, energetic particles) occurring alongside main channel collider scale Q

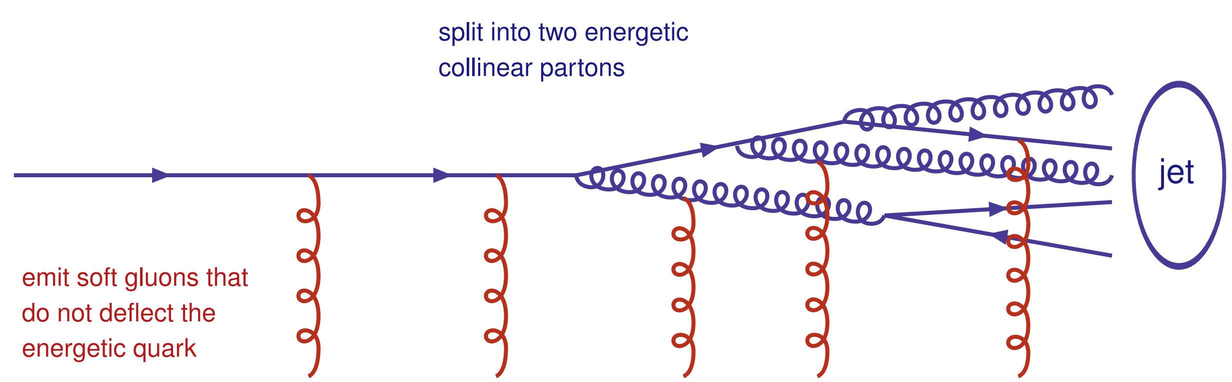


$$\alpha_s^n \ln^m \left(\frac{\mu_1}{\mu_2}\right)$$





$\Rightarrow$ jet of collinear particles	$m_J^2 \ll E_J^2$
soft large-angle radiation	$E_s \ll E_J$



- SCET permits the precision resummation of large logs of these scales via renormalization group evolution!

Introducing SCET: factorization

Original SCET papers: [0005275] [0011336] [0109045] [0206152]

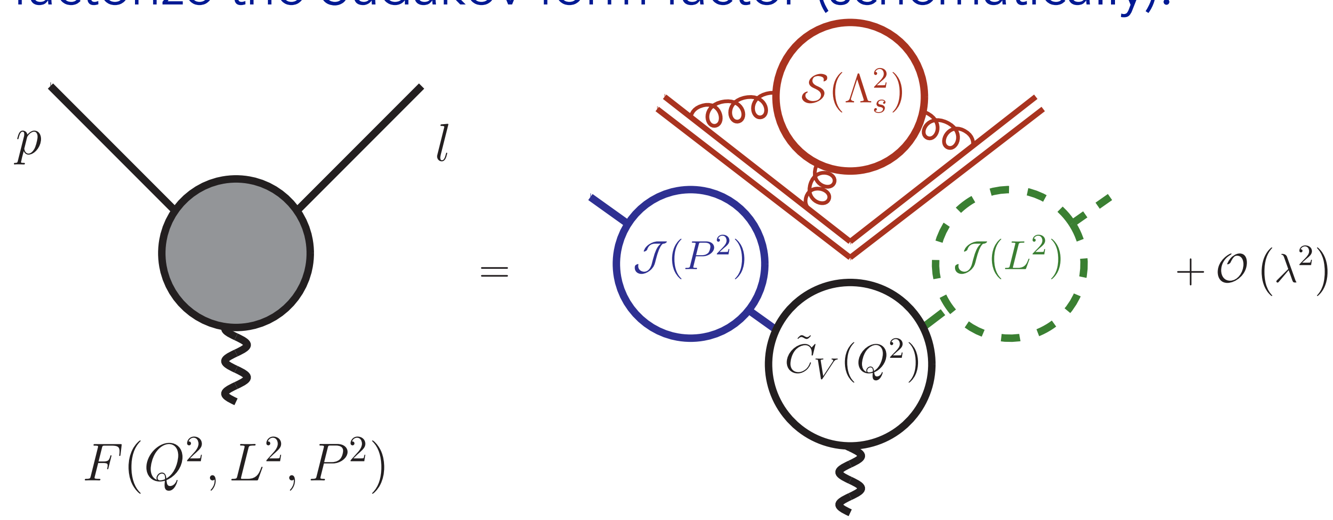
- Begin with fundamental QCD fields and split into soft and collinear components:

$$A^\mu(x) \to A_c^\mu(x) + A_s^\mu(x) \qquad \Psi^\mu(x) \to \Psi_c^\mu(x) + \Psi_s^\mu(x)$$

- Further project collinear fermion into two components, and determine scaling of correlators:

$$\zeta(x) = \frac{\not{n}\not{\bar{n}}}{4}\Psi_c(x), \quad \eta(x) = \frac{\not{\bar{n}}\not{n}}{4}\Psi_c(x) \qquad \langle 0|\{\zeta(x)\bar{\zeta}(0)\}|0\rangle \sim \lambda^2 \Rightarrow \zeta(x) \sim \lambda \qquad (\eta(x) \sim \lambda^2)$$

- After **multipole expansion**, **hard-collinear factorization**, and **soft decoupling** (achieved with Wilson lines), one can factorize the Sudakov form factor (schematically):

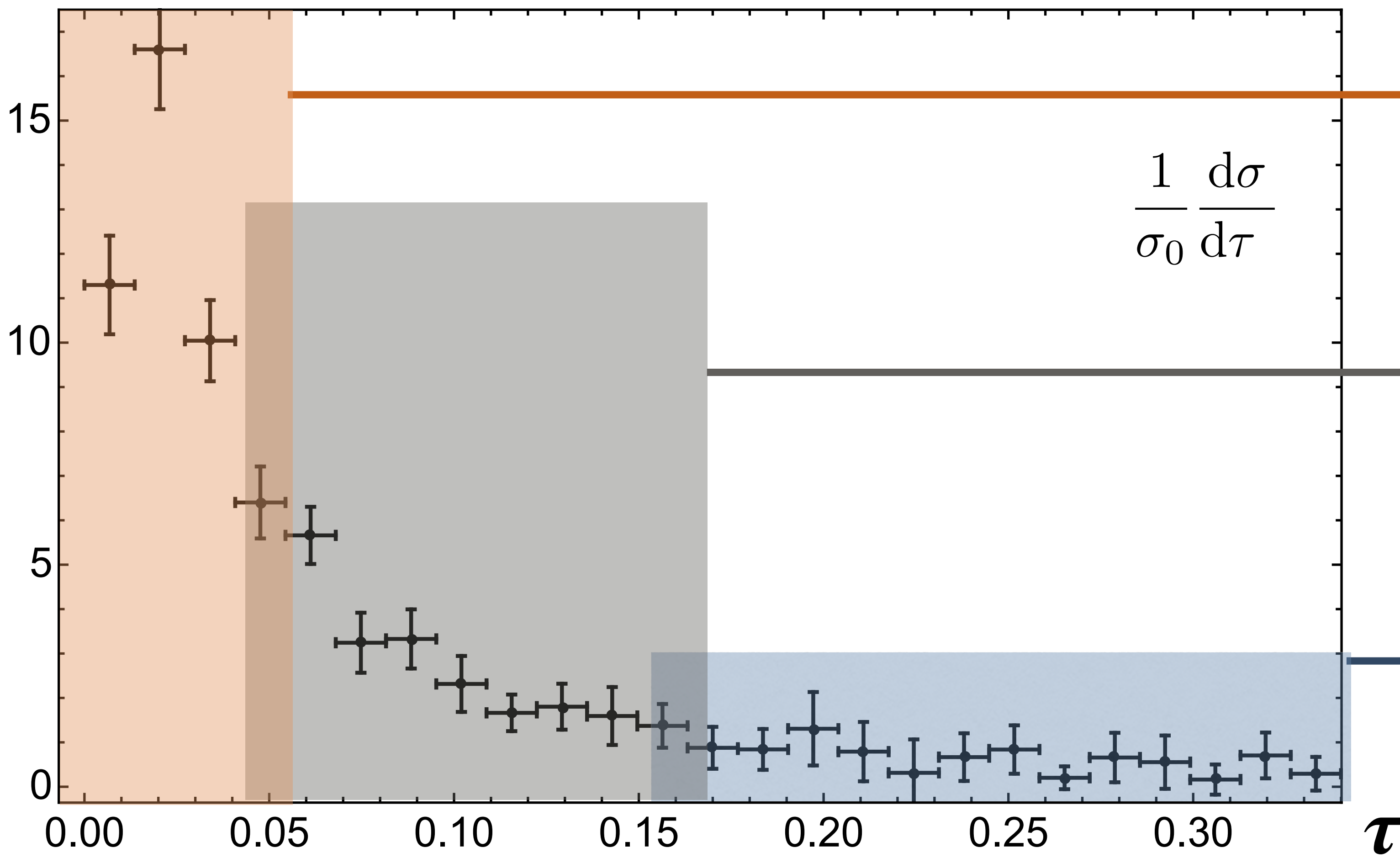


- Obviously, **dijet event shape factorization in e+e-** closely related, with addition of explicit measurements on soft and collinear momenta: $\lambda = \sqrt{\tau}$ (thrust)

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{de} = H(Q;\mu)\int de_n\, de_{\bar{n}}\, de_s\, J_n(e_n;\mu)\, J_{\bar{n}}(e_{\bar{n}};\mu)\, S(e_s;\mu)\, \delta(e - e_n - e_{\bar{n}} - e_s)$$

e = τ (Thrust)

Dissecting dijets — constructing the curve



$d\sigma \sim H \cdot \mathcal{J} \otimes \mathcal{J} \otimes \mathcal{S}$

'Far Tail' Region: fixed-order, multi-jet region. QCD MATCHING

'Tail' Region: resummation region. PERTURBATIVE SCET PREDICTIONS

'Peak' Region: non-perturbative, soft region. NON-PERTURBATIVE MODELING

See e.g. 'Intro to SCET' Becher, Broggio, & Ferroglia
or EFT Lecture Notes, I. Stewart

SCETching thrust: resummation

- H, J^i, and S contain logs of the form (respectively):

$$d\sigma \sim H \cdot \mathcal{J} \otimes \mathcal{J} \otimes \mathcal{S} \qquad \ln\frac{\mu^2}{Q^2}, \quad \ln\frac{\mu^2}{\tau Q^2}, \quad \ln\frac{\mu^2}{\tau^2 Q^2}$$

- We evaluated H, J^i, and S at a common scale. Yet there are 'natural' scales at which the logarithms are no longer large:

$$\mu_h \sim Q \quad \mu_j \sim Q\sqrt{\tau} \quad \mu_s \sim Q\tau$$

- We thus wish to RG run our functions up to their natural scales. Take H as a simple example:

$$H(Q^2,\mu) = H(Q^2,\mu_h)\ U_h(\mu_h,\mu)$$

- Where the function U is a solution to the RG equation for the hard function:

$$\frac{dH(Q^2,\mu)}{d\ln\mu} = \left[2\Gamma_{cusp}\ln(\frac{Q^2}{\mu^2}) + 4\gamma_H(\alpha_s)\right] H(Q^2.\mu)$$

- Which, at LL approximation, has the following form:

$$H(Q^2,\mu) = exp\left[\frac{4\pi\Gamma_0}{\beta_0^2}\frac{1}{\alpha_s(Q)}\left(1-\frac{1}{r}-\ln r\right)\right] \qquad r = \frac{\alpha_s(\mu)}{\alpha_s(Q)}$$

- Similar for jet and soft functions…

SCETching thrust: resummation and profiles

- Evolving all scales to/from their 'natural' settings, one arrives at, for the $\mathrm{SCET_I}$ cumulant:

[0801.4569]
[0901.3780]

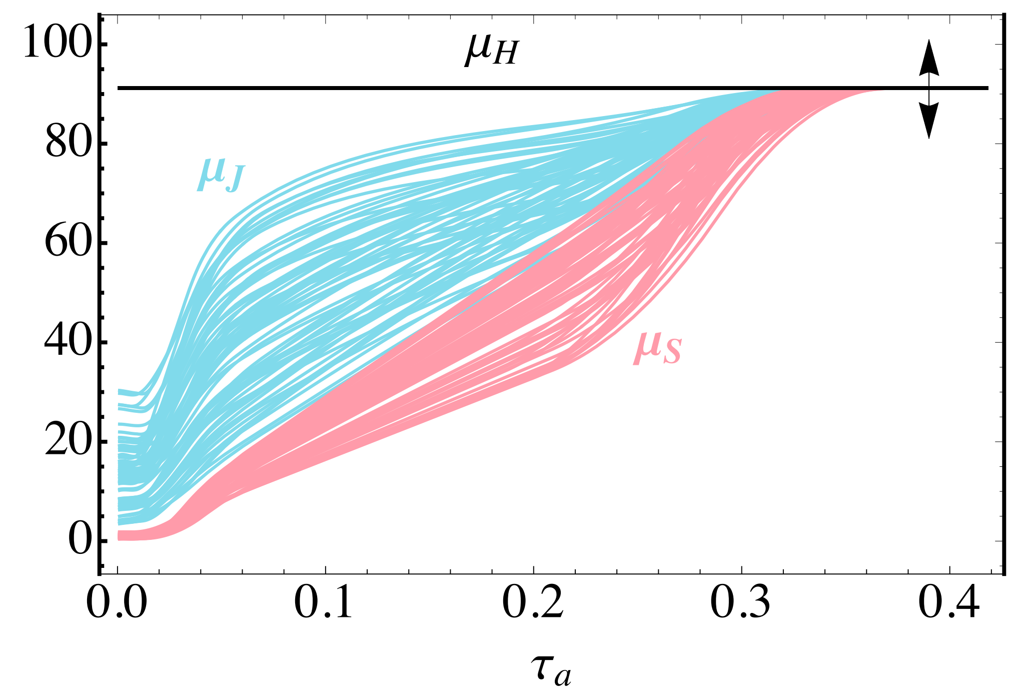
$$\sigma_c(\tau_a) = \frac{1}{\sigma_0}\int_0^{\tau_a} d\tau_a' \frac{d\sigma}{d\tau_a'}$$

$$= e^{K(\mu,\mu_H,\mu_J,\mu_S)}\left(\frac{\mu_H}{Q}\right)^{\omega_H(\mu,\mu_H)}\left(\frac{\mu_J^{2-a}}{Q^{2-a}\tau_a}\right)^{2\omega_J(\mu,\mu_J)}\left(\frac{\mu_S}{Q\tau_a}\right)^{\omega_S(\mu,\mu_S)}$$

$$\times H(Q^2,\mu_H)\widetilde{J}\Big(\partial_\Omega + \ln\frac{\mu_J^{2-a}}{Q^{2-a}\tau_a},\mu_J\Big)^2\widetilde{S}\Big(\partial_\Omega + \ln\frac{\mu_S}{Q\tau_a},\mu_S\Big)\frac{e^{\gamma_E\Omega}}{\Gamma(1-\Omega)}$$

Accuracy	$\Gamma_{\rm cusp}$	$\gamma_F, \gamma_\Delta^\mu, \gamma_R$	β	$H, \tilde{J}, \tilde{S}, \delta_a$
LL	α_s	1	α_s	1
NLL	α_s^2	α_s	α_s^2	1
NNLL	α_s^3	α_s^2	α_s^3	α_s
$\mathrm{N^3LL}$	α_s^4	α_s^3	α_s^4	α_s^2

This cookbook changes at 'primed' accuracies, and of course when considering matching to QCD!



- Note that there also is freedom in scale-setting choices -> '*profiles*'

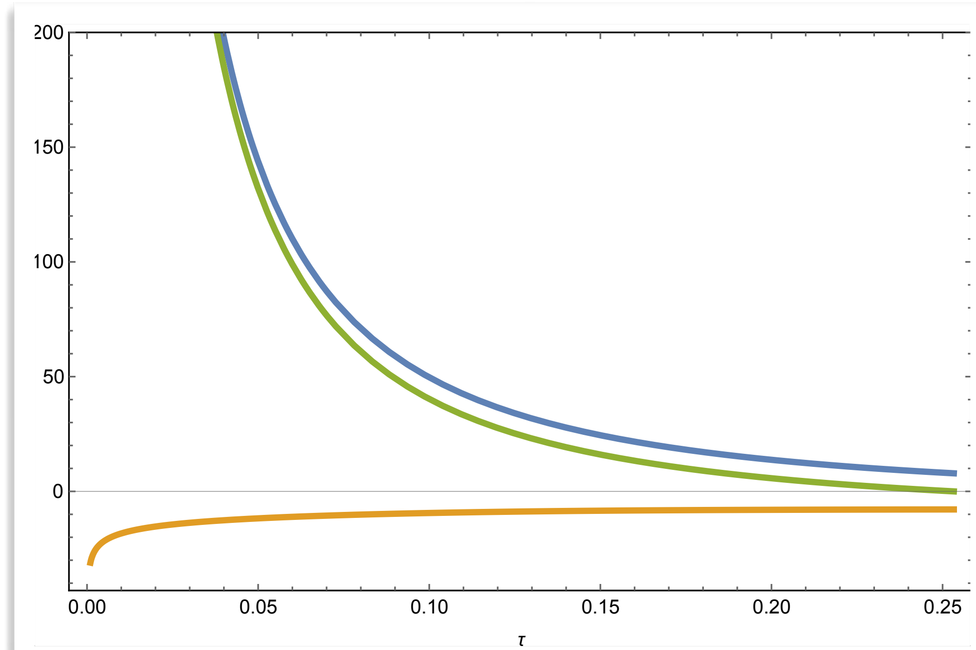
SCETching thrust: matching to QCD

- SCET predicts the singular component of the cross section. One must then match to QCD:

$$\underbrace{\frac{\sigma_c(\tau_a)}{\sigma_0}}_{\text{QCD distribution}} - \underbrace{\frac{\sigma_{c,\text{sing}}(\tau_a)}{\sigma_0}}_{\text{SCET distribution}} = r_c(\tau_a) = \theta(\tau_a)\left\{\frac{\alpha_s(Q)}{2\pi}\underbrace{r_c^1(\tau_a)}_{\text{Remainder coefficients}} + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 \underbrace{r_c^2(\tau_a)}_{\text{Remainder coefficients}}\right\} + \ldots$$

At $O(\alpha_s)$, for example, the matching restricts the distribution to the domain $\tau \in \{0, \tau_{max}\}$:

Unmatched **Remainder** **Matched**



hep-ph/0901.3780

- Results for $O(\alpha_s^{(2,3)})$ matching, obtained from **EVENT2 / EERAD3**, included.

SCETching thrust: non-perturbative corrections

- A treatment of **non-perturbative effects** is critical in *e^+e^- -> hadrons*…
- When dominant power corrections come from the soft function, NP effects can be parameterized into a shape function f_{mod}:

$$S(k,\mu) = \int dk'\, S_{\rm PT}(k-k',\mu)\, f_{\rm mod}(k'-2\overline{\Delta}_a) \qquad f_{\rm mod}(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} b_n\, f_n\left(\frac{k}{\lambda}\right)\right]^2$$

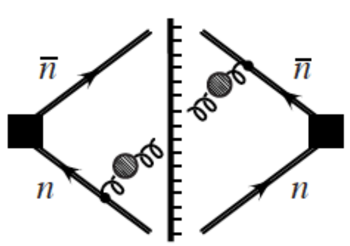
[0709.3519]
[0807.1926]

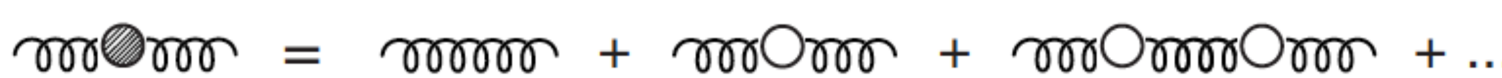
- The leading impact of this shape function correction is to shift the overall perturbative distribution:

[9408222]
[9504219]
[9806537]
[9902341]
[0611061]

a = 0 (Thrust) $$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow[\rm NP]{} \frac{d\sigma}{d\tau_a}\left(\tau_a - c_{\tau_a}\frac{\overline{\Omega}_1}{Q}\right) \qquad \frac{2\overline{\Omega}_1}{1-a} = 2\overline{\Delta}_a + \int dk\, k\, f_{\rm mod}(k)$$

- However, both the gap parameter Δ and the soft function S_PT have a renormalon ambiguity!





see e.g. Beneke (9807443)

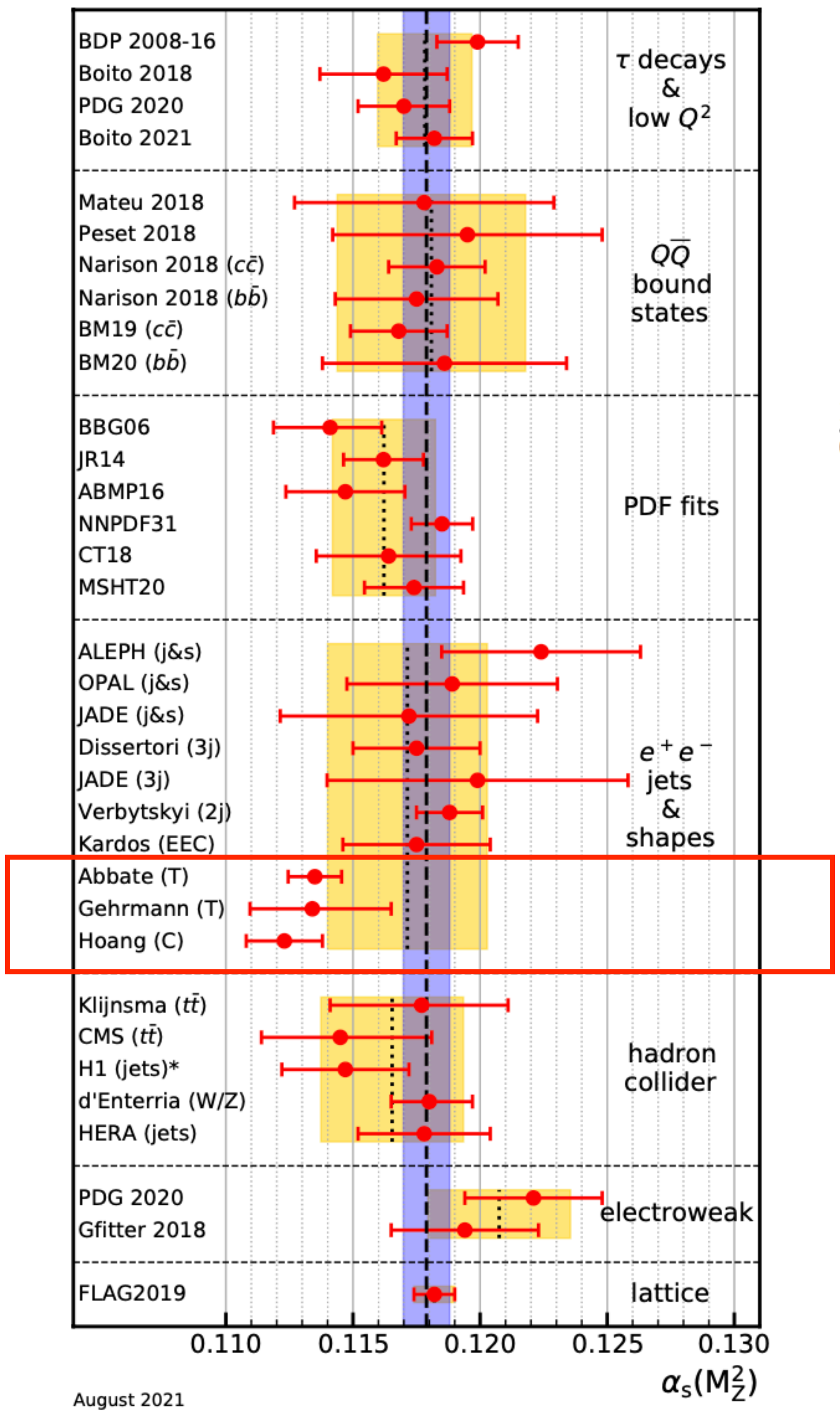
- Solution: subtract a series with a compensating/cancelling ambiguity:

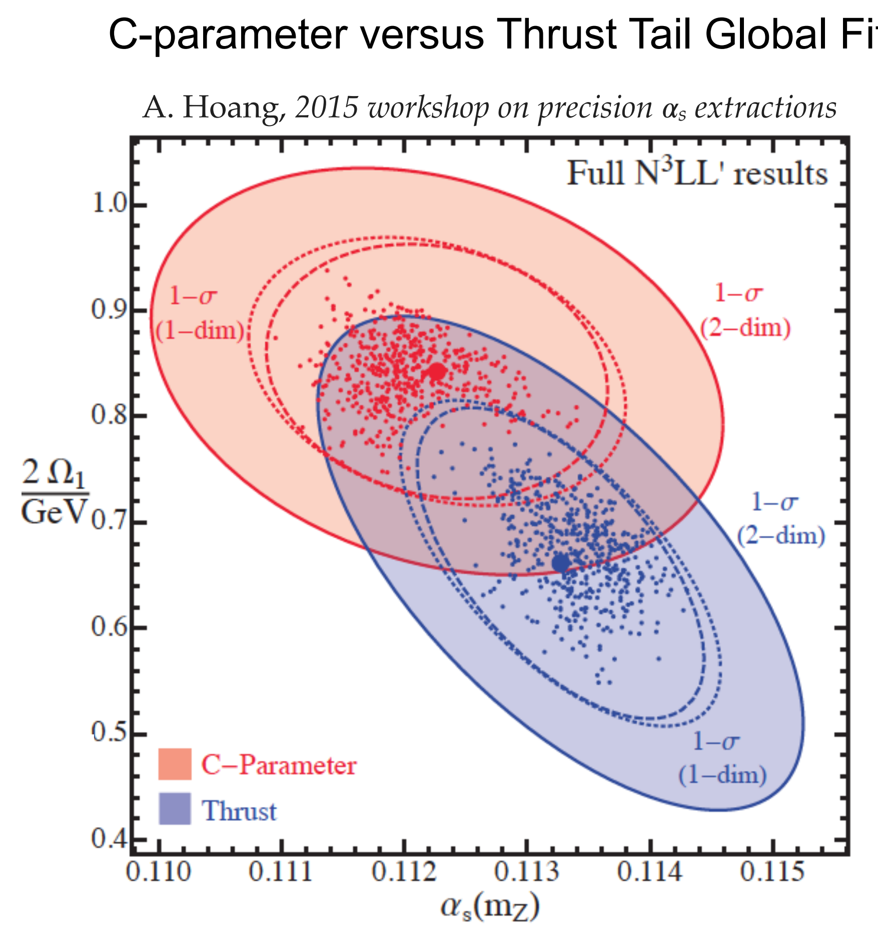
[0003179]
[0709.3519]
[0803.4214]
[0806.3852]
[0801.4743]
[0908.3189]

$$\overline{\Delta}_a = \Delta_a(\mu) + \delta_a(\mu) \longrightarrow \widetilde{S}(\nu,\mu) = \left[e^{-2\nu\Delta_a(\mu)}\widetilde{f}_{\rm mod}(\nu)\right]\left[e^{-2\nu\delta_a(\mu)}\widetilde{S}_{\rm PT}(\nu,\mu)\right]$$

- The highest precision SCET extractions have done so with a very particular scheme…

SCET extractions @ N^{3}LL + O(α^3) accuracy





2022 PDG world average: .1179 +- .0009

hep-ph/0803.0342 (BS)
hep-ph/1006.3080 (AFHMS)
hep-ph/1501.04111 (HKMS)

(Q1) Why are SCET results so discrepant with PDG?

(Q2) What can break the α_s - Ω degeneracy?

Thrust fits revisited

Our collaboration's goals

- We want to **surgically** review the extraction of $\{\alpha_s, \Omega_1\}$ from e^+e^- event shapes, focusing on thrust.
- This includes (1) a **dedicated, independent crosscheck** of prior work, but also (2) looking at all of the **assumptions** made in prior literature:

non-perturbative physics *MC event generation*

theory uncertainty estimation *(perturbative) power corrections*

missing three-loop ingredients *binning techniques*

+ … *fit windows* *observables & datasets*

- Furthermore, we do so at the highest order achievable with current theory inputs:

Accuracy	$\Gamma_{\rm cusp}$	$\gamma_F, \gamma_\Delta^\mu, \gamma_R$	β	$H, \tilde{J}, \tilde{S}, \delta$
LL	α_s	1	α_s	1
NLL	α_s^2	α_s	α_s^2	1
NNLL	α_s^3	α_s^2	α_s^3	α_s
$\mathrm{N^3LL}$	α_s^4	α_s^3	α_s^4	α_s^2

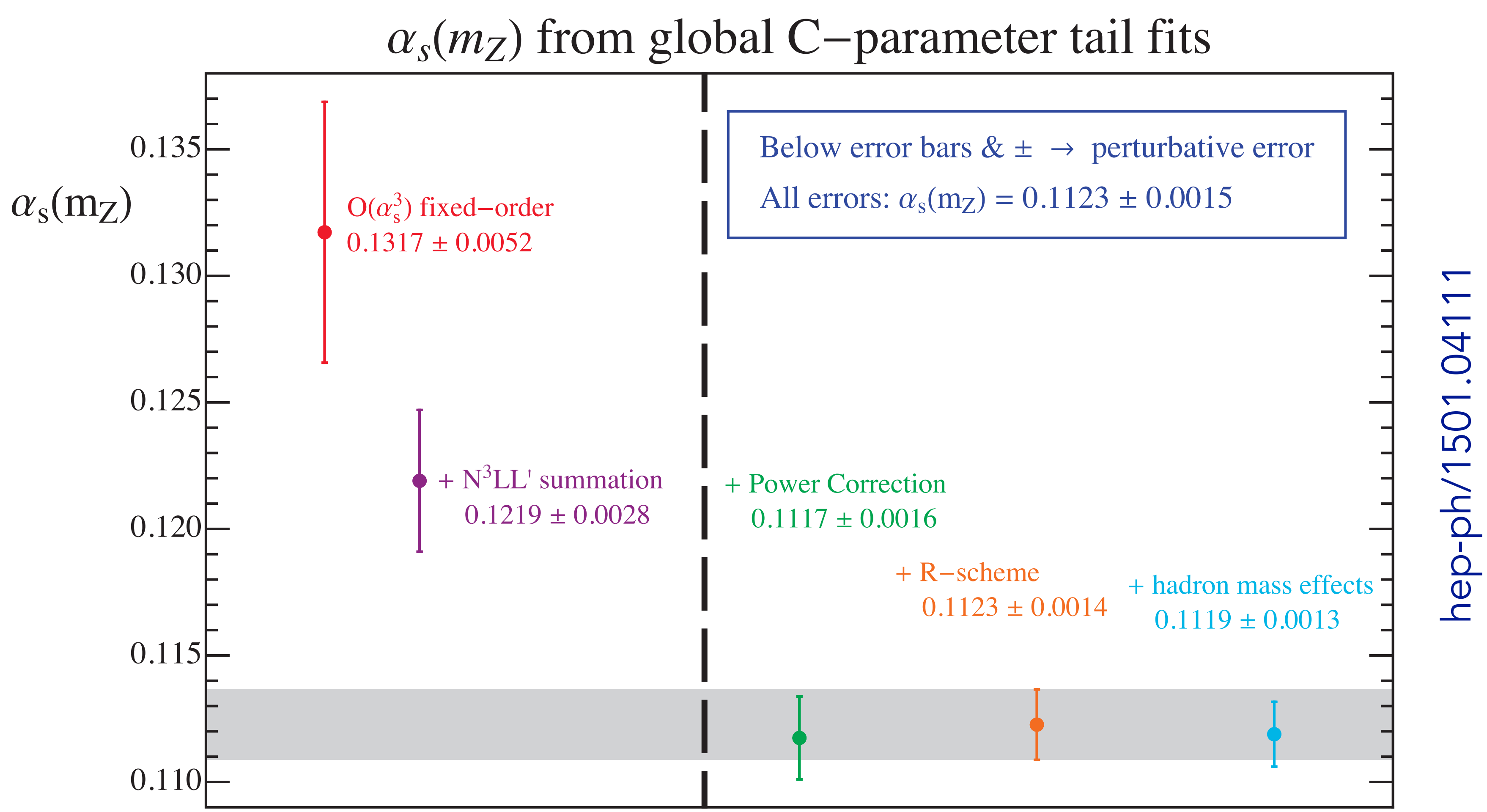
Accuracy	$H, \tilde{J}, \tilde{S}, \delta$
NLL$'$	α_s
NNLL$'$	α_s^2
$\mathrm{N^3LL}'$	α_s^3

Matching	$r^n(\tau)$
$+\mathcal{O}(\alpha_s)$	α_s
$+\mathcal{O}(\alpha_s^2)$	α_s^2
$+\mathcal{O}(\alpha_s^3)$	α_s^3

- Time allowing, I'll comment on the missing three-loop ingredient, c_s^3

Renormalon corrections and α_s

- We have estimates of the effects of the shape function and renormalon corrections from prior analyses:



order	$\alpha_s(m_Z)$ (with $\overline{\Omega}_1$)	$\alpha_s(m_Z)$ (with $\Omega_1(R_\Delta, \mu_\Delta)$)
NLL$'$	0.1071(60)(05)	0.1059(62)(05)
N^{2}LL$'$	0.1102(32)(06)	0.1100(33)(06)
N^{3}LL$'$ (full)	0.1117(16)(06)	**0.1123(14)(06)**

order	$\overline{\Omega}_1$ [GeV]	$\Omega_1(R_\Delta, \mu_\Delta)$ [GeV]
NLL$'$	0.533(154)(18)	0.582(134)(16)
N^{2}LL$'$	0.443(119)(19)	0.457(83)(19)
N^{3}LL$'$ (full)	0.384(91)(20)	**0.421(60)(20)**

This matters! So let's perhaps look here first to see what's happening…

R scheme

[0803.4214]
[0806.3852]
[0801.4743]
[0908.3189]

- Recall that we reorganize the soft sector via a redefinition of the gap parameter:

$$\overline{\Delta}_a = \Delta_a(\mu) + \delta_a(\mu) \xrightarrow{\text{Laplace space}} \widetilde{S}(\nu,\mu) = \Big[e^{-2\nu\Delta_a(\mu)}\widetilde{f}_{\mathrm{mod}}(\nu)\Big]\Big[e^{-2\nu\delta_a(\mu)}\widetilde{S}_{\mathrm{PT}}(\nu,\mu)\Big]$$

- Then, choosing the **R scheme** to cancel the leading renormalon,

$$Re^{\gamma_E}\frac{d}{d\ln\nu}\Big[\ln\widehat{S}_{\mathrm{PT}}(\nu,\mu)\Big]_{\nu=1/(Re^{\gamma_E})} = 0 \longrightarrow \delta_a(\mu,R) = \frac{1}{2}Re^{\gamma_E}\frac{d}{d\ln\nu}\Big[\ln\widetilde{S}_{\mathrm{PT}}(\nu,\mu)\Big]_{\nu=1/(Re^{\gamma_E})},$$

$$\widehat{S}_{\mathrm{PT}}(\nu,\mu) = e^{-2\nu\delta_a(\mu)}\widetilde{S}_{\mathrm{PT}}(\nu,\mu)$$

All of these objects can be defined perturbatively!

- and accounting for R and μ evolution,

$$\frac{d}{dR}\Delta_a(R,R) = -\frac{d}{dR}\delta_a(R,R) \equiv -\gamma_R[\alpha_s(R)]\,, \qquad \mu\frac{d}{d\mu}\Delta_a(\mu,R) = -\mu\frac{d}{d\mu}\delta_a(\mu,R) \equiv \gamma_\Delta^\mu[\alpha_s(\mu)]\,.$$

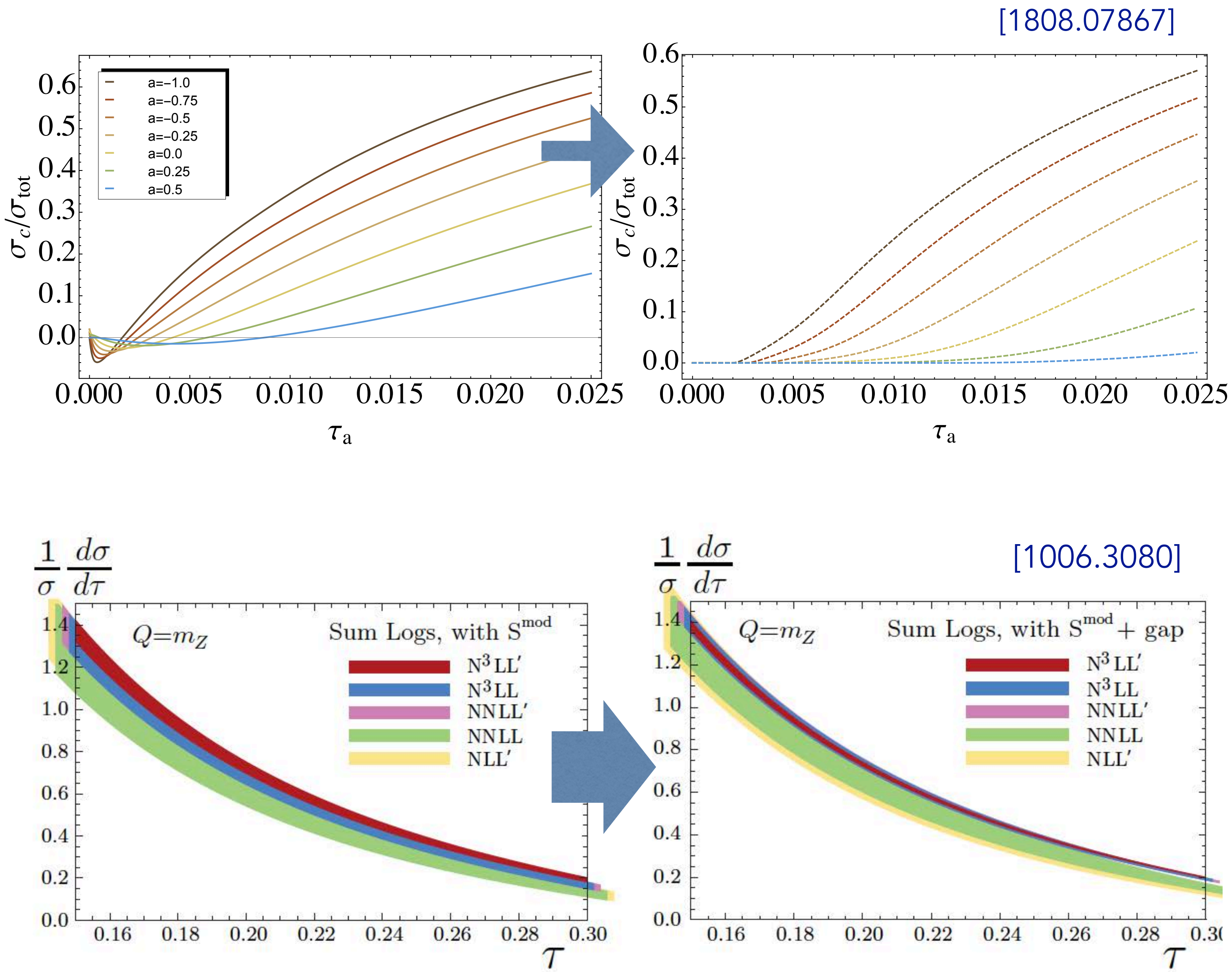
- one obtains the final soft function -> cross section:

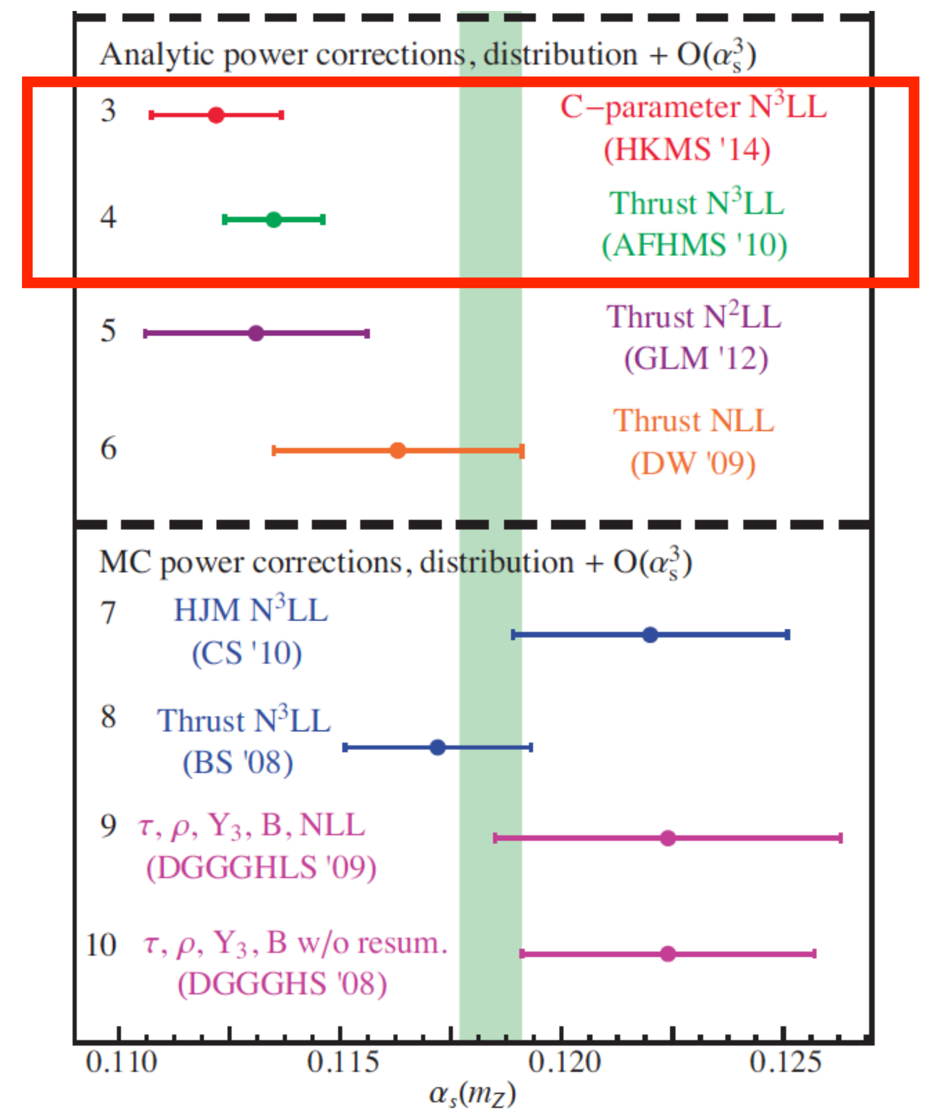
Final cross section is expanded order-by-order in bracketed term

$$\frac{1}{\sigma_0}\sigma(\tau_a) = \int dk\,\sigma_{\mathrm{PT}}\Big(\tau_a - \frac{k}{Q}\Big)\Big[e^{-2\delta_a(\mu_S,R)\frac{d}{dk}}f_{\mathrm{mod}}(k-2\Delta_a(\mu_S,R))\Big]$$

R Scheme phenomenology

- R scheme removes unphysical effects in cross-section predictions and gives good qualitative agreement with data:





see A. Hoang, 2015 workshop on precision α_s

- How non-perturbative effects are implemented (clearly) affects the extraction of the strong coupling!

Effective non-perturbative shifts

- Before considering gapped renormalons, the leading-order NP effect is a constant shift:

$$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow[\text{NP}]{} \frac{d\sigma}{d\tau_a}\left(\tau_a - c_{\tau_a}\frac{\Omega_1}{Q}\right) \qquad \Omega_1 = \frac{1}{N_C}\mathrm{Tr}\,\langle 0|\,\overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T\,(0)\,Y_n \overline{Y}_{\bar{n}}\,|0\rangle$$

- But what is the 'effective shift' of the distribution in the R scheme?

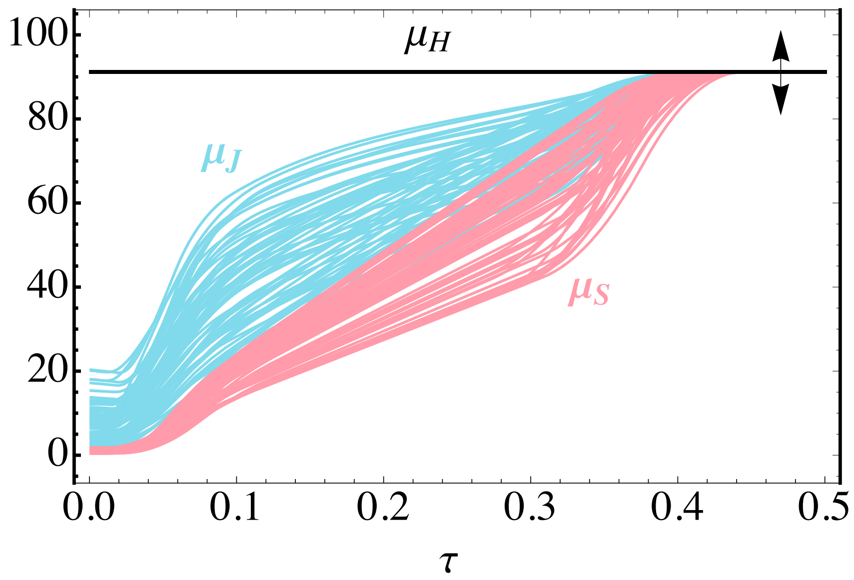
$$\int dk\, k\, e^{-2\delta_a(\mu_S,R)\frac{d}{dk}} f_{\mathrm{mod}}\left(k - 2\Delta_a\left(\mu_S, R\right)\right) = \int dk\, k \left[\sum_i f_{\mathrm{mod}}^{(i)}\left(k - 2\Delta_a\left(\mu_S, R\right)\right)\right]$$

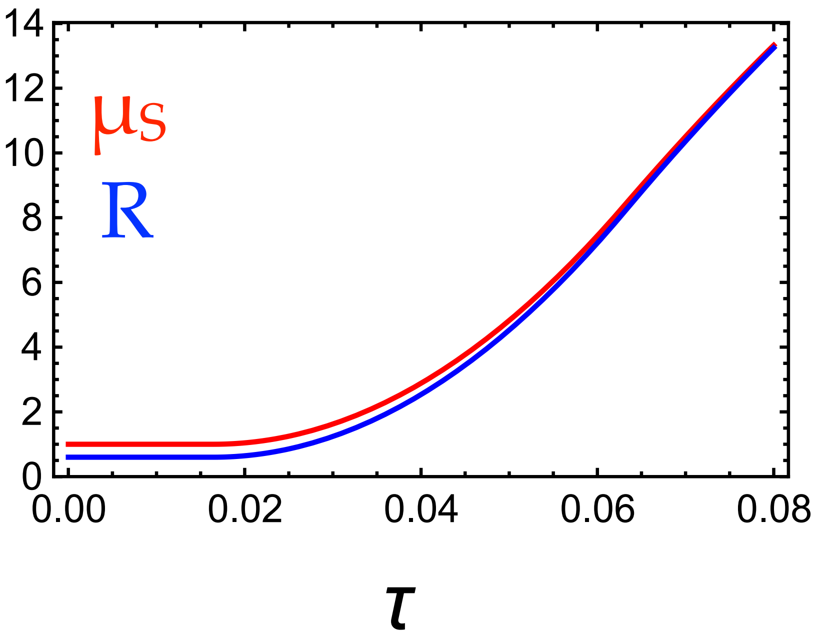
- Shape function expanded order-by-order depending on logarithmic accuracy:

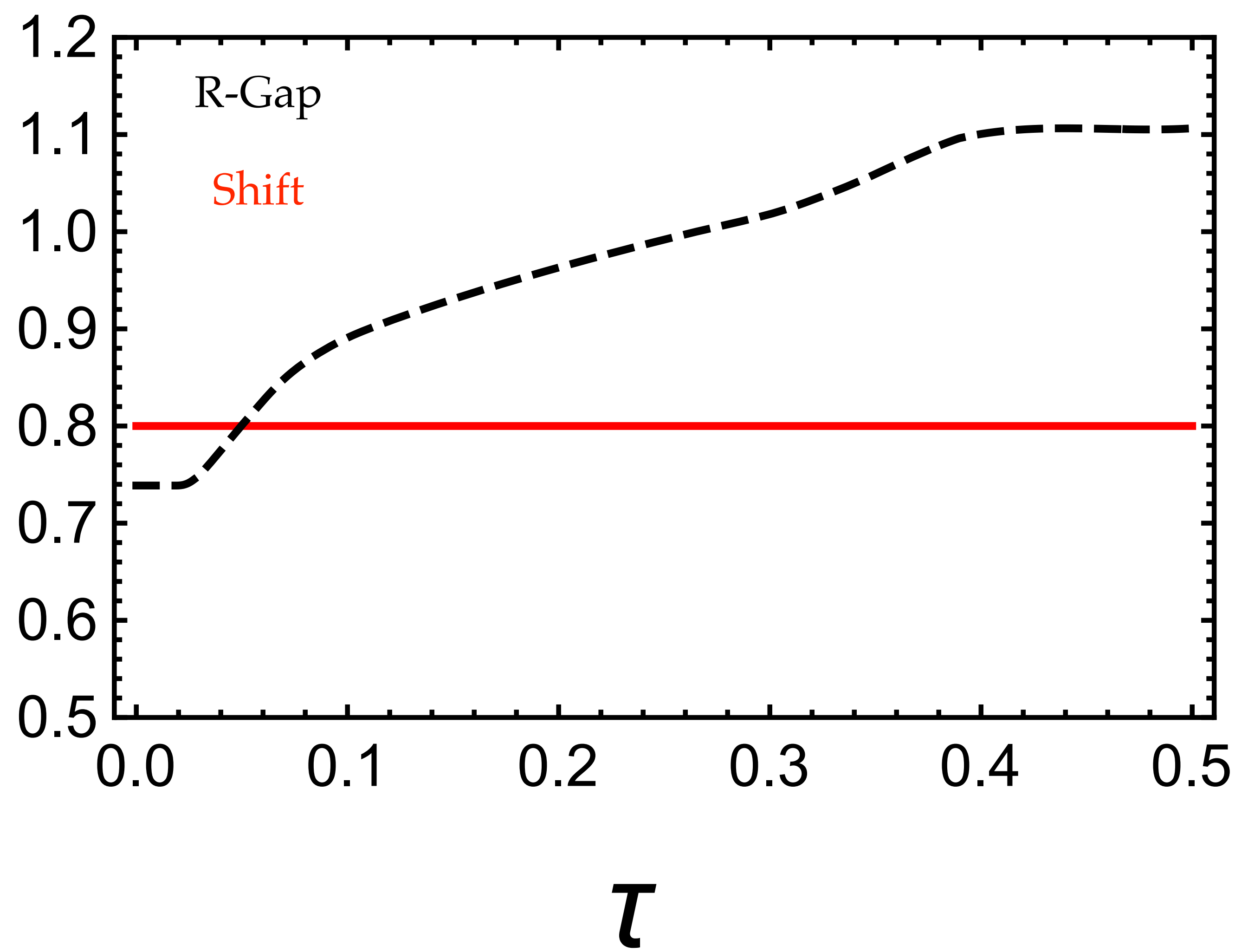
$$\begin{aligned}
f_{\mathrm{mod}}^{(0)}(k - 2\Delta_a(\mu_S, R)) &= f_{\mathrm{mod}}(k - 2\Delta_a(\mu_S, R))\,, \\
f_{\mathrm{mod}}^{(1)}(k - 2\Delta_a(\mu_S, R)) &= -\frac{\alpha_s(\mu_S)}{4\pi}\, 2\delta_a^1(\mu_S, R) R e^{\gamma_E} f'_{\mathrm{mod}}(k - 2\Delta_a(\mu_S, R))\,, \\
f_{\mathrm{mod}}^{(2)}(k - 2\Delta_a(\mu_S, R)) &= \left(\frac{\alpha_s(\mu_S)}{4\pi}\right)^2 \Big[-2\delta_a^2(\mu_S, R) R e^{\gamma_E} f'_{\mathrm{mod}}(k - 2\Delta_a(\mu_S, R)) \\
&\qquad + 2(\delta_a^1(\mu_S, R) R e^{\gamma_E})^2 f''_{\mathrm{mod}}(k - 2\Delta_a(\mu_S, R))\Big]\,,
\end{aligned}$$

Effective non-perturbative shifts

- Distributional shifts at NNLL′ accuracy (central profile scales):







*Why does the effect **grow** as one moves toward the fixed-order regime?*

A naive way to limit the shift…

- Obvious solution is to simply limit the growth of the renormalon scale:

$$\gamma_R \to \theta(R_{\max} - R)\gamma_R \qquad\qquad R = R(\tau)$$

need: $$\frac{d}{dR}\delta_a(R,R) = \gamma_R[\alpha_s(R)]\theta(R_{\max} - R)$$

recall: $$\delta_a(R,R) = Re^{\gamma_E}\Big[\frac{\alpha_s(R)}{4\pi}\delta_a^1(R,R) + \Big(\frac{\alpha_s(R)}{4\pi}\Big)^2\delta_a^2(R,R) + \cdots\Big]$$

- Simple solution is to simply set a max value for the R scale:

$$R^* \equiv \begin{cases} R & R < R_{\max} \\ R_{\max} & R \geq R_{\max} \end{cases}$$

$$\delta_a^1(\mu,R) = \Gamma_S^0 \ln\frac{\mu}{R}\,,$$

$$\delta_a^2(\mu,R) = \Gamma_S^0\beta_0\ln^2\frac{\mu}{R} + \Gamma_S^1\ln\frac{\mu}{R} + \frac{\gamma_S^1(a)}{2} + c_S^1(a)\beta_0$$

Turns off the R-scale at a given (fixed) Rmax (good)

Potentially large logs of μ/R ! (bad)

R*: a new scheme

- Generalized renormalon cancellation schemes can be defined: [2012.12304]

$$\delta_a(\mu) = \frac{R}{2\xi}\frac{d^n}{d(\ln v)^n}\ln\tilde{S}(v,\mu)\Big|_{v=\xi/R} \longrightarrow \boxed{\delta_a^*(R) = \frac{1}{2}R^* e^{\gamma_E}\frac{d}{d\ln\nu}\left[\ln S_{\rm PT}(\nu,\mu=R^*)\right]_{\nu=1/(R^* e^{\gamma_E})}}$$

R* Scheme:

(n, ξ, μ) = (1, exp(-γ_E), R*)

we are not forced to set $\mu = \mu_S$ in the subtraction series, we can pick $\mu = R$

- Anomalous dimensions, subtractions, turn on at one higher order:

$$\delta_a^\star(R^\star) = \frac{R^\star e^{\gamma_E}}{2}\left[\frac{\alpha_s(R^\star)}{4\pi}\cdot 0 + \left(\frac{\alpha_s(R^\star)}{4\pi}\right)^2\left(\gamma_S^1 + 2c_{\tilde{S}}^1\beta_0\right) + \mathcal{O}(\alpha_s^3)\right]$$

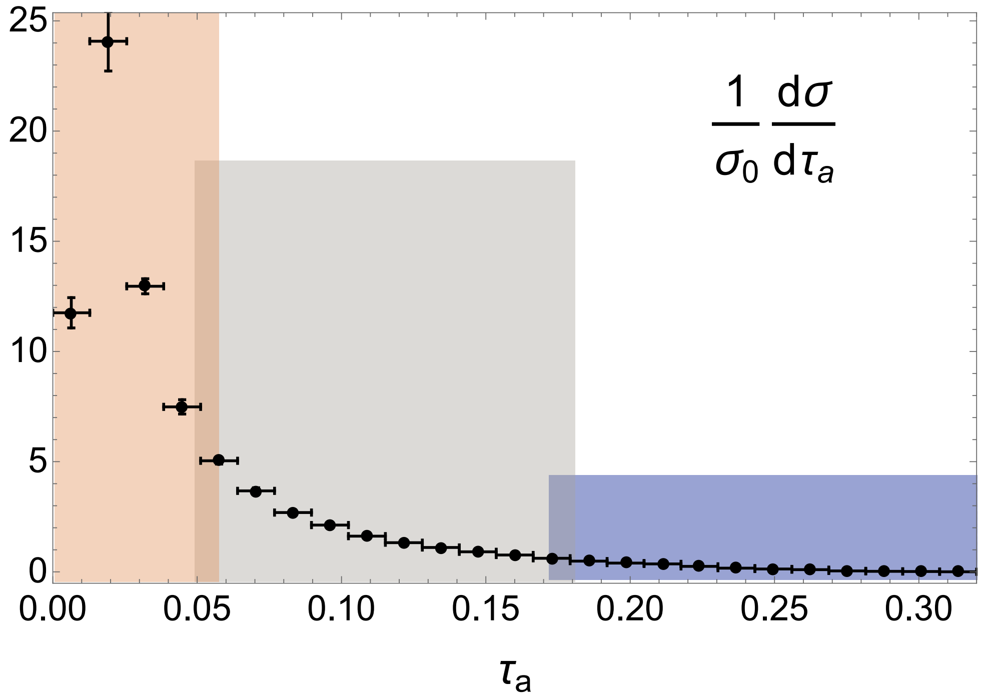
$$\gamma_R^\star = e^{\gamma_E}\left[\frac{\alpha_s(R^\star)}{4\pi}\cdot 0 + \left(\frac{\alpha_s(R^\star)}{4\pi}\right)^2\left(\gamma_S^1 + 2c_{\tilde{S}}^1\beta_0\right) + \mathcal{O}(\alpha_s^3)\right]$$

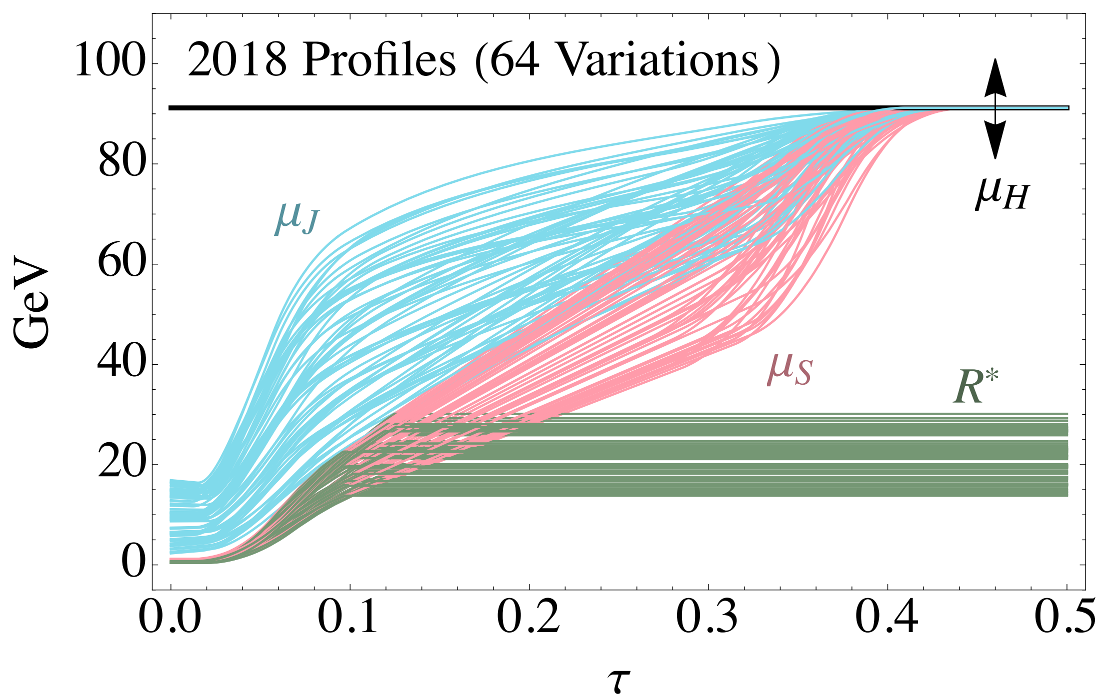
- This scheme is just a **choice**!
- But then again, so is our choice of **theory profiles**, which also drive the effective shift…

Profiling a fit window

hep-ph/1808.07867

- How can we identify a region sensitive to Ω_1 and α_s, and for which our best theory curves are reliable? Look to the profiles!

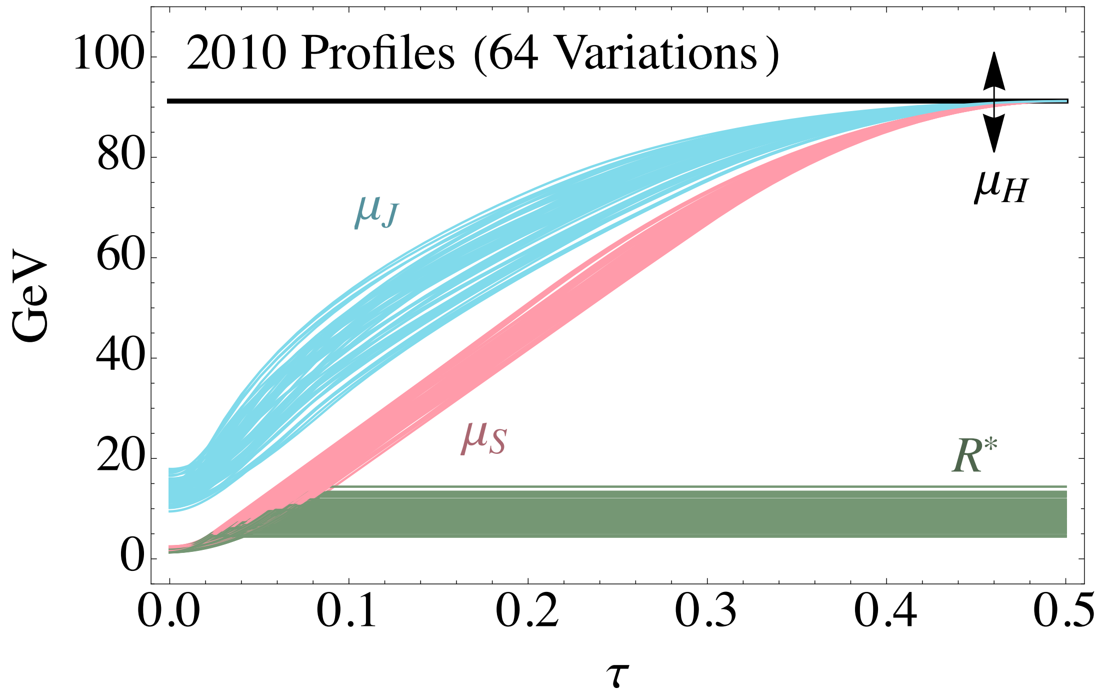




[1006.3080]

- Profiles trace scale hierarchies through different regimes of a given distribution:

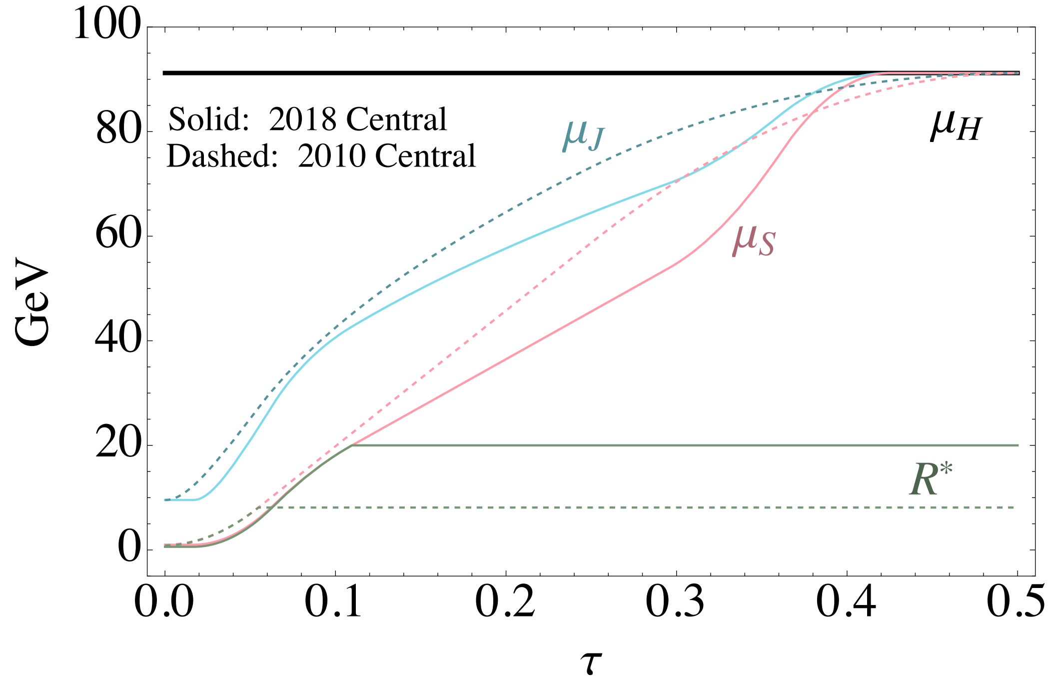
Peak	$\mu_H \gg \mu_J \gg \mu_S \sim \Lambda_{QCD}$
Tail	$\mu_H \gg \mu_J \gg \mu_S \gg \Lambda_{QCD}$
Far Tail	$\mu_H = \mu_J = \mu_S \gg \Lambda_{QCD}$

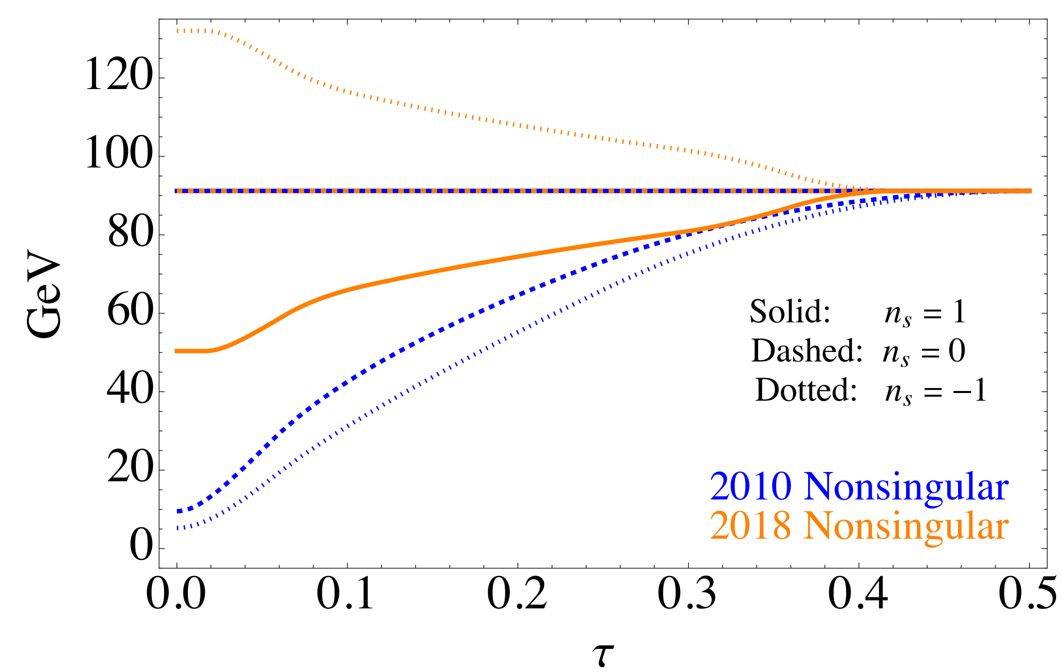


[1808.07867]

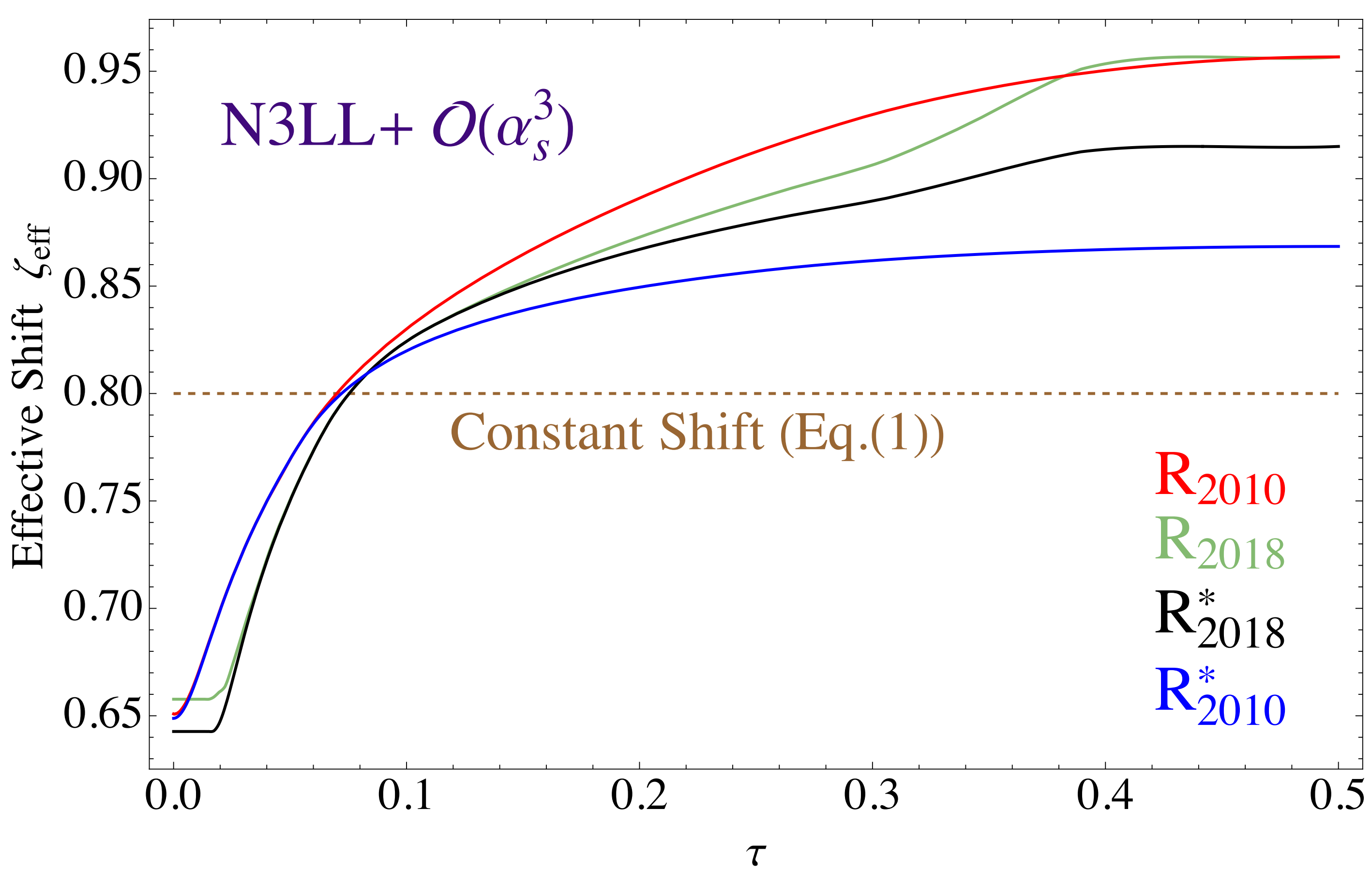
- A default fit window will be between [6/Q, 0.33].

R* Scheme: profiles and shifts





Effective non-perturbative shift flattened, as desired/ expected…



What else can we vary?

- Generalized cancellation schemes also depend on derivative rank and normalization params:

$$\frac{d^n}{d(\ln v)^n} \ln\left[\tilde{S}(v,\mu_{SUB})e^{-2v\delta(\mu_{SUB})}\right]_{v=\xi/\mu_R} = 0$$

- Let's also try to define a scheme with **n=0**. We've calculated the required subtraction terms and anomalous dimensions to three-loop order.

$$\mathbf{R^0\ Scheme:}\quad \{n,\xi,\mu_{SUB},\mu_R\} = \{0, 2\pi, \mu_S, R\}$$

$$\begin{aligned}\delta^3(\mu_s) = {} & \frac{2}{3}\Gamma_s^0\beta_0^2\,L^4 + \frac{2}{3}\left(2\Gamma_s^1\beta_0 + \Gamma_s^0\beta_1 + 4\Gamma_s^0\beta_0^2\ln\frac{\mu_s}{\mu}\right)L^3 \\ & + \left(\Gamma_s^2 + 2\gamma_s^1\beta_0 + 4c_s^1\beta_0^2 + 2\ln\frac{\mu_s}{\mu}\left(2\Gamma_s^0\beta_0^2\ln\frac{\mu_s}{\mu} + \Gamma_s^0\beta_1 + 2\Gamma_s^1\beta_0\right)\right)L^2 \\ & + \left(\gamma_s^2 + 2c_s^1\beta_1 + 4c_s^2\beta_0 - 2(c_s^1)^2\beta_0 + 4\beta_0\ln\frac{\mu_s}{\mu}\left(\gamma_s^1 + 2c_s^1\beta_0\right)\right)L \\ & + 2\ln\frac{\mu_s}{\mu}\left(2c_s^1\beta_0^2\ln\frac{\mu_s}{\mu} + c_s^1\beta_1 + 2c_s^2\beta_0 - (c_s^1)^2\beta_0\right) - c_s^1c_s^2 + \frac{1}{3}(c_s^1)^3 + c_s^3\,,\end{aligned}$$

Cancellation (and evolution terms) sensitive to logs @ one higher power, with respect to n=1 schemes…

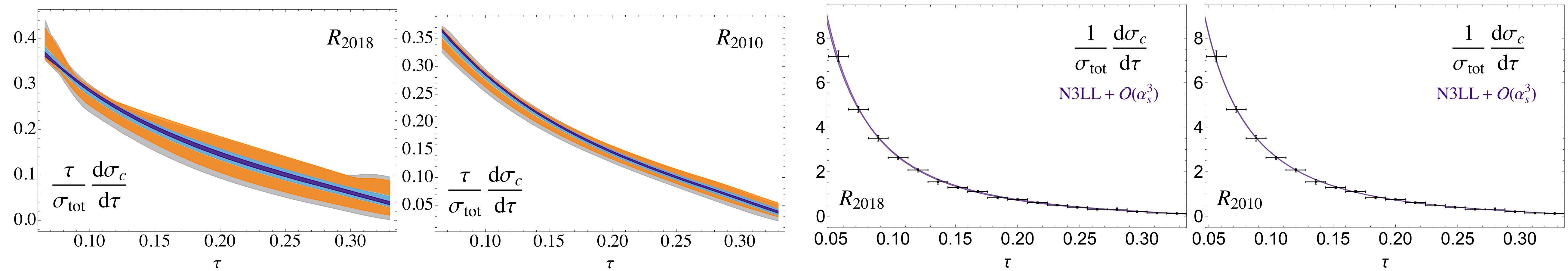
Cancellation (and evolution terms) explicitly sensitive to missing three-loop finite soft constant…

Preliminary!

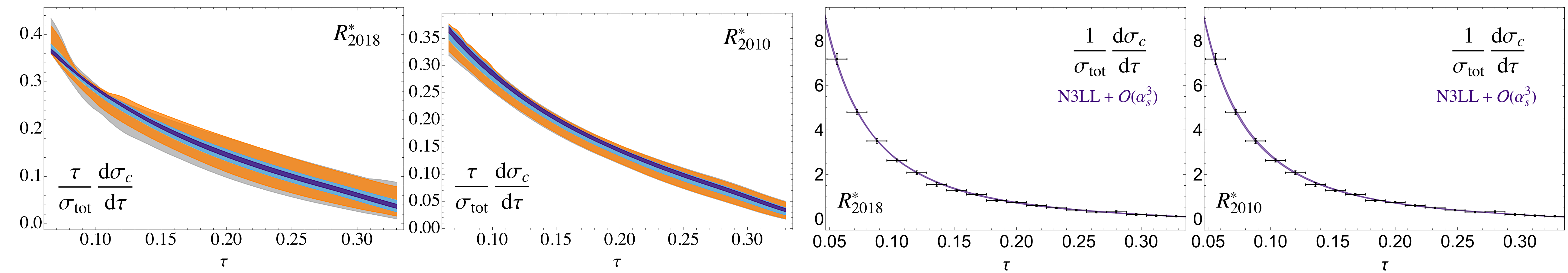
Convergence & data comparison

N3LL + $O(\alpha_s^3)$
N3LL + $O(\alpha_s^2)$
NNLL' + $O(\alpha_s^2)$
NNLL + $O(\alpha_s)$
NLL' + $O(\alpha_s)$

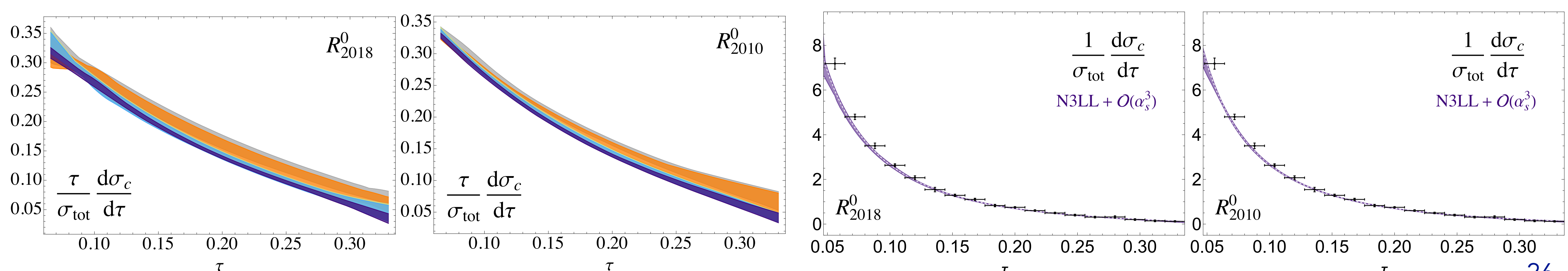
R Scheme



R* Scheme



R⁰ Scheme



Fit technique

- We perform a $\chi^2_{d.o.f.}$ analysis at the level of binned theory predictions:

$$\chi^2 \equiv \sum_{i,j} \Delta_i V_{ij}^{-1} \Delta_j$$

$$\overline{\tau} \equiv (\tau_1 + \tau_2)/2$$

$$\Delta_i \equiv \left(\frac{1}{\sigma}\frac{d\sigma}{d\tau}(\tau_i)|^{\rm exp} - \frac{1}{\sigma}\frac{d\sigma}{d\tau}(\tau_i)|^{\rm th} \right)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}(\tau_i)\Big|^{\rm th}_{MP} \equiv \frac{1}{\sigma_{tot}} \frac{\sigma_c(\tau_2, \mu_a(\overline{\tau})) - \sigma_c(\tau_1, \mu_a(\overline{\tau}))}{\tau_2 - \tau_1}$$

- Experimental errors (stat. and syst.) accounted for with 'minimal overlap model':

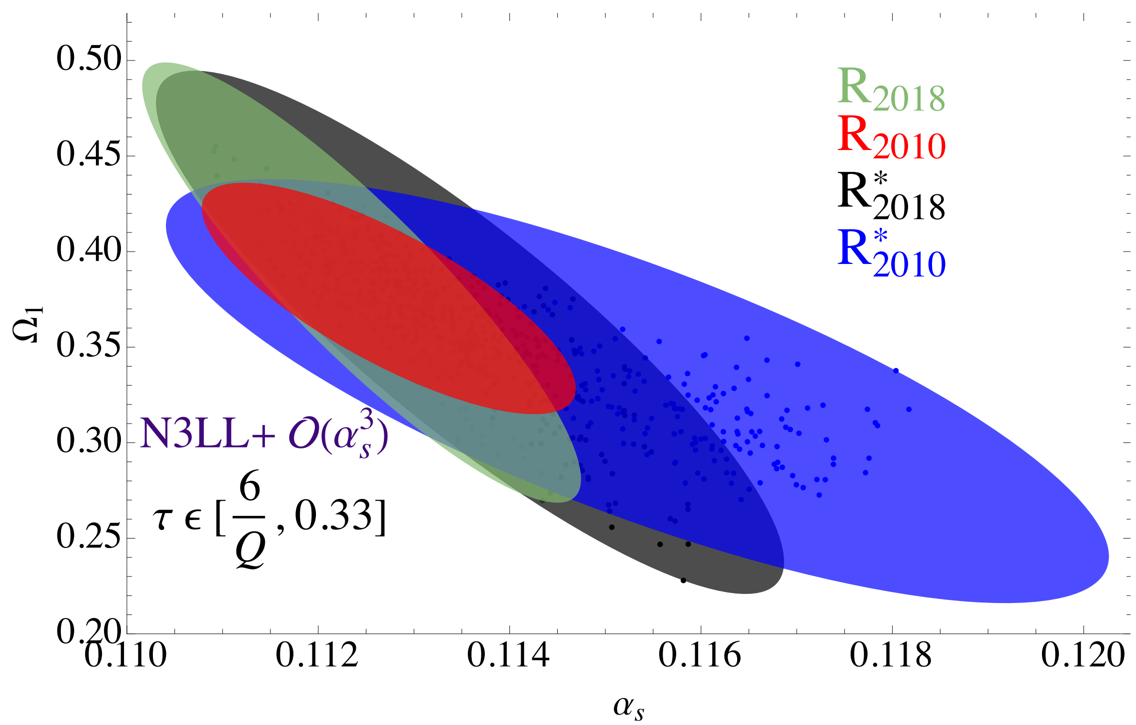
$$V_{ij}\Big|_{\rm MOM} = (e_i^{stat.})^2 \delta_{ij} + \min(e_i^{sys}, e_j^{sys})^2$$

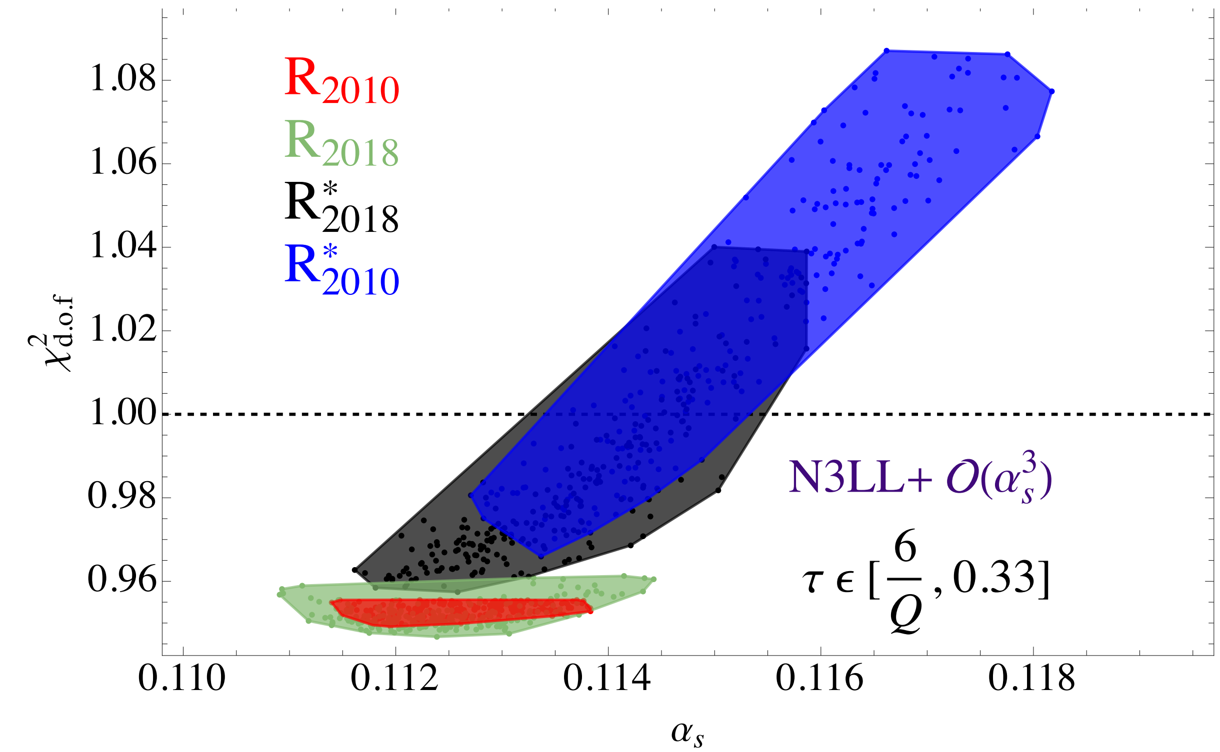
- Theory errors are conveniently parameterized in terms of an error ellipse *K*:

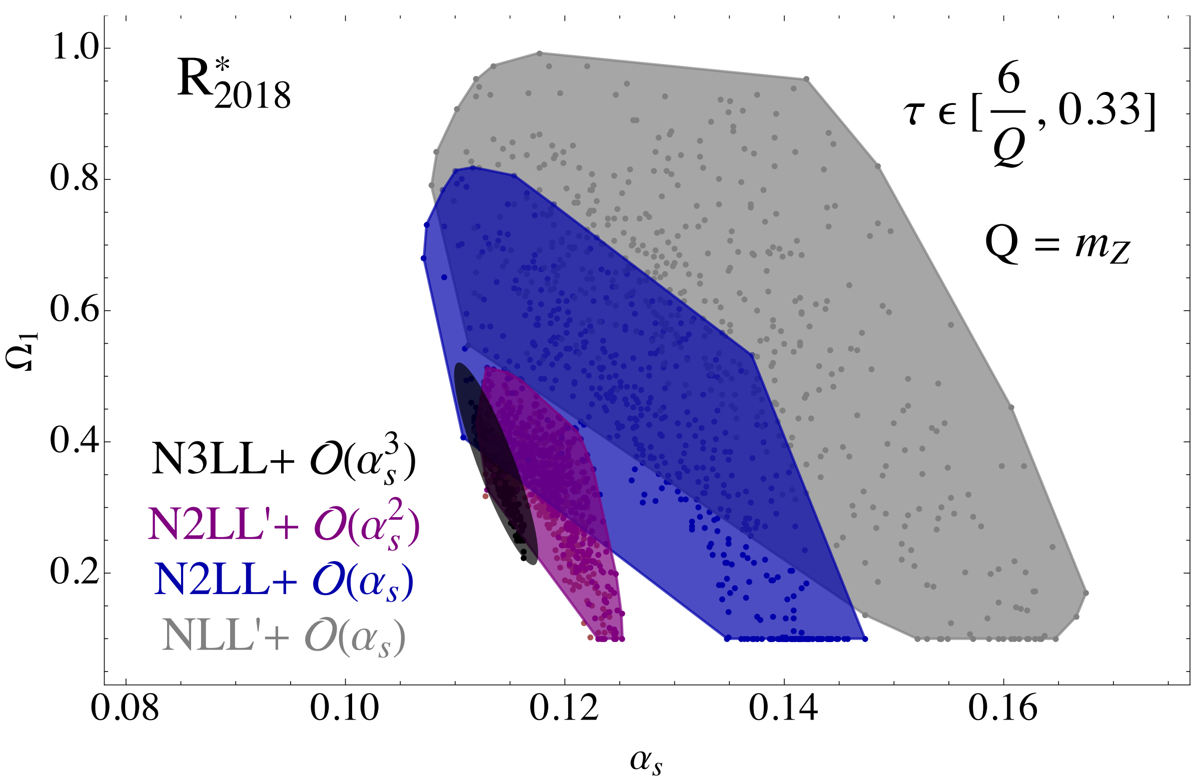
$$1 = \boldsymbol{X}^T K^{-1}_{theory} \boldsymbol{X} \qquad \boldsymbol{X}^T = \{\alpha_s,\ \Omega_1\} - \{\mu_\alpha, \mu_\Omega\} \qquad K_{theory} = \begin{pmatrix} \sigma_\alpha^2 & \rho_{\alpha\Omega}\,\sigma_\alpha\sigma_\Omega \\ \rho_{\alpha\Omega}\,\sigma_\alpha\sigma_\Omega & \sigma_\Omega^2 \end{pmatrix}$$

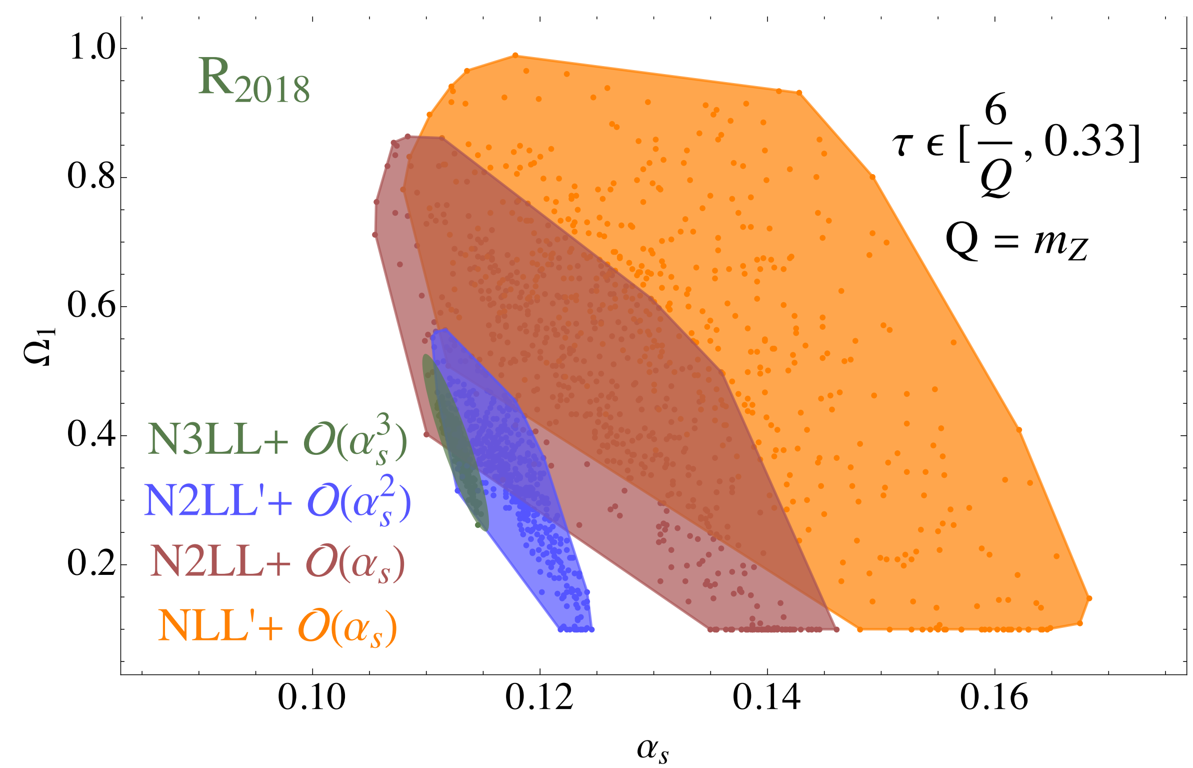
N3LL$^{(\prime)}$ + O(α_s^3) accuracy fits to α_s and Ω_1 *Preliminary!*

- Global fits include 488 bins of data with c.o.m energies between 35-207 GeV.

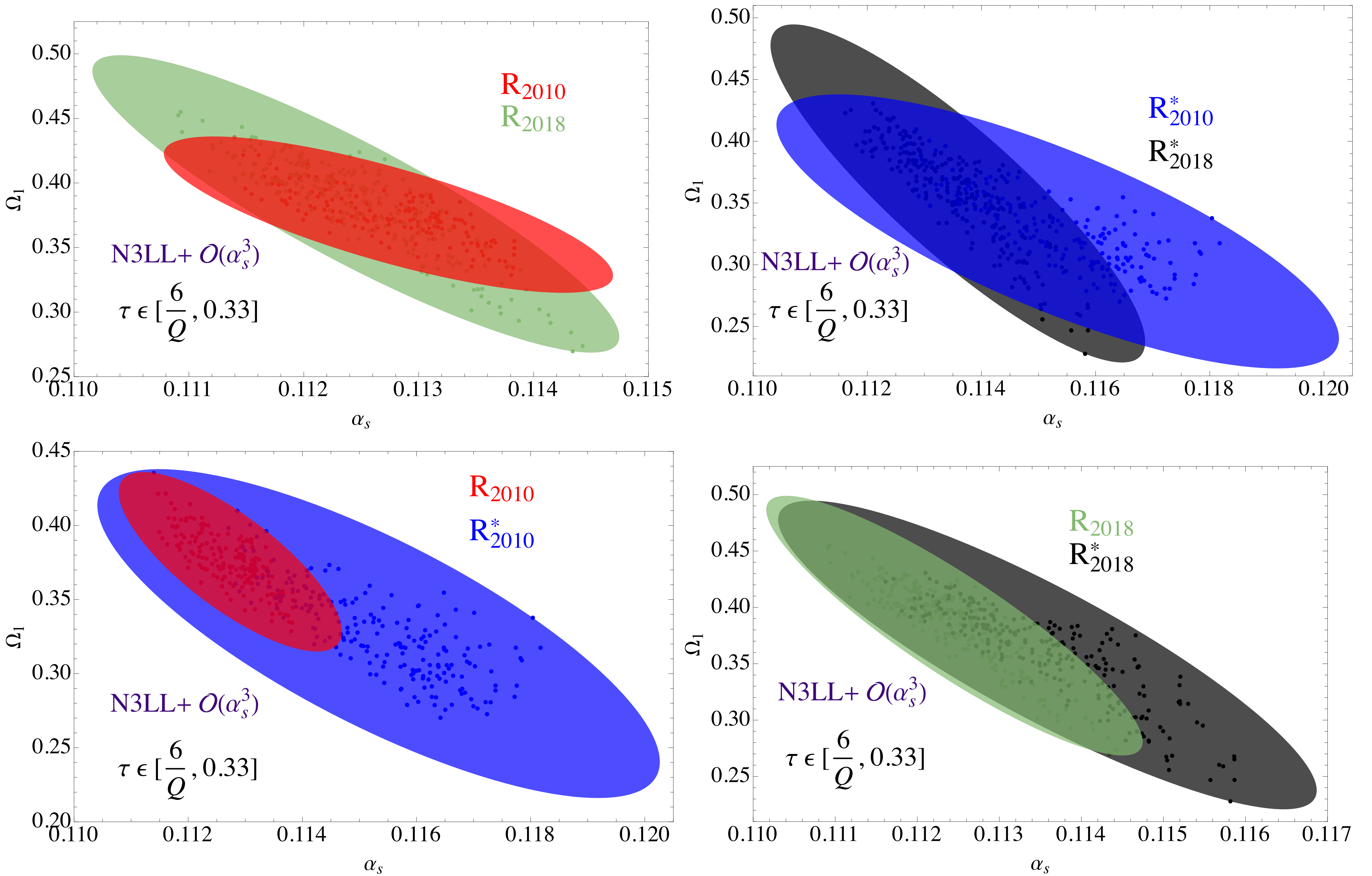




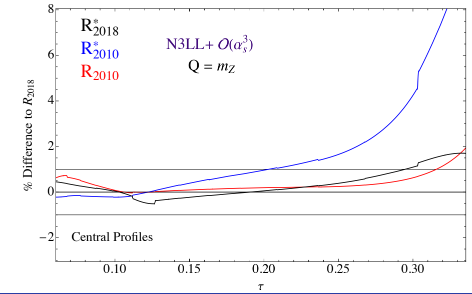




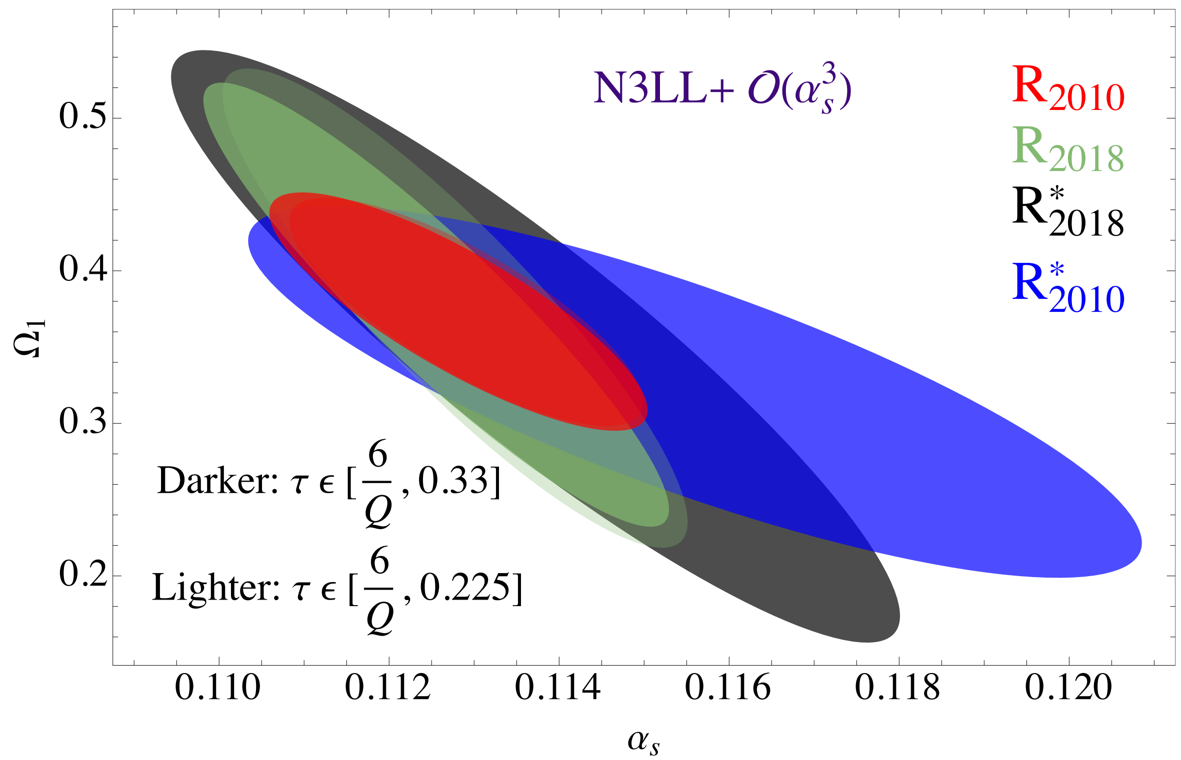
N3LL$^{(\prime)}$ + O(α_s^3) accuracy fits to α_s and Ω_1 *Preliminary!*

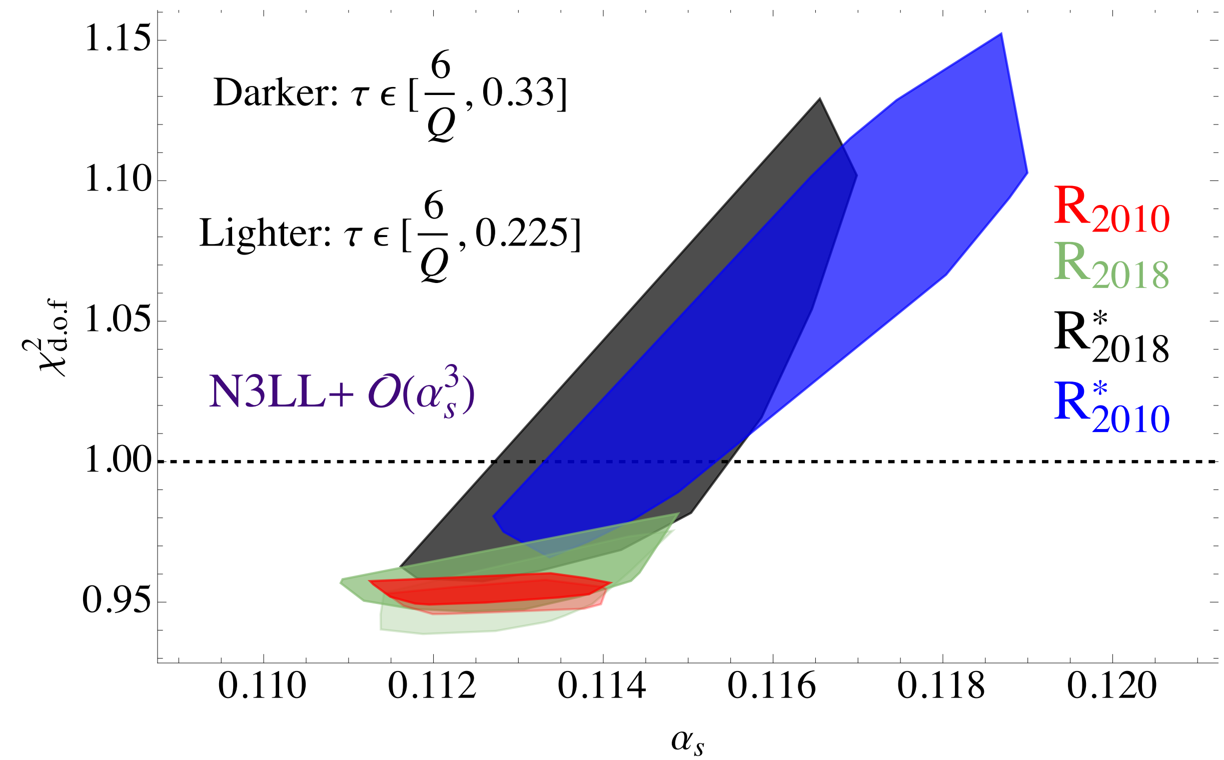


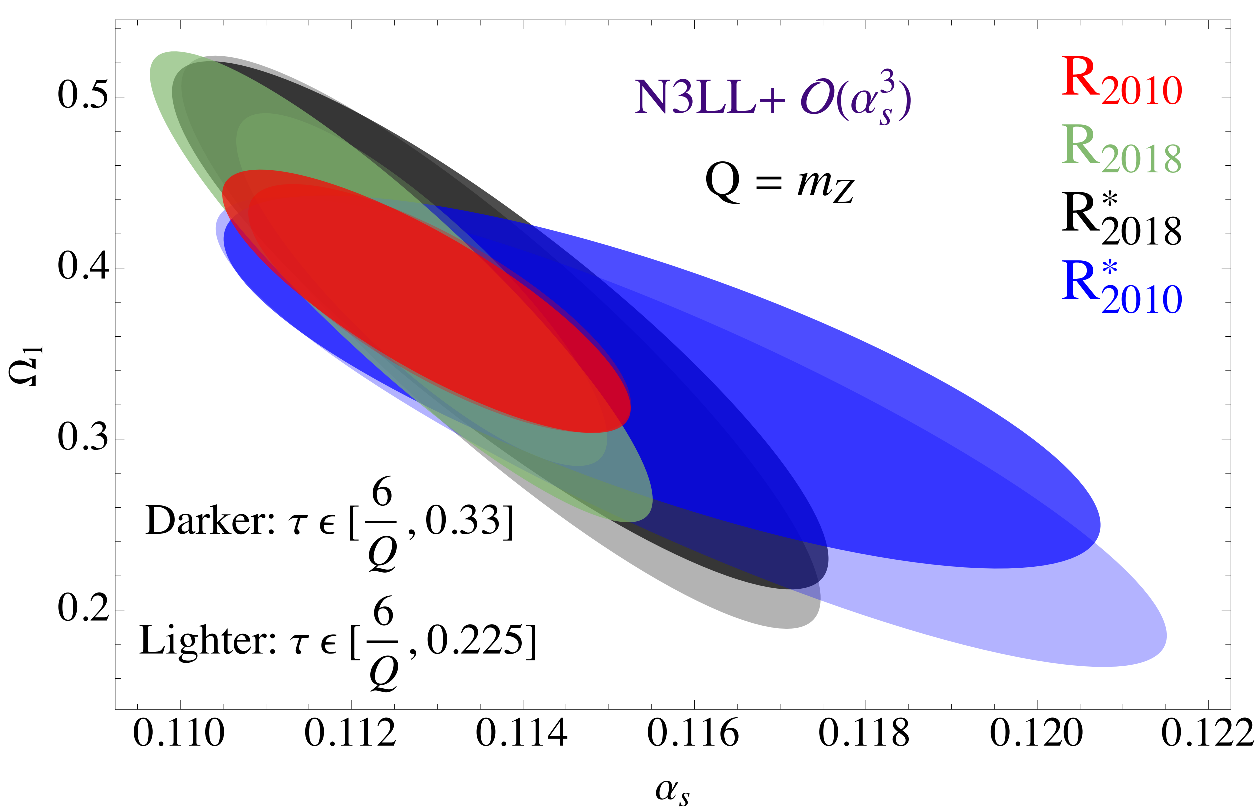
Fit windows

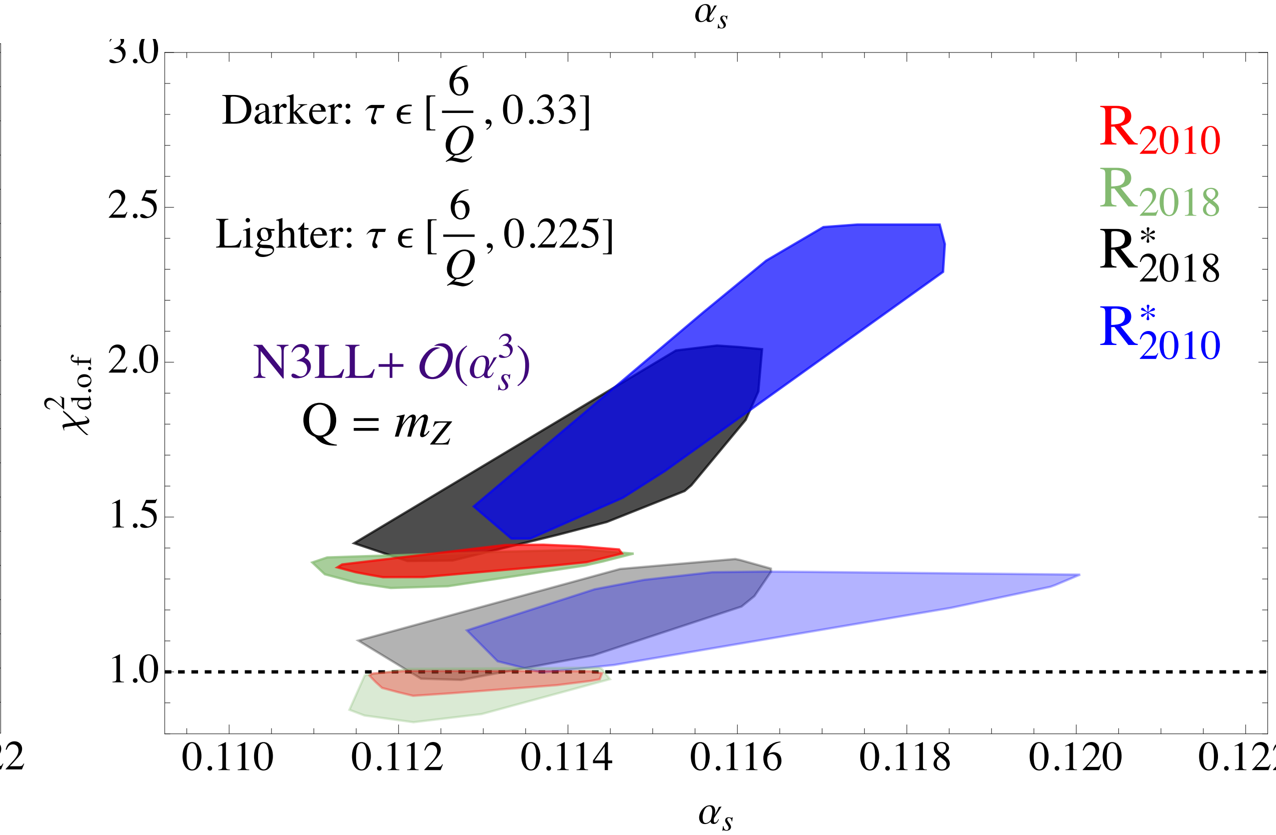


Preliminary!



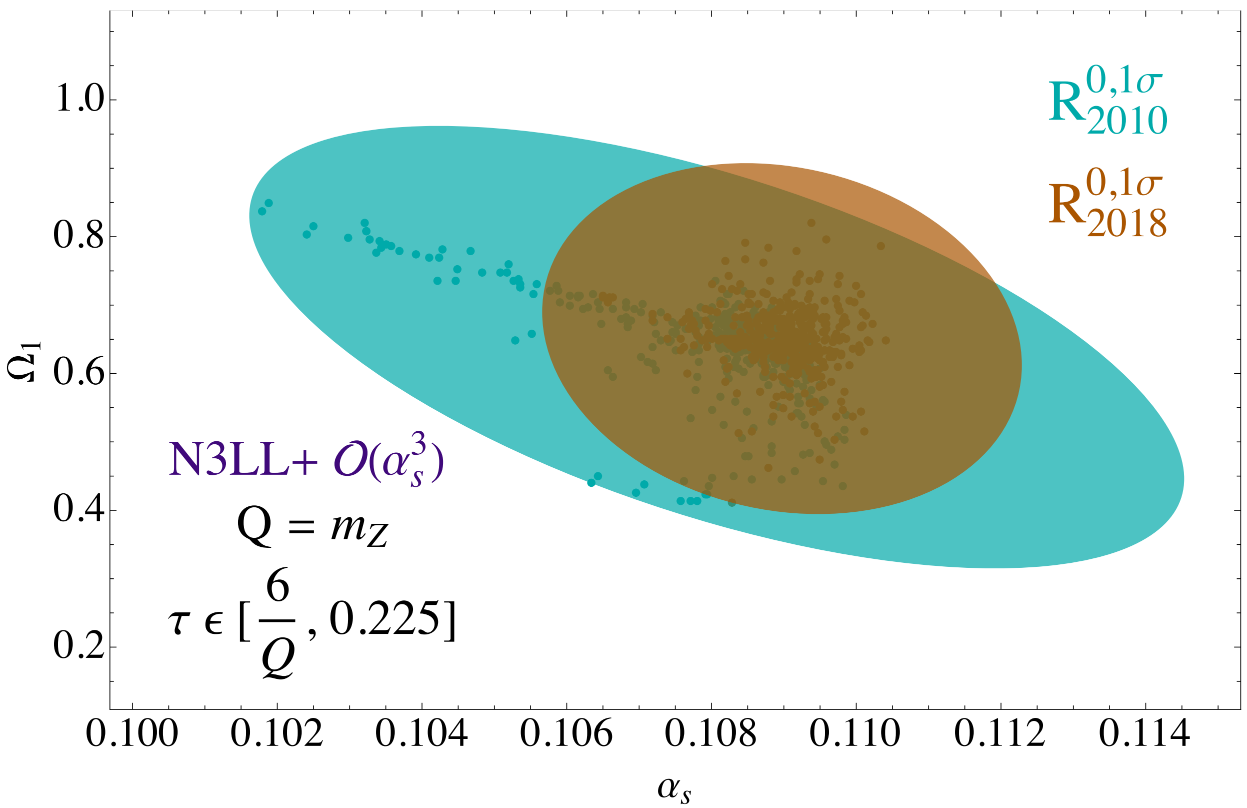


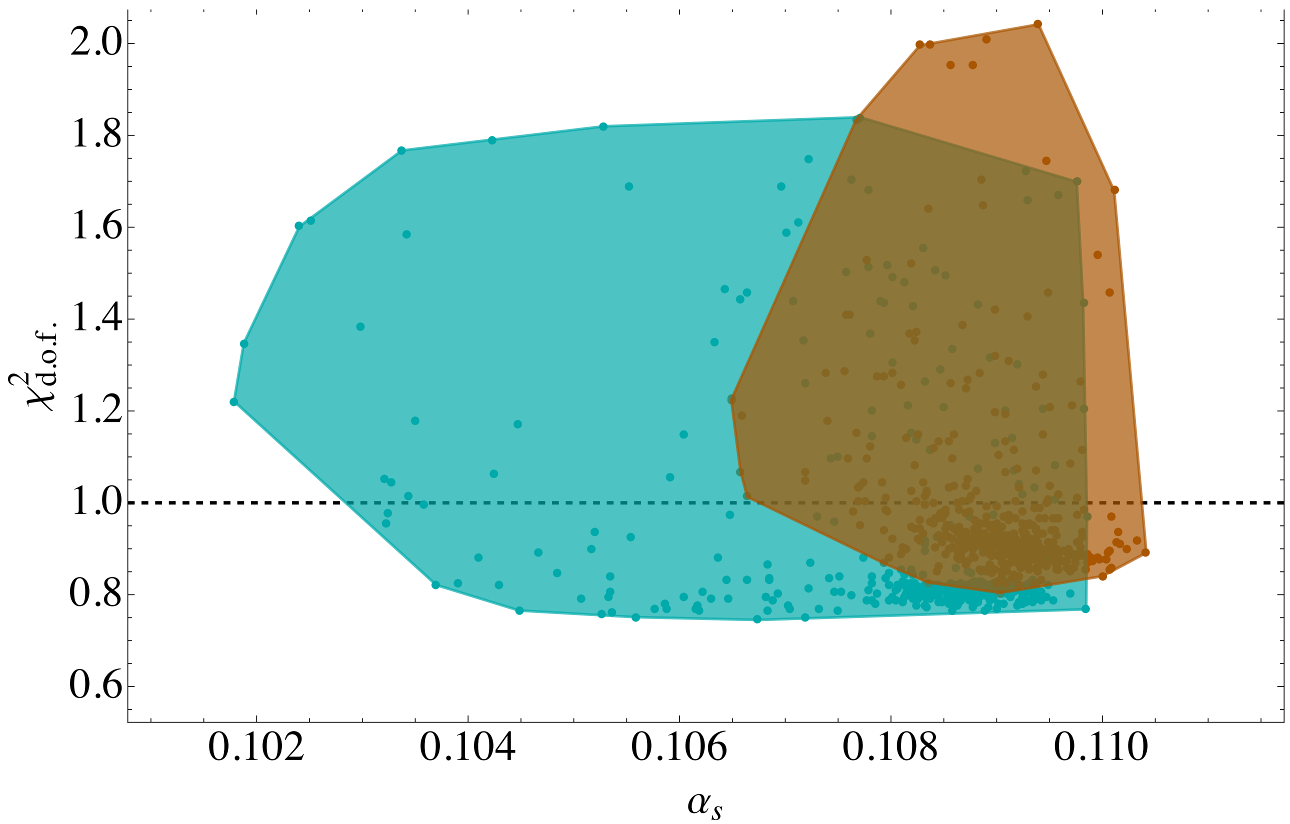


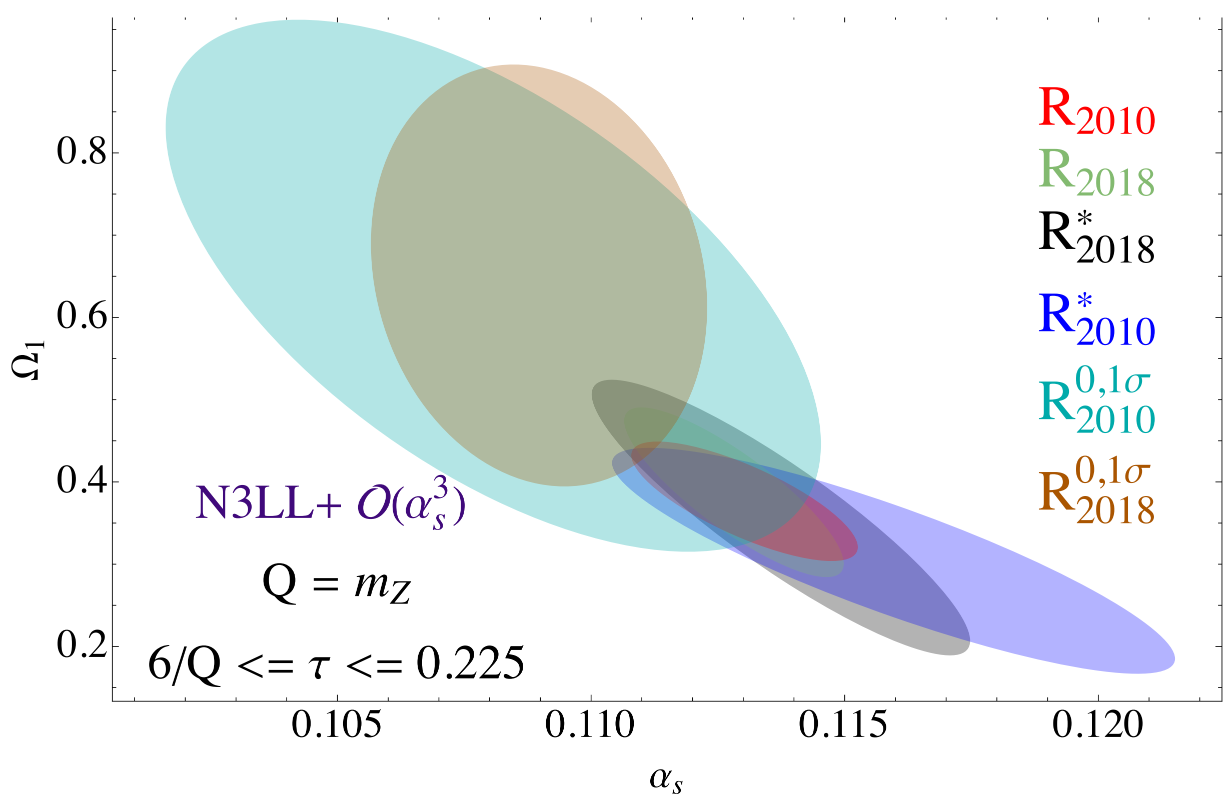


R^0 sneak peeks

Preliminary!







Currently generating remaining datasets…

Missing fixed-order ingredients

Preliminary!

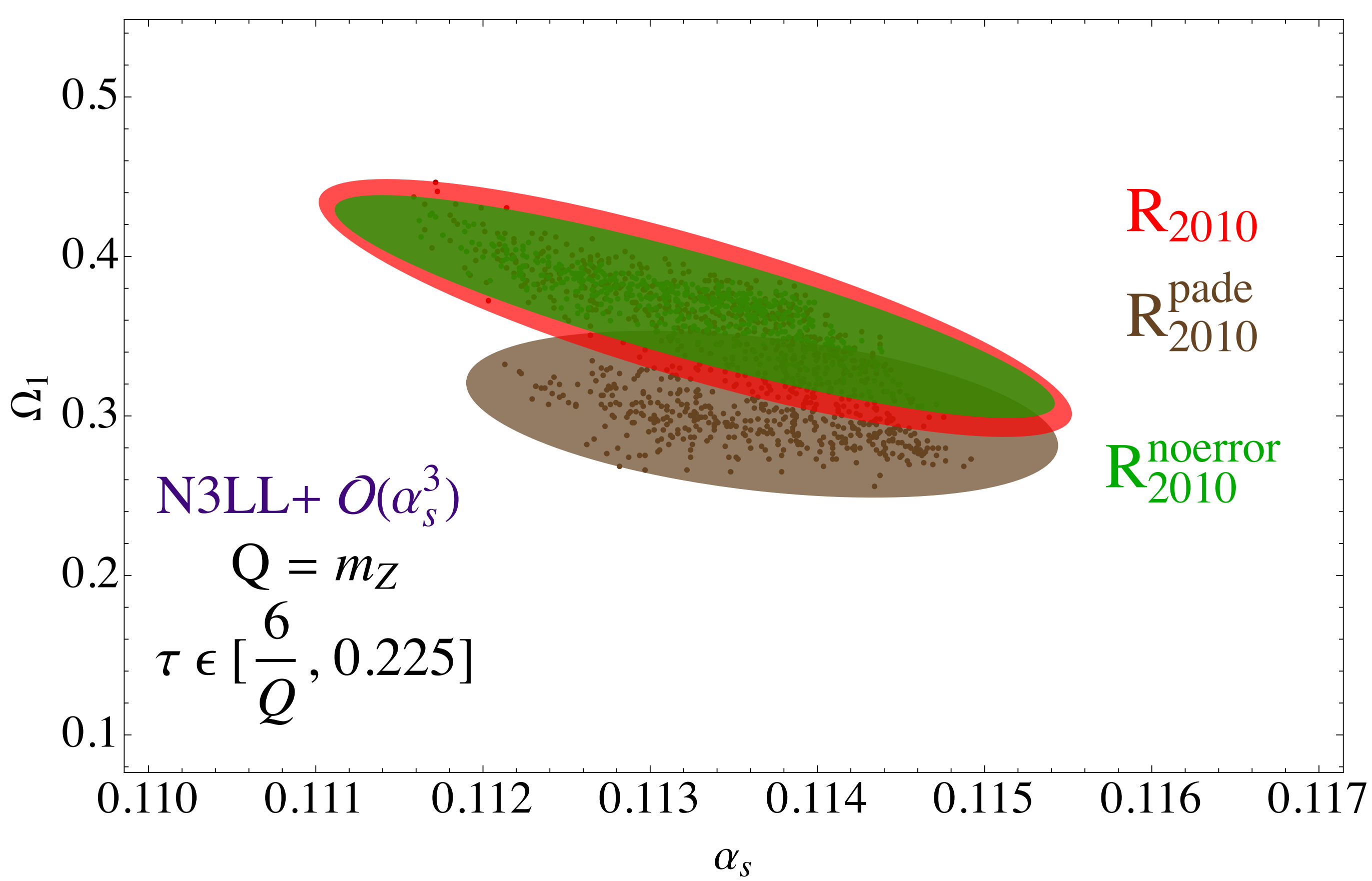
- Our extractions are sensitive, to more or less degrees, on calculations / simulations related to missing fixed-order ingredients.
- We have extracted the three-loop remainder function, and have attempted to extract the three-loop finite singular constant (as have others).

[1804.09722]

$$c_3^S = 2s_3 + 691 = -19988 \pm 1440\,(\text{stat.}) \pm 4000\,(\text{syst.}) \qquad \textbf{vs.} \qquad c_s^3\big|_{Padé} \simeq 691 \pm 1000$$

This object (and especially its central value) clearly matters at the accuracies we consider.

This would be especially true for n=0 schemes…



- We believe there is more to say here….

Binning technique

Preliminary!

- Even the technique with which we bin our theory distributions matters for the extractions!

'Midpoint' (Default) Scheme

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}(\tau_i)\Big|_{MP}^{\mathrm{th}} \equiv \frac{1}{\sigma_{tot}}\frac{\sigma_c(\tau_2,\mu_a(\overline{\tau})) - \sigma_c(\tau_1,\mu_a(\overline{\tau}))}{\tau_2-\tau_1}$$

$$\overline{\tau} \equiv (\tau_1+\tau_2)/2$$

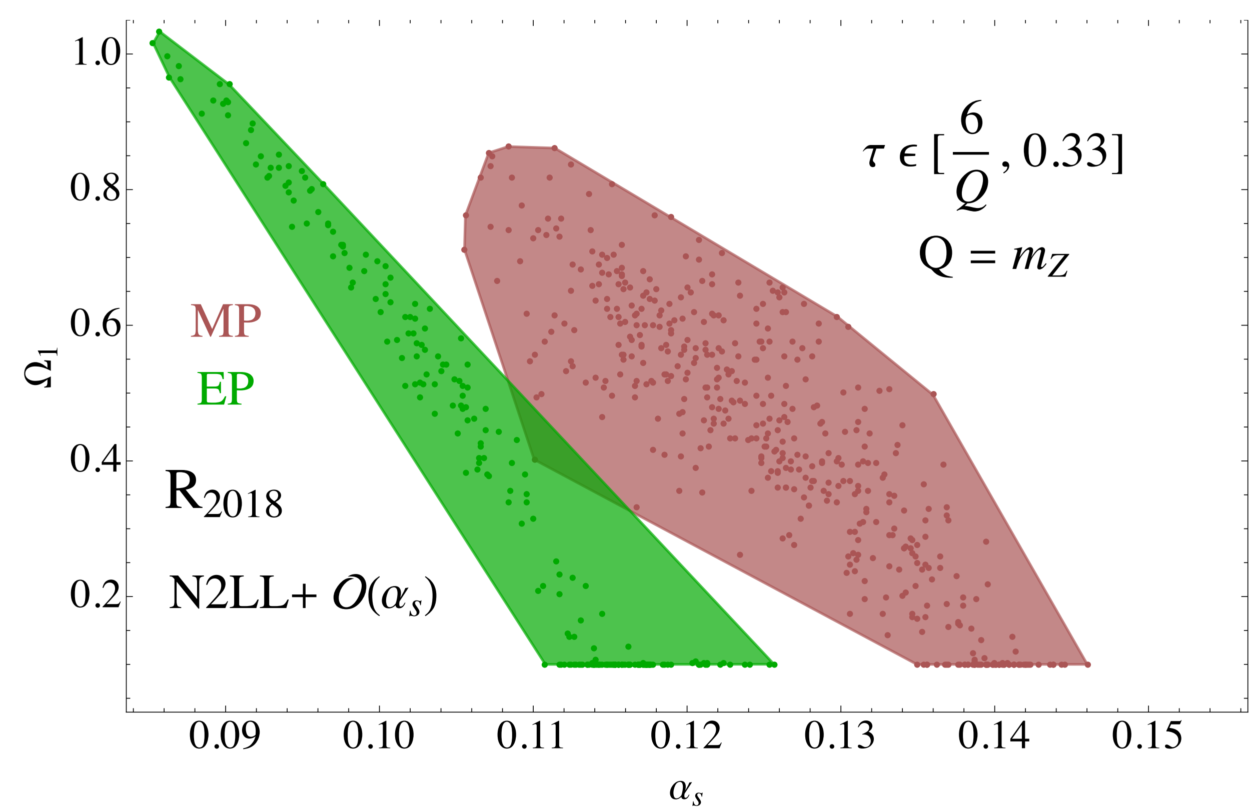
Endpoint Scheme

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}(\tau_i)\Big|_{EP}^{\mathrm{th}} \equiv \frac{1}{\sigma_{tot}}\frac{\sigma_c(\tau_2,\mu_a(\tau_2)) - \sigma_c(\tau_1,\mu_a(\tau_1))}{\tau_2-\tau_1}$$

- The endpoint scheme is argued to feature spurious contributions associated to the τ-dependence of the profile scales. Regardless, it is also a reasonable approximation to a differential distribution.

Note: NNLL accuracy plots for today…

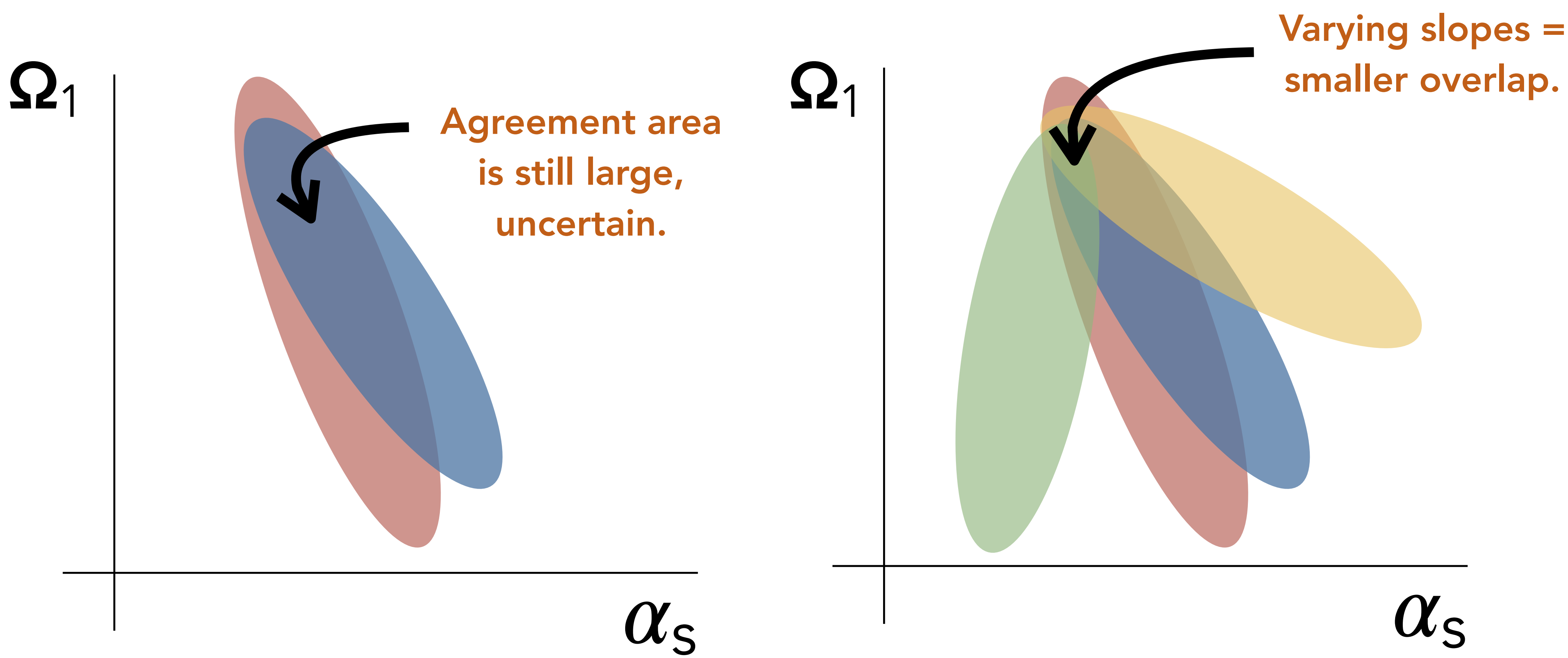
There's probably more to say here as well…

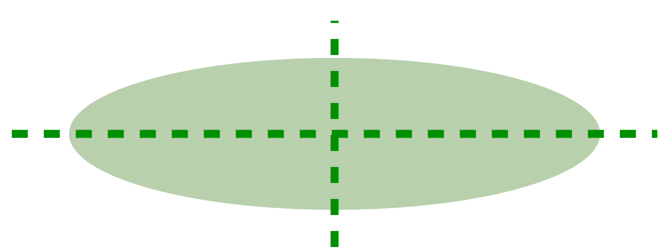


Towards Generalized Angularity Fits

Global two-parameter fits

- Multiple data sets can disentangle the strong coupling constant from non-perturbative parameter(s):





$$\frac{\Omega_1}{(-slope)} = \alpha_s^{PT}(m_Z) - \alpha_s(m_Z)$$

- The slope of the ellipse is **Q-dependent** for all event shapes, and also depends on the strength of **non-perturbative effects**. Global fits over many different data sets necessary for extraction.

Angularities: from τ to b

[0303051]
[0801.4569]
[0901.3780]
[1808.07867]

- Consider *Angularities*, which can be defined in terms of the of the rapidity and p_T of a final state particle 'i', with respect to the thrust axis:

IR safe for a ∈ {-∞, 2}!

$$\tau_a = \frac{1}{Q}\sum_i |\mathbf{p}_\perp^i|\, e^{-|\eta_i|(1-a)}$$

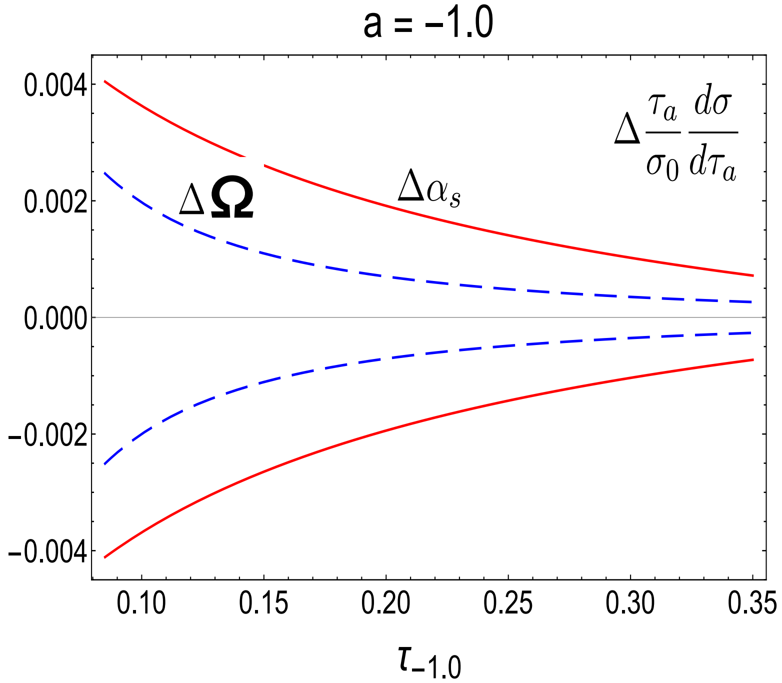
a = 0 <-> \`Thrust'

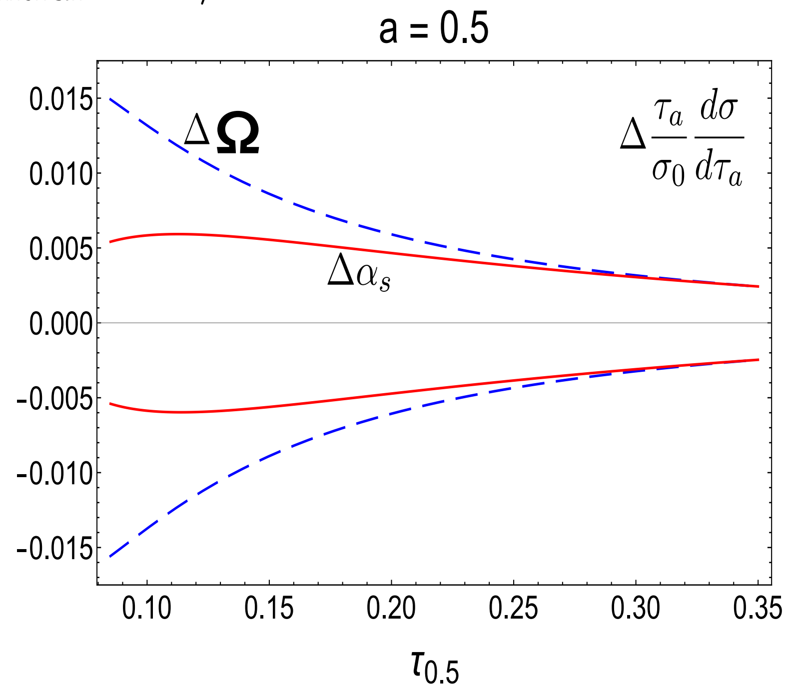
a = 1 <-> \`Jet Broadening'

- Leading NP effect is also an (a-dependent (!)) shift of the perturbative distribution:

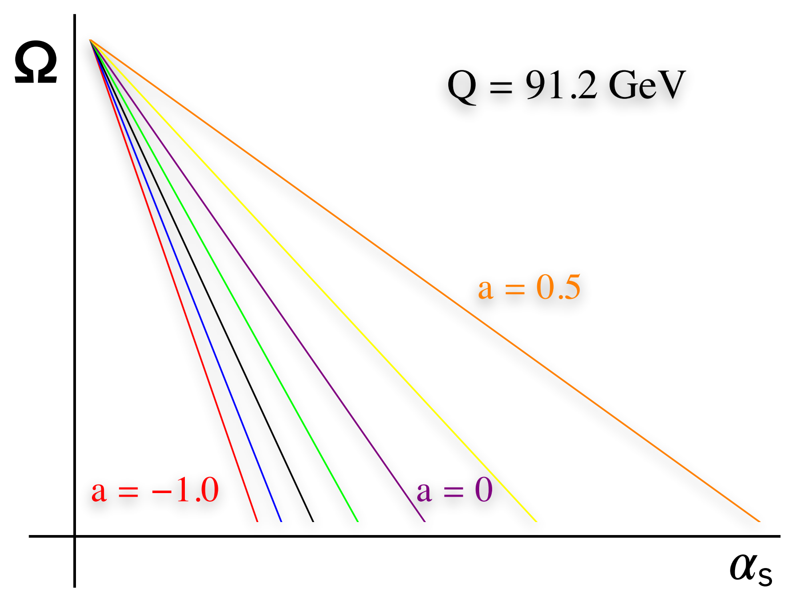
$$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow[\text{NP}]{} \frac{d\sigma}{d\tau_a}\left(\tau_a - c_{\tau_a}\frac{\Omega_1}{Q}\right) \qquad c_{\tau_a} = \frac{2}{1-a}$$

$(d\sigma/d\tau_a)_{\text{central}} - d\sigma/d\tau_a$





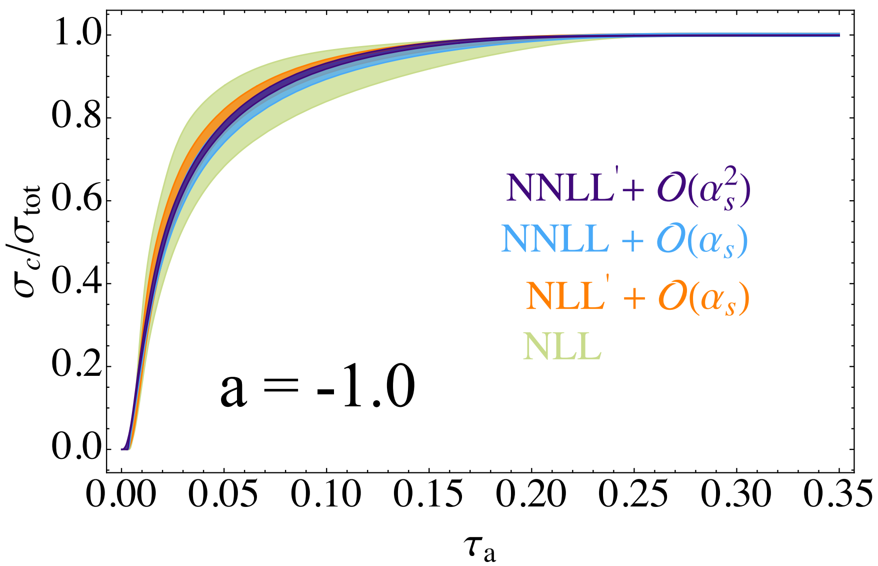
$\Delta slope$( )



- Varying **Ω** between 35 and 207 GeV generates same difference as varying a ∈ {-2.0, 0.5} (~6)!!

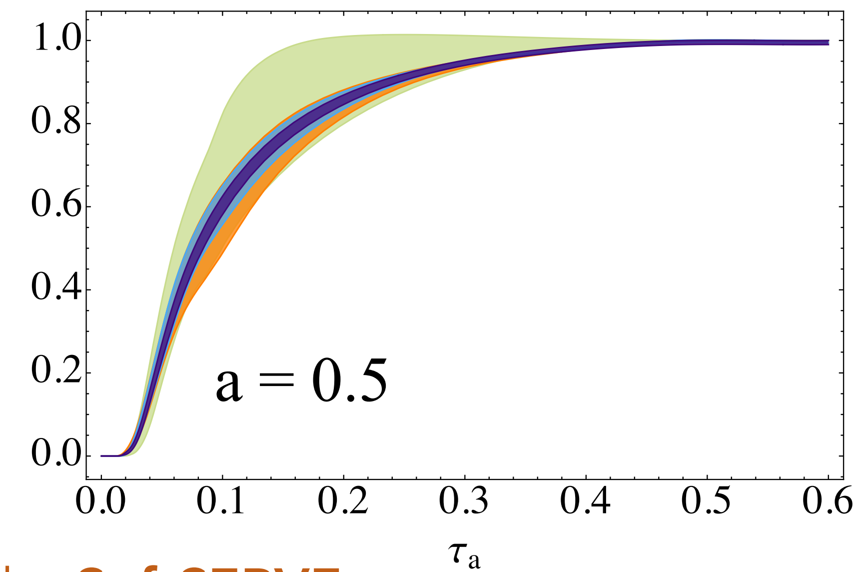
[0901.3780]
[1805.12414]
[1808.07867]
[1812.08690]
[2004.08396]

2018 progress: NLL′ to NNLL′

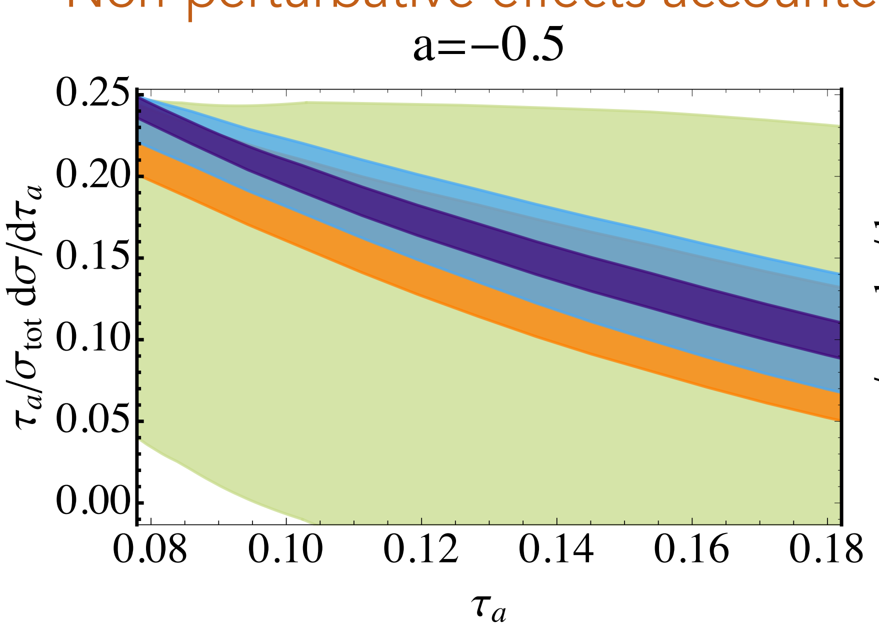


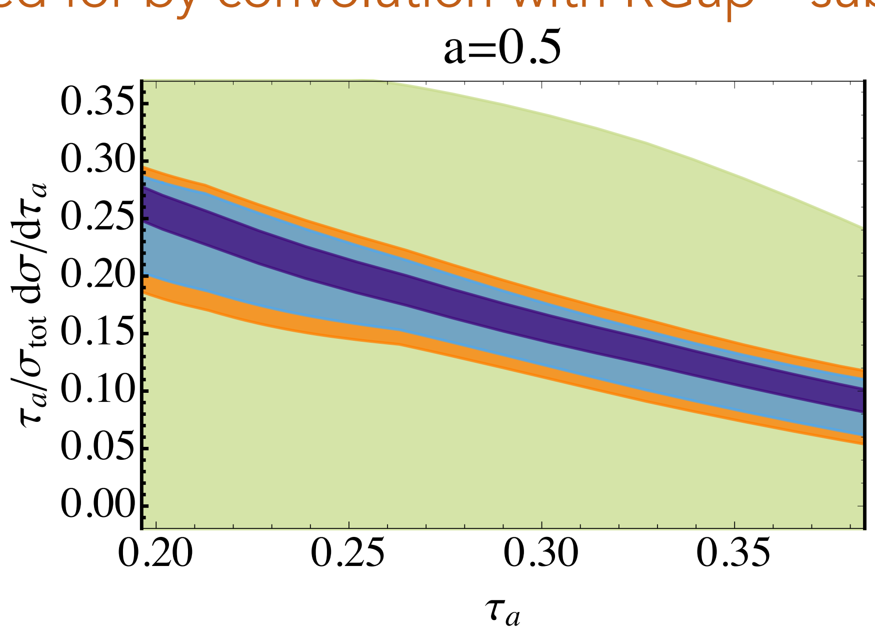
softserve.hepforge.org

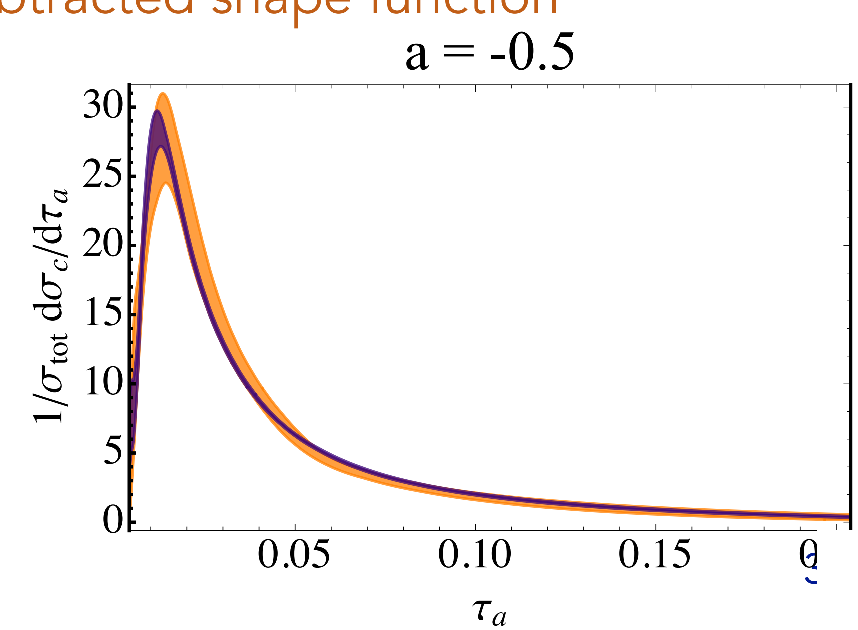
Bell, Rahn & Talbert



- Two-loop soft anomalous dimensions and singular constants provided by **SoftSERVE**
- Two-loop jet anomalous dimension obtained from consistency relations
- Two-loop singular jet constants extracted from **EVENT2 (though now calculable)** Bell, Hornig, Lee & Talbert
- Matching to QCD at $\mathcal{O}(\alpha_s^2)$ extracted from **EVENT2 ***
- Includes set of H,J,S, & non-sing. profile scales, tuned for a-dependence, and varied with a random scan over parameters
- Non-perturbative effects accounted for by convolution with RGap—subtracted shape function







The (only) dataset

Generalized event shape and energy flow studies in e^+e^- annihilation at $\sqrt{s} = 91.2\text{-}208.0\,\text{GeV}$

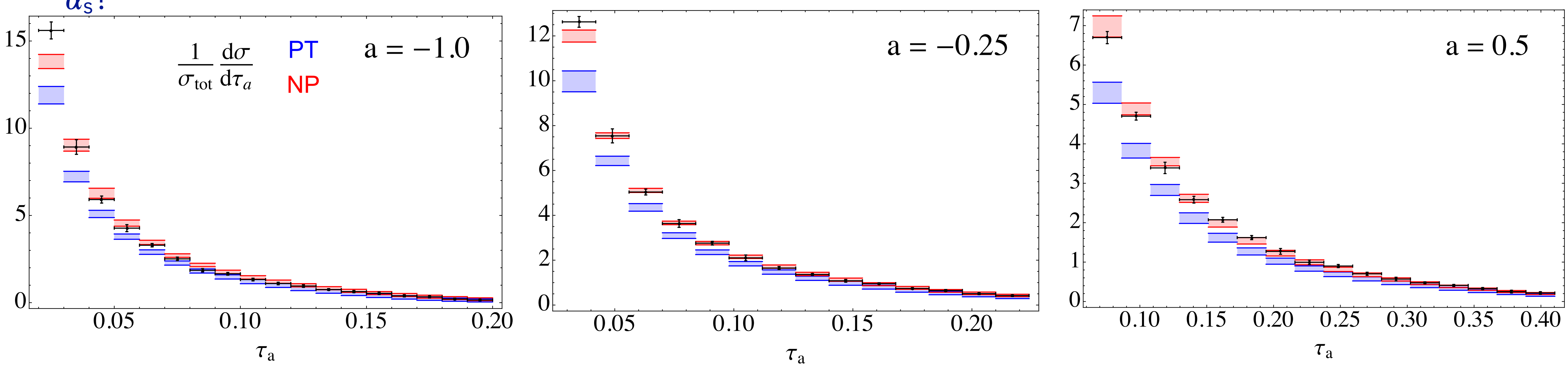
L3 Collaboration

JHEP 10 (2011) 143

RECEIVED: *May 12, 2009*
REVISED: *May 3, 2011*
ACCEPTED: *August 24, 2011*
PUBLISHED: *October 31, 2011*

Also see thesis by Pratima Jindal, Panjab University, Chandigarh

- Data for a = {**-1.0**, **-0.75**. **-0.5**, **-0.25**, **0.0**, **0.25**, **0.5**, 0.75} at **91.2** and 197 GeV
- Total number of bins = (bins per a) x (number of a) = 25 x 7 = 175 bins @ Q = 91.2 GeV
- Compare to 404 bins **included** in 2015 C-Parameter fit (across all Q considered)…
- Early theory predictions look good against the data, but what does this translate to for **Ω** and α_s?



BLUE: NNLL′ + $O(\alpha_s^2)$ **RED**: NNLL′ + $O(\alpha_s^2)$ + NP

Summary and outlook

- We have presented results demonstrating the impact of (non-)perturbative physics on a global SCET extraction of the strong coupling from the Thrust e+e- event shape.
- Our results are valid at **N3LL$^{(\prime)}$ + $O(\alpha_s^3)$**. Represents improvement over our prior results.
- We have also shown how Thrust fit values are sensitive to the **fit window** chosen, as well as to **profile parameters** associated to scale setting.
- When the effective shift of the distribution, due to non-perturbative physics, grows less in the multi-jet window, the value of the strong coupling from Thrust approaches the PDG average. Regardless, an **additional systematic uncertainty** is clearly present in EFT extractions…
- Analytic control over **multi-jet power corrections** would clearly be valuable (also see Luisoni et al., Nason, Zanderighi e.g.), as would control of **next-to-leading power corrections** in the EFT.

- In addition, we have argued that by studying the **Angularities** class of event shapes, one may have the opportunity to further disentangle two-parameter fits.
- However, only one L3 dataset exists. **More data**, at more values of ***Q*** and ***a***, could permit an unambiguous **disentangling** of leading non-perturbative effects.

THANK YOU!

Backup Slides

Data sets

For thrust:

```
ALEPH-2004: 133. GeV (7)
ALEPH-2004: 161. GeV (7)
ALEPH-2004: 172. GeV (7)
ALEPH-2004: 183. GeV (7)
ALEPH-2004: 189. GeV (7)
ALEPH-2004: 200. GeV (6)
ALEPH-2004: 206. GeV (8)
ALEPH-2004: 91.2 GeV (26)
AMY-1990: 55.2 GeV (5)
DELPHI-1999: 133. GeV (7)
DELPHI-1999: 161. GeV (7)
DELPHI-1999: 172. GeV (7)
DELPHI-1999: 89.5 GeV (11)
DELPHI-1999: 93. GeV (12)
DELPHI-2000: 91.2 GeV (12)
DELPHI-2003: 183. GeV (14)
DELPHI-2003: 189. GeV (15)
DELPHI-2003: 192. GeV (15)
DELPHI-2003: 196. GeV (14)
DELPHI-2003: 200. GeV (15)
DELPHI-2003: 202. GeV (15)
DELPHI-2003: 205. GeV (15)
DELPHI-2003: 207. GeV (15)
DELPHI-2003: 45. GeV (5)
DELPHI-2003: 66. GeV (8)
DELPHI-2003: 76. GeV (9)
JADE-1998: 35. GeV (5)
JADE-1998: 44. GeV (7)
L3-2004: 130.1 GeV (11)
L3-2004: 136.1 GeV (10)
L3-2004: 161.3 GeV (12)
L3-2004: 172.3 GeV (12)
L3-2004: 182.8 GeV (12)
L3-2004: 188.6 GeV (12)
L3-2004: 194.4 GeV (12)
L3-2004: 200. GeV (11)
L3-2004: 206.2 GeV (12)
L3-2004: 41.4 GeV (5)
L3-2004: 55.3 GeV (6)
L3-2004: 65.4 GeV (7)
L3-2004: 75.7 GeV (7)
L3-2004: 82.3 GeV (8)
L3-2004: 85.1 GeV (8)
L3-2004: 91.2 GeV (10)
OPAL-1997: 161. GeV (7)
OPAL-2000: 172. GeV (8)
OPAL-2000: 183. GeV (8)
OPAL-2000: 189. GeV (8)
OPAL-2005: 133. GeV (6)
OPAL-2005: 177. GeV (8)
OPAL-2005: 197. GeV (8)
OPAL-2005: 91. GeV (5)
SLD-1995: 91.2 GeV (6)
TASSO-1998: 35. GeV (4)
TASSO-1998: 44. GeV (5)

------ Summary ------
Totlal: 516
Q > 95 : 345
Q < 88 : 89
Q ~ MZ : 82
```

For angularities:

Generalized event shape and energy flow studies in e^+e^- annihilation at $\sqrt{s} = 91.2\text{-}208.0\,\mathrm{GeV}$

L3 Collaboration

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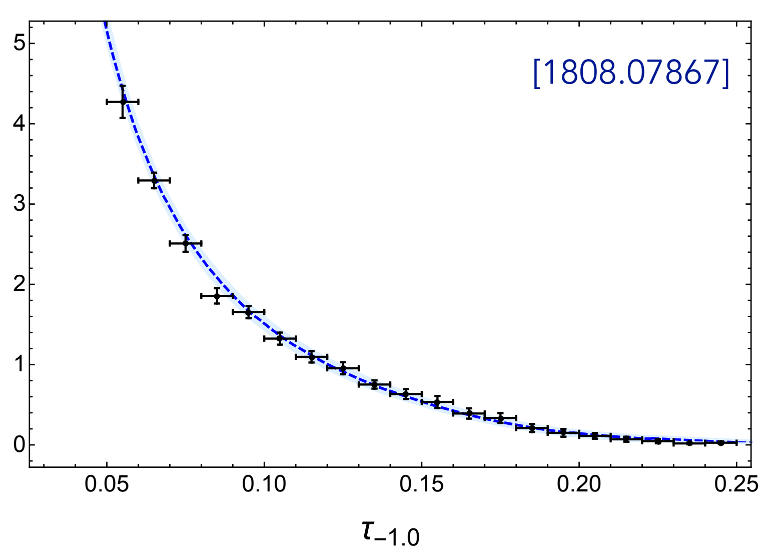
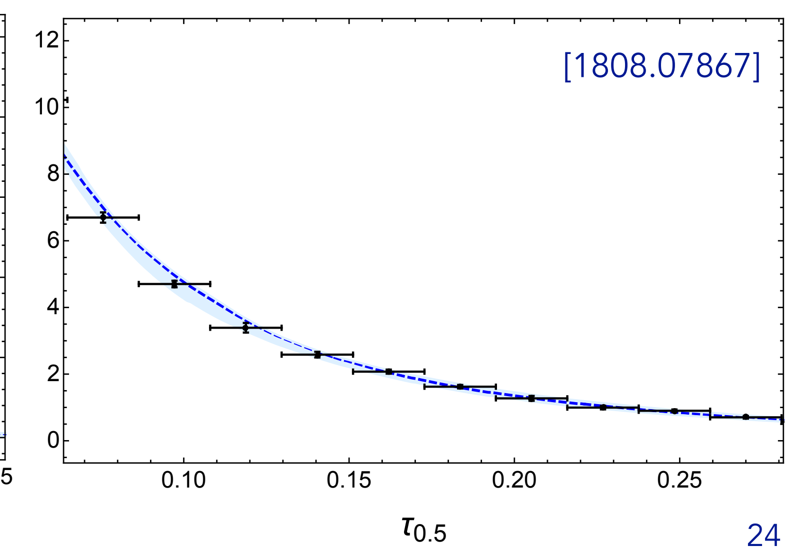
Also see thesis by Pratima Jindal, Panjab University, Chandigarh

- Data for a = {**-1.0**, **-0.75**. **-0.5**, **-0.25**, **0.0**, **0.25**, **0.5**, 0.75} at **91.2** and 197 GeV
- Total number of bins = (bins per a) x (number of a) = 25 x 7 = 175 bins @ Q = 91.2 GeV
- e.g. a = -1 and 0.5, Q = 91.2 GeV, compared to our NNLL' prediction:





Introducing SCET: dijet factorization

- Begin with fundamental QCD fields and split into soft and collinear components:

$$A^\mu(x) \to A^\mu_c(x) + A^\mu_s(x) \qquad \Psi^\mu(x) \to \Psi^\mu_c(x) + \Psi^\mu_s(x)$$

- Further project collinear fermion into two components, and determine scaling of correlators:

$$\zeta(x) = \frac{\not{n}\not{\bar{n}}}{4}\Psi_c(x), \quad \eta(x) = \frac{\not{\bar{n}}\not{n}}{4}\Psi_c(x)$$

$$\langle 0|\{\zeta(x)\bar{\zeta}(0)\}|0\rangle \sim \lambda^2 \Rightarrow \zeta(x) \sim \lambda \qquad (\eta(x) \sim \lambda^2)$$

- Now, integrate out momentum suppressed modes. Note, this is not a traditional EFT! Let's consider the factorization at the level of the current. Two critical steps. *"Hard-Collinear factorization"* (1) & *"Soft-decoupling"* (2):

$$(1) \quad \bar{\Psi}(0)\ \gamma^\mu\ \Psi(0) \to \int dsdt\ C_V(s,t)\ (\bar{\zeta}_{\bar{n}} W_{\bar{n}})(sn)\ \gamma^\mu_\perp\ (W^\dagger_n \zeta_n)(t\bar{n})$$

- **Wilson lines** necessary for gauge invariance: $W_c = Pexp\left(ig\int_{-\infty}^{0} ds\bar{n}\cdot A_c(x+s\bar{n})\right)$

$$(2) \quad \bar{\Psi}(0)\ \gamma^\mu\ \Psi(0) \to \int dsdt\ C_V(s,t)\ \bar{\zeta}^0_{\bar{n}}\ W^{0,\dagger}_{\bar{n}} S^\dagger_{\bar{n}}\ \gamma^\mu_\perp\ W^0_n S_n\ \zeta^0_n$$

$$\zeta_n(x) = S_n(x_-)\zeta^0_n(x)$$

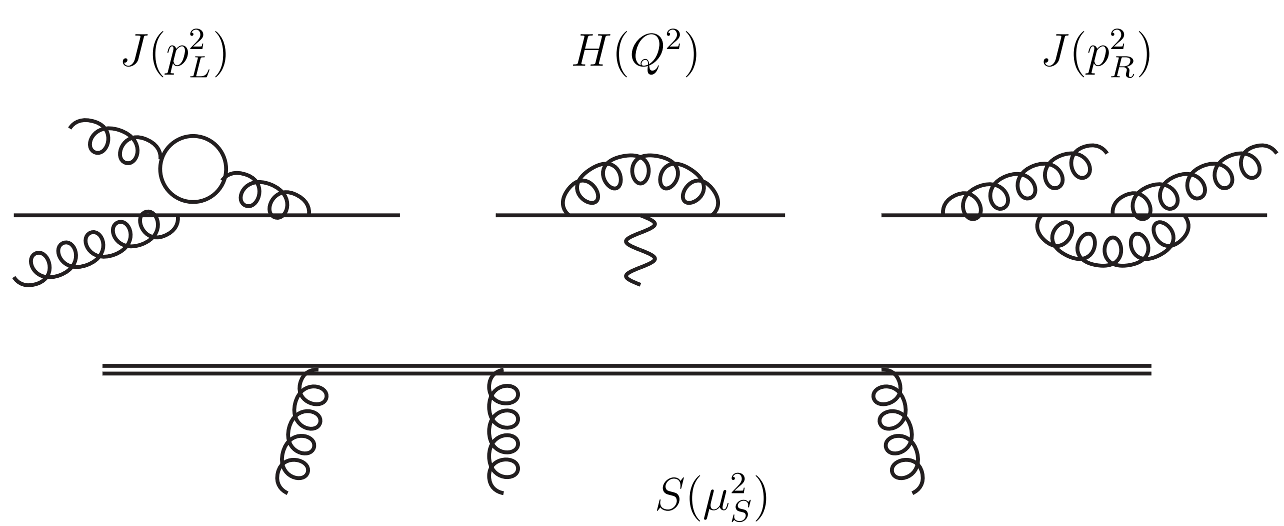
Introducing SCET: dijet factorization

- We can thus **factorize** our matrix element for the dijet, two-fermion operator quite simply:

$$|C_V|^2 \sum_X |\langle 0|\mathcal{O}_{n\bar{n}}|X\rangle|^2$$

$$= |C_V|^2 \; \langle 0| \left[\bar{\zeta}^0_{\bar{n}} W^{0,\dagger}_{\bar{n}}\right] \left[\bar{\zeta}^0_{\bar{n}} W^{0,\dagger}_{\bar{n}}\right]^\dagger |0\rangle \; \langle 0| \left[W^0_n \zeta^0_n\right] \left[W^0_n \zeta^0_n\right]^\dagger |0\rangle \; \langle 0| \left[S^\dagger_{\bar{n}} S_n\right] \left[S^\dagger_{\bar{n}} S_n\right]^\dagger |0\rangle$$

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{de} = H(Q;\mu) \int de_n \, de_{\bar{n}} \, de_s \, J_n(e_n;\mu) \, J_{\bar{n}}(e_{\bar{n}};\mu) \, S(e_s;\mu) \, \delta(e - e_n - e_{\bar{n}} - e_s)$$



"soft-decoupling"