Constraining parameters of initial condition in isobar collisions

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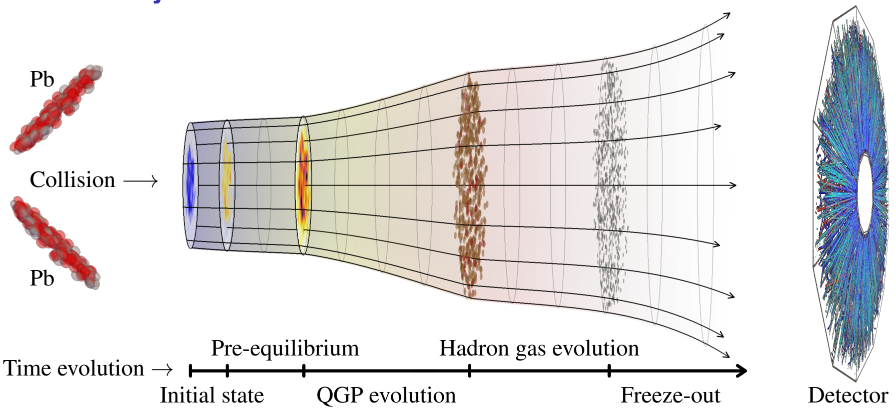
In collaboration with J. Jia

INT program: Intersection of nuclear structure and high - energy nuclear collisions
Seattle February 2st, 2023

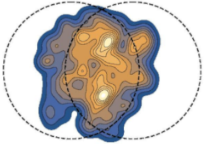
State-of-the-art heavy-ion collision models



Hybrid Model $=$	IP-Glasma $\mathrm{T_RENTo}$ MC-Glauber MC-KLN ⋮	$\otimes$	$\mathbb{1}$ Free-streaming KøMPøST Gauge/Gravity ⋮	$\otimes$	VISH2+1 MUSIC Trajectum VH2+1 ⋮	$\otimes$	UrQMD SMASH B3D ⋮

[References in the backup slides]

An intuitive picture



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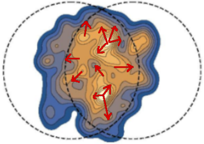

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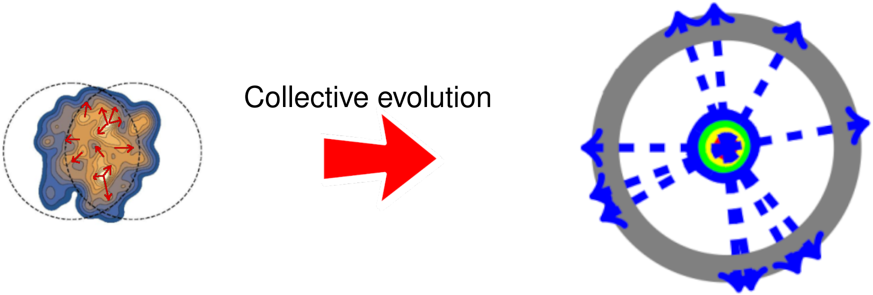

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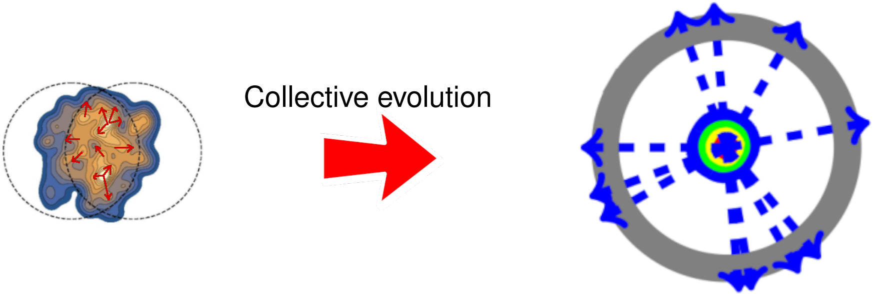
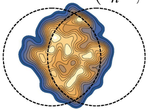


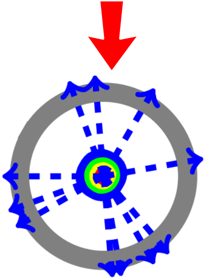
Initial spatial anisotropy $\longrightarrow$ *Final momentum anisotropy*

Collective evolution from experiment

$(\varepsilon_n^{(1)}, r_{\mathrm{rms}}^{(1)}, \ldots)$



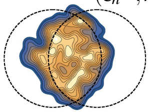
collective evolution



Collective evolution from experiment

$$\hat{\varepsilon}_{n,m} = -\frac{\{\rho^n e^{im\varphi}\}}{\{\rho^n\}}, \quad r_n = \{\rho^n\} \qquad (\varepsilon_n \equiv |\hat{\varepsilon}_{n,n}|, \quad r^2_{\text{rms}} \equiv r_2)$$

$$\frac{d^2N}{p_T dp_T d\varphi} = N(p_T)\left[1 + \sum_{n=1}^{\infty} 2 v_n \cos\left[n(\varphi - \psi_n)\right]\right]$$



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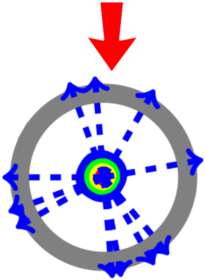

Flow harmonics, (v_n, ψ_n), depend on the initial state parameters, transport coefficients $(\eta/s,\ \zeta/s, \ldots)$, ...

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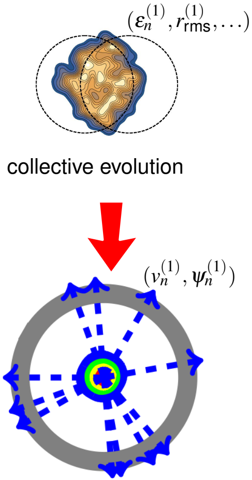

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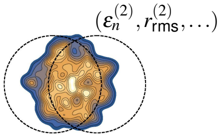

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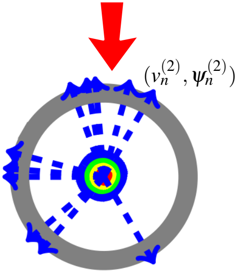

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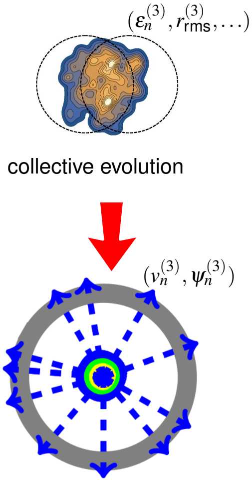

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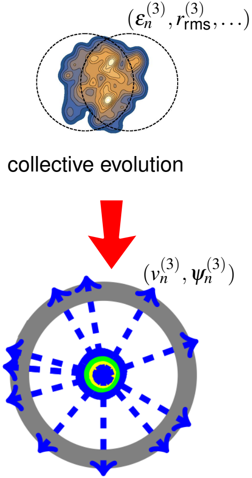

Flow harmonics, $(\boldsymbol{v}_n, \psi_n)$, depend on the initial state parameters, transport coefficients $(\eta/s$, $\zeta/s, \ldots)$, ...

$$p([p_T], v_n, \psi_n, \ldots)$$

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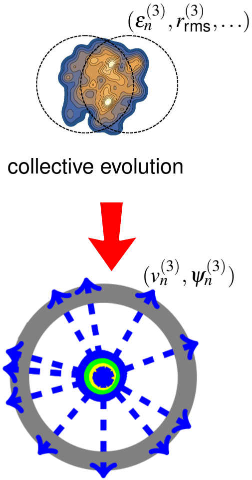
$$\frac{d^2N}{p_T dp_T d\varphi} = N(p_T)\left[1 + \sum_{n=1}^{\infty} 2 {\color{red}v_n} \cos\left[n(\varphi - {\color{blue}\psi_n})\right]\right]$$



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$$p([p_T], v_n, \psi_n, \ldots)$$

$$v_n\{2\} \equiv (\langle v_n^2\rangle)^{1/2}, \qquad v_n\{4\} \equiv \left(-\langle v_n^4\rangle + 2\langle v_n^2\rangle^2\right)^{1/4}, \qquad \cdots$$

[Borghini, Dinh, Ollitrault, PRC, 64, 054901 (2001)]

$$\rho_n = \frac{\langle [p_T] v_n^2\rangle - \langle [p_T]\rangle \langle v_n^2\rangle}{\sigma_{p_T} \sigma_{v_n^2}}$$

[Piotr Bozek, PRC (2016) 93, 044908]

Theoretical models Vs experimental data

Initial state parameters

$N(\sqrt{s_{NN}})$	Overall normalization
p	Entropy deposition parameter
w	Gaussian nucleon width
⋮	

Pre-equilibrium parameters

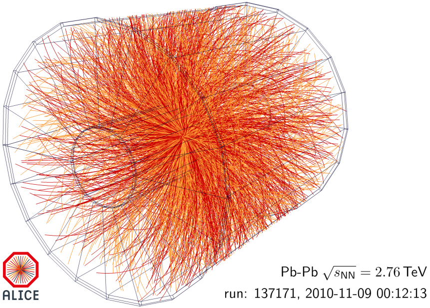
τ_{fs}	Free-streaming time
⋮	

QGP evolution parameters

$\eta/s(T_c)$	Minimum $\eta/s(T)$
$(\eta/s)_{slope}$	Slope of $\eta/s(T)$ above T_c
$(\eta/s)_{curve}$	Curvature of $\eta/s(T)$ above T_c
⋮	

Hadronic gas evolution parameters

⋮



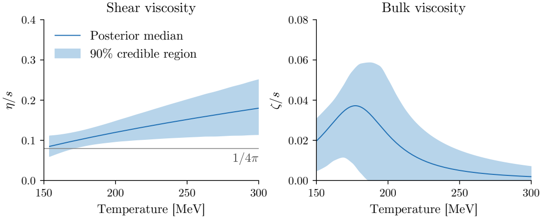
Experimental observables

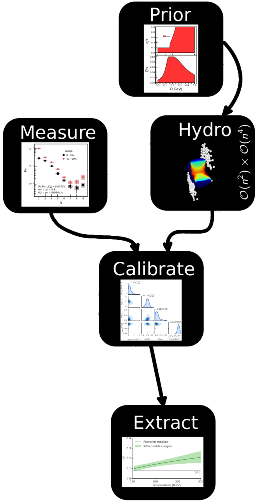
dN/dy	Particle yields, $\pi^{\pm}$, $k^{\pm}$, ...
$\langle [p_T] \rangle$	Mean transverse momentum, $\pi^{\pm}$, $k^{\pm}$, ...
$v_n\{2\}$	Anisotropic flow two-particle correlation
$v_n\{4\}$	Anisotropic flow four-particle correlation
⋮	

Model in the light of experimental data

Bayes's theorem: $P(\textit{Theory}|\textit{Data}) \propto P(\textit{Data}|\textit{Theory})P(\textit{Theory})$

Ref. [1]:





- Theoretical developments: collectivity **[2]**, jet-quenching **[3]**, nucleon substructure **[4]**

[1] Bernhard, PhD Thesis, arXiv: 1804.06469; Bernhard, Moreland, Bass, Nature Phys. 15 (2019) 11, 1113-1117
[2] Auvinen, et al., PRC 102 (2020) 044911, Nijs et al., PRL 126 (2021) 202301, JETSCAPE, PRC 103 (2021) 054904
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[4] Mäntysaari, Schenke, Shen, Zhao, arXiv: 2202.01998

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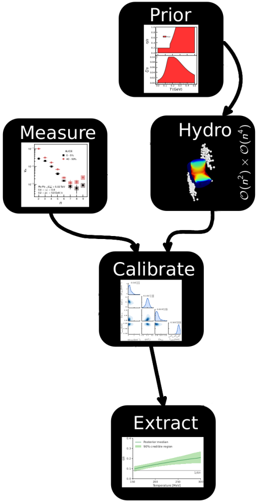

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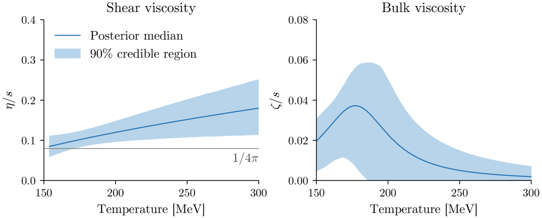

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Initial state, T$_R$ENTo model [Moreland, Bernhard, Bass, PRC 92 (2015), 011901; Moreland, Bernhard, Bass, PRC 101 (2020), 024911]

- Distribute nucleons based on the Woods-Saxon distribution,

$$WS(r,\theta,\phi) = \frac{n_0}{1+e^{[r-R(\theta,\phi)]/a_0}}, \qquad R(\theta,\phi) = R_0\left(1+\beta_2 Y_2^0+\beta_3 Y_3^0+\beta_4 Y_4^0+\cdots\right)$$

- We impose a constraint on the minimum distance that two nucleons can have d_{min}.
- It is assumed that nucleons have a Gaussian shape $\rho_N(\vec{r}) \propto \exp\left[-\frac{r^2}{2w^2}\right]$ with width w.
- Two nucleons *participate* in a collision with probability

$$P_{\text{coll}} = 1-\exp\left[-\sigma_{gg}\int dxdy\int dz\rho_{N_1}(\vec{r})\int dz\rho_{N_1}(\vec{r})\right].$$

  σ_{gg} is fixed by the nucleon nucleon total cross section measurements.
- Adding up participants to make $\rho_{A,B}(\vec{r})$, participant thickness function is defined as $T_{A,B}(x,y) = \int dz\rho_{A,B}(\vec{r})$
- The deposited entropy into the collision region is obtained via

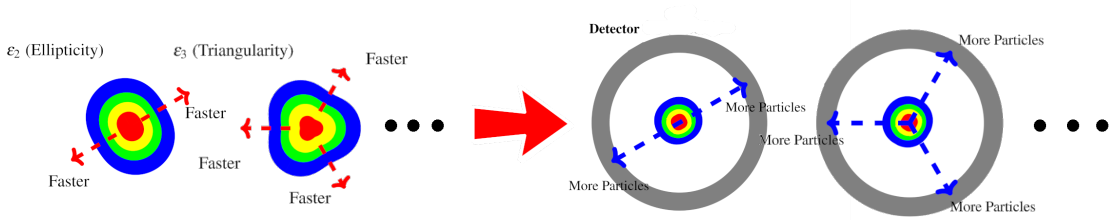
$$T_R(p,T_A,T_B) = \left(\frac{T_A^p+T_B^p}{2}\right)^{1/p}, \qquad \langle N_{\text{ch}}\rangle \propto \int dxdy\, T_R(x,y).$$

p qualitatively controls the mechanism of entropy production during the collision.

The parameters: $w,\ d_{min},\ p,\ R_0,\ a_0,\ \beta_2,\ \beta_3,\ \ldots$

(and some more: σ_{fluc}: subnucleonic structure, ...)

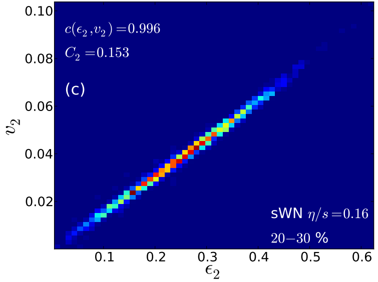
Linear response hydrodynamics



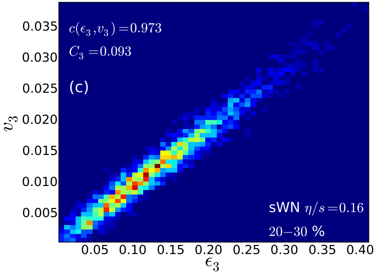
v_2 (Elliptic flow)

v_3 (Triangular flow)

$v_2 \approx k_2 \varepsilon_2$



$v_3 \approx k_3 \varepsilon_3$



So we can estimate

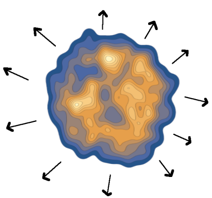
$$v_n\{2\} \approx k_n \varepsilon_n\{2\}$$

for $n = 2, 3$.

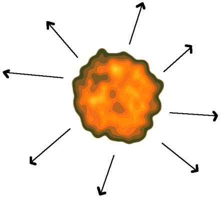
[Niemi, Denicol, Holopainen, Huovinen, PRC 87 (2013) 5, 054901]

Transverse momentum linear response [1,2]

At fixed entropy S



Larger r_{rms}^2, smaller $[p_T]$



Smaller r_{rms}^2, larger $[p_T]$

For an observable O, define event-by-event deviation and variance $\delta O \equiv O - \langle O \rangle, \quad \sigma_O^2 = \langle O^2 \rangle - \langle O \rangle^2$.

Considering the overlap region $A_\perp = \pi r_{rms}^2 \sqrt{1-\varepsilon_2^2}$, one can define a predictor $d_\perp = \sqrt{N_{\text{part}}/A_\perp}$.

Linear approximation for deviation: $\frac{\delta [p_T]}{\langle [p_T] \rangle} \approx k_p \frac{\delta d_T}{\langle d_T \rangle}$

Linear approximation for standard deviation: $\frac{\sigma_{[p_T]}}{\langle [p_T] \rangle} \approx k_0 \frac{\sigma_{d_T}}{\langle d_T \rangle}$

Linear approximation for average? $\langle [p_T] \rangle \approx k_0' \langle d_\perp \rangle$

[1] Broniowski, Chojnacki, Obara PRC, 80 (2009), 051902; Bozek, Broniowski PRC 96 (2017), 014904
[2] Schenke, Shen, Teaney, PRC, 102 (2020), 034905

Isobar Ratios, a good choice for the linear response approximations

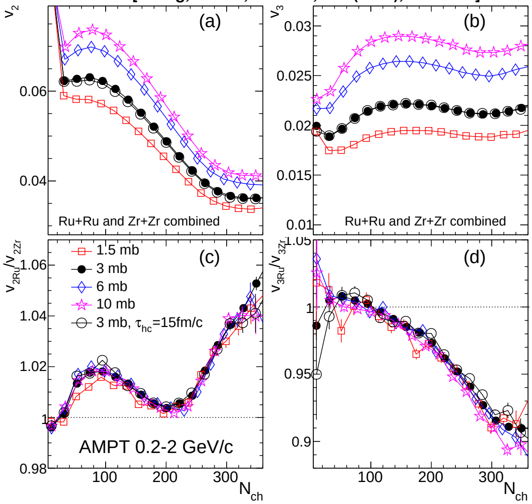
$^{96}_{40}Ru - ^{96}_{40}Ru \quad Vs \quad ^{96}_{44}Zr - ^{96}_{44}Zr$

- The ratio of observables should have very low sensitivity to the collective evolution.

If we are lucky enough:

- the linear response coefficients, $k_2,\ k_3,\ k_0,\ k'_0,$ are constants at least in some range of multiplicity

[Zhang, Bhatta, Jia PRC, 106 (2022), L031901]



$$\frac{v_n\{2\}|_{Ru}}{v_n\{2\}|_{Zr}} \approx \frac{\varepsilon_n\{2\}|_{Ru}}{\varepsilon_n\{2\}|_{Zr}}, \qquad \frac{\sigma_{[p_T]}/\langle[p_T]\rangle|_{Ru}}{\sigma_{[p_T]}/\langle[p_T]\rangle|_{Zr}} \approx \frac{\sigma_{d_\perp}/\langle d_\perp\rangle|_{Ru}}{\sigma_{d_\perp}/\langle d_\perp\rangle|_{Zr}}, \qquad \frac{\langle[p_T]\rangle|_{Ru}}{\langle[p_T]\rangle|_{Zr}} \approx \frac{\langle d_\perp\rangle|_{Ru}}{\langle d_\perp\rangle|_{Zr}}$$

Setup

Scanning the parameter space:

$$w,\ d_{min},\ p,\ R_0,\ a_0,\ \beta_2,\ \beta_3$$

Nuclei size and shape:

	R_0[fm]	a_0[fm]	β_2	β_3
Ru96	5.09	0.46	0.162	0
RuII	5.09	0.46	0.06	0
RuIII	5.09	0.46	0.06	0.2
RuIV	5.09	0.52	0.06	0.2
Zr96	5.02	0.52	0.06	0.2

Initial state internal structure:

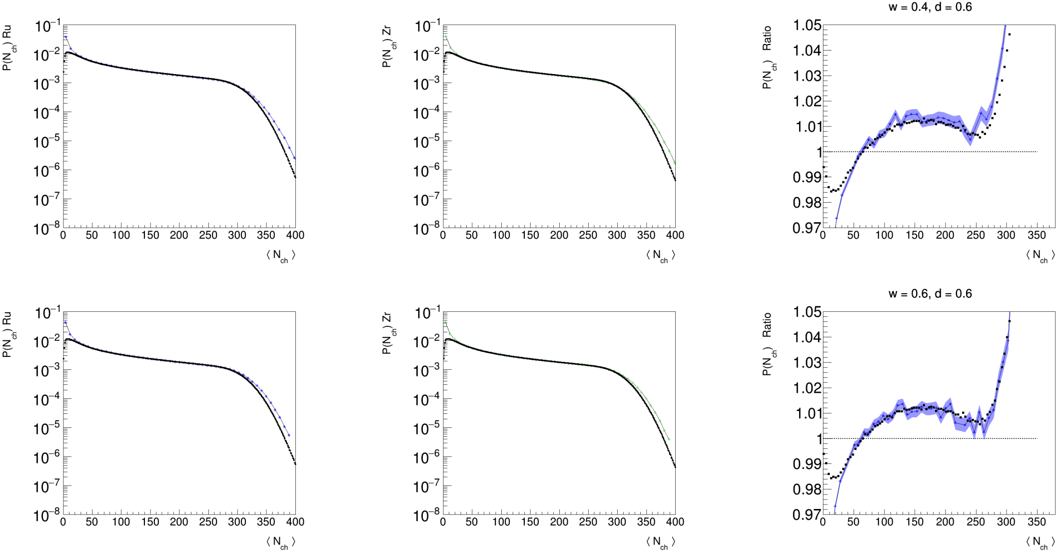
$$(w, d_{\text{min}}, p) \in \{0.2, 0.3, \ldots, 0.9\} \otimes \{0., 0.2 \ldots, 1.0\} \otimes \{-1, 0, 1\}.$$

30M minimum bias events per each point, 240 points in total.

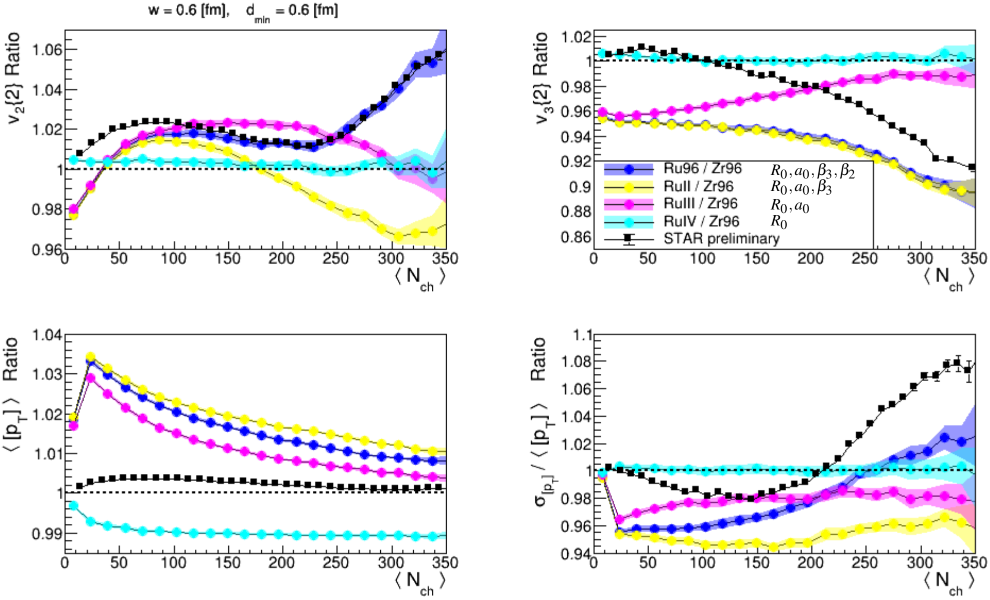
Multiplicity distribution ratio

$\langle N_{\text{ch}} \rangle = \text{N} \int dxdy\, T_R(x,y)$

Finding overall normalization using experimental measurements.

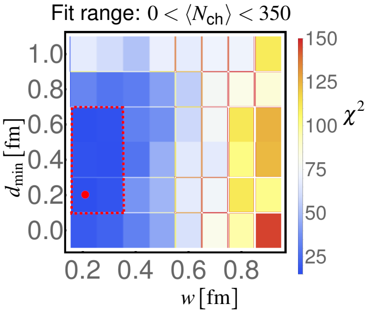


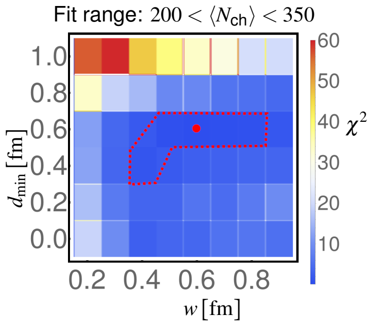
Results:

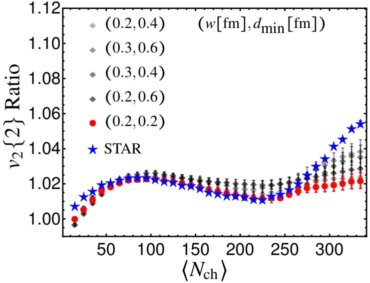


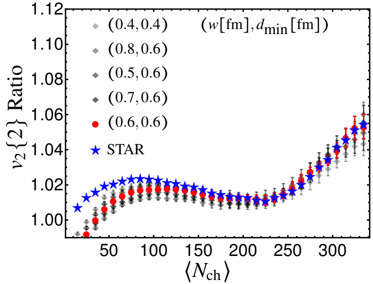
w = 0.6 [fm], d_min = 0.6 [fm]
v_2{2} Ratio
v_3{2} Ratio
⟨[p_T]⟩ Ratio
σ_[p_T] / ⟨[p_T]⟩ Ratio
⟨N_ch⟩
Ru96 / Zr96 $R_0, a_0, \beta_3, \beta_2$
RuII / Zr96 R_0, a_0, β_3
RuIII / Zr96 R_0, a_0
RuIV / Zr96 R_0
STAR preliminary

Best fit with χ^2, only $v_2\{2\}$

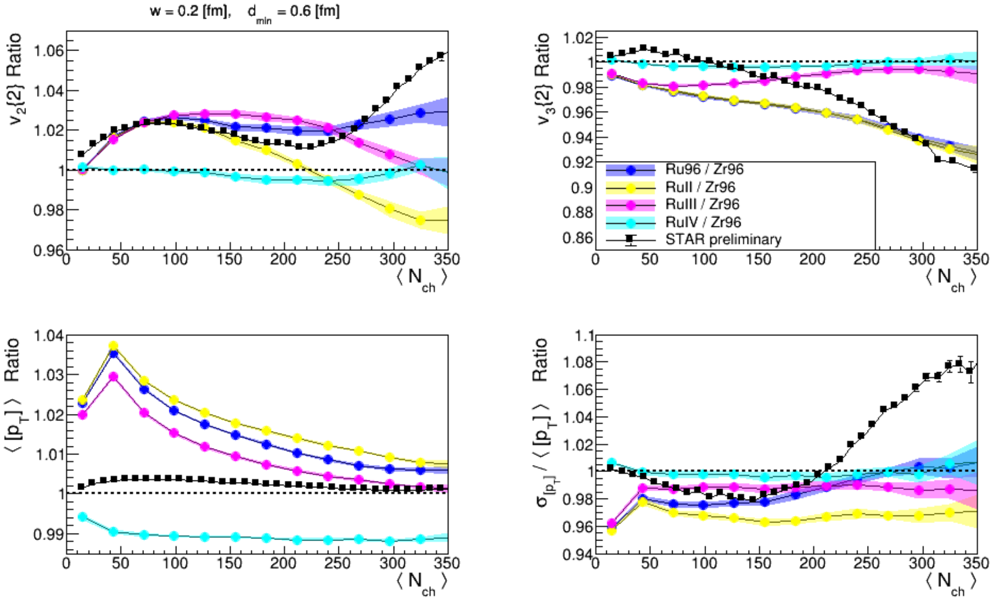








Ratio sensitivity to w and ρ,



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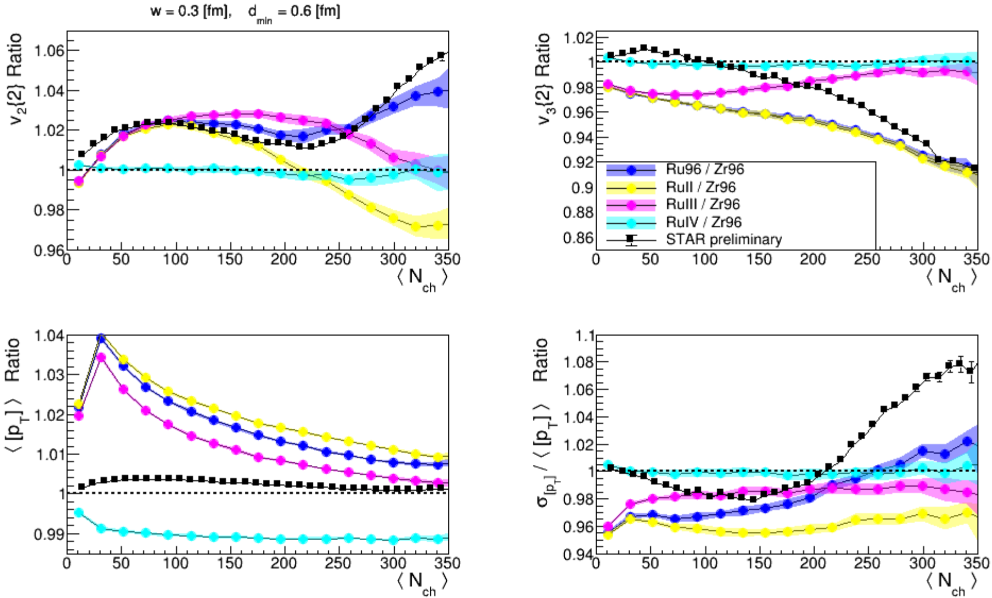

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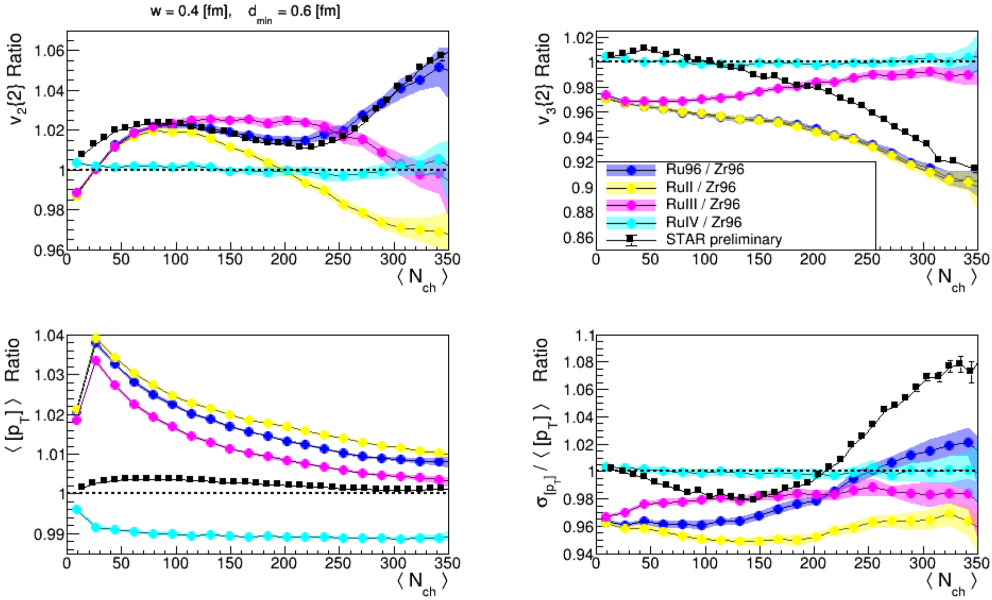

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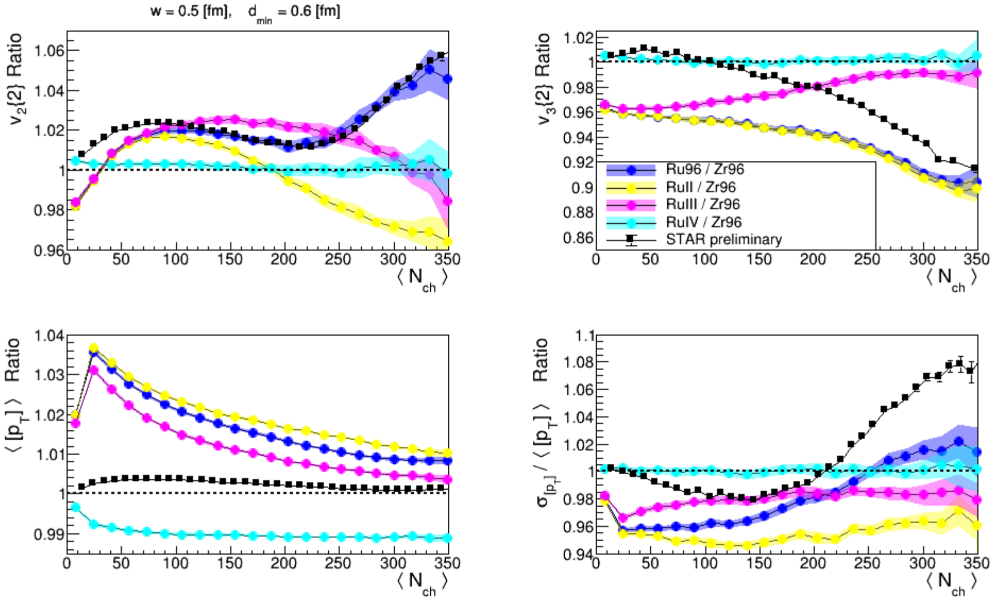

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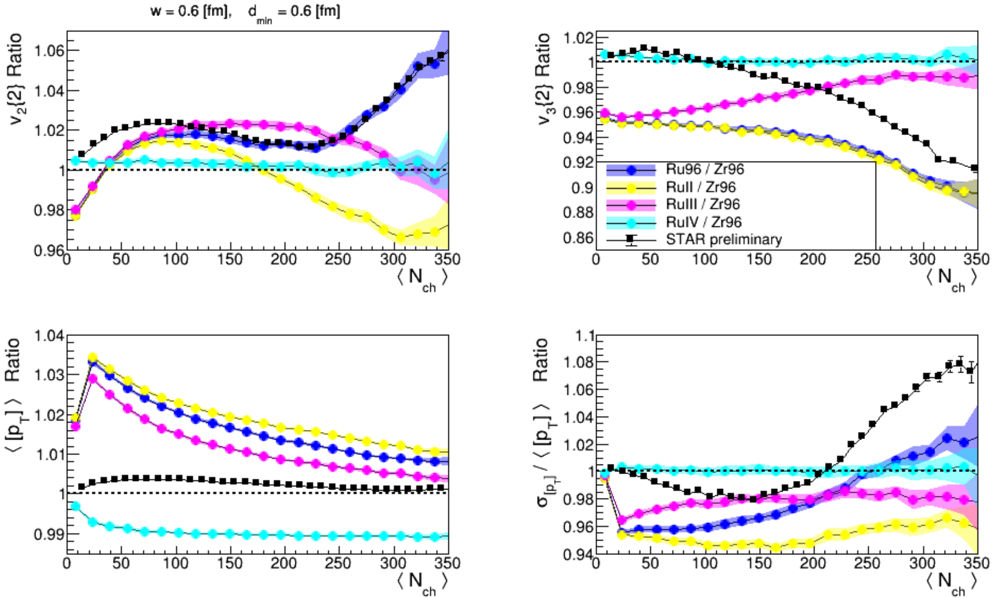

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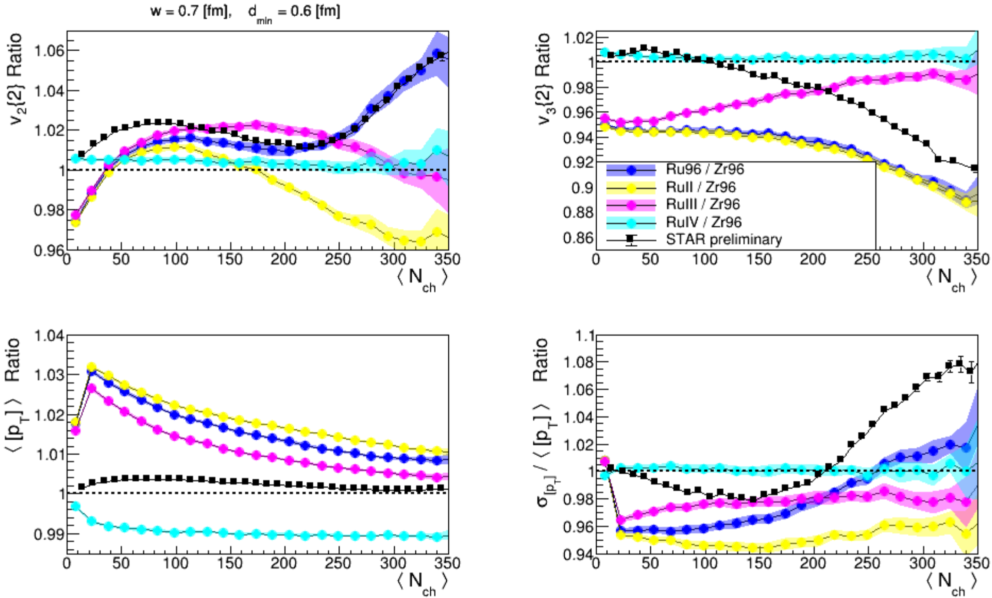
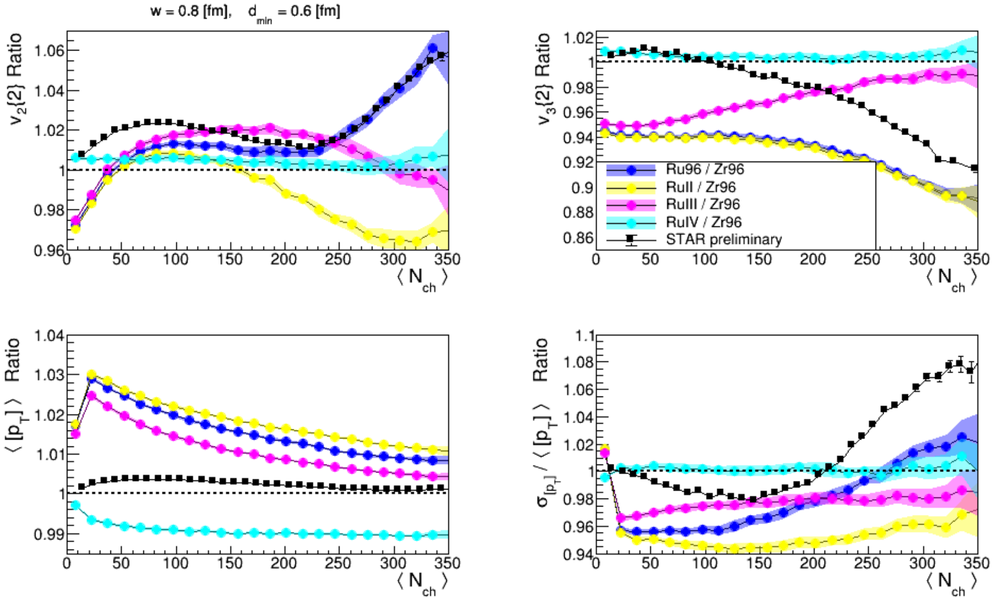


Ratio sensitivity to w and ρ,



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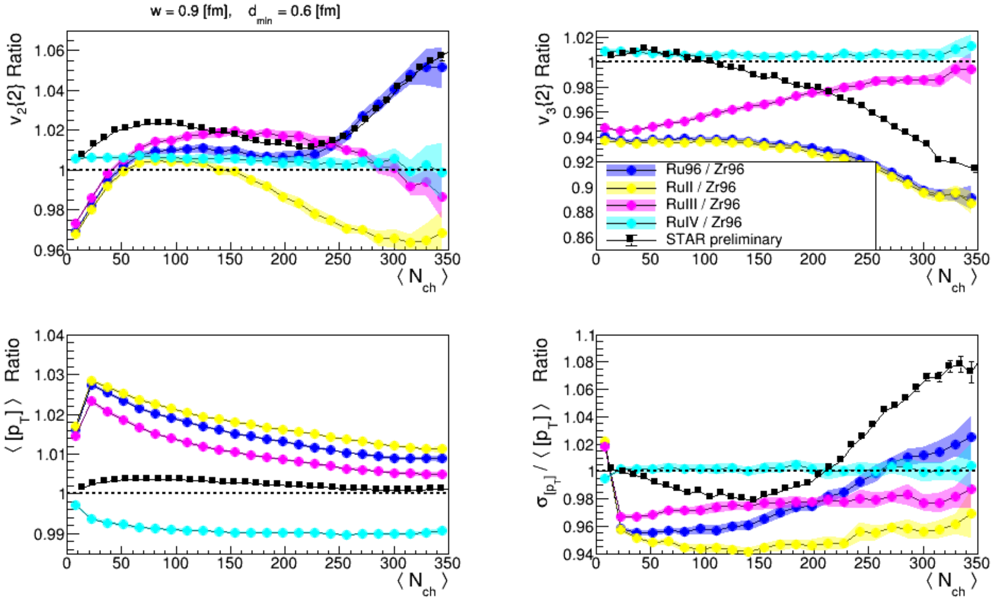

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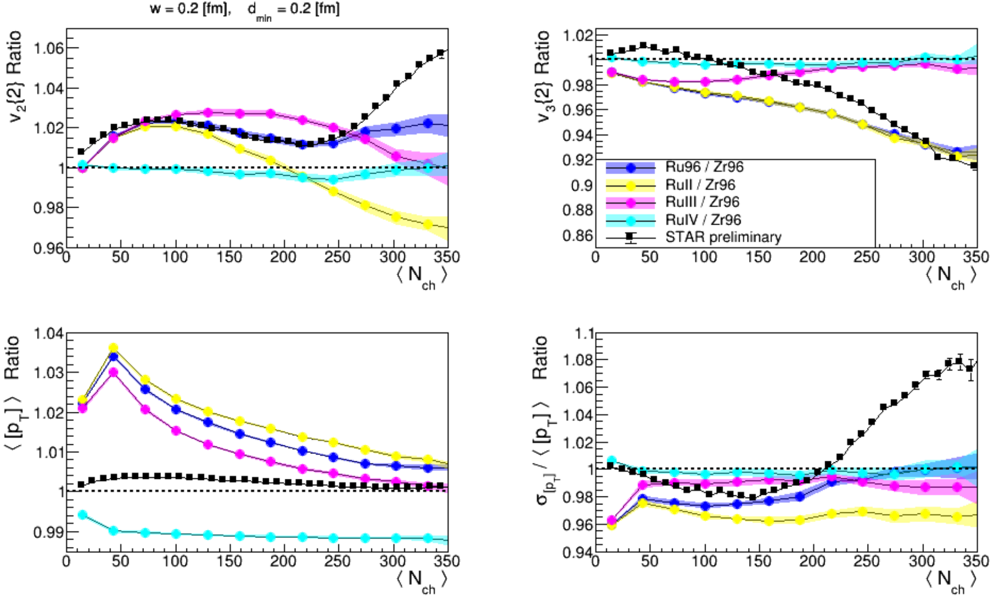

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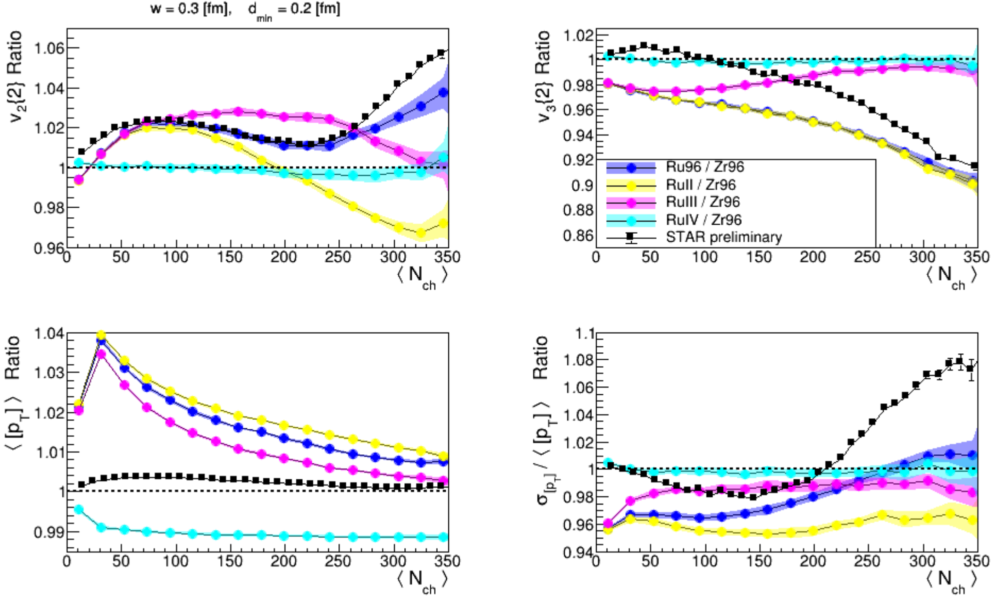

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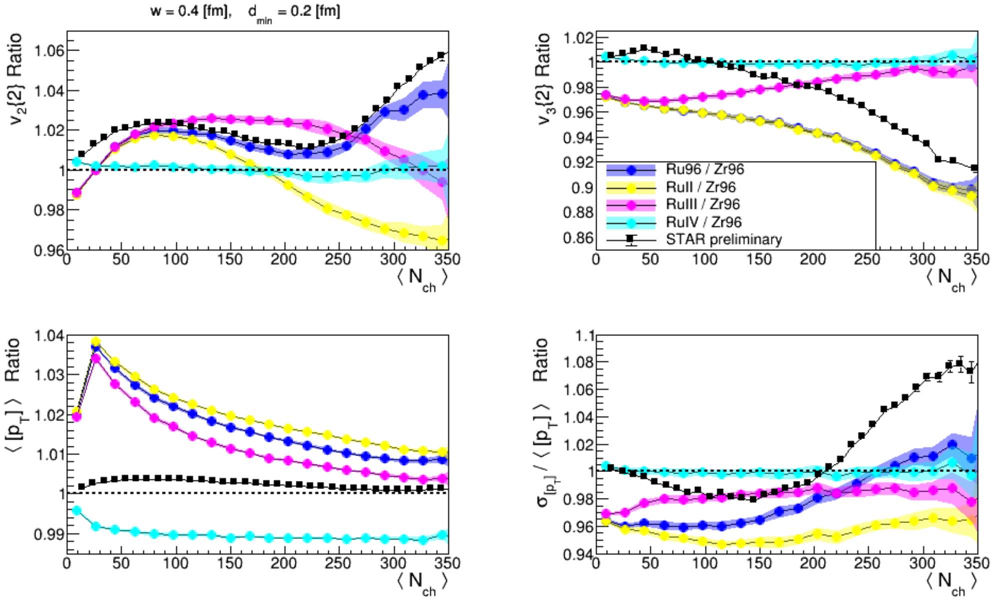

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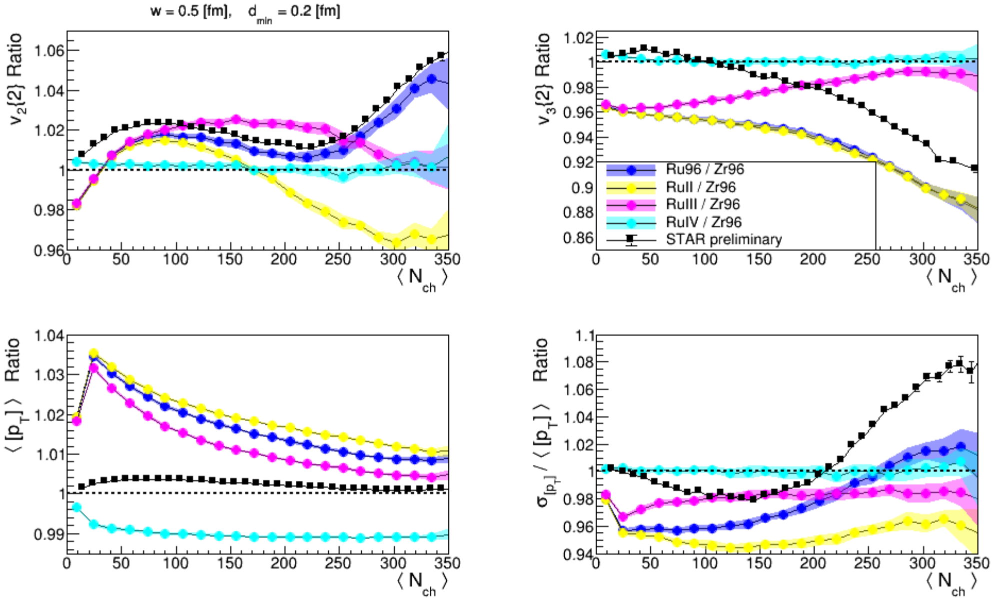

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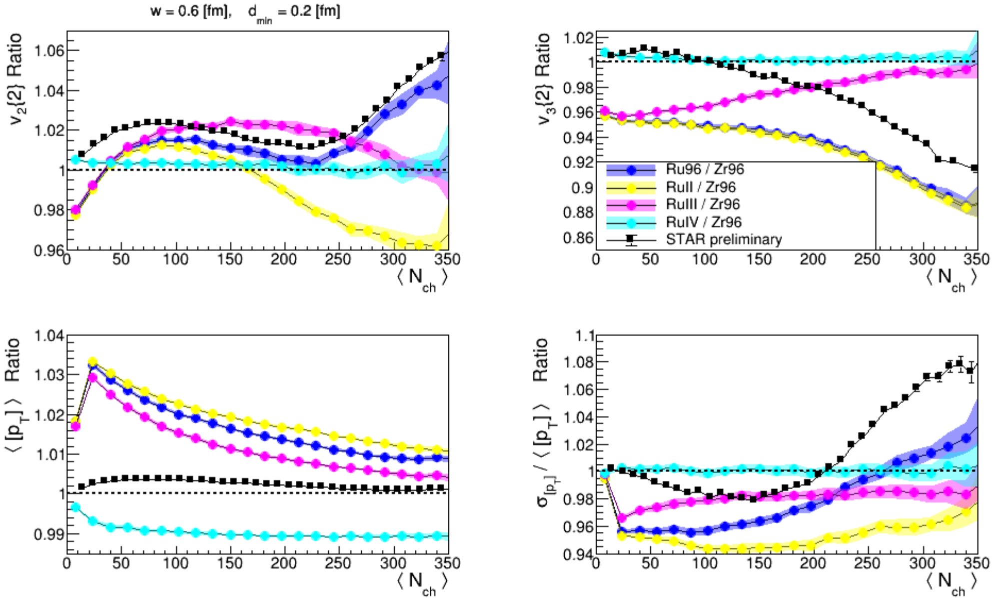

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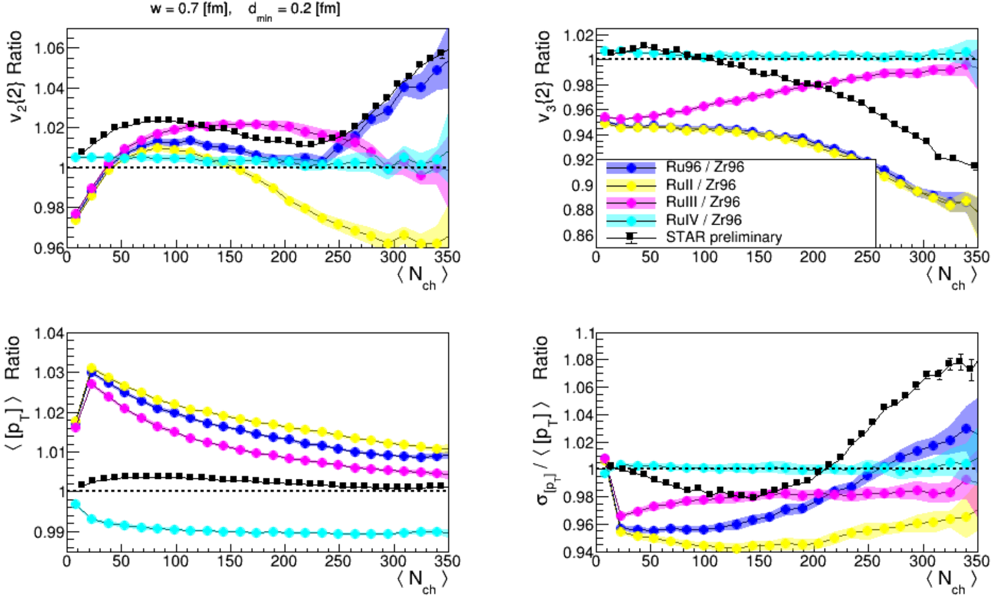

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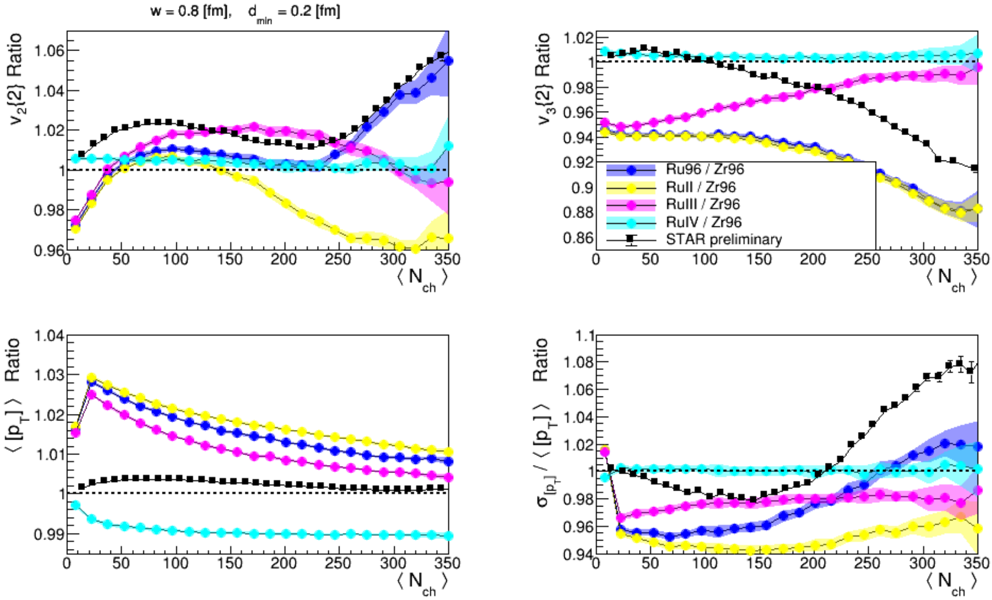

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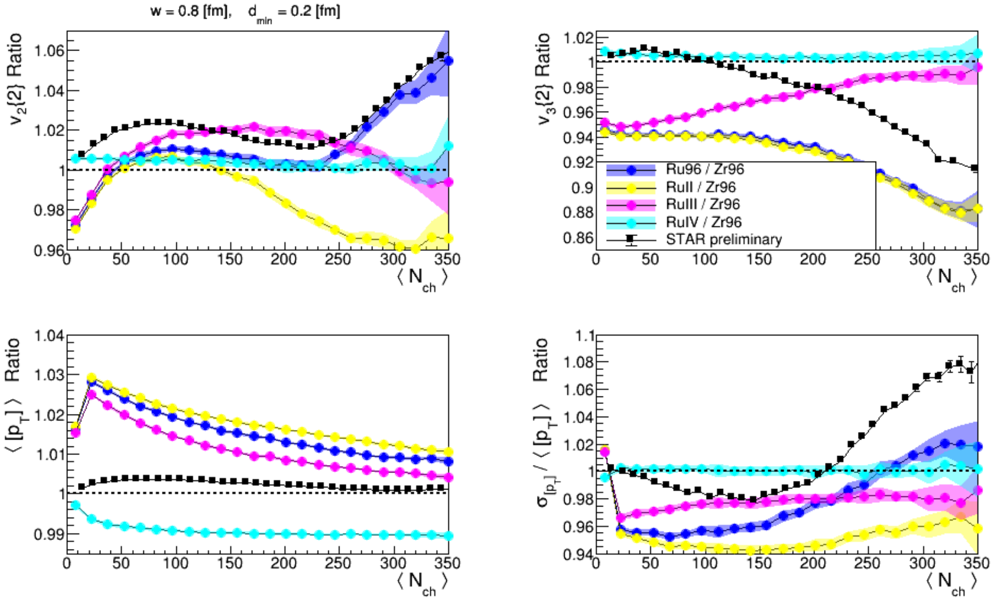

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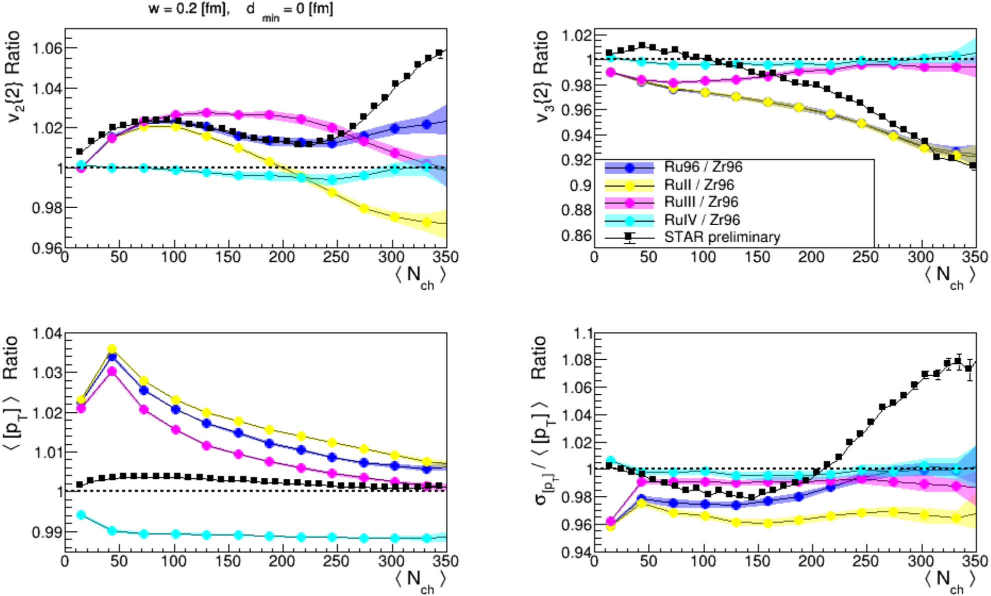

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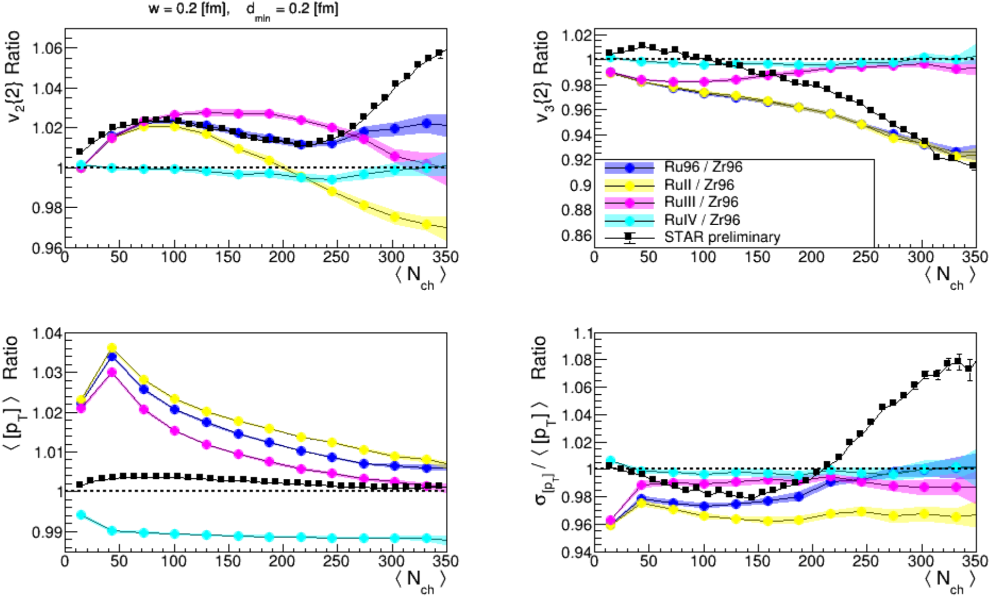

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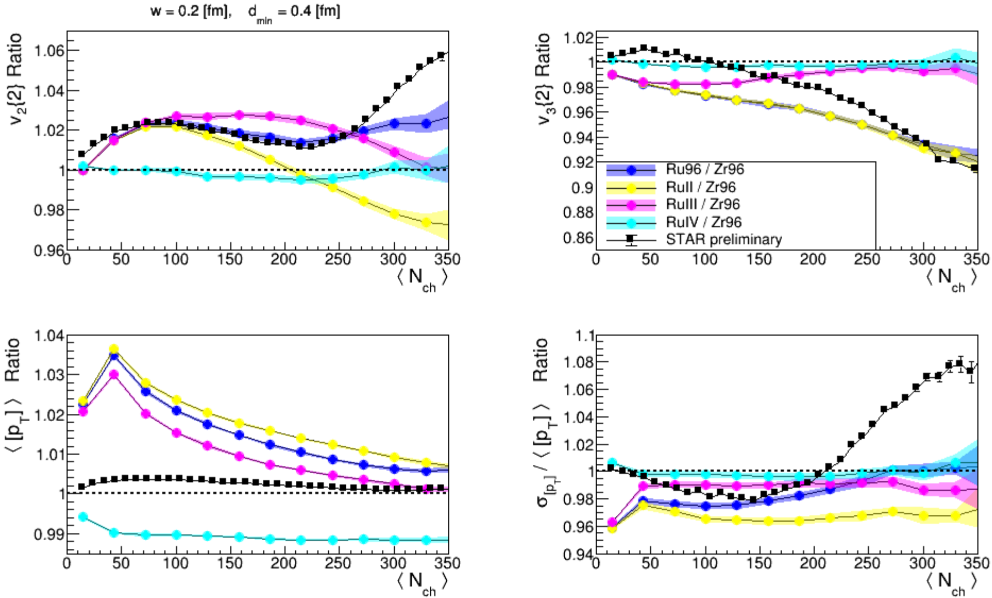

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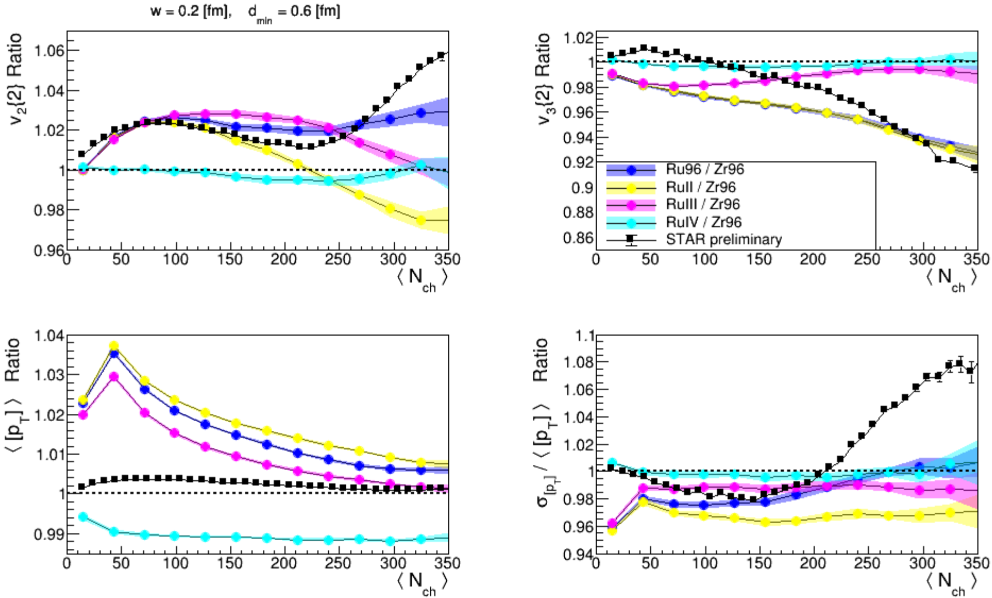

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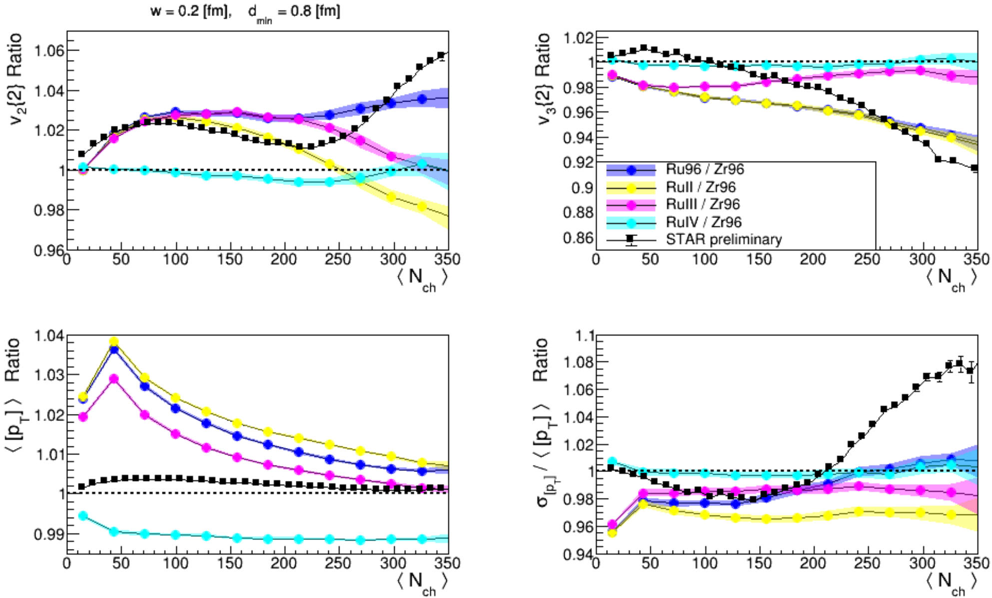

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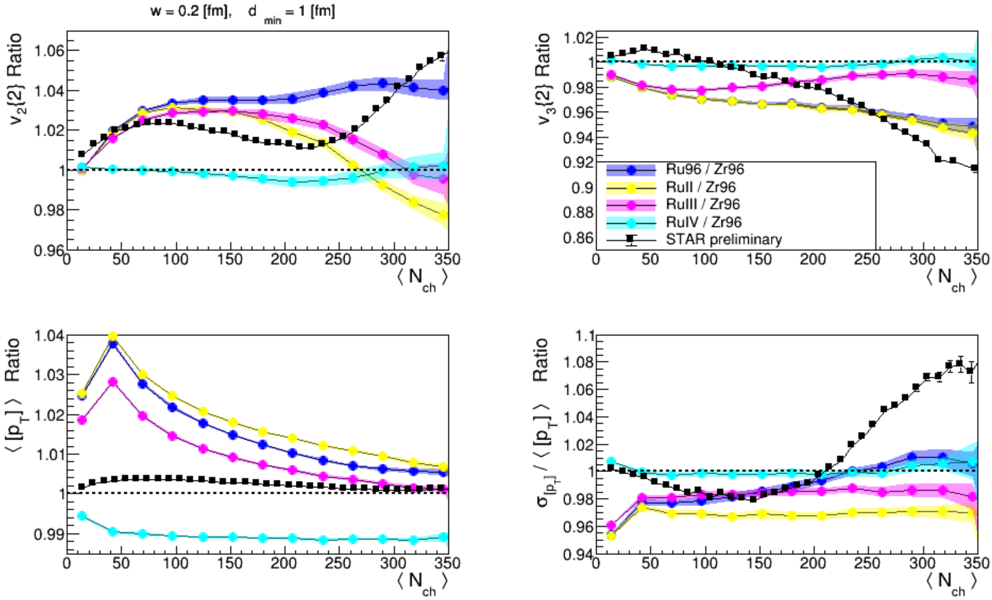

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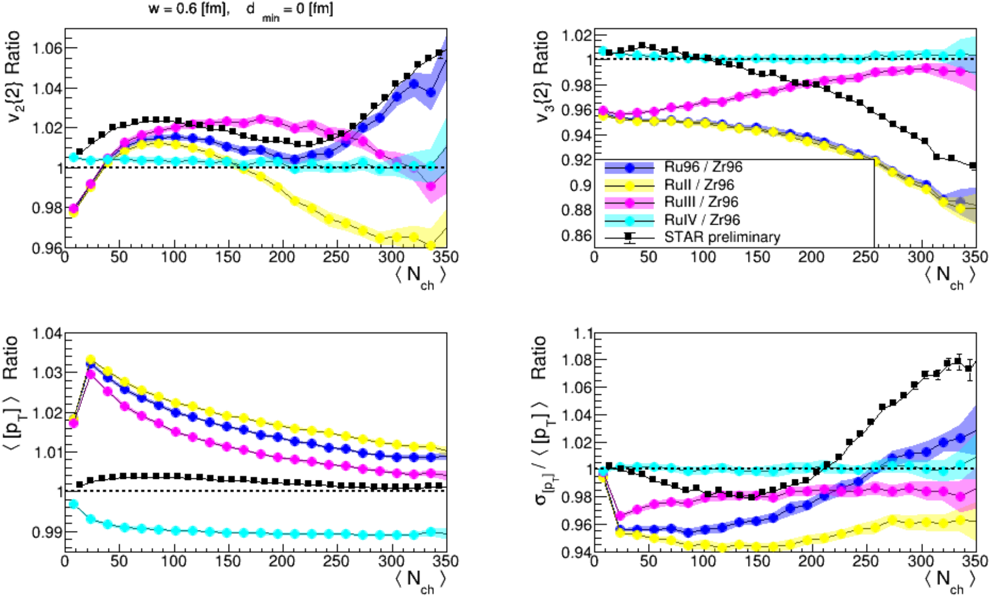

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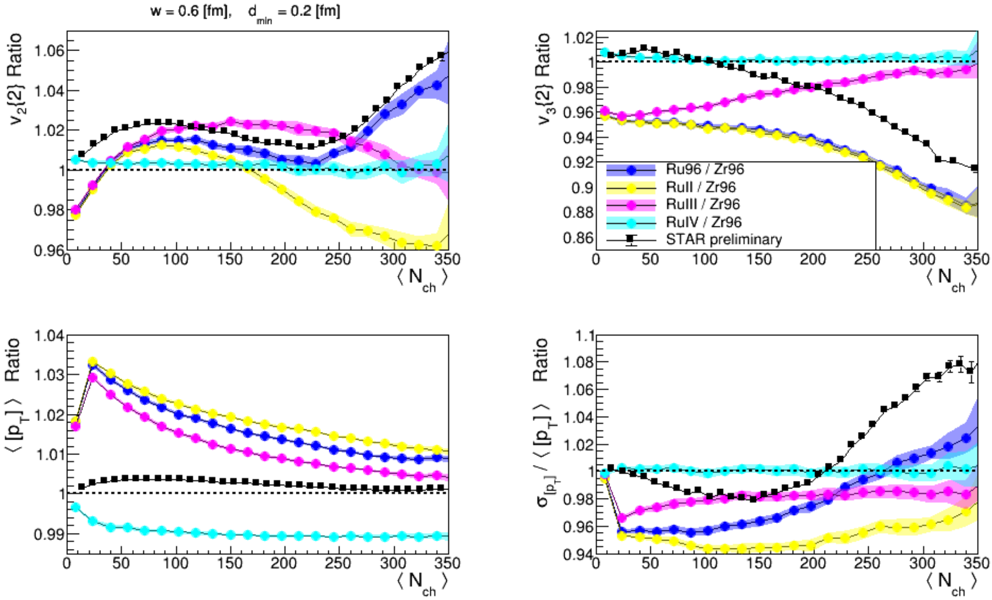

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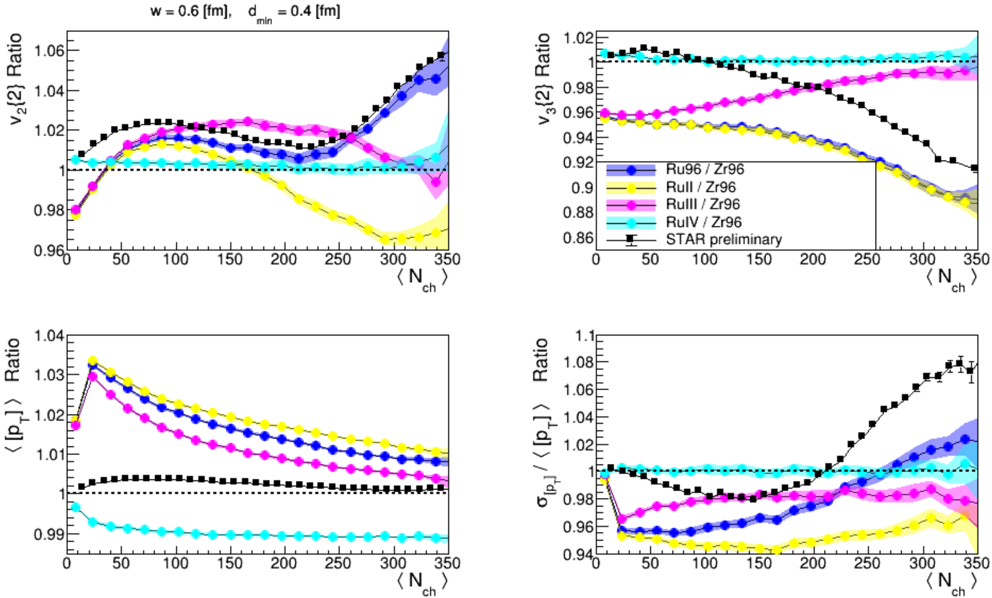

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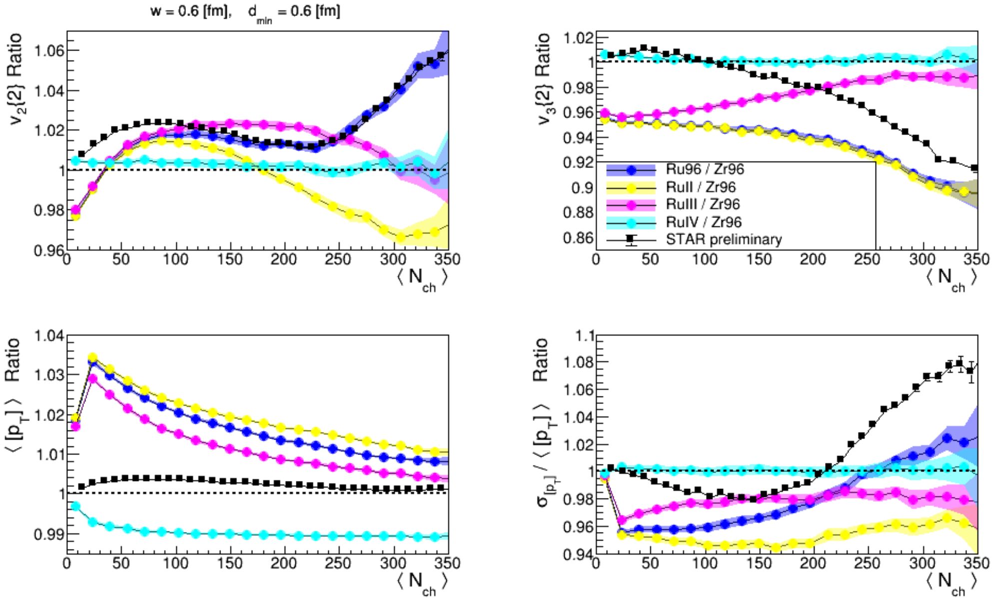

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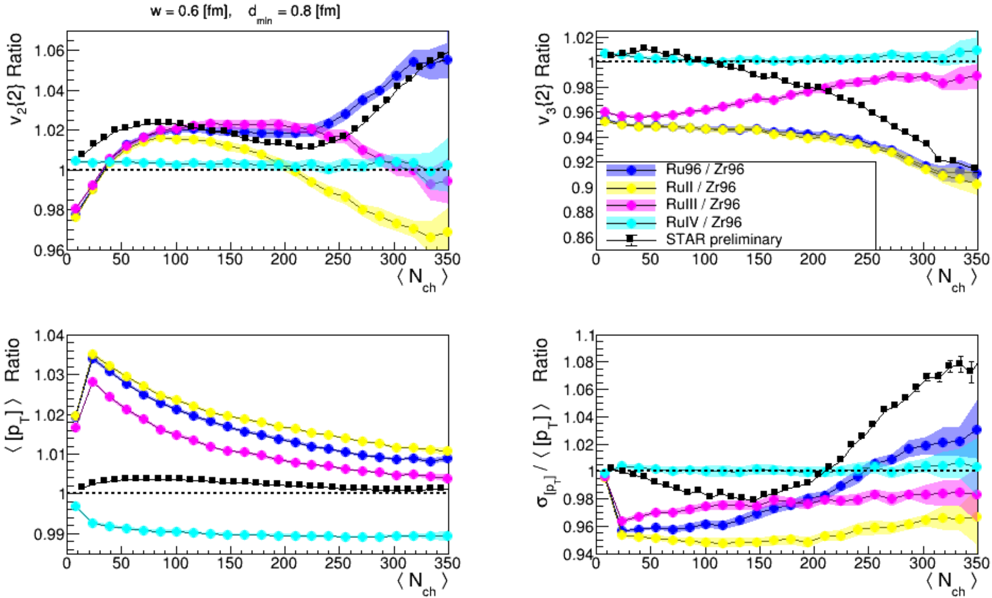

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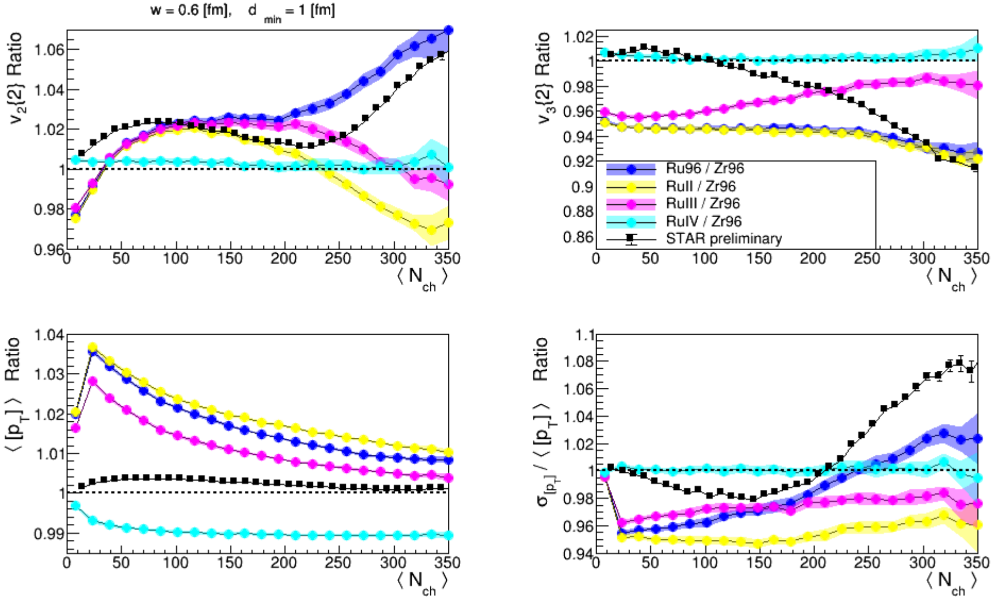

Conclusion / Summary / Outlook

$v_2\{2\}$ *isobar ratio is sensitivity to* w, d_{min} *initial state parameters.*

- The best fit point for high multiplicity events:

$$w = 0.6\,[\text{fm}], \qquad d_{\text{min}} = 0.6\,[\text{fm}]$$

- Nucleon minimum distance: $0.4 \lesssim d_{\text{min}} \lesssim 0.6\,[\text{fm}]$
- Nucleon width: $0.4 \lesssim w \lesssim 0.8\,[\text{fm}]$
- Nucleon size and nucleon minimum distance from other studies:

	w [fm]	d_{min} [fm]	Ref(s).
Bayesian analysis	~ 0.8 to ~ 1.0	~ 0.5 to ~ 1.5	[1]
$\rho_n([p_T], v_n^2)$	~ 0.5	-	[2]
nucleus-nucleus cross-section	~ 0.7	-	[3]

- The effect of other parameters need to be explored.
- Hydrodynamic calculation increase the prediction power by including lower multiplicity events.

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Thank You!

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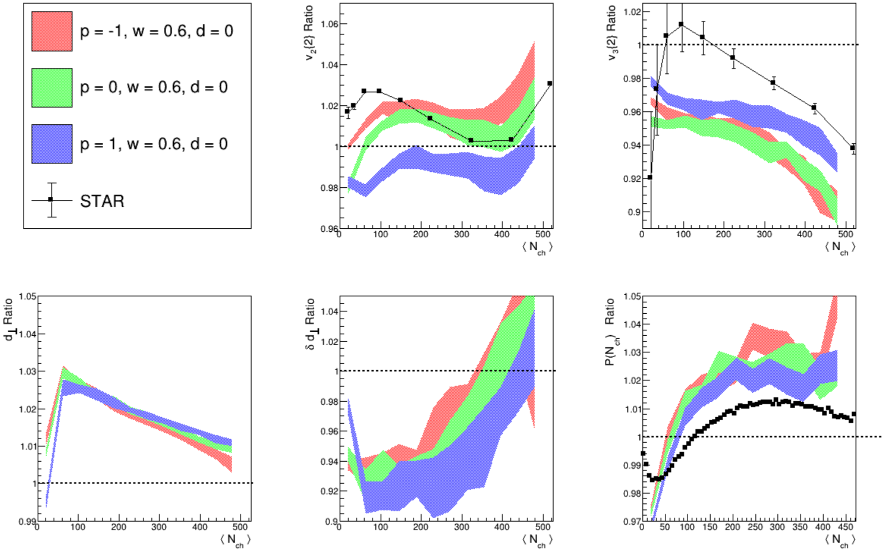
Add p dependent



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