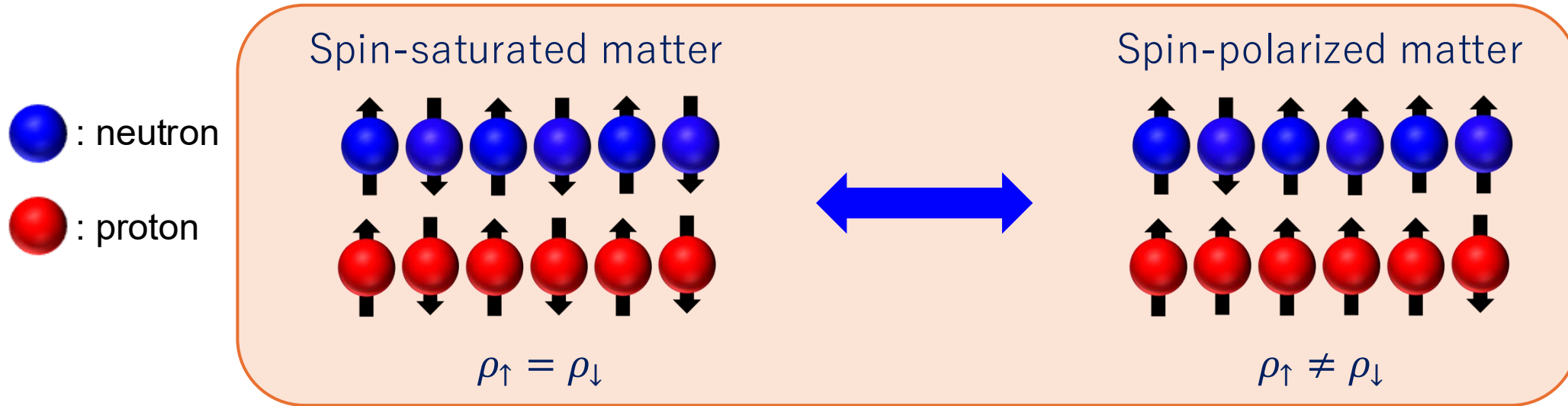


A New “Skin Thickness” in High-spin Isomers as a Probe for EOS of Spin-polarized Matter

Toi Tachibana, Department of Physics, Kyoto University

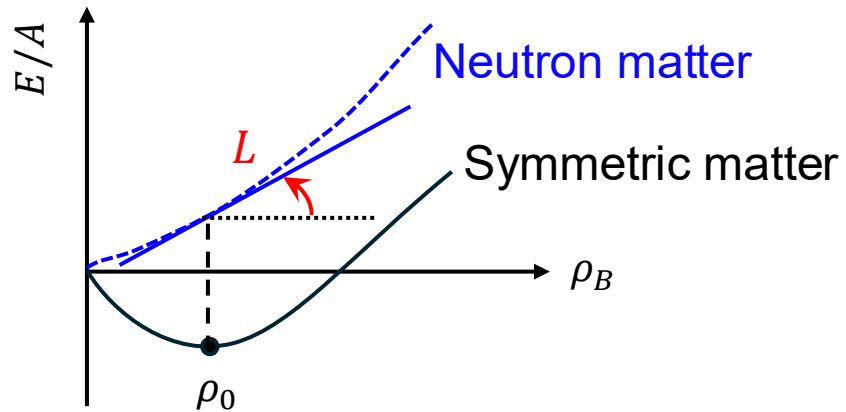
with K. Hagino (Kyoto Univ.), K. Yoshida (RCNP), Q. Zhao (CIAE)



- The ferromagnetic phase in high-density region ▶ Origin of the magnetic field of **magnetar**
I. Vidana, I. Bombaci, Phys. Rev. C 66, 045801 (2002)
T. Maruyama, T. Tatsumi, Nucl. Phys. A 693 (2001) 710-730
- Strong magnetic field in the remnant of neutron star merger ▶ Possible formation of **spin-polarized matter**
B. D. Metzger, et al., Astrophys. J. Lett. 856, 101 (2018)
- Changes in particle fraction due to spin polarization ▶ Affecting **neutron star cooling**.
N. H. Khoa, N. H. Tan, D. T. Khoa, Phys. Rev. C 105, 065802 (2022)

▶ EOS for spin-polarized matter has been increasingly important.

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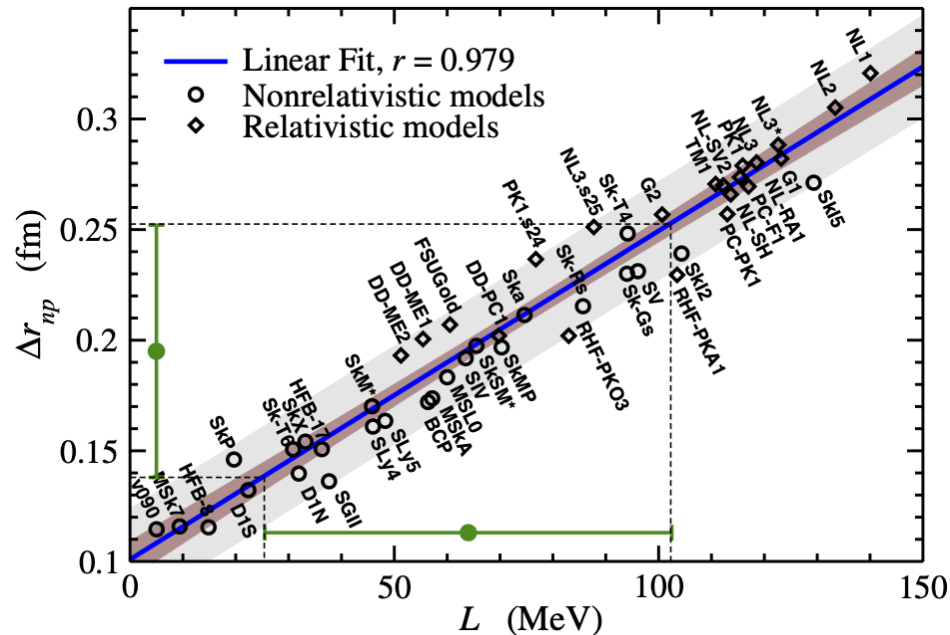


A diagram of a blue sphere representing a ^{208}Pb nucleus. To the left of the sphere, a vertical dashed blue line with a horizontal double-headed arrow indicates the radius of the nucleus.

$$N = 126$$

D. Adhikari, et al.,
Phys. Rev. Lett. 126,
172502 (2021)

J. Zenihiro, et al.,
Phys. Rev. C 82,
044611 (2010)



Neutron skin thickness is an experimental probe to constrain the EOS for asymmetric nuclear matter.

Almost no analogous probes for constraining the EOS for **spin-polarized matter** has been considered.

Our goals

- Proposing a method to constrain the EOS for spin-polarized matter by terrestrial nuclear experiments.

This Study

- Calculation of the EOS for spin-polarized matter in the relativistic model, which can naturally treat the spin d.o.f. of nucleons
- Exploring a **new skin “thickness”** correlated with **the spin slope parameter**

Slope parameter

$$\frac{E}{A}(\rho, \delta) \cong \frac{E}{A}(\rho, \delta = 0) + \underline{S(\rho)}\delta^2$$

Symmetry energy

$$L = 3\rho_0 \left. \frac{\partial S(\rho)}{\partial \rho} \right|_{\rho=\rho_0}$$

Spin slope parameter

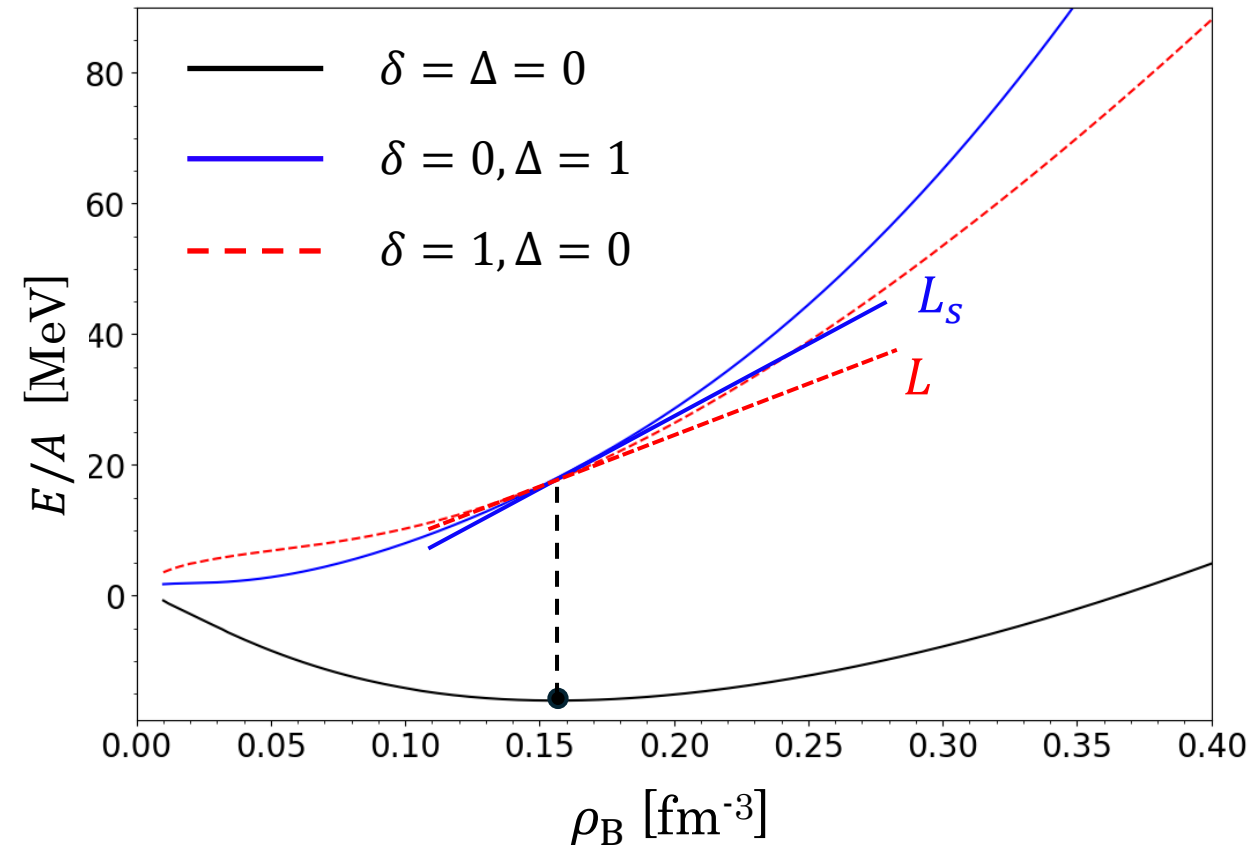
$$\frac{E}{A}(\rho, \Delta, \delta) \cong \frac{E}{A}(\rho, \Delta = 0, \delta) + \underline{W(\rho, \delta)}\Delta^2$$

Spin symmetry energy

$$L_s = 3\rho_0 \left. \frac{\partial W(\rho)}{\partial \rho} \right|_{\rho=\rho_0}$$

$$\delta \equiv (\rho_n - \rho_p)/\rho_B, \quad \Delta_\tau \simeq (\rho_{\uparrow,\tau} - \rho_{\downarrow,\tau})/\rho_\tau$$

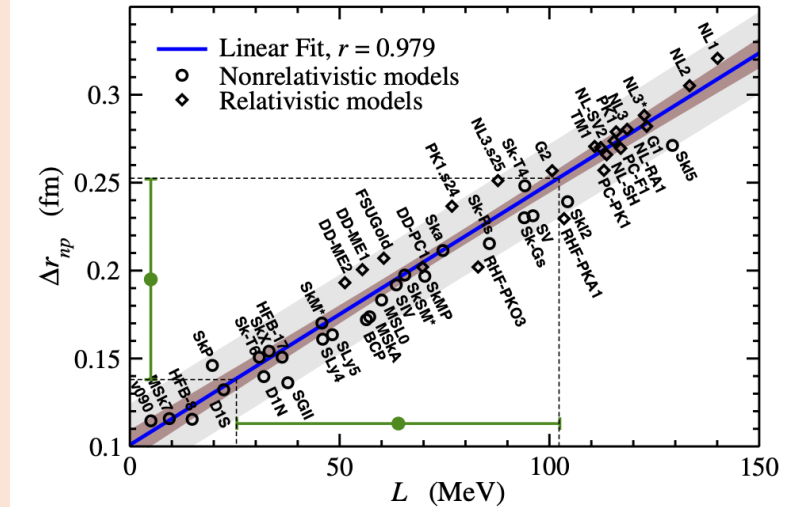
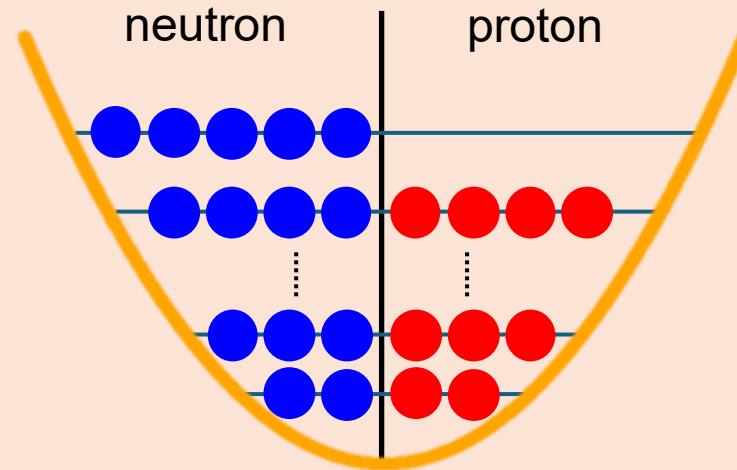
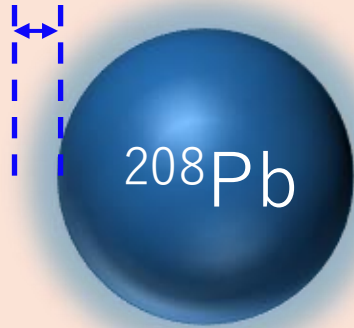
$$\Delta \equiv |\Delta_n| = |\Delta_p|$$



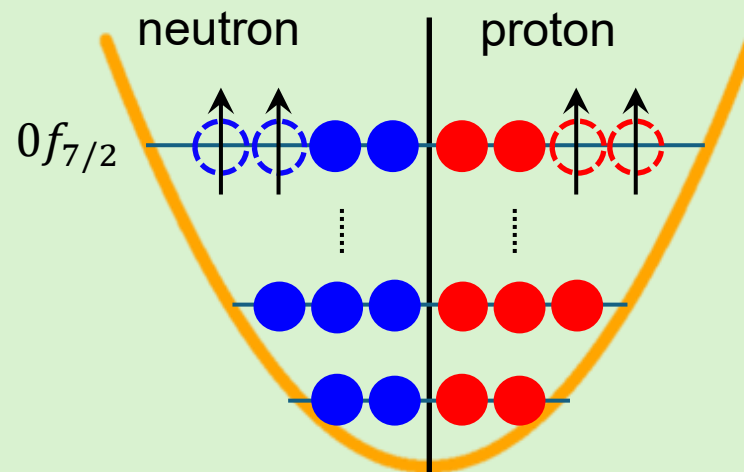
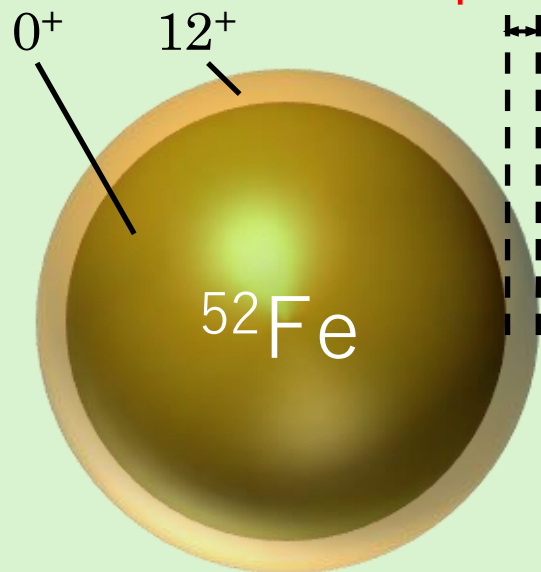
Possible New “Skin Thickness” in High-spin Isomers

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Neutron skin thickness

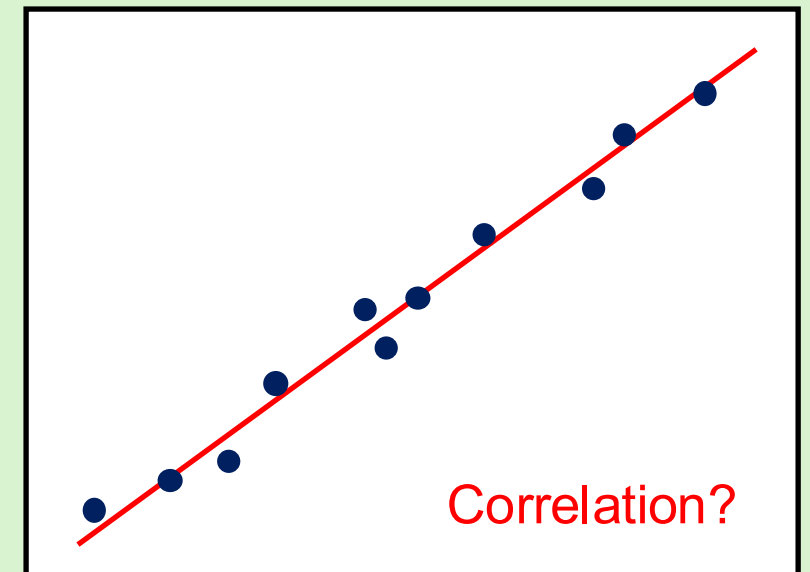


Spin skin thickness?

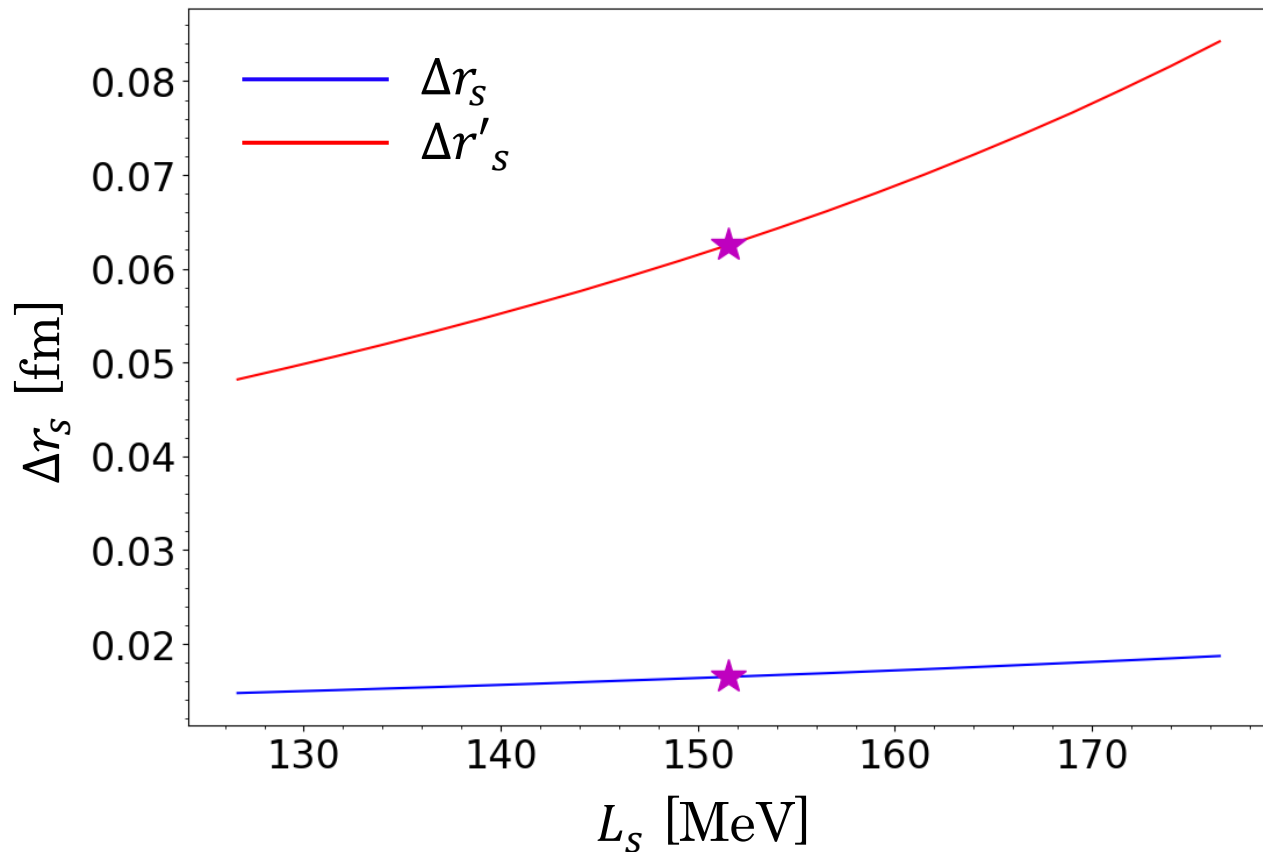


Spin skin thickness

Spin slope parameter



L_s : spin slope parameter



Two definitions of the "spin skin thickness"

1. $\Delta r_s = (\text{rad. of } 12^+ \text{ isomer}) - (\text{rad. of } 0^+ \text{ g. s.})$

2. $\Delta r'_s = r_+ - r_-$

$r_{\pm}$: rms radius of $\rho_{\pm} = \langle \psi^\dagger (1 \pm \Sigma_3) \psi \rangle / 2$,
the spin-up (-down) nucleon density.

$\Delta r'_s$ is determined only by the properties of 12^+ state of ^{52}Fe .

Δr_s : positive but weak correlation

cf. $\Delta R_{\text{exp}} \approx 0.003 \text{ fm}$

A. R. Vernon, et al., PRL 134 252501 (2025)

$\Delta r'_s$: positive and stronger correlation

However, how to observe?

Back Up

$$\begin{aligned}\mathcal{L}_{\text{int}} = & -\frac{1}{2}\alpha_S(\bar{\psi}\psi)^2 - \frac{1}{2}\alpha_{tS}(\bar{\psi}\vec{\tau}\psi)^2 - \frac{1}{2}\alpha_V(\bar{\psi}\gamma^\mu\psi)^2 - \frac{1}{2}\alpha_{tV}(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 - \frac{1}{2}\alpha_{PS}(\bar{\psi}\gamma_5\psi)^2 - \frac{1}{2}\alpha_{tPS}(\bar{\psi}\gamma_5\vec{\tau}\psi)^2 \\ & - \frac{1}{2}\alpha_{PV}(\bar{\psi}\gamma_5\gamma^\mu\psi)^2 - \frac{1}{2}\alpha_{tPV}(\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2 - \frac{1}{2}\alpha_T(\bar{\psi}\sigma^{\mu\nu}\psi)^2 - \frac{1}{2}\alpha_{tT}(\bar{\psi}\sigma^{\mu\nu}\vec{\tau}\psi)^2\end{aligned}$$

Mean Field approx.

$$\psi = \sum_{\mathbf{p},s,t} [u(\mathbf{p},s,t)a_{\mathbf{p},s,t}e^{-i\mathbf{p}\cdot\mathbf{x}} + \cancel{v(\mathbf{p},s,t)b_{\mathbf{p},s,t}^\dagger e^{i\mathbf{p}\cdot\mathbf{x}}}]\quad (\text{no sea approximation})$$

$$\begin{aligned}\langle [\bar{\psi}(\mathcal{O}\Gamma)_i\psi]^2 \rangle &= \sum_{\alpha,\beta} \int_{V_{p_F}} dp^3 dq^3 [\bar{u}(\mathbf{p},\alpha)(\mathcal{O}\Gamma)_i u(\mathbf{p},\alpha) \times \bar{u}(\mathbf{q},\beta)(\mathcal{O}\Gamma)_i u(\mathbf{q},\beta) \quad \mathcal{O} \in \{1, \tau_3\} \\ &\quad - \bar{u}(\mathbf{p},\alpha)(\mathcal{O}\Gamma)_i u(\mathbf{q},\beta) \times \bar{u}(\mathbf{q},\beta)(\mathcal{O}\Gamma)_i u(\mathbf{p},\alpha)] \quad \Gamma \in \{1, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}\} \\ &\quad \text{Fierz transf.} \quad \blacktriangleright \Lambda_{ij} \bar{u}(\mathbf{p},\alpha)(\mathcal{O}\Gamma)_j u(\mathbf{p},\alpha) \times \bar{u}(\mathbf{q},\beta)(\mathcal{O}\Gamma)_j u(\mathbf{q},\beta) \\ &= (\delta_{ij} - \Lambda_{ij}) \left[\sum_{\alpha} \int_{V_{p_F}} dp^3 \bar{u}(\mathbf{p},\alpha)(\mathcal{O}\Gamma)_j u(\mathbf{p},\alpha) \right]^2\end{aligned}$$

Total Energy Density

$$\mathcal{E} = \langle \mathcal{H} \rangle = \mathcal{E}_{\text{kin}} + \frac{1}{2} \alpha_S \rho_S^2 + \frac{1}{2} \alpha_{ts} \rho_{ts}^2 + \frac{1}{2} \alpha_V \rho_V^2 + \frac{1}{2} \alpha_{tV} \rho_{tV}^2 - \frac{1}{2} \alpha_{PV} \rho_{PV}^2 - \frac{1}{2} \alpha_{tPV} \rho_{tPV}^2 + \alpha_T \rho_T^2 + \alpha_{tT} \rho_{tT}^2$$

$$\rho_S = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} \bar{u}(p, s, t) u(p, s, t) , \quad \rho_V = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p, s, t) u(p, s, t)$$

$$\rho_{PV} = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p, s, t) \gamma^0 \gamma^3 \gamma_5 u(p, s, t) = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p, s, t) \Sigma_3 u(p, s, t) ,$$

$$\Sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

$$\rho_T = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p, s, t) \gamma_0 \sigma^{12} u(p, s, t) = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p, s, t) \gamma_0 \Sigma_3 u(p, s, t) ,$$

$u(p, s, t)$: positive-energy plane wave spinor

Numerical Results (EOS)

EOS for symmetric nuclear matter (using PCF-PK1)

Q. Zhao, et. al., Phys. Rev. C 106, 034315 (2022)

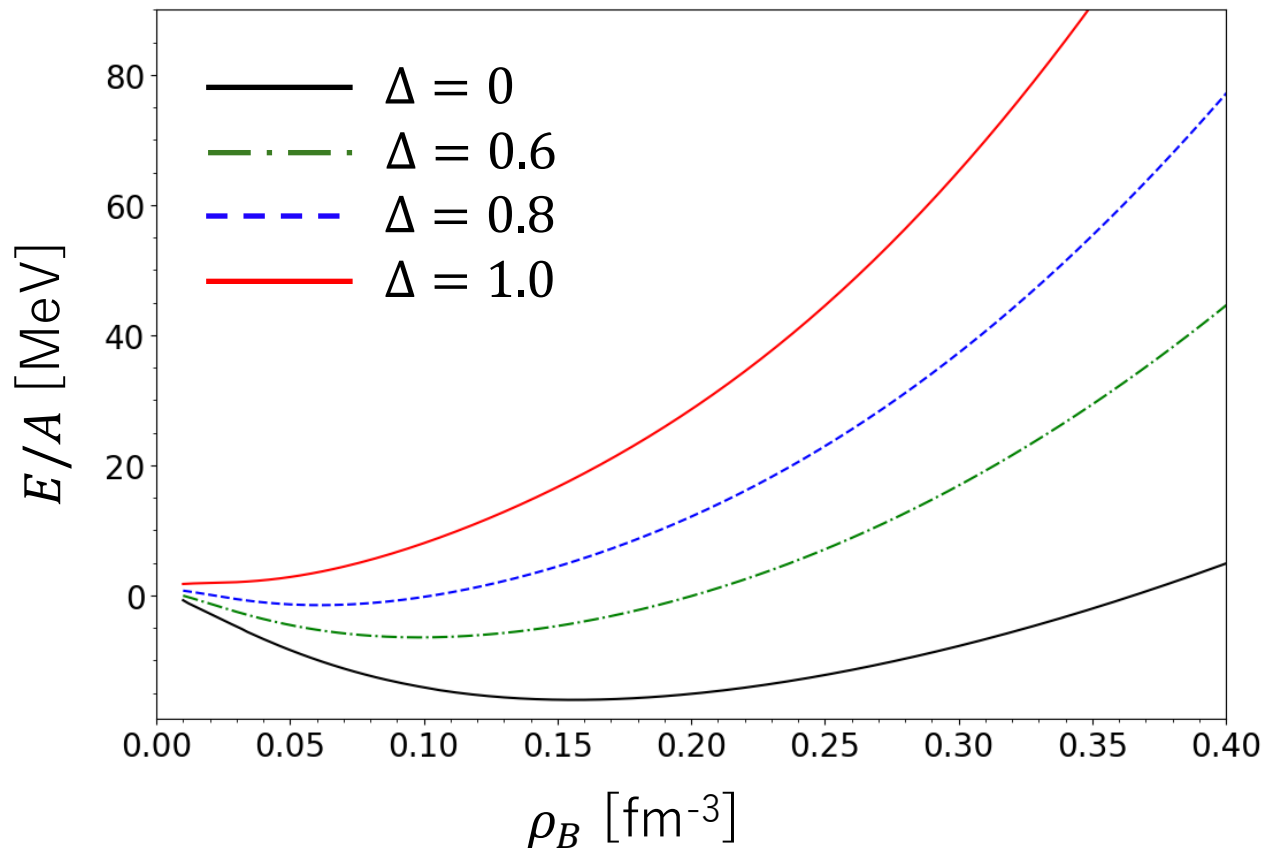
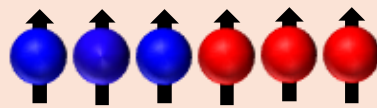
$$\delta \equiv (\rho_n - \rho_p)/\rho_B, \quad \Delta_t \simeq (\rho_{\uparrow,t} - \rho_{\downarrow,t})/\rho_t$$

For simplicity, we assume $|\Delta_n| = |\Delta_p|$

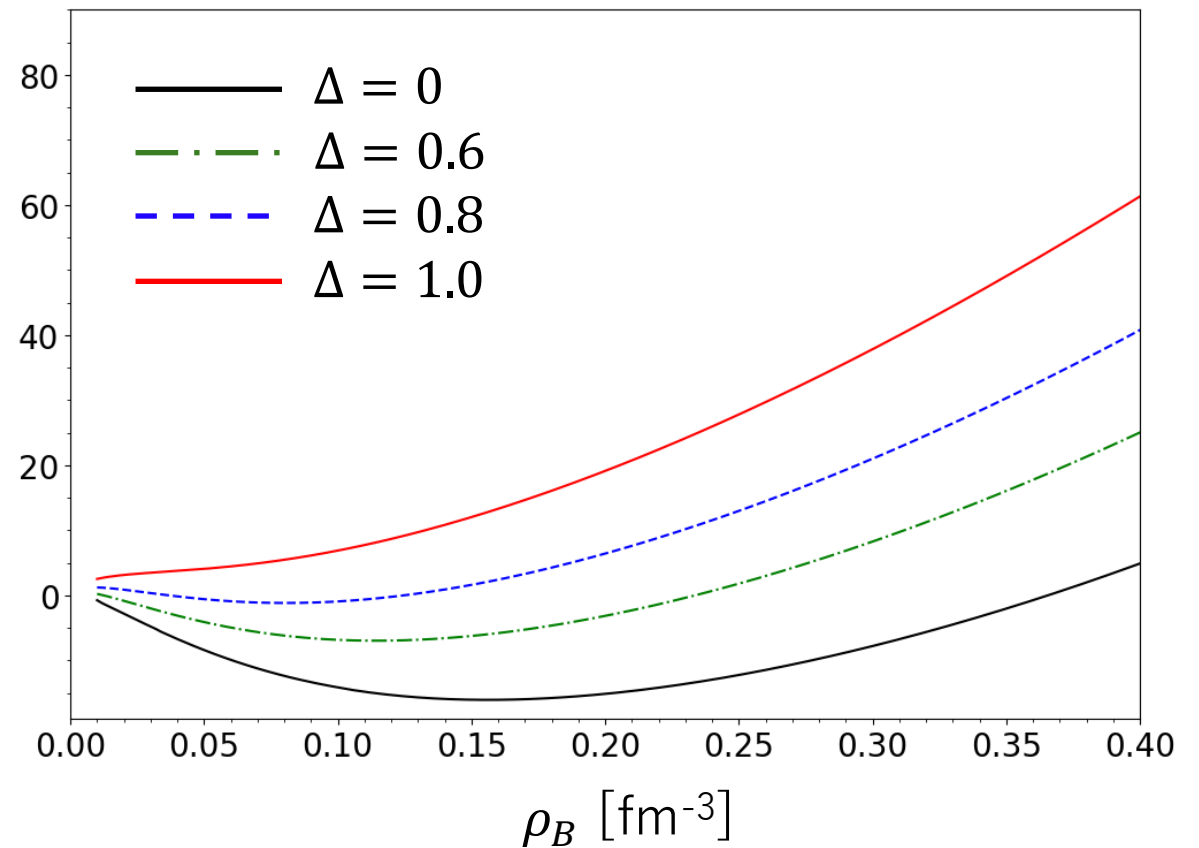
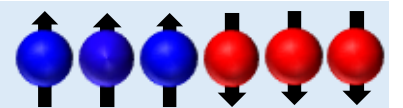


$$E(\rho_{\uparrow,n}, \rho_{\downarrow,n}, \rho_{\uparrow,p}, \rho_{\downarrow,p}) = E(\rho_B, \delta, \Delta)$$

IS(Isoscalar): $\Delta_n = \Delta_p = \Delta$



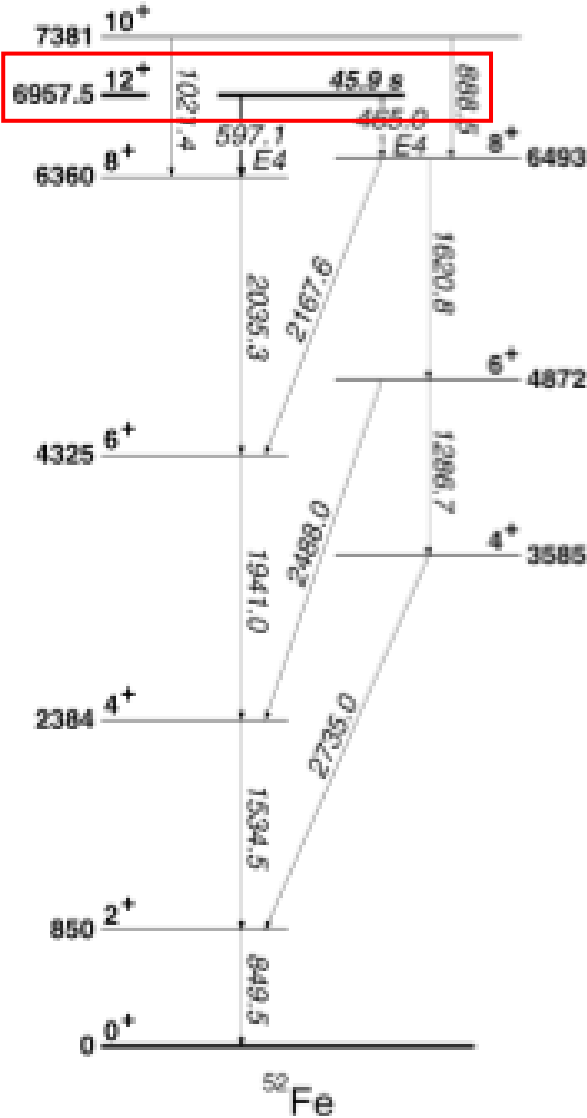
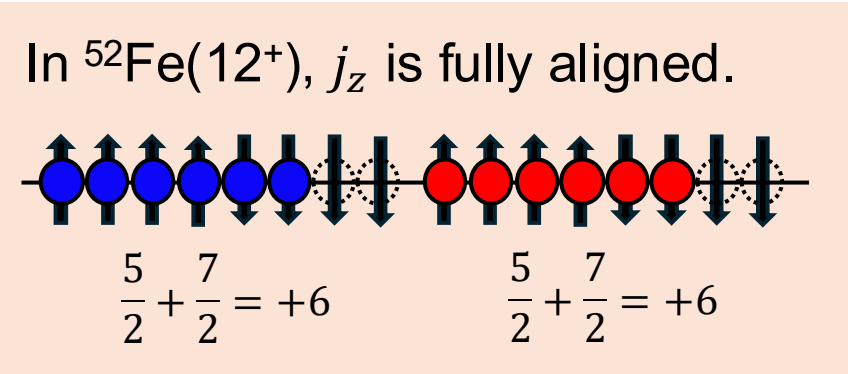
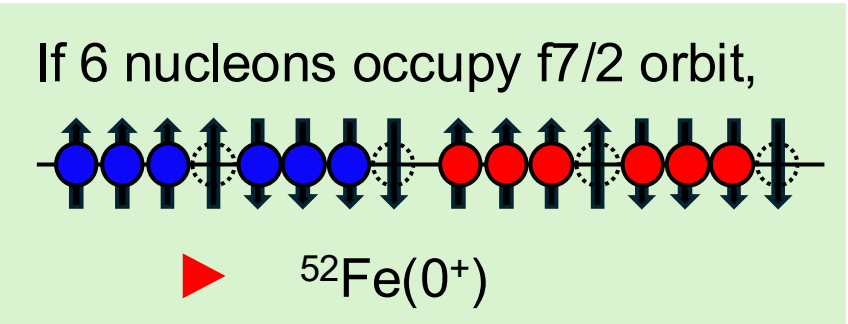
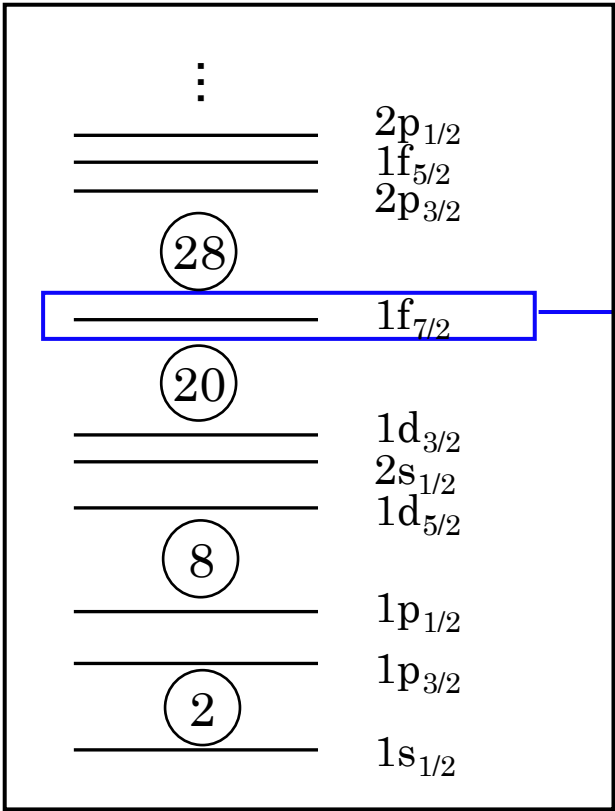
IV(Isovector): $\Delta_n = -\Delta_p = \Delta$



Nuclear Isomer and 12⁺ Excited State of ⁵²Fe (*N* = *Z* = 26)

Nuclear isomer : excited state with a long lifetime

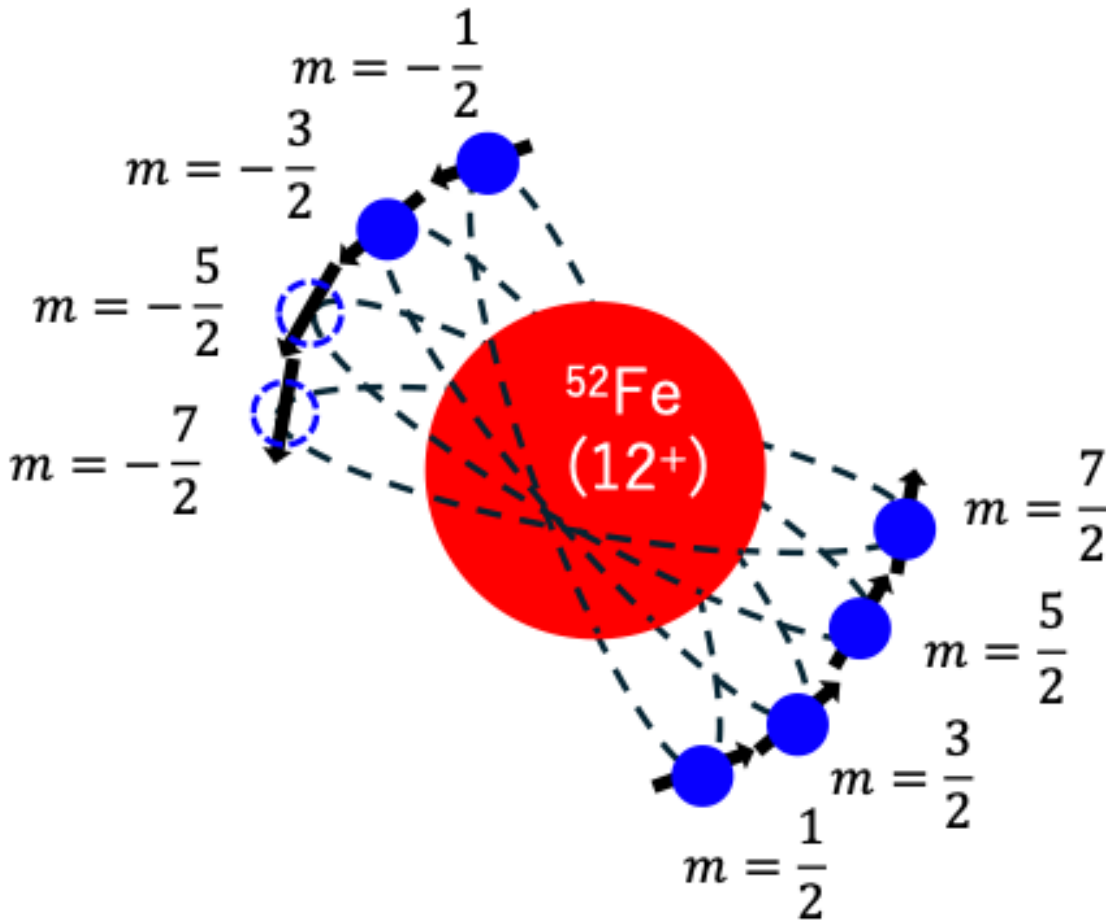
Lifetime of 12⁺ excited state of ⁵²Fe is 45.9s. ► Isomer with high spin



Calculation of 0^+ g.s. and 12^+ isomer of ^{52}Fe

0^+ ground state : spherical HFB calculation with separable pairing interaction

12^+ isomer state : HF calculation for ^{52}Fe with the fixed configuration



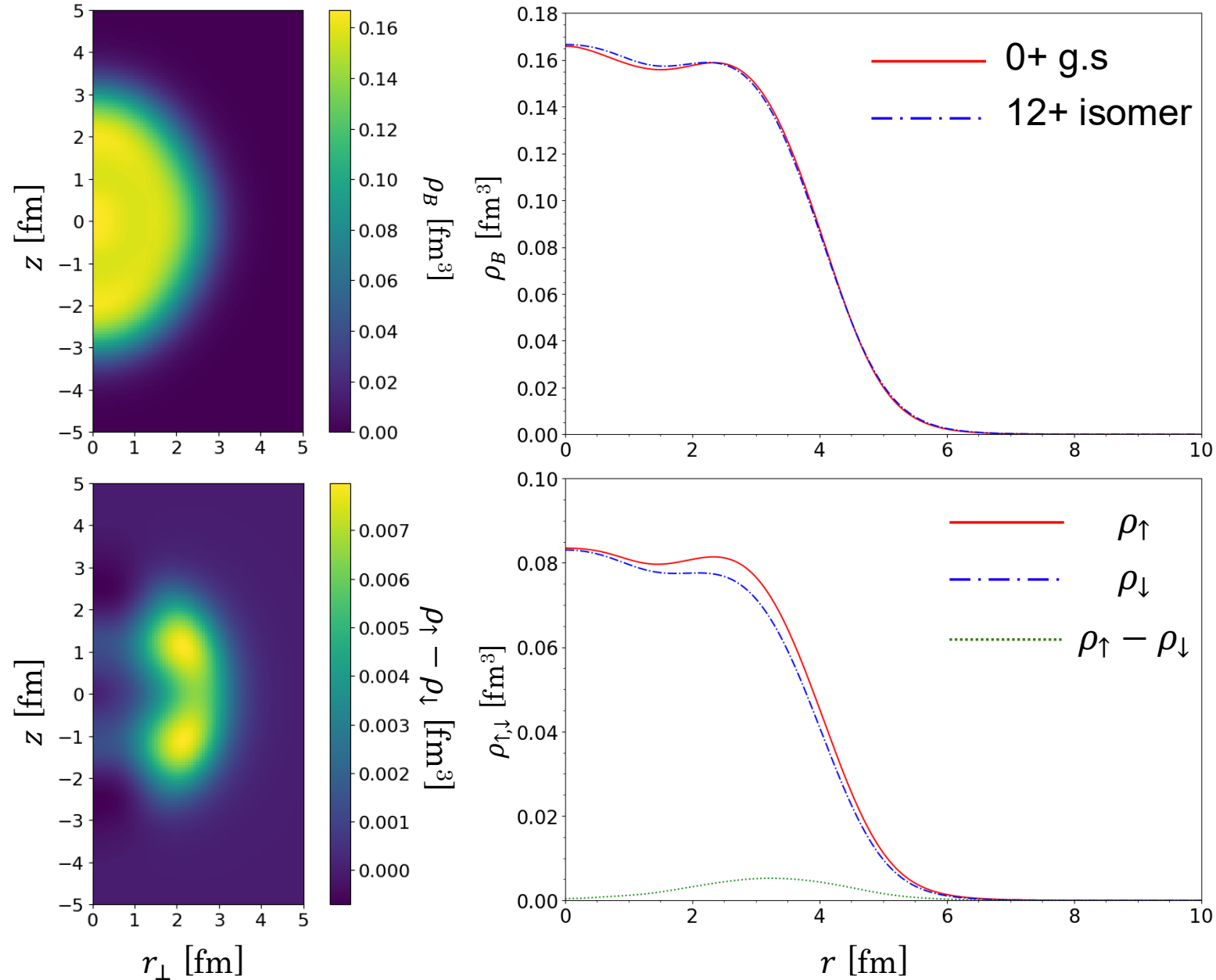
- The configurations in $f_{7/2}$ orbit are fixed by blocking $m = -5/2$ and $m = -7/2$ orbits.



The total angular momentum is 6 for both neutrons and protons.

- Pairing correlations are neglected in calculation of 12^+ isomer state.

Nucleon and Spin Density of $^{52}\text{Fe}(12+)\text{ State}$



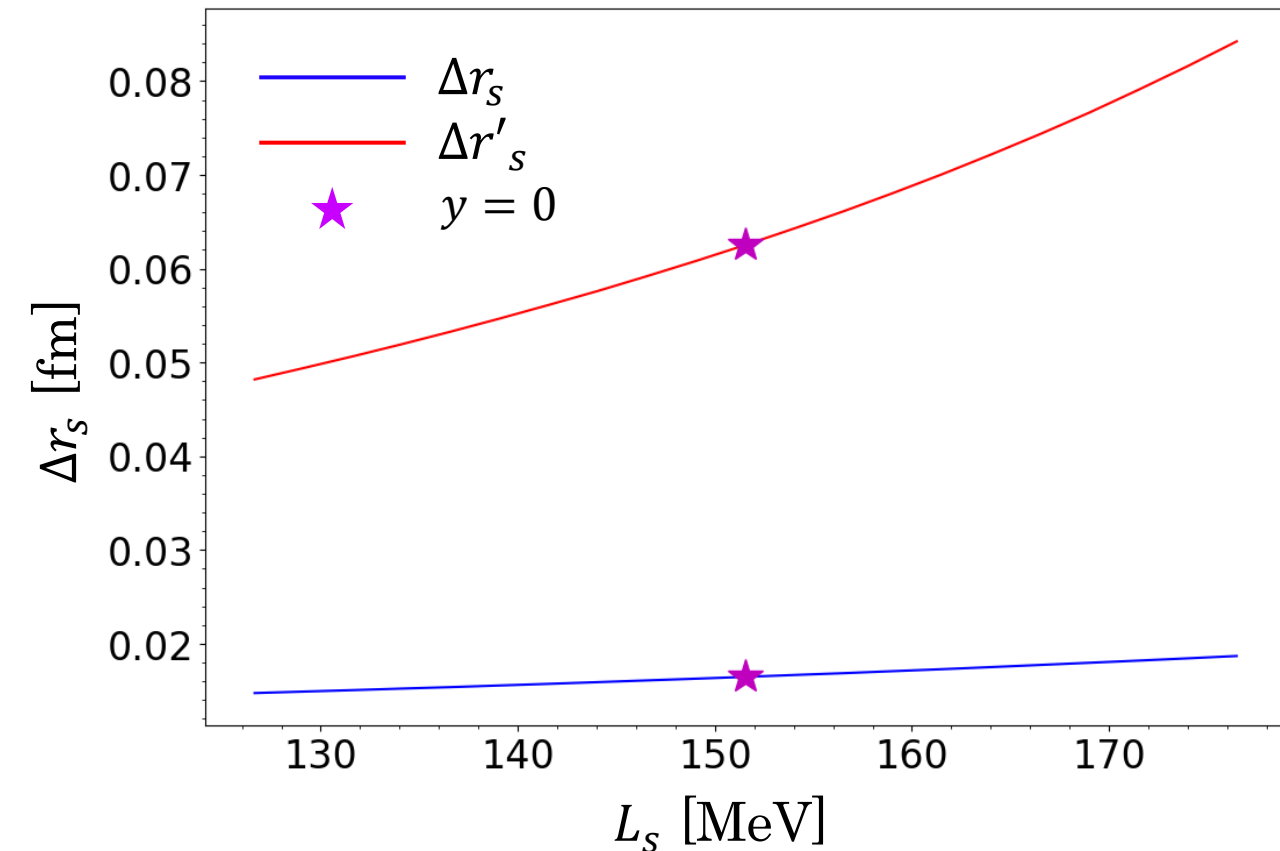
Spin densities are localized near the surface!

Correlation between spin skin thickness and spin slope parameter

Introducing a new variable y :

$$\alpha_{tV}(\rho) \rightarrow (1 - y)\alpha_{tV}(\rho) + y\alpha_{tV}(\rho_0)$$

cf. T. Inakura, H. Nakada, Phys. Rev. C 92, 064302 (2015)



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