Calculations of Nucleon EDMs on a Lattice with Background Field

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INT 23-1b Program
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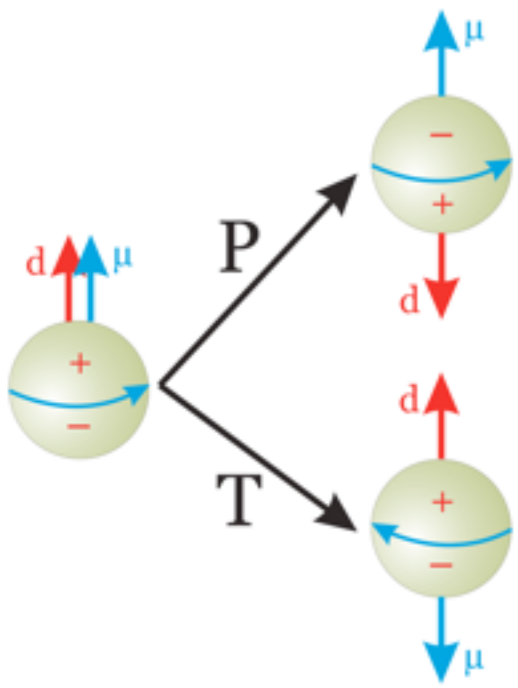
Outline

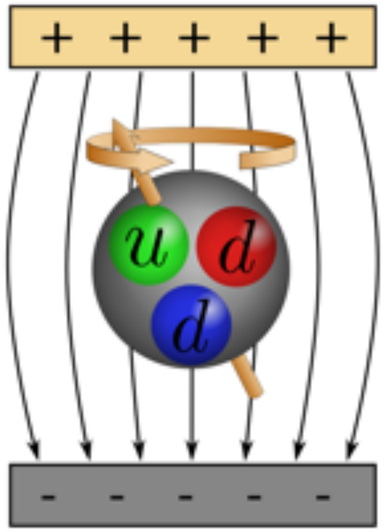
- Introduction
  - *Motivation, status & outlook*
  - *Methods and challenges of lattice calculations*
- Nucleon EDM induced by $\theta_{\rm QCD}$–term
  - *Topological charge on a lattice*
  - *Discussion of recent results*
- Computing EDMs with background electric field
  - *EDM from Feynman-Hellman theorem*
  - *Analysis of gradient-flowed correlators*
- Summary & Outlook

Nucleon Electric Dipole Moments



$$\vec{d}_N = d_N \frac{\vec{S}}{S}$$

$$\mathcal{H} = -\vec{d}_N \cdot \vec{E}$$



EDMs are the most sensitive probes of CPv:

- Signals for beyond SM physics

  (SM = 10^{-5} of the current exp.bound)
- Prerequisite for Baryogenesis
- Strong ~~CP~~ problem : θ_{QCD}-induced EDM?

A.Sakharov's conditions for baryon asymmetry in the Universe [JETP letters, 1967]

- ~~P~~, ~~CP~~ symmetry violation
- Baryon number violation
- non-equilibrium transition

$$\langle N_{p'} | J^\mu | \bar{N}_p \rangle_{\not{CP}} = \bar{u}_{p'} \left[F_1 \gamma^\mu + (F_2 + i F_3 \gamma_5) \frac{\sigma^{\mu\nu}(p'-p)_\nu}{2 m_N} \right] u_p$$

F_1 → Dirac; F_2 → Pauli (anom.magnetic); F_3 → Electric dipole

Experimental Outlook

Current nEDM limits:

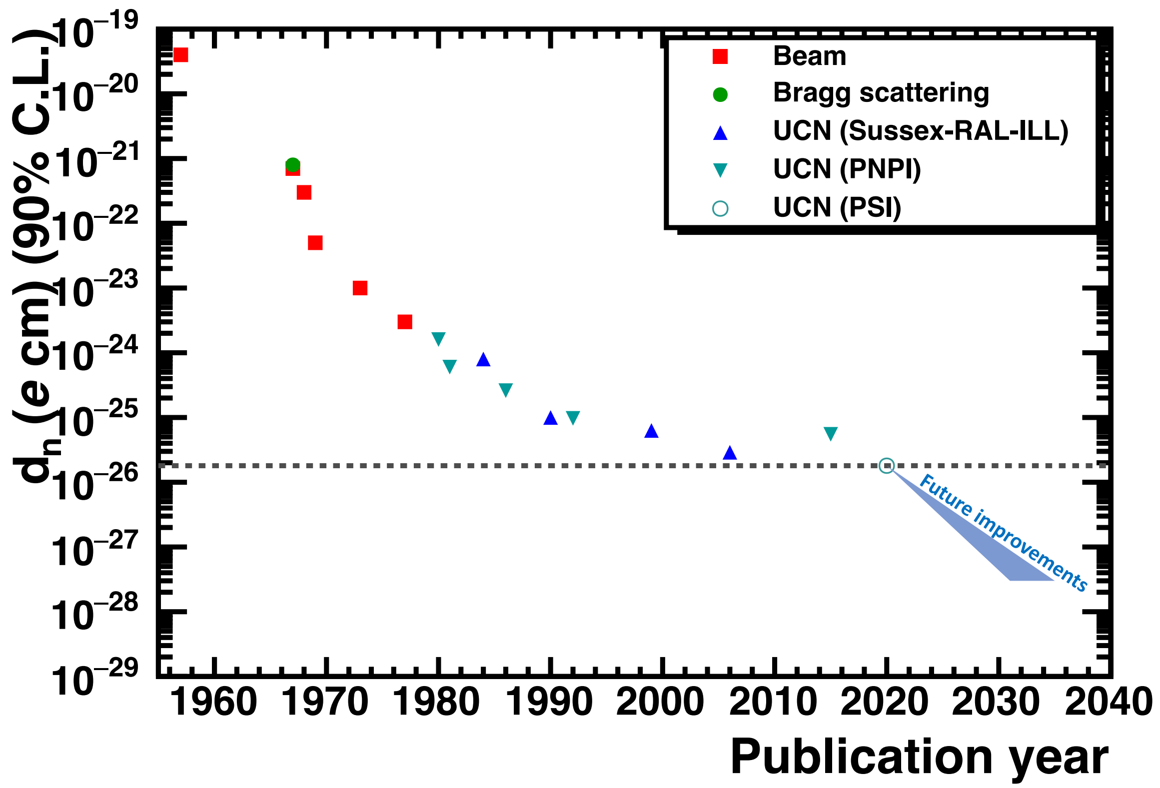
- $|d_n| < 2.9 \times 10^{-26}\, e \cdot \mathrm{cm}$ (stored UC neutrons) [Baker et al, PRL97: 131801(2006)]
- $|d_n| < 1.6 \times 10^{-26}\, e \cdot \mathrm{cm}$ (^{199}Hg) [Graner et al, PRL116:161601(2016)]

Future nEDM sensitivity :

- 1–2 years : next best limit?
- 3–4 years : x10 improvement
- 7-10 years : x100 improvement

	10^{-28} e cm
CURRENT LIMIT	<300
Spallation Source @ORNL	**< 5**
Ultracold Neutrons @LANL	~30
PSI EDM	<50 (I), **<5 (II)**
ILL PNPI	<10
Munich FRMII	**< 5**
RCMP TRIUMF	<50 (I), **<5 (II)**
JPARC	**< 5**
Standard Model (CKM)	< *0.001*

[Snowmass EDM workshop report, arXiv:2203.08103]



Nucleon EDMs: a Window into New Physics

- Effective quark-gluon CPv interactions: dimension $\Longleftrightarrow$ scale of BSM physics

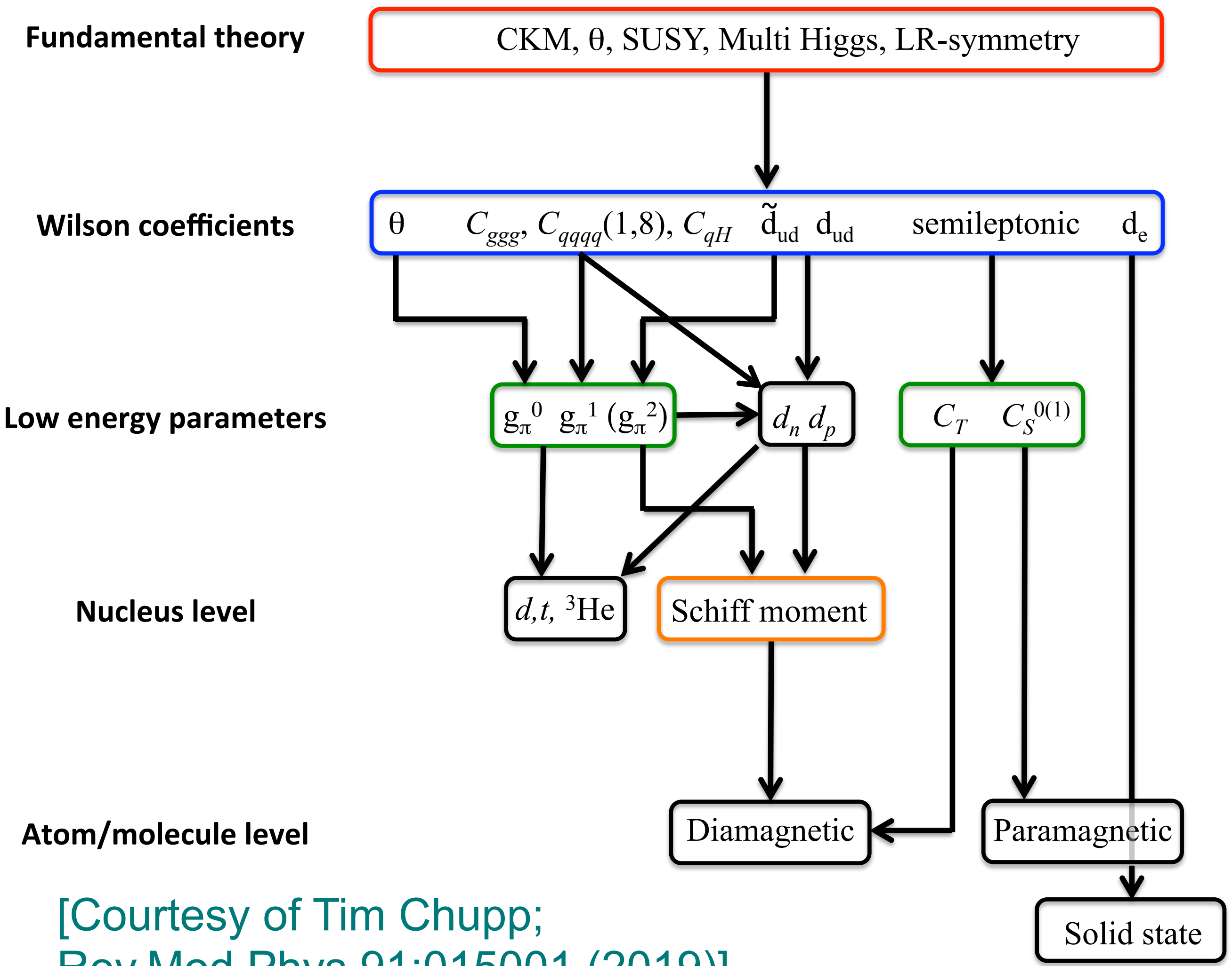
  [Engel, Ramsey-Musolf, van Kolck, Prog.Part.Nucl.Phys. 71:21 (2013)]

$$\mathcal{L}_{eff} = \sum_i \frac{c_i}{[\Lambda_{(i)}]^{d_i-4}} \mathcal{O}_i^{[d_i]}$$

d=4 : θ_{QCD}

d=5(6) : quark EDM, chromo-EDM

d=6 : 4-fermion CPv, 3-gluon (Weinberg)



[Courtesy of Tim Chupp; Rev.Mod.Phys 91:015001 (2019)]

$$d_{n,p} = d^{\theta}_{n,p}\theta_{\rm QCD} + d^{cEDM}_{n,p} c_{cEDM} + \dots$$

$$c_i \Longleftrightarrow d_{n,p} \quad ?$$

- Nonperturbative QCD on a Lattice:

  Quark-gluon CPv interactions $\Longrightarrow$ nucleon EDMs , CPv πNN couplings

Adding CPv Interactions to Lattice QCD

- Linear response to θ_{QCD}-term
  [Aoki et al (2005); Berruto et al (2005); Shindler et al (2015) ; Alexandrou et al (2015) ; Shintani et al (2016); Dragos et al(2019); Alexandrou et al(2020); Bhattacharya et al (2021) ;Liang et al (2023)]

$$\langle \mathcal{O} \ldots \rangle_{\not{C}\not{P}} = \langle \mathcal{O} \ldots \rangle_{CP-even} - i\theta \langle Q \cdot \mathcal{O} \ldots \rangle_{CP-even} + O(\theta^2)$$

$\not{C}\not{P}$ coupling

$\not{C}\not{P}$ operator: $G\tilde{G}$, cEDM, $GG\tilde{G}$(Weinberg), 4-quark

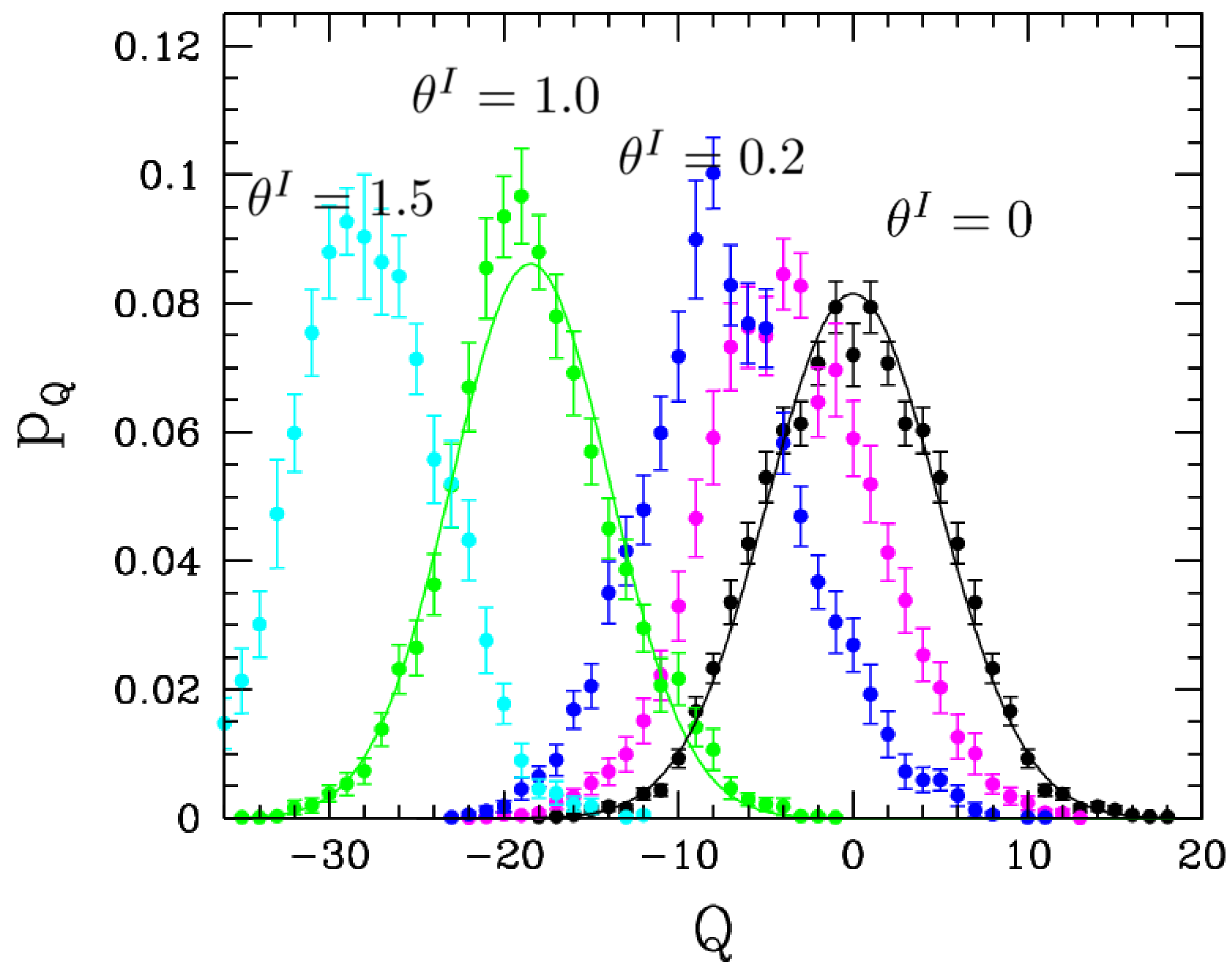
(n) → (n+1) correlation function, e.g.

$$\langle N J \bar{N} \rangle \quad \longrightarrow \langle Q N J \bar{N} \rangle$$

- Finite (imaginary) CPv: θ^I_{QCD}
  [R.Horsley et al (2008) ; F.K.Guo et al (2015)]

$$\langle \mathcal{O} \ldots \rangle_\theta \sim \int \mathcal{D}U \, e^{-S-\theta^I Q} \, (\mathcal{O} \ldots)$$

require dedicated QCD simulation
⟹ better sampling of Q≠0 sectors



Determination of Nucleon EDM

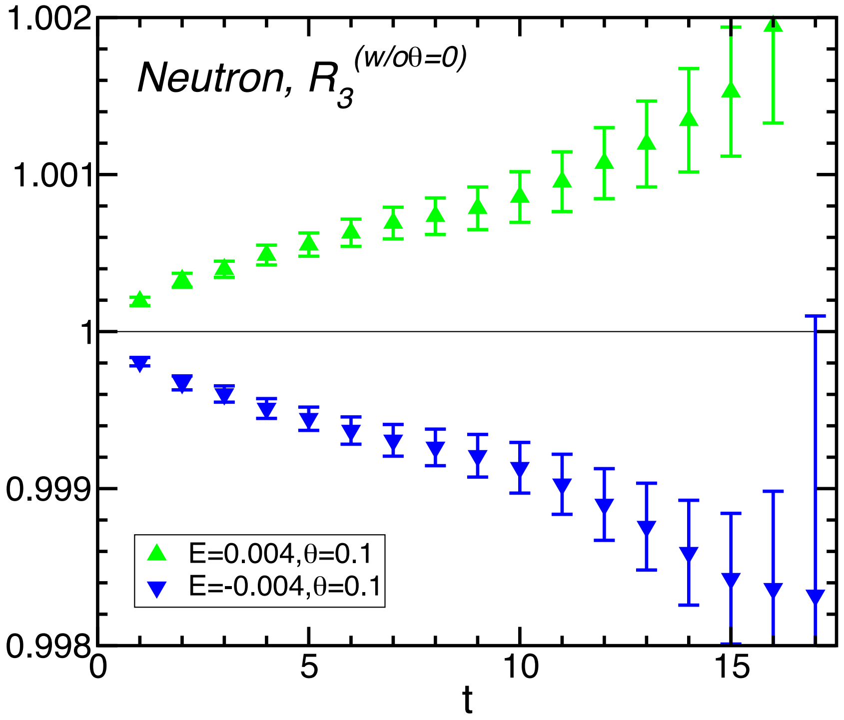
- ***"Energy-Shift method"*** (uniform electric field)
  [S.Aoki et al '89 ; E.Shintani et al '06;
  E.Shintani et al, PRD75, 034507(2007)]

$$\langle N(t)\bar{N}(0)\rangle_{\theta,\vec{E}} \sim e^{-(E\pm\vec{d}_N\cdot\vec{E})t}$$

Euclidean lattice:

Real-valued **E** $\Longrightarrow$ violate time-BC

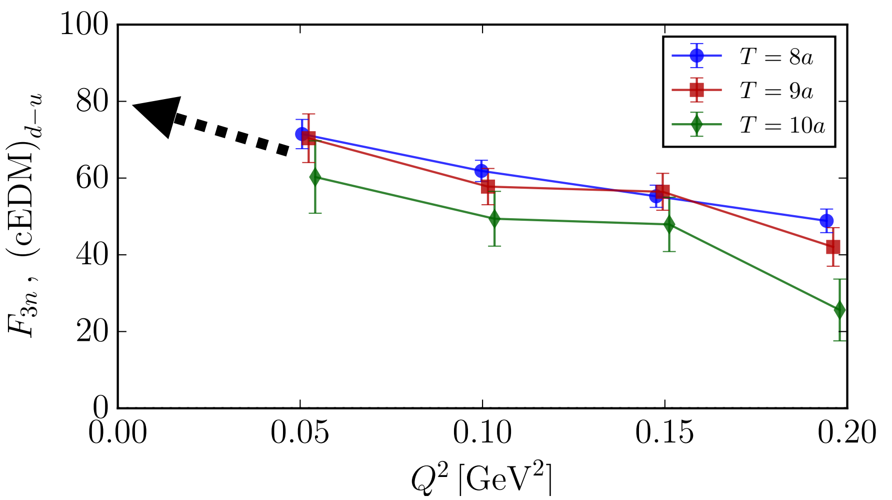
Imag-valued **E** $\Longrightarrow$ imaginary shift in m_N



- ***"Form-Factor method"*** : $d_N = F_3(Q^2 \to 0) / (2m_N)$
  [(everybody else, almost)]

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\not{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + i\boxed{F_3}\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p$$

- pre-2017 : spurious $\mu_n \leftrightarrow d_n$ mixing
- Dragos et al(2019) $d_n / \theta = -0.0015(7)\ e{\cdot}\mathrm{fm}$
- Alexandrou et al(2020) $d_n / \theta = 0.0009(24)\ e{\cdot}\mathrm{fm}$
- Bhattacharya et al (2021) $|d_n / \theta| \lesssim 0.01\ e{\cdot}\mathrm{fm}$
- Liang et al (2023) $d_n / \theta = -0.0015(1)(3)\ e{\cdot}\mathrm{fm}$



Need extrapolation to forward-limit $F_3(Q^2 \to 0)$

Nucleon "Parity Mixing"

Any CPv interaction induces a chiral phase in nucleon spinor on a lattice

$$\langle \mathrm{vac}|N|p,\sigma\rangle_{\not{CP}} = e^{i\alpha\gamma_5}u_{p,\sigma} = \tilde{u}_{p,\sigma}$$

$N \to u\,[u^T C\gamma_5 d]$

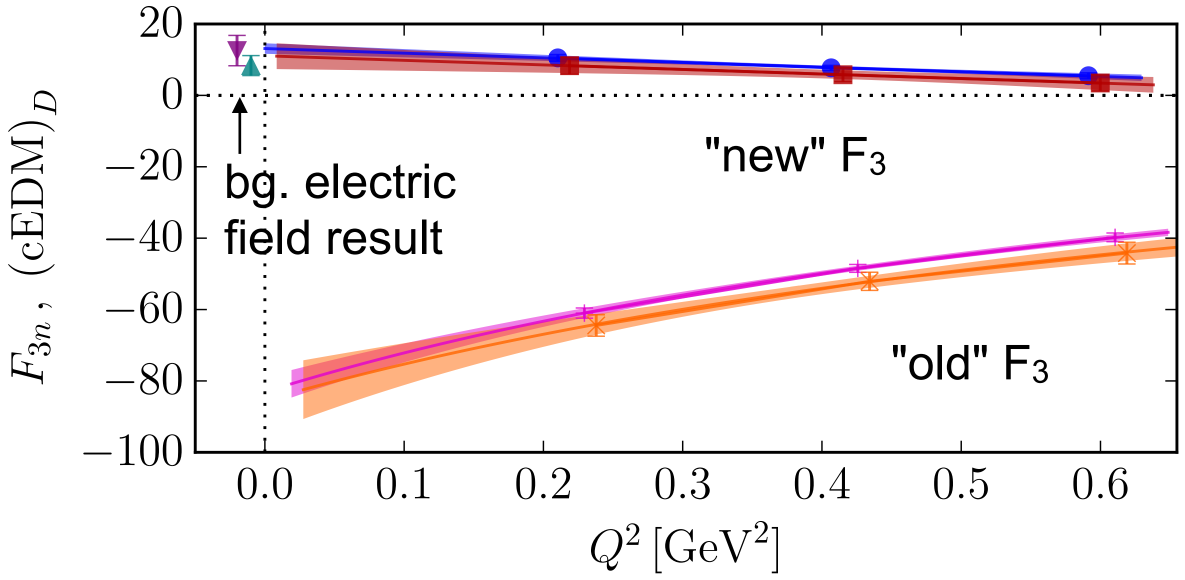
$\tilde{u}_{p,\sigma} \to \sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma} \sim \left(-i\not{p}_{\mathcal{E}} + m_N e^{2i\alpha\gamma_5}\right)$

[M.Abramczyk, S.Aoki, S.N.S, et al (2017) arXiv:1701.07792]

EDM and MDM are defined with positive-parity spinors

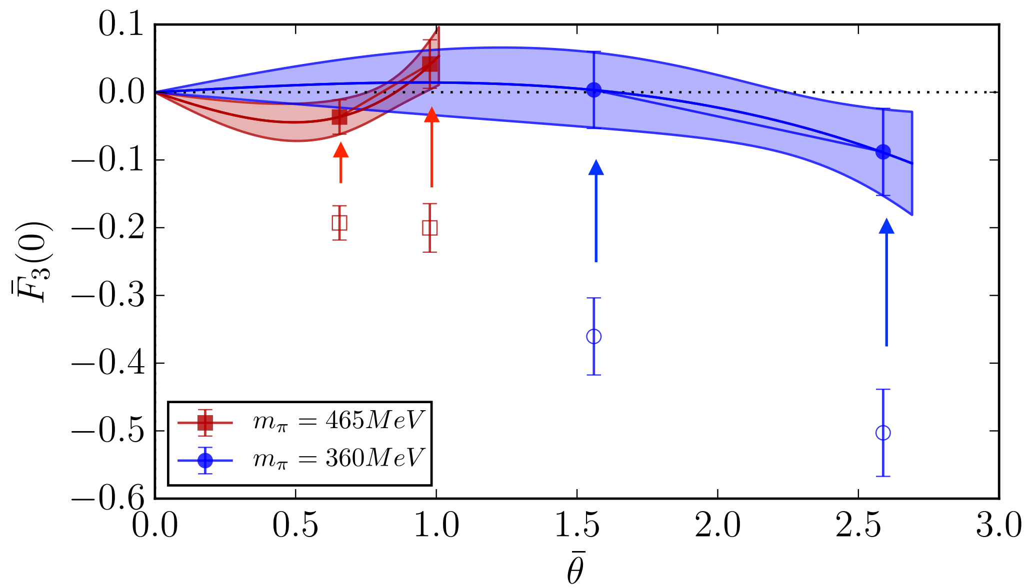
$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\not{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p \quad , \quad \text{with} \quad \begin{matrix}\gamma_4 u = +u \\ \bar{u}\gamma_4 = +\bar{u}\end{matrix}$$

check: cEDM-induced EDM / EDFF comparison of form factor F_3 to energy shift in *background* $\bar{E}$*=const*



Pre-2017 results: mixing correction

$$\text{“}d_{n,p}\text{”} \approx [d_{n,p}]_{\mathrm{true}} - 2\alpha\frac{\kappa_{n,p}}{2m_N}$$



Nucleon "Parity Mixing"

CPv interaction induces a chiral phase in nucleon wave functions on a lattice

$$\langle \mathrm{vac}|N|p,\sigma\rangle_{\not{CP}} = e^{i\alpha\gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma}$$

$N \to u\,[u^T C\gamma_5 d]$

$\tilde{u}_{p,\sigma} \to \sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma} \sim \left(-i\not{p}_{\mathcal{E}} + m_N e^{2i\alpha\gamma_5}\right)$

[M.Abramczyk, S.Aoki, S.N.S, et al (2017) arXiv:1701.07792]
EDM and MDM are defined with positive-parity spinors

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\not{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p \quad , \quad \text{with} \quad \begin{array}{l}\gamma_4 u = +u \\ \bar{u}\gamma_4 = +\bar{u}\end{array}$$

Θ-nEDM

		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
[ETMC 2016]	n	373	1.216(4)	$-1.50(16)^a$	$-0.217(18)$	$-0.555(74)$	0.094(74)
[Shintani et al 2005]	n	530	1.334(8)	$-0.560(40)$	$-0.247(17)^b$	$-0.325(68)$	$-0.048(68)$
	p	530	1.334(8)	0.399(37)	$-0.247(17)^b$	0.284(81)	0.087(81)
[Berruto et al 2006]	n	690	1.575(9)	$-1.715(46)$	$-0.070(20)$	$-1.39(1.52)$	$-1.15(1.52)$
	n	605	1.470(9)	$-1.698(68)$	$-0.160(20)$	0.60(2.98)	1.14(2.98)
[Guo et al 2015]	n	465	1.246(7)	$-1.491(22)^c$	$-0.079(27)^d$	$-0.375(48)$	$-0.130(76)^d$
	n	360	1.138(13)	$-1.473(37)^c$	$-0.092(14)^d$	$-0.248(29)$	$0.020(58)^d$

After removing the spurious contribution,

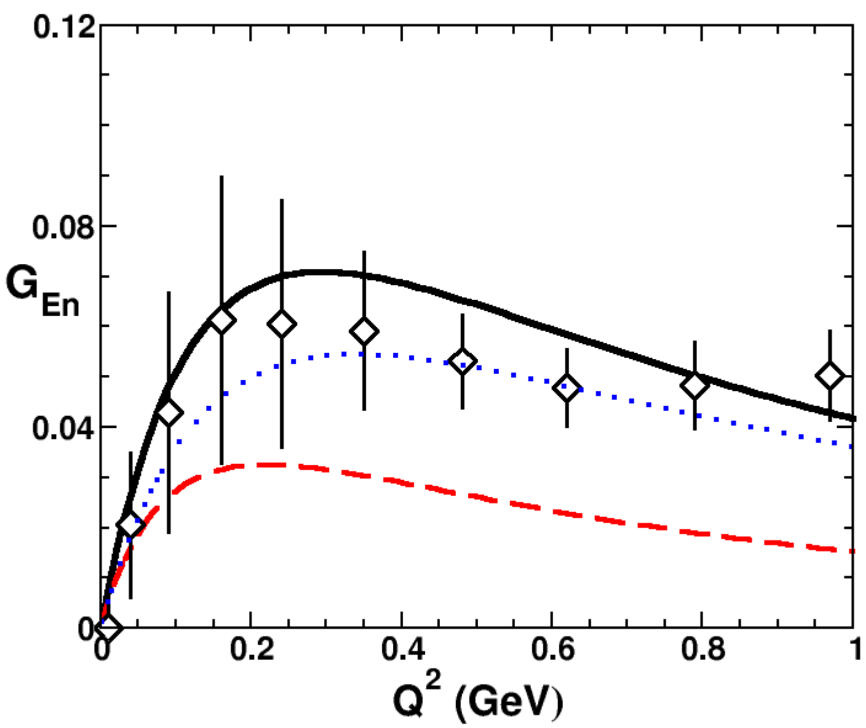
- *no lattice signal for θ_{QCD}-induced nEDM*
- *RESOLVED conflict with pheno. values, lack of $d_N \sim m_q$ scaling*

Importance of "Parity Mixing" Correction

Exact value of α_5 is critical for correct determination of EDM:

$$F_3^{\rm lat}(Q^2) \approx \underbrace{\frac{m}{q_3}\langle N_\uparrow(0)|\bar{q}\gamma_4 q|N_\uparrow(-q_3)\rangle_{\not{CP}}}_{\text{CPv matrix element}} - \underbrace{\alpha_5 G_E(Q^2)}_{\text{Sachs form factor subtraction}}$$

- Proton ($G_{Ep}(0)=1$) : Correction ~ α_5
- Neutron ($G_{En}(0)=0$) : No correction at $Q^2=0$
  However, $Q^2 \to 0$ extrapolation may be skewed by neutron electric form factor $\sim\alpha_5\, G_{En}(Q^2)$



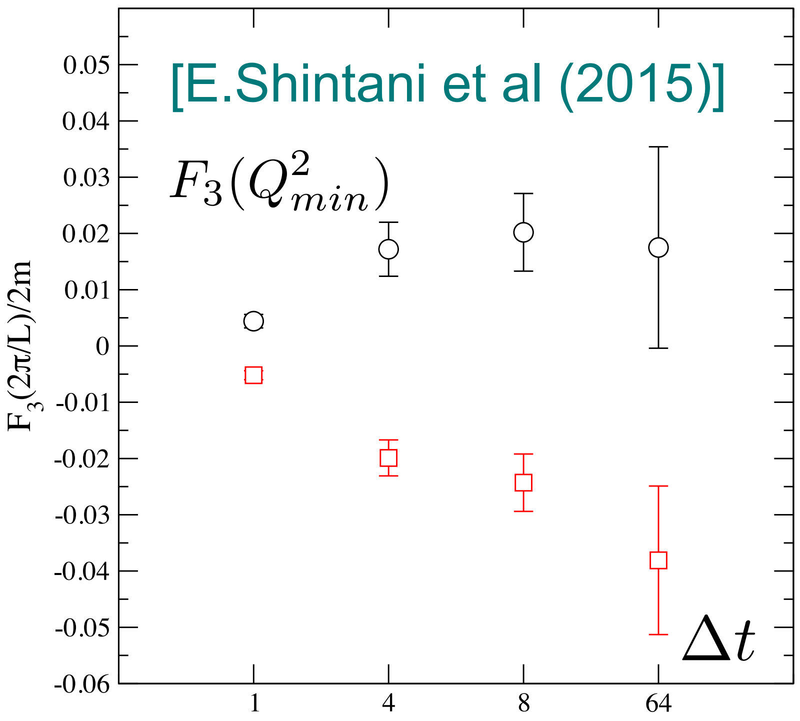
[Punjabi et al, 1503.01452]

Noise from θ-Term in nEDM from

Variance of lattice θ-induced nEDM signal ~ (*Volume*)$_{4d}$:

$$d_N \sim \langle Q \cdot (N J_\mu \bar{N}) \rangle$$

Top. charge $Q \sim \int_{V_4} (G\tilde{G})$, with $\langle |Q|^2 \rangle \sim V_4$



Constrain Q sum to the fiducial volume

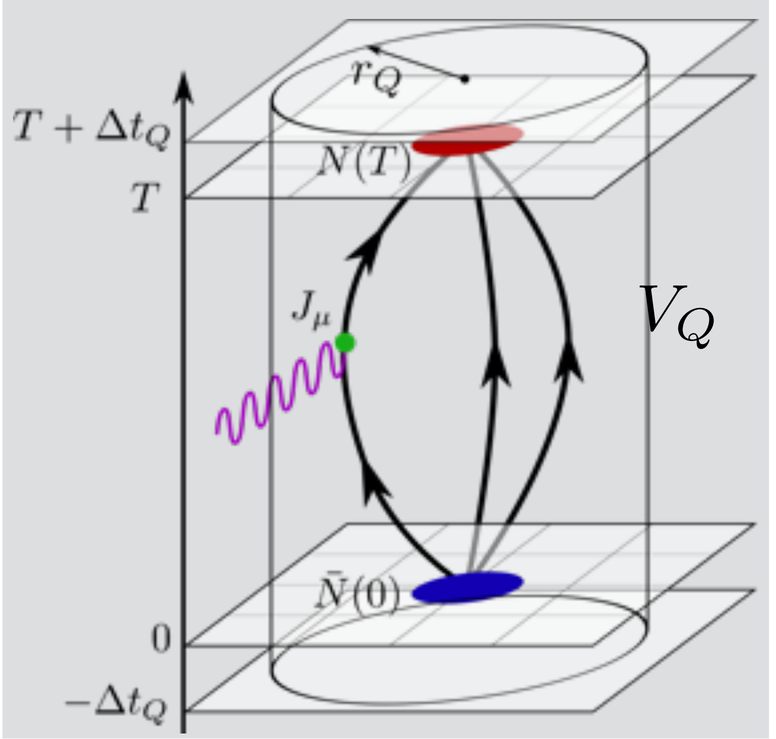
- in time around current, $|t_Q - t_J| < \Delta t$ [E.Shintani et al (2015); Yoon et al (2019)]
- in time around source, $|t_Q - t_{source}| < \Delta t$ [Dragos et al (2019)]
- 4-d sphere around sink, $|x_Q - x_{sink}| < R$ [K.-F. Liu et al (2023)]:

$$Q \approx \int_{V_Q} d^4 z\, q(z)$$

Proper treatment of nucleon parity mixing is critical for correct determination of F_3
⟹ nucleon must "settle" in the new θ≠0 vacuum

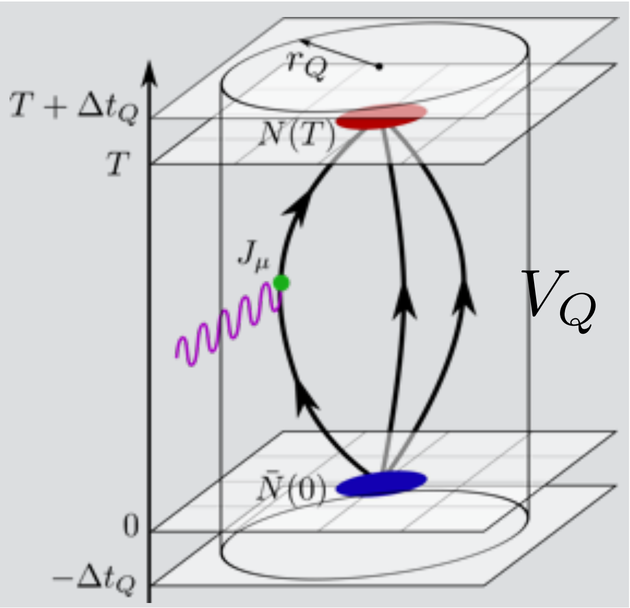
$$N^{(+)} \to \tilde{N}^{(+)} \approx N^{(+)} + i\alpha N^{(-)}$$
$$N^{(-)} \to \tilde{N}^{(-)} \approx N^{(-)} - i\alpha N^{(+)}$$

⟹ *constrain time and space differently :*
4d "cylinder" $V_Q \; : \; |\vec{z}| < r_Q , \quad -\Delta t_Q < z_0 < T + \Delta t_Q$

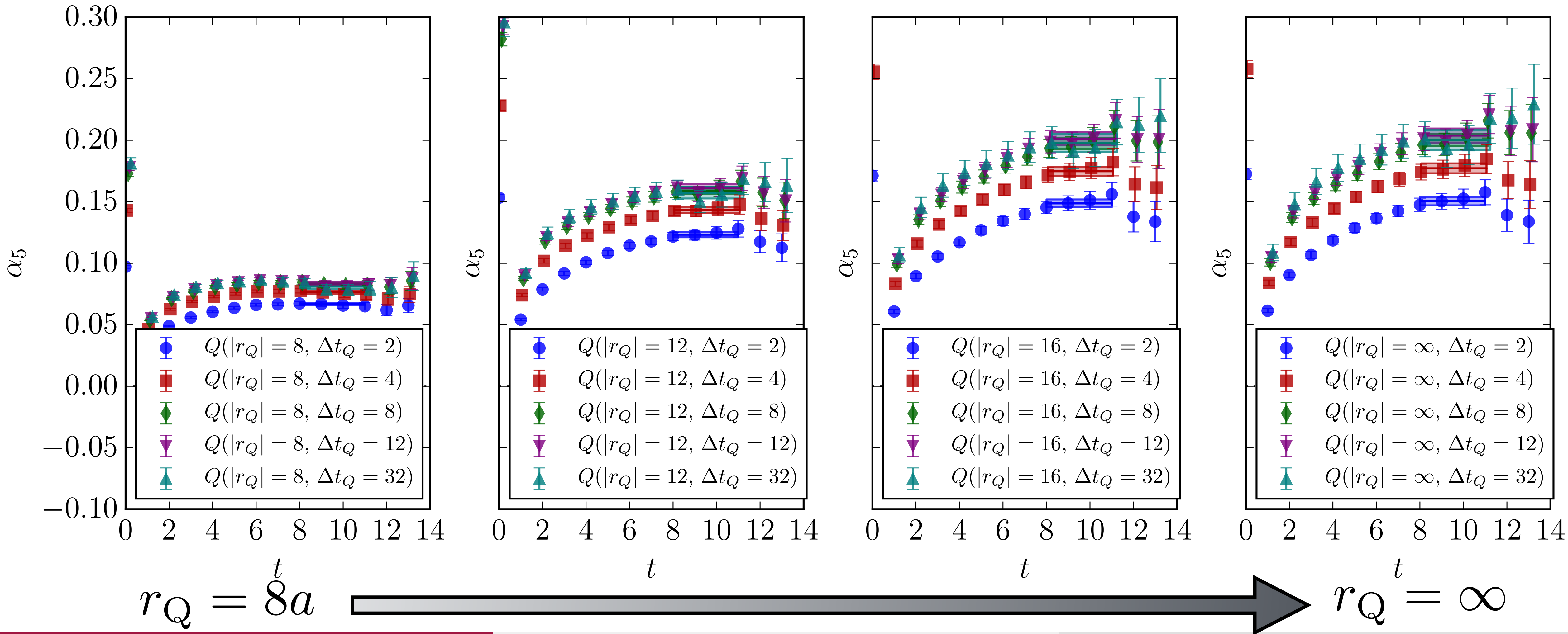


Effect of $G\tilde{G}$ cuts: Parity Mixing Angle

- 24^3x64 a = 0.114 fm mπ=330 MeV (N_f=2+1 chiral-symmetric quarks)
- 1400 confiigs $\Longrightarrow$ 89.6k stat.
- $G\tilde{G}$: Wilson-flowed (t=8a^2) gauge links [M.Luscher, 1006.4518]
  5-loop improved $G\tilde{G}$ [P. de Forcrand et al '97]
- Cuts in space $r \leq r_Q$, time Δt_Q



parity mixing angle α : $\langle \text{vac}|N|p,\sigma\rangle_{\not{CP}} = e^{i\alpha\gamma_5} u_{p,\sigma}$



Effect of $G\tilde{G}$ cuts : nEDM Form Factor F_3

- 24^3x64 a = 0.114 fm m_π=330 MeV (N_f=2+1 chiral-symmetric quarks)
- 1400 confiigs $\Longrightarrow$ 89.6k stat.
- $G\tilde{G}$: Wilson-flowed ($t=8a^2$) gauge field [M.Luscher, 1006.4518]
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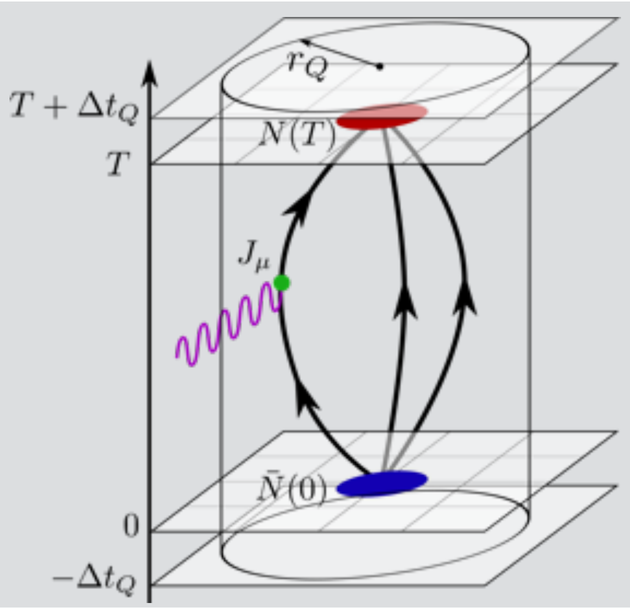
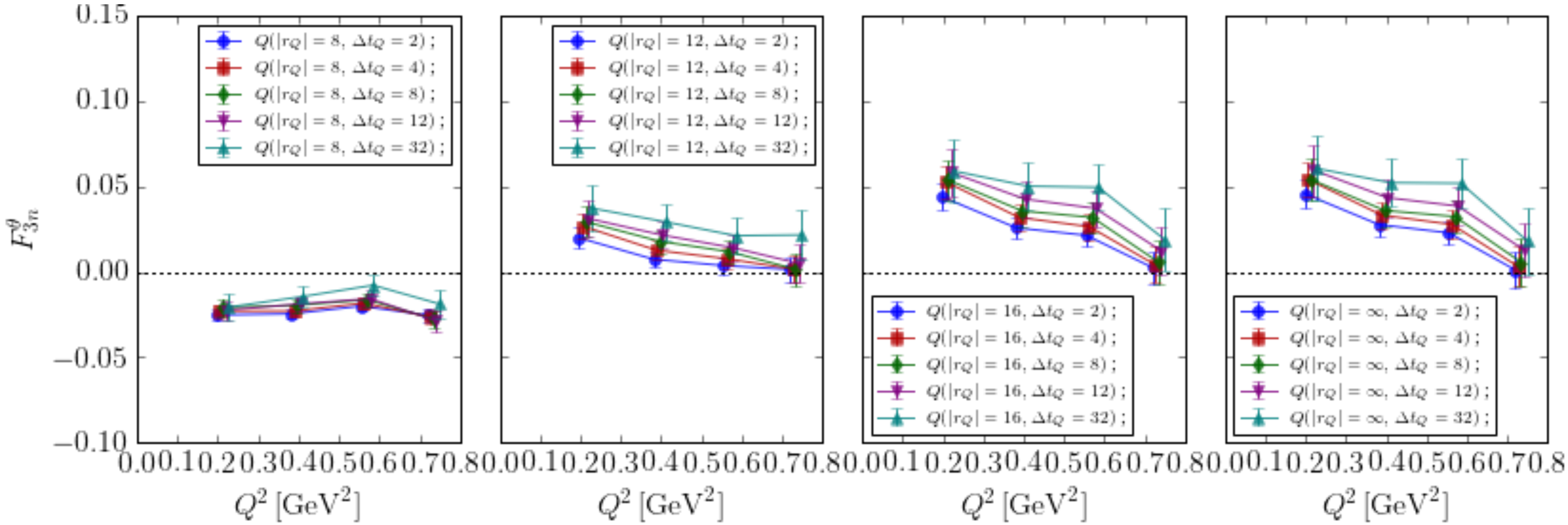


$$F_3^{\rm lat}(Q^2) \approx \frac{m}{q_3} \underbrace{\langle N_\uparrow(0)|\bar{q}\gamma_4 q|N_\uparrow(-q_3)\rangle_{\not{CP}}} - \underbrace{\alpha_5 G_E(Q^2)}$$



$r_Q = 8a$ ⟶ $r_Q = \infty$

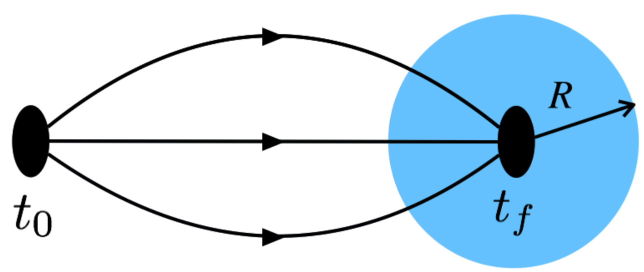
Another Definition of Top.Charge Density

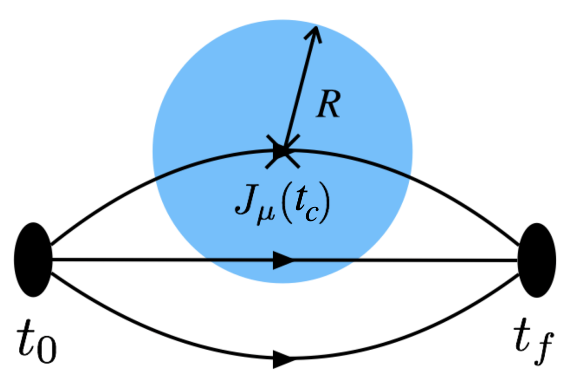
[Liang et al ; 2301.04331]

- top.charge from *chiral* fermion modes (better than gradient-flowed $G\tilde{G}$?)

$$q(x) = \frac{1}{2}\mathrm{Tr}[\gamma_5 D_{ov}(x,x)]$$

- space-time cuts around nucleon field OR vector current

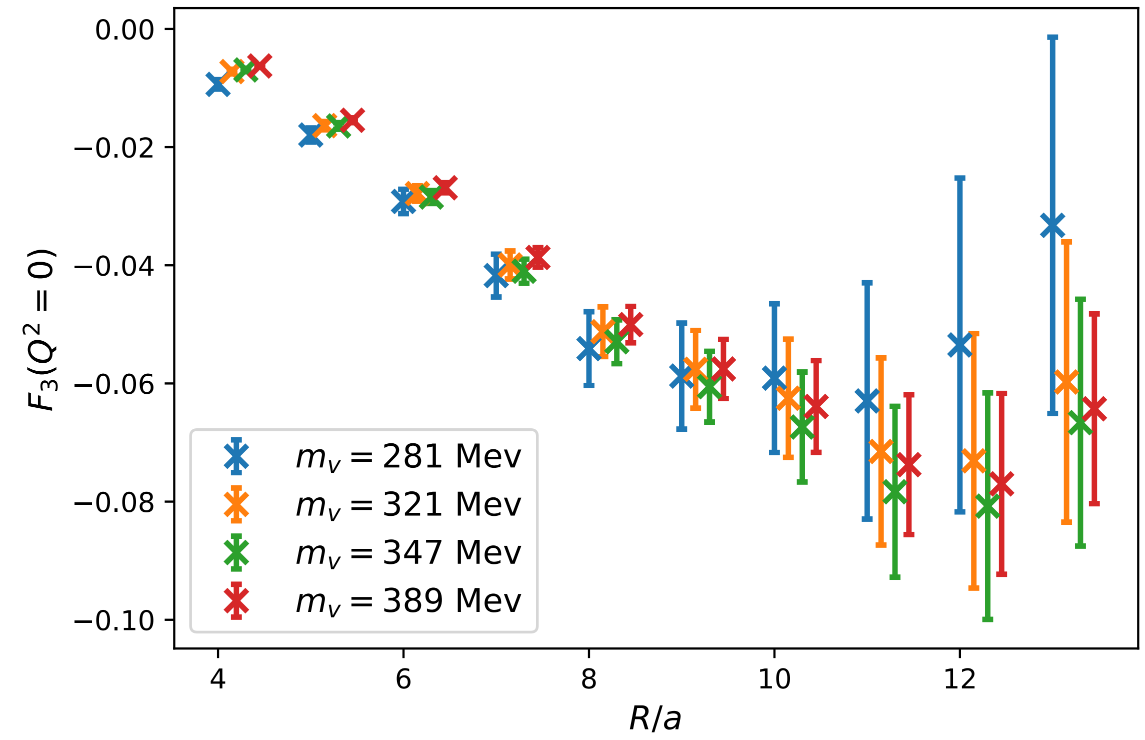


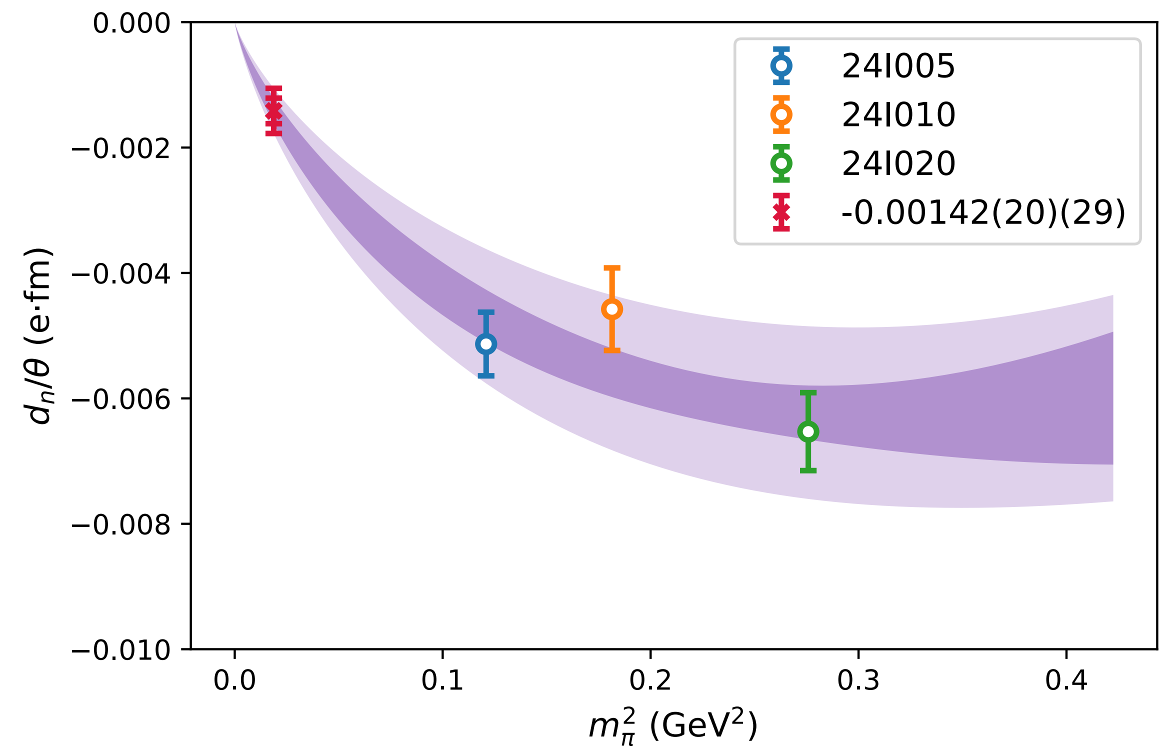


- best definition of d_n to date?

$$d_n^\theta/\theta = -0.0015(1)(3)\ e\cdot\mathrm{fm}$$

(chiral extrapolation)

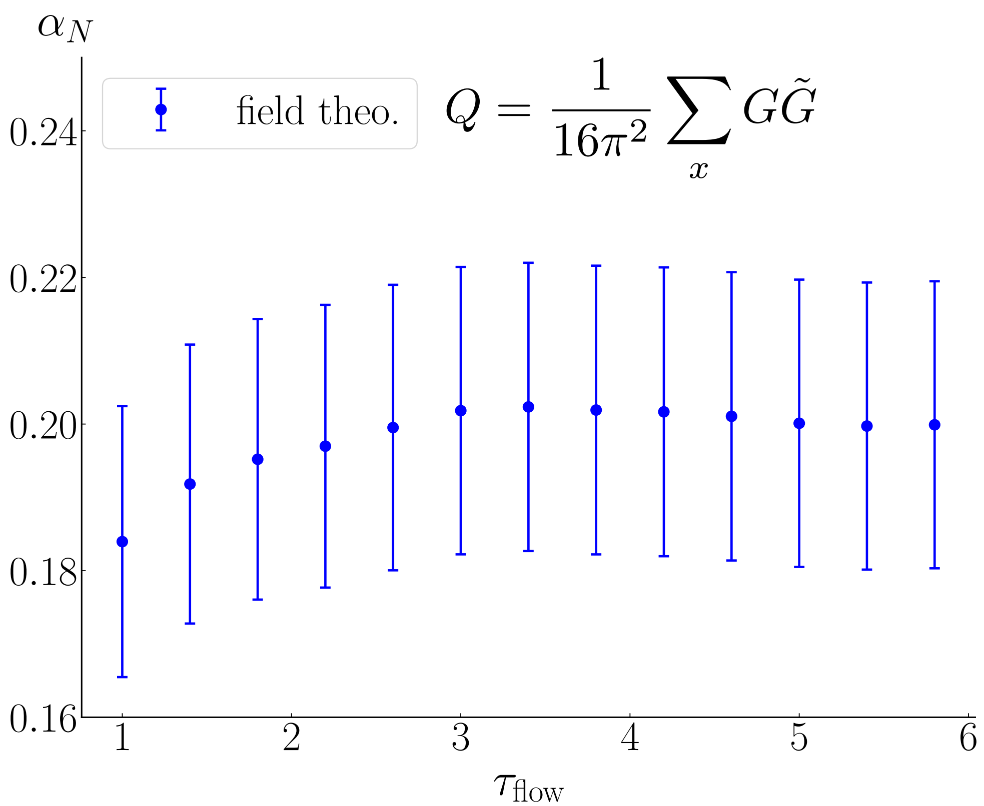


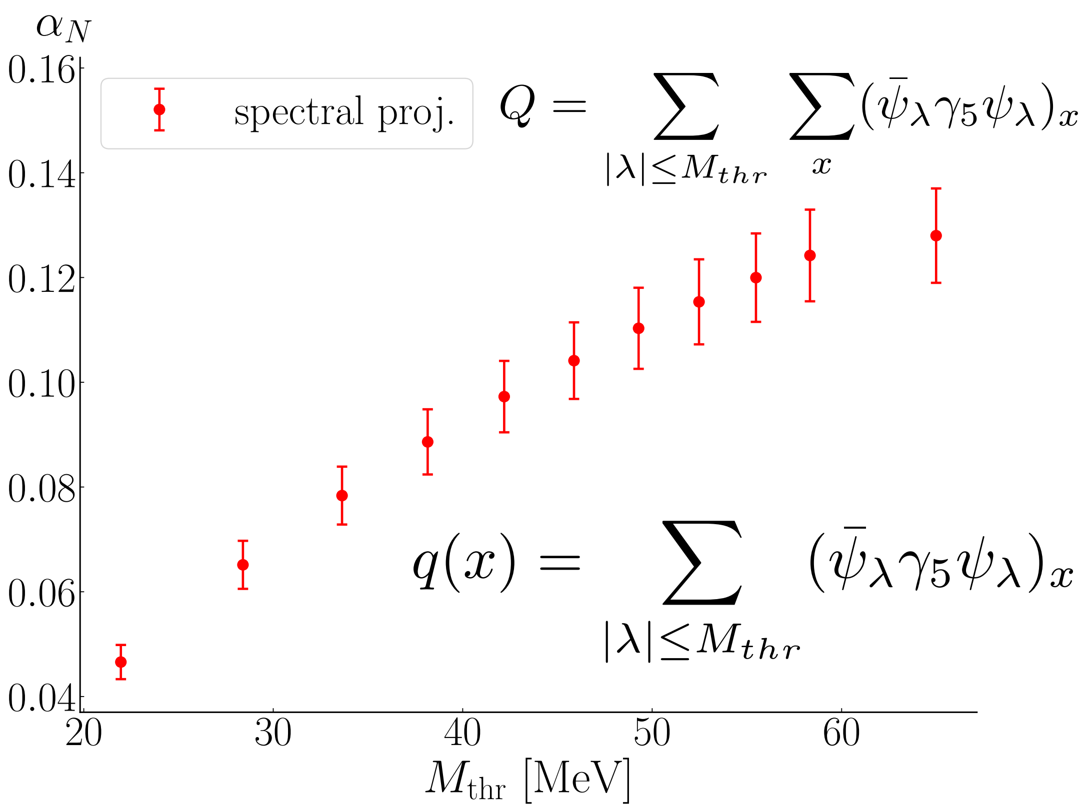


EDM from Full Q at Physical Point

[Alexandrou et al, PRD(202) ; 2011.01084]

- comparison of EDM from $Q_G=\int G\tilde{G}$ vs. fermion*-mode Q_F





$d_n / \theta = \; 0.0018(56)\; e\cdot\text{fm}$

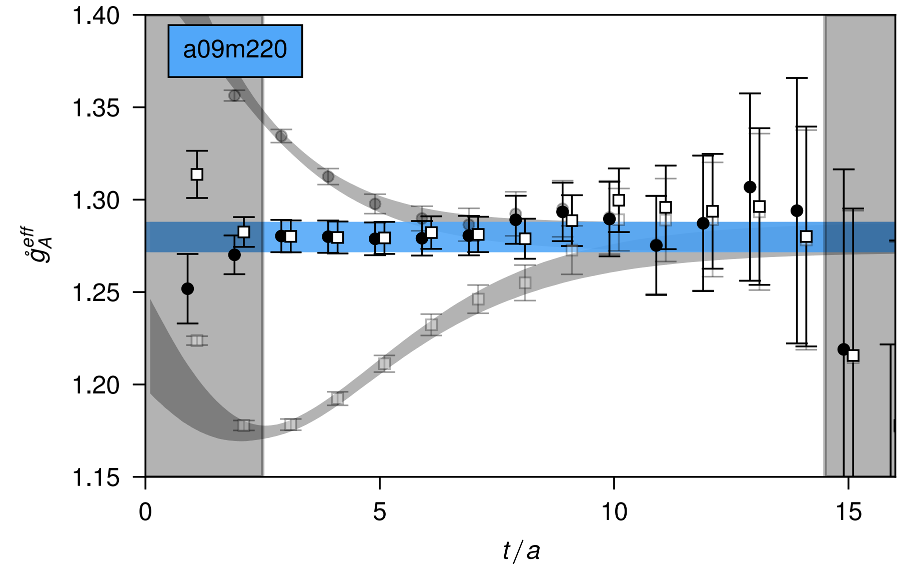
$d_n / \theta = \; 0.0009(24)\; e\cdot\text{fm}$

Alternative: EDM from Feynman-Hellman Thm

- FH theorem :
  Perturbation's matrix element $\Longleftrightarrow$ Energy shift

$$\frac{\partial E_\lambda}{\partial \lambda} = \left\langle \phi_\lambda \middle| \frac{\partial \hat{H}_\lambda}{\partial \lambda} \middle| \phi_\lambda \right\rangle$$

(used successfully to compute g_A to sub-% precision) and other forward hadron matrix elements



[Chang et al (CalLat), Nature 558:91 (2018)]

Nucleon EDM from FH:

- θ-term as a linear perturbation induces EDM $d_n \sim \theta$:

$$\delta\mathcal{L} = \frac{\theta g^2}{32\pi^2} \int dt \int d^3x G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}$$

- background electric field leads to a <u>energy shift</u> of a polarized-nucleon

$$m'_N = m_N - (d_N^\theta \theta)\, \vec{\Sigma} \cdot \vec{\mathcal{E}}$$

- FH : relation between EDM and matrix element of <u>local top.charge density</u>

$$d_N^\theta \propto \left\langle N_\uparrow \middle| \int d^3x\, G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu} \middle| N_\uparrow \right\rangle_{\mathcal{E}_z}$$

Advantage: precise d_n from $G\tilde{G}$ on only one time slice $\Longrightarrow$ noise reduction

EDM ⇔ Density of Top.charge in Polarized Nucleon

$$d_N^\theta \propto \left\langle N_\uparrow \middle| \int d^3x\, G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu} \middle| N_\uparrow \right\rangle_{\mathcal{E}_z}$$

Nonzero in CP-even vacuum only if both ***spin***- and ***charge***-polarized
(can be simulated with background E || B fields)

(**permanent) EDM** $\Longleftrightarrow$ **correlation** of *spin* and *charge*

- *charge polarization* in CPv vacuum — $\langle N|\, \mathbf{d}\, |N\rangle_{CPv} \sim \mathbf{E}$
  OR
- *spin polarization* in CPv vacuum — $\langle N|\, \mathbf{\Sigma}\, |N\rangle_{CPv} \sim \mathbf{E}$
  OR
- *"topological" polarization* in CP-even vacuum — $\langle N|\, G\tilde{G}\, |N\rangle_{CP\text{-}even} \sim \mathbf{S}\cdot\mathbf{E}$

Normally $\langle N|\, G\tilde{G}\, |N\rangle_{CP\text{-}even}$ is zero

$$\left.\begin{aligned} \mathcal{P}\big(|N_\uparrow^{(+)}\rangle\big) &= +|N_\uparrow^{(+)}\rangle \\ \mathcal{P}\big(G\tilde{G}\big) &= -G\tilde{G} \end{aligned}\right\}$$

unless background **E** field breaks parity,
polarizes N into a mixed-parity state:

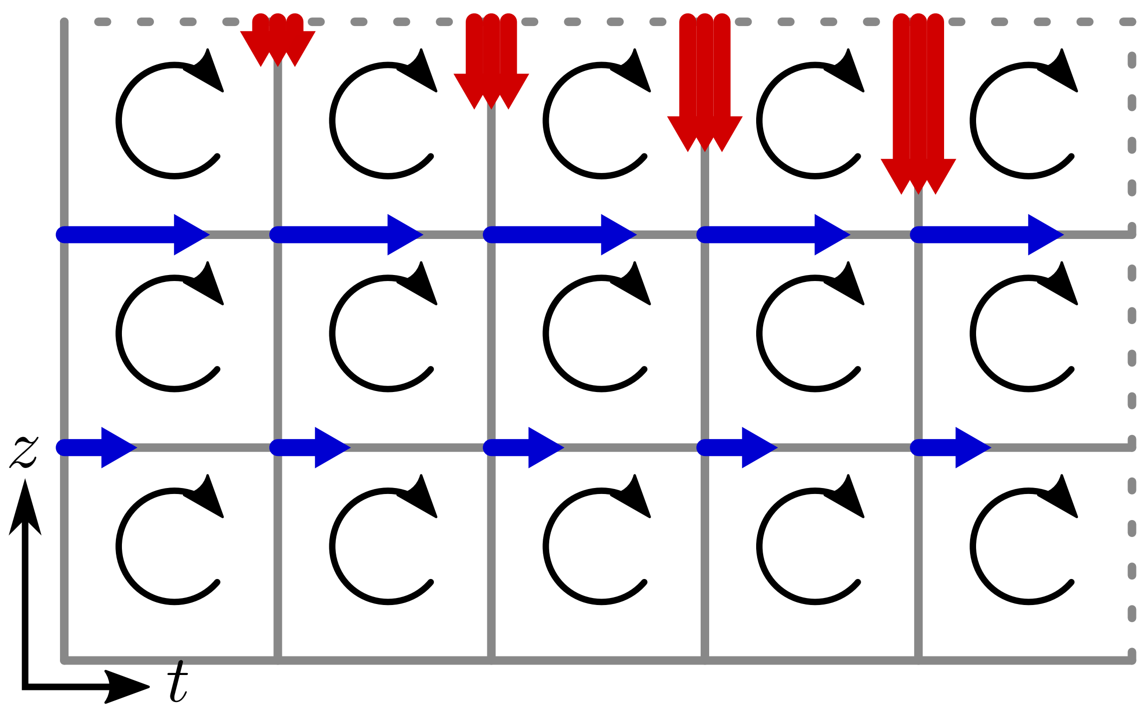
$$|N\rangle_{\mathcal{E}_z} = |N^{(+)}\rangle + O(\mathcal{E}_z)|N^{(-)}\rangle$$

used in

> $\langle N_\uparrow|\, G\tilde{G}\, |N_\uparrow\rangle_{E=0}$ *vanishes*
> ⇩
> *sensitive to EDM directly*
> *(no subtractions)*

Background Electric Field

Accessing magnetic and electric moments at Q^2=0
Imag.Minkowski/Real Euc. electric field on a lattice
[W.Detmold et al (2009)] : calculation of hadron polarizabilities



$$U_\mu \to e^{iqA_\mu} U_\mu$$

$$A_z(z,t) = n\,\mathcal{E}_{\min} \cdot t$$

$$A_t(z, t = L_t - 1) = -n\,\mathcal{E}_{\min} \cdot L_t z$$

Full flux through the "side" of the periodic box

$$= q\Phi = 2\pi \cdot n$$

Constant Electric field has to be quantized,

$$\mathcal{E}_{\min} = \frac{1}{|q_d|}\frac{2\pi}{L_x L_t}$$

Electric field on a 24^3x64 lattice

$$\mathcal{E} = \frac{6\pi}{L_x L_t} \approx 0.037\ \mathrm{GeV}^2$$

$$\approx 186\ \mathrm{MV/fm}$$

Unambiguous determination of EDM from the energy shift
Straightforward for neutron with Q=0

Topological Charge with Gradient Flow

[M.Luscher, JHEP08:071; 1006.4518]

Gradient flow: covariant *4D-diffusion* of quantum fields with "G.F." time t_{GF}:

$$\frac{d}{dt_{\rm GF}} B_\mu(t_{\rm GF}) = D_\mu G_{\mu\nu}(t_{\rm GF})\,, \quad B_\mu(0) = A_\mu$$

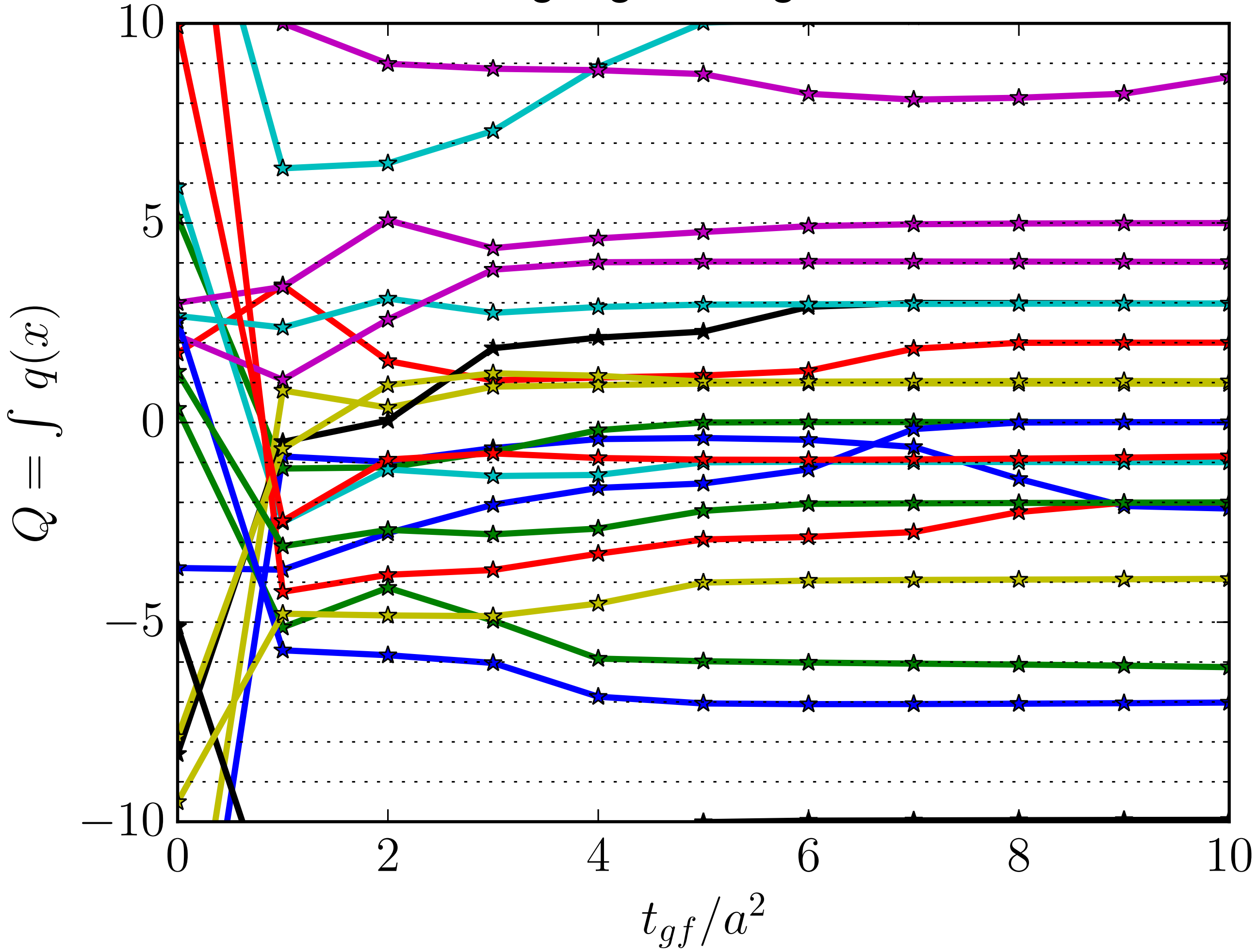
Tree-level:

$$B_\mu(x, t_{\rm GF}) \propto \int d^4y \, \exp\left[-\frac{(x-y)^2}{4t_{\rm GF}}\right] A_\mu(y)$$

Gradient-flowed topological charge:

$$\tilde{Q}(t_{\rm GF}) = \int d^4x \, \frac{g^2}{32\pi^2} \left[G_{\mu\nu}\tilde{G}_{\mu\nu}\right]\Big|_{t_{\rm GF}}$$

total top. charge on 20 randomly chosen gauge configurations



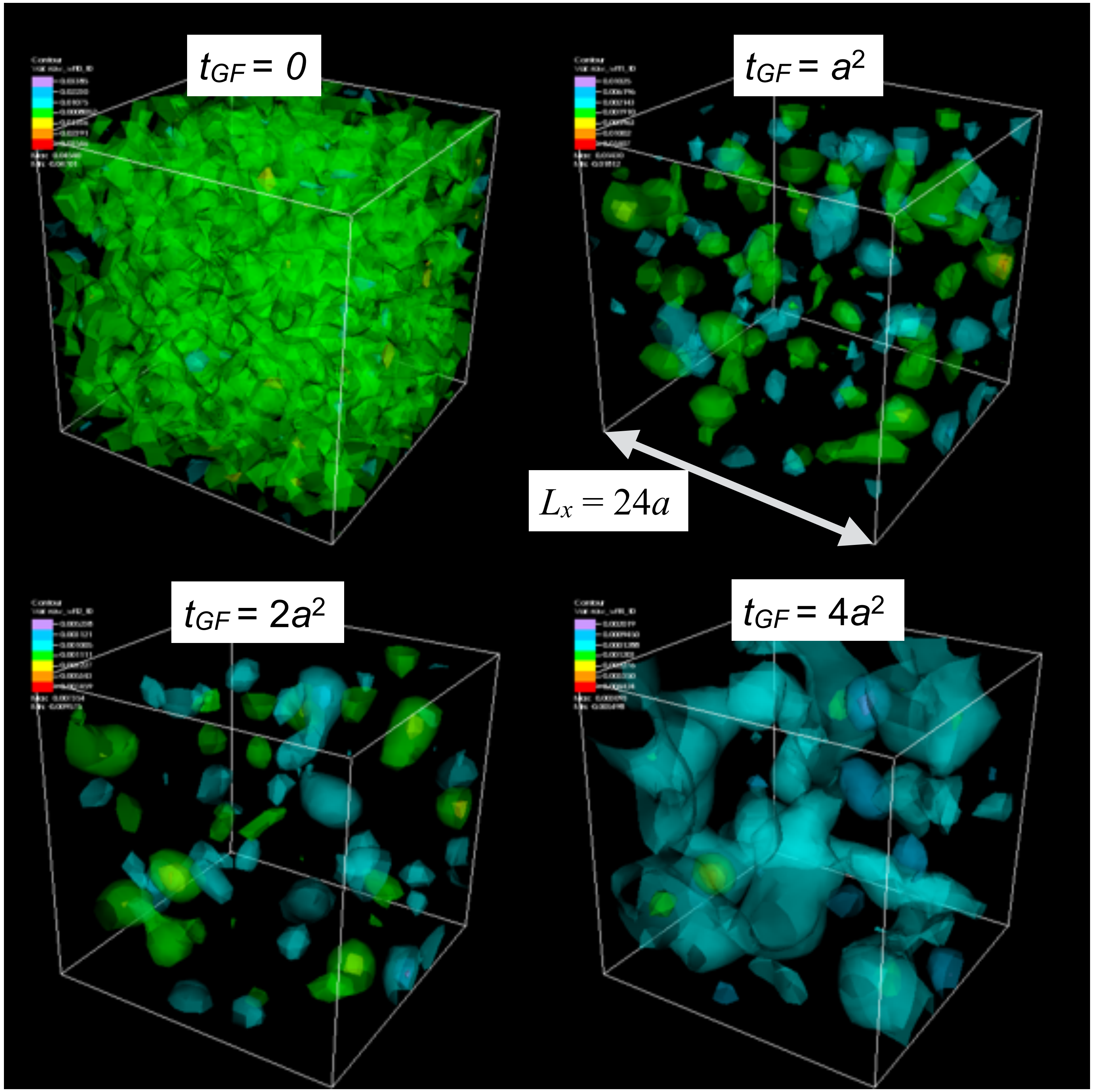
- effective scale $\Lambda_{\rm UV} \to (t_{\rm GF})^{-1/2}$
- smooth fields (reduce $|G_{\mu\nu}|$)

  $\Longleftrightarrow$ continuous "cooling"
- remove $G_{\mu\nu}$ dislocations

  $\Rightarrow$dynamical separation of top. sectors

  [M.Luscher, JHEP08:071; 1006.4518]
- diffusion of top.charge density

Gradient-Flowed Topological Charge Density



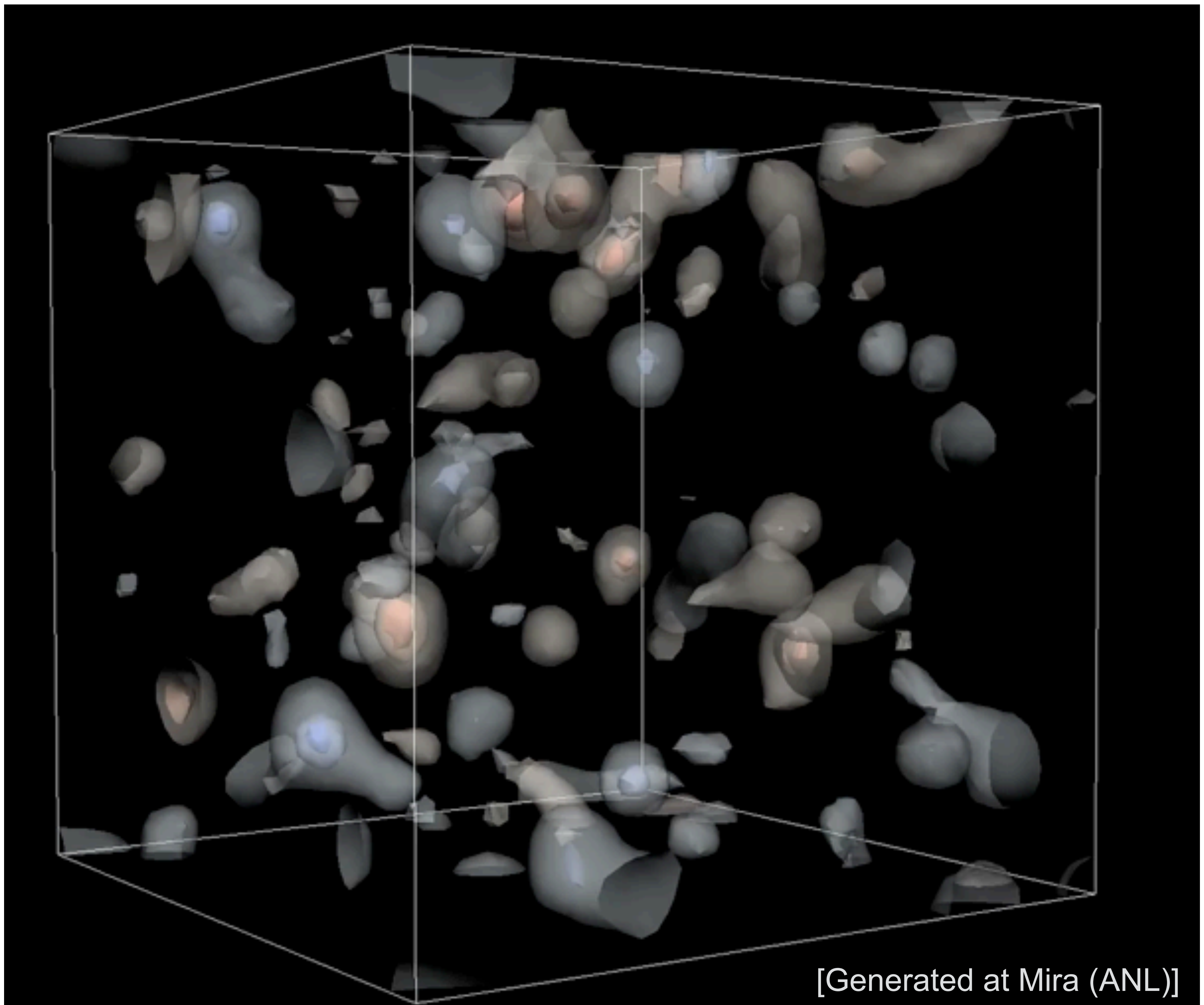
$24^3 \times 64$ lattice, $m\pi \approx 340\ MeV$

$$q(x) = \frac{g^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}$$
$$\approx \frac{1}{16\pi^2} \frac{1}{a^4} \mathrm{Tr}\left[G^{\mathrm{lat}}_{\mu\nu} \widetilde{G}^{\mathrm{lat}}_{\mu\nu}\right]$$
$$\propto (\mathbf{E} \cdot \mathbf{H})_{\mathrm{color}}$$

Gradient flow:

- effective scale $(\Lambda_{UV})^{-1} \rightarrow (t_{GF})^{1/2}$
- make fields smooth (reduce $|G_{\mu\nu}|$)
- remove dislocations⇒dynamical separation of topological sectors [M.Luscher, JHEP08:071; 1006.4518]
- 4D-diffusion (including time) of $q(x)$ $\langle\, q(x)q(0)\, \rangle \sim exp[\, -(x-y)^2 / 8t_{GF}\,]$

Tunneling Between Topology Sectors



5 fm=5·10^{-15}m

6 s video =5 fm / c =1.7·10^{-23} s real time

[Lattice QCD at the physical point]

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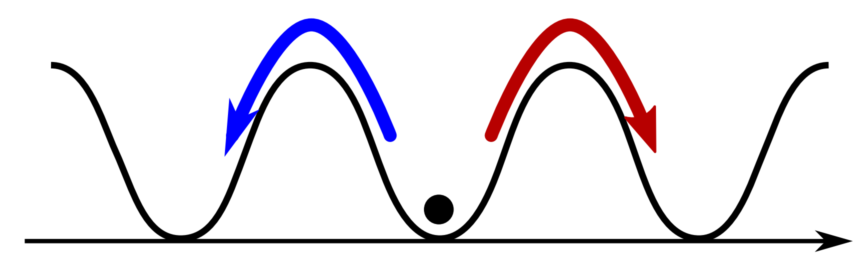
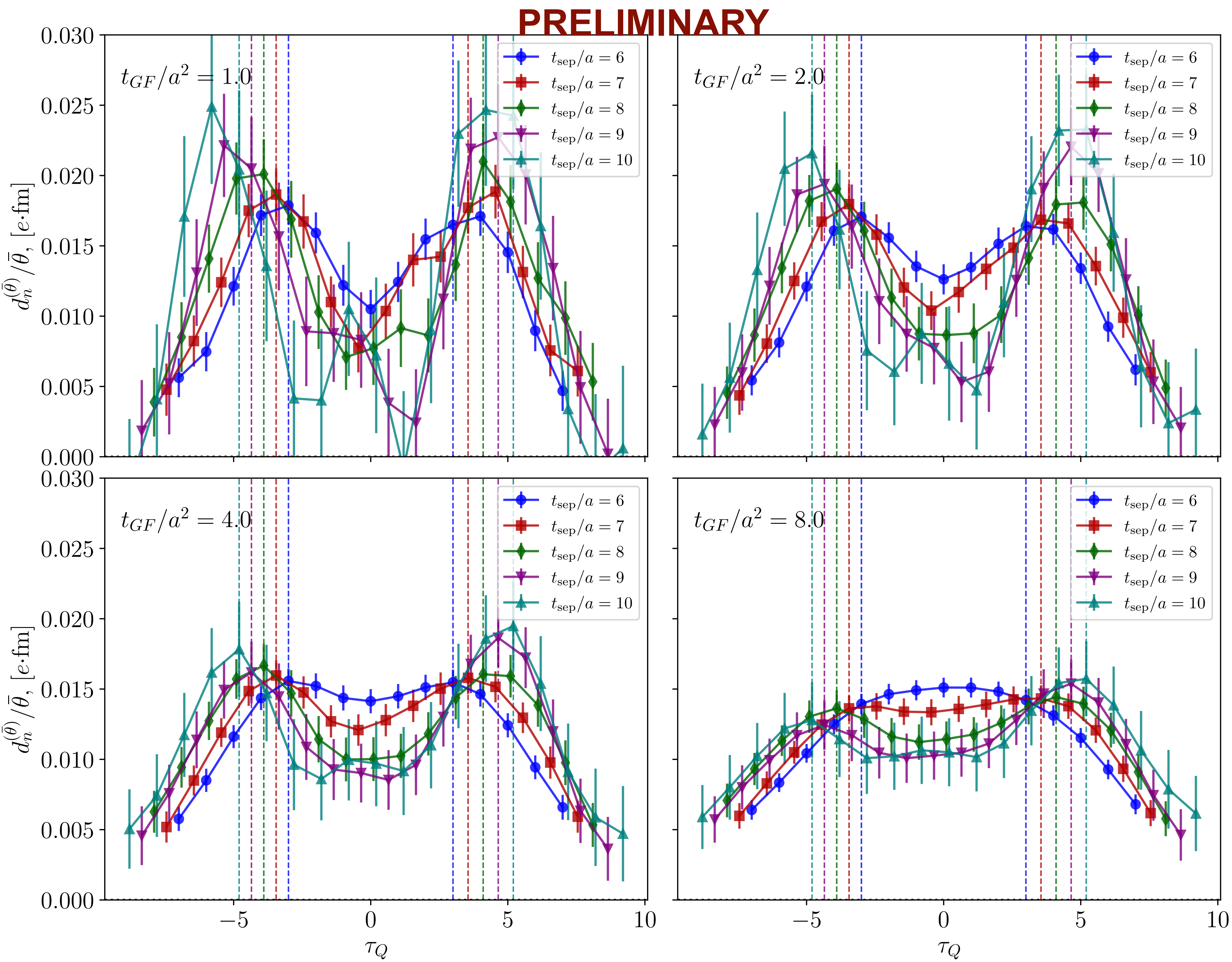
Instantons and Anti-Instantons :
Quantum tunneling of gluon field between topological sectors



CPv-QCD Θ-Vacuum :

$$|vac\rangle_\theta = \sum_Q e^{i\theta Q}\,|Q\rangle$$

Matrix Elements of $G\tilde{G}$ vs. Gradient Flow Time



Two effects observed:

1. Convergence to ground state matrix el.
2. Diffusion of top.charge for $t_{sep} \lesssim 7a$

PRELIMINARY estimates

$2md_n = F_3(0) \approx 0.11 \,..\, 0.13$

agree with form factor

Analysis of (τ_Q, t_{GF}) required to detangle

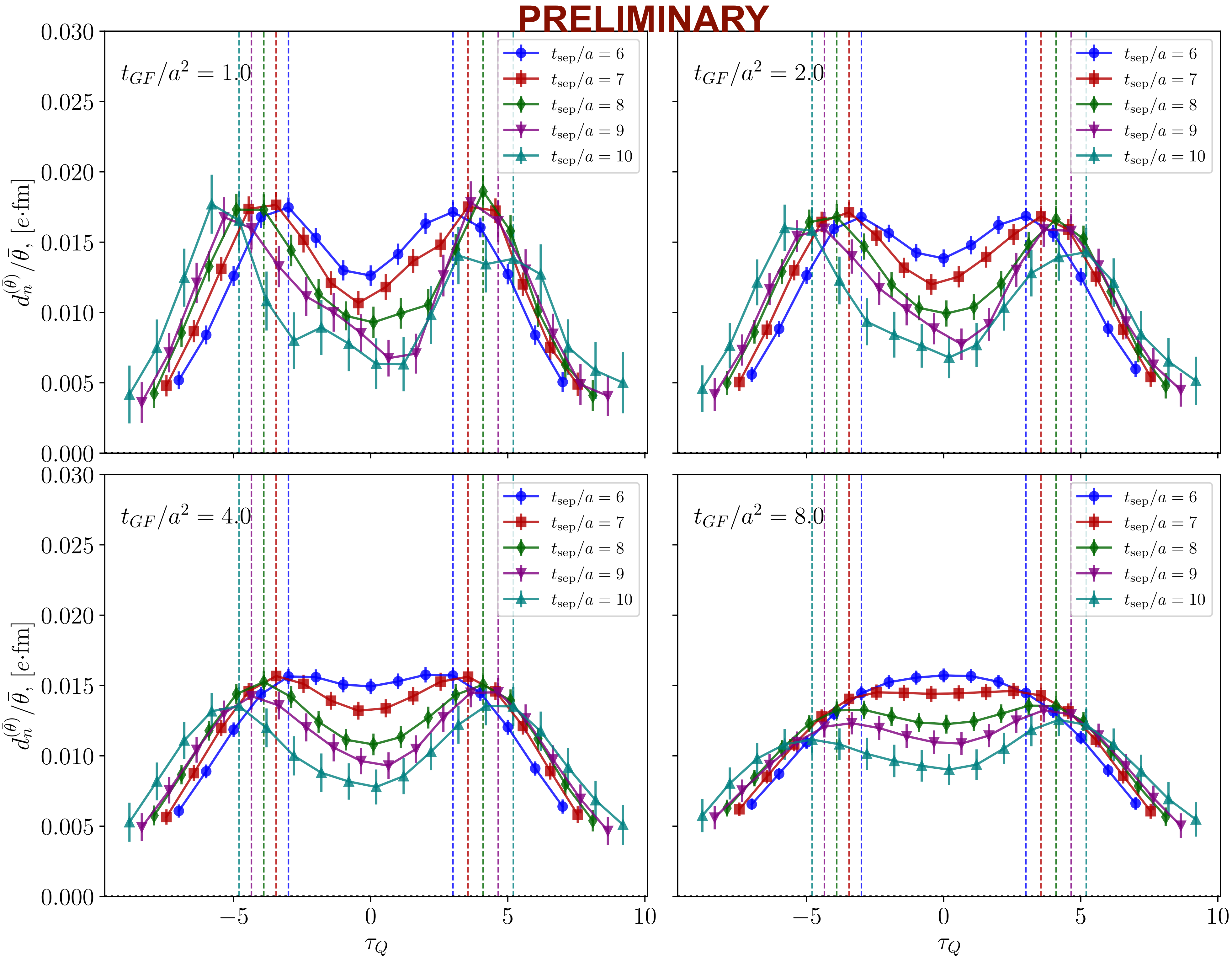
$$\langle N|G\widetilde{G}|N\rangle \,,$$

$$\langle N|G\widetilde{G}|N\rangle_{\rm exc} \,,$$

$$\langle {\rm vac}|G\widetilde{G}|N\bar{N}\rangle \,,$$

...

Matrix Elements of GG (Low-mode Improved)



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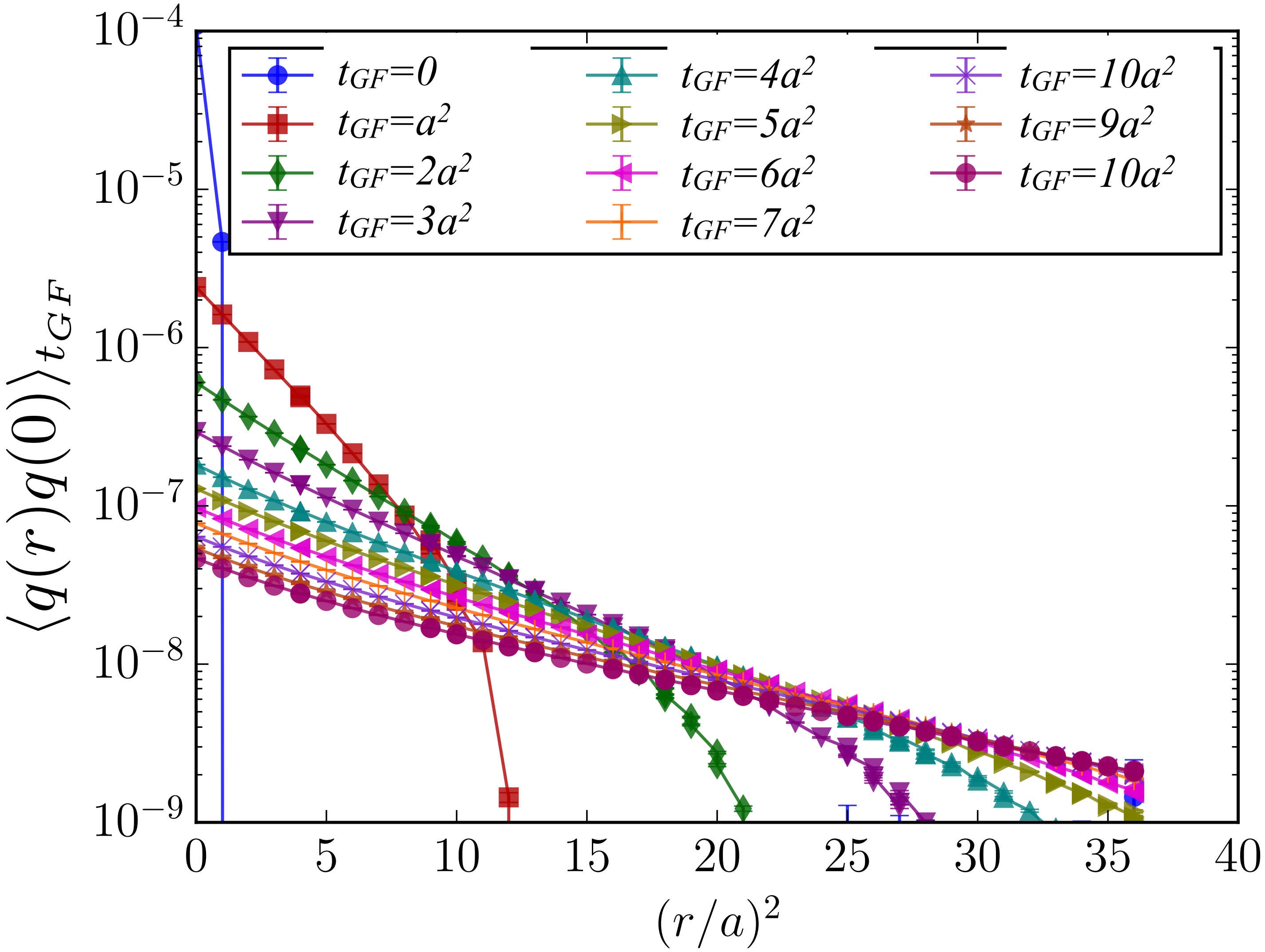
Gradient Flow as "Diffusion" of Top.Charge



Empirically for $r, \sqrt{t_{\rm GF}} \gg m_{\eta'}^{-1}$

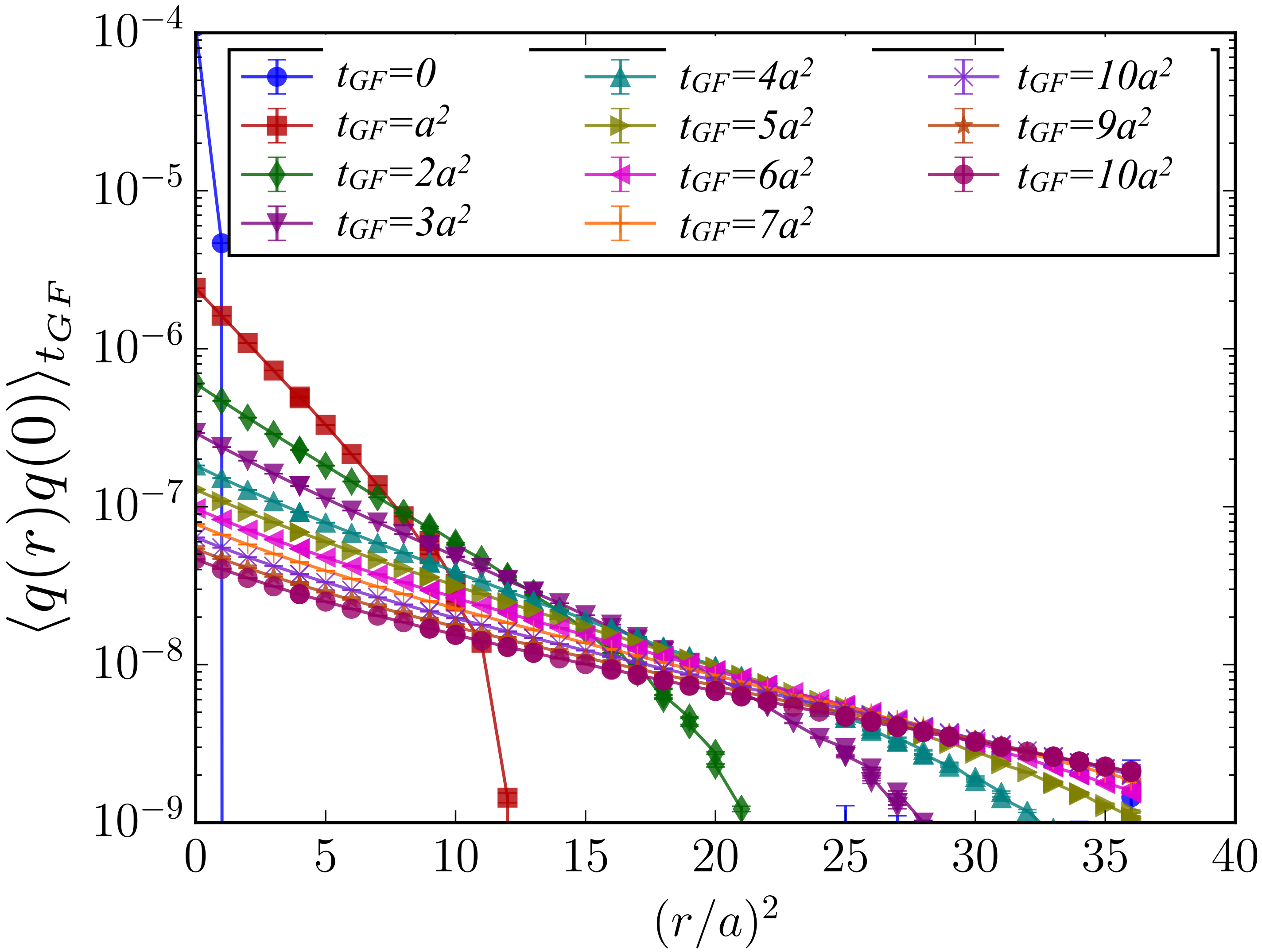
$$\langle\widetilde{q}(r)\widetilde{q}(0)\rangle \propto \exp\left[-\frac{r^2}{4r_Q^2(t_{\rm GF})}\right]$$

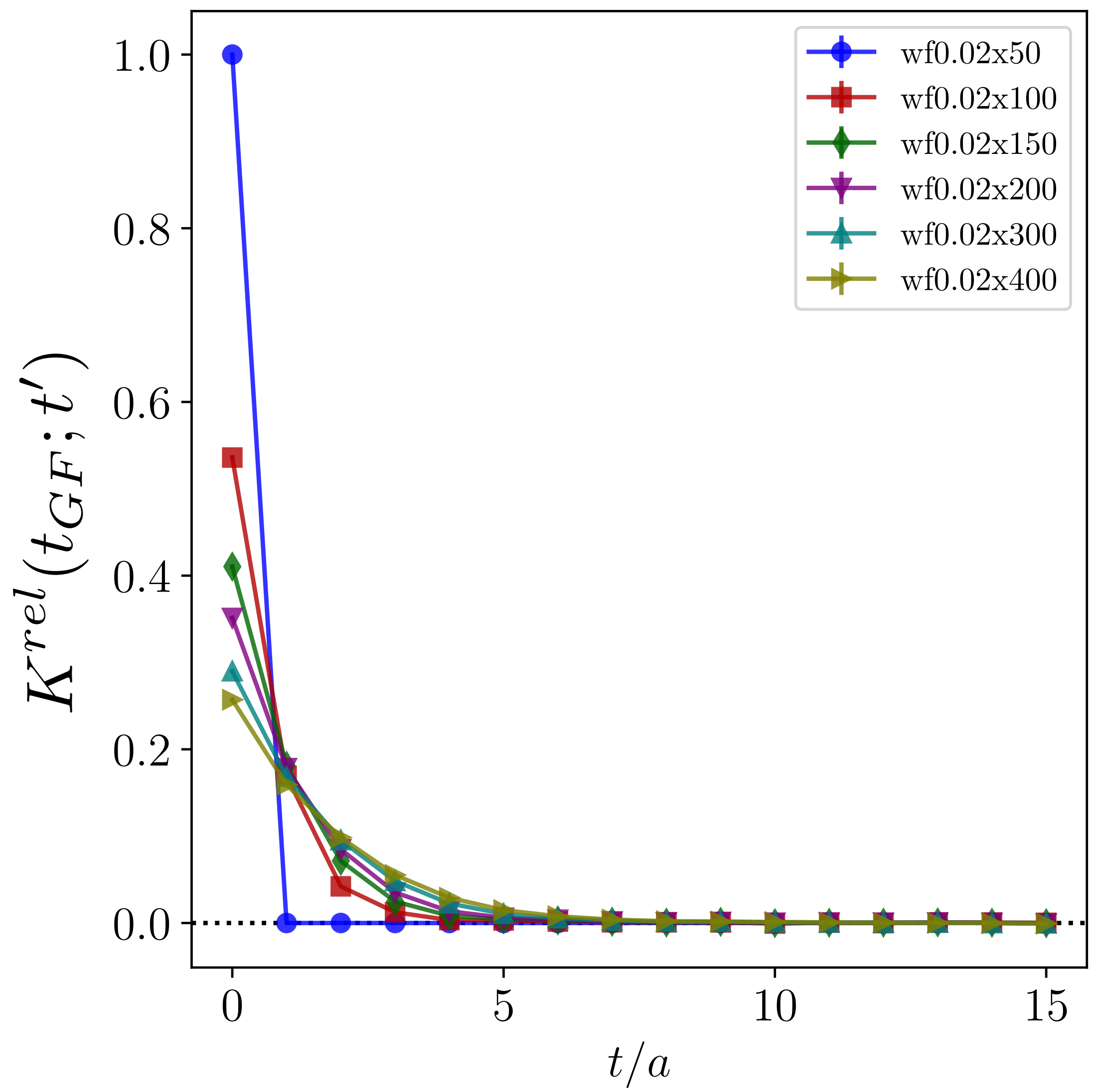
Diffusion of q(x) in Euclidean (lattice) time:

$$q(t_{GF};t) = \sum_{t'} K(t_{GF};t-t')q(t')$$

complications for matrix element analysis

Gradient Flow as "Diffusion" of Top.Charge





Diffusion of q(x) in Euclidean (lattice) time:

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complications for matrix element analysis

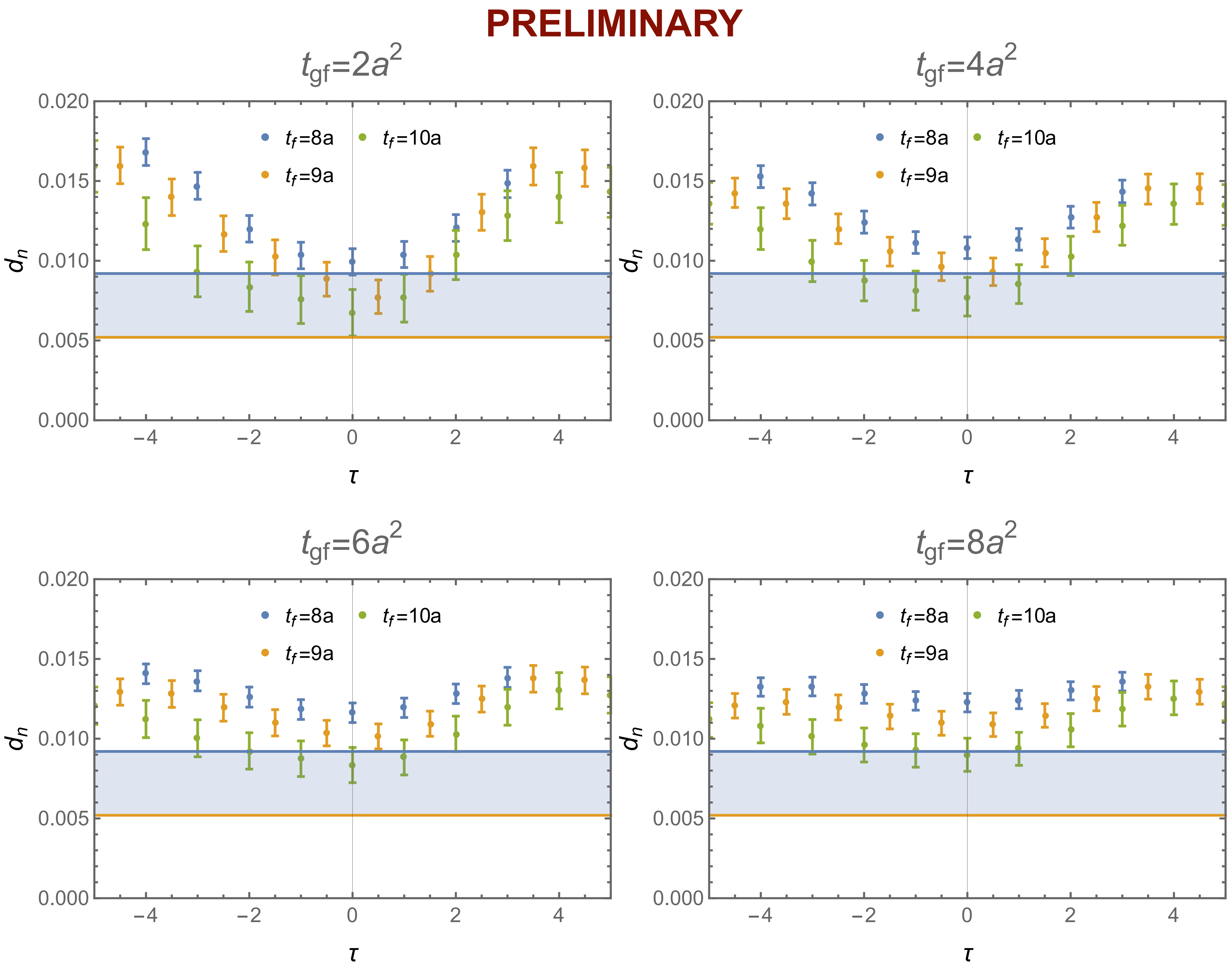
Extract kernel K from lattice data

$$\tilde{K}^{rel}(t_{GF};\omega) = \sqrt{\frac{\tilde{\chi}(t_{GF};\omega)}{\tilde{\chi}(t_{GF0};\omega)}}$$

where

$$\chi(t_{GF};t_2-t_1) = \langle q(t_{GF};t_2)q(t_{GF};t_1)\rangle$$

Combined Fit: Euclidean Time & Gradient Flow

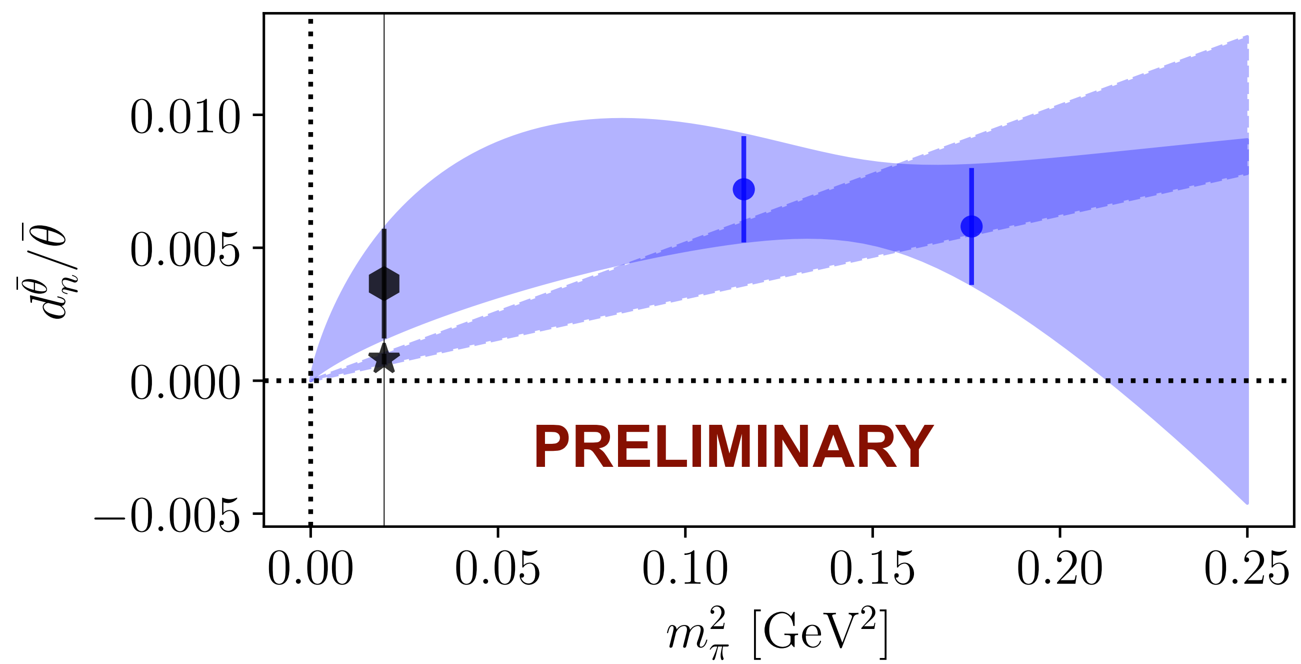


Combined Analysis of (τ_Q, t_{GF}) dependence:

- ground state $\langle N|G\widetilde{G}|N\rangle$
- excited state(s) $\langle N|G\widetilde{G}|N\rangle_{\rm exc}$
- "contact" amplitudes $\langle N\,(G\widetilde{G})|N\rangle$
- N$\bar{\text{N}}$ annihilation by G$\tilde{\text{G}}$ $\langle \text{vac}|G\widetilde{G}|N\bar{N}\rangle$
- grey band: "summation analysis"

Fit $\langle N(t_{sep})\;q(t_{GF},\tau_Q)\;\bar{N}(0)\rangle \quad\sim\quad K(t_{GF},|\tau_Q-\tau_Q'|)\otimes\langle N(t_{sep})\;q(\tau_Q)\;\bar{N}(0)\rangle$

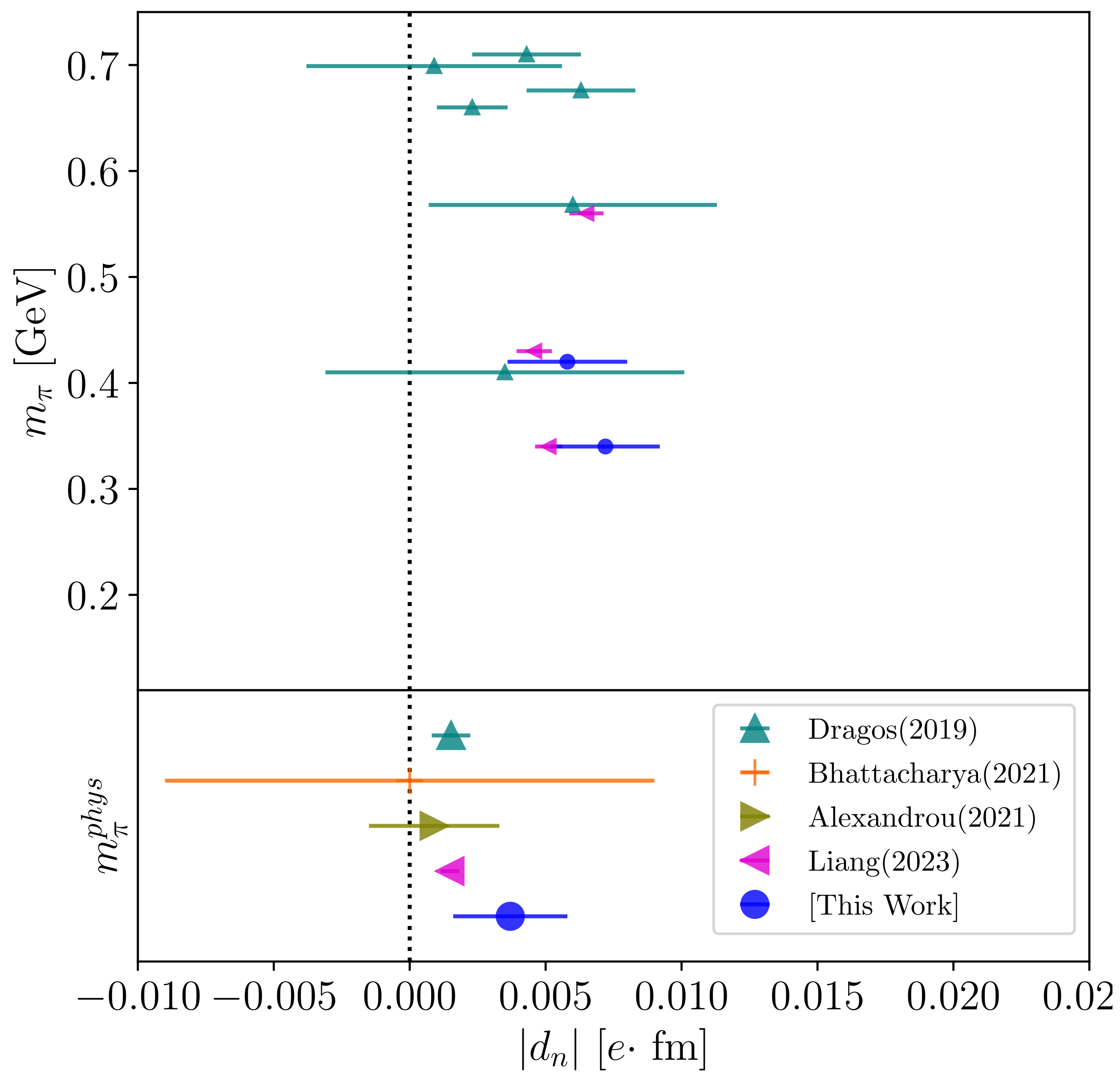
Extrapolation to the Physical Point



Chiral extrapolation
[Hockings, van Kolck (2005)]

$$d_n(m_\pi) = C_1 m_\pi^2 + C_2 m_\pi^2 \log \frac{m_\pi^2}{m_N^2}$$

(Only multiplicative O(a^2) corrections with chiral-symmetric lattice fermions)



Summary of neutron θ-EDM
from Lattice QCD

Summary

- Novel method to compute nEDM from local topological charge
  - *Results consistent with earlier works (and also with zero)*
  - *Potential method of choice for physical-point calculations with large V_4*
- Important cross-check for E.D. form-factor calculations
  - *Controllable space cut-off of "disconnected" CPv interaction*
- Current results compatible with zero;
  more statistics, additional pion-mass point needed
  - *Potential method of choice for physical-point calculations with large V_4*

Outlook: nEDM from other CPv Operators

- EDM of the Proton : background field requires only energy shift (acc. cancels out)
- Background field method may reduce errors in calculations of nEDM from other "disconnected" CPv interactions: Weinberg, isoscalar (strange quark cEDM?)
- Simplified contractions for 4-quark CPv operators (L-R, SUSY)

$$\mathcal{O}^{(1)}_{\varphi ud} = \frac{1}{3}(\bar{u}u)\,(\bar{d}\gamma_5 d) + (\bar{u}T^A u)\,(\bar{d}\gamma_5 T^A d) - [u \leftrightarrow d]$$

$$\mathcal{O}^{(1)}_{quqd} = (\bar{u}\gamma_5 u)\,(\bar{d}d) + (\bar{u}u)\,(\bar{d}\gamma_5 d) - [(\bar{u}u)(\bar{d}d) \leftrightarrow (\bar{u}d)(\bar{d}u)]$$

$$\mathcal{O}^{(8)}_{quqd} = (\bar{u}\gamma_5 T^A u)\,(\bar{d}T^A d) + (\bar{u}T^A u)\,(\bar{d}\gamma_5 T^A d) - [(\bar{u}u)(\bar{d}d) \leftrightarrow (\bar{u}d)(\bar{d}u)]$$