

INT Workshop 07/12/2022

# Spinodal Enhancement of Light Nuclei Yield Ratio in Relativistic Heavy-Ion Collisions

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Ref.: K. J. Sun, W. H. Zhou, L. W. Chen, C. M. Ko, and F. Li, R. Wang, and J. Xu, arXiv:2205.11010(2022)



# Contribution to this workshop

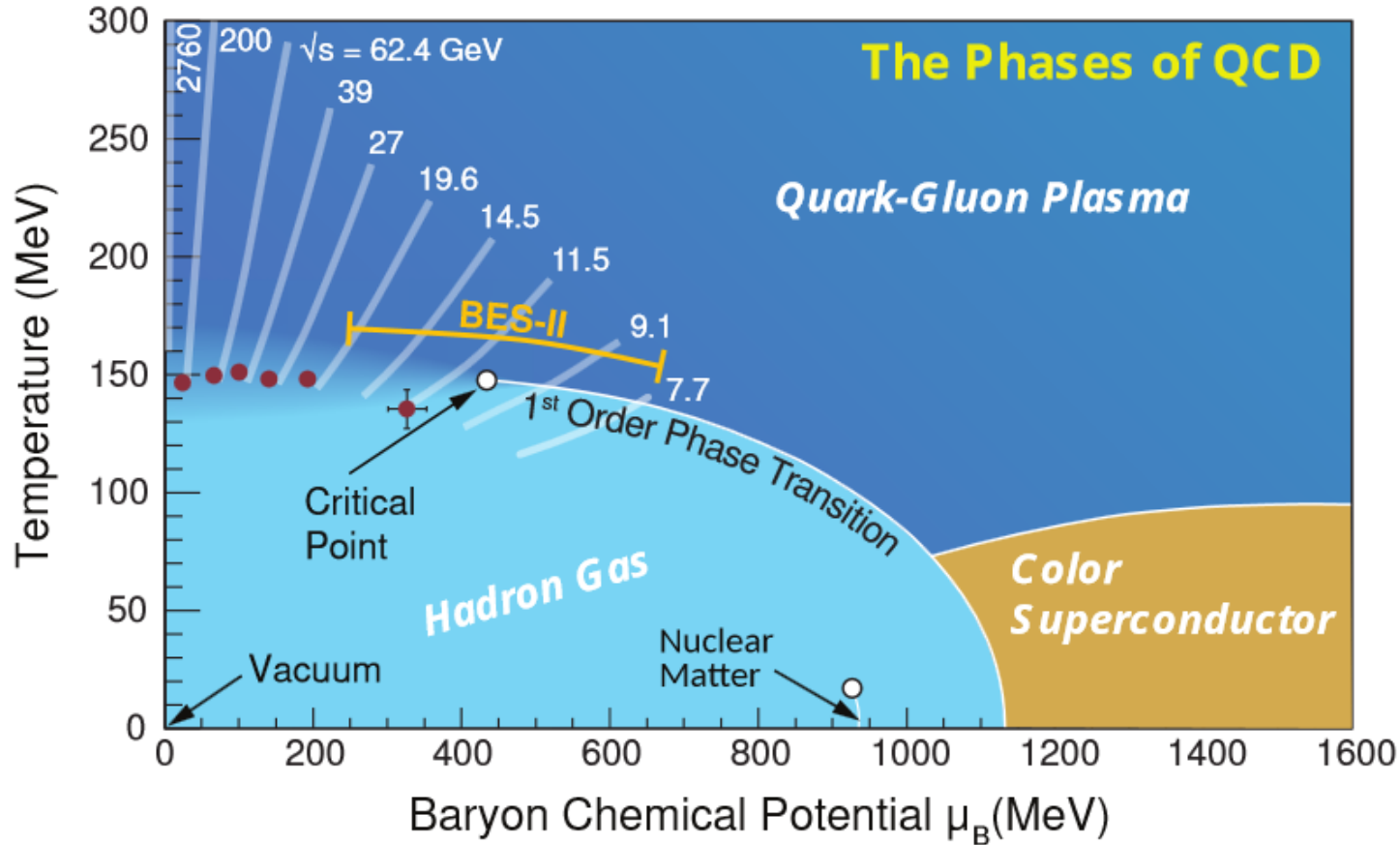
1. A flexible EoS of quark matter is developed and applied to relativistic heavy-ion collisions at 3-200 GeV.
2. The effect of a first-order QCD phase transition on light nuclei production is studied

# Outline

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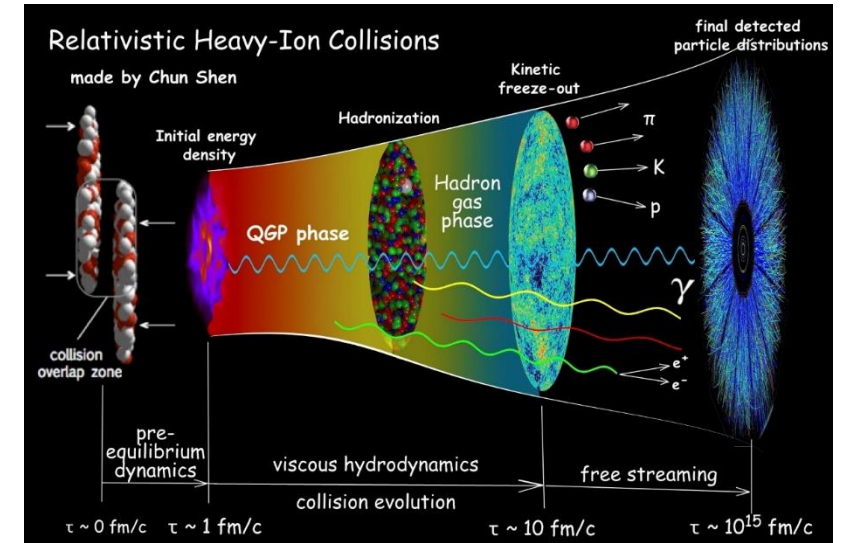
1. Motivation: Why light nuclei? Why  $N_t N_p / N_d^2$  ( $tp/d^2$ )?
2. Spinodal enhancement of  $tp/d^2$  from the first-order QCD phase transition
3. Summary and Outlook

# 1.1 QCD phase transition & light nuclei production (1)



**Critical Point: long-range correlation**

**First-order Phase Transition: Spinodal instability**



**Neutron star merger**



From Wiki.

X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017) A. Bzdak et al., Phys. Rept. 853, 1 (2020);

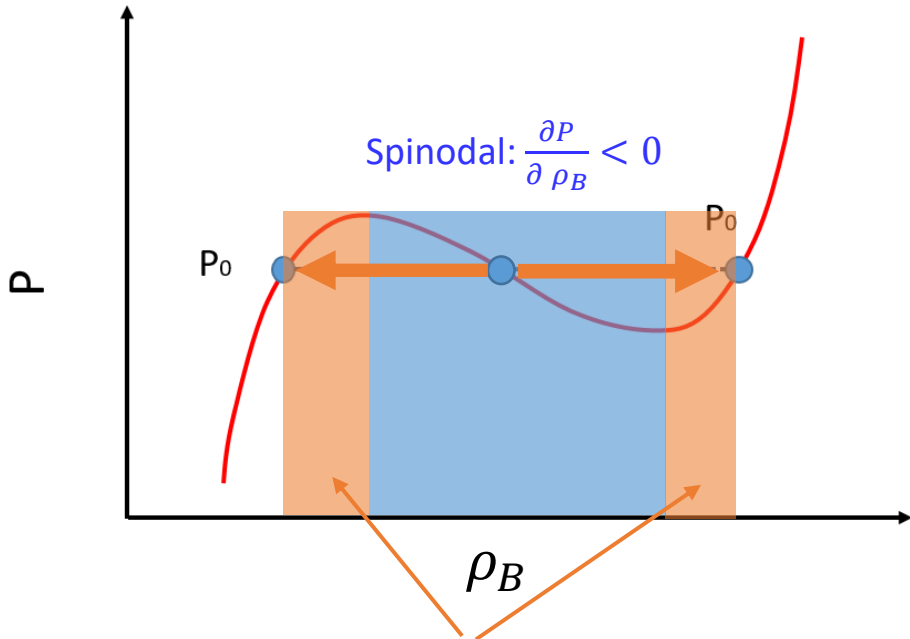
W. J. Fu, J. M. Pawłowski, F. Renneke, Phys. Rev. D101, 054032 (2020); LIGO & VIRGO, Phys. Rev. Lett. 119, 161101 (2017)

# 1.2 1<sup>st</sup> order QCD phase transition & light nuclei production (2)

P. Chomaz, M. Clonna, and J. Randrup, Phys. Rep. 389, 263 (2004)

Phase separation, spinodal decomposition(SD)

See talk by Maria Colonna



Metastable

Density matrix formulation (coalescence)

$$N_d \propto \langle d | \hat{\rho}_s | d \rangle \quad N_t \propto \langle t | \hat{\rho}_s | t \rangle$$



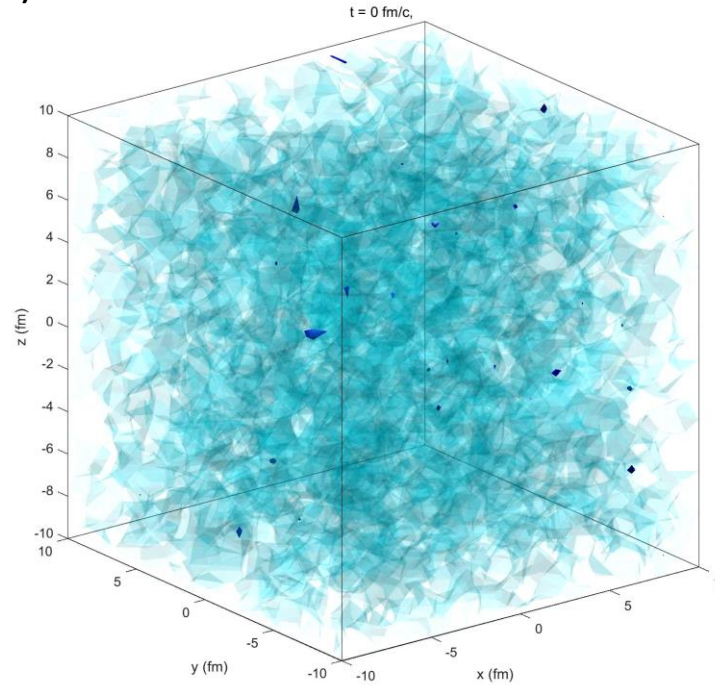
$$\frac{N_t N_p}{N_d^2} \propto \Delta \rho_n$$

$$\frac{3 \text{ pairs}}{2 \text{ pairs}} \sim 1 \text{ pair}$$

$$\Delta \rho_n = \frac{\int dx (\delta \rho(x))^2}{\int dx \rho_0^2}$$

characterizes density inhomogeneity

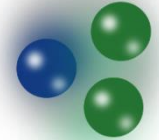
K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu, Phys. Lett. B 774, 103 (2017)



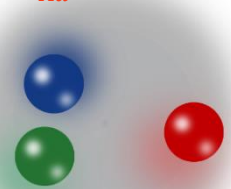
d (np)  
B=2.2 MeV



<sup>3</sup>He (npp)  
B=7.7 MeV



<sup>3</sup>H (npΛ)  
B<sub>Λd</sub> = 0.41 MeV



See [PLB 816, 136258 (2021)] for critical effects on  $N_t N_p / N_d^2$

# 1.3 QCD phase transition & light nuclei production

(3)

light nuclei production & QCD phase transition

2012 2014 2015 2016 2017 2018 2019 2020 2021 2022

Other works

First-order phase transition & composite particle production

- ▶ J. Steinheimer et al. PRC 87, 054903 (2013)
- ▶ PRL 109, 212301 (2012)(Hydrodynamics)
- ▶ JHEP 12, 122(2019)(Machine learning)

Baryon clustering near the critical point

- ▶ E. Shuryak, J.M.Torres-Rincon et al., PRC 100, 024903(2019)
- ▶ PRC 101,034914(2020)
- ▶ EPJA 56, 241(2020)
- ▶ PRC 104,024908(2021)

Background effects

- ▶ S. Wu et al.,PRC 106,034905(2022)

Our works

Probing QCD phase transition with light nuclei production

- ▶ PLB 774, 103 (2017)  
K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu

$$\frac{N_t N_p}{N_\alpha^2} \approx \frac{1}{2\sqrt{3}} [1 + \Delta\rho_n]$$

- ▶ PLB 781, 499 (2018)
- ▶ PLB 816, 136258 (2021)(criticality)

1st-order QCD phase transition

- ▶ PRD 103, 014006 (2021)
- ▶ EPJA 57, 313 (2021)(Transport)
- ▶ **arxiv:2205.11010 (Transport)**  
**(First-order PT in BES)**

2. Spinodal enhancement of  $tp/d^2$  from the first-order QCD phase transition

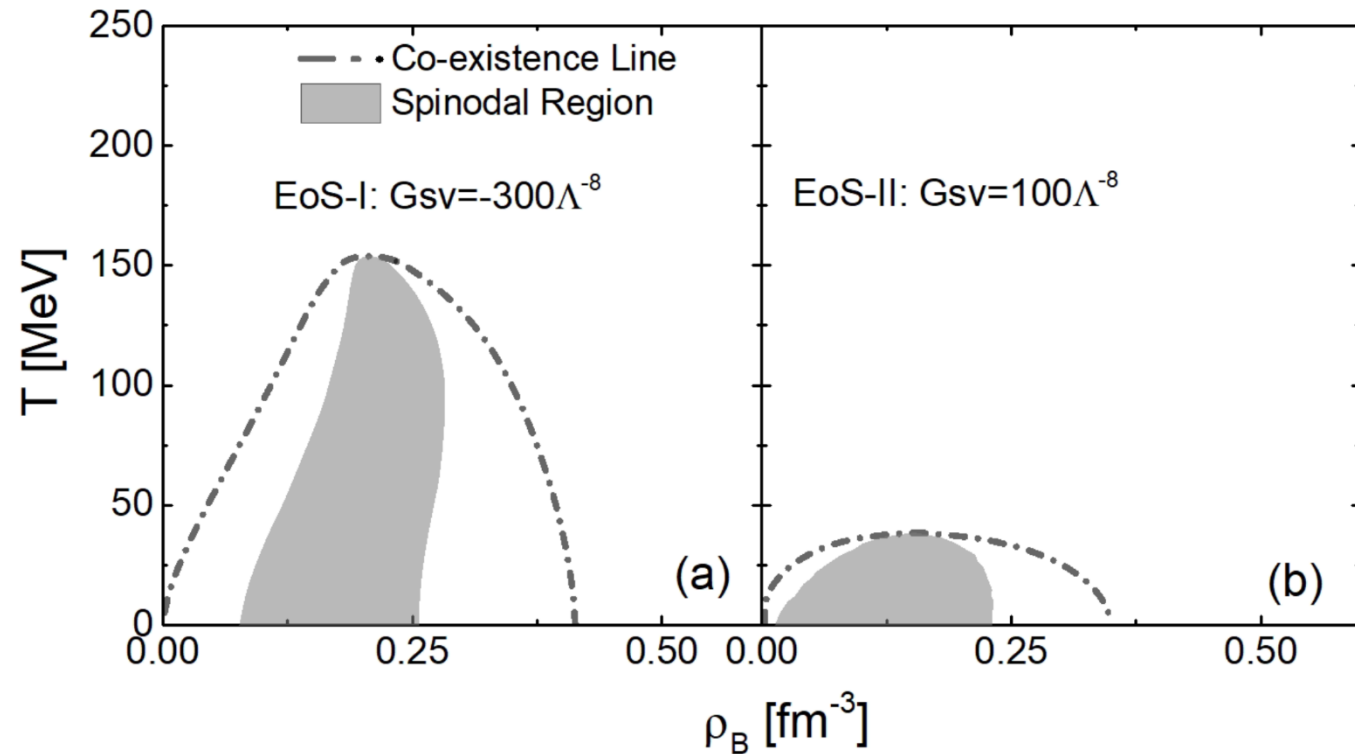
# 2.1 Equation of State (extended NJL model) (4)

The eNJL provides a flexible equation of state (EoS). The critical temperature can be easily changed by varying the strength of the scalar-vector interaction without affecting the vacuum properties.

Lagrangian density for eNJL

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m})\psi + G_S \sum_{a=0}^3 [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] \\ & - K \{ [\det[\bar{\psi}(1 + \gamma_5)\psi] + \det[\bar{\psi}(1 - \gamma_5)\psi]] \} \\ & + G_{SV} \left\{ \sum_{a=1}^3 [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] \right\} \\ & \times \left\{ \sum_{a=1}^3 [(\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\lambda^a\psi)^2] \right\}, \end{aligned}$$

$\Lambda$ [MeV]	602.3	$M_{u,d}$ [MeV]	367.7
$G\Lambda^2$	1.835	$M_s$ [MeV]	549.5
$K\Lambda^5$	12.36	$(\langle\bar{u}u\rangle)^{1/3}$ [MeV]	-241.9
$m_{u,d}$ [MeV]	5.5	$(\langle\bar{s}s\rangle)^{1/3}$ [MeV]	-257.7
$m_s$ [MeV]	140.7		



M. Buballa, Phys. Rept. 407, 205 (2005)

K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)



# 2.2 Box Simulation

(5)

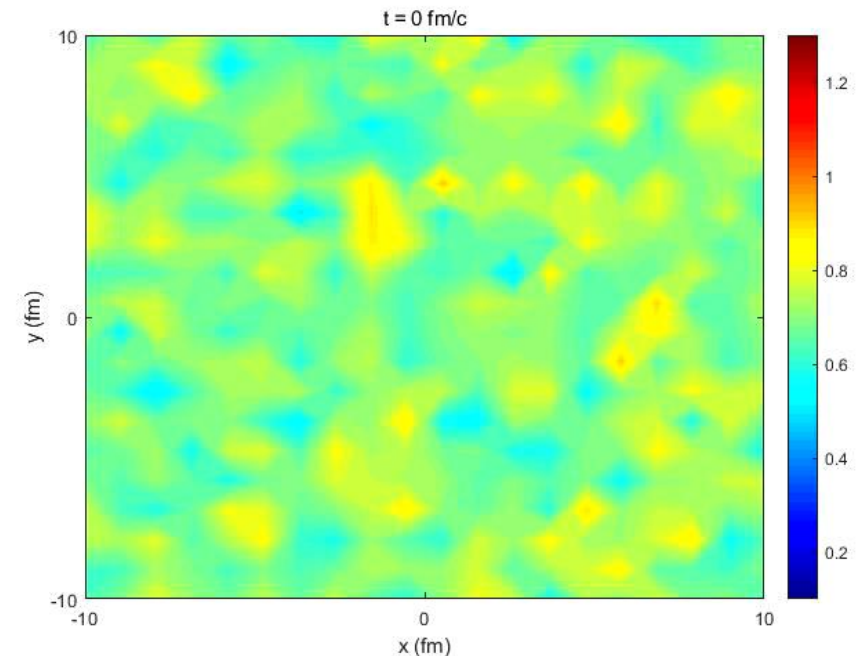
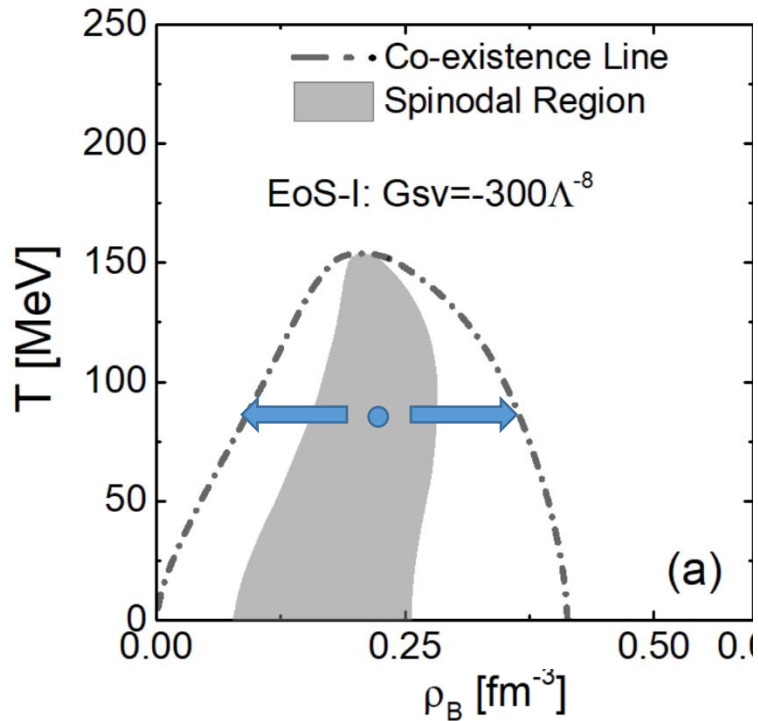
Effective mass:

$$\begin{aligned}
 M_u &= m_u - 4G_S\phi_u + 2K\phi_d\phi_s \\
 &\quad - 2G_{SV}(\rho_u + \rho_d)^2(\phi_u + \phi_d), \\
 M_d &= m_d - 4G_S\phi_d + 2K\phi_u\phi_s \\
 &\quad - 2G_{SV}(\rho_u + \rho_d)^2(\phi_u + \phi_d), \\
 M_s &= m_s - 4G_S\phi_s + 2K\phi_u\phi_d
 \end{aligned}
 \quad
 \begin{aligned}
 \phi_i &= -2N_c \int_0^\Lambda \frac{d^3p}{(2\pi\hbar)^3} \frac{M_i}{E_i} (1 - f_i - \bar{f}_i) \\
 \rho_i &= 2N_c \int_0^\Lambda \frac{d^3p}{(2\pi\hbar)^3} (f_i - \bar{f}_i)
 \end{aligned}$$

Test-particle method: [J. Xu, arXiv:1904.00131 \(2019\)](#)

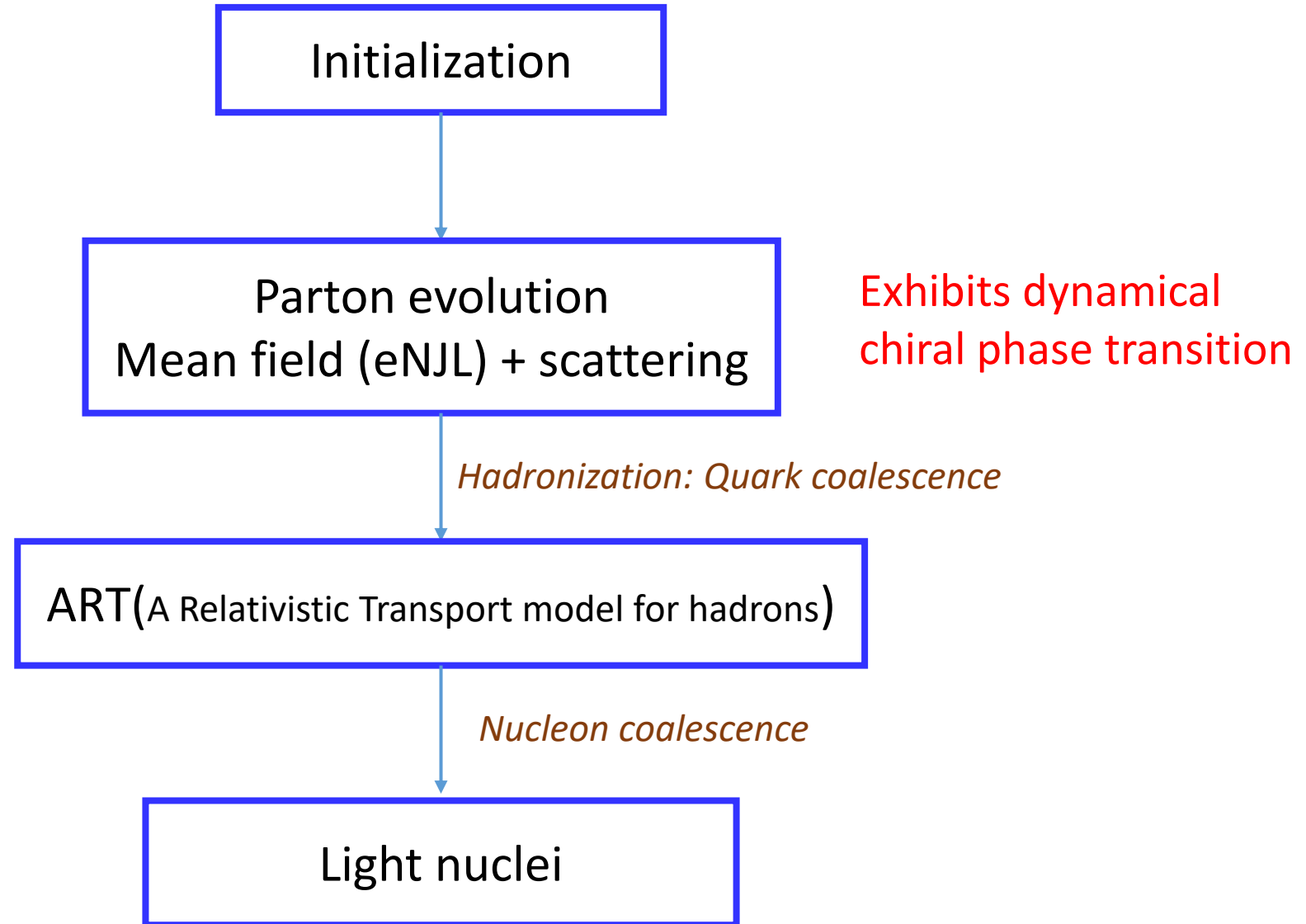
$$\begin{aligned}
 \frac{d\mathbf{r}}{dt} &= \mathbf{v}, \\
 \frac{d\mathbf{p}}{dt} &= -\frac{M}{E^*} \nabla_r M \pm \mathbf{E} \pm \mathbf{v} \times \mathbf{B}.
 \end{aligned}$$

Strong EM fields



# 2.3 Relativistic Heavy-Ion Collisions

(6)



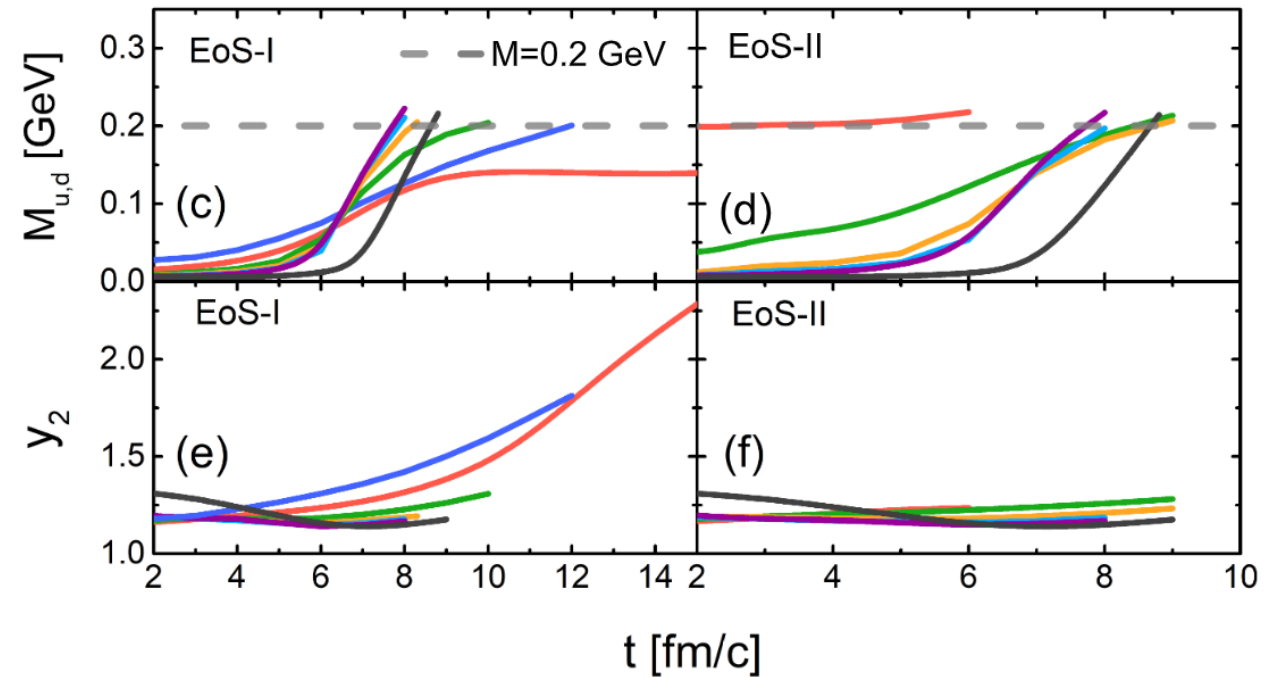
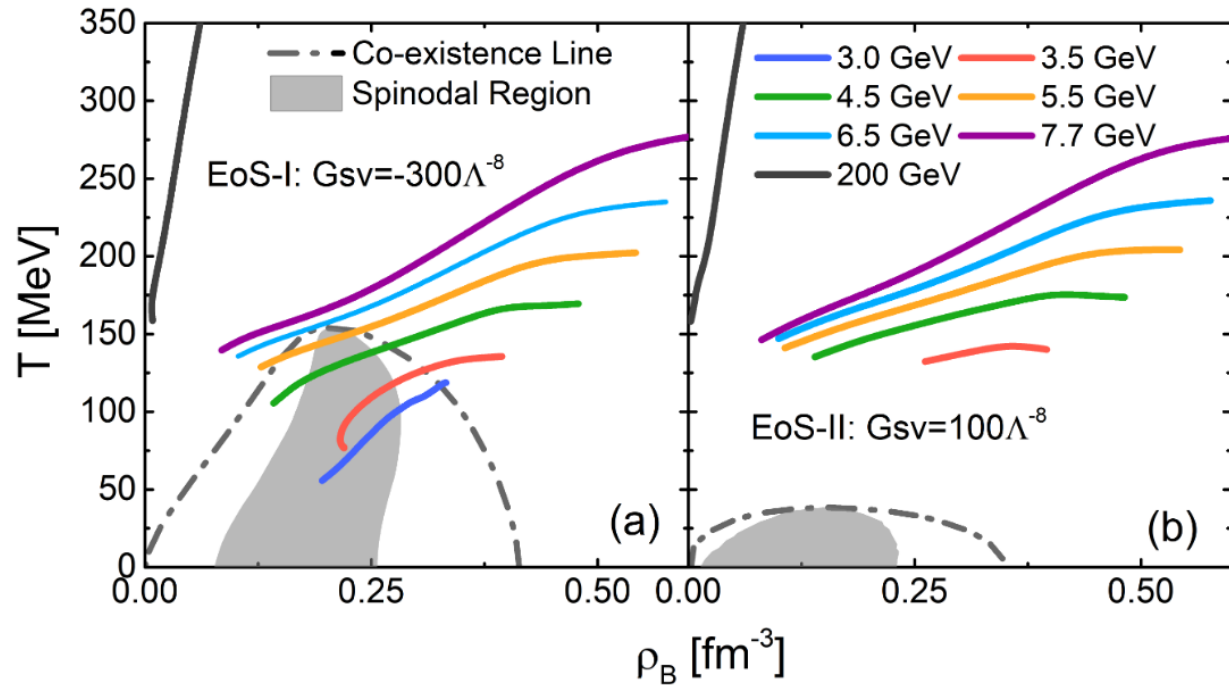
# 2.4 Trajectories in the phase diagram

(7)

Phase trajectories of central cells in the phase diagram

$$\overline{\rho^N} = \frac{\int d\mathbf{x} \rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x} \rho(\mathbf{x})}$$

$$y_2 = \frac{[\int d\mathbf{x} \rho(\mathbf{x})][\int d\mathbf{x} \rho^3(\mathbf{x})]}{[\int d\mathbf{x} \rho^2(\mathbf{x})]^2}$$

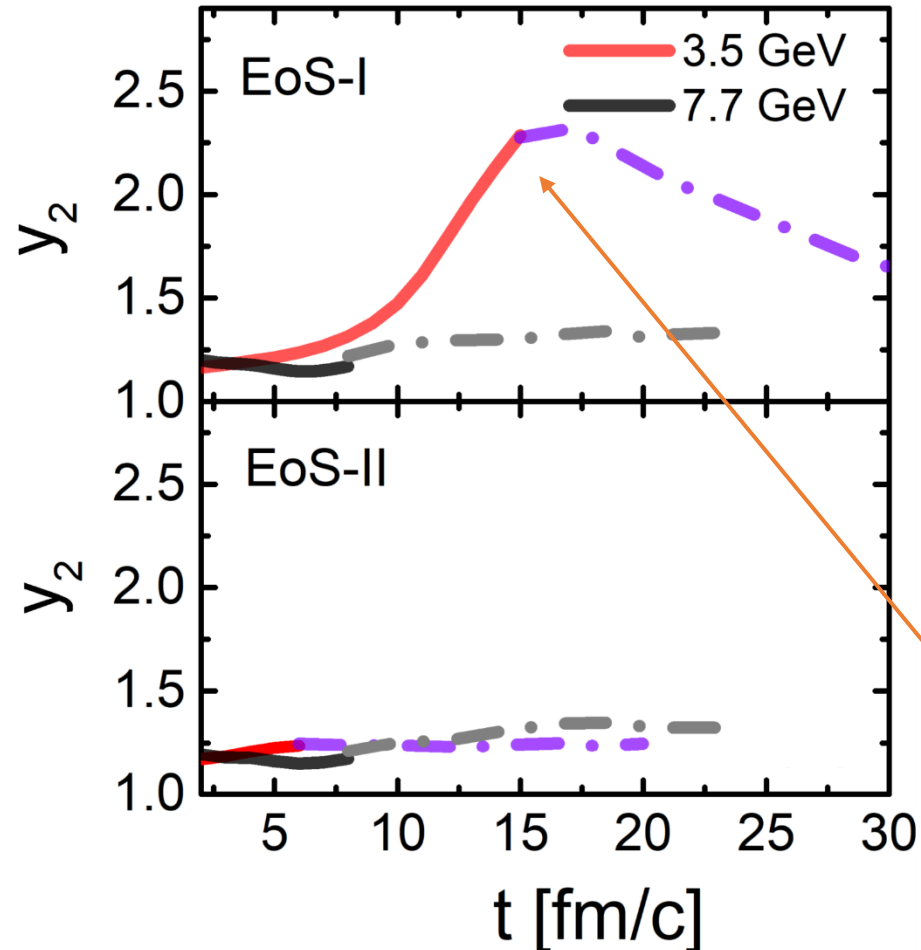


## 2.5 Memory Effects

(8)

### Survival of density fluctuation in an expanding fireball

#### Off-equilibrium effects



Density moment:

$$\overline{\rho^N} = \frac{\int d\mathbf{x} \rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x} \rho(\mathbf{x})}$$

$$y_2 = \frac{[\int d\mathbf{x} \rho(\mathbf{x})][\int d\mathbf{x} \rho^3(\mathbf{x})]}{[\int d\mathbf{x} \rho^2(\mathbf{x})]^2}$$

If the expansion is self-similar or scale invariant

$$\rho(\lambda(t)x, t) = \alpha(t)\rho(x, t_h)$$

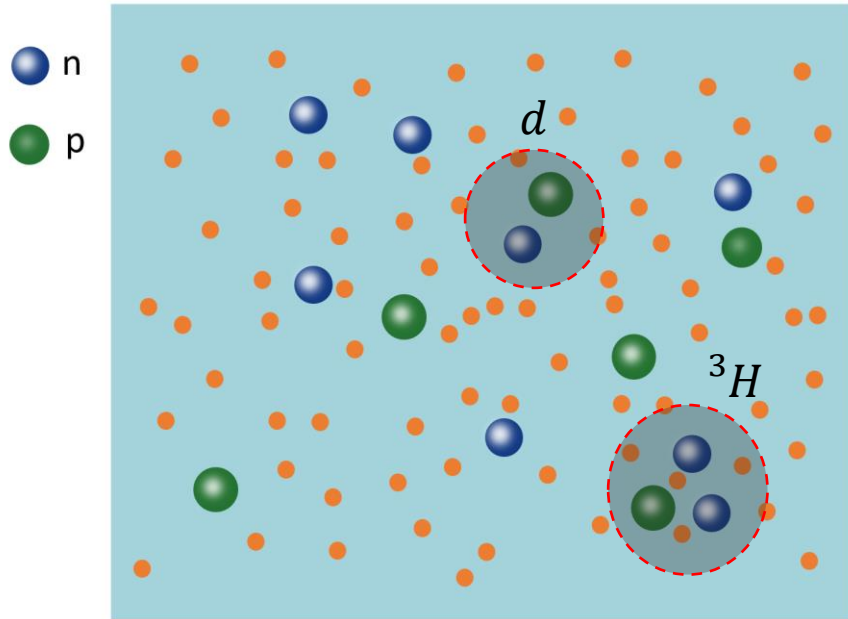
then  $y_2(t) = y_2(t_h)$ , i.e., remains a constant

K.J.Sun et al., Eur.Phys.J.A 57 (2021) 11, 313

'Memory effects': Large density inhomogeneity survives to kinetic freezeout

# 2.6: light nuclei production

(9)



$$N_d \propto W_d\left(\frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}}\right)$$

$$N_t \propto W_t\left(\frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}}, \frac{x_1 + x_2 - 2x_3}{\sqrt{6}}, \frac{p_1 + p_2 - 2p_3}{\sqrt{6}}\right)$$

Wigner function:  $W_d(r, k) = 8 \exp\left(-\frac{r^2}{\sigma_d^2} - \sigma_d^2 k^2\right) \quad \sigma_d \approx 2.26 \text{ fm}$

$$W_t(\rho, \lambda, k_\rho, k_\lambda) = 8^2 \exp\left(-\frac{\rho^2}{\sigma_t^2} - \frac{\lambda^2}{\sigma_t^2} - \sigma_t^2 k_\rho^2 - \sigma_t^2 k_\lambda^2\right) \quad \sigma_t \approx 1.59 \text{ fm}$$

R. Scheibl and U. W. Heinz, Phys. Rev. C59. 1585(1999)

some references on light nuclei production:

J. Chen et al., Phys. Rept. 760,1 (2018)

P. Braun-Munzinger and B. Donigus, Nucl. Phys. A987, 144(2019)

B. Donigus, Int. J. Mod. Phys. E29, 2040001 (2020)

D. Oliinychenko, arXiv:2003,05476(2020)

S. Bazak et al., Mod. Phys. Lett. A3, 1850142 (2018)

W. Zhao et al., Phys. Rev. C98,054905 (2018)

F. Bellini et al., Phys. Rev. C99,054905 (2019)

K.J.Sun and C. M. Ko, Phys. Lett. B792, 132(2019)

D. Oliinychenko et al., Phys. Rev. C99, 044907(2019)

X. Xu and R. Rapp, Eur.Phys.J. A55,68(2019)

Y. Cai et al., Phys.Rev. C100 , 024911 (2019)

V. Vovchenko et al., arXiv:2004.04411(2020)

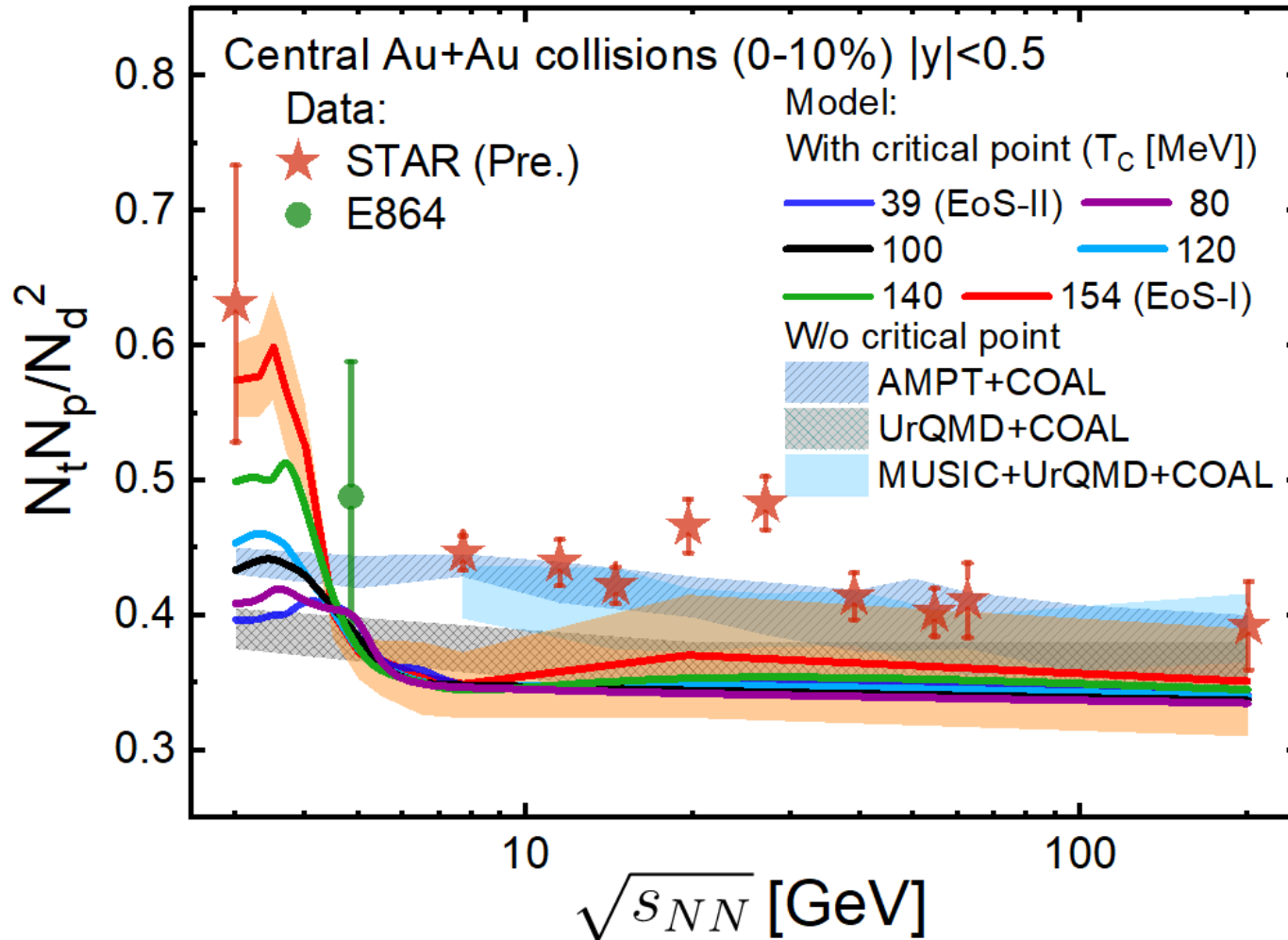
S. Mrowczynski, arXiv:2004.07029(2020)

K. Blum and M. Takimoto, Phys.Rev.C99,

044913(2019) .....

# 2.7 Collision energy dependence

(10)



1. Without a critical point:  
The energy dependence of  $tp/d^2$  is almost flat.
2. With a first-order phase transition:  
The spinodal instability induced enhancement of  $tp/d^2$  during the first-order phase transition increases as increasing the critical temperature.

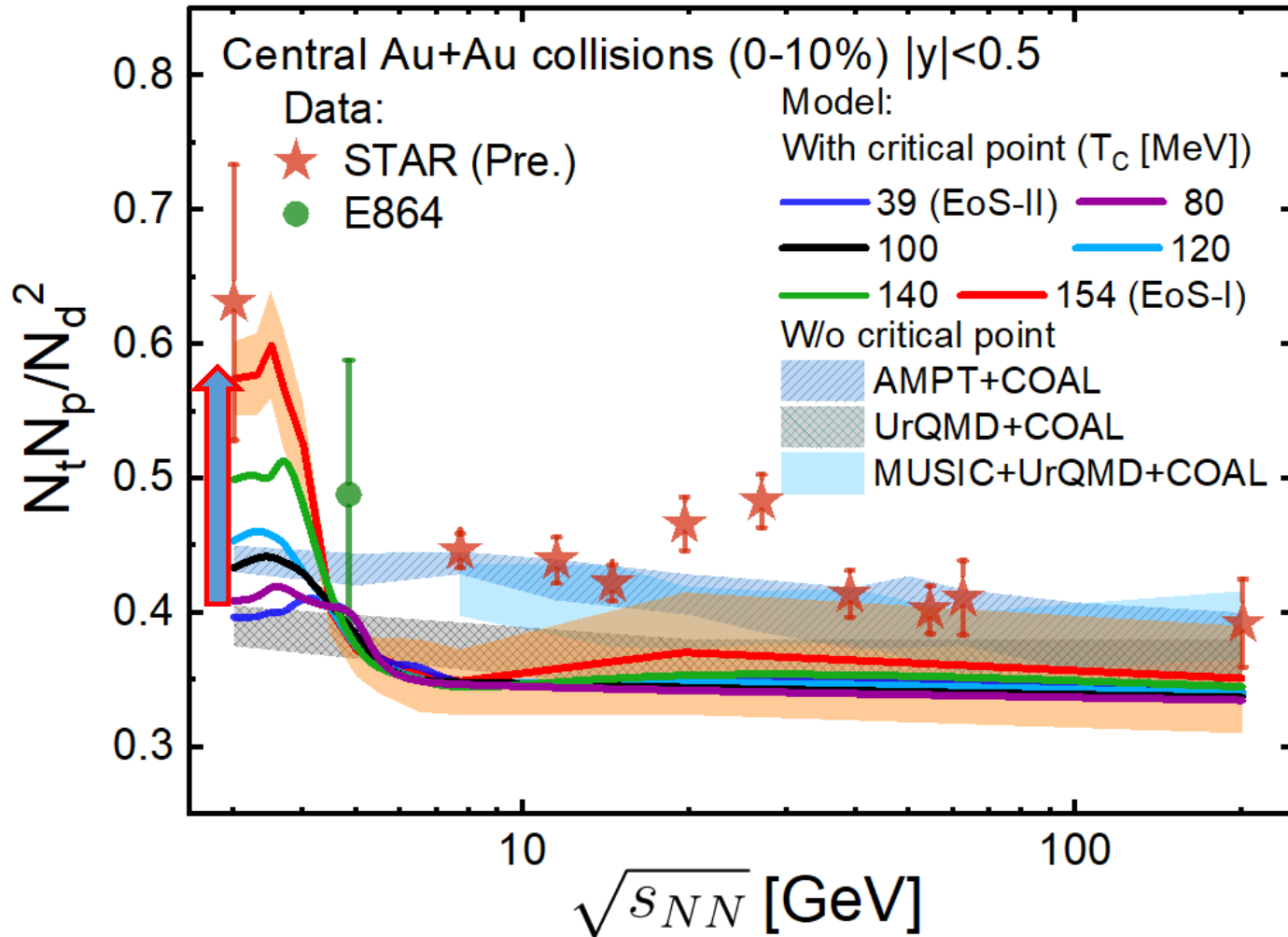
STAR, arXiv:2209.08058(2022)

Hui Liu (STAR), QM2022

T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).

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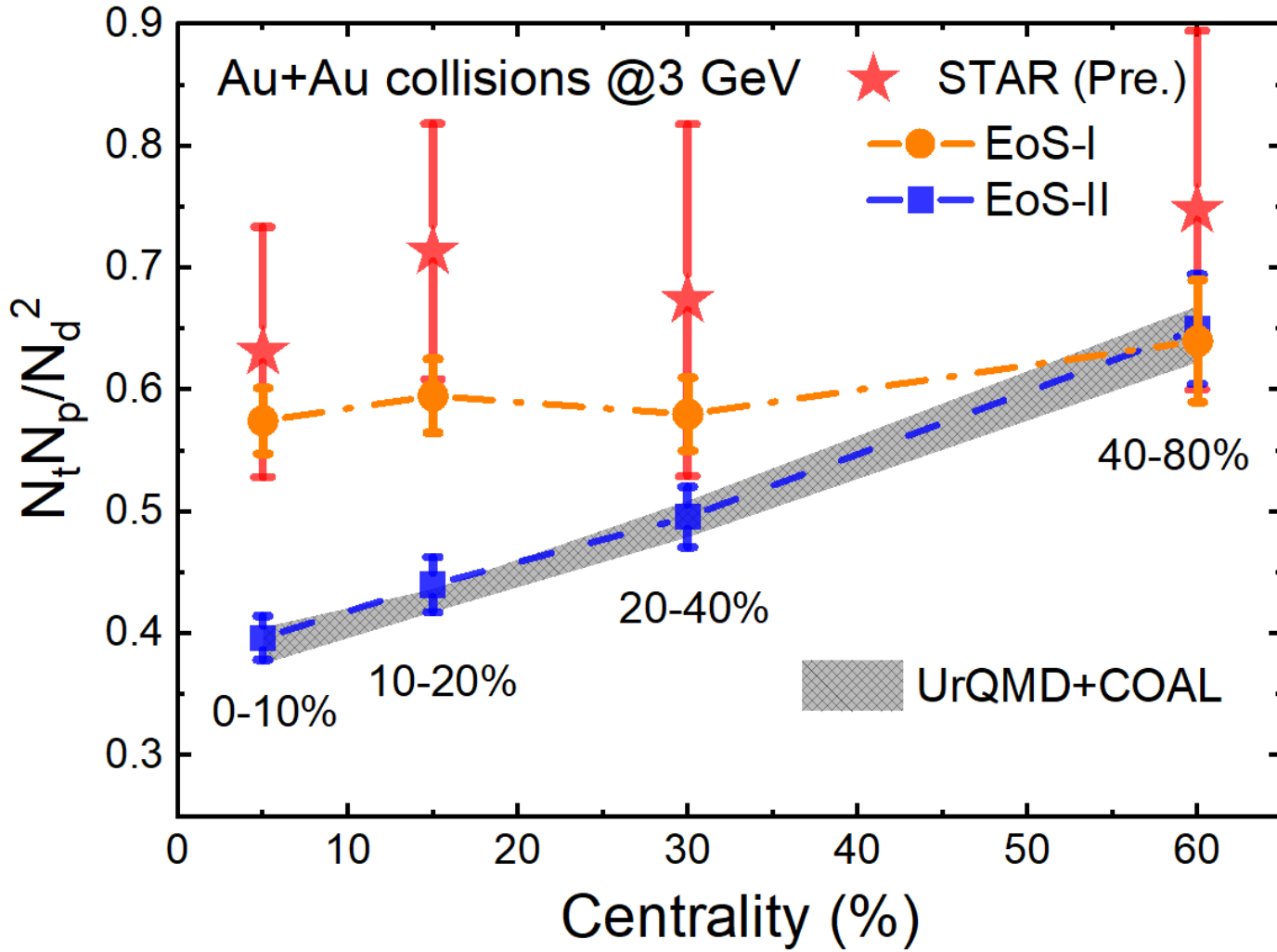
STAR, arXiv:2209.08058(2022)

Hui Liu (STAR), QM2022

T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).

# 2.8 Centrality dependence

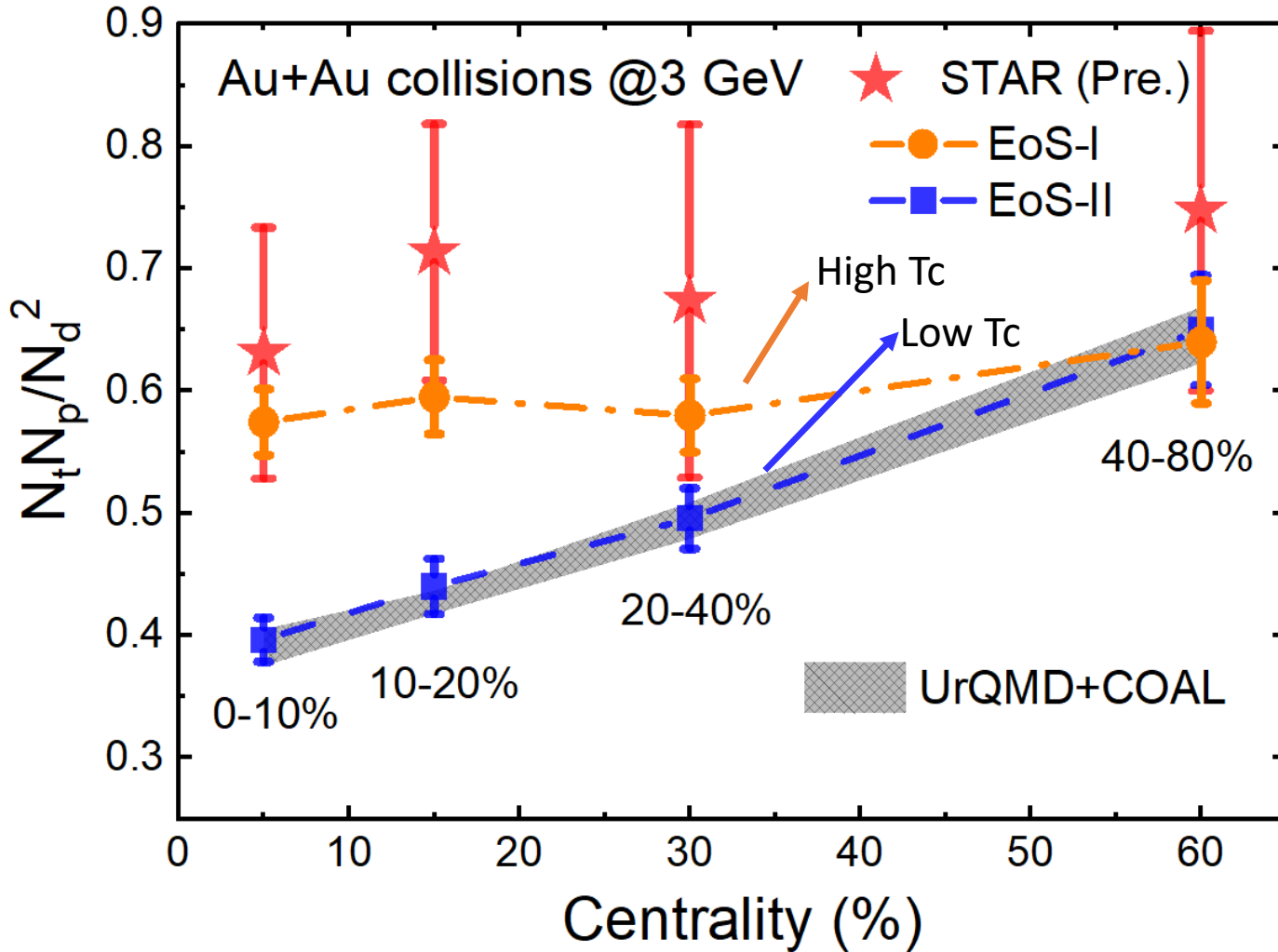
(11)





# 2.8 Centrality dependence

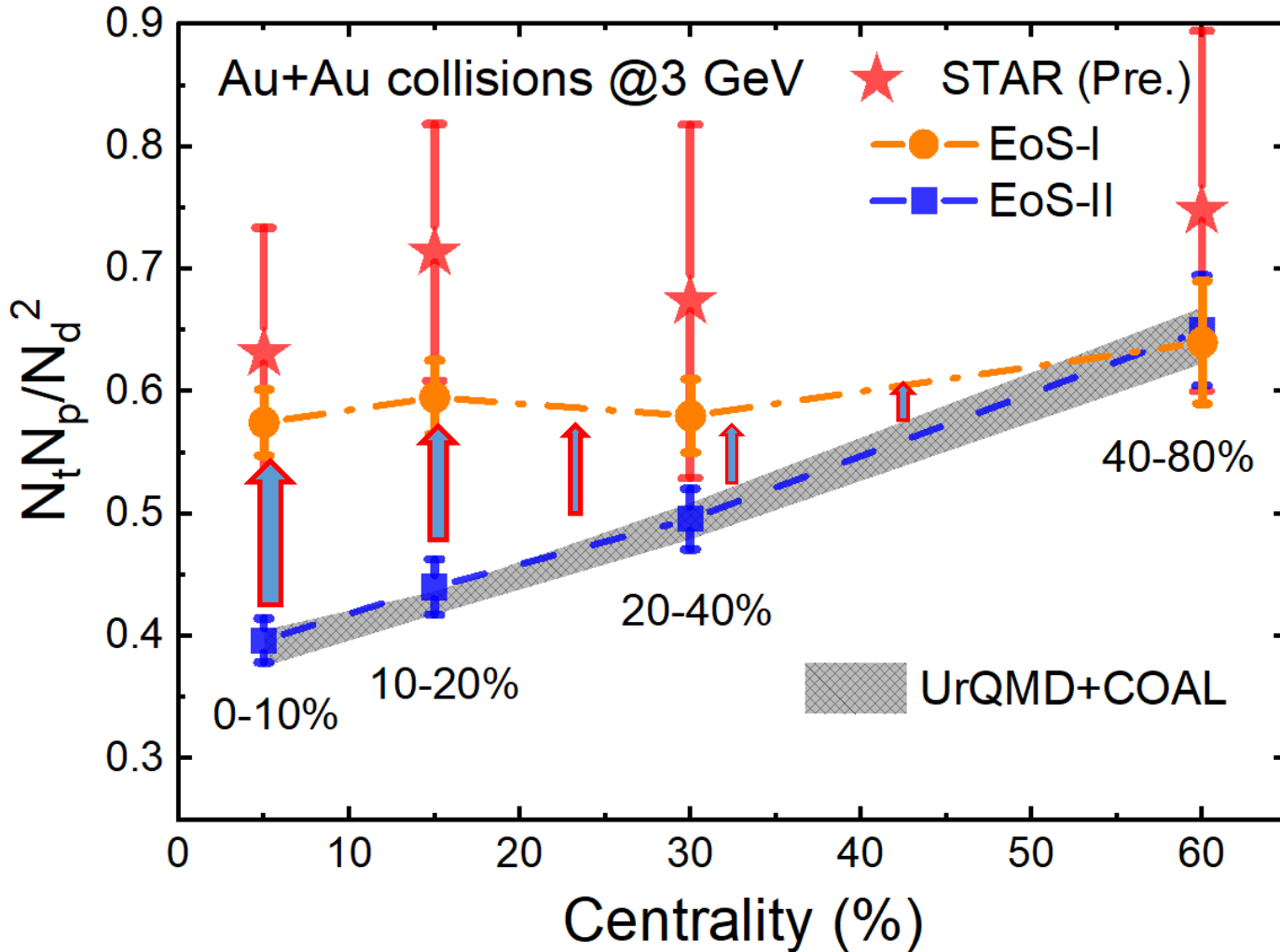
(11)



The spinodal enhancement of  $tp/d^2$  subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.

# 2.8 Centrality dependence

(11)



The spinodal enhancement of  $tp/d^2$  subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.

The slope with EoS-I is 5 times smaller

# 3. Summary and Outlook

(13)

## Main findings:

1. With scans of the collision energy and centrality as well as the equation of state using a novel transport model, we find that large density inhomogeneities generated by the spinodal instability during the first-order QCD phase transition can survive the fast expansion of the subsequent hadronic matter and lead to an enhanced  $tp/d^2$  in central Au+Au collisions at  $\sqrt{s_{NN}} = 3 - 5$  GeV for  $T_c \geq 80$  MeV, which is in accordance with the STAR measurements.
2. We also find that the spinodal enhancement of  $tp/d^2$  subsides with increasing collision centrality because of the shortening of fireball lifetime, and this effect results an almost flat centrality dependence of  $tp/d^2$  at  $\sqrt{s_{NN}} = 3$  GeV, which can also be used as a signal for the occurrence of a first-order phase transition.

## Future developments:

1. Incorporation of Polyakov loop
2. Inclusion of long-range correlation