New Bayesian Framework with Mass-Radius-Space Priors to Constrain the Neutron Star EOS

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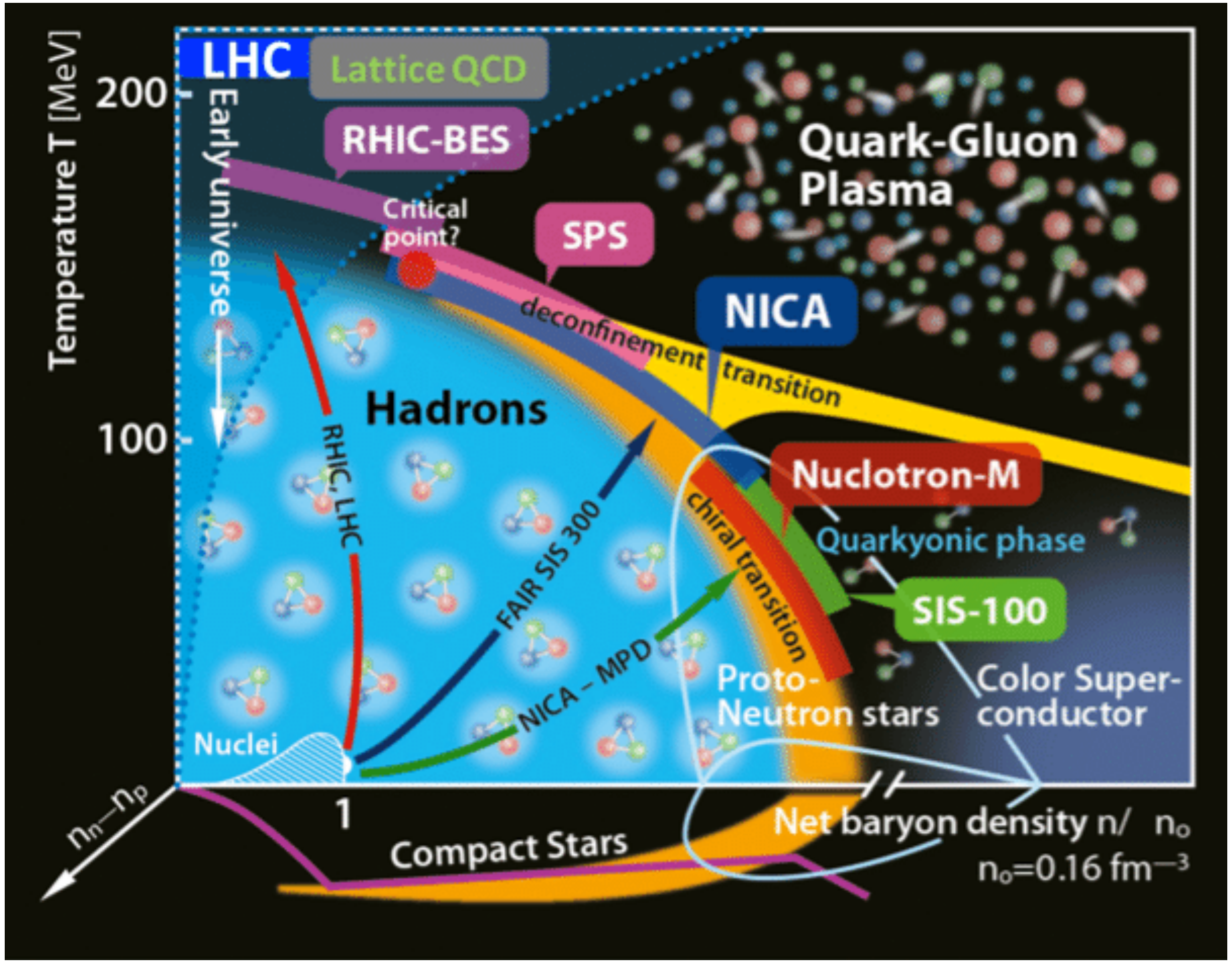
Background

We aim to understand the EOS of cold ultra-dense matter (degenerate neutron matter)

However,

- ➢ It's impossible to access in terrestrial experiments currently (or near future)
- ➢ It's a non-perturbative QCD problem

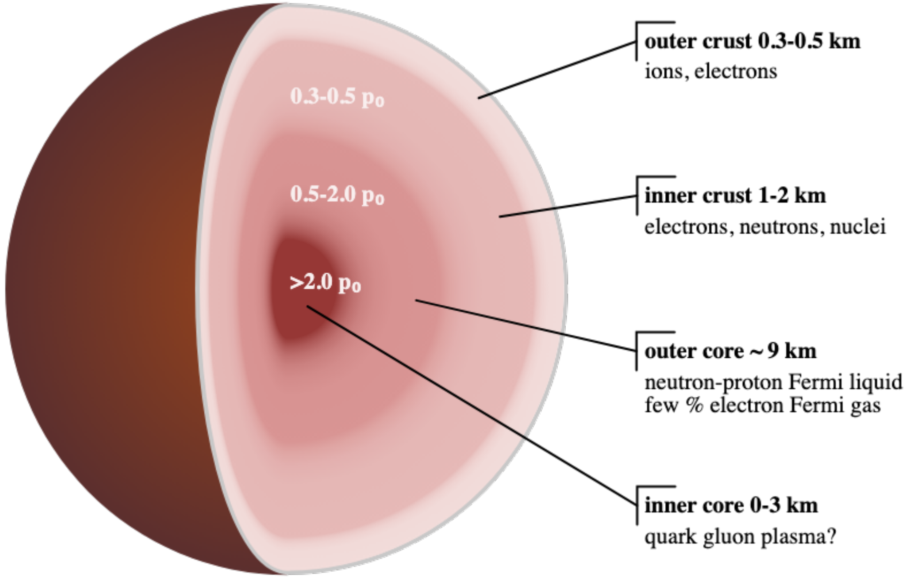
So, what to do?

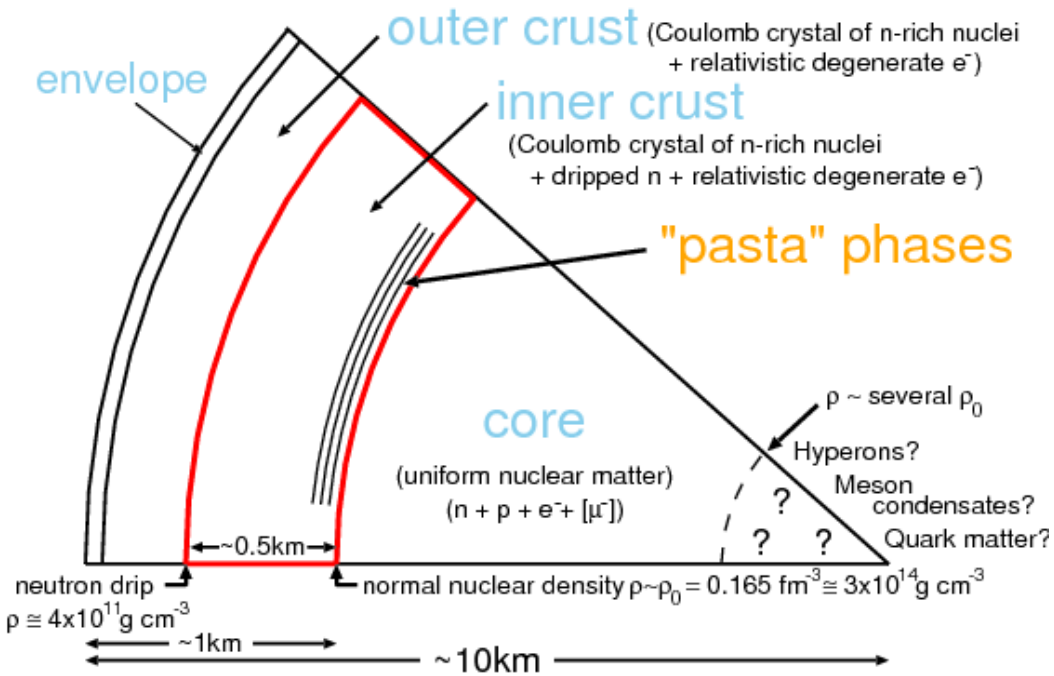


https://doi.org/10.48550/arXiv.2201.00202

Background

Neutron stars are natural labs to probe (cold) ultra-dense matter

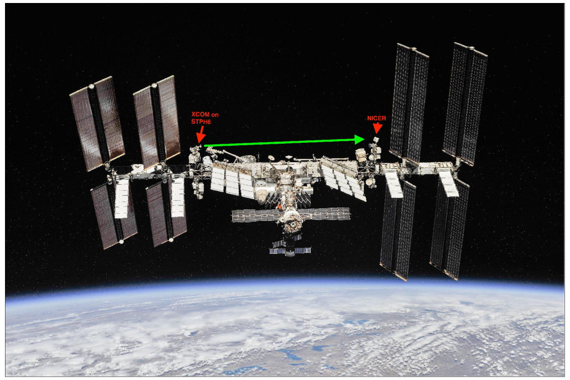




https://doi.org/10.48550/arXiv.1109.3511

Background

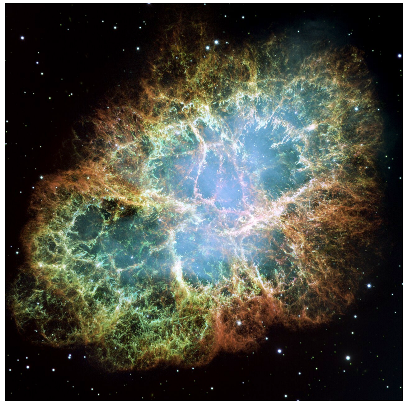
We now have multi-messenger observations of neutron stars



X-ray Pulsar Timing: Mass and Radius

GW detections: Tidal deformability

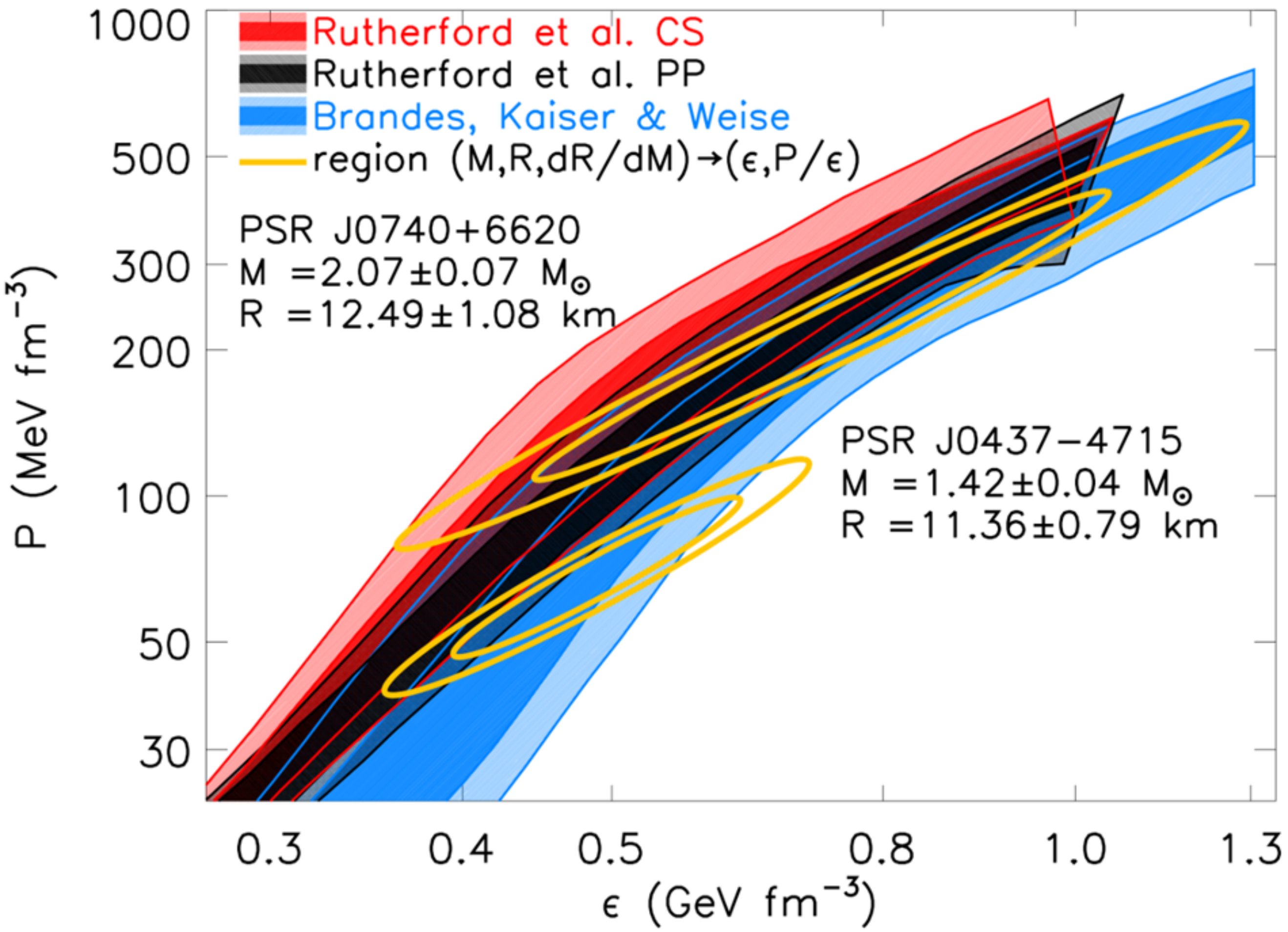
Radio Pulsar Timing: Moment of Inertia



Supernova explosion: Binding energy

Background

From observations to EOS: Bayesian approach



Bayes' Theorem:

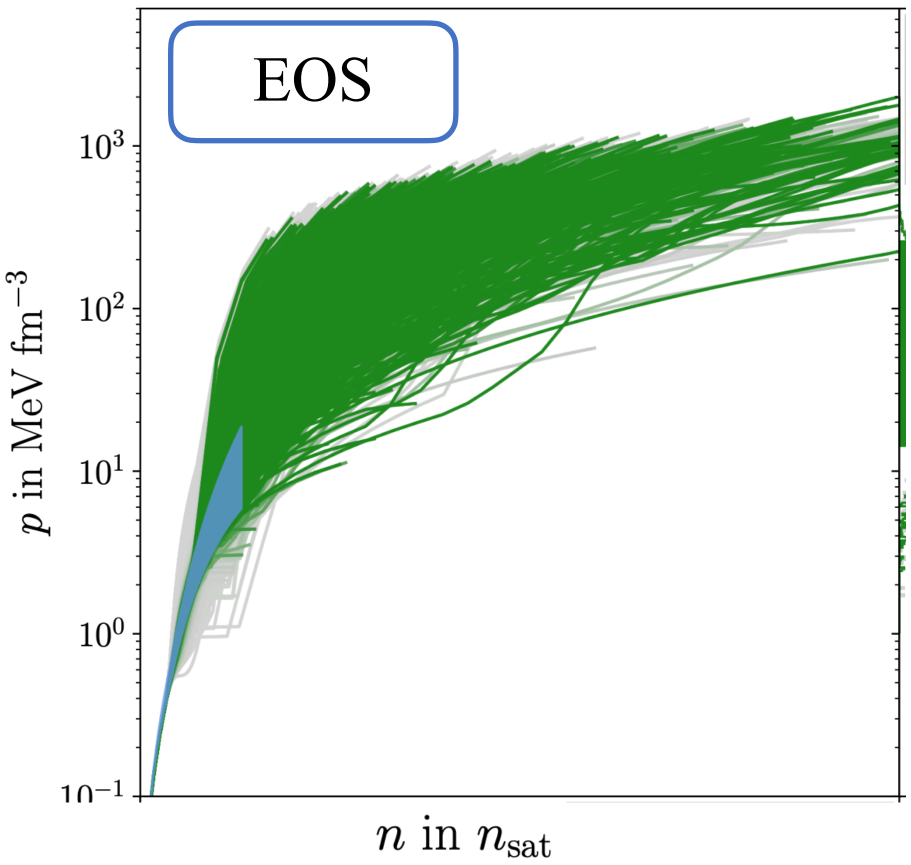
$$\mathscr{P}(\mathscr{M}(\theta,\mathscr{E})\,|\,\mathscr{D}) = \frac{\mathscr{P}(\mathscr{D}\,|\,\mathscr{M}(M,R))\mathscr{P}(\mathscr{M}(\theta,\mathscr{E}))}{\sum \mathscr{P}(\mathscr{D}\,|\,\mathscr{M}(M,R))\mathscr{P}(\mathscr{M}(\theta,\mathscr{E}))},$$

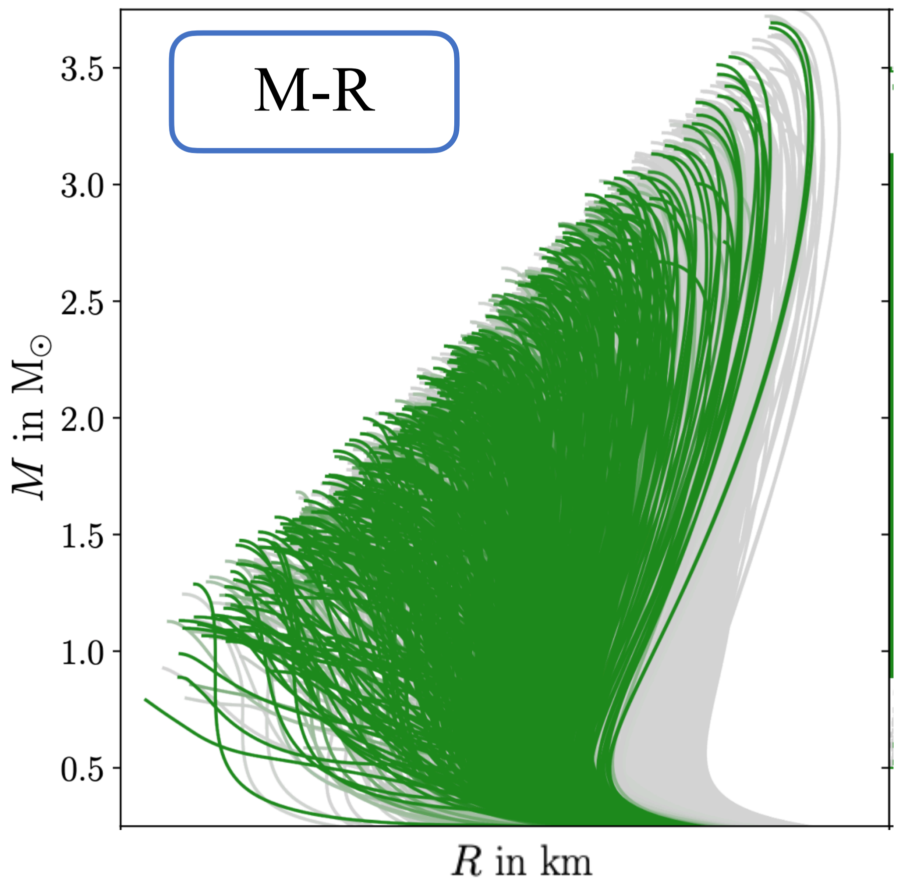
B. Sun and J. M. Lattimer. Correlations between the Neutron Star Mass–Radius Relation and the Equation of State of Dense Matter. ApJ ***984*** *30 (2025)*

Background

Bayes' Theorem:

$$\mathscr{P}(\mathscr{M}_i(\theta,\mathscr{E})\,|\,\mathscr{D}) = \frac{\mathscr{P}(\mathscr{D}\,|\,\mathscr{M}_i(M,R))\mathscr{P}(\mathscr{M}_i(\theta,\mathscr{E}))}{\sum_j \mathscr{P}(\mathscr{D}\,|\,\mathscr{M}_j(M,R))\mathscr{P}(\mathscr{M}_j(\theta,\mathscr{E}))},$$





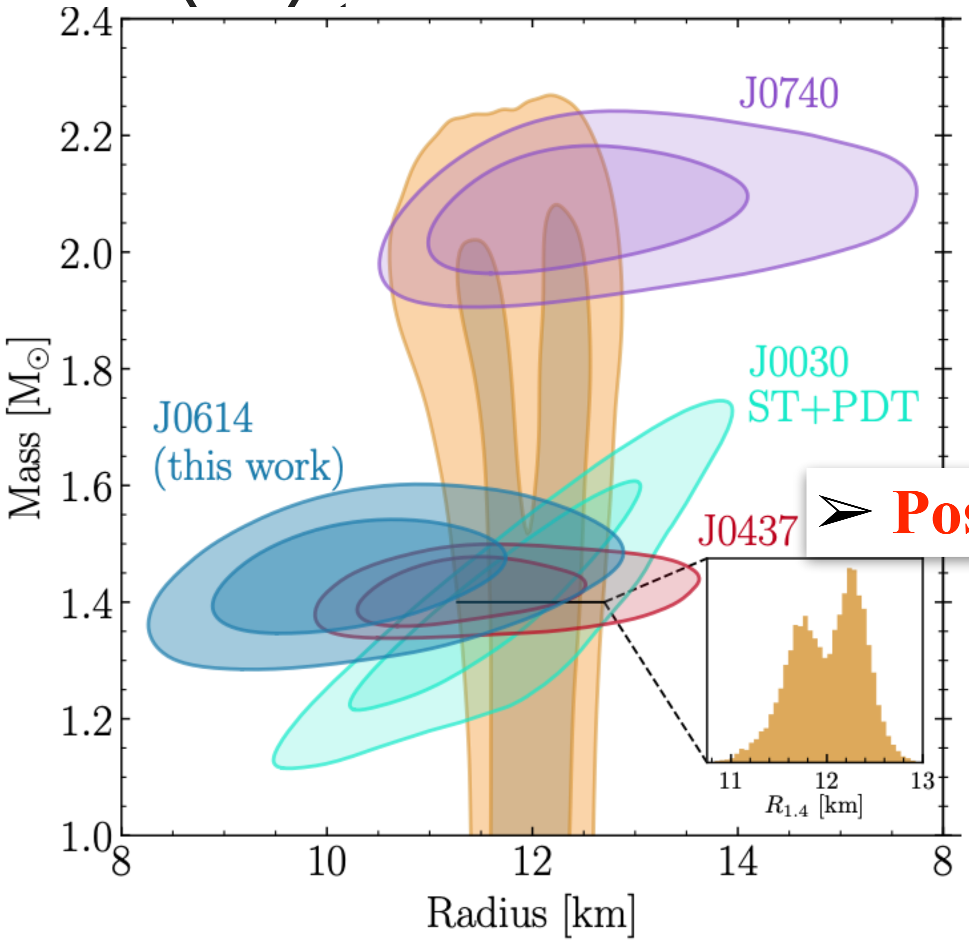
Koehn et al. Phys. Rev. X ***15****, 021014 (2025)*

➢ **Prior distribution:** $\mathscr{P}(\mathscr{M}_i(\theta,\mathscr{E}))$**, where** $\mathscr{M}_i(\theta,\mathscr{E})$ **refers to a certain model**

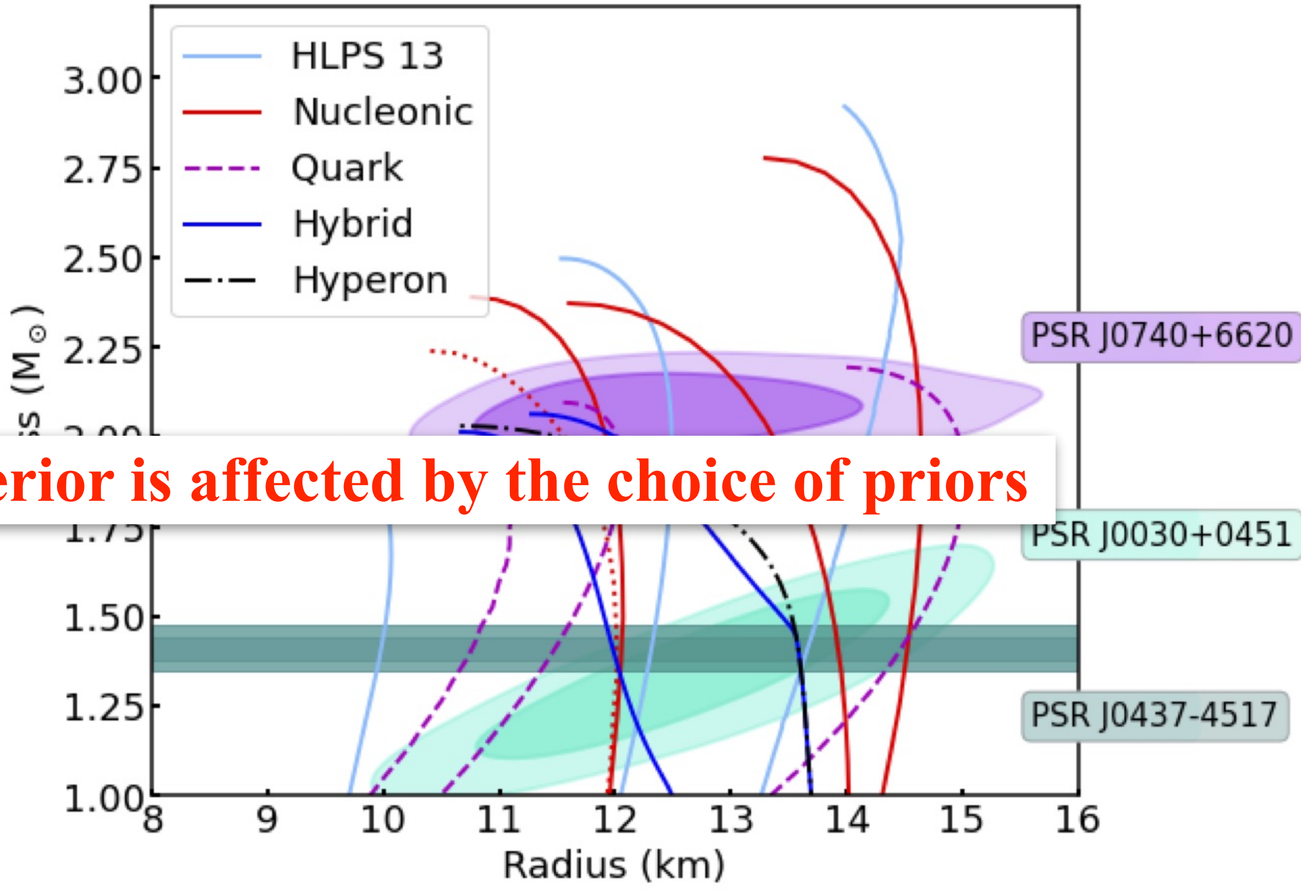
Background

Bayes' Theorem:

$$\mathscr{P}(\mathscr{M}_i(\theta, \mathscr{E}) \mid \mathscr{D}) = \frac{\mathscr{P}(\mathscr{D} \mid \mathscr{M}_i(M,R))\mathscr{P}(\mathscr{M}_i(\theta, \mathscr{E}))}{\sum_j \mathscr{P}(\mathscr{D} \mid \mathscr{M}_j(M,R))\mathscr{P}(\mathscr{M}_j(\theta, \mathscr{E}))},$$



Mauviard et al. ApJ 995 60 (2025)



- **Posterior is affected by the choice of priors**

https://www.jinaweb.org/news/nicer-measures-radius-most-massive-neutron-star

- **Observational distribution:** $\mathscr{D}$
- **Likelihood:** $\mathscr{P}(\mathscr{D} \mid \mathscr{M}_i(M,R))$

Motivation

How to define a good prior for Bayesian analysis?

Let's take a look at Alan's fish:

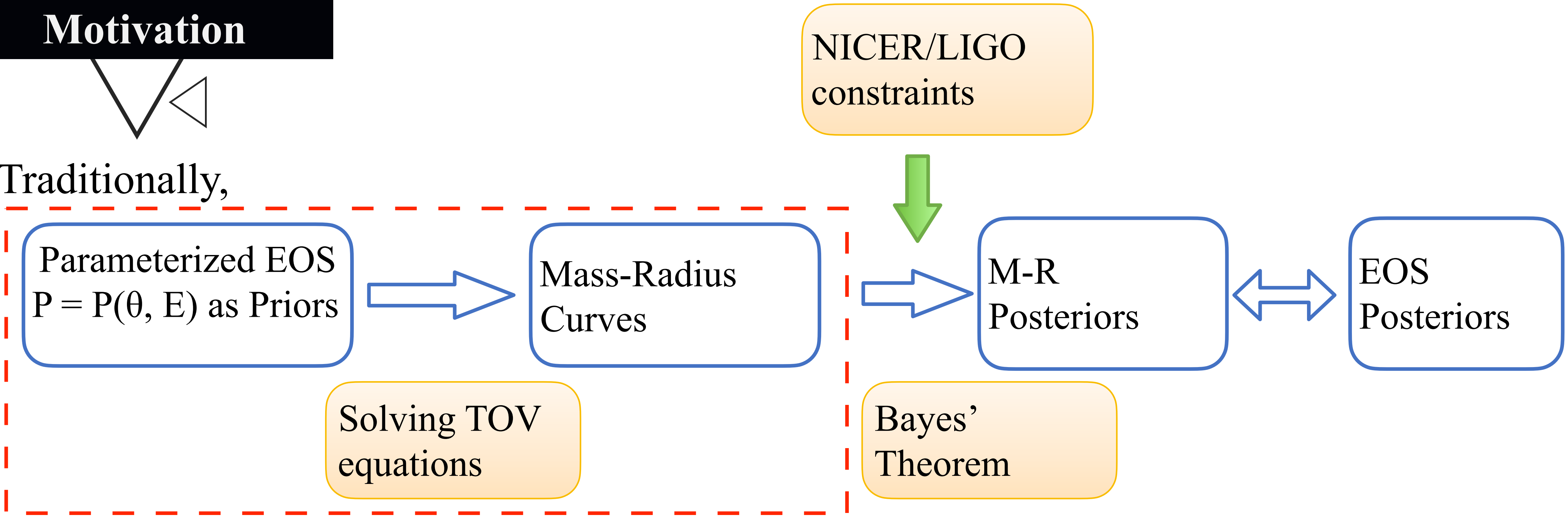
Two independent observations for the fish:
8 AM: Alive   8 PM: Dead

Question: When did the fish die?

➢ **The ideal prior may uniformly cover time from 8 AM to 8 PM.**

Motivation

Traditionally,



Prior and observations are in two different spaces (EOS vs M-R)

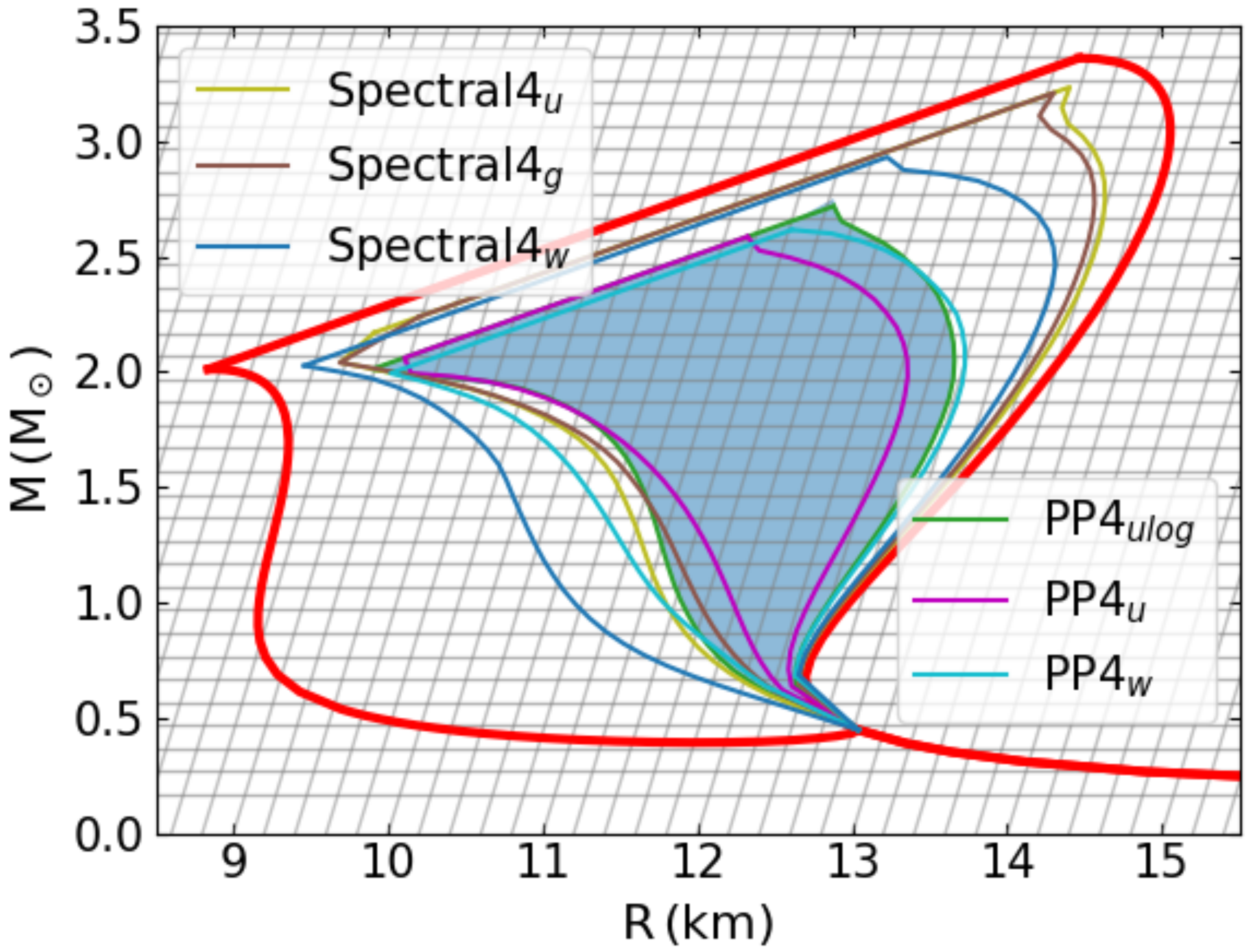
➢ **May have a concentration in M-R space; May be more sensitive to Prior choices…**

Motivation

Red boundary: physically allowed boundary for neutron star M-R curves in terms of causality and thermodynamic stability

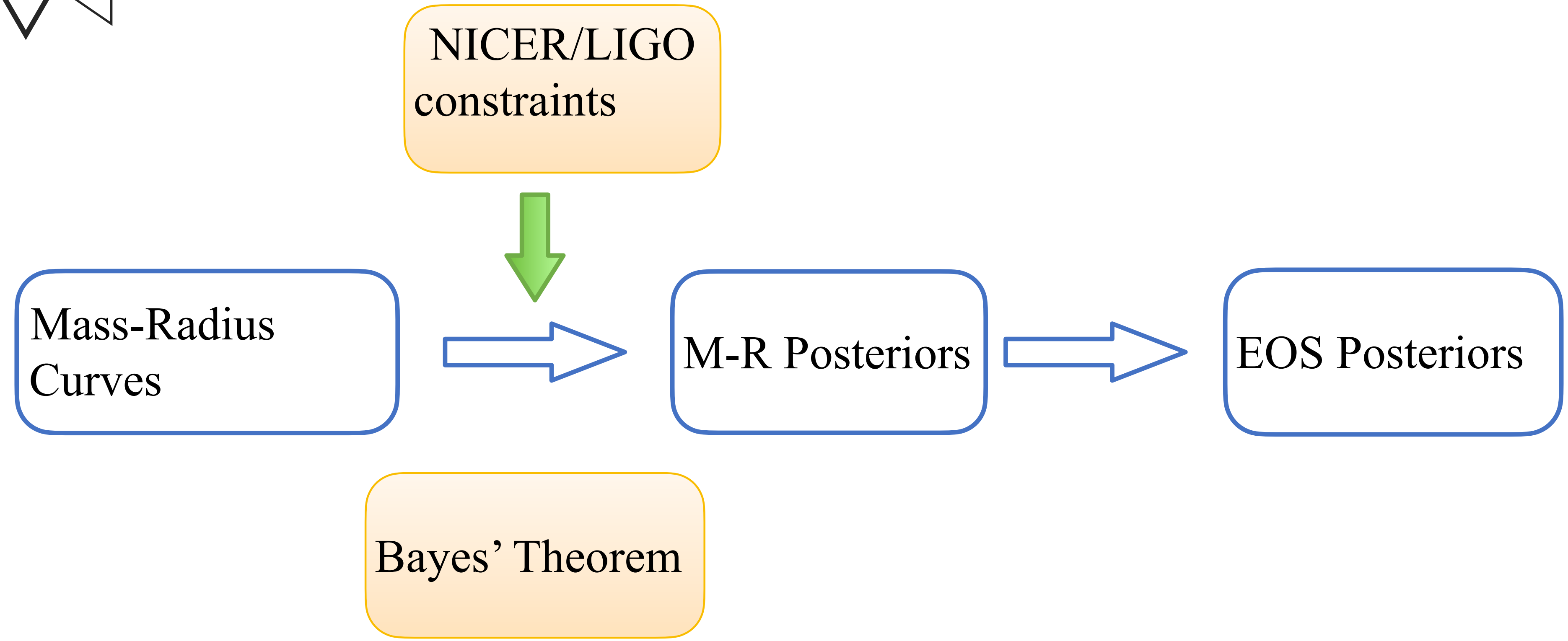
Boundaries inside with various colors: different traditional priors

➢ **They tend to concentrate in larger-radius region**



Motivation

A more Natural way?



Both priors and observations are in M-R space; No need to solve TOV equations

Motivation

To accomplish this scheme, two main issues should be solved:

1, How to parameterize M-R curves directly?

2, How to map (M, R) to its underlying (P, E)?

⇩

We have developed the direct, highly-accurate inversion formula

Motivation

We developed a method fitting with maximum mass and two radii at certain fractional mass points:

$$G_f = a_{Gf} \left(\frac{M_{max}}{M_\odot}\right)^{b_{Gf}} \left(\frac{R_g}{10\ \mathrm{km}}\right)^{c_{Gf}} \left(\frac{R_h}{10\ \mathrm{km}}\right)^{d_{Gf}}$$

where G ∈ [Ɛ, P, c_s/c, μ, n]

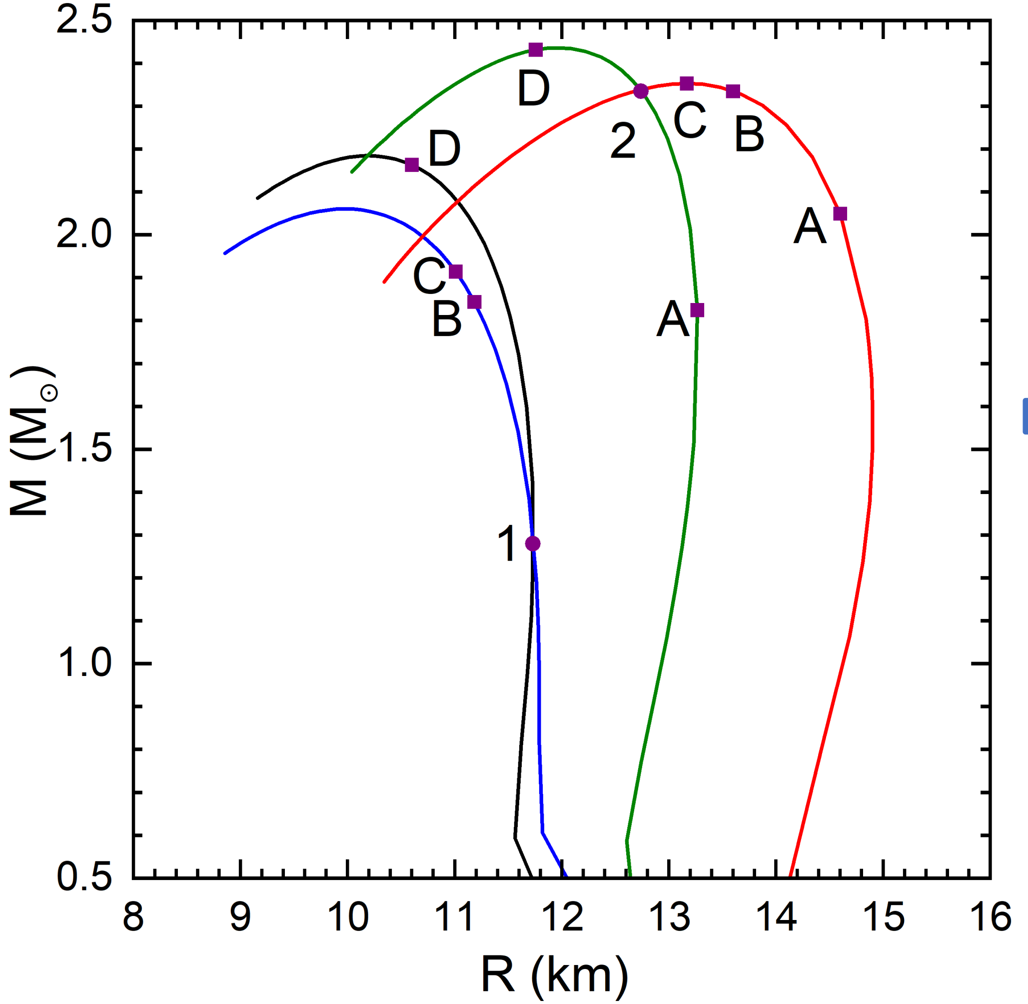
f ∈ [1, 0.95, 0.9, 0.85, 0.8, 0.75, 2/3, 3/5, 1/2, 2/5, 1/3] is the grid we chose,

[a, b, c, d, g, h] are fitting parameters (g and h tell which radii to use)

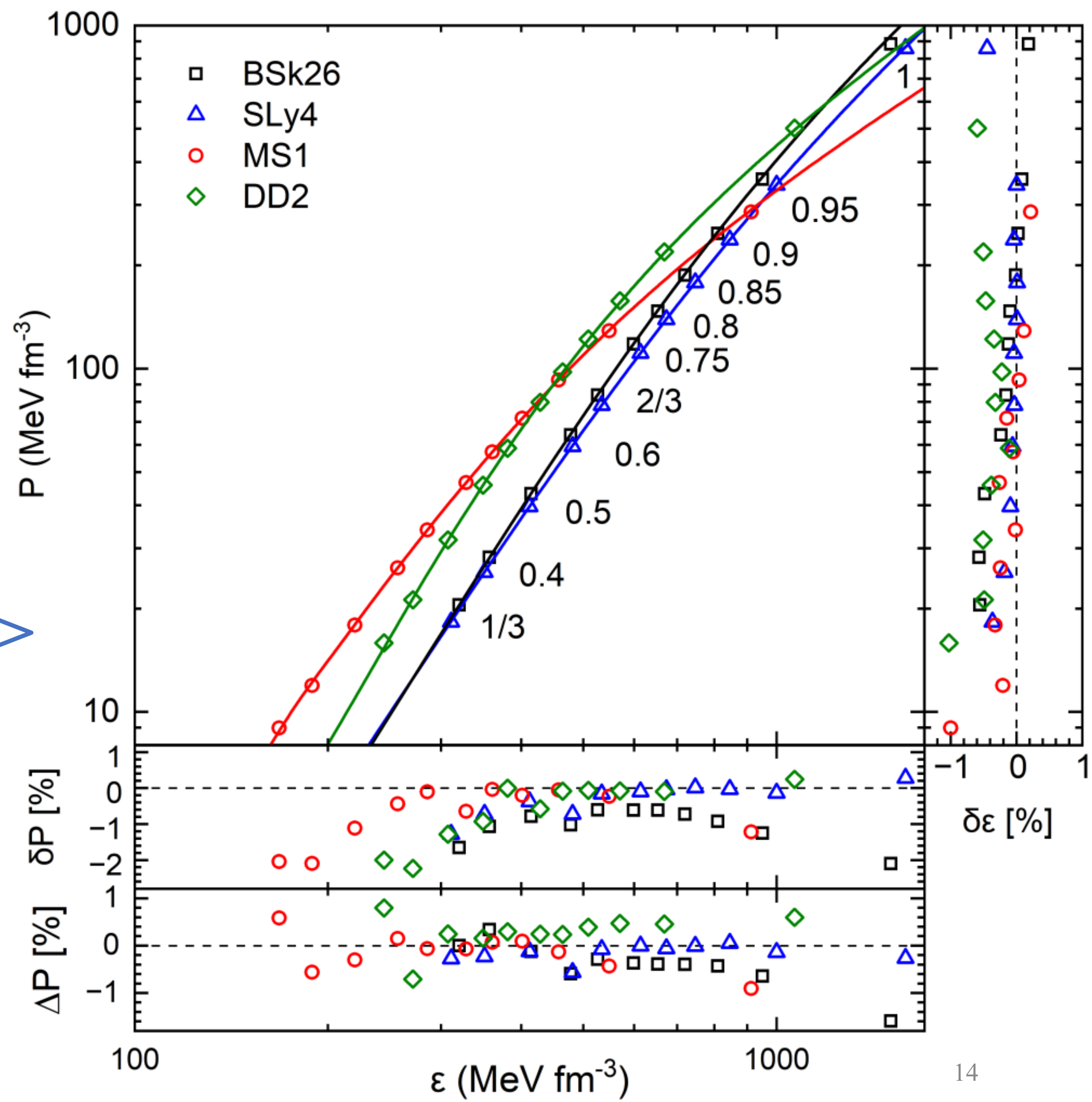
So, matrix of a total of 6*11=66 parameters

The fitting accuracy (RMS error) is about 0.5% for both Ɛ and P

Motivation



M ($M_\odot$)
R (km)
A
B
C
D
1
2



BSk26
SLy4
MS1
DD2
P (MeV fm^{-3})
ε (MeV fm^{-3})
δP [%]
ΔP [%]
δε [%]
1/3
0.4
0.5
0.6
2/3
0.75
0.8
0.85
0.9
0.95
1

Motivation

Moreover, the fitting error should be taken into consideration:

We assumed a Gaussian-like error, the probability density distribution for EOS is

$$f(\mathscr{E}, P) = \sum_i \frac{\mathscr{P}_i}{2\pi\sigma_{P_i}\sigma_{\mathscr{E}_i}} \exp\left[-\frac{(\mathscr{E}-\mu_{\mathscr{E}_i})^2}{2\sigma_{\mathscr{E}_i}^2} - \frac{(P-\mu_{P_i})^2}{2\sigma_{P_i}^2}\right],$$

where i refers to the grid point in one curve.

The total density distribution is the sum of all curves

PMR method

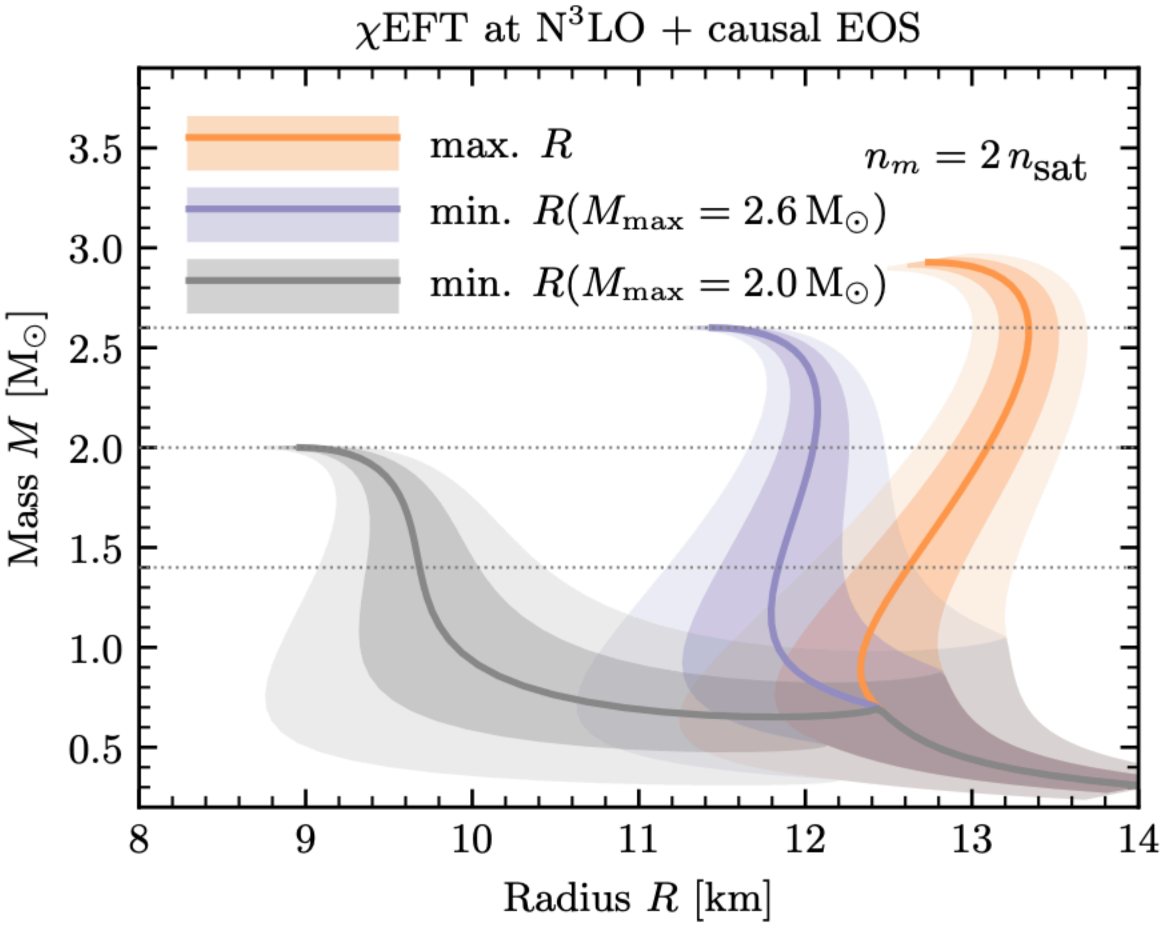
Now it comes to the main question:

How to parametrize the mass-radius space directly?

➢ **We propose a simple but effective approach, namely PMR method (Parameterized Mass-Radius method)**

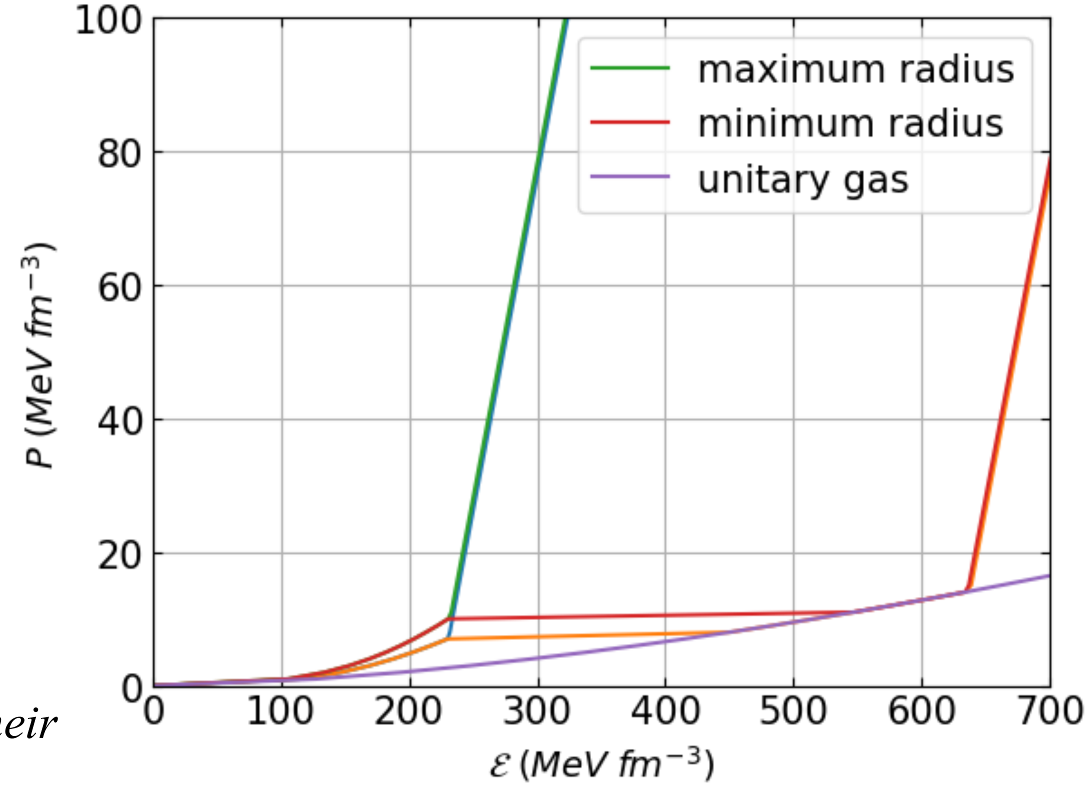
PMR method

Step 1: Establishing the M-R boundary



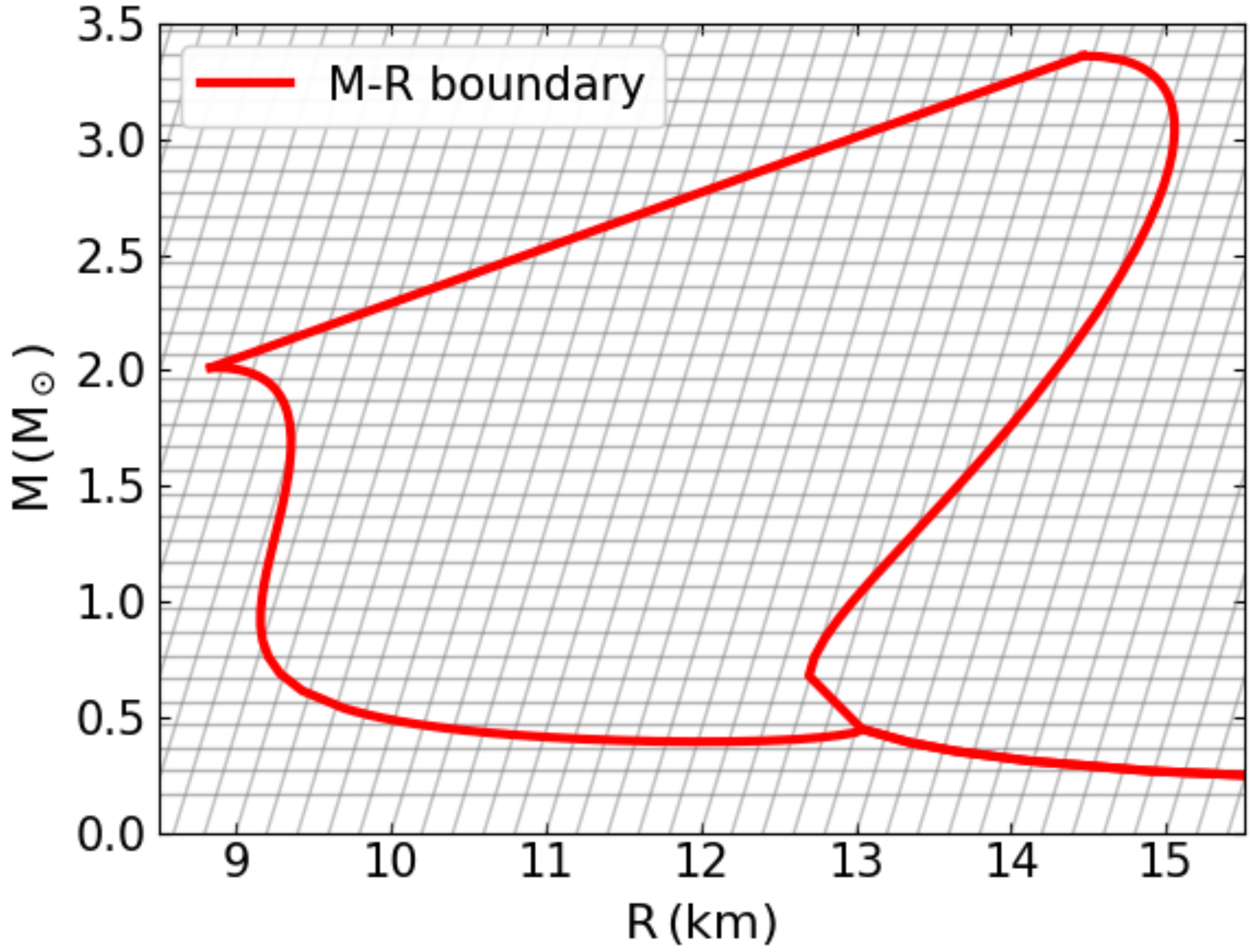
C. Drischer. Limiting masses and radii of neutron stars and their implications, Phys. Rev. C 103, 045808 (2021)

- ➢ Right boundary: constrained by causality
- ➢ Left boundary: A sharp 1st order phase transition followed by causality, achieving the assumed minimum M_{max}



PMR method

Step 2: Constructing the M-R mesh



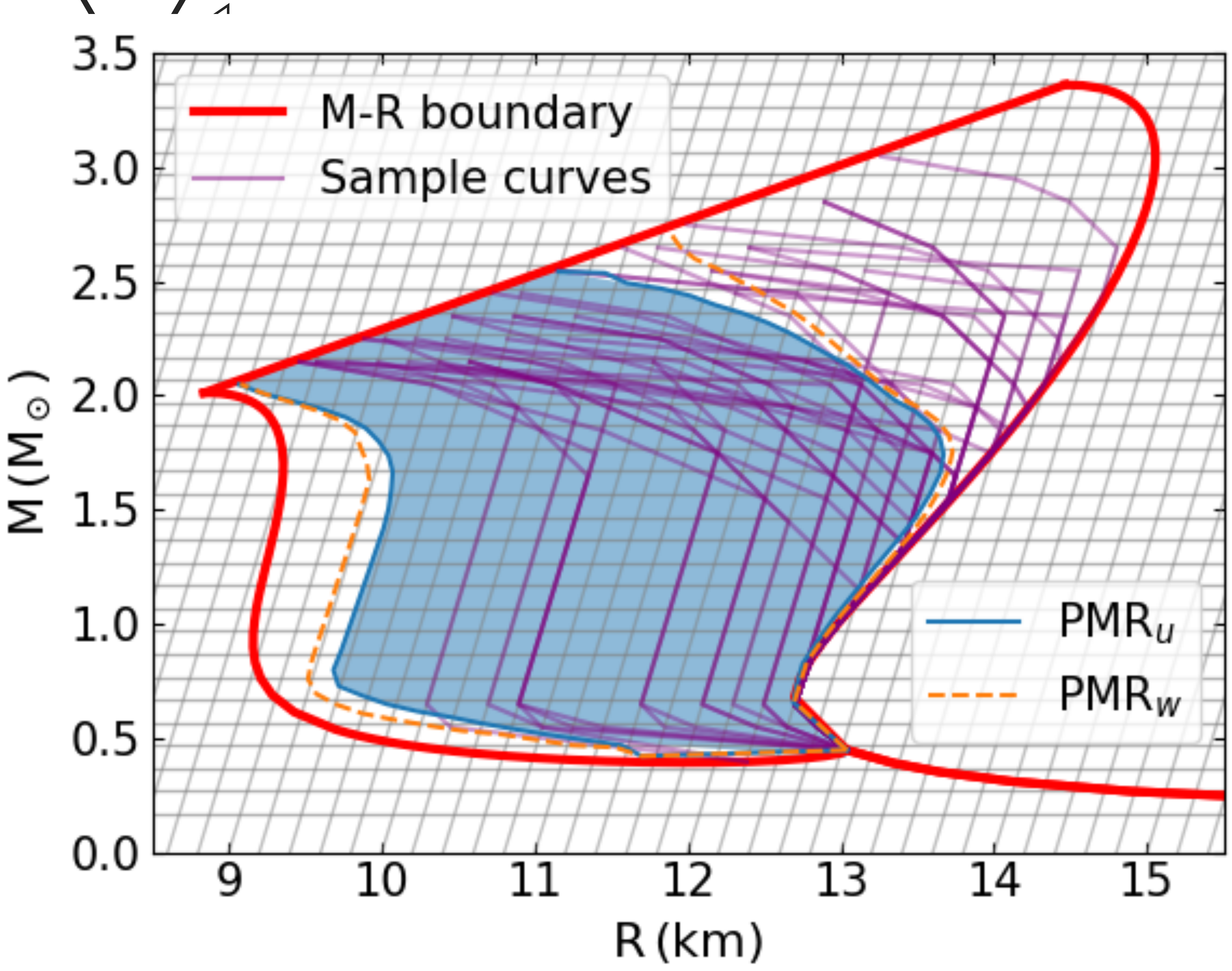
We used SLy4 crust EOS + N3LO central value of 1 sigma band up to 1.5 n_s

A uniform M-R mesh with sloped radius lines is adopted to improve computing efficiency

$\Delta M = 0.1\ M_\odot$,
$\Delta R = 0.2\ km$

PMR method

Step 3: Generating M-R curves



$\Delta M = +1$ unit;
extend to the same or smaller radial grid lines

Satisfying:

dR/dM non-increasing
OR
Along the boundaries

Transition ⇩ point

dR/dM non-decreasing
OR
Terminate if M > $2M_\odot$

PMR method

For comparison, different priors are chosen:

PMR:

➢ Uniform sampling in M – R Space: Generate all curves and sampled randomly (PMR_u)

➢ Uniform distribution of $R_{1.4}$ values: Group curves by M_{max}, then make $R_{1.4}$ uniform in each one (PMR_w)

➢ Variation of the N3LO boundaries: Upper/center/lower bounds of 1σ of N3LO (PMR_p and PMR_m)

Traditional EOS parameterizations:

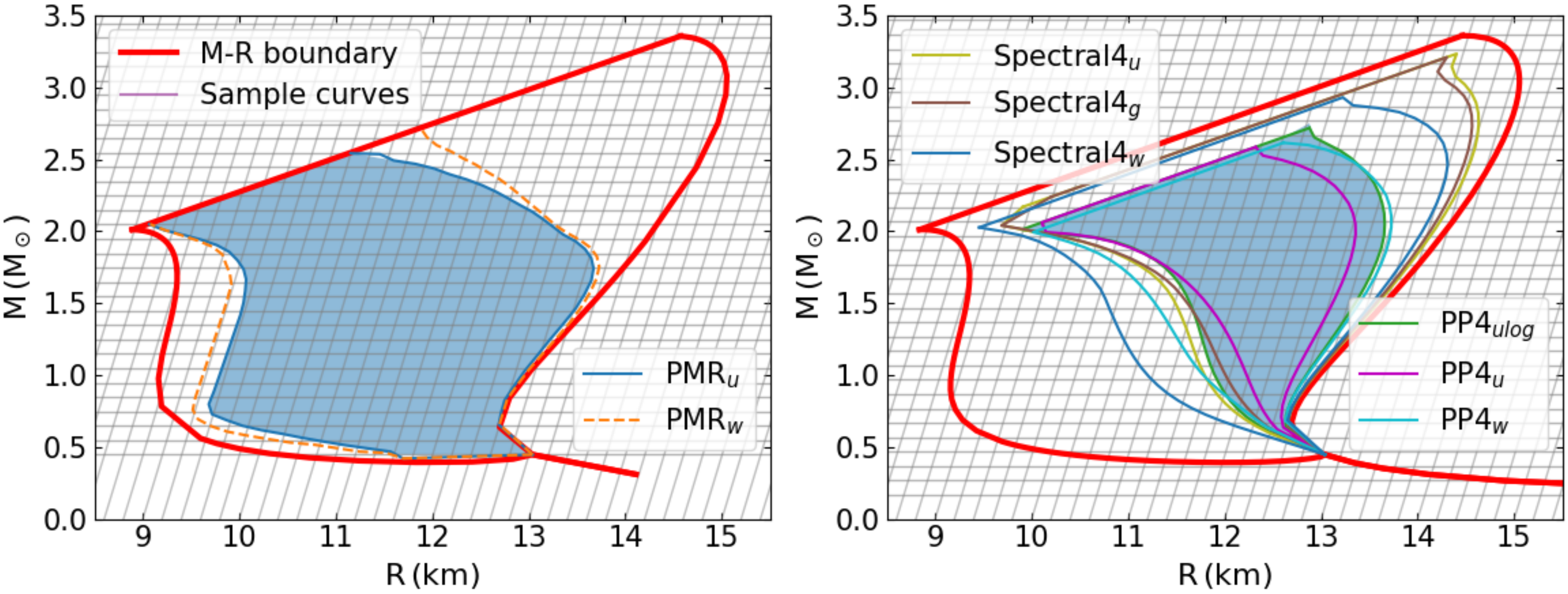
➢ 4-parameter Piecewise Polytropic (PP4), choosing parameters uniformly or log-uniformly, or weighted ($PP4_u$ / $PP4_{ulog}$ / $PP4_w$)

➢ 4-parameter Spectral representation, choosing parameters uniformly or in Gaussian, or weighed ($Spectral4_u$ / $Spectral4_g$ / $Spectral4_w$)

➢ Variation of the N3LO boundaries

Results

Prior distributions: PMR vs Tradition methods



Results

All Bayesian methods are affected by the choice of priors

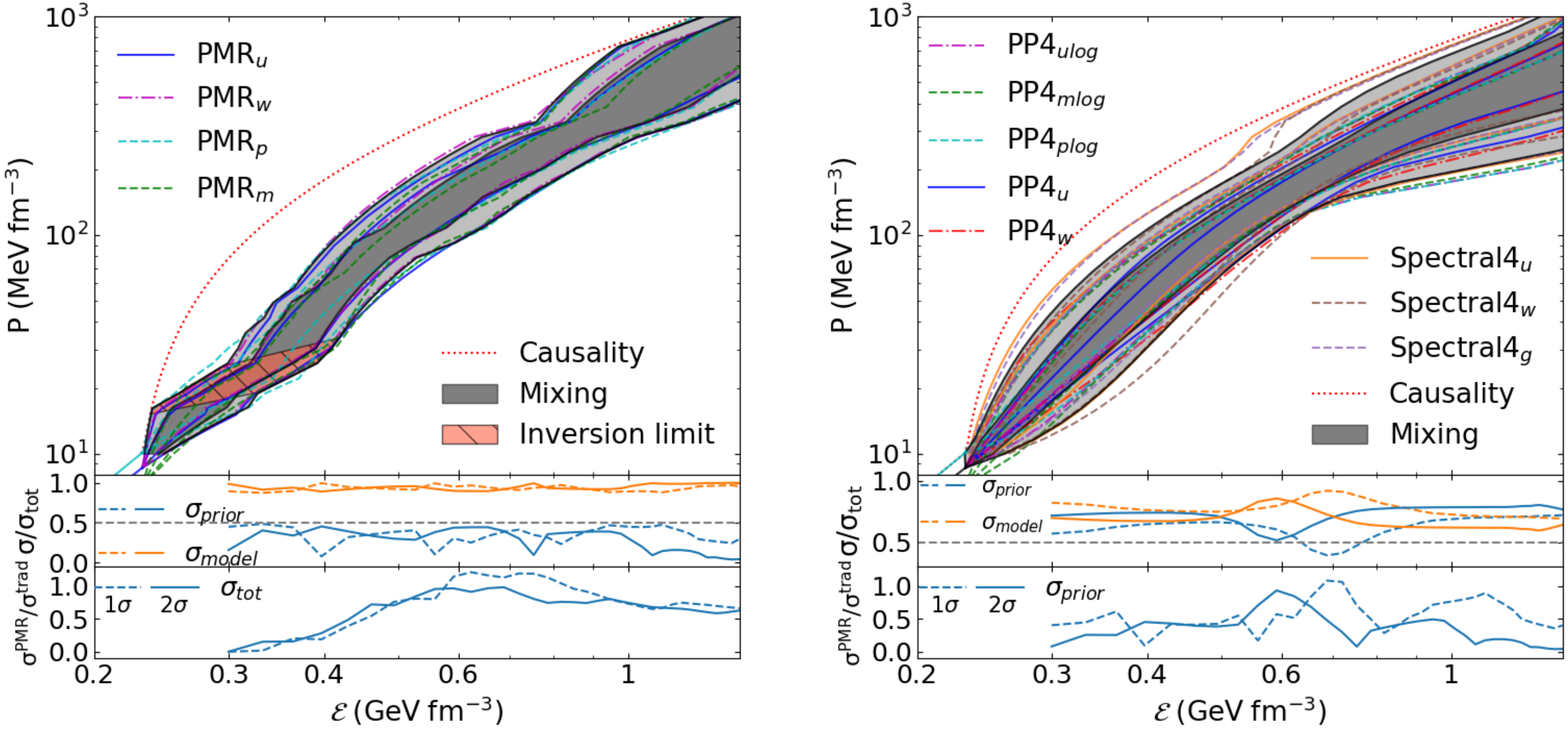
We want to examine:
(1). the relative importance of prior modeling uncertainties relative to the observational uncertainties ($\sigma_{prior}/\sigma_{tot}$)

(2). the relative accuracies of the two approaches when confronted with the same astrophysical data (PMR/traditional)

We define the σ_{prior} by model mixing and
$\sigma^2_{tot}(\mathscr{E}) = \sigma^2_{model}(\mathscr{E}) + \sigma^2_{prior}(\mathscr{E})$

Results

Posterior distributions: PMR vs Tradition methods



Conclusion

We conclude the following potentials of PMR method:

1, Providing a new framework that’s more direct and intuitive in terms of observations

2, Enhanced computing efficiency with the current inversion formula

3, The new prior has a lower concentration at large radius region

4, Posteriors exhibit lower sensitivity to prior assumptions

5, Generally has a smaller total uncertainty when confronted with the same observational data

Outlook

We are working on

Using PMR method in the M-R space only to get posterior as updated M-R distribution

Then, combining with traditional Bayesian approaches to deal with the EOS inference

In short, this is basically a two-step Bayesian iteration process, and we want to see what will happen

Outlook

In a nutshell,

We proposed a new idea to do Bayesian analysis to infer neutron star EOS.

Hopefully there will be many more studies following this natural framework.

https://arxiv.org/abs/2601.04294

THANK YOU