Towards determining polarized gluon distribution in the nucleon from Lattice QCD

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INT 22-83W Parton Distributions and Nucleon Structure

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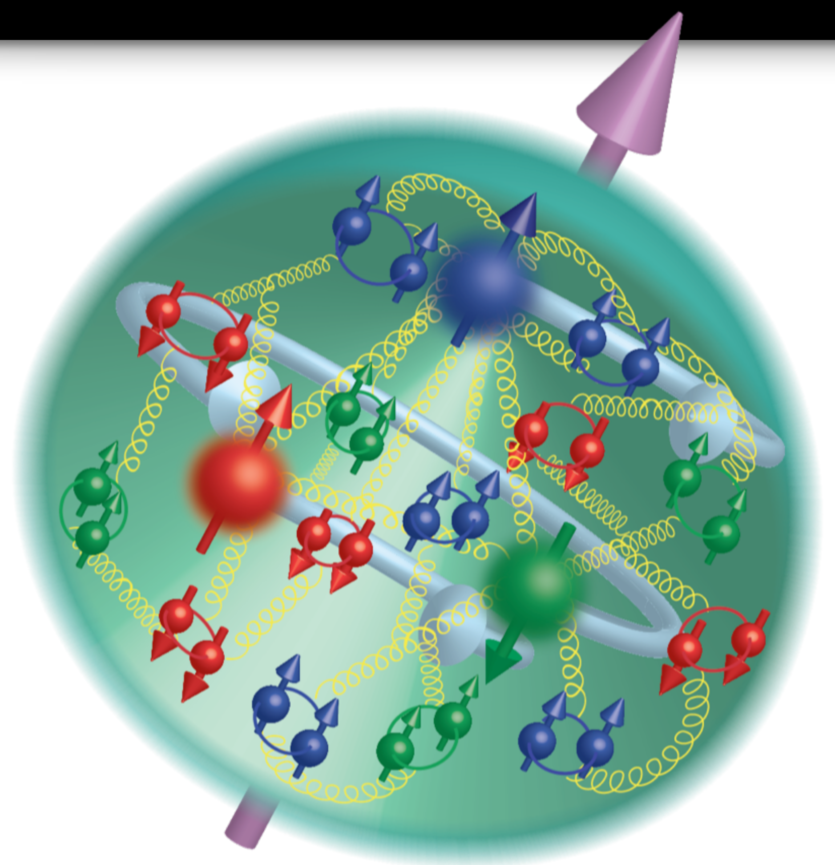
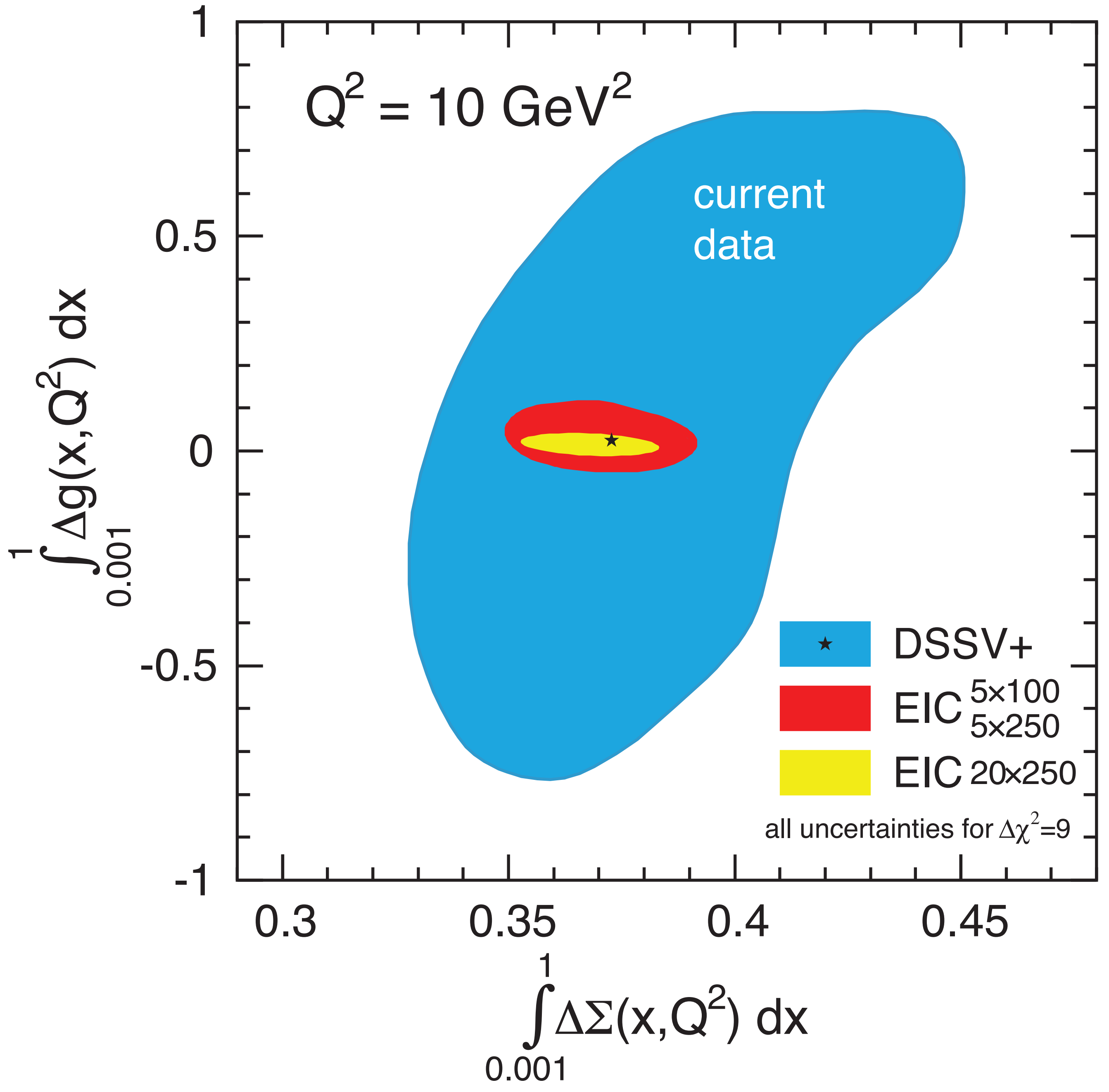
Where does the proton spin come from and how?

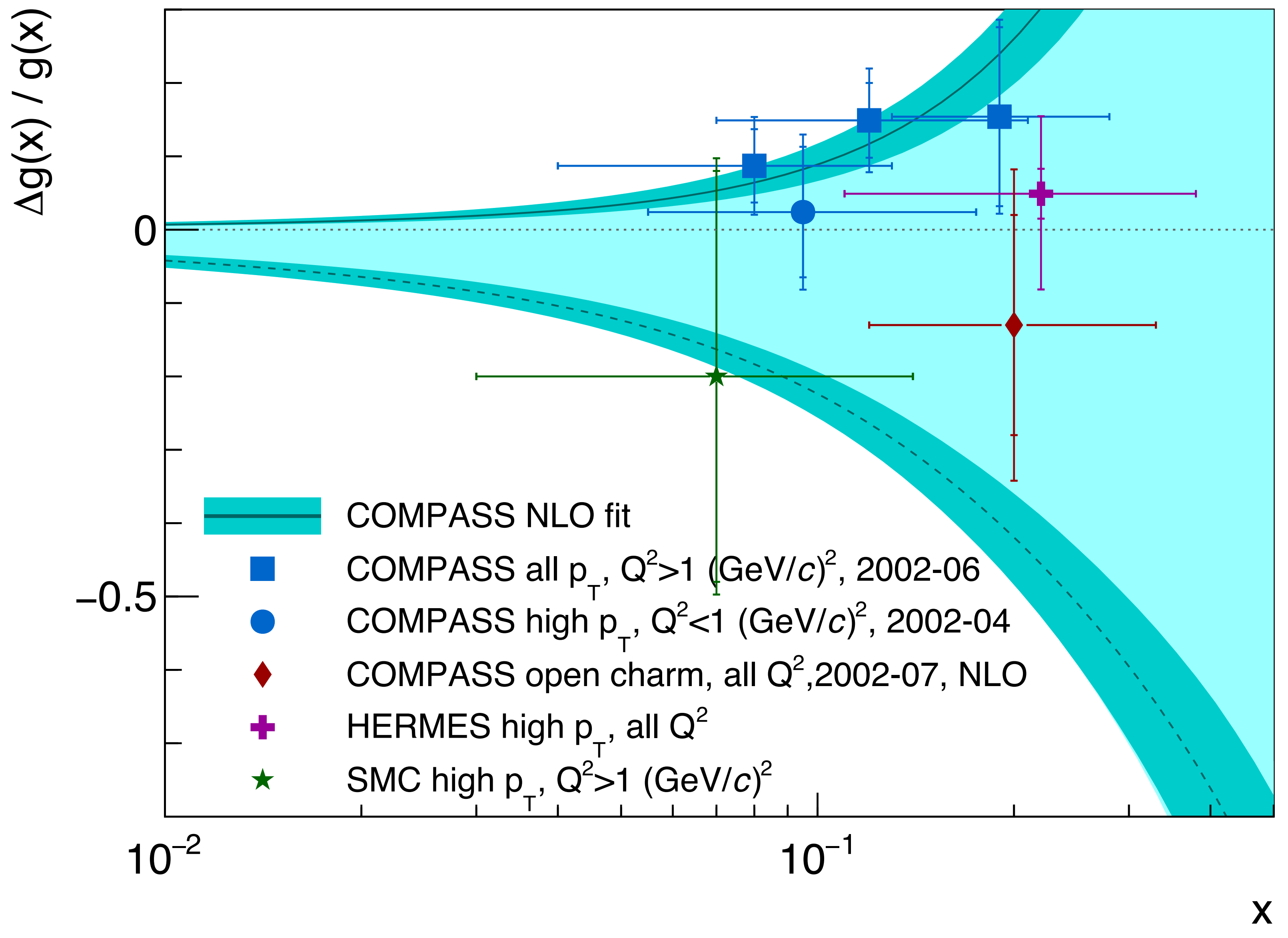


$$\frac{1}{2} = \boxed{\text{spin of all quarks} \sim 30\%} + \boxed{\text{glue spin}} + \boxed{\text{quark orbital angular momentum}} + \boxed{\text{gluon orbital angular momentum}}$$

- Gluon helicity distribution is not well-constrained from experimental data

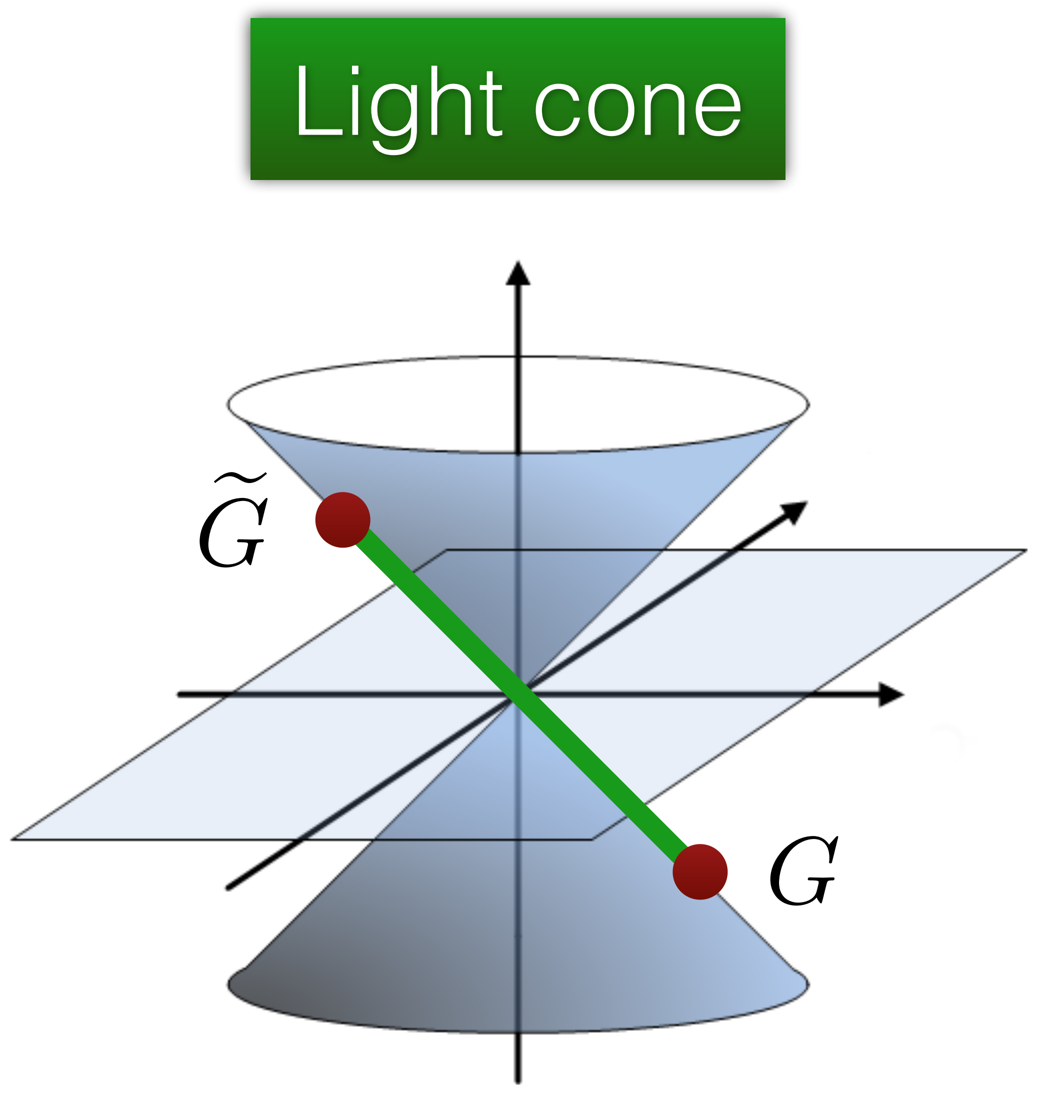


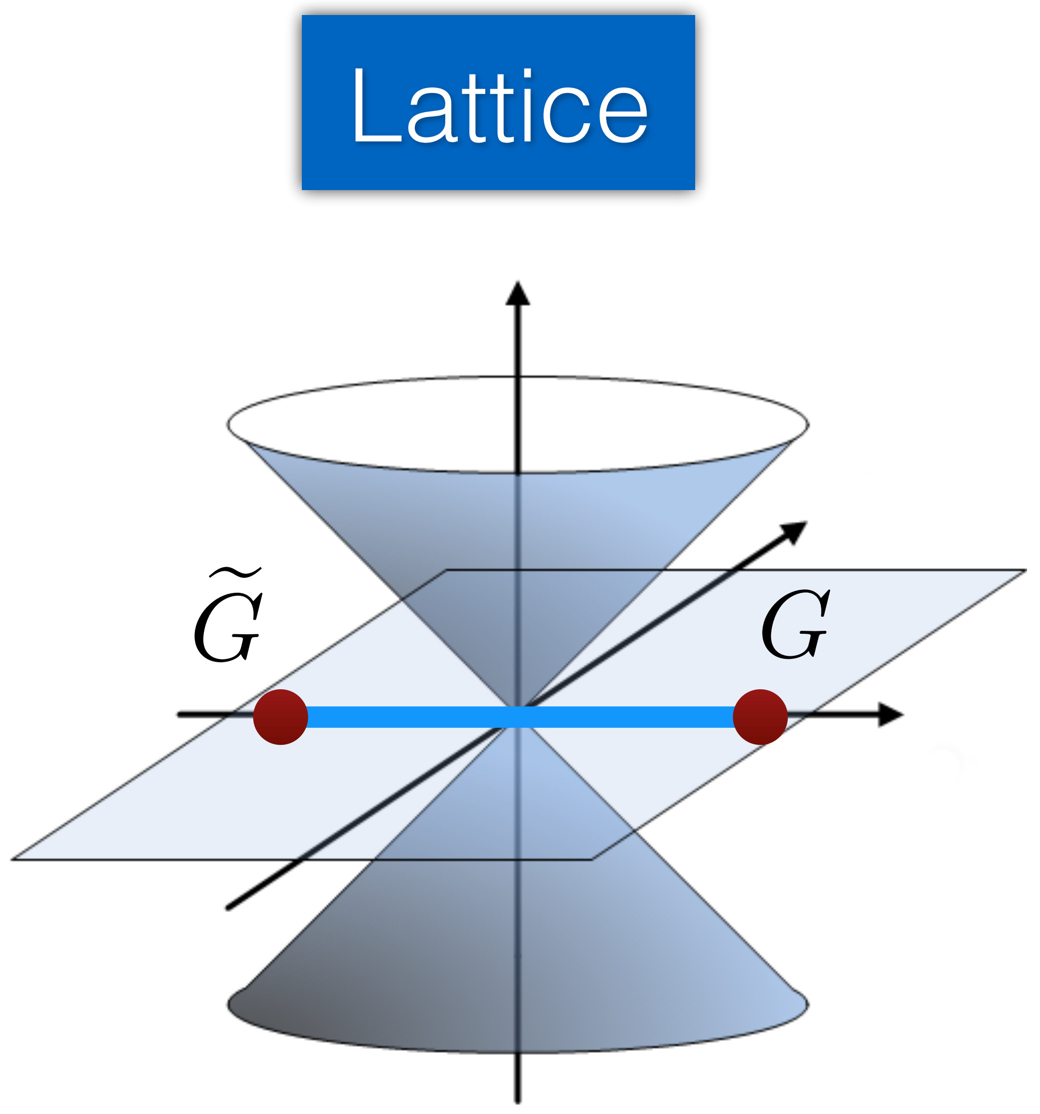
EIC white paper: EPJA (2016)



COMPASS Collaboration: PRD (2018)

LQCD formalism for calculating gluon PDFs





- On the lattice, calculate spatial correlation in coordinate space
  - ▶ **For polarized gluon PDF:** X. Ji [PRL 2013]

$$\widetilde{M}_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p,s|\, G_{\mu\alpha}(z)\, W[z,0]\, \widetilde{G}_{\lambda\beta}(0)\, |p,s\rangle$$

- Appropriate combination for this LQCD calculation

$$\widetilde{M}_{00}(z,p_z) \equiv [\widetilde{M}_{ti;ti}(z,p_z) + \widetilde{M}_{ij;ij}(z,p_z)] \qquad (i,j=x,y)$$

Balitsky et al [JHEP 2022]

- Multiplicative renormalizability

Zhang, et al [PRL 2019], Li, et al [PRL 2019]

▶ Renormalization:

$$\widetilde{\mathfrak{M}}(z, p_z) \equiv i \frac{[\widetilde{M_{00}(z, p_z)/p_z p_0}]/Z_{\rm L}(z/a_L)}{M_{00}(z, p_z = 0)/m_p^2}$$

"Pseudo-PDF" - Radyushkin [PRD 2017]
Balitsky et al [JHEP 2022]

- $$\widetilde{\mathfrak{M}}(\nu, z^2) \xrightarrow[\textbf{matching}]{\textbf{perturbative}} \widetilde{\mathcal{I}}_p(\nu, \mu^2) = \frac{i}{2} \int_{-1}^{1} \mathrm{d}x \, e^{-ix\nu} \, x \, \Delta g(x, \mu^2)$$

Lattice distribution

Light-cone distribution

Gluon helicity distribution

Ioffe time, $\nu = p_z z$ (convention from Braun, et al [PRD 1995])

LQCD matrix elements for polarized gluon distribution

- What we want is the light-cone Ioffe-time distribution:

$$\widetilde{\mathcal{I}}_p(\nu) \equiv i\,[\widetilde{\mathcal{M}}^{(+)}_{ps}(\nu) - \nu\widetilde{\mathcal{M}}_{pp}(\nu)]$$

  - Gluon helicity: $\Delta G(\mu^2) = \int_0^\infty \mathrm{d}\nu\ \widetilde{\mathcal{I}}_p(\nu,\mu^2) = \int_0^1 \mathrm{d}x\ \Delta g(x,\mu^2)$

Braun, et al [PRD 1995]

- What we get from the lattice calculation:

$$\widetilde{\mathfrak{M}}(\nu,z^2) = [\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu,z^2) - (1+m_p^2/p_z^2)\nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2)]$$

$$= \left[\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu,z^2) - \nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2)\right] - \frac{m_p^2}{p_z^2}\nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2)$$

Lattice QCD calculation

- Lattice details:
  - $L \times T = 32^3 \times 64$ $a \approx 0.094\,\mathrm{fm}$ $m_\pi = 358\,\mathrm{MeV}$
- Nucleon correlation function using Distillation

Peardon, et al [PRD 2009]

- Gluonic operator using Wilson flow

Luscher, JHEP 2010

- Solutions of summed generalized eigenvalue problem (sGEVP) for estimators of matrix elements

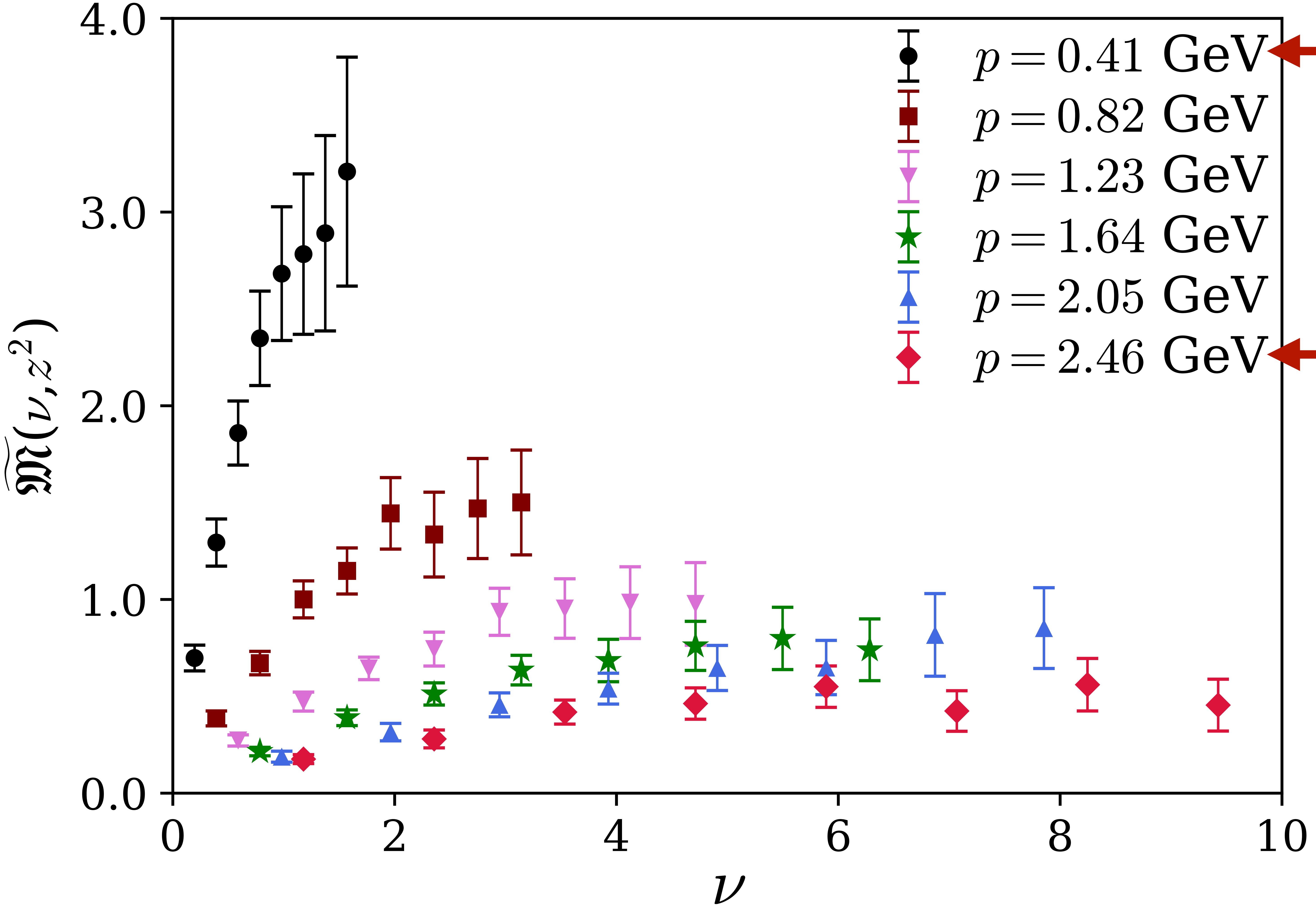
Bulava, et al, JHEP 2012

  - $C\exp(-\Delta E t/2)$ (GEVP)
  - $D\,t\exp(-\Delta E t)$ (sGEVP)
- Similar numerical techniques as in **Khan, *RSS*, et al (HadStruc Collaboration) : PRD 2021**
  - Also see, C. Egerer's talk today]

Ioffe time pseudo-distribution in the zero flow time limit

- Contamination term present in LQCD matrix element dominates

$$\widetilde{\mathfrak{M}}(\nu, z^2) = [\widetilde{\mathcal{M}}_{sp}^{(+)}(\nu, z^2) - (1 + m_p^2/p_z^2)\nu\widetilde{\mathcal{M}}_{pp}(\nu, z^2)]$$



$m_p^2/p^2 \sim 7.3$

$m_p^2/p^2 \sim 0.2$

grows with ν

- Correction through fits using moments

$$\widetilde{\mathfrak{M}}(\nu, z^2) = \left[\widetilde{\mathcal{M}}_{sp}^{(+)}(\nu, z^2) - \nu\widetilde{\mathcal{M}}_{pp}(\nu, z^2)\right] - \frac{m_p^2}{p_z^2}\nu\widetilde{\mathcal{M}}_{pp}(\nu, z^2)$$

- $\widetilde{\mathcal{M}}_{sp}^{(+)}$: odd in ν
- $\widetilde{\mathcal{M}}_{pp}$: even in ν

$$\widetilde{\mathfrak{M}}(\nu) = \sum_{i=0} \frac{(-1)^i}{(2i+1)!} a_i \nu^{2i+1} + \nu \frac{m_p^2}{p^2} \sum_{j=0} \frac{(-1)^j}{(2j)!} b_j \nu^{2j}$$

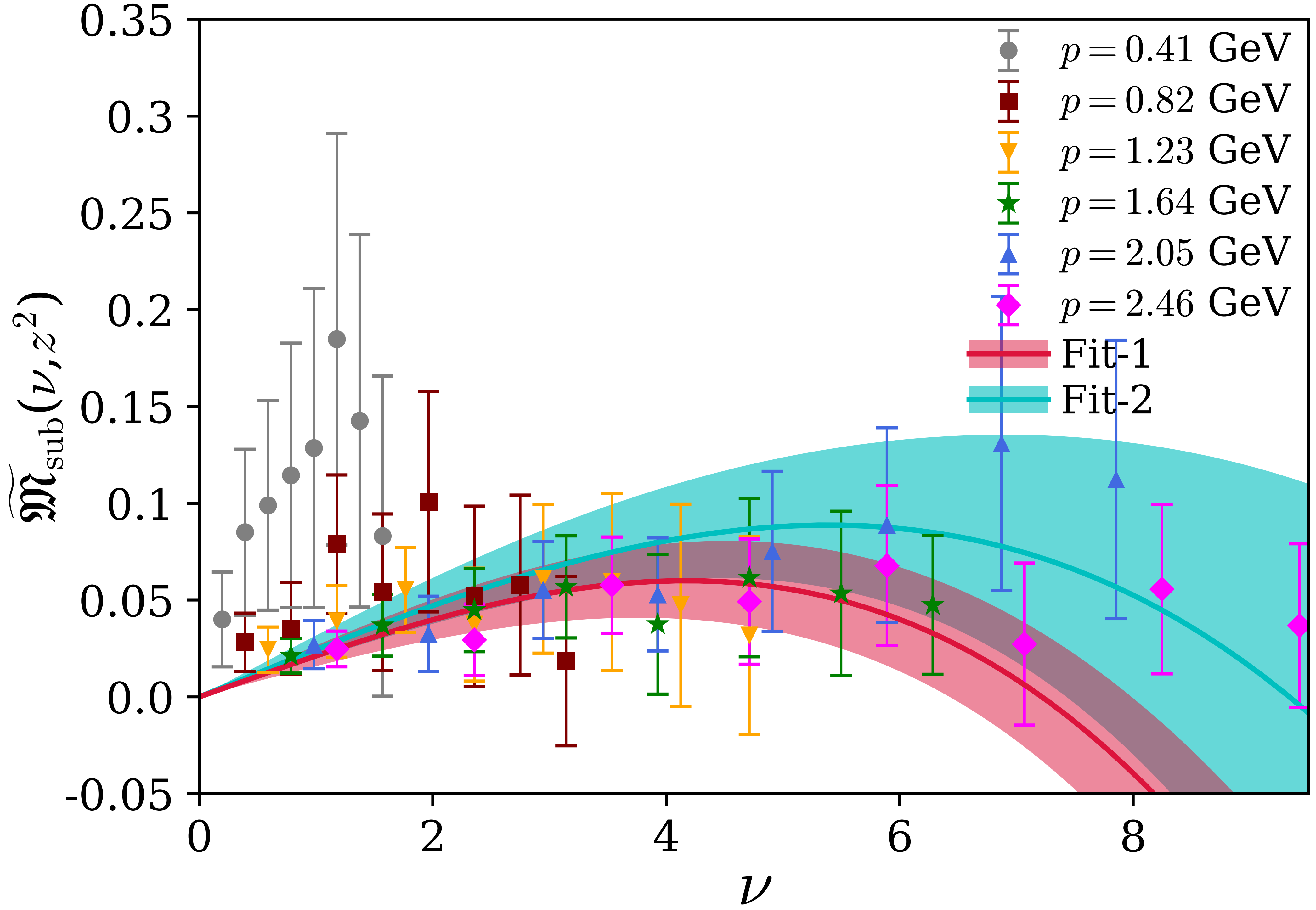
- Correction by subtracting zero momentum matrix elements

$$\widetilde{M}_{0i;0i}(z,p_z)+\widetilde{M}_{ij;ij}(z,p_z)=-2p_zp_0\widetilde{\mathcal{M}}_{sp}^{(+)}(\nu,z^2)+\boxed{2p_0^3z\widetilde{\mathcal{M}}_{pp}(\nu,z^2)}$$

non-vanishing at $p_z=0$

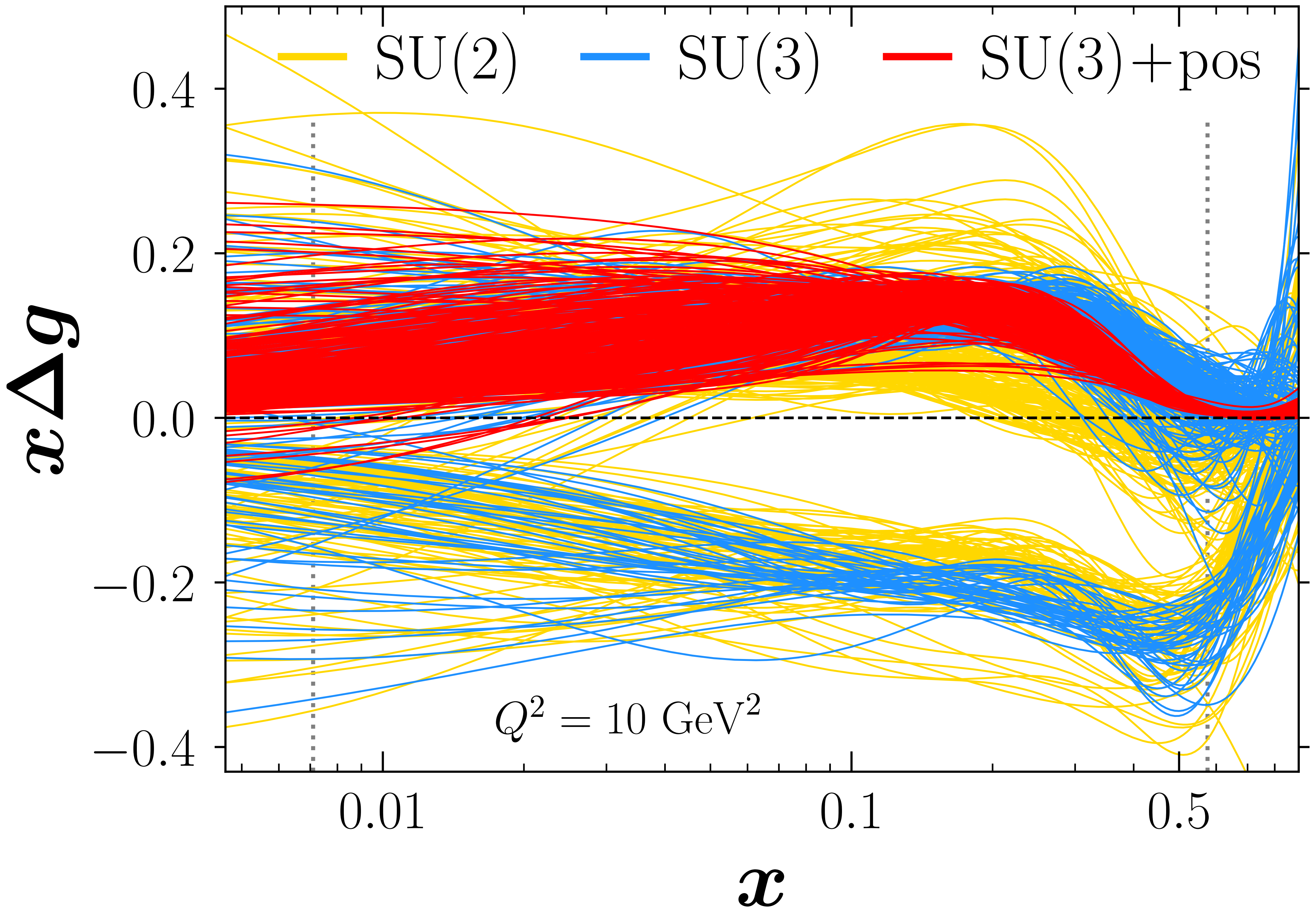
- Proposed subtraction :

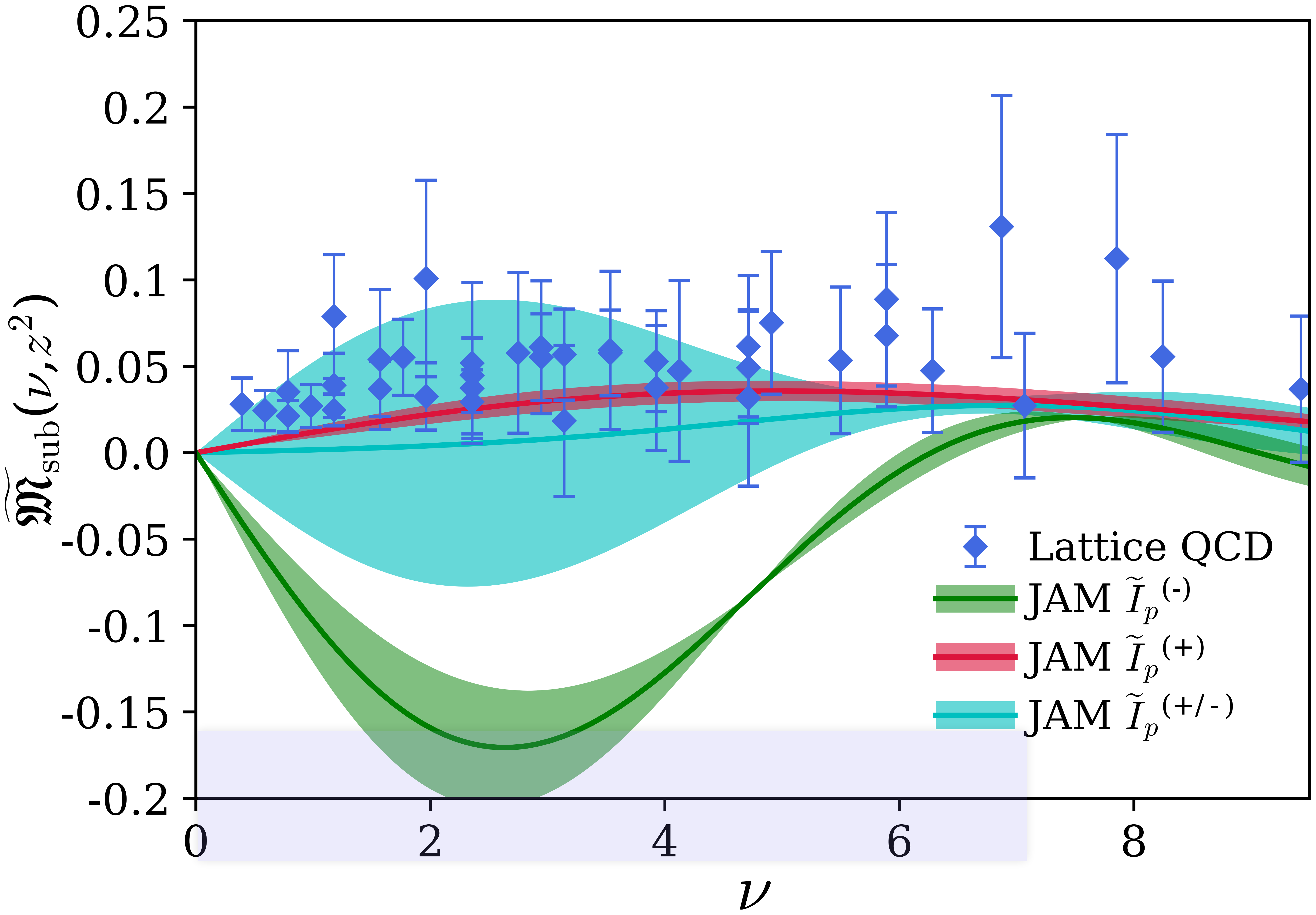
$$\widetilde{\mathcal{M}}_{\rm sub}(z,p_z)=\widetilde{\mathcal{M}}_{sp}^{(+)}(\nu,z^2)-\nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2)-\nu\frac{m_p^2}{p_z^2}\left[\widetilde{\mathcal{M}}_{pp}(\nu,z^2)-\widetilde{\mathcal{M}}_{pp}(\nu=0,z^2)\right]$$



- Sign of gluon helicity distribution is unsettled from global analyses of experimental data

Zhao, Sato, Melnitchouk
[PRD 2022]

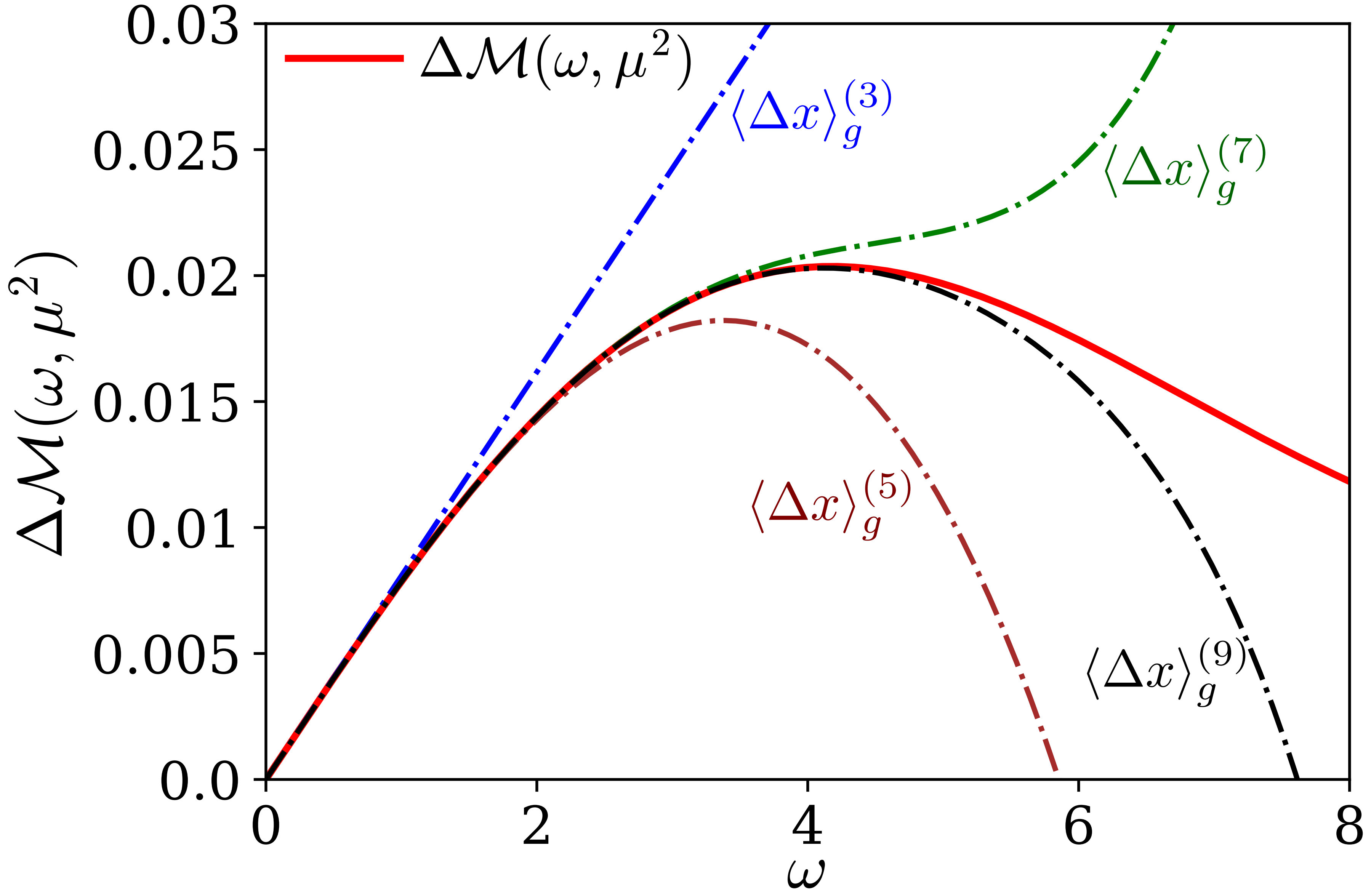


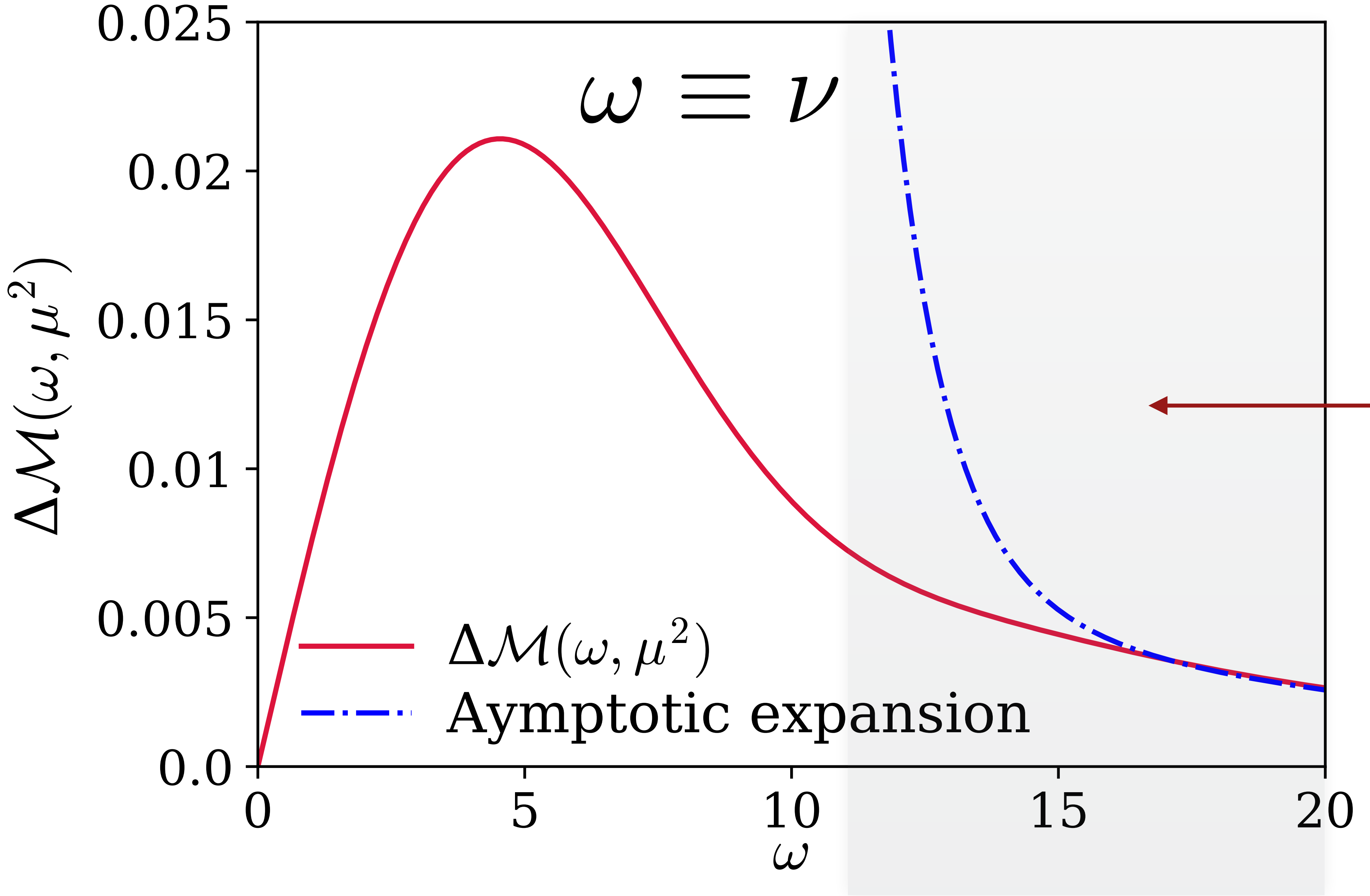


***RSS*, Khan, Karthik, et al (HadStruc Collaboration) : 2207.08733**

Challenge for LQCD calculation of x-dependent gluon helicity distribution

- Lattice data in a limited range of ν
- Available lattice data is sensitive up to first few moments





Long-term mission for LQCD community

- Asymptotic form: $$\mathcal{M}(\omega,\mu^2) = A\left[\left(C_R(\alpha,4+\beta;\omega) + \gamma\, C_R(\alpha+1/2,4+\beta;\omega) + \delta\, C_R(\alpha+1,4+\beta;\omega)\right) + (\beta\to\beta+2)\right] + B\left[\beta\to\beta+1\right] + \mathcal{O}(1/\omega^{a+R+1})$$

RSS, Liu, Paul
PRD 2021

Question: How best to extract x-dependent gluon helicity distribution from LQCD

- Need two calculations to properly constrain $x\Delta g(x)$ from LQCD :

- LQCD determination of gluon spin from local matrix element:

$$\vec{S}_g \to \vec{E} \times \vec{A}_{\text{phys}}$$

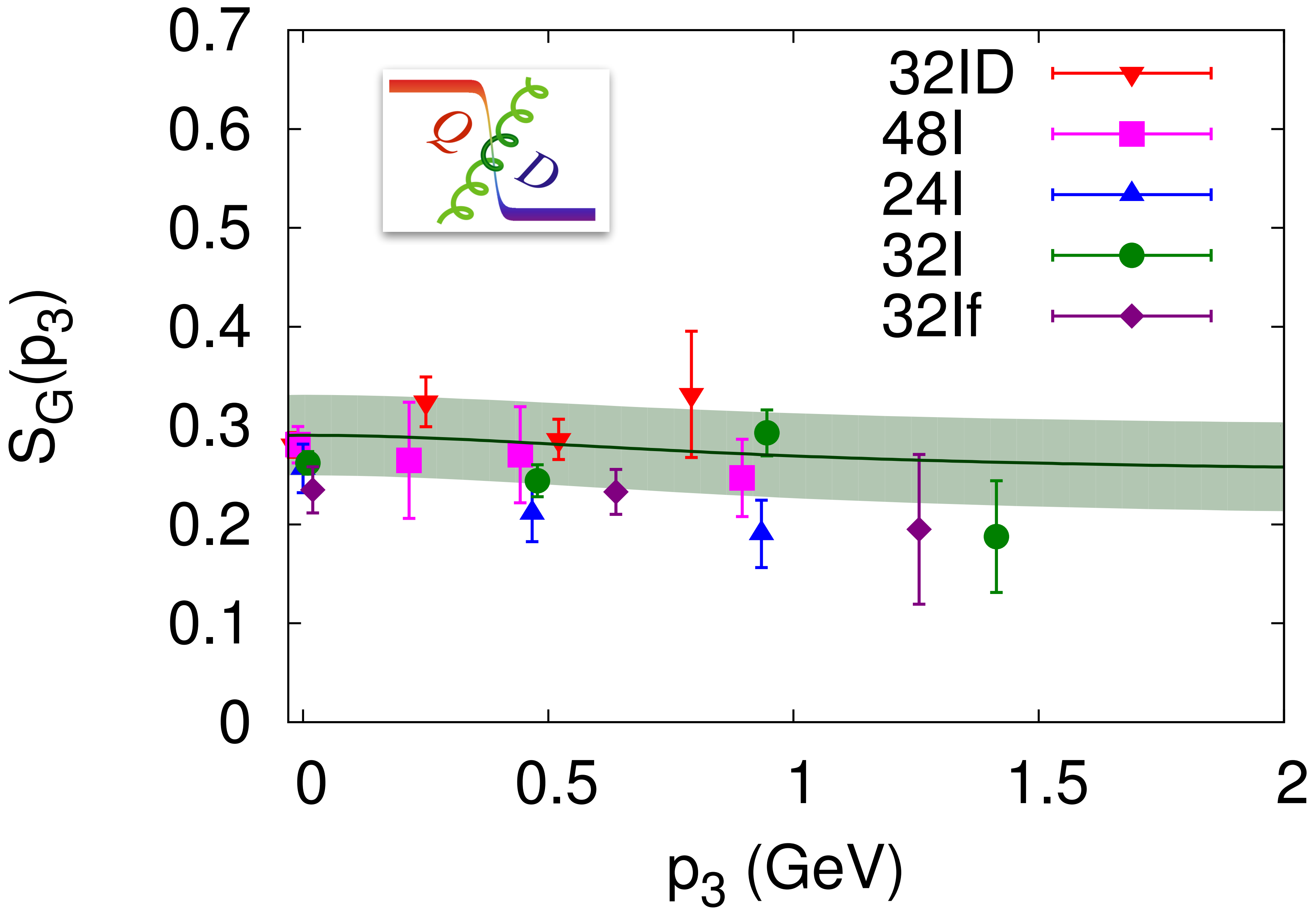
$\downarrow$

LaMET matching
[Ji, Sci. China Phys 2014]

$\downarrow$

$$\Delta G$$

Ji, Zhang, Zhao [PRL 2013]



RSS, Glatzmaier, Yang, Liu, et al
PoS LATTICE(2014)
Yang, *RSS*, et al (PRL 2017)

- After 1-loop matching

$$\boxed{\Delta G(\mu^2 = 10\,\text{GeV}^2) = 0.251(47)(16)}$$

Question: How best to extract x-dependent gluon helicity distribution from LQCD

- Recall: $\widetilde{\mathcal{I}}_p(\nu) = \frac{i}{2}\int_{-1}^{1} \mathrm{d}x\, e^{-ix\nu}\, x\,\Delta g(x)$

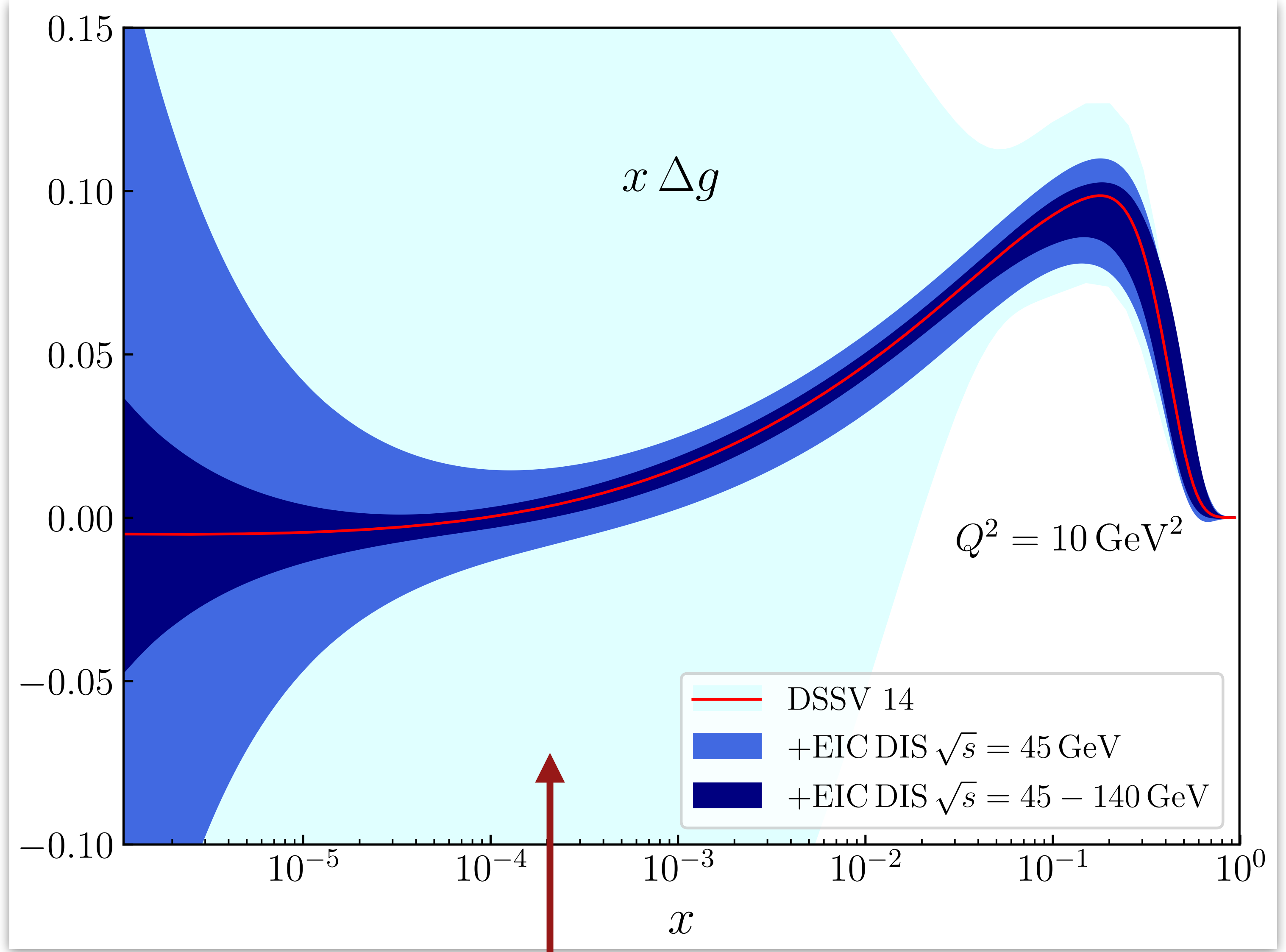
$$\Delta G = \int_0^\infty \mathrm{d}\nu\, \widetilde{\mathcal{I}}_p(\nu) = \int_0^1 \mathrm{d}x\, \Delta g(x)\,.$$

LQCD calculation of local mat. elem $E \times A_{phys}$ + matching

Nonlocal LQCD mat. elem + factorization

- Combine ΔG and $\widetilde{\mathcal{I}}_p(\nu)$ to determine $\Delta g(x,\mu^2)$ from LQCD

Impact of projected EIC data (EIC Yellow Report)



A_{LL} always limited to $x \in [x_{\min}, 1]$

Kovchegov, et al [JHEP 2018]

Constraints from LQCD & theory of small-x needed

Summary & Outlook

- First LQCD determination of polarized gluon Ioffe-time distribution

- Future calculation:

  - With precise LQCD matrix elements, perform pQCD matching to obtain light-cone Ioffe-time distribution

  - Consider mixing with singlet quark distribution

- Goal: determination of gluon contribution to proton spin & x-dependent helicity distribution from LQCD

Thank you!