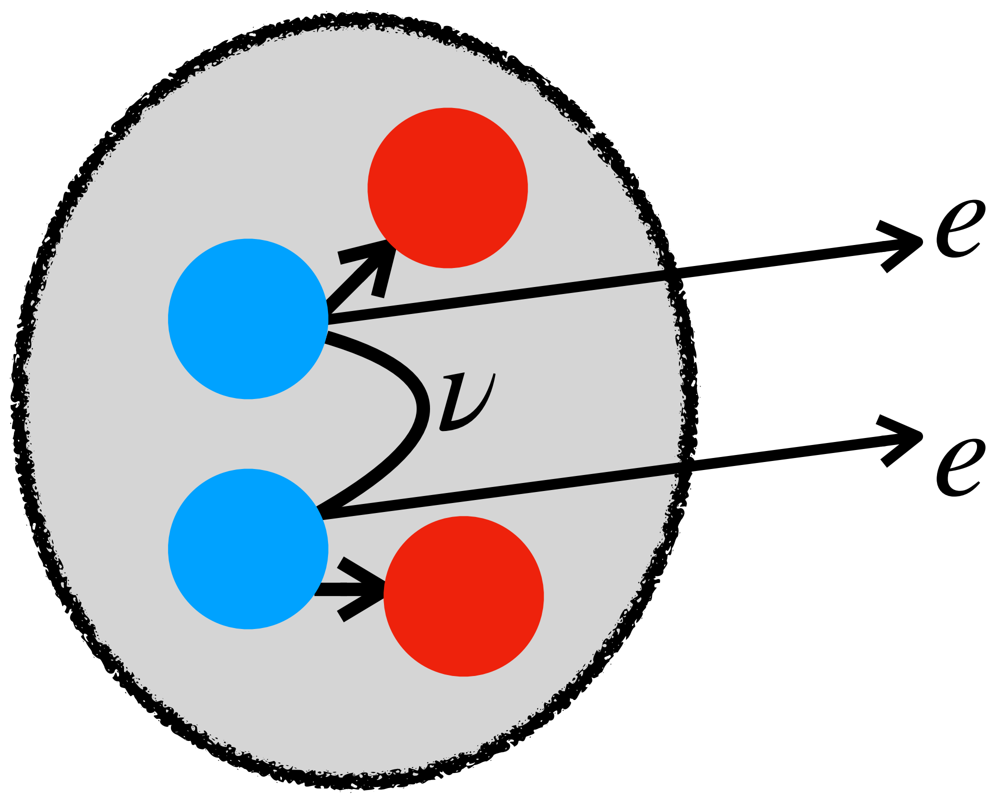
Estimating IMSRG truncation errors for $0\nu\beta\beta$ decay



Ragnar Stroberg
University of Notre Dame

INT 26-2a "Recommended Values for Neutrinoless Double-Beta Decay Nuclear Matrix Elements"
Seattle, July 1, 2026

Featuring work done with/by Antoine Belley, Bingcheng He, Jason Holt, Taiki Shickele, Zhonghao Sun, Alex Todd, Victor Vaida, and the NDB Collaboration.

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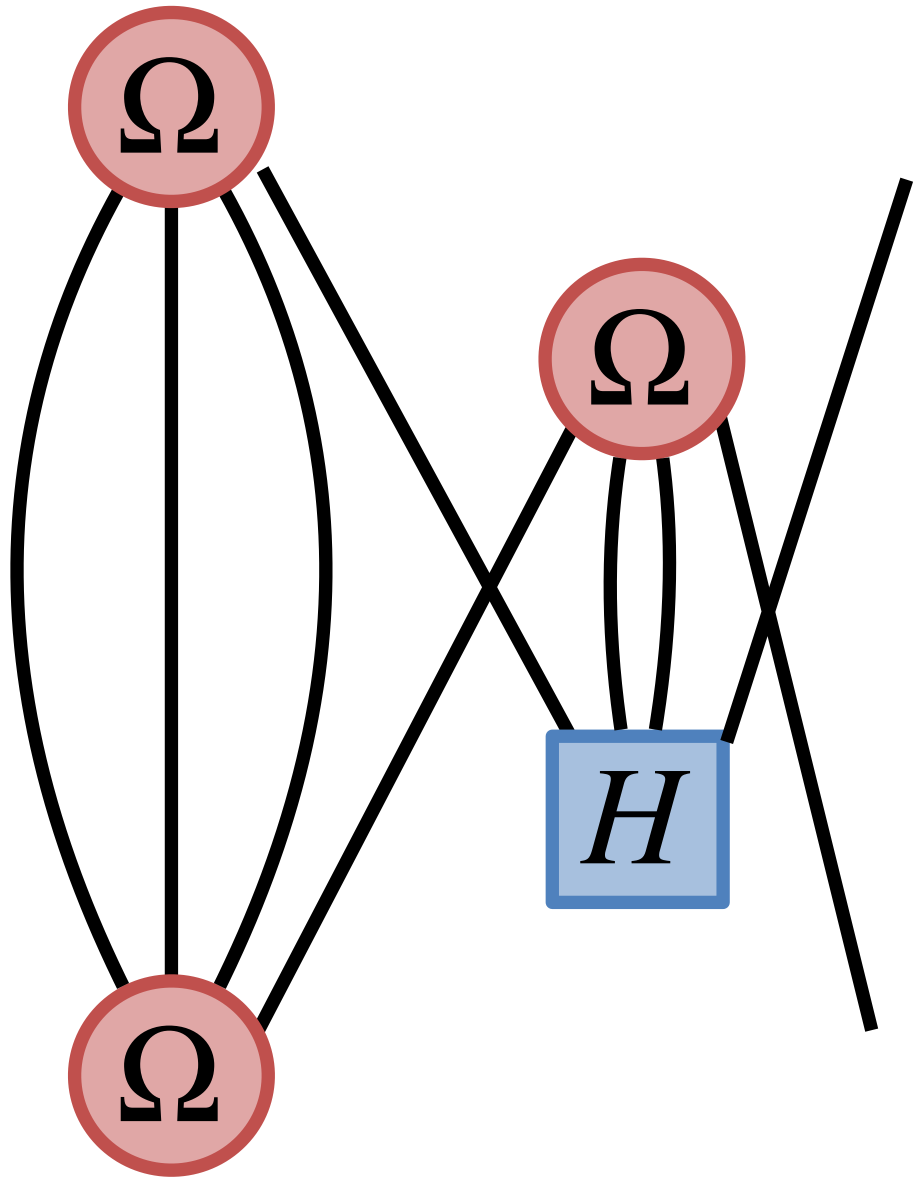

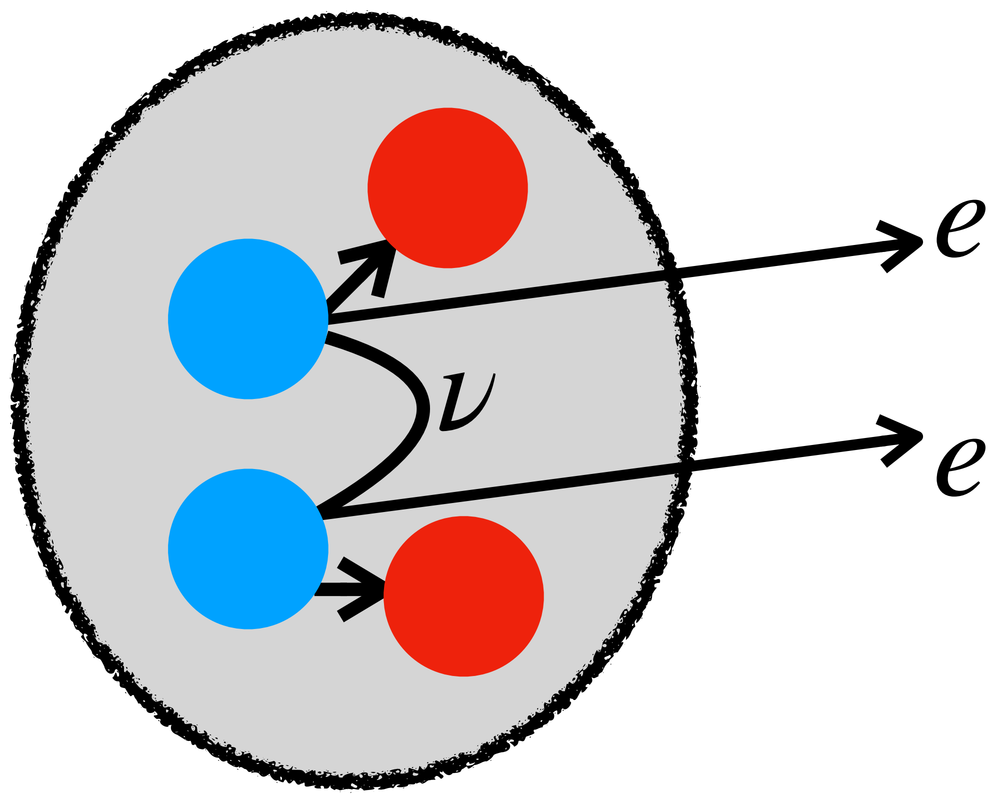

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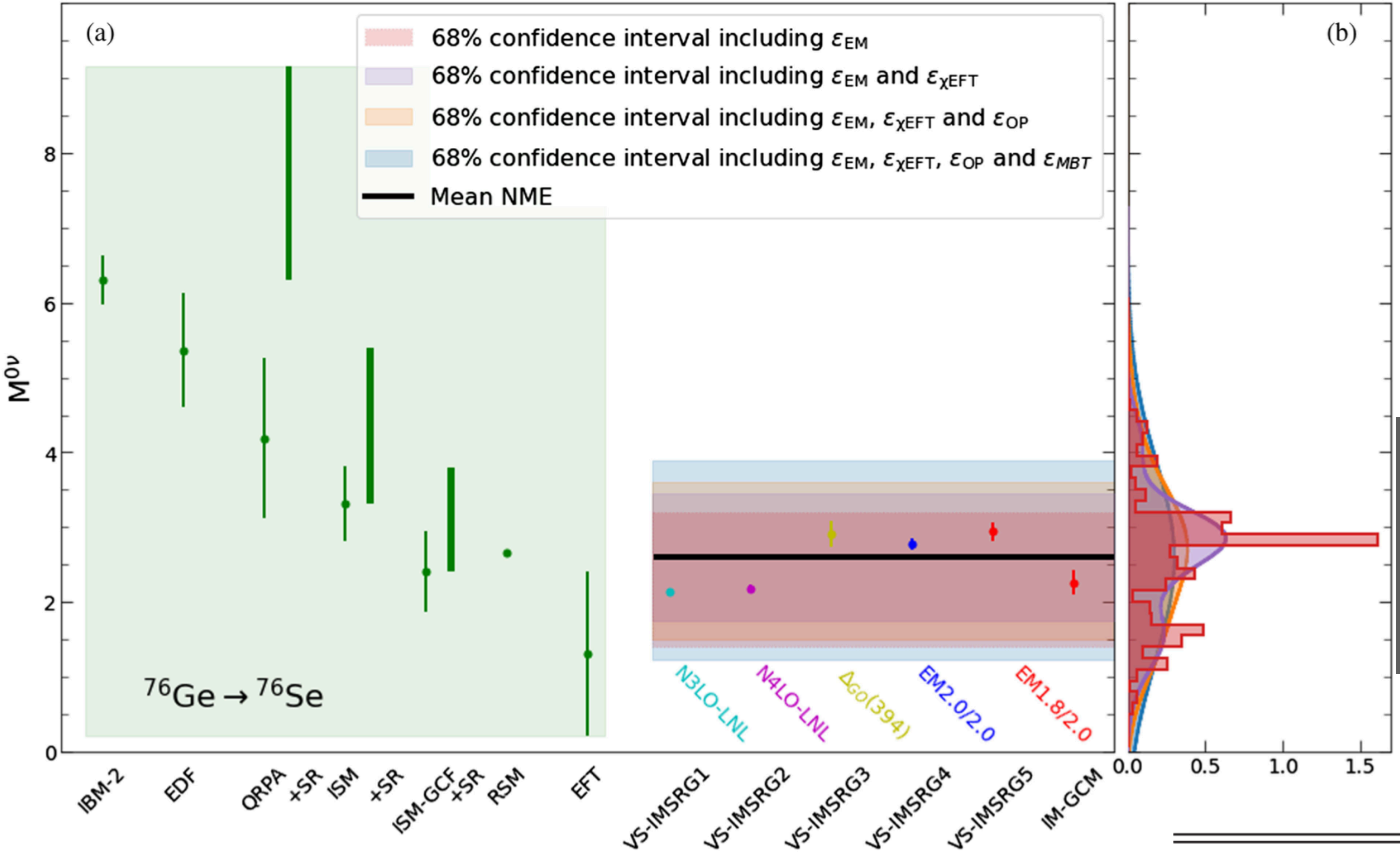


Many-body truncation is the biggest, and least understood source of uncertainty.

$M^{0\nu}$	ϵ_{LEC}	$\epsilon_{\chi EFT}$	ϵ_{MBT}	ϵ_{OP}	ϵ_{EM}
$2.60^{+1.28}_{-1.36}$	0.75	0.3	0.88	0.47	< 0.06

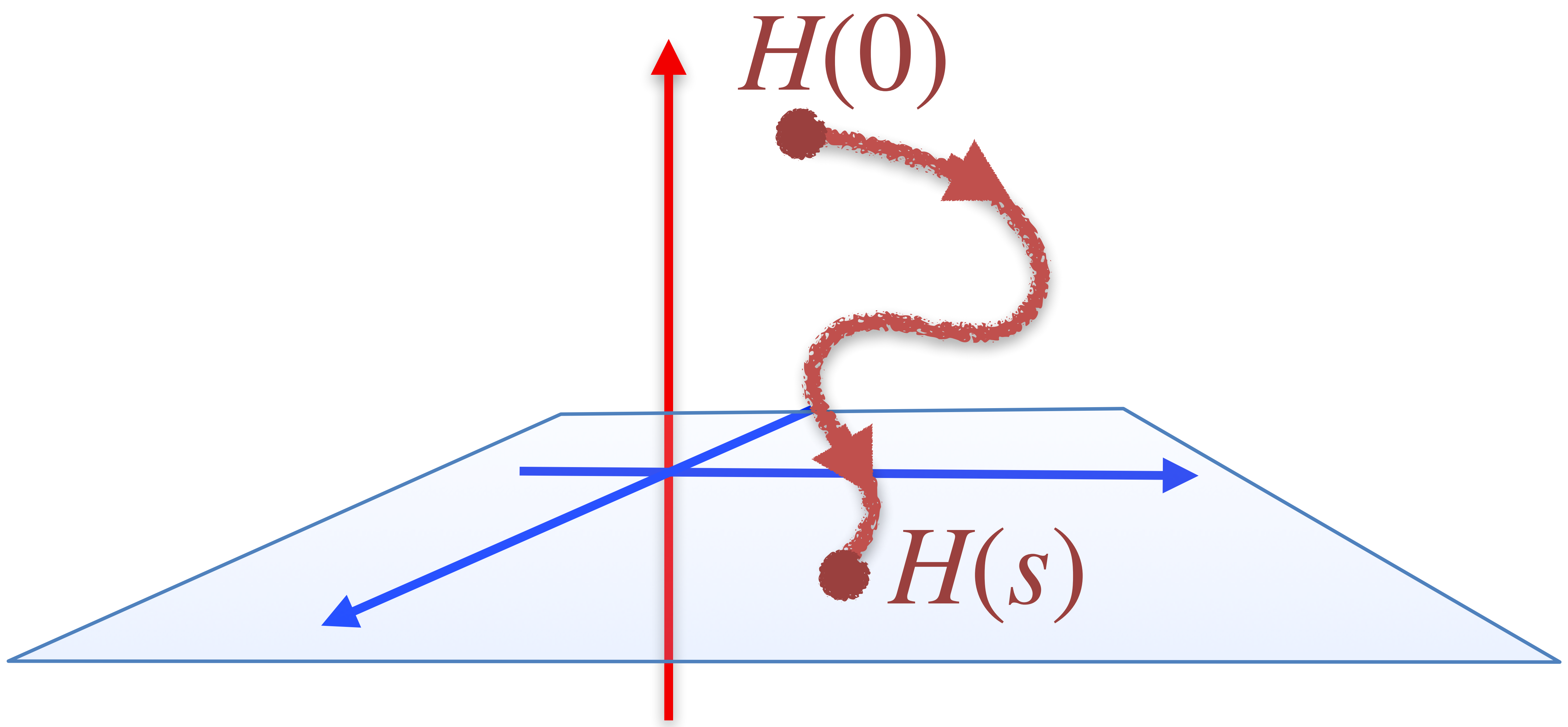
In-Medium Similarity Renormalization Group (IMSRG)

unitary transformation

$$H(s) = U(s)HU^\dagger(s)$$

SRG flow equation

$$\frac{dH}{ds} = [\eta(s), H(s)]$$



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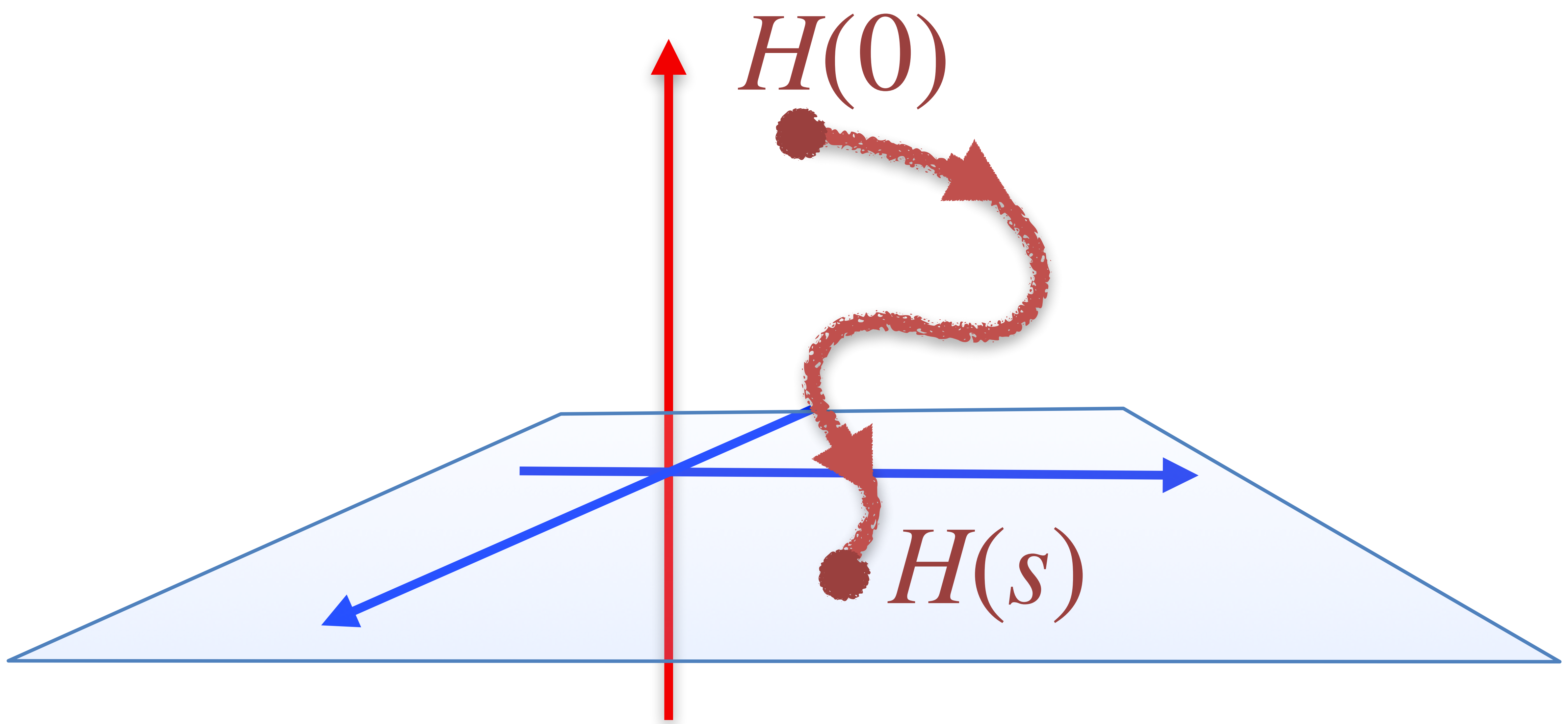
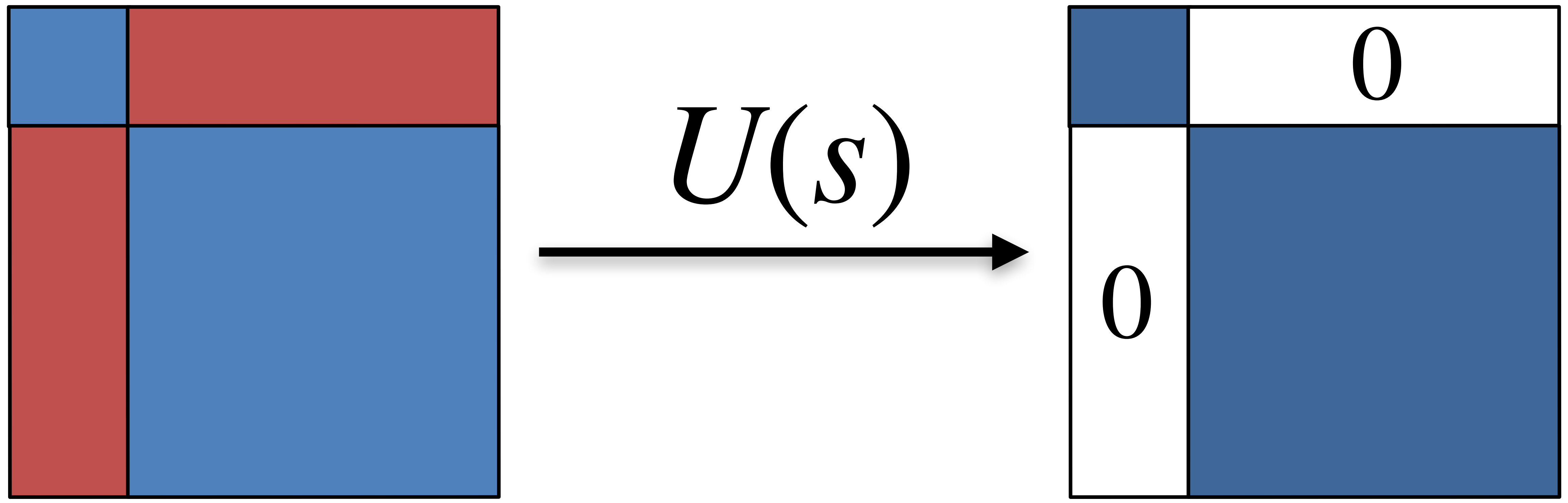


$$H = H^{\mathrm{d}} + H^{\mathrm{od}} \qquad H^{\mathrm{od}}(s) \rightarrow 0$$



In-Medium Similarity Renormalization Group (IMSRG)

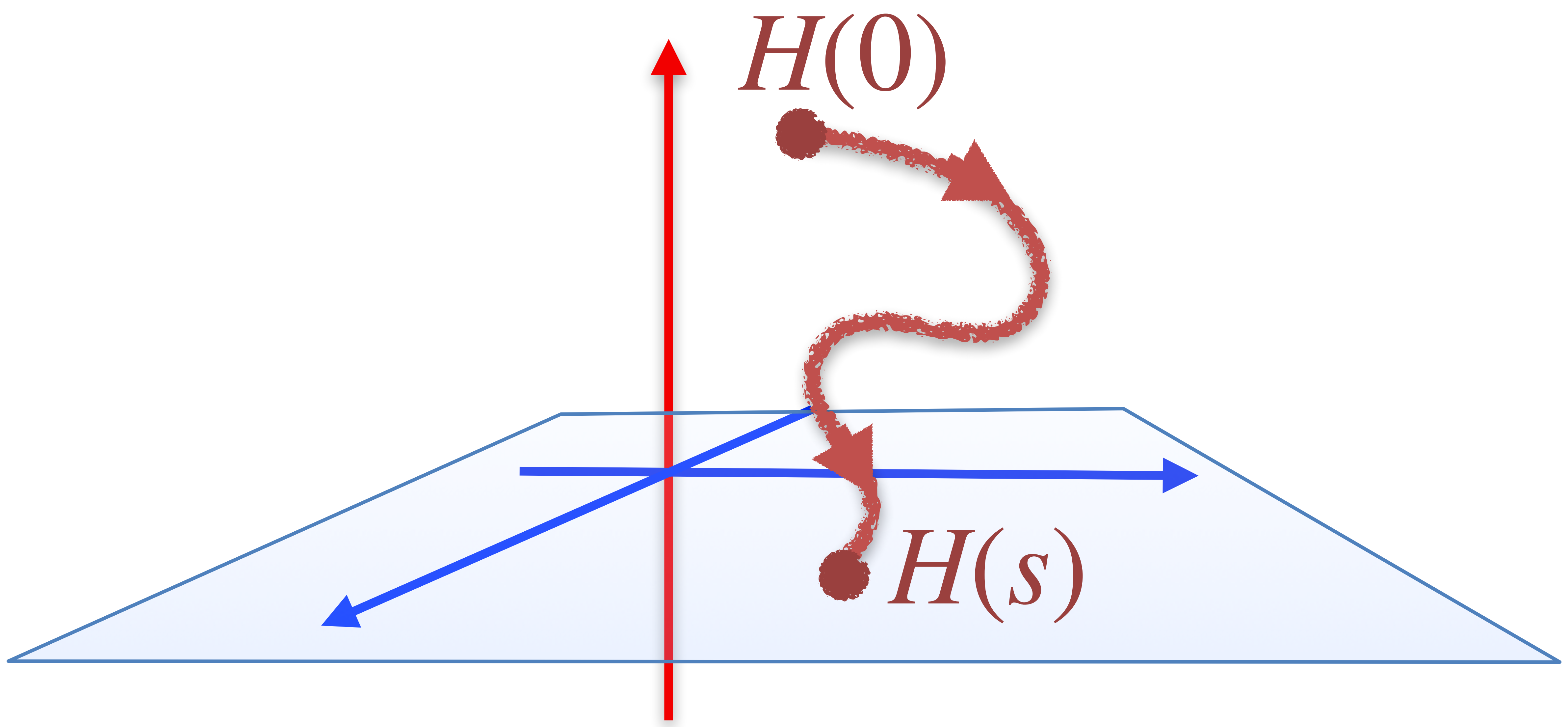
unitary transformation $H(s) = U(s) H U^\dagger(s)$

SRG flow equation $$\frac{dH}{ds} = [\eta(s), H(s)]$$

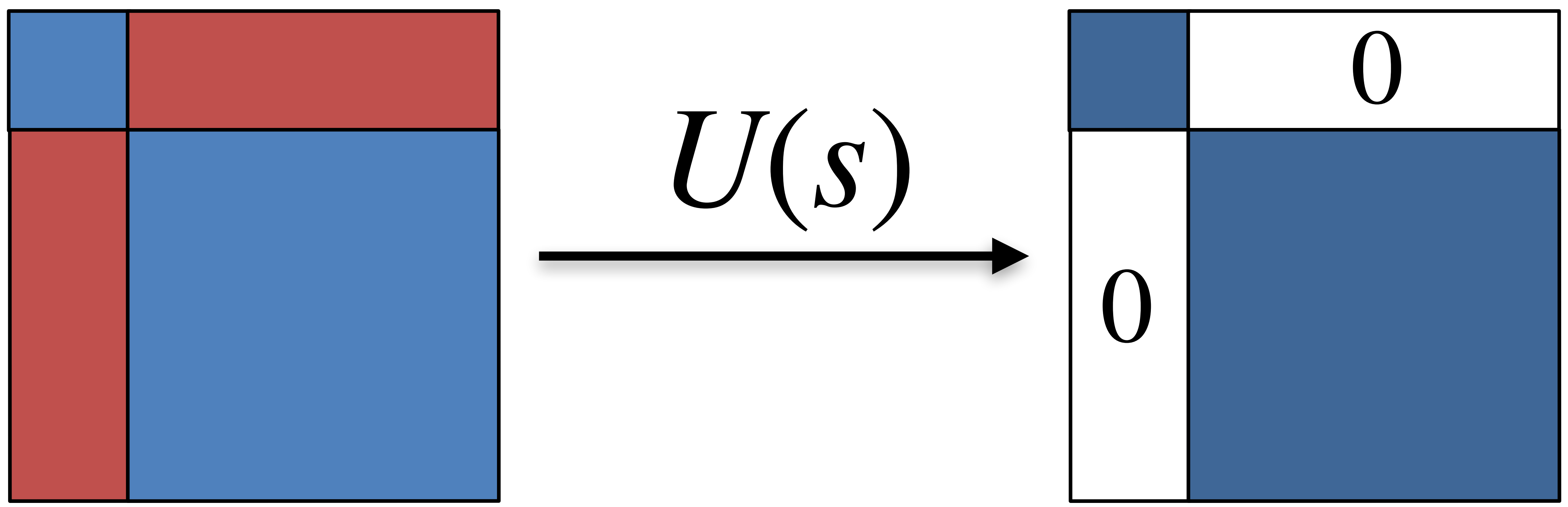
Magnus $U(s) = e^{\Omega(s)}$

$$H(s) = e^{\Omega} H e^{-\Omega}$$
$$= H + [\Omega, H] + \frac{1}{2}[\Omega, [\Omega, H]] + \dots$$

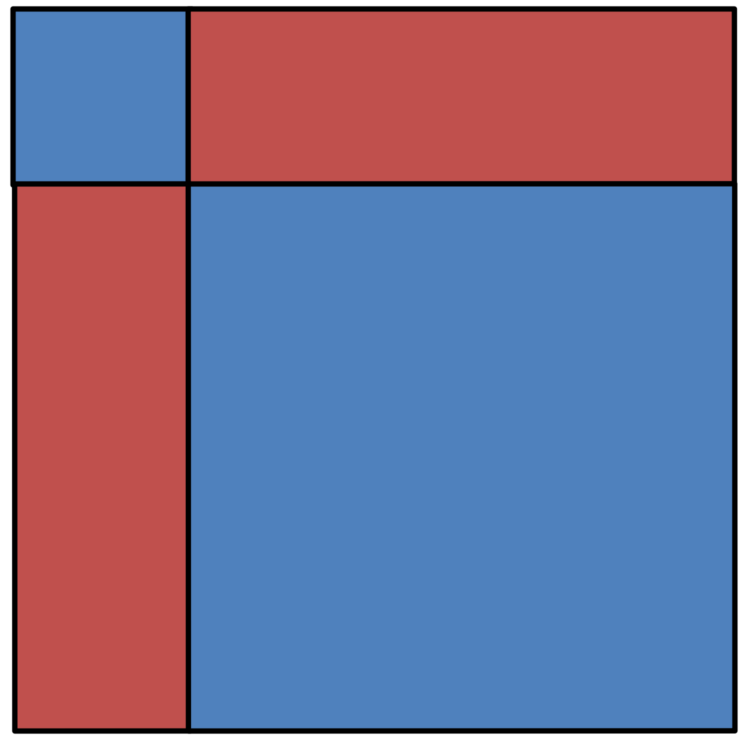
Find Ω by iteration, or flow equation



$H = H^{\mathrm{d}} + H^{\mathrm{od}}$ $H^{\mathrm{od}}(s) \to 0$



Magnus IMSRG



$$U(s) = e^{\Omega(s)} \qquad \Omega(0) = 1$$

Flow equation for Magnus operator:

$$\frac{d\Omega}{ds} = \sum_k \frac{B_k}{k!}[\Omega, \eta]^{(k)}$$

$$= \eta + \frac{1}{2}[\eta, \Omega] + \frac{1}{12}[\Omega, [\Omega, \eta]] + \dots$$

White's generator: $\eta = \frac{H^{\mathrm{od}}}{\Delta}$

$$\langle a|\eta|b\rangle = \frac{\langle a|H^{\mathrm{od}}|b\rangle}{\langle a|H^d|a\rangle - \langle b|H^d|b\rangle}$$

$$\eta = -\eta^\dagger$$

$$\Omega = -\Omega^\dagger$$

$$\langle ab|\eta|cd\rangle = \frac{\langle ab|H^{\mathrm{od}}|cd\rangle}{\langle ab|H^d|ab\rangle - \langle cd|H^d|cd\rangle}$$

Magnus IMSRG

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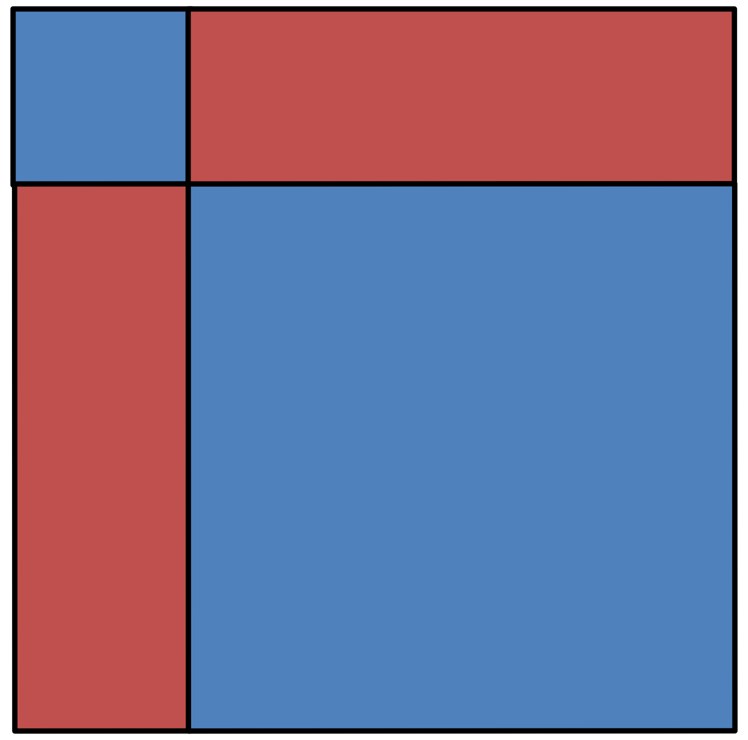

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Magnus IMSRG

$$H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$$

$$= H + [\Omega, H] + \frac{1}{2!}[\Omega, [\Omega, H]] + \frac{1}{3!}[\Omega, [\Omega, [\Omega, H]]] + \dots$$

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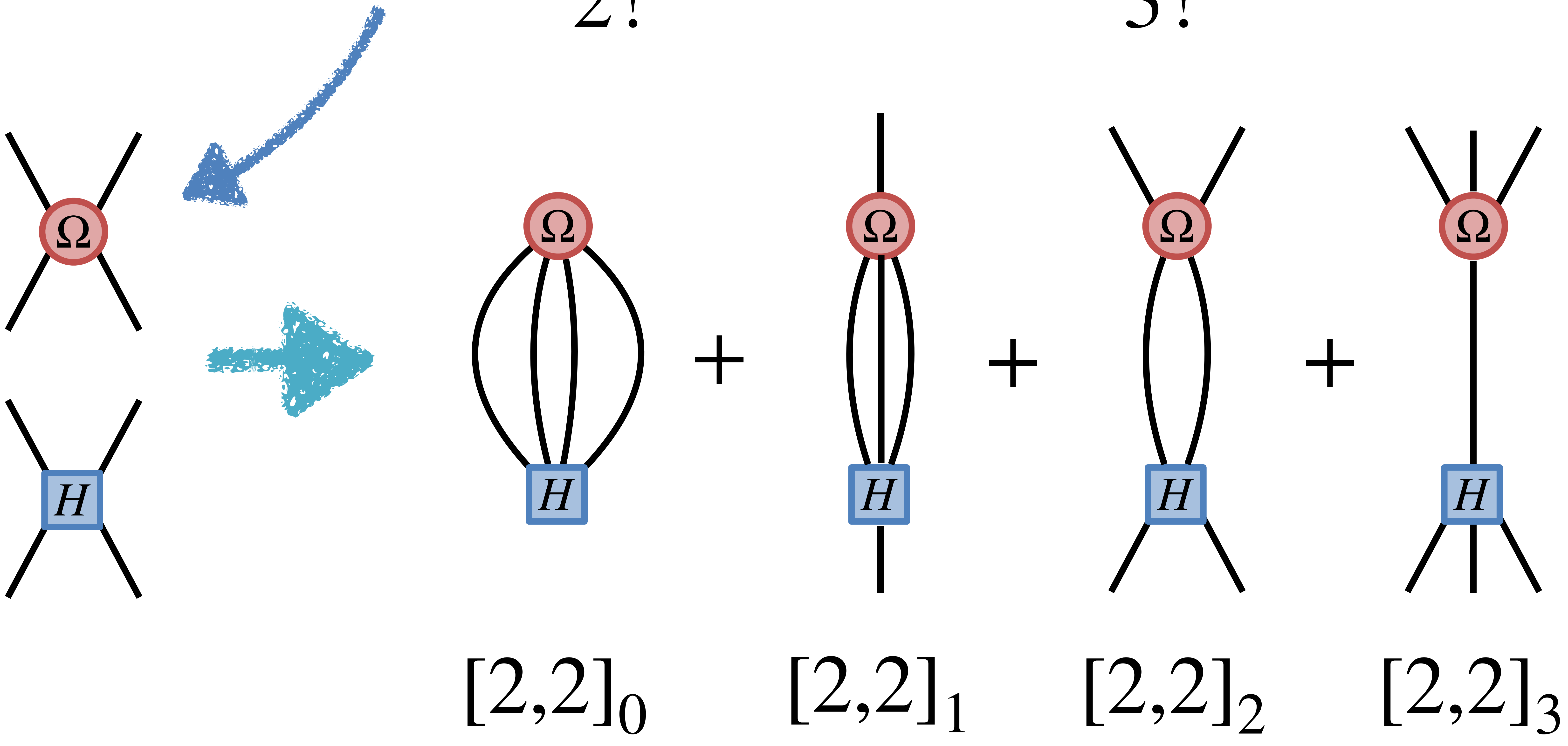

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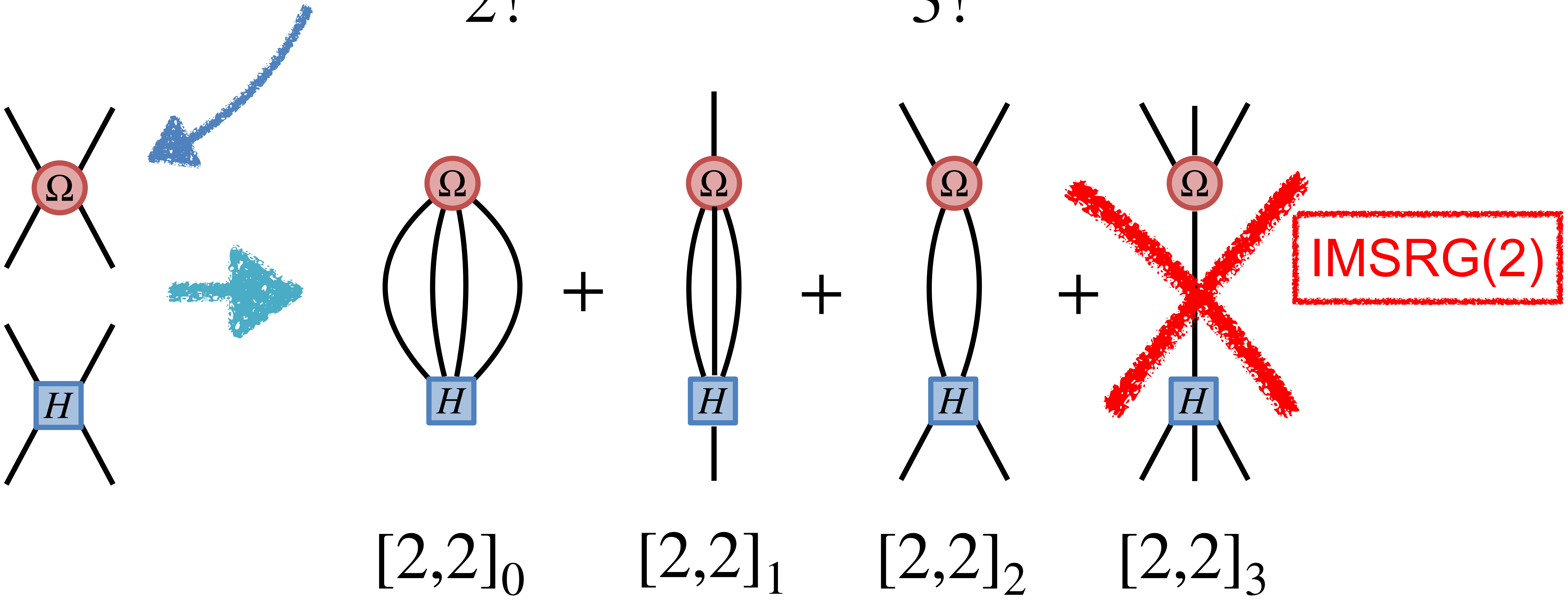
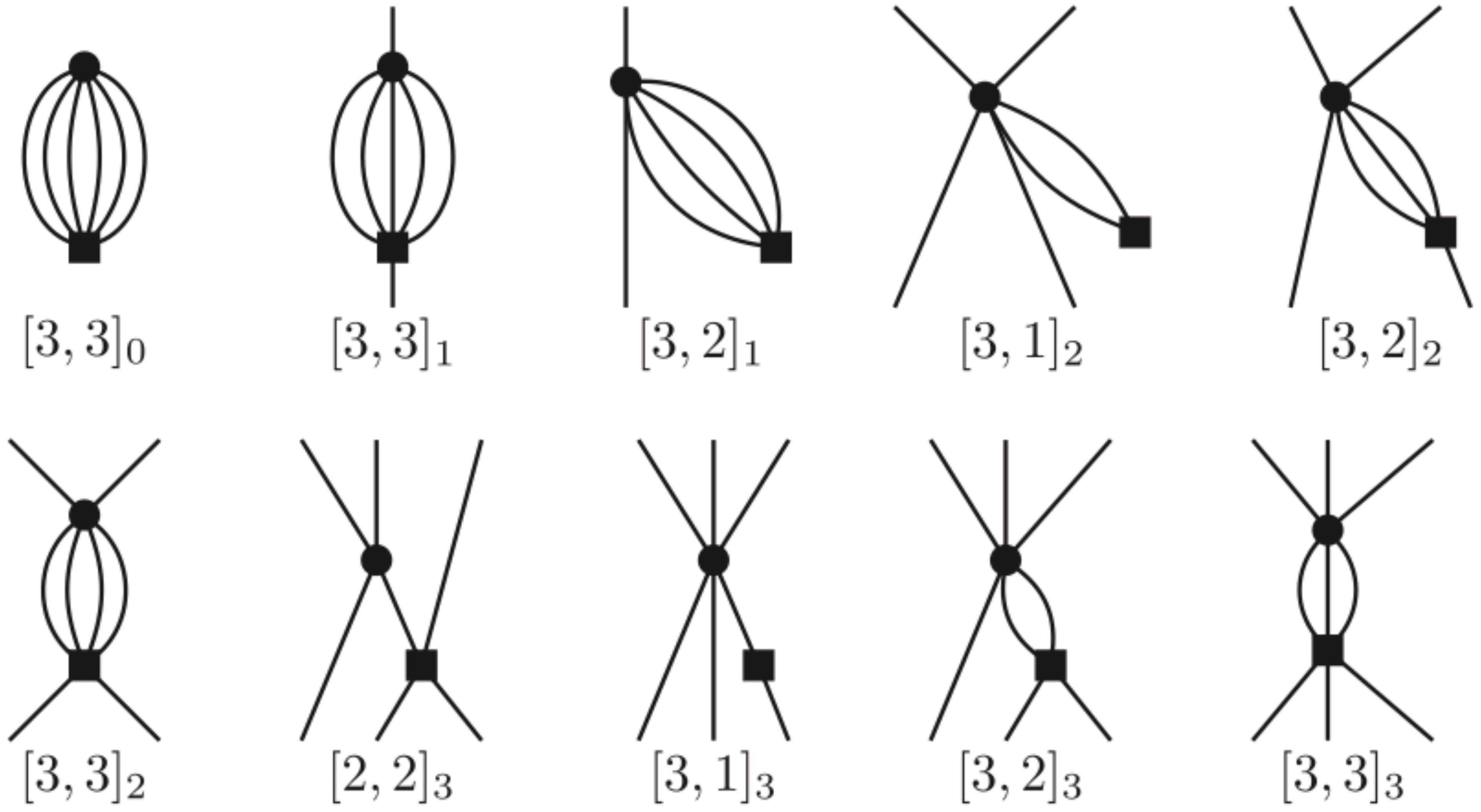
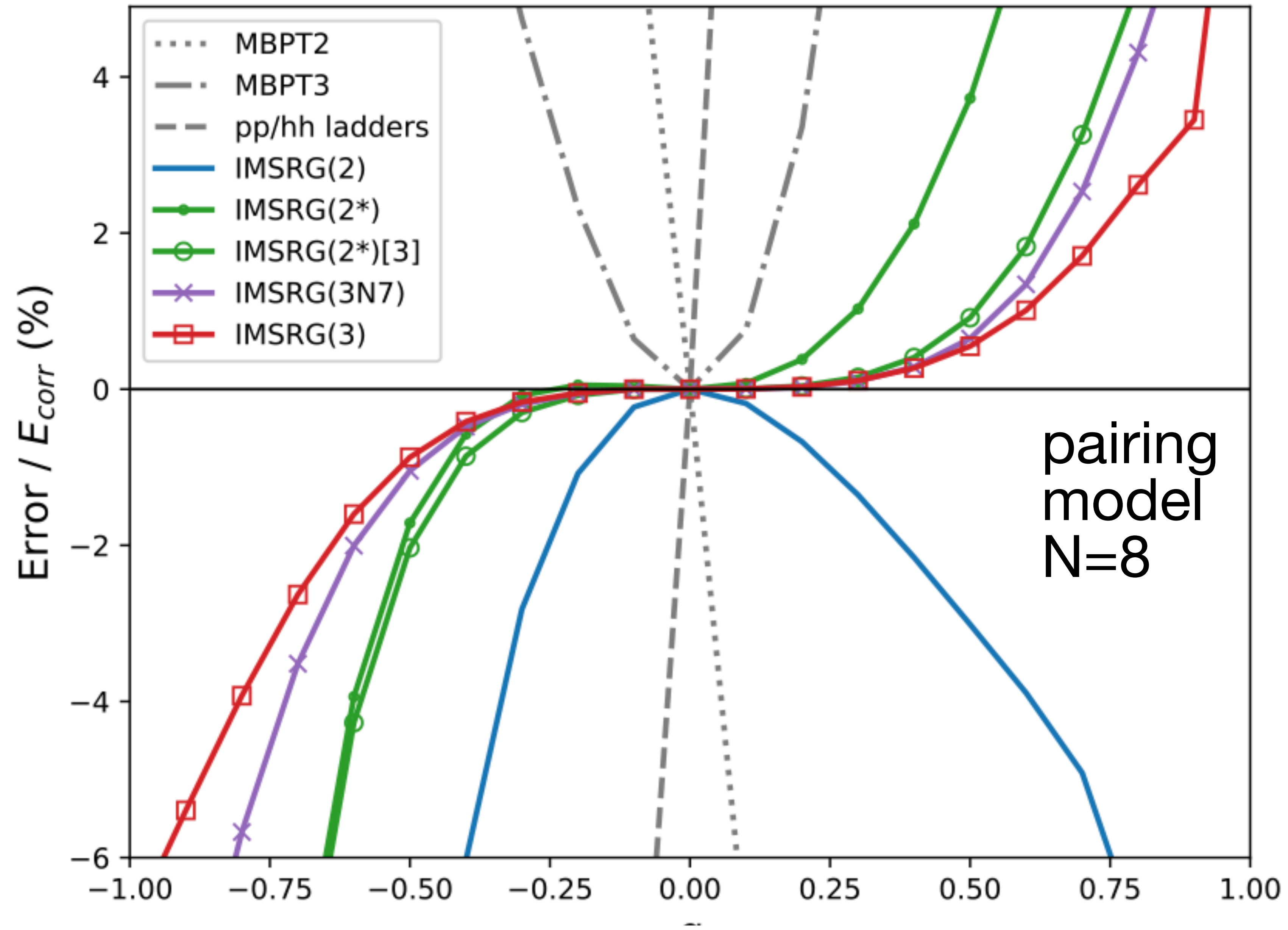
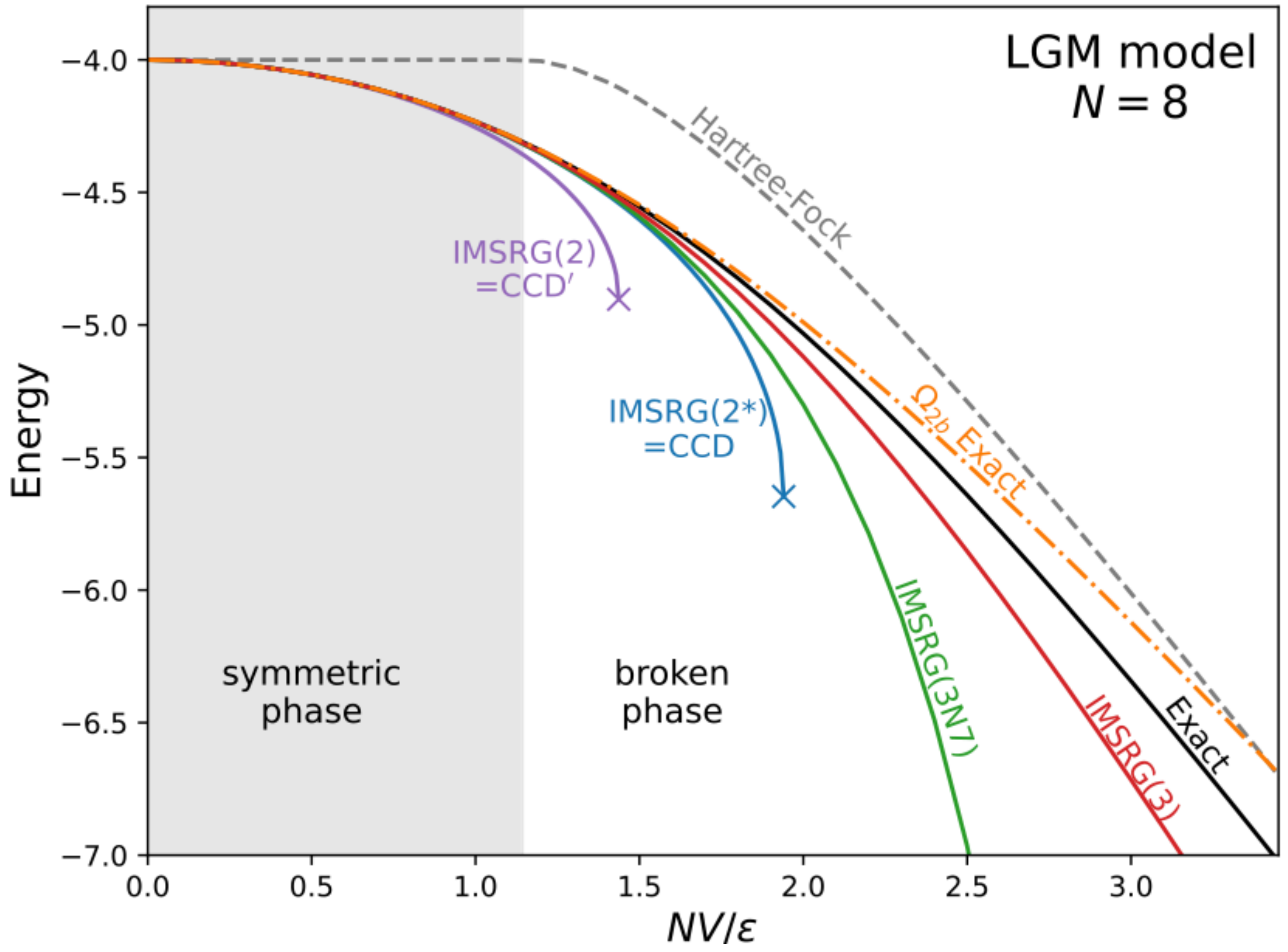


IMSRG(3)







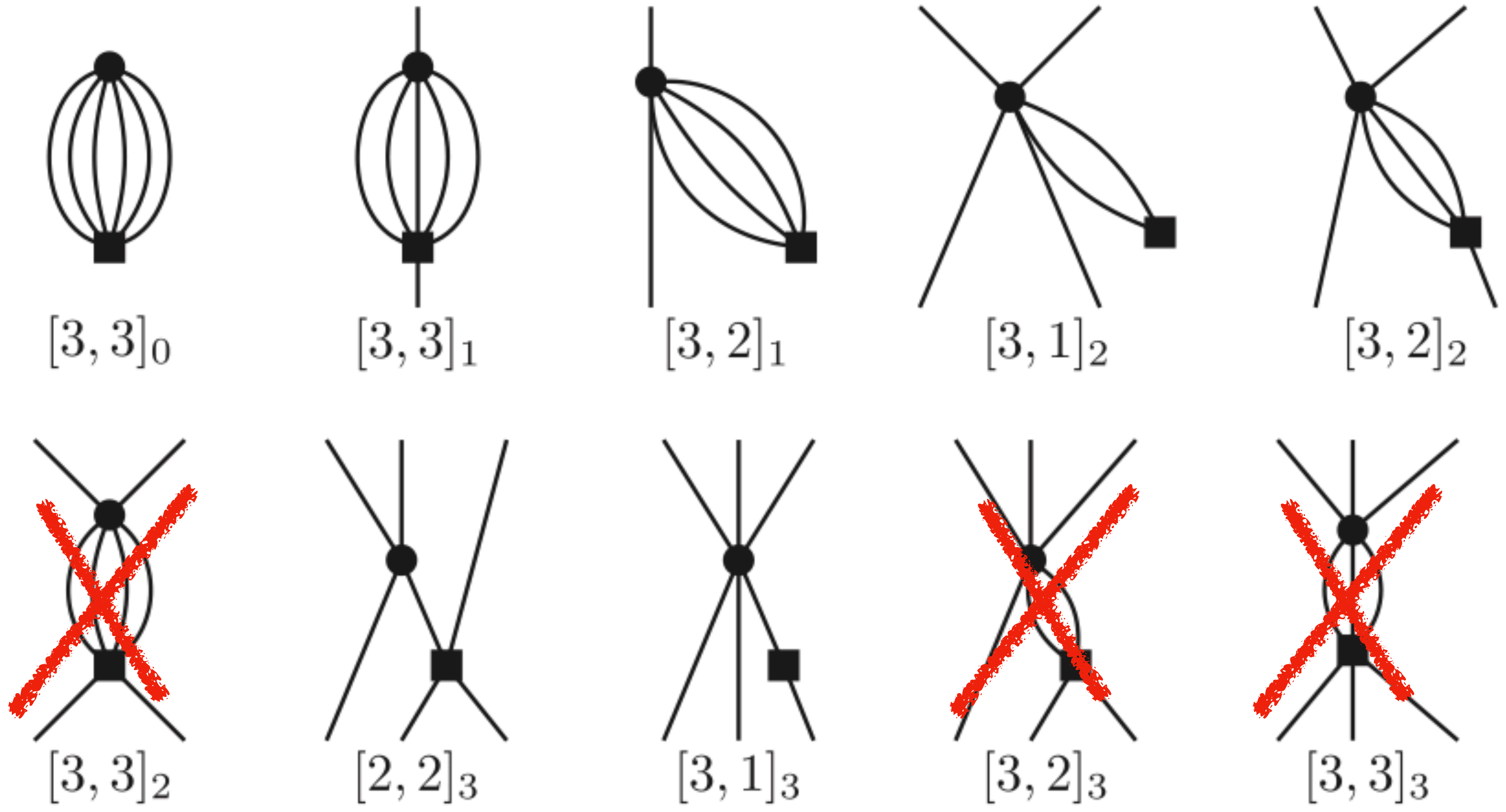
Scaling

IMSRG(2) $\sim N^6$

IMSRG(3N7) $\sim N^7$

IMSRG(3) $\sim N^9$

IMSRG(3)



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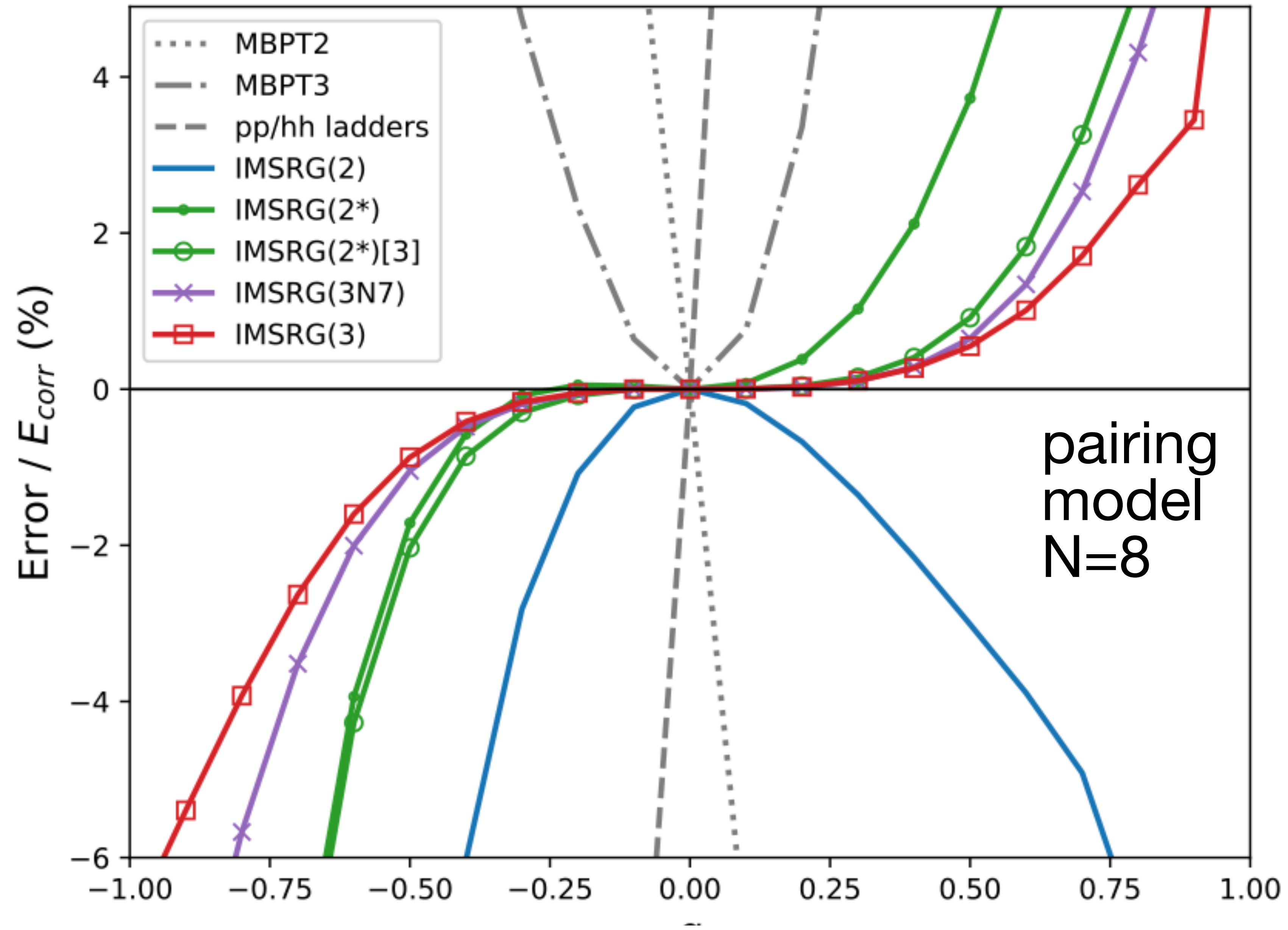

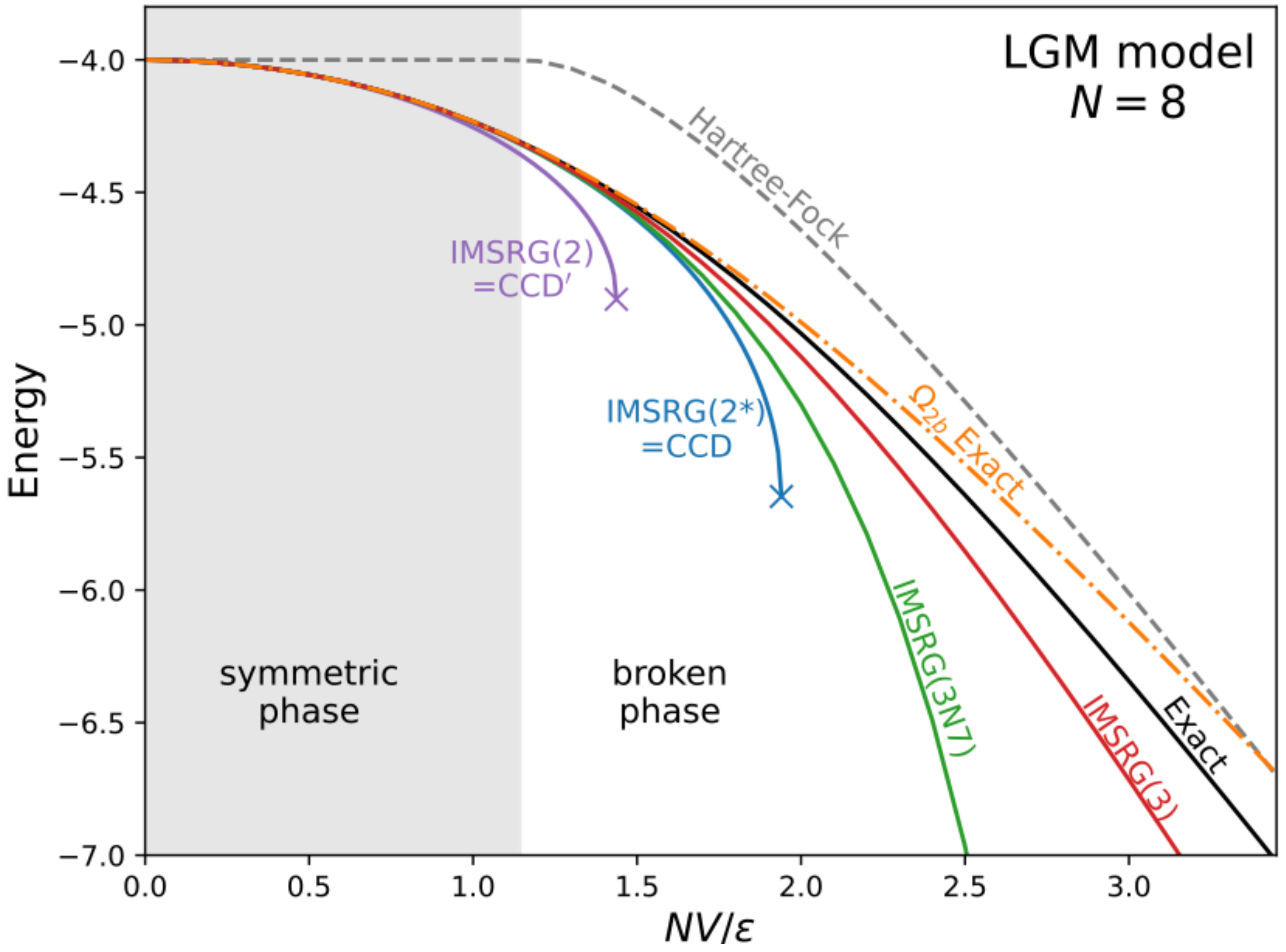

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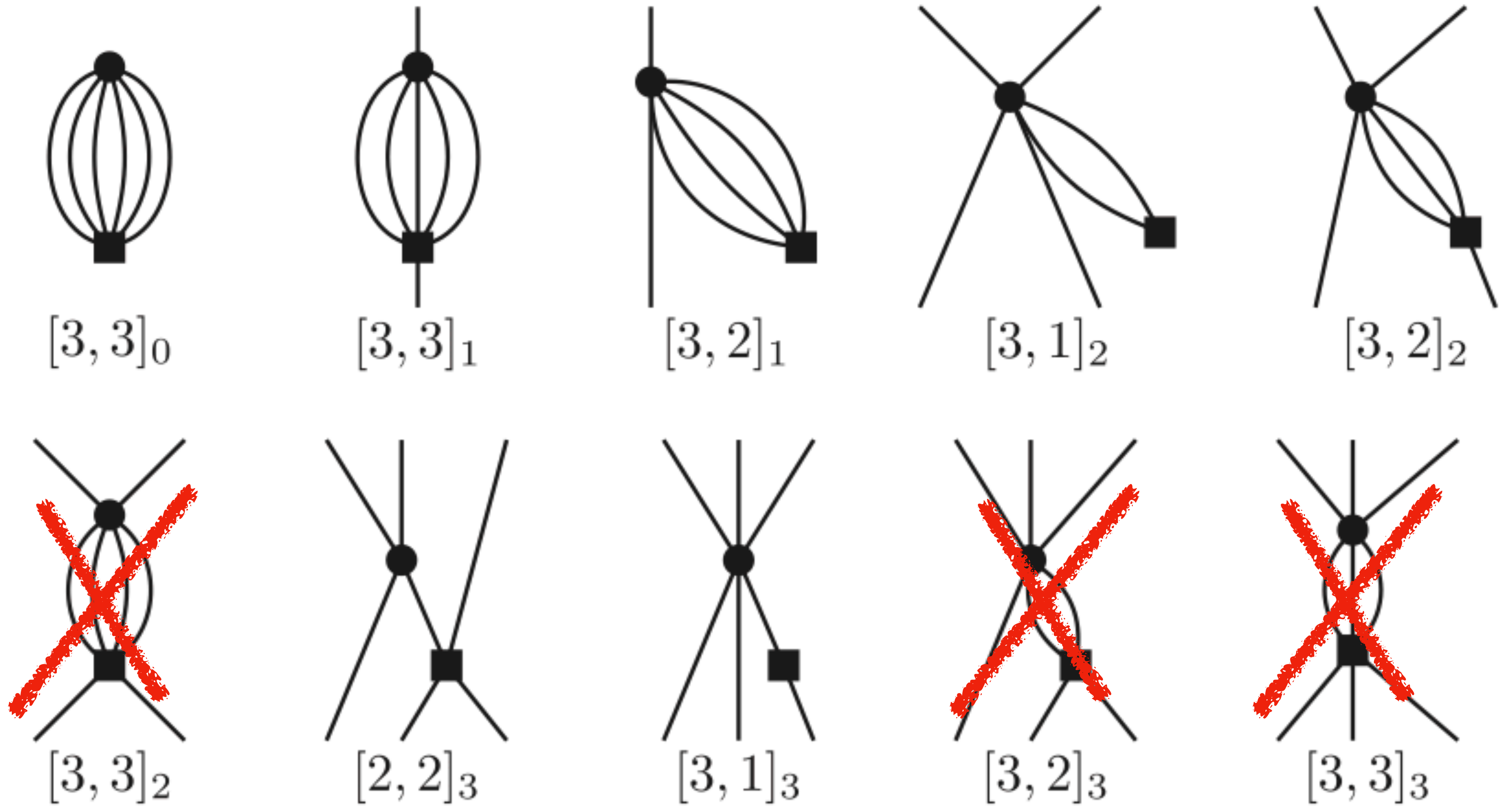
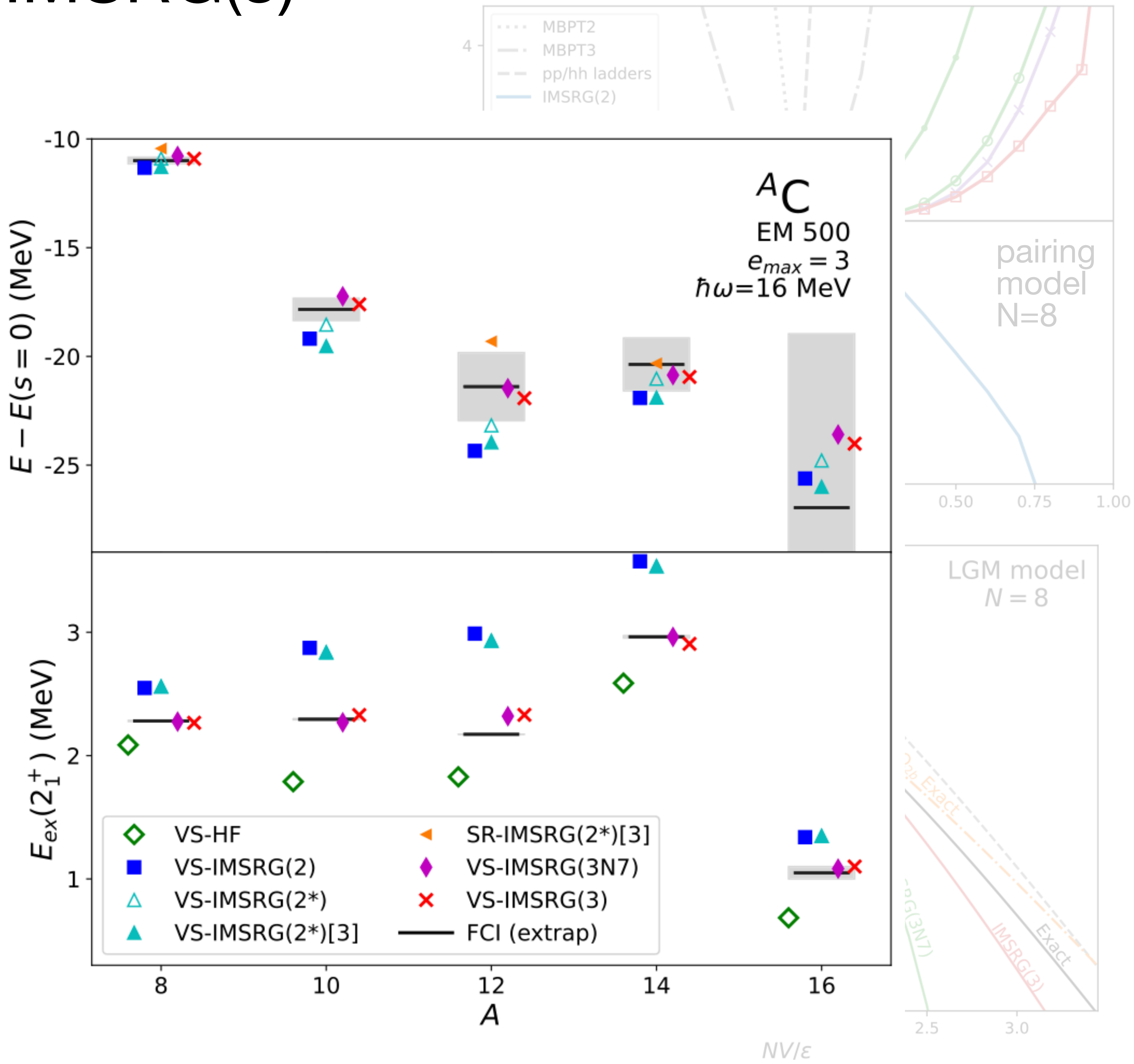




IMSRG(3N7)

Scaling

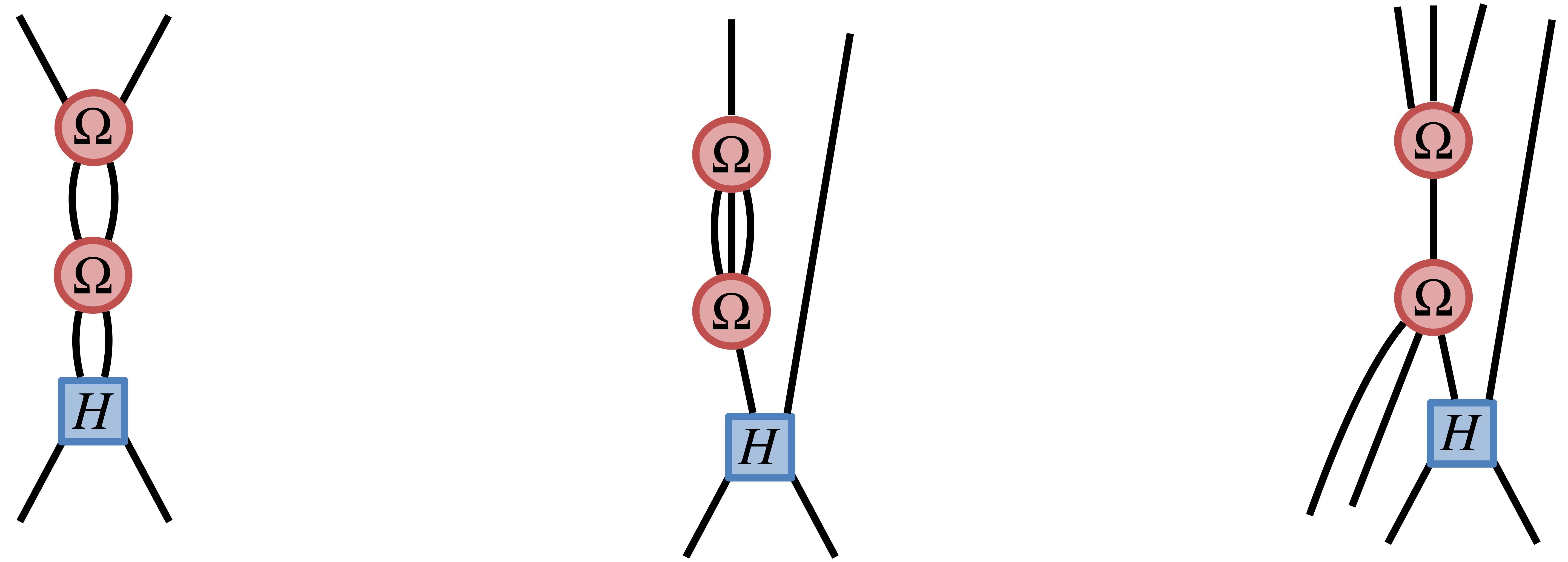
IMSRG(2) $\sim N^6$

IMSRG(3N7) $\sim N^7$

IMSRG(3) $\sim N^9$

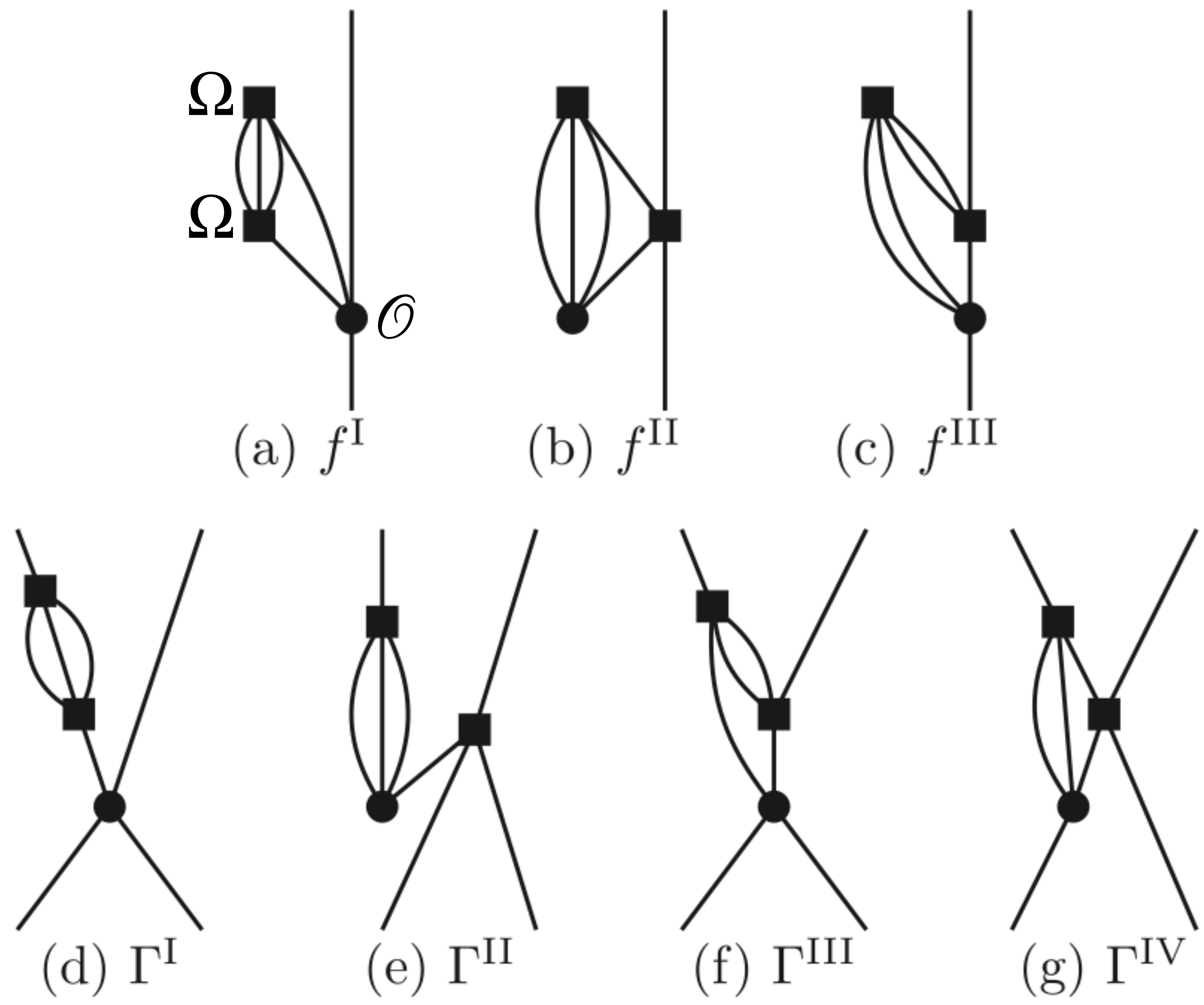
The IMSRG($3f_2$) approximation

$$[\Omega,[\Omega,H]] = \underbrace{[\Omega,[\Omega,H]_{1,2}]_{0,1,2}}_{\text{IMSRG(2)}} + \underbrace{[\Omega,[\Omega,H]_{3}]_{1,2}}_{\text{IMSRG}(3f_2)} + \cancel{[\Omega,[\Omega,H]_{3}]_{3,4}}$$

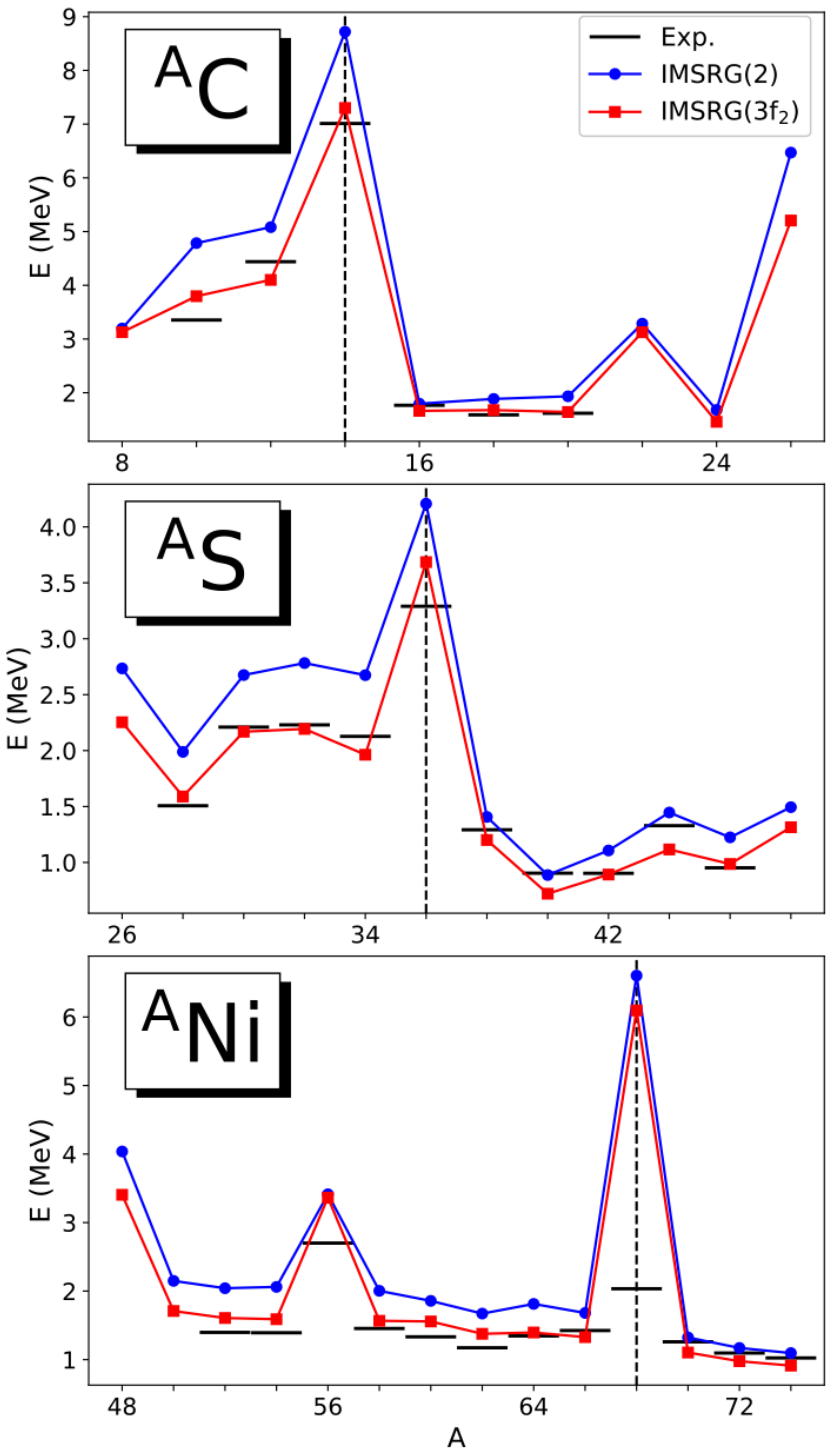


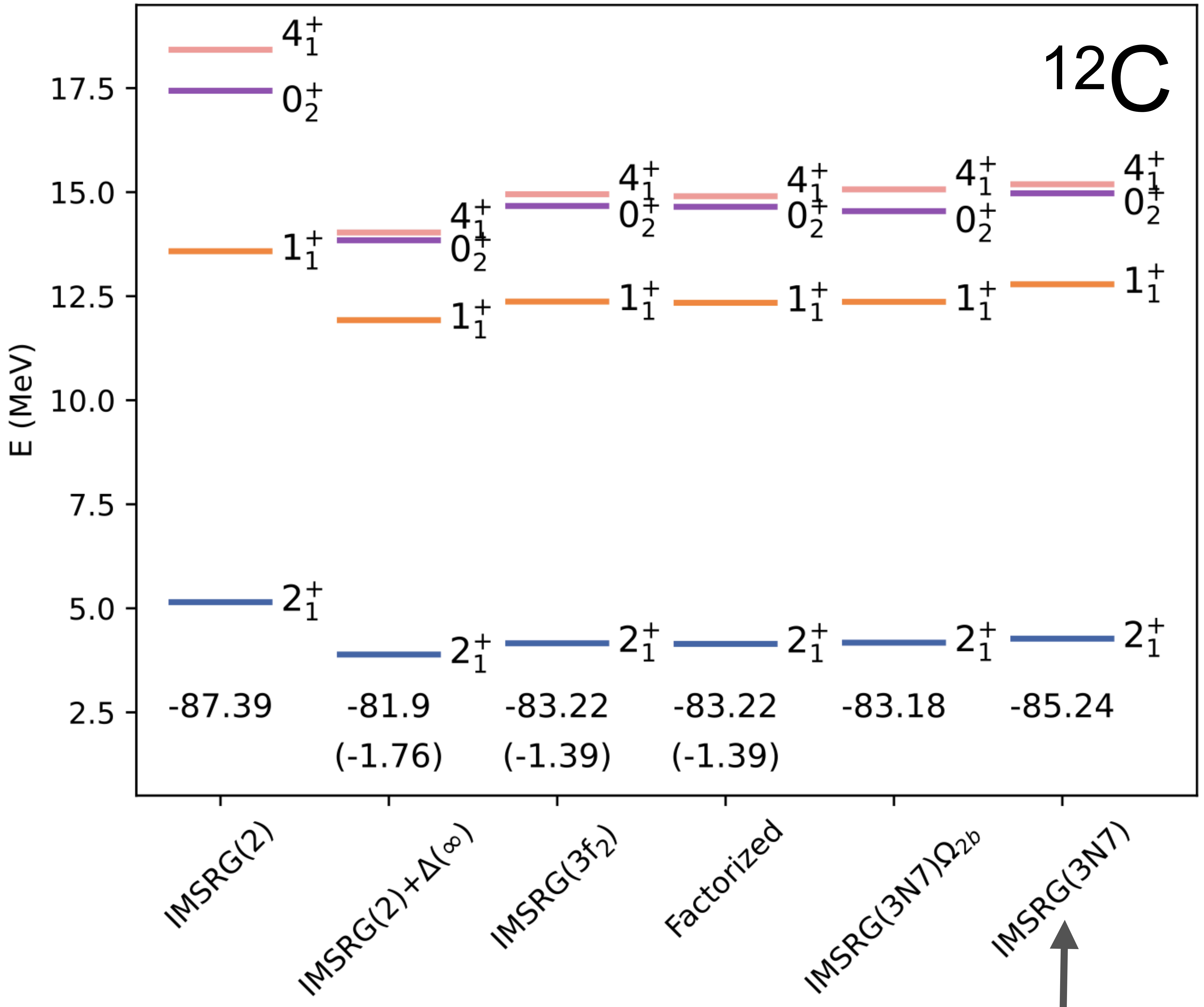
$\sim N^6$ scaling

$\geq N^7$ scaling


Ω
Ω
$\mathcal{O}$
(a) f^{I}
(b) f^{II}
(c) f^{III}
(d) Γ^{I}
(e) Γ^{II}
(f) Γ^{III}
(g) Γ^{IV}

The IMSRG($3f_2$) approximation

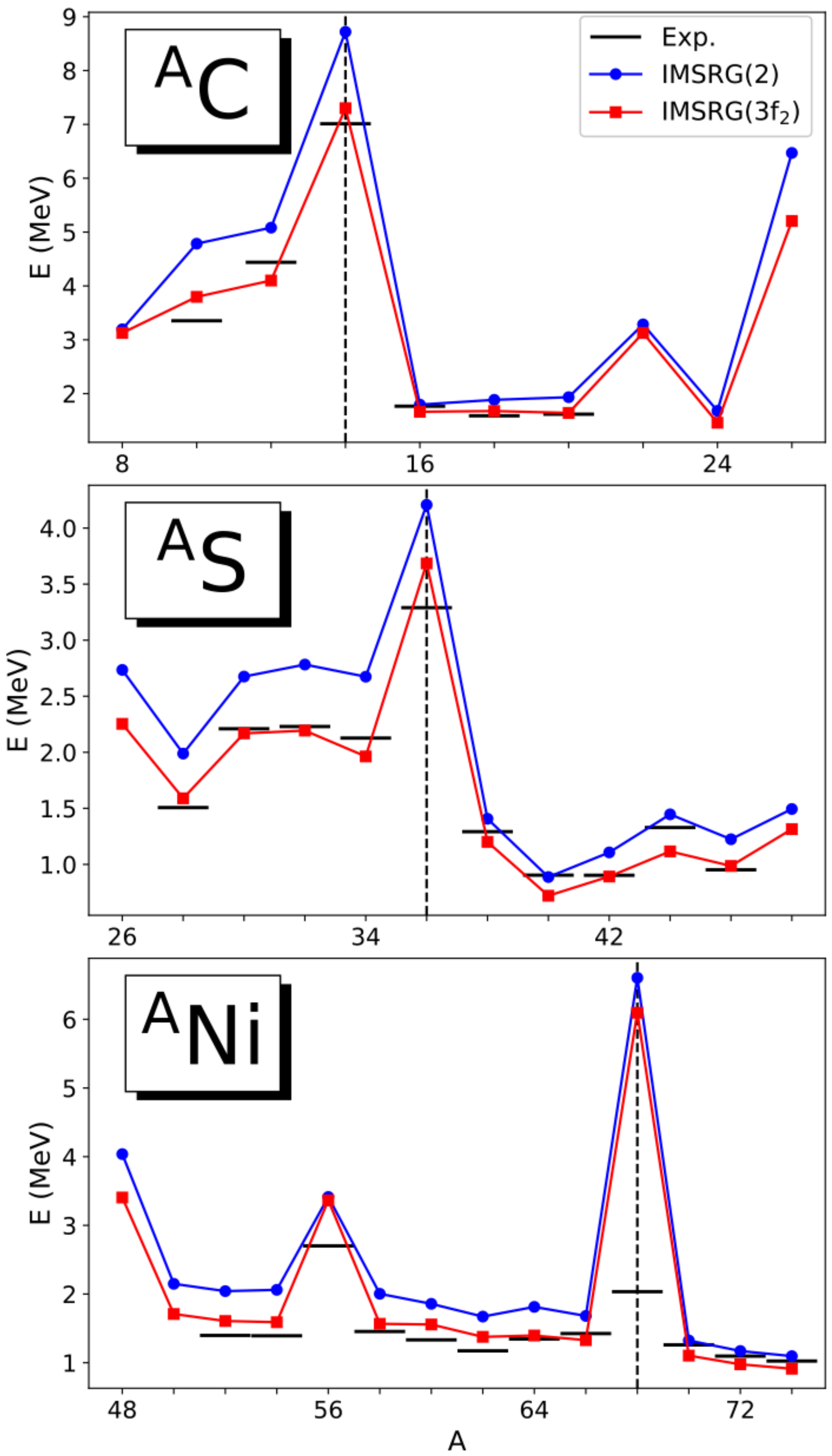


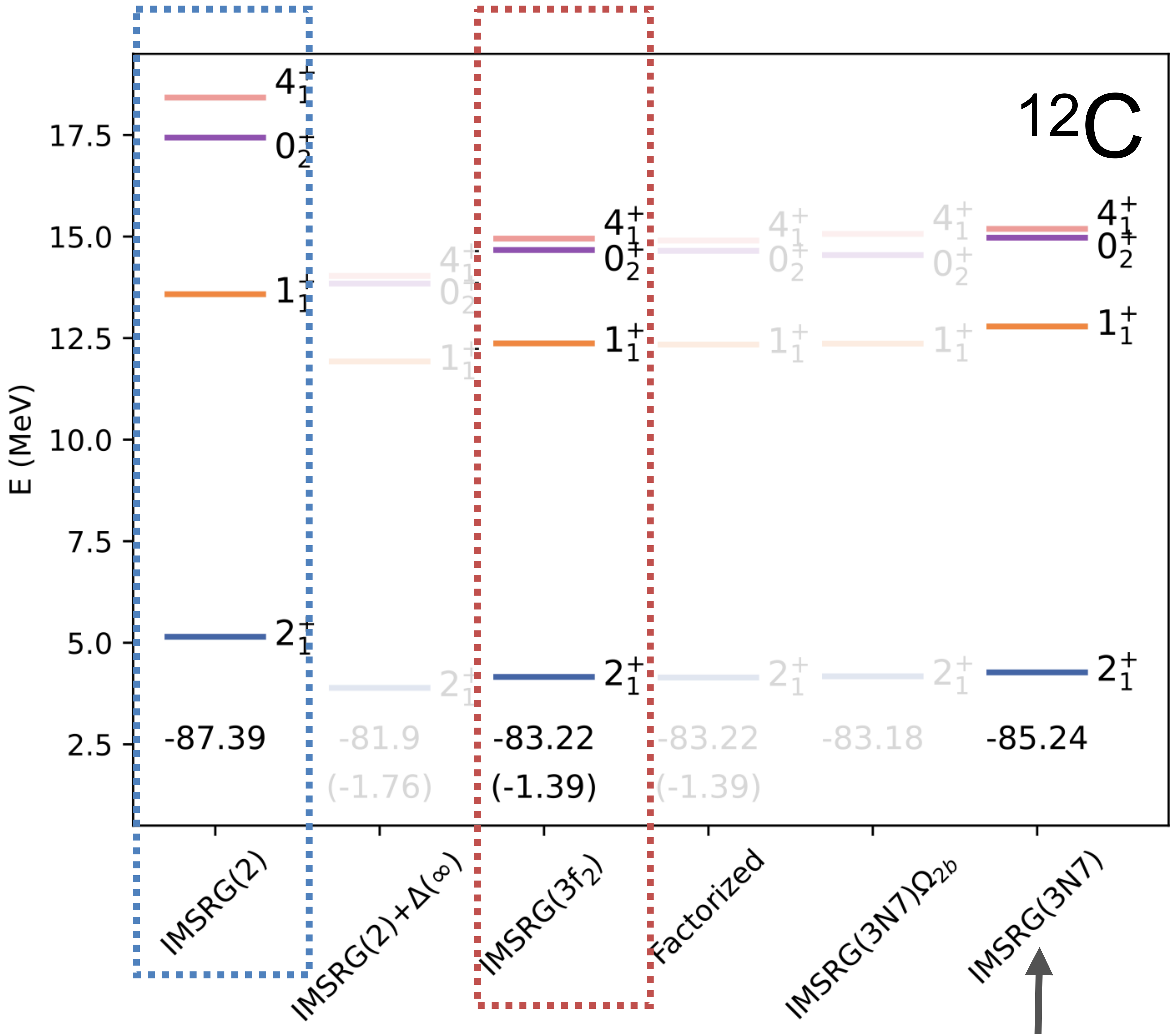


B.C. He and SRS PRC 110 044317 (2024)

best calculation

The IMSRG($3f_2$) approximation

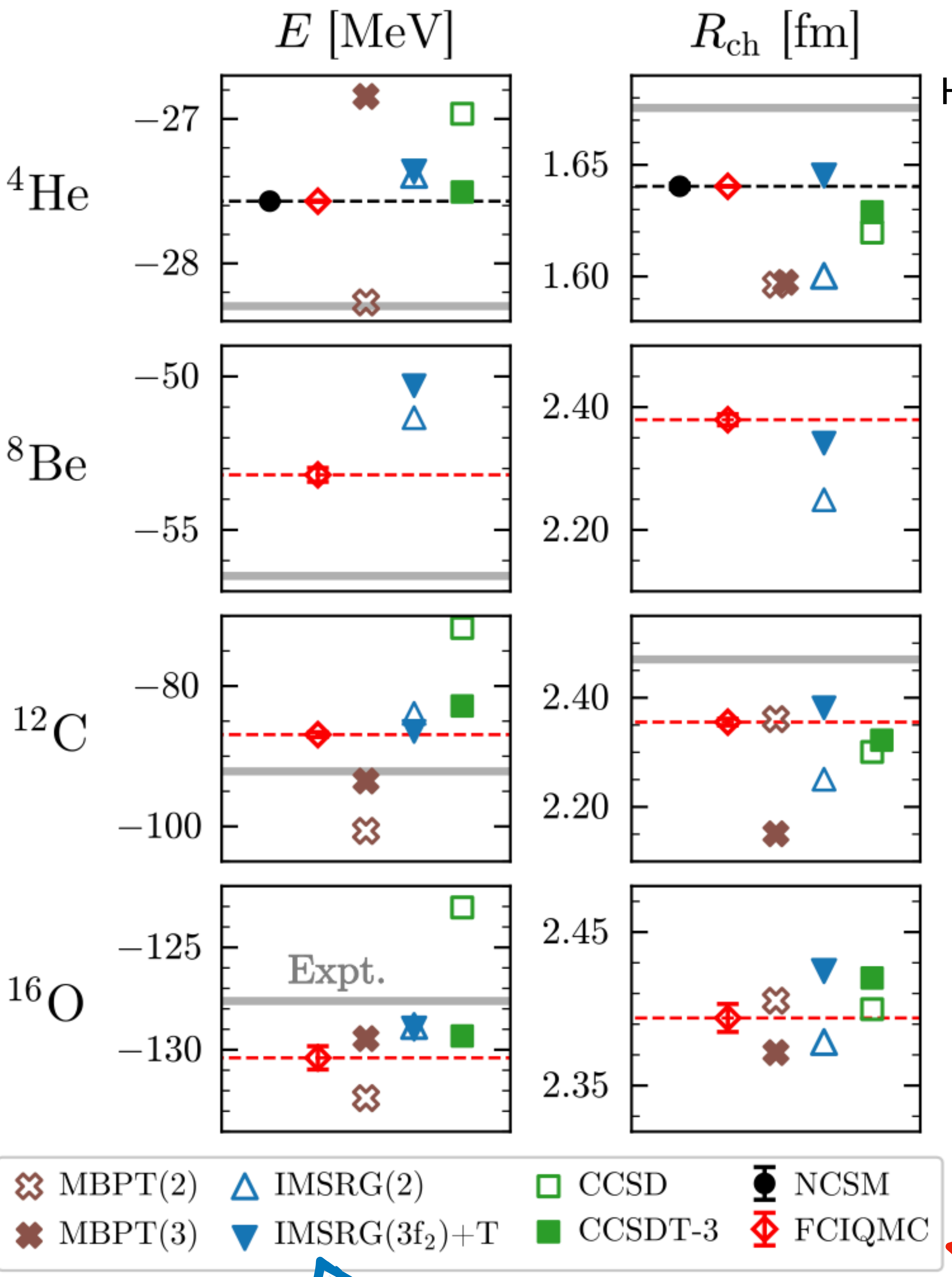




B.C. He and SRS PRC 110 044317 (2024)

best calculation

Theory to theory benchmark in a large-space calculation



Hu+ arXiv 2606.22341

NNLO$_{opt}$ e_{max}=10

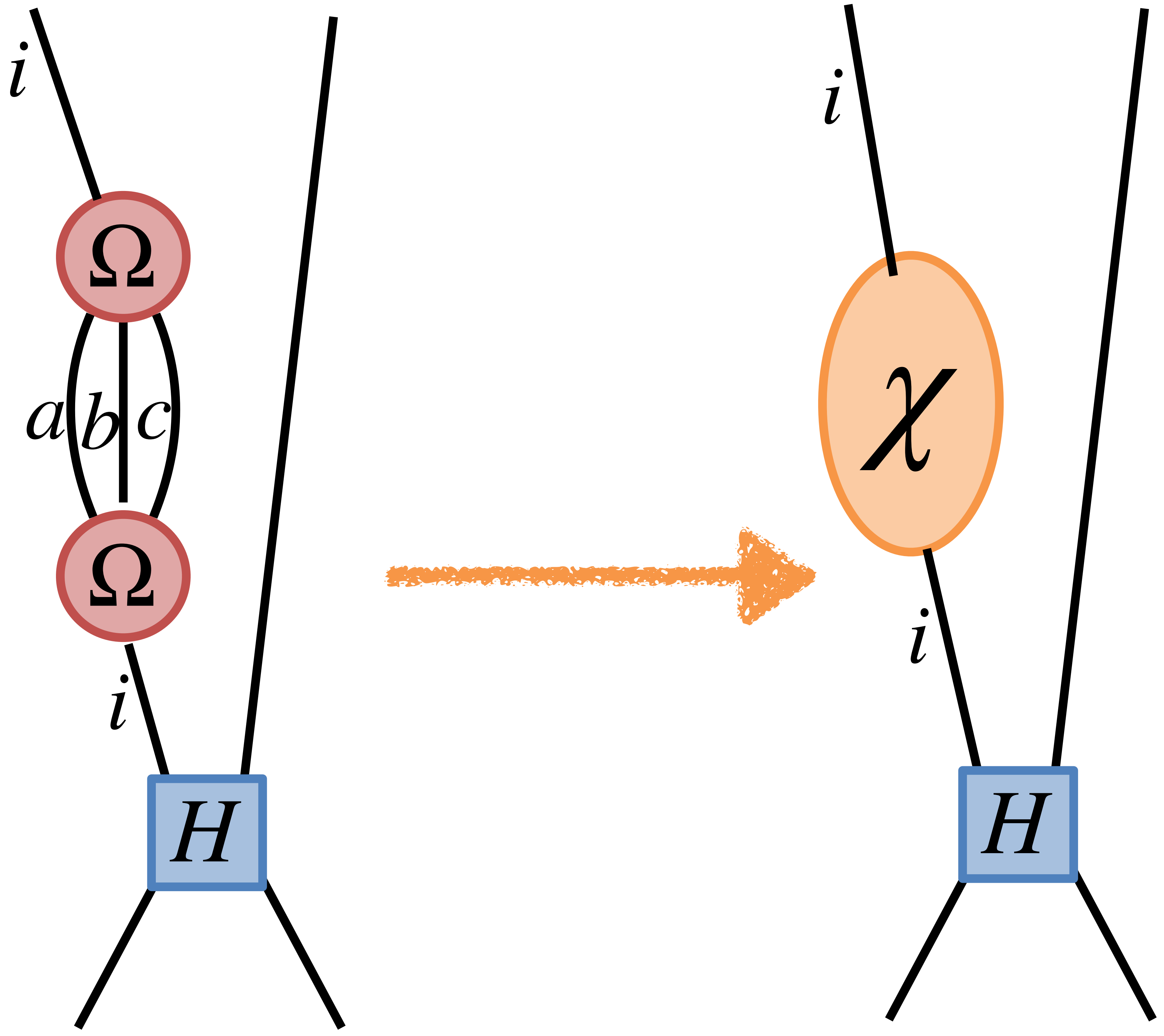
~10^6 CPU hr

~200 CPU hr

But what is the $3f_2$ correction doing?

Topological coherence

⇒enhance triple connections

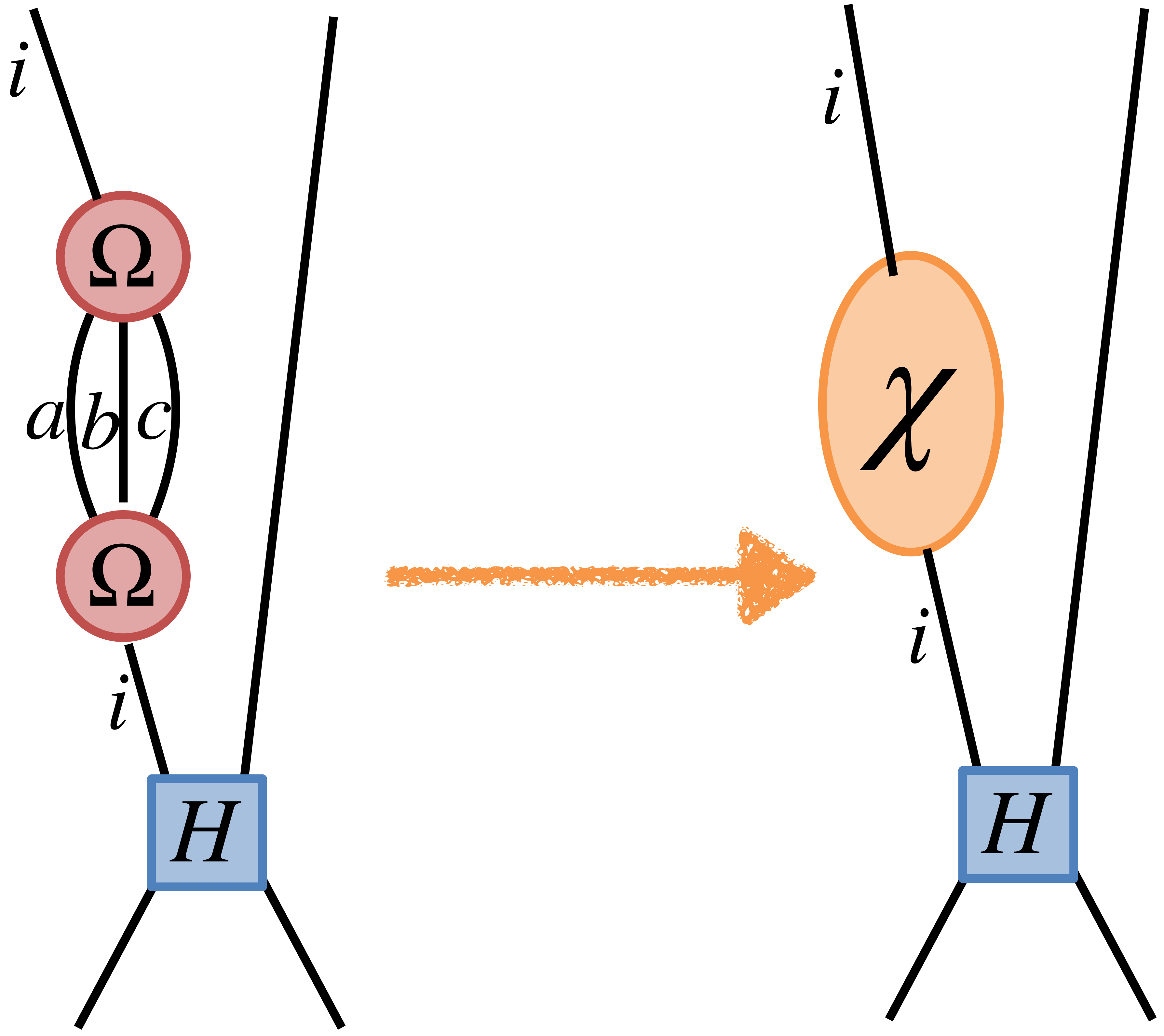


$$\chi_{ii} \sim \sum_{abc} \Omega_{iabc}\Omega_{bcai} = -\sum_{abc} |\Omega_{iabc}|^2$$

coherent

Topological coherence

⇒enhance triple connections

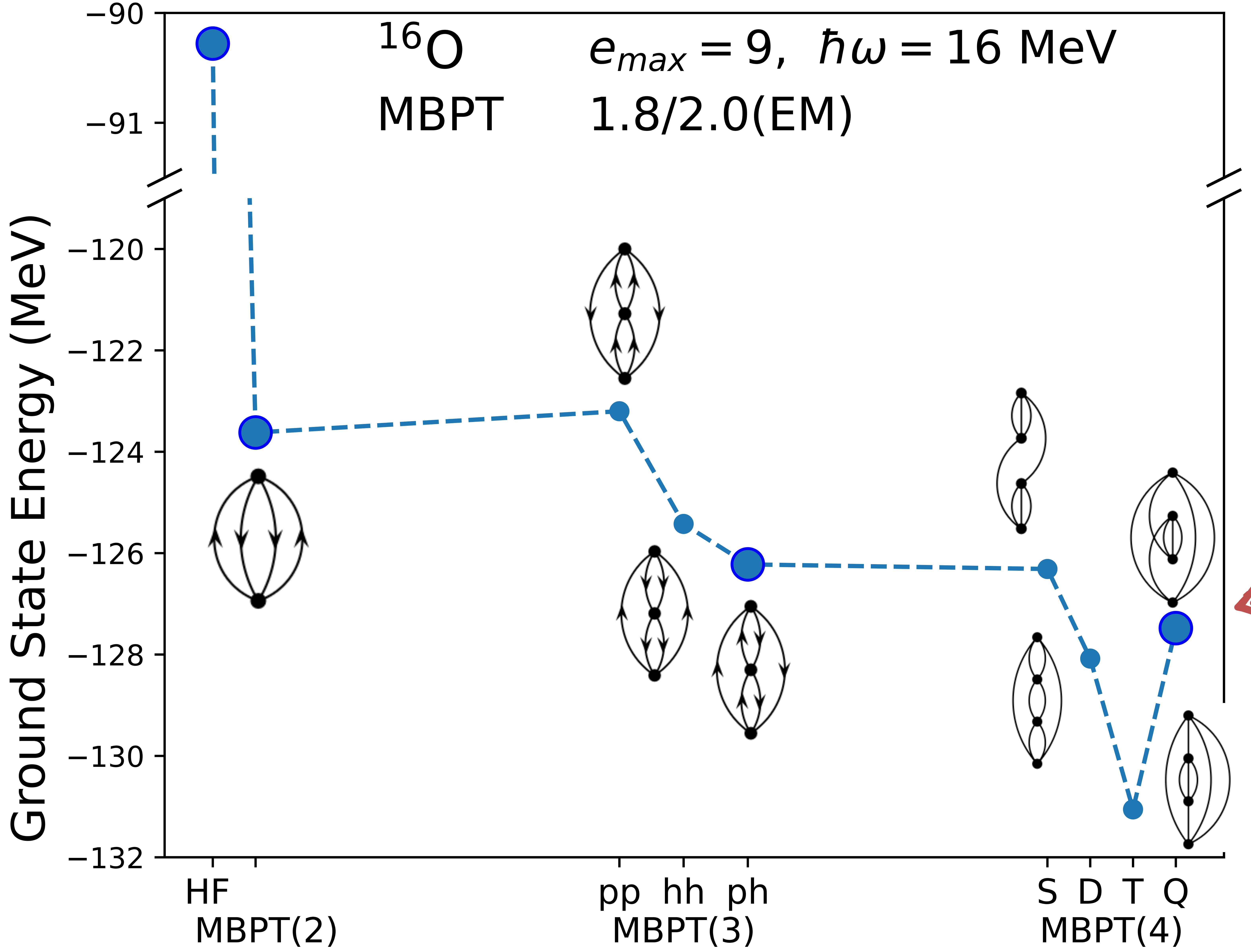


$$\chi_{ii} \sim \sum_{abc} \Omega_{iabc}\Omega_{bcai} = -\sum_{abc} |\Omega_{iabc}|^2$$

coherent

Will this help us predict *other* important diagrams?

Can we understand this pattern?



Nominal expansion parameter:

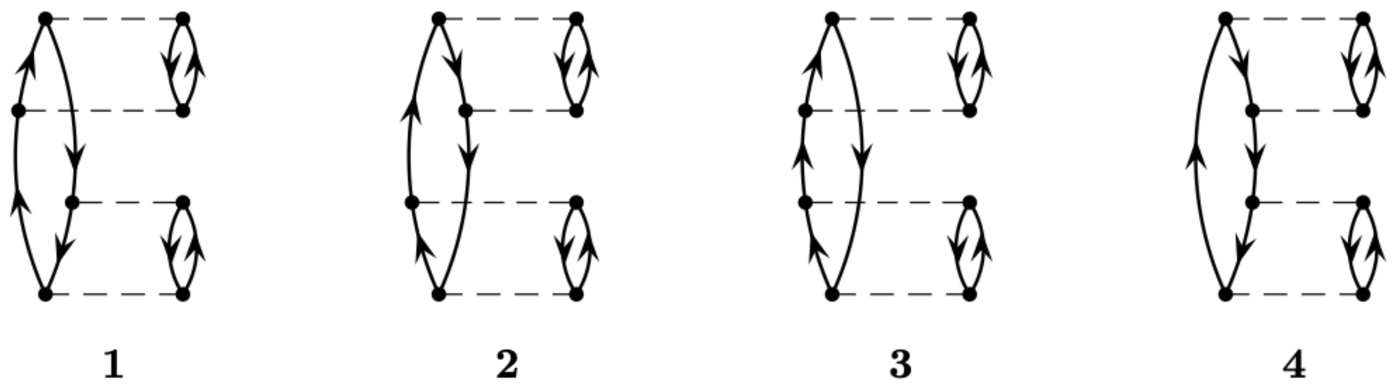
$$\langle \phi \left| \frac{V}{E_0 - H_0} \right| \phi' \rangle$$

Smallness of 4th order correction appears to depend on approximate cancellation of terms that are not small.

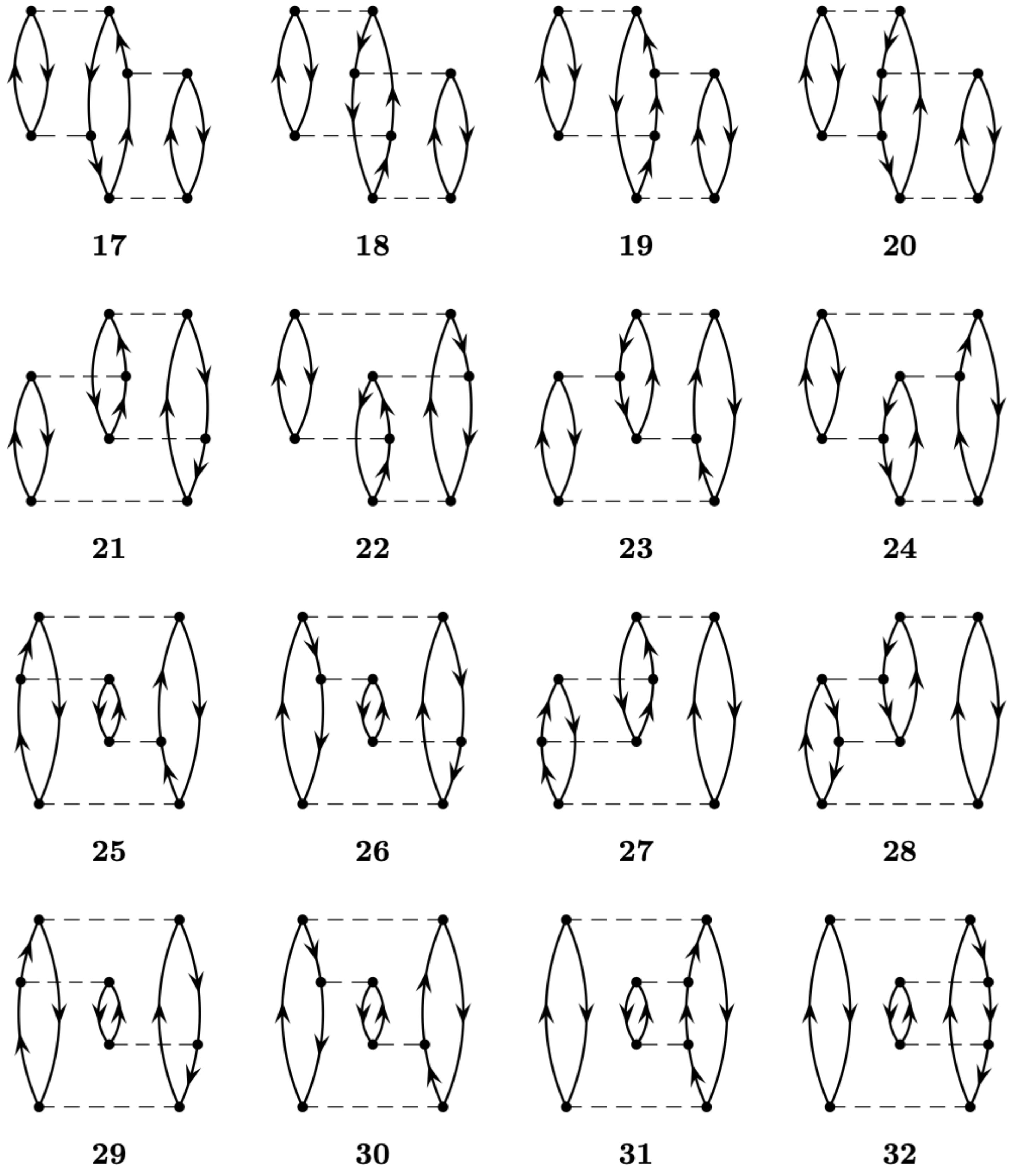
(39 distinct 4th order diagrams)

MBPT(4) Diagrams

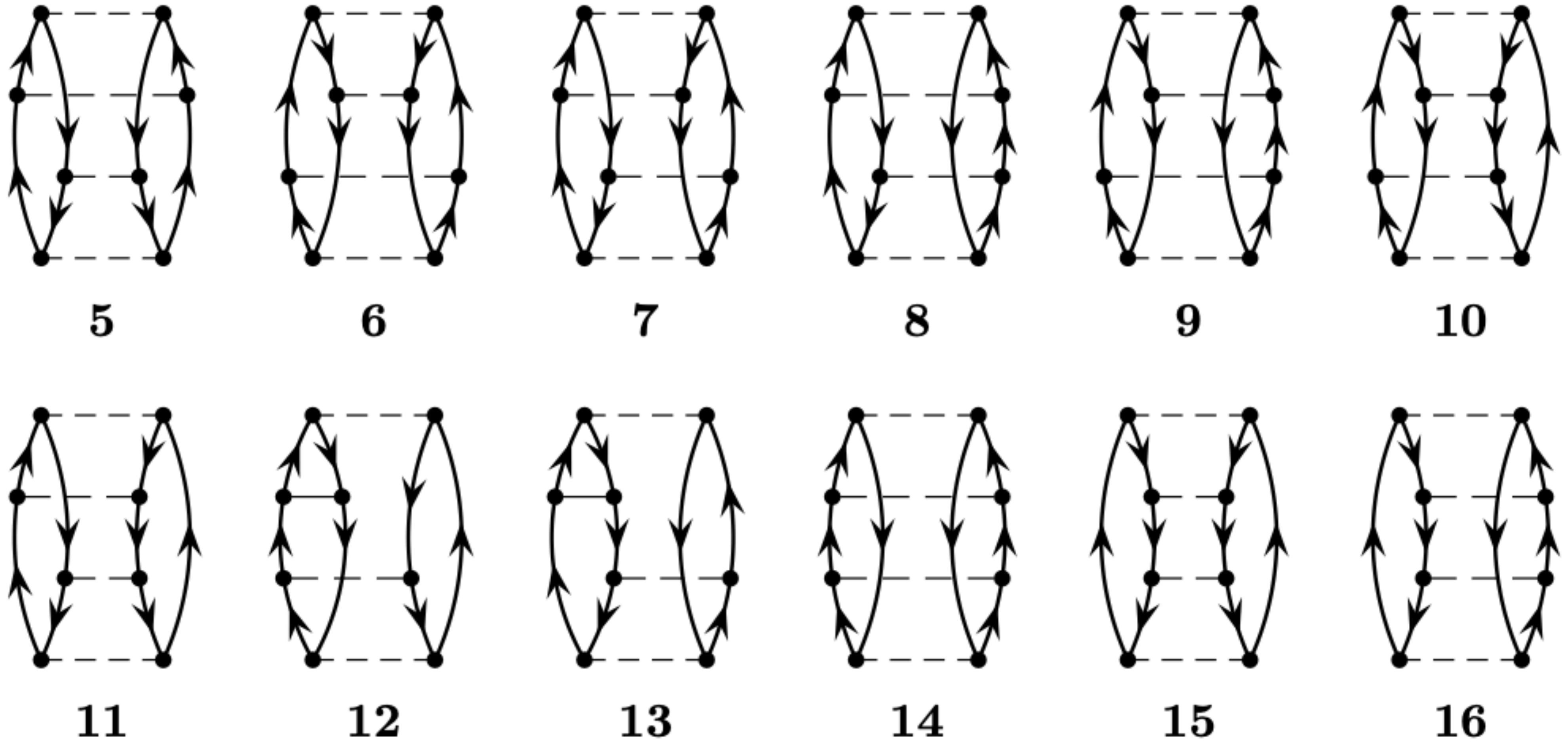
Singles



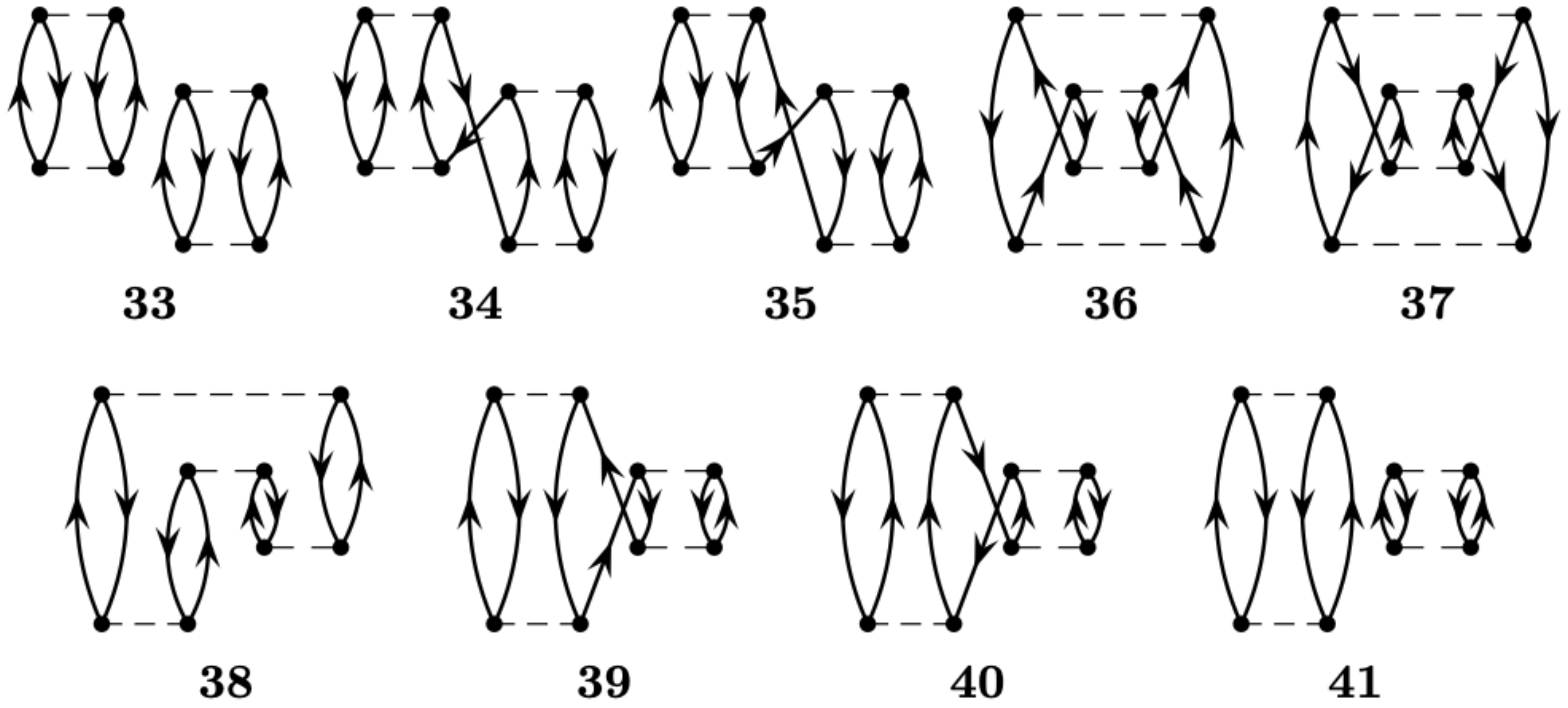
Triples



Doubles

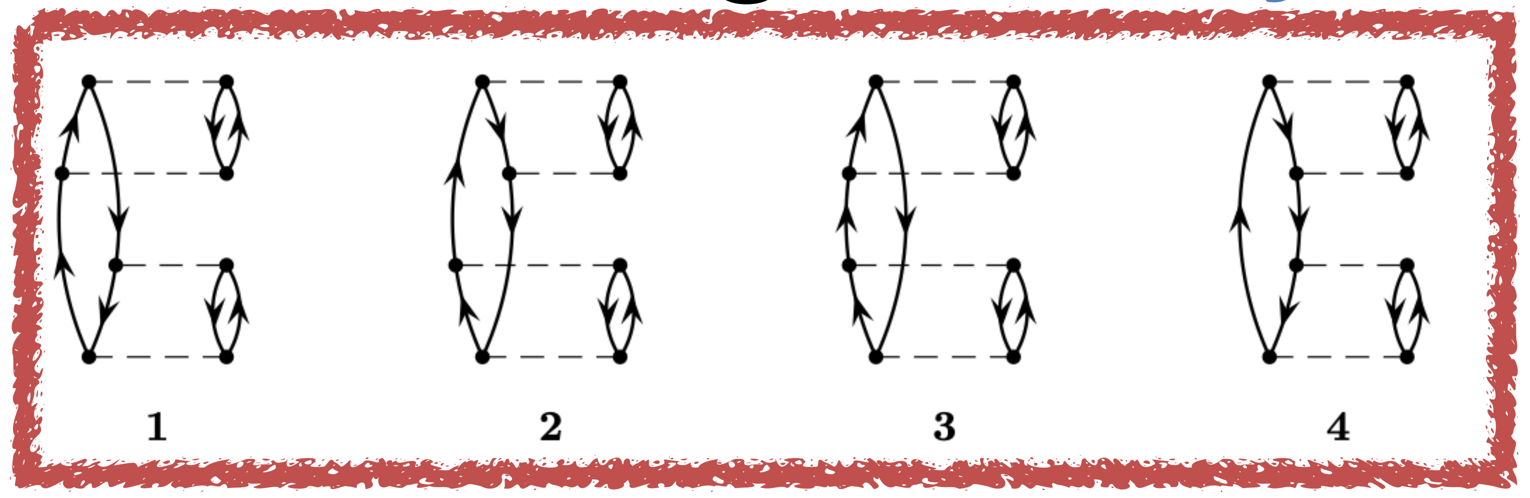


Quadruples

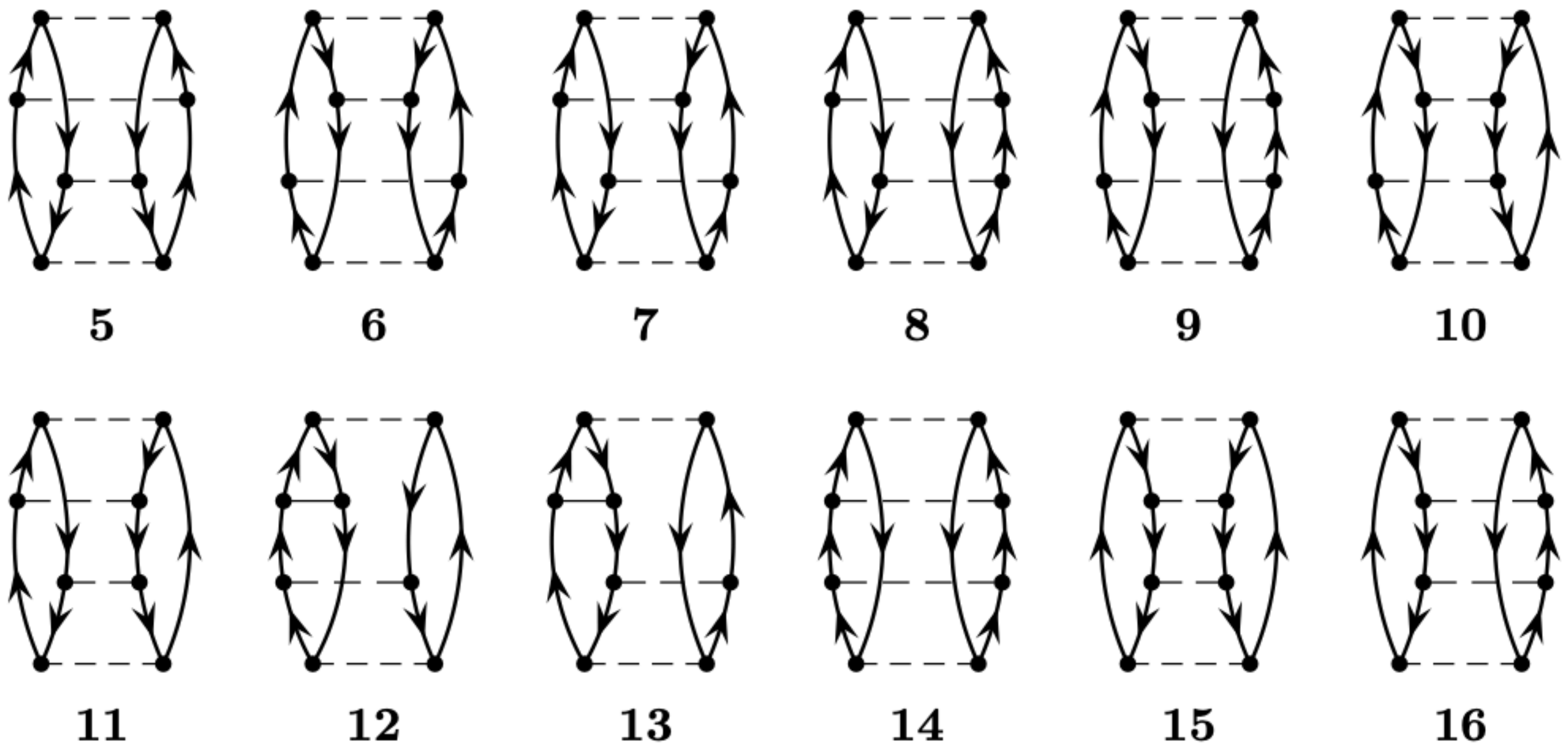


MBPT(4) Diagrams

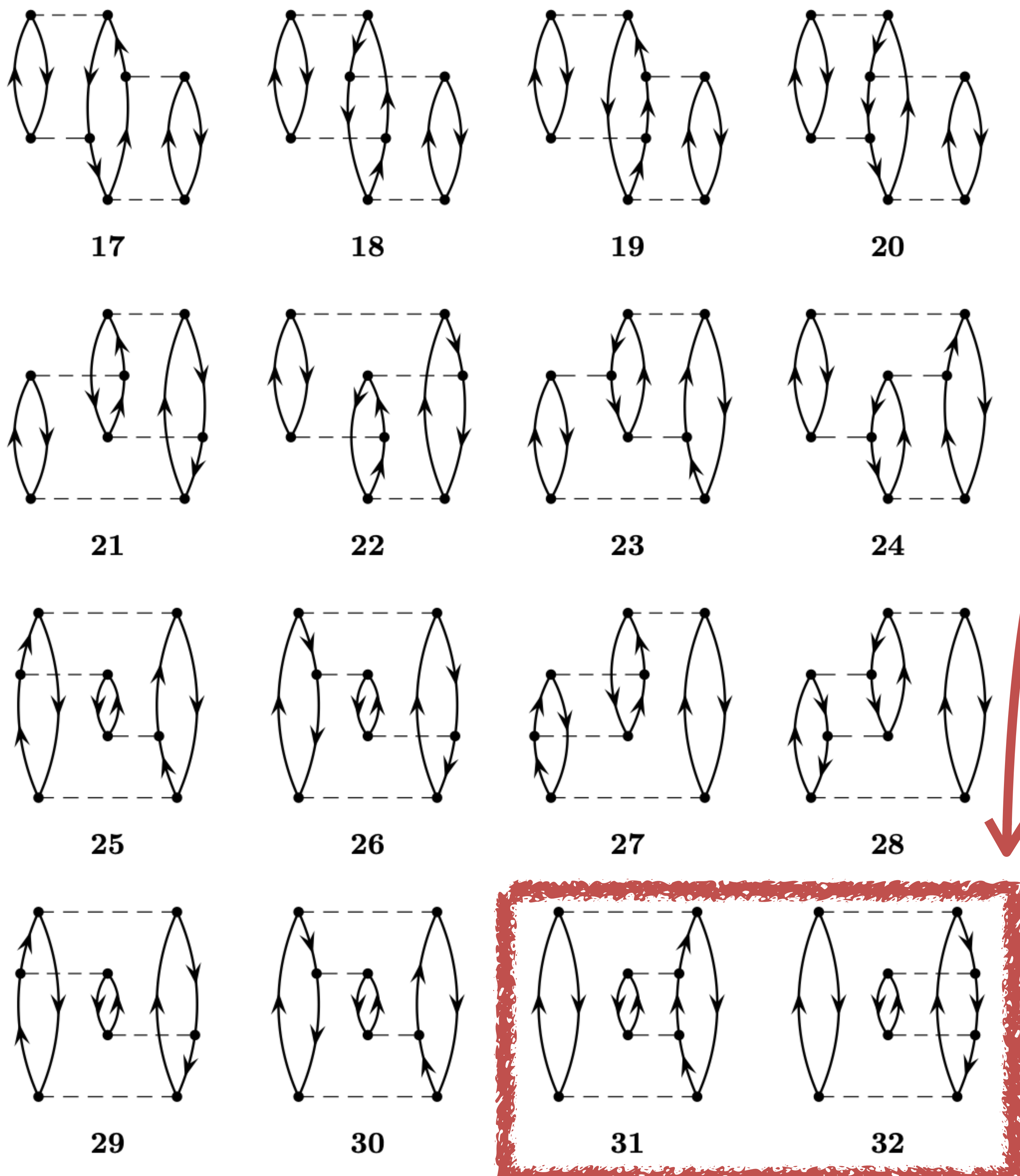
Singles



Doubles

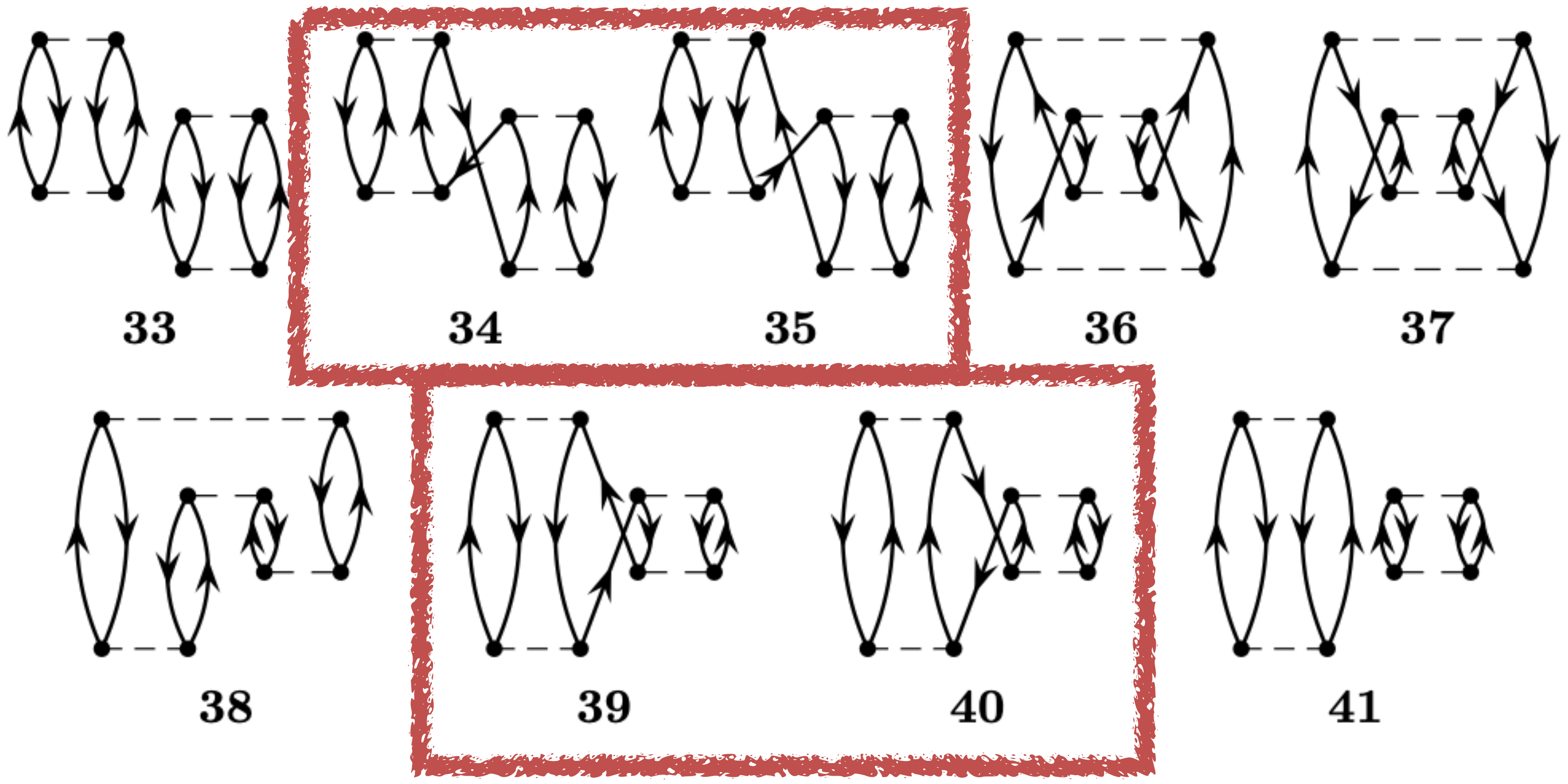


Triples



diagrams with triple contractions

Quadruples



MBPT(4) Diagrams

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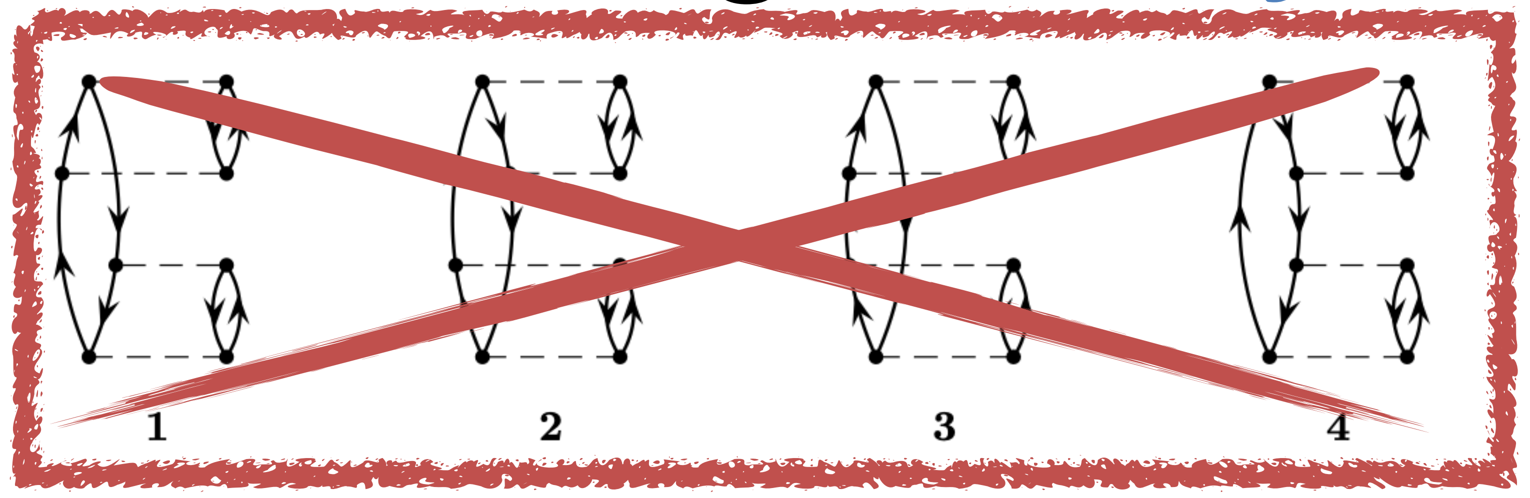
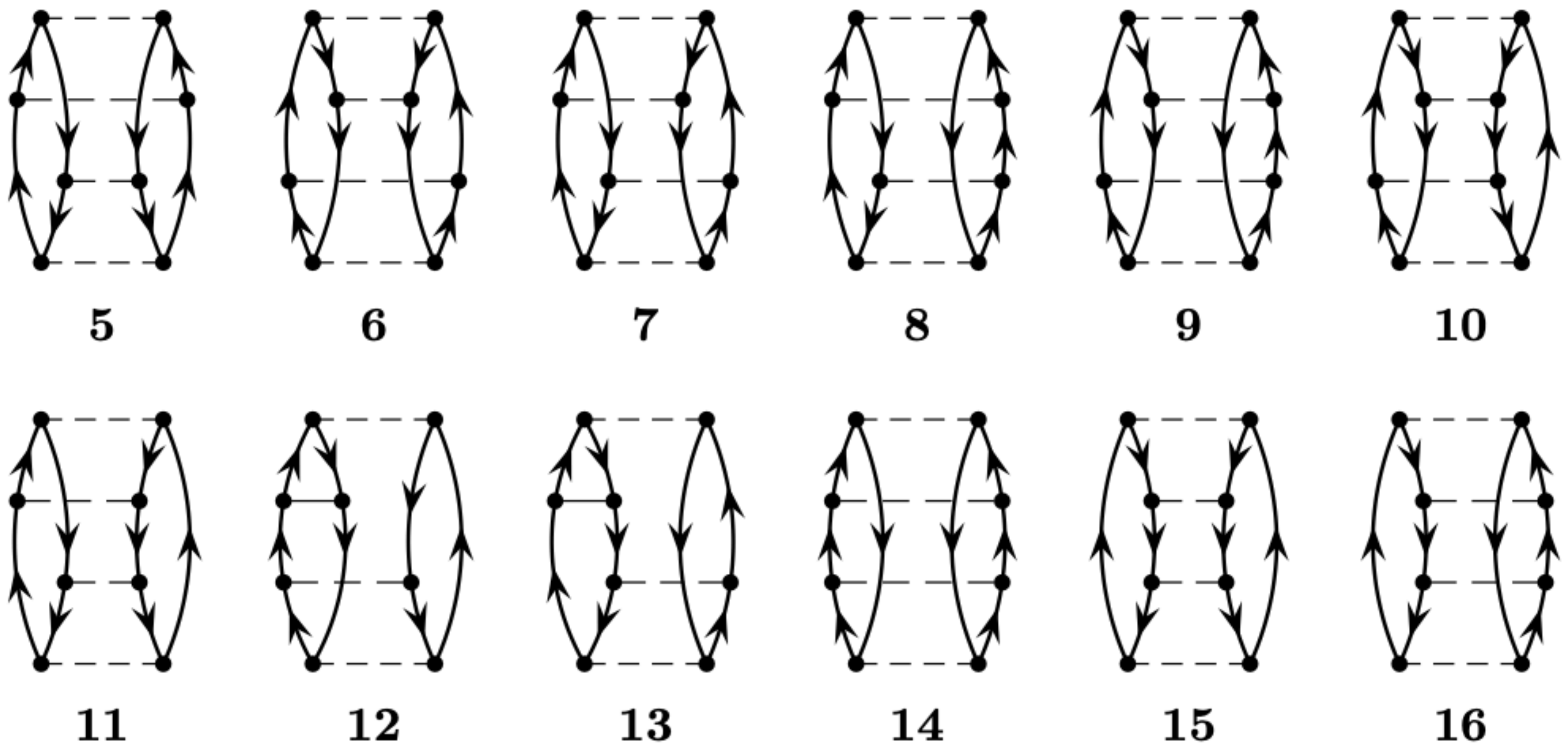
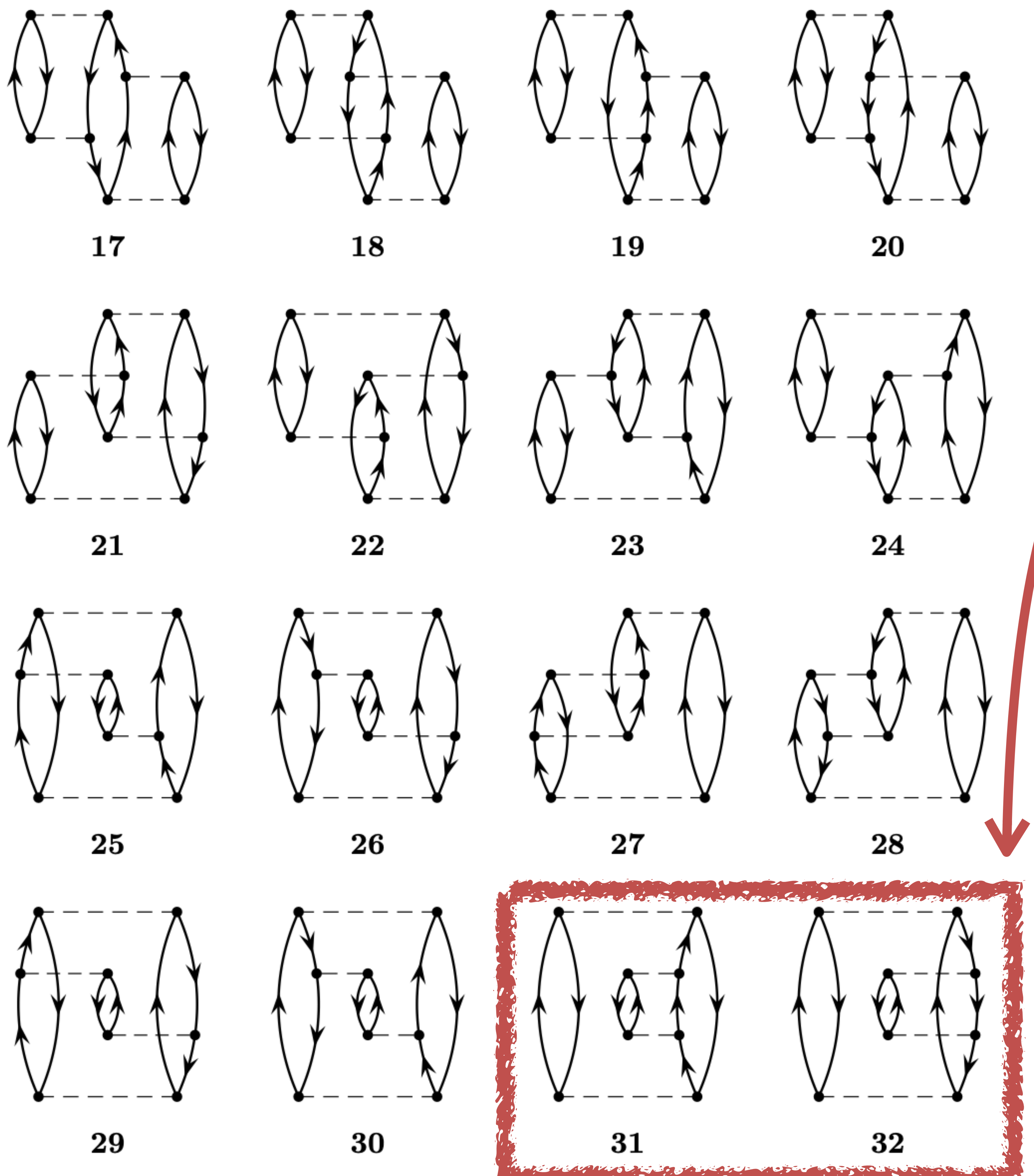


Doubles



Triples



diagrams with triple contractions

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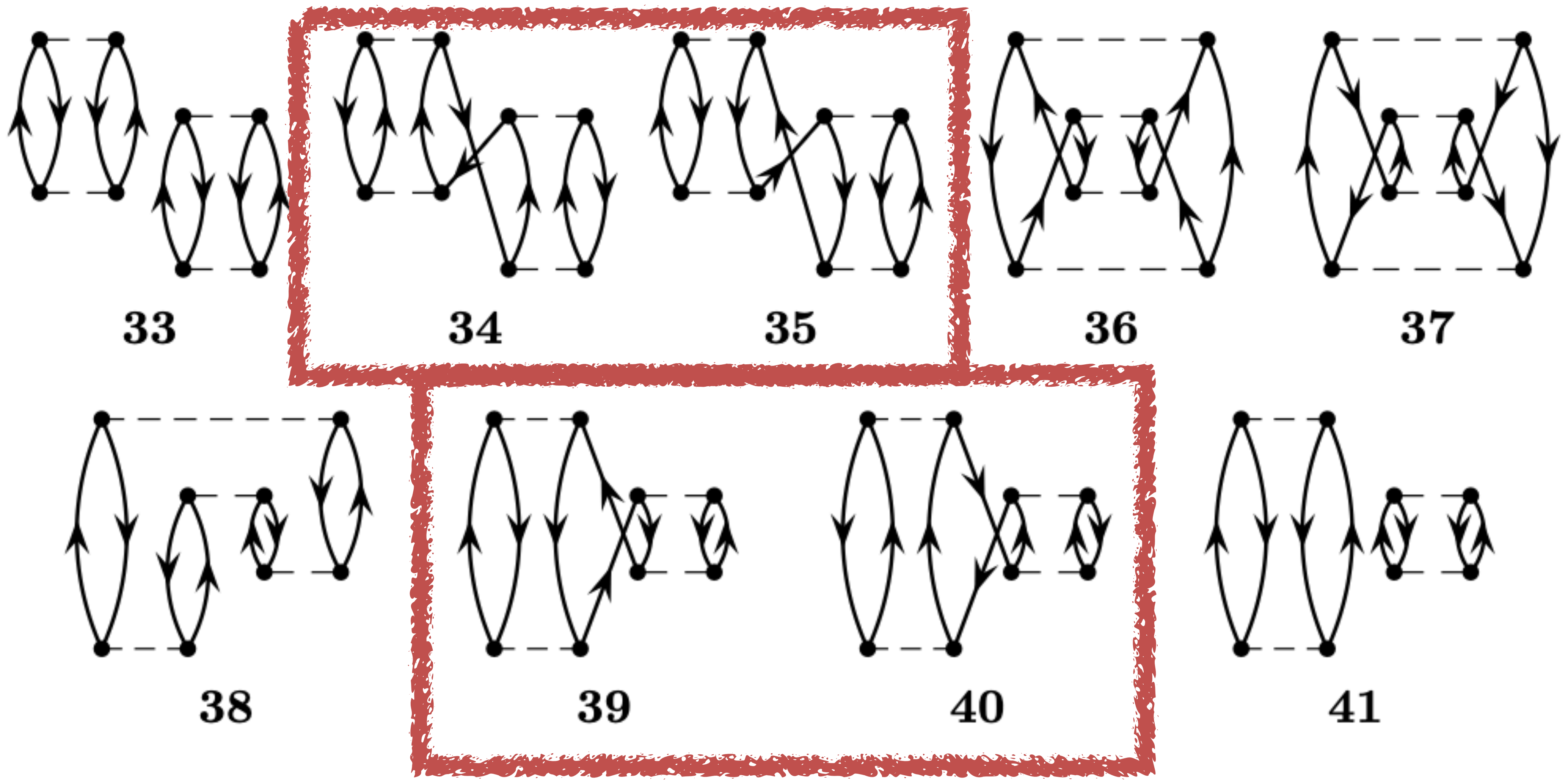


Singles: -0.0906
Doubles: -1.7675
Triples: -2.9749
Quadruples: 3.5752
^{16}O, $e_{max} = 9$
MBPT2 Energy = -33.340854
MBPT3: pp = 0.419302, hh = -2.223272, ph = -0.798104
Diagram contribution (MeV)
4th order diagram number
31
32
34
35
38
39
40


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Triples dominated by 2 of the 16 diagrams
…and we can understand *why*.
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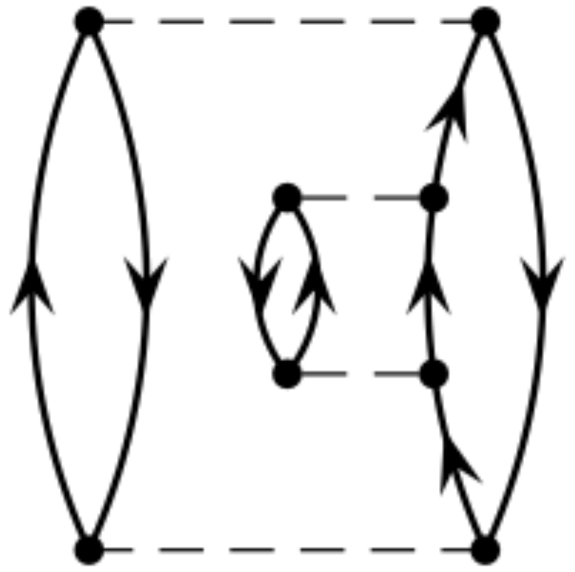
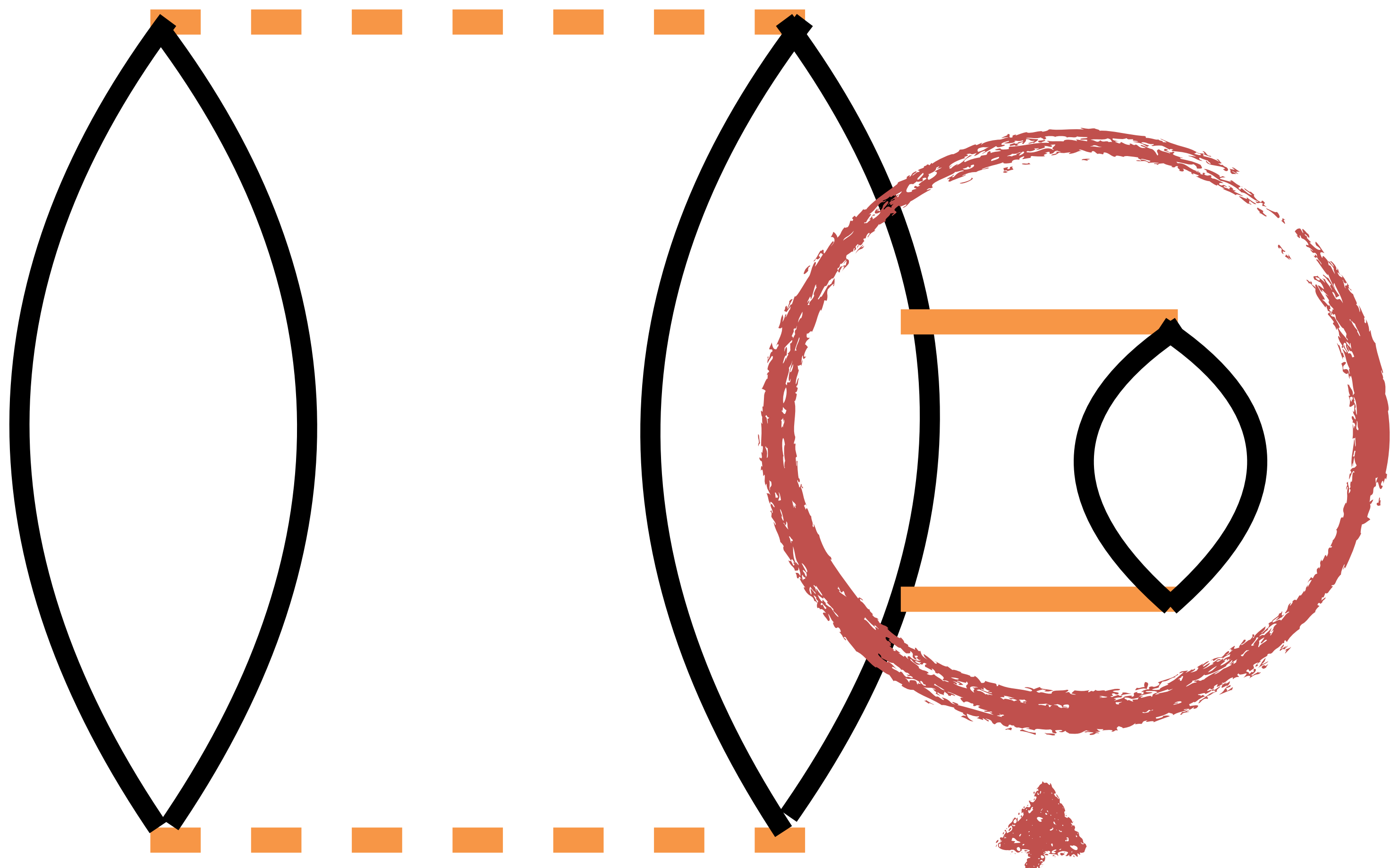
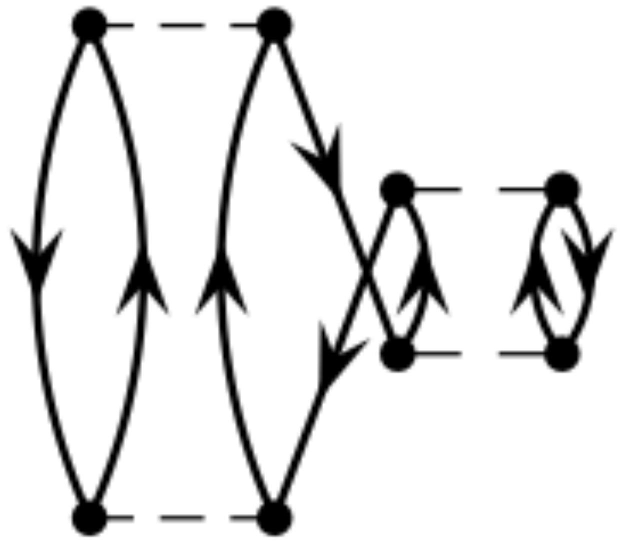
Triples dominated by 2 of the 16 diagrams
…and we can understand *why*.
Large cancellation between big triples and big quadruples. Can we understand that?

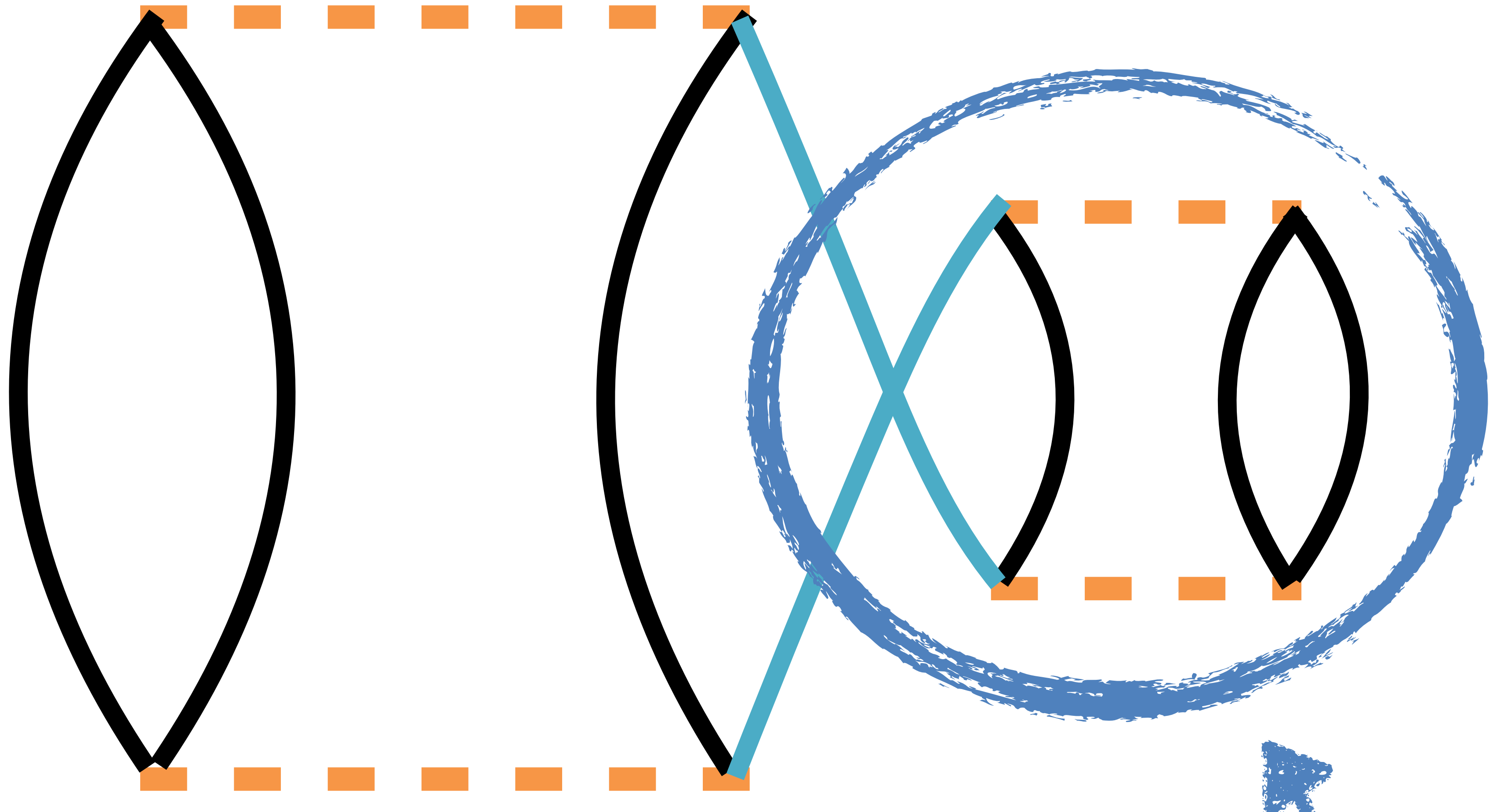


Triples

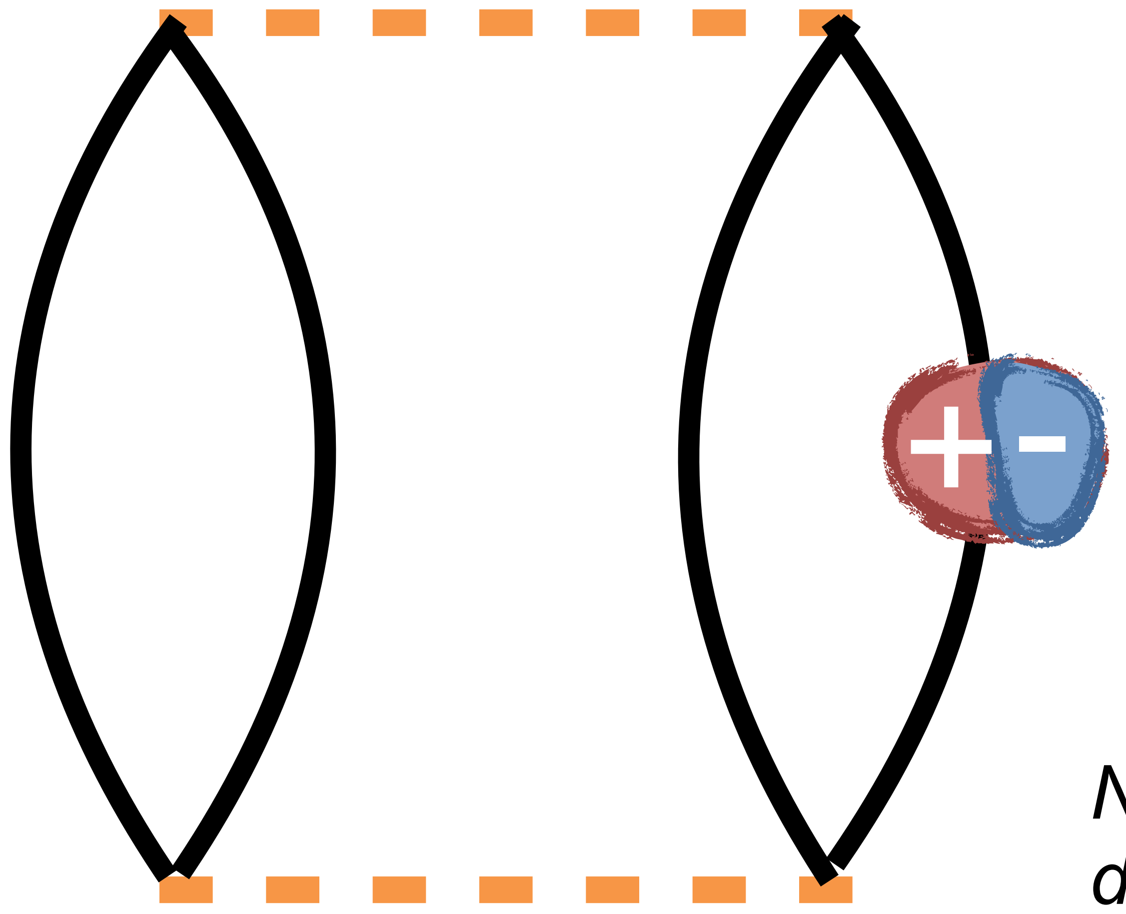


Quadruples





2nd order 1b propagator.
Coherently **enhances**
MBPT(2) excitations.



Pauli-violating diagram.
Coherently **suppresses**
MBPT(2) excitations.

NB: In infinite matter, these two topologies are individually divergent at the fermi surface, and must be combined to yield a finite result. For $\Delta \neq 0$ they do not formally cancel.

Magnus IMSRG

$$H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$$

$$= H + [\Omega, H] + \frac{1}{2!}[\Omega, [\Omega, H]] + \frac{1}{3!}[\Omega, [\Omega, [\Omega, H]]] + \dots$$

Largest contribution to H_{1b}:

Magnus IMSRG

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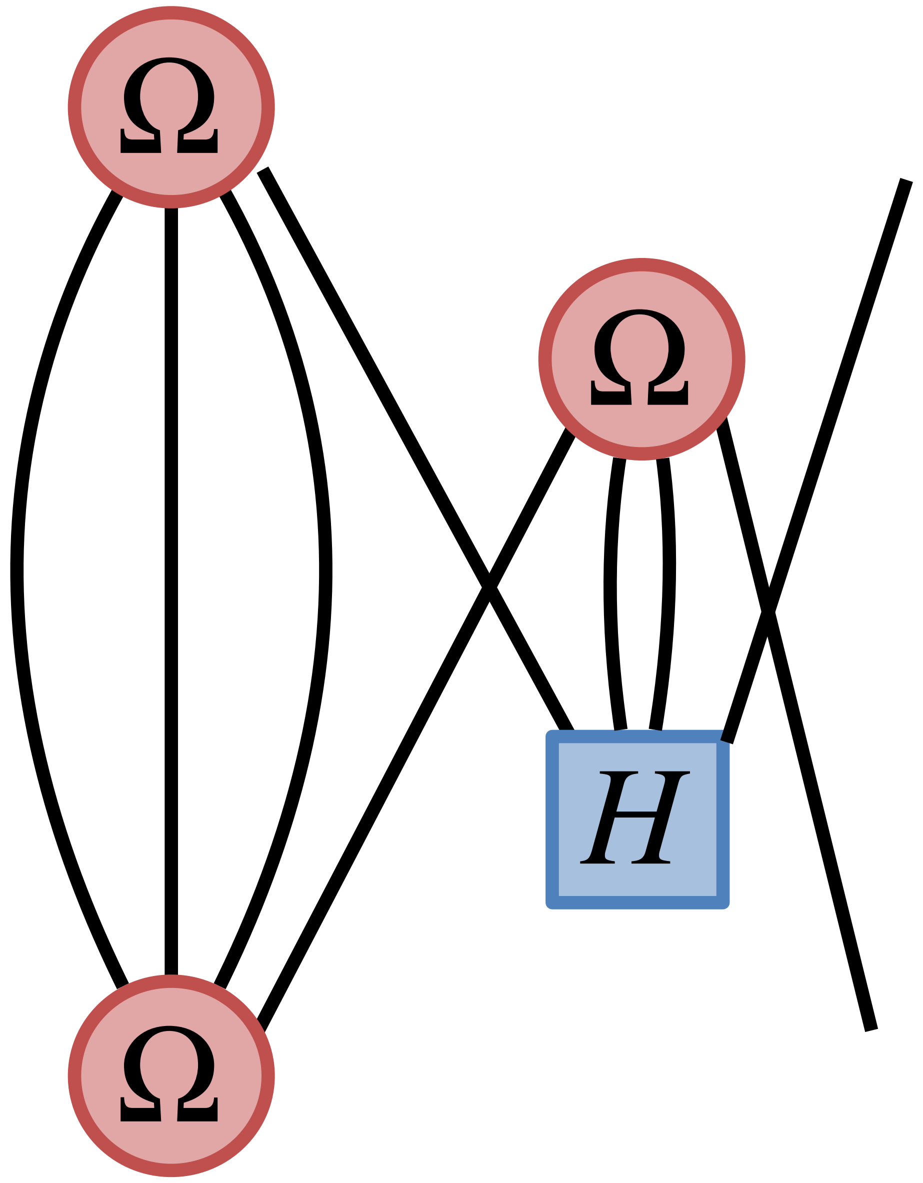
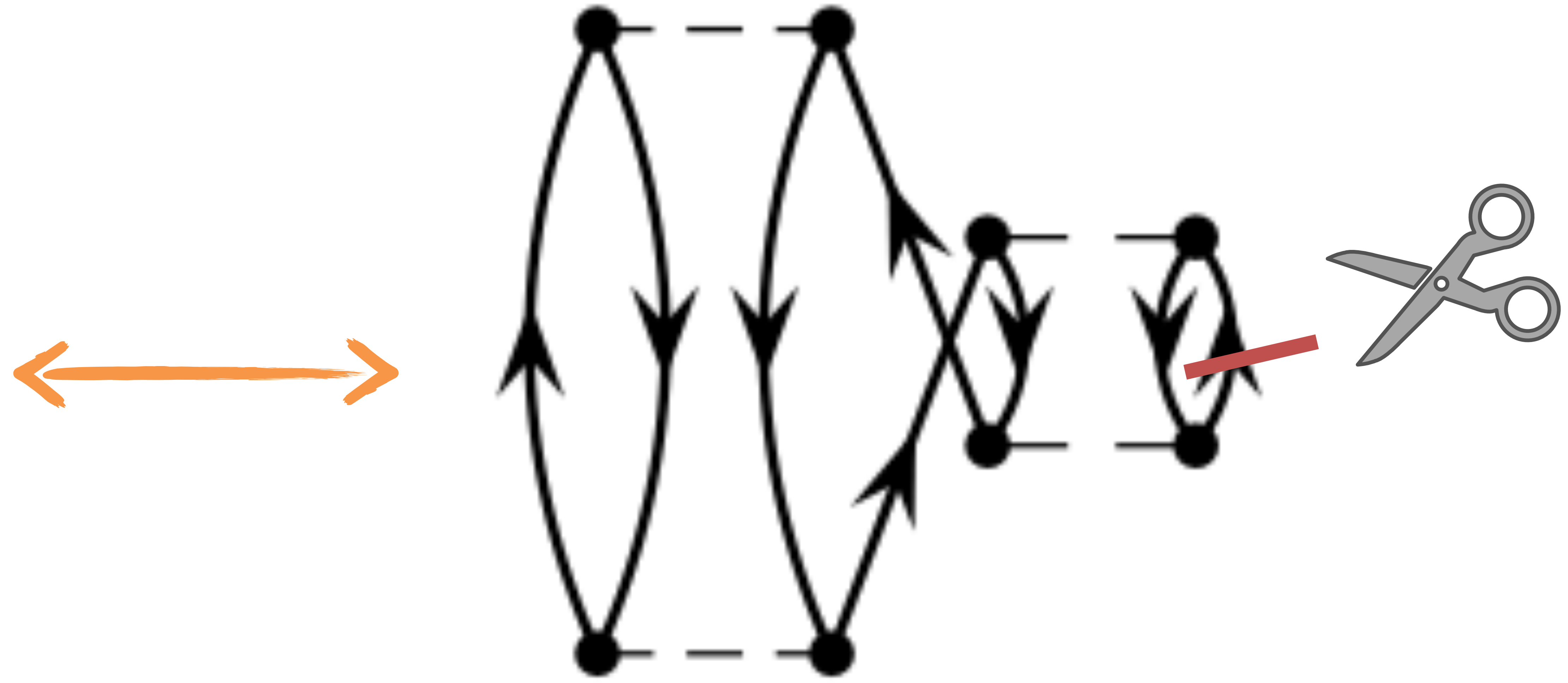
Largest contribution to H_{1b}:



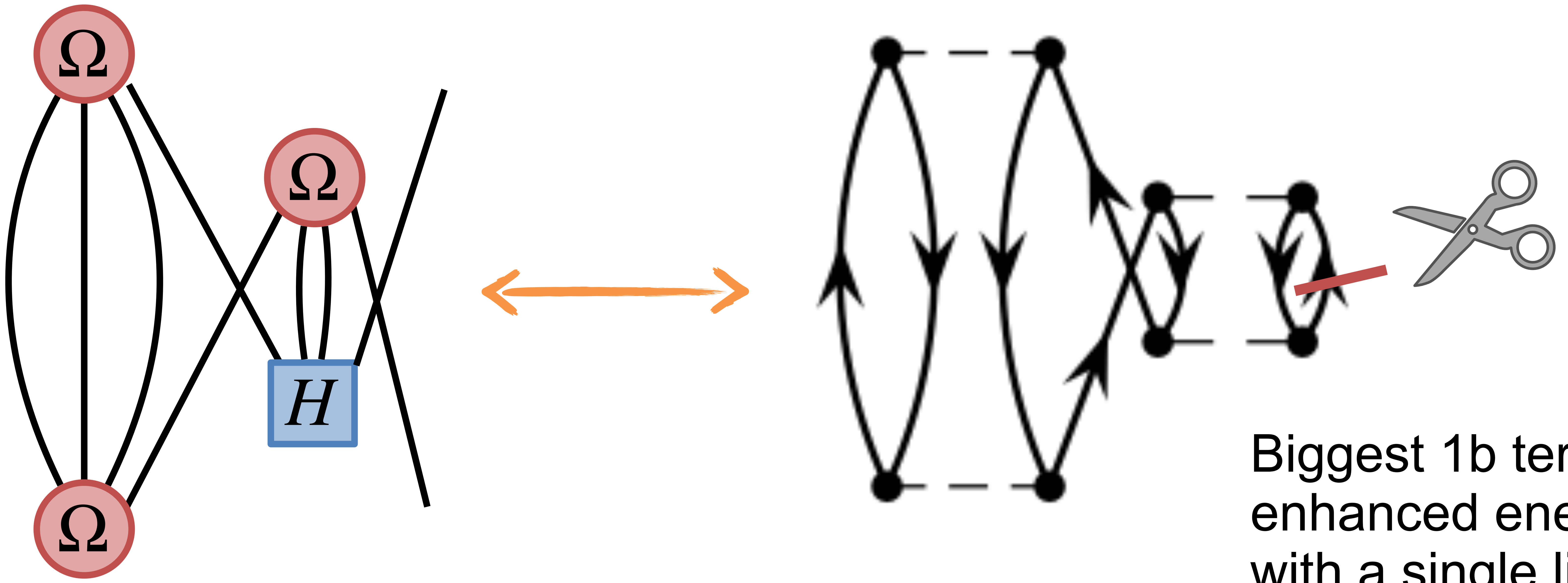


Magnus IMSRG

$$H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$$

$$= H + [\Omega, H] + \frac{1}{2!}[\Omega, [\Omega, H]] + \frac{1}{3!}[\Omega, [\Omega, [\Omega, H]]] + \ldots$$

Largest contribution to H_{1b}:

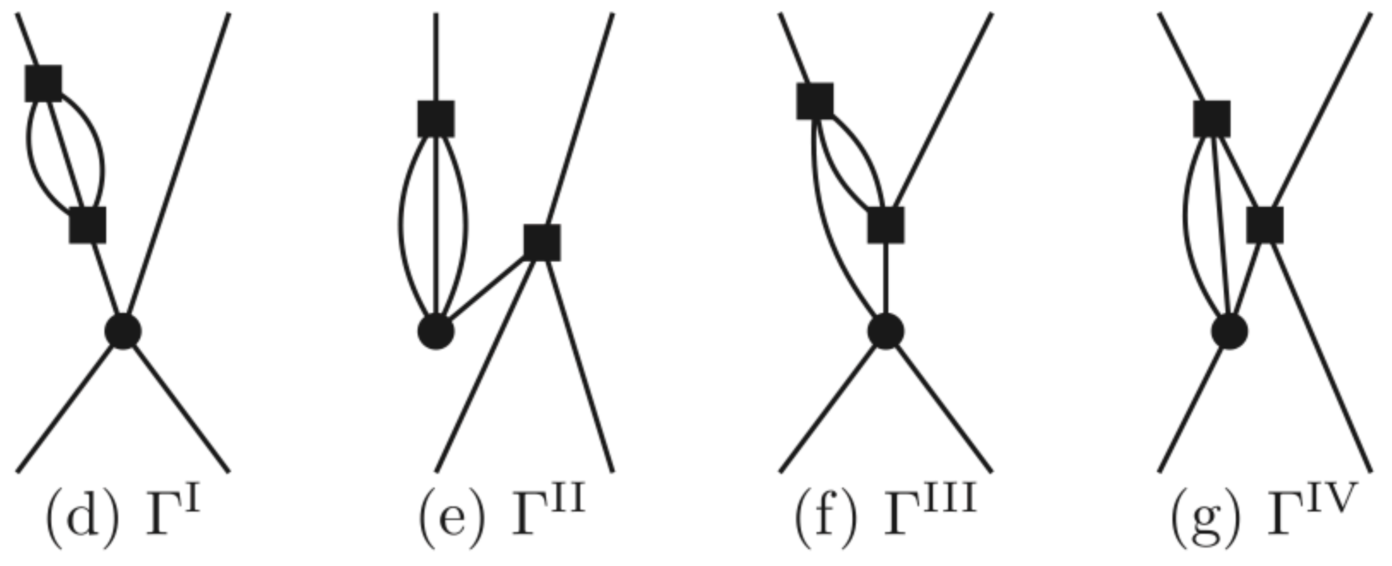


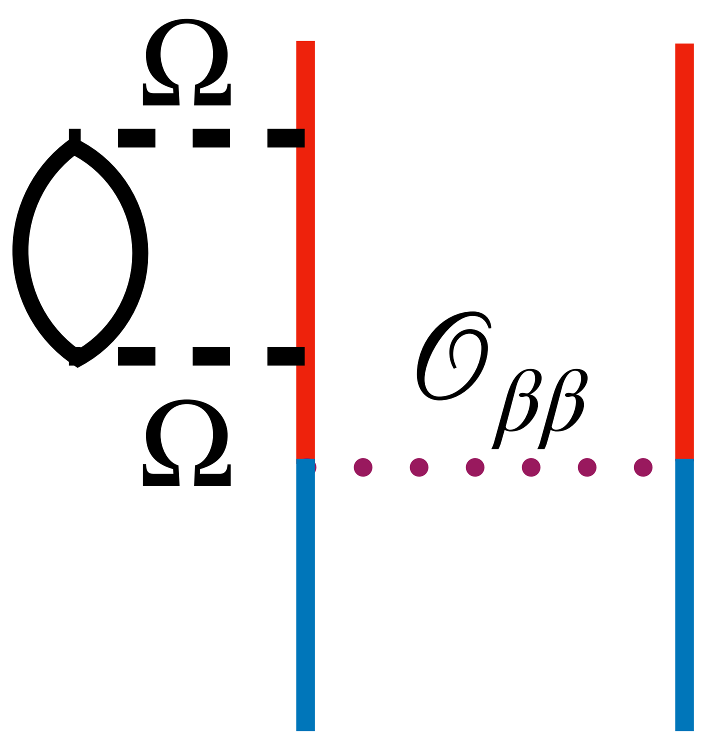
Biggest 1b terms correspond to enhanced energy diagrams with a single line cut.

$0\nu\beta\beta$ decay

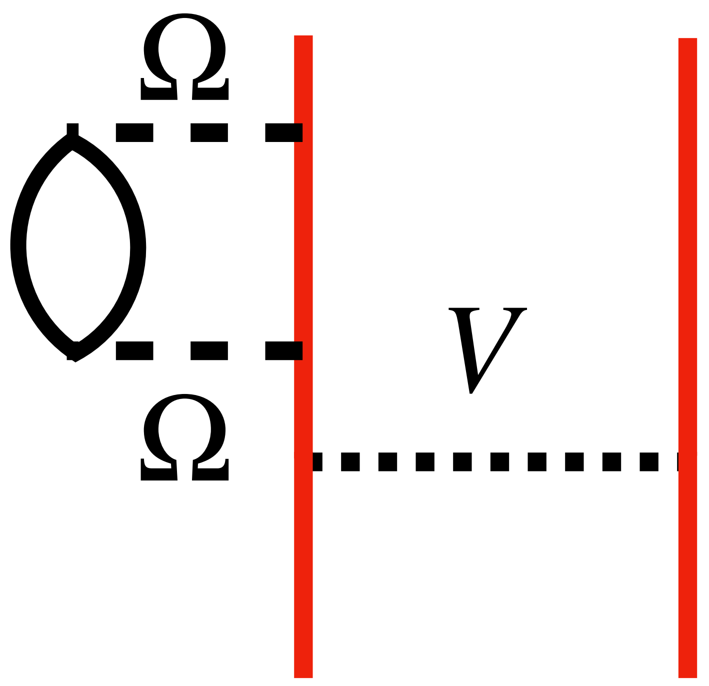
$$\mathcal{O}_{\beta\beta}(s) = \mathcal{O}_{\beta\beta} + [\Omega, \mathcal{O}_{\beta\beta}] + \frac{1}{2}[\Omega, [\Omega, \mathcal{O}_{\beta\beta}]] + \dots$$

What should we expect from the 3f2 correction?





$\Rightarrow$ reduce $\mathcal{O}_{\beta\beta}$

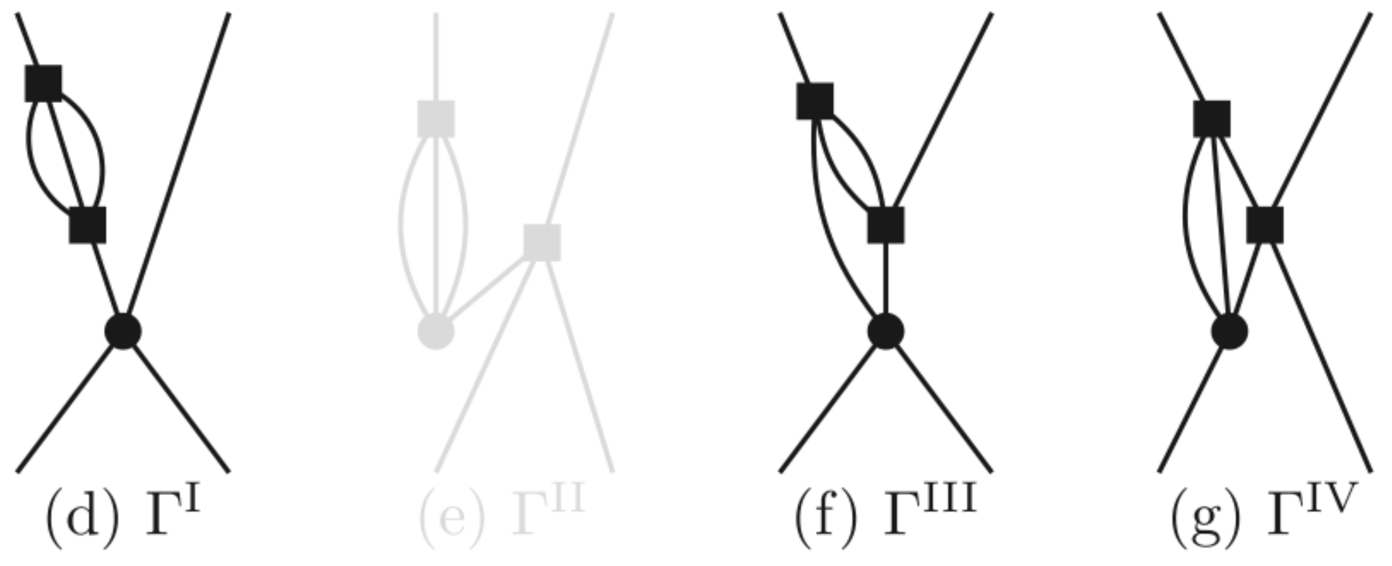


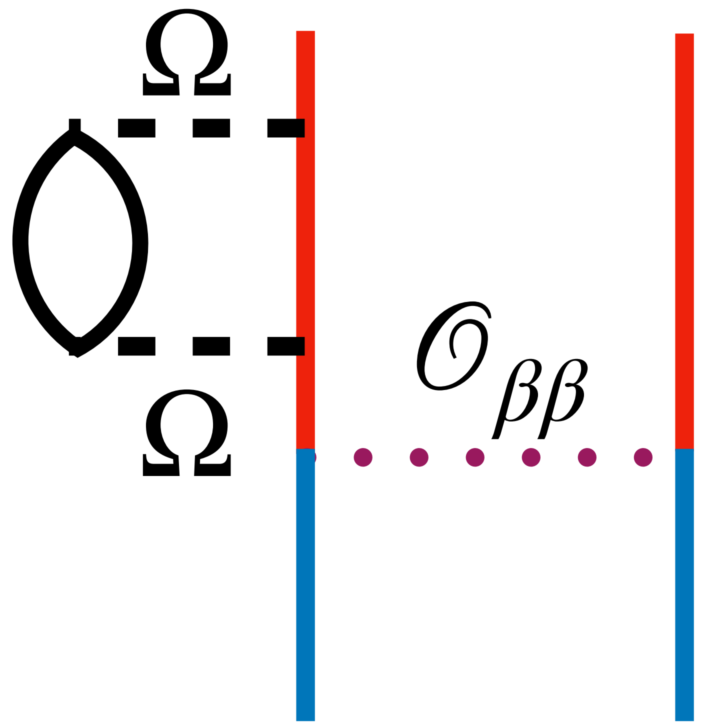
$\Rightarrow$ reduce correlations, including pairing

$0\nu\beta\beta$ decay

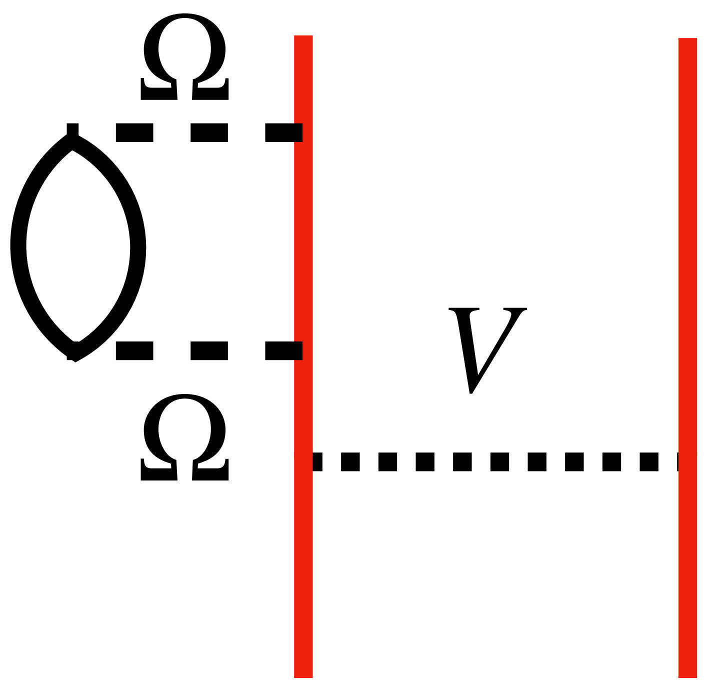
$$\mathcal{O}_{\beta\beta}(s) = \mathcal{O}_{\beta\beta} + [\Omega, \mathcal{O}_{\beta\beta}] + \frac{1}{2}[\Omega, [\Omega, \mathcal{O}_{\beta\beta}]] + \ldots$$

What should we expect from the 3f2 correction?



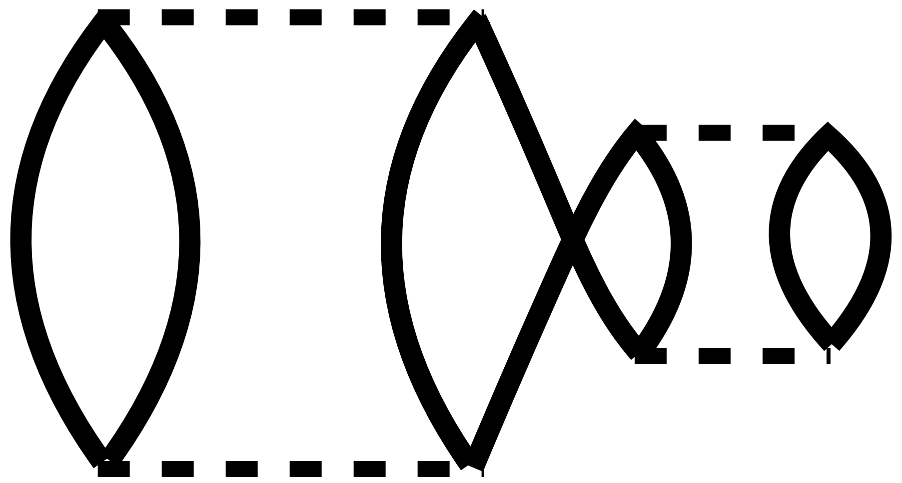


$\Rightarrow$ reduce $\mathcal{O}_{\beta\beta}$

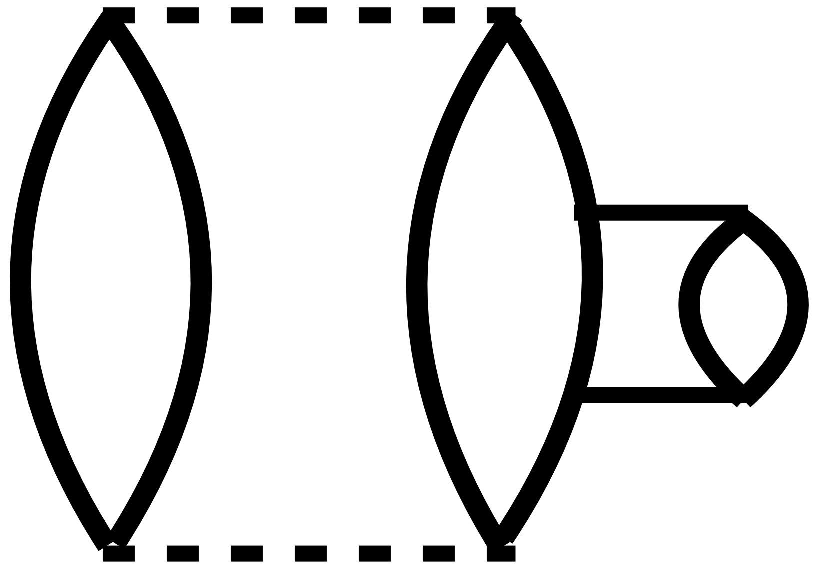


$\Rightarrow$ reduce correlations, including pairing

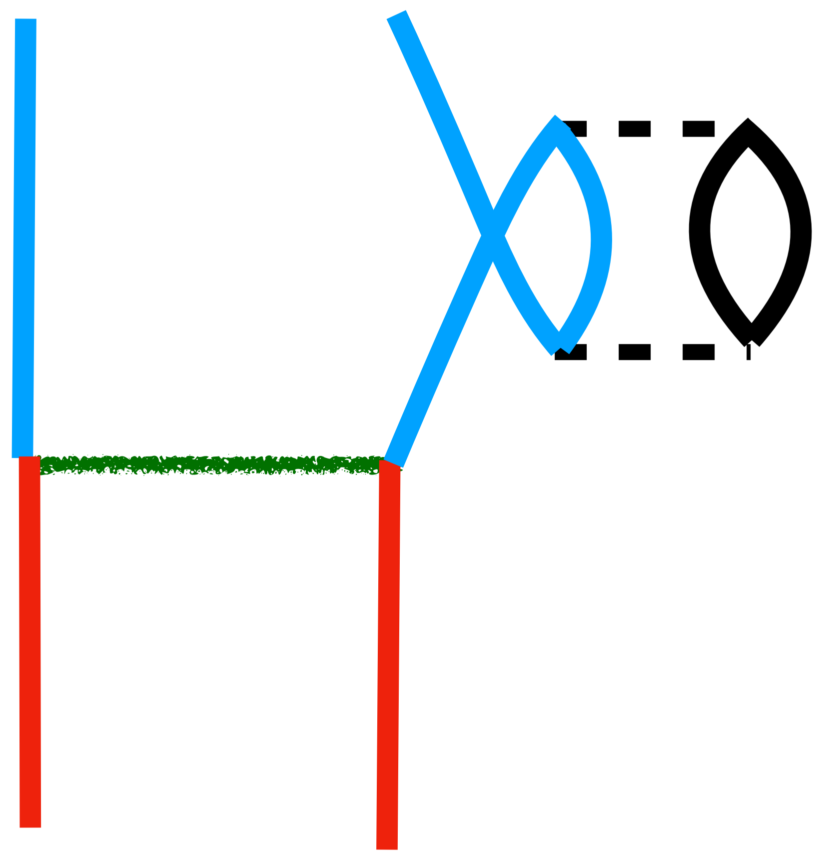
In the 4th order energy, the reduced binding from the 3f2 correction is approximately offset by the binding from triple-excitations.



$[\Omega_3, H_3]_0$

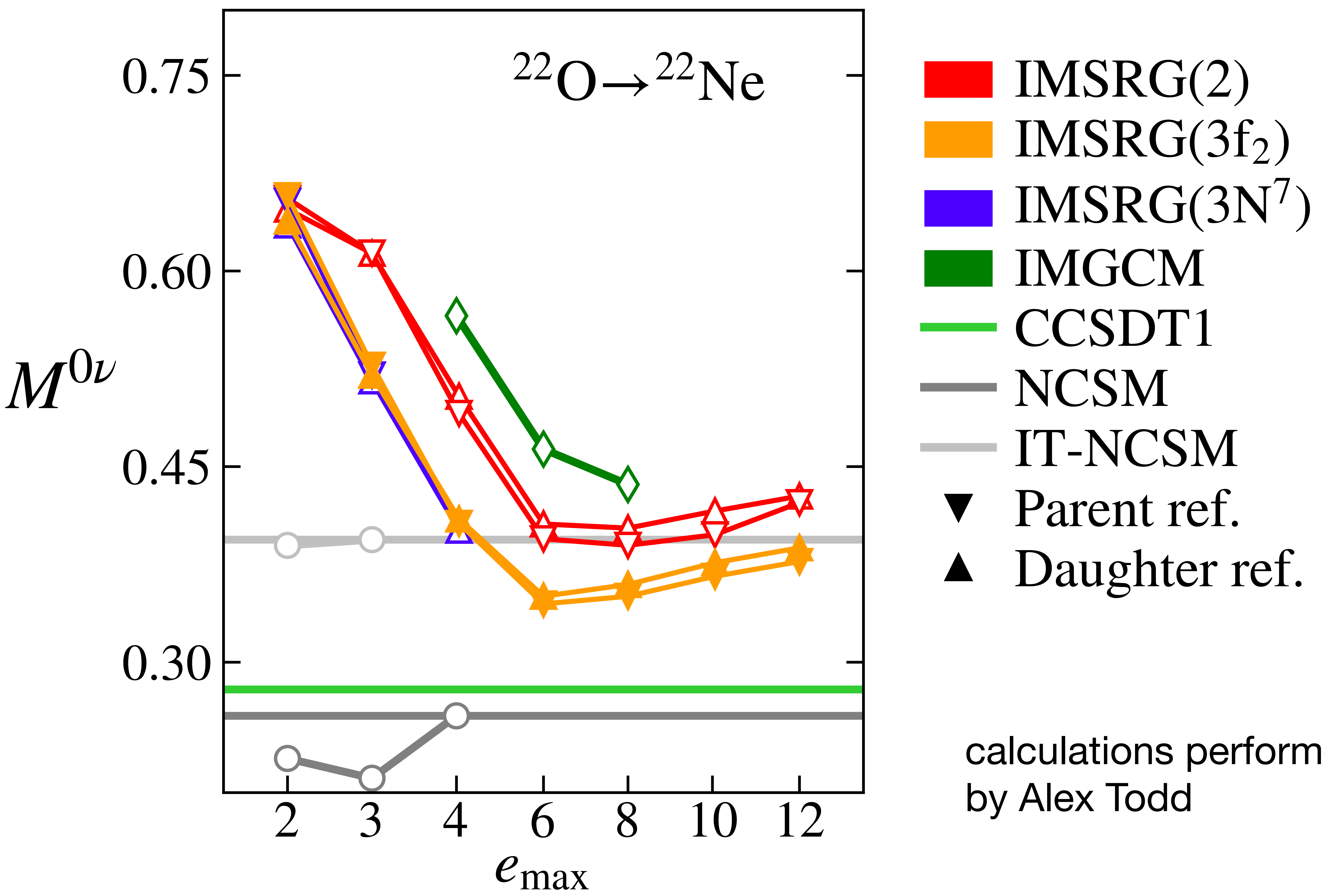


Will there be be a similar effect in $0\nu\beta\beta$?

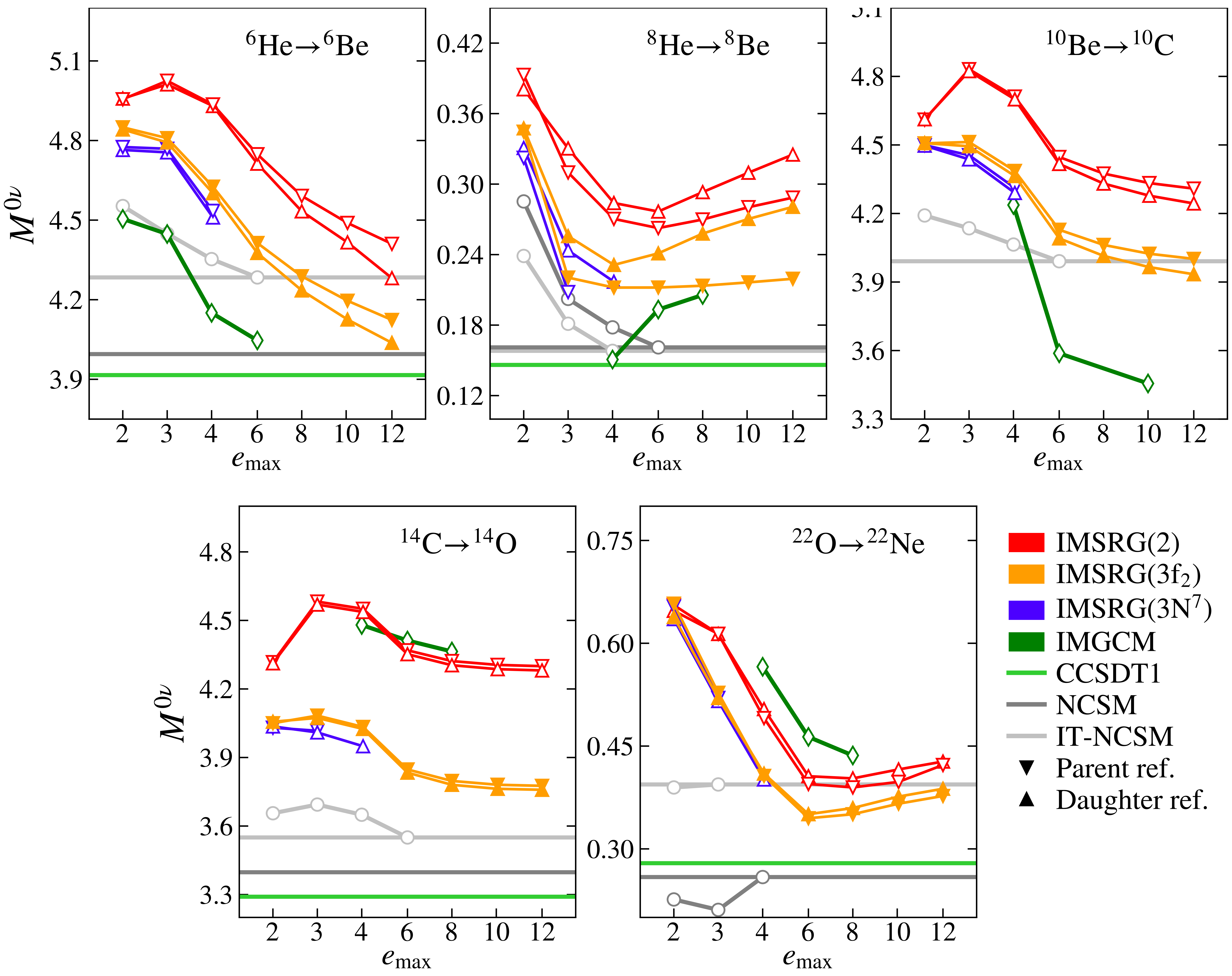


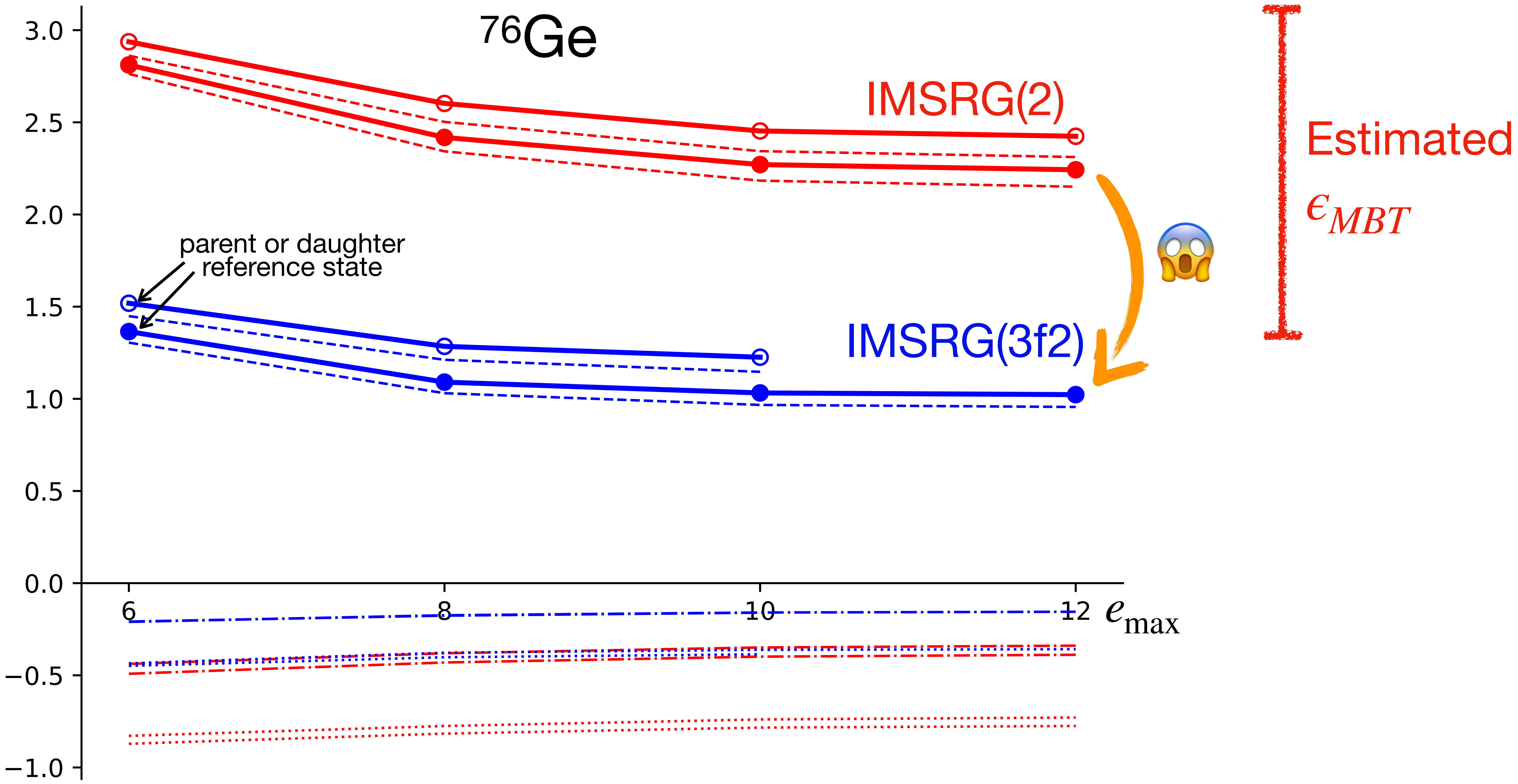
Maybe not. $\mathcal{O}_{\beta\beta}$ does not contribute to Ω.
H is special in that way.

Benchmarking in light nuclei

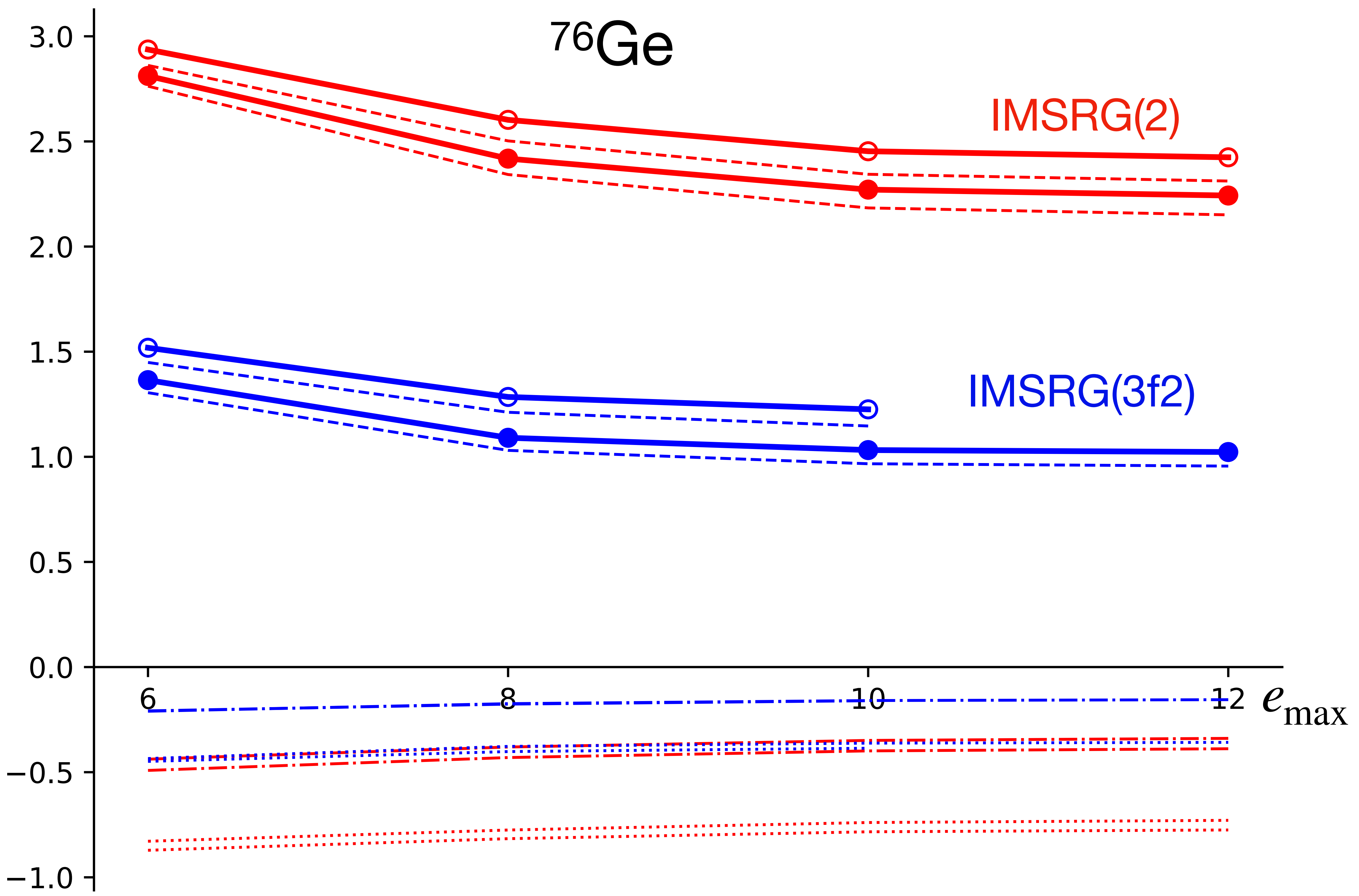


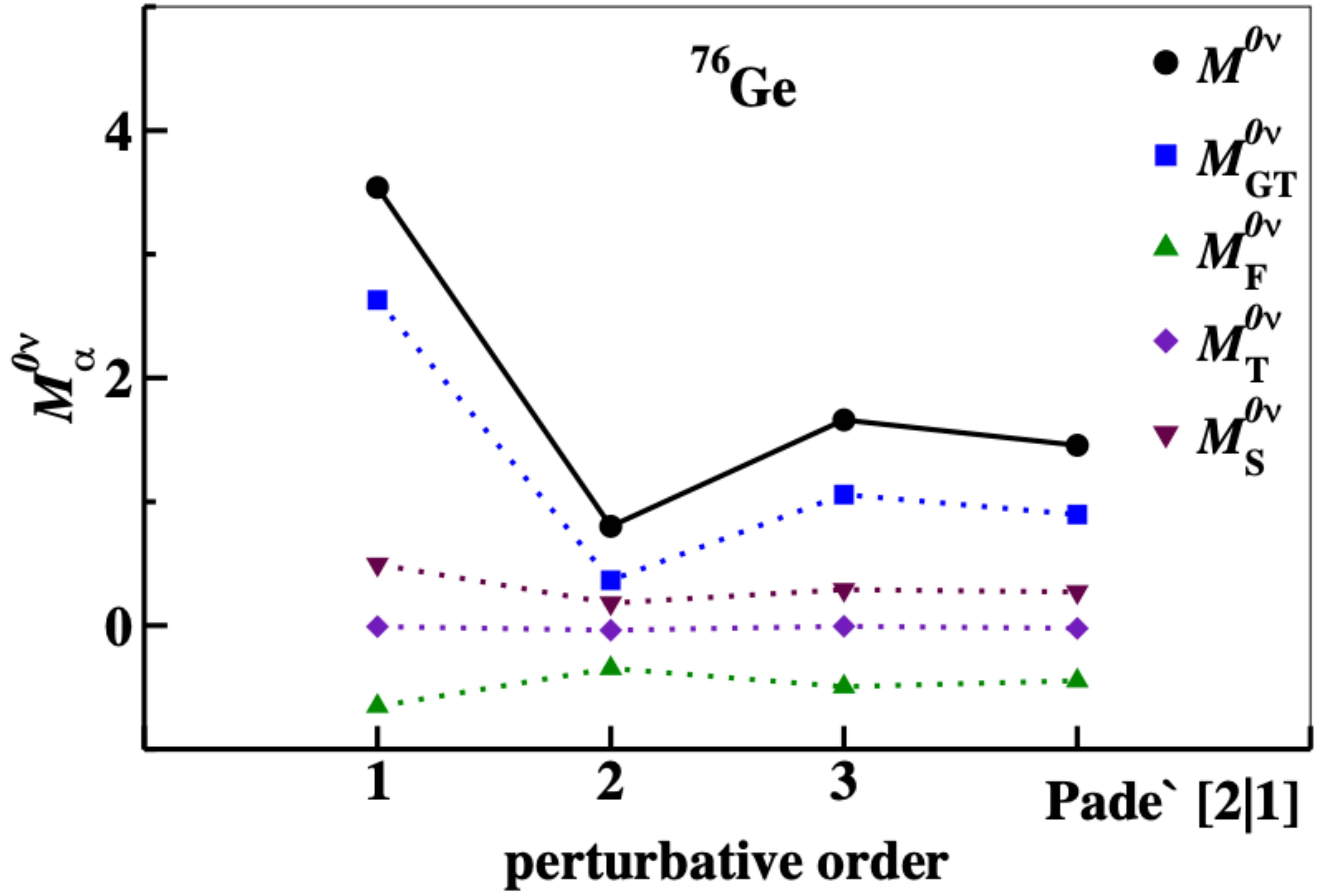
calculations performed by Alex Todd


^{6}He→^{6}Be
^{8}He→^{8}Be
^{10}Be→^{10}C
^{14}C→^{14}O
^{22}O→^{22}Ne
$M^{0\nu}$
e_{max}
IMSRG(2)
IMSRG(3f$_2$)
IMSRG(3N^7)
IMGCM
CCSDT1
NCSM
IT-NCSM
Parent ref.
Daughter ref.

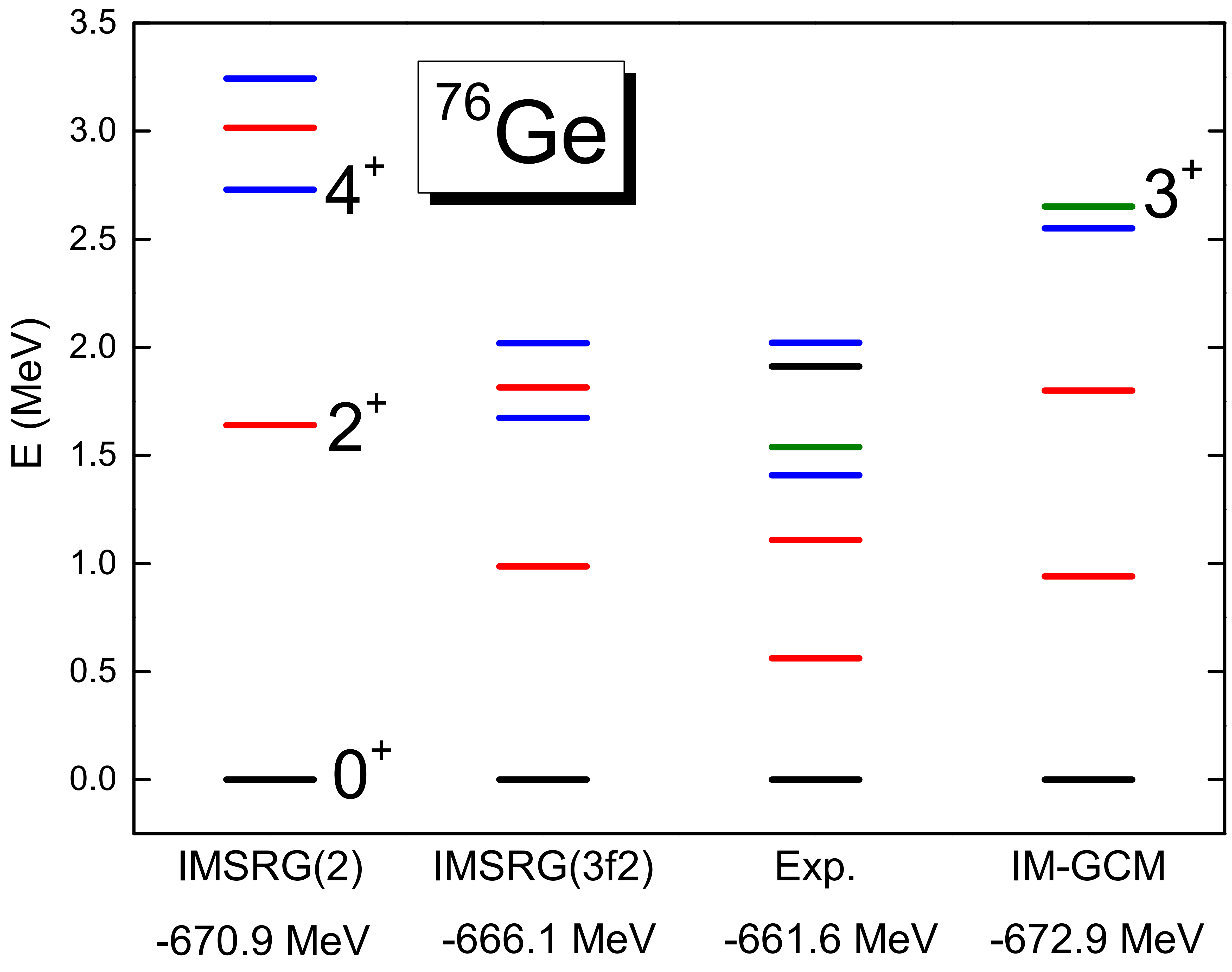


^{76}Ge
IMSRG(2)
IMSRG(3f2)
parent or daughter
reference state
Estimated
ϵ_{MBT}
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
6
8
10
12
$e_{\max}$

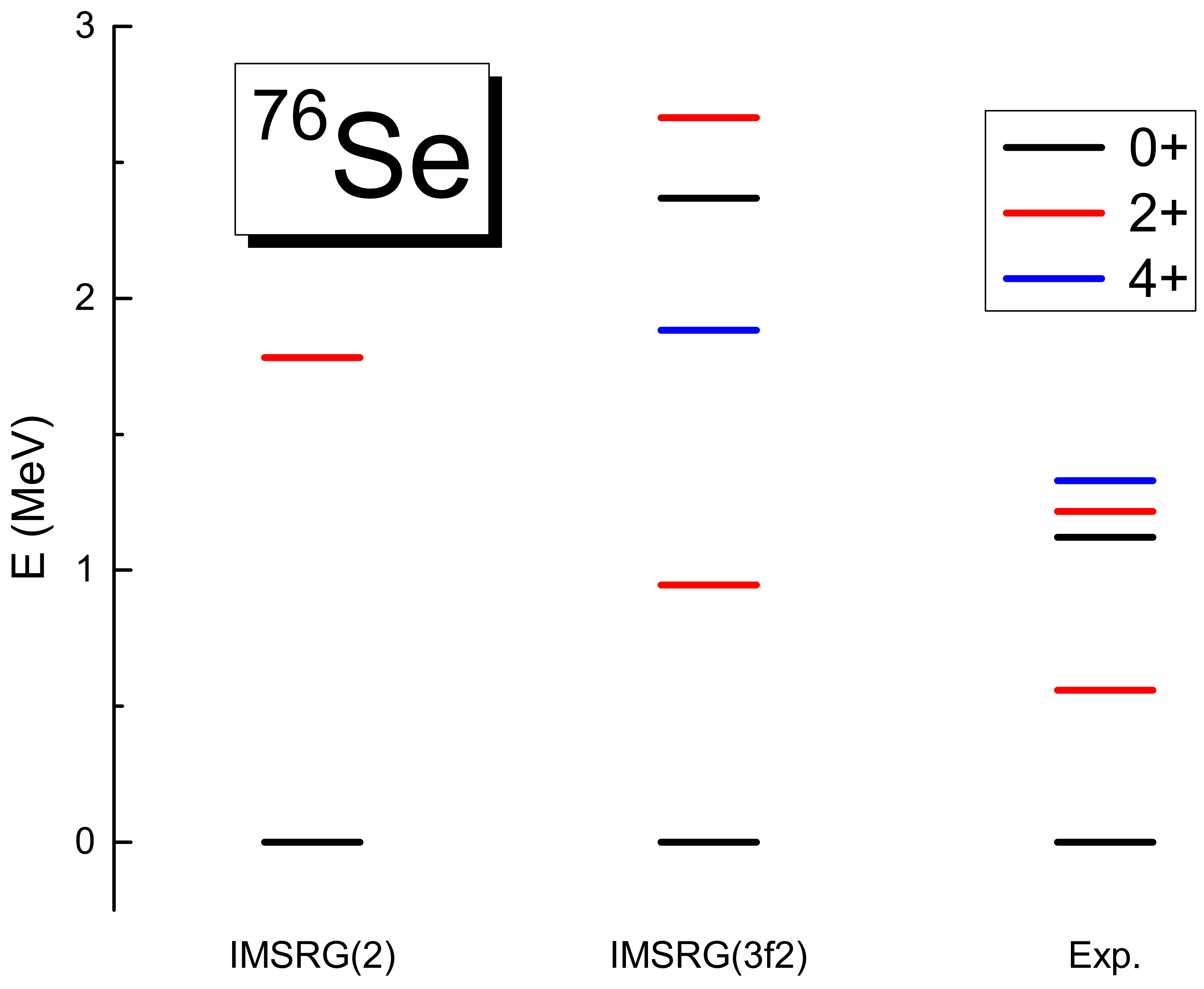




Corragio+ arXiv:2606.25486



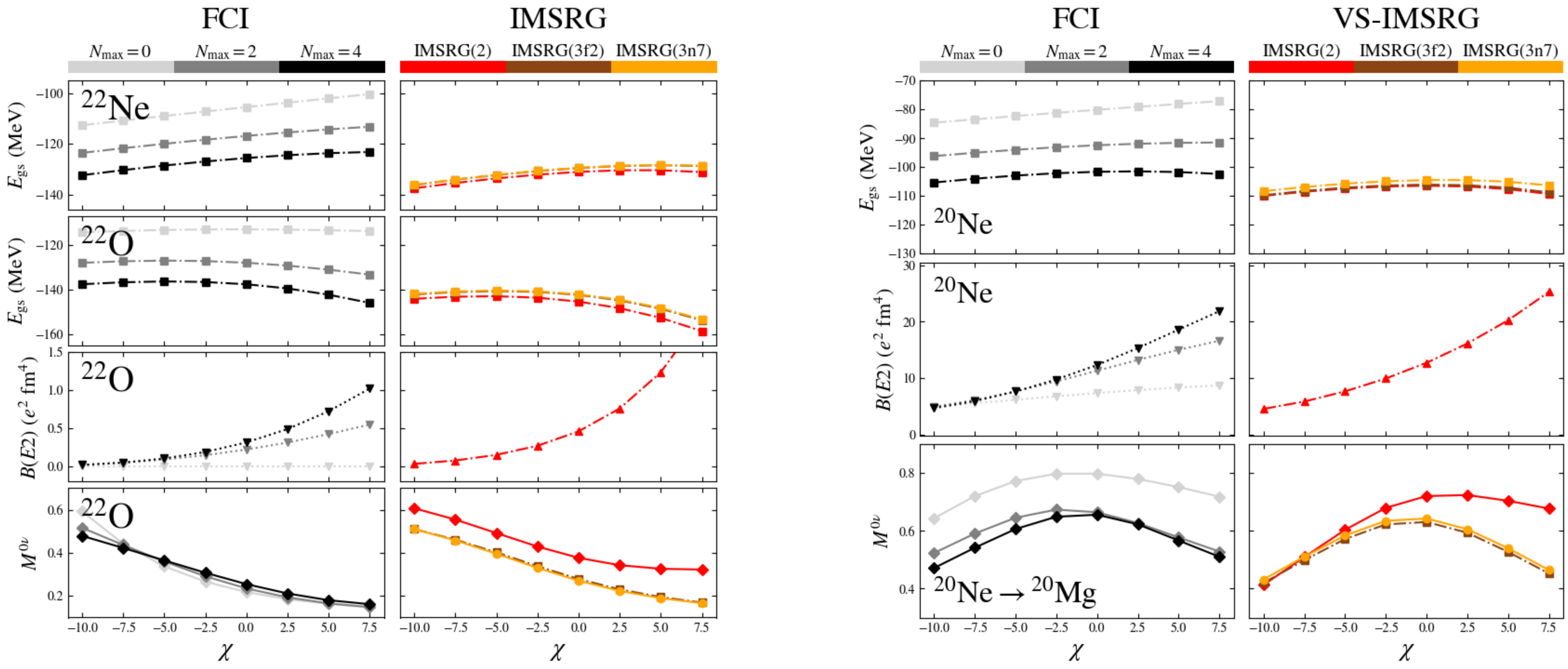
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
E (MeV)
^{76}Ge
4^+
2^+
0^+
3^+
IMSRG(2)
IMSRG(3f2)
Exp.
IM-GCM
-670.9 MeV
-666.1 MeV
-661.6 MeV
-672.9 MeV



3
2
1
0
E (MeV)
^{76}Se
0+
2+
4+
IMSRG(2)
IMSRG(3f2)
Exp.

Are we capturing deformation effects?

$$H \rightarrow H + \chi Q \cdot Q$$



Summary

- We have a systematically improvable method, though we'd like to improve more efficiently.

- We predicted what part of 3f2 would be the biggest, and what its sign would be. The size of the correction is slightly larger than our rough estimate based on comparison with IM-GCM.

- A more thorough investigation is ongoing, decomposing how the pieces of 3f2 act for $0\nu\beta\beta$, and how further corrections might act.

- Next step: valence 3N operators.