

Bottomonium suppression at NLO in the ratio of the binding energy over temperature

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N. Brambilla, M.-A. Escobedo, A. Islam, MS, A. Tiwari, A. Vairo, P. Vander Griend, 2205.10289
JHEP 2022, 303

Heavy Flavor Production in Heavy-Ion and Elementary Collisions
Institute for Nuclear Theory, University of Washington
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Bottomonium Suppression

- In a high temperature quark-gluon plasma we expect **weaker color binding** (Debye screening + asymptotic freedom)

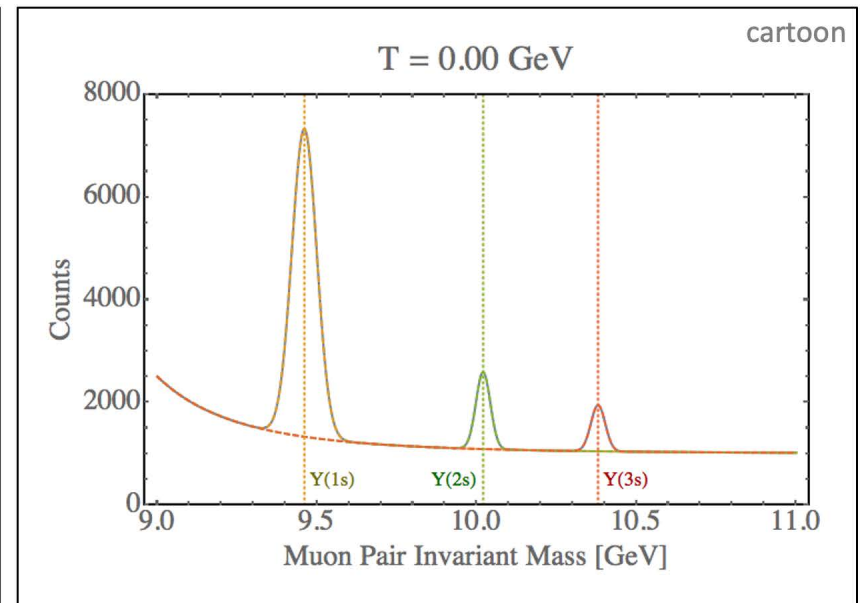
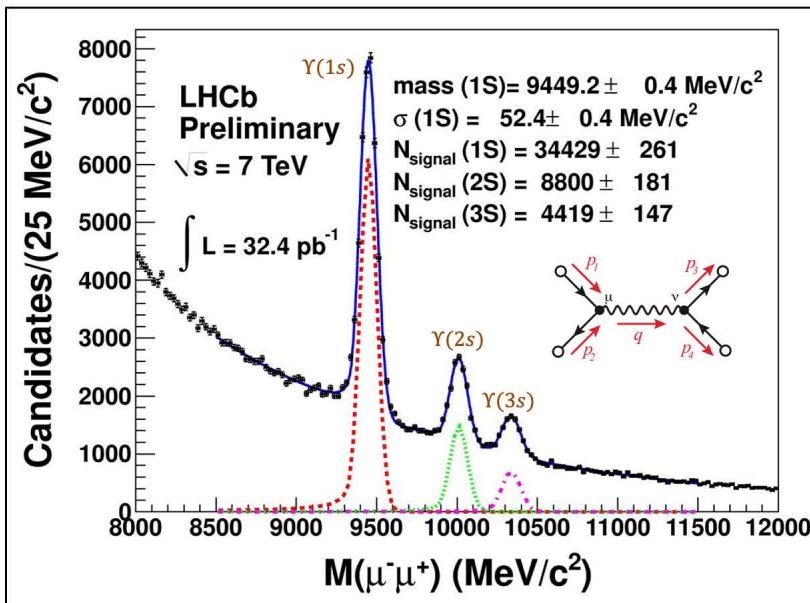
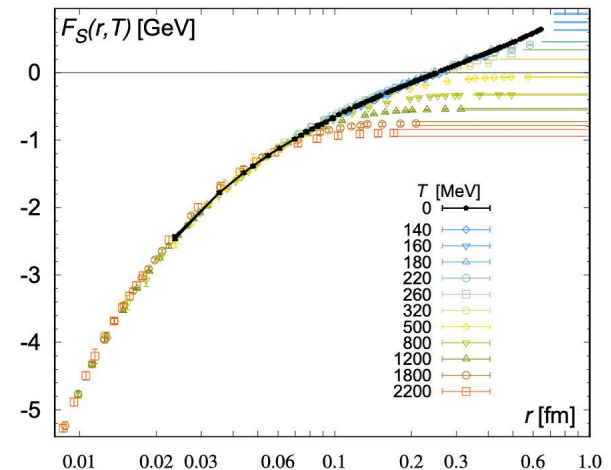
E. V. Shuryak, Phys. Rept. 61, 71–158 (1980)

T. Matsui, and H. Satz, Phys. Lett. B178, 416 (1986)

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- Also, high energy plasma particles which slam into the bound states cause them to have shorter lifetimes → **larger spectral widths**

TUMQCD Collaboration, 1804.10600



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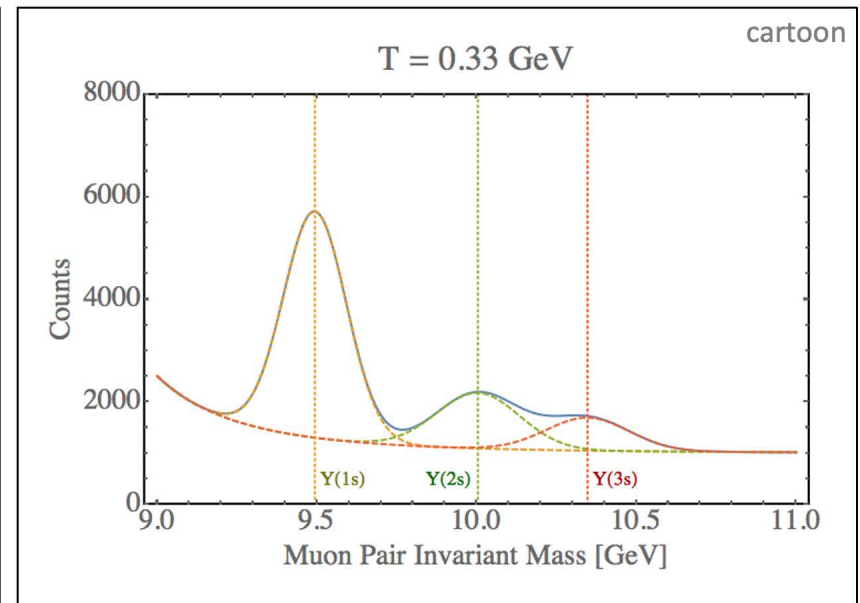
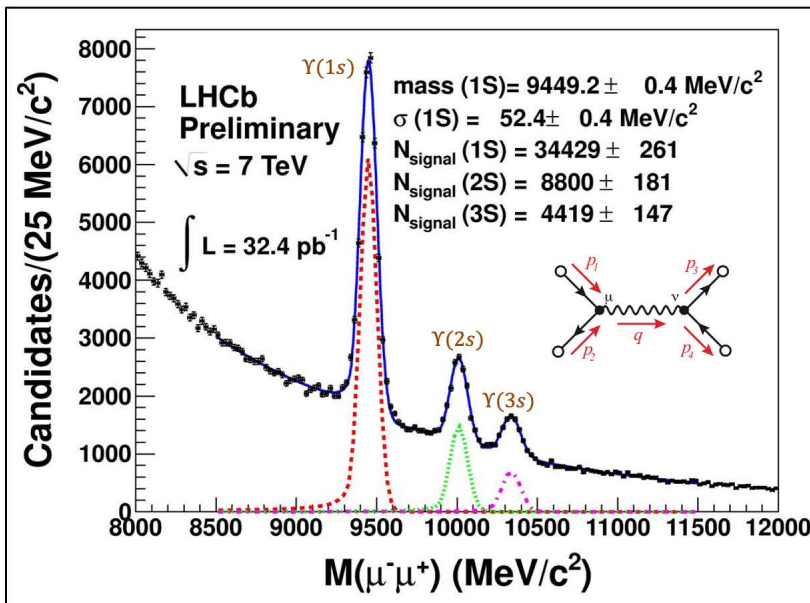
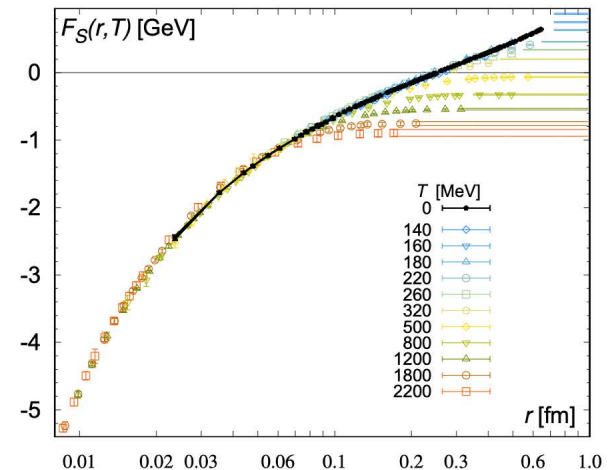
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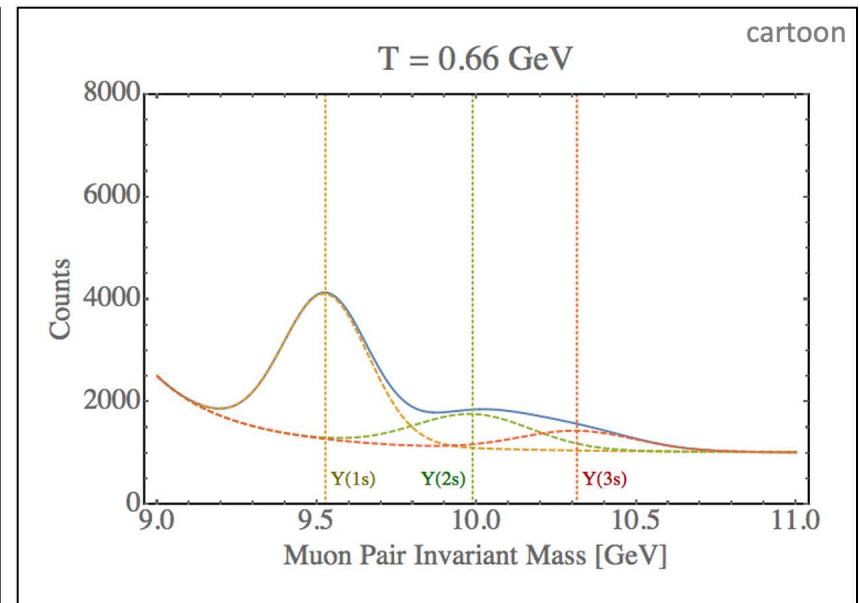
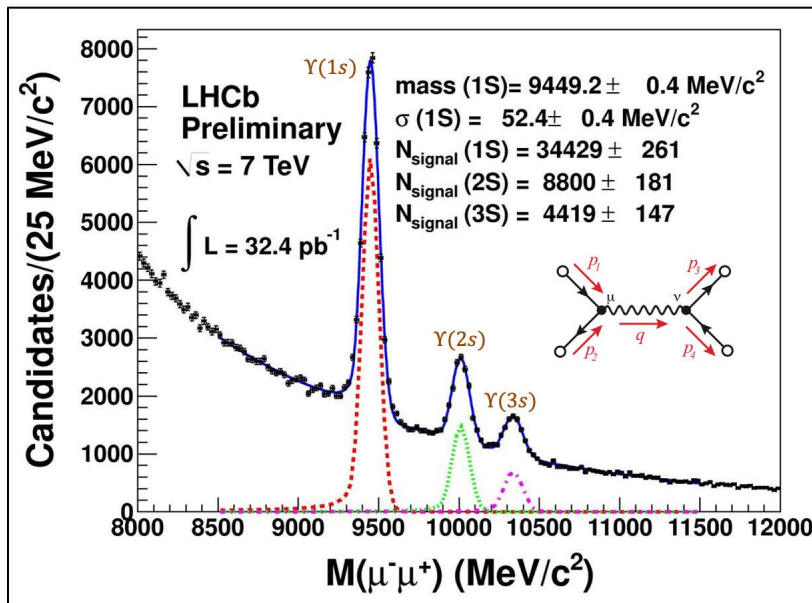
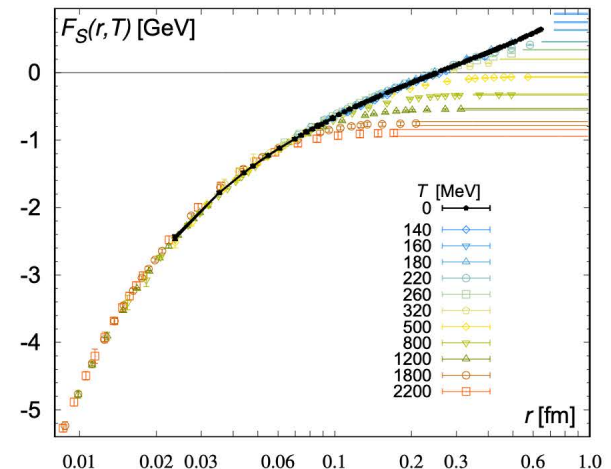
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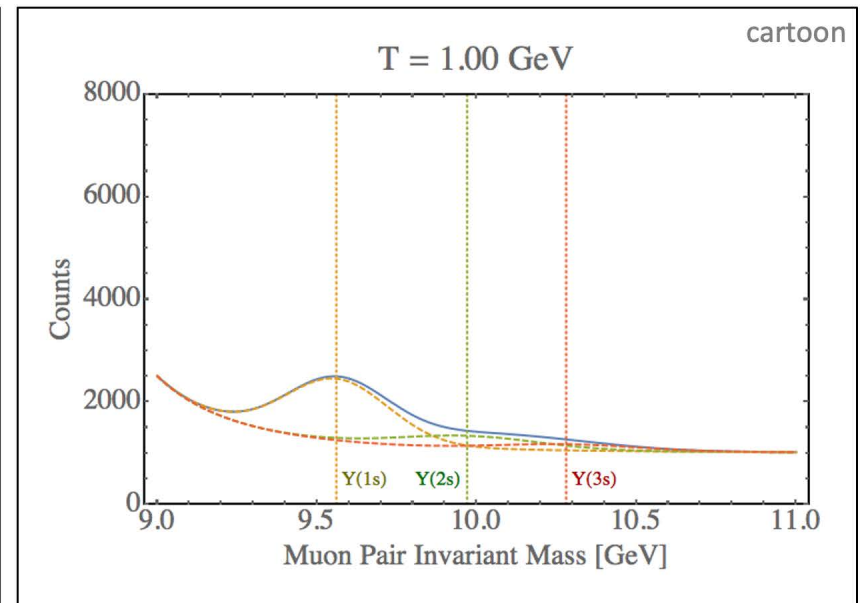
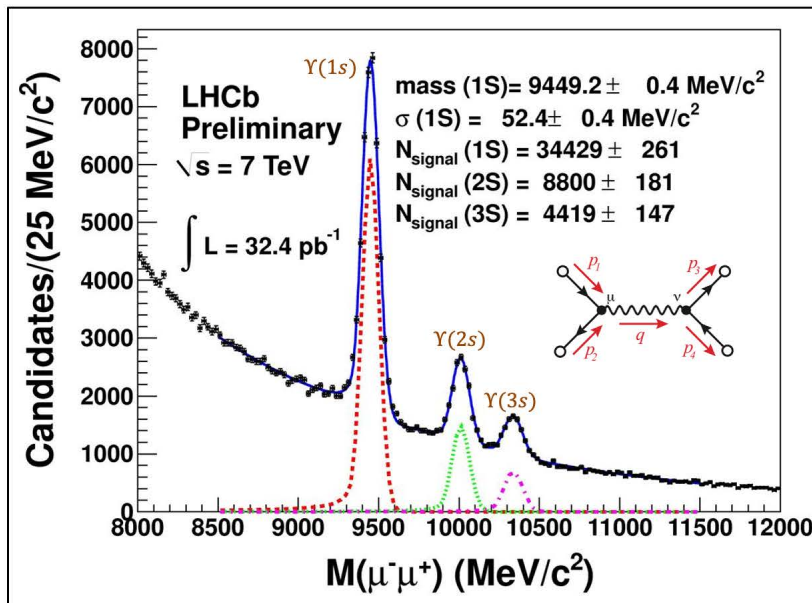
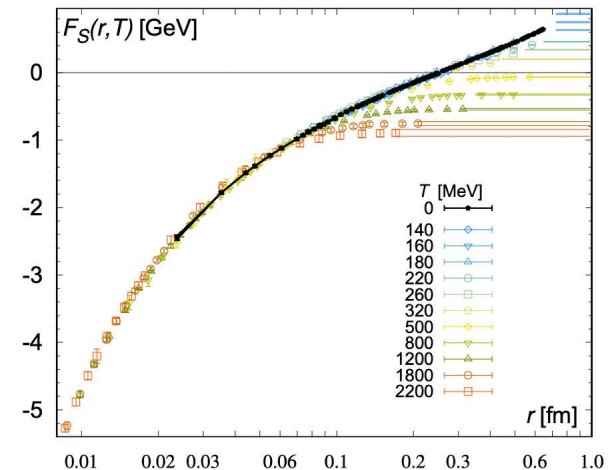
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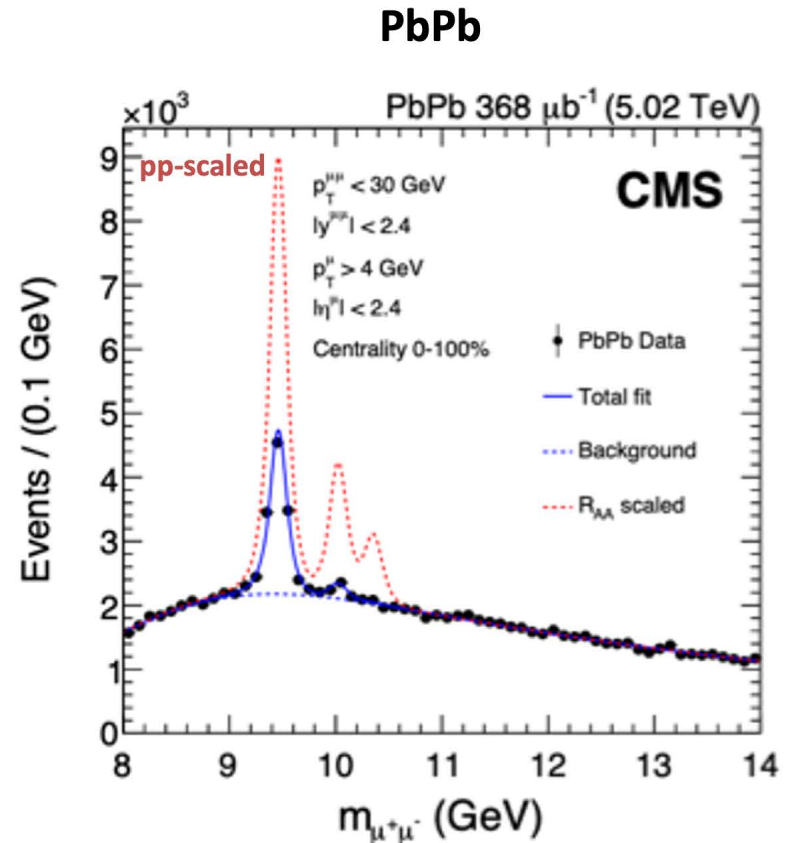
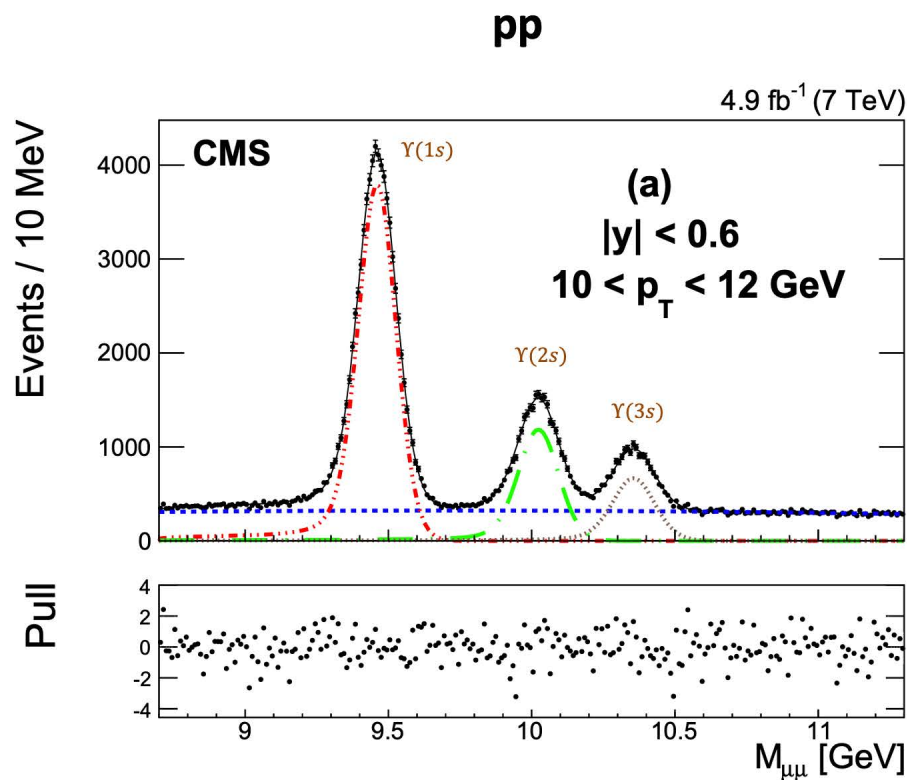
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TUMQCD Collaboration, 1804.10600



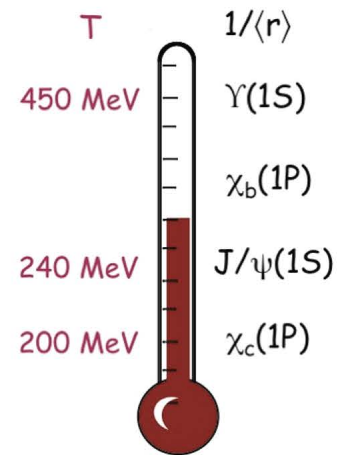
Experimental data – 5.02 TeV Dimuon Spectra

The **CMS**, **ALICE**, and **ATLAS** experiments have measured bottomonium production in both pp and Pb-Pb collisions at 5.02 TeV. Here I show CMS results.



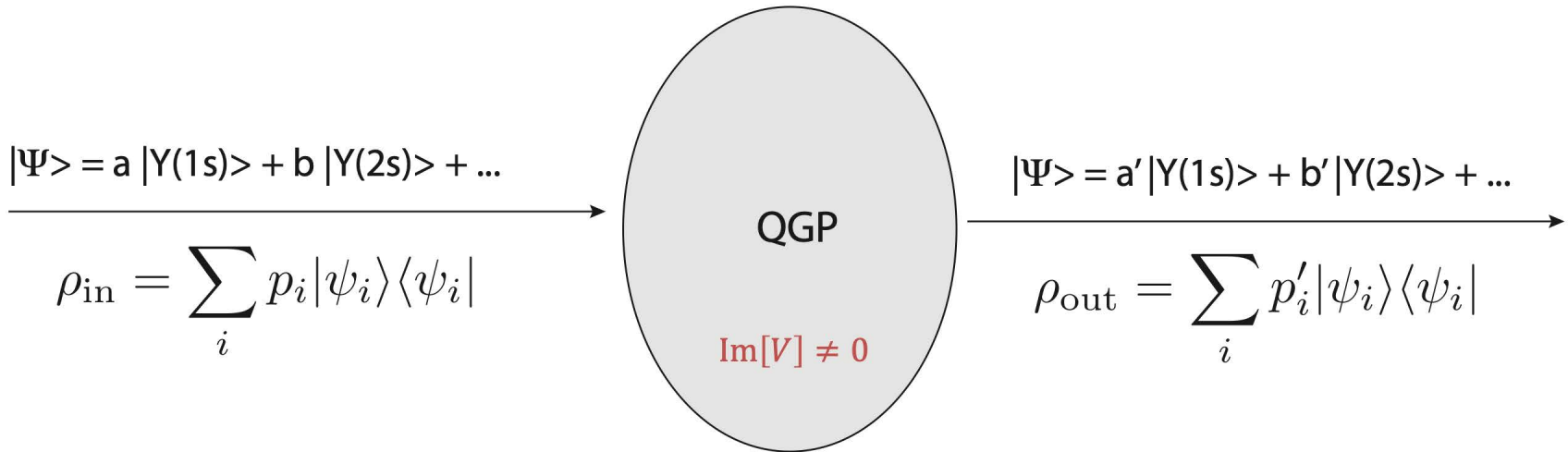
Why focus on bottomonia?

- Can trust heavy quark effective theory more.
- Cold nuclear matter (CNM) effects in AA decrease with increasing quark mass.
- The masses of bottomonia ($m \sim 10$ GeV) are much higher than the temperature generated in HICs ($T < 1$ GeV) $\rightarrow$ bottomonia production dominated by initial hard scatterings.
- Since bottom quarks and anti-quarks are relatively rare in RHIC and LHC HICs, the probability for regeneration of bottomonia through statistical recombination is much smaller than for charm quarks. [see e.g. E. Emerick, X. Zhao, and R. Rapp, arXiv:1111.6537 and others]



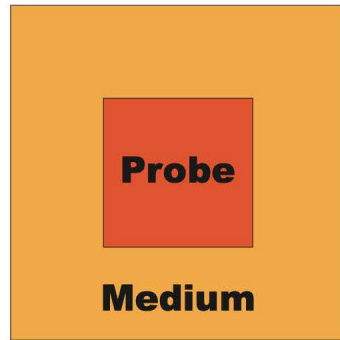
A. Mocsy, P. Petreczky,
and MS, 1302.2180

Conceptual problem



- Bottomonium states are produced locally (hard processes) at early times in hard collisions ($t < 1 \text{ fm}/c$).
- They then propagate through the plasma and interact with the medium.
- Bound states can break up and potentially re-form due to in-medium transitions induced by in-medium gluon absorption and emission.

Open quantum system (OQS) approach



Probe = heavy-quarkonium state

Medium = light quarks and gluons that comprise the QGP

- Can treat heavy quarkonium states propagating through QGP using an open quantum system approach

$$H_{\text{tot}} = H_{\text{probe}} \otimes I_{\text{medium}} + I_{\text{probe}} \otimes H_{\text{medium}} + H_{\text{int}}$$

- Total density matrix

$$\rho_{\text{tot}} = \sum_j p_j |\psi_j\rangle \langle \psi_j| \longrightarrow \frac{d}{dt} \rho_{\text{tot}} = -i[H_{\text{tot}}, \rho_{\text{tot}}]$$

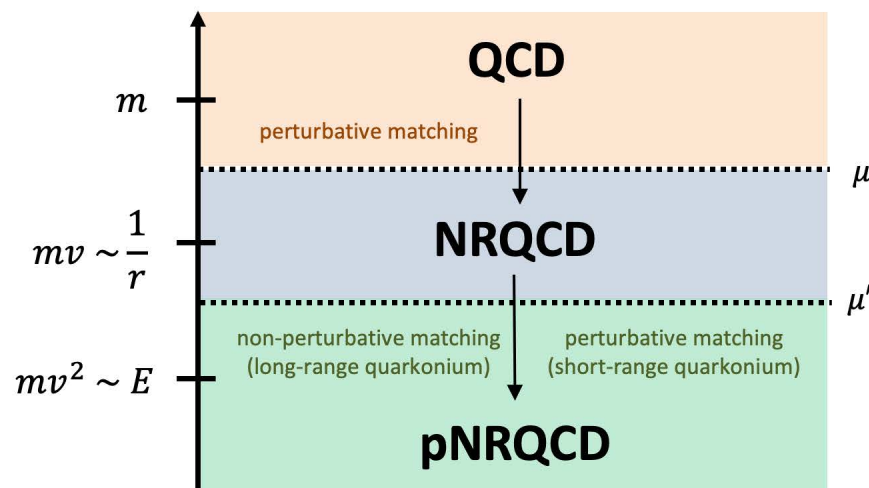
- Reduced density matrix (medium DOF integrated out)

$$\rho_{\text{probe}} = \text{Tr}_{\text{medium}}[\rho_{\text{tot}}] \longrightarrow \text{“Master equation”}$$

OQS + pNRQCD $\rightarrow$ Lindblad equation

- What are the relevant scales?

- Temperature T
- Bound state mass $m \gg T$
- Bound state size $r \sim 1/mv \sim a_0$ (Bohr radius)
- Debye mass m_D
- Binding energy $E \sim mv^2$



- Separation of time scales

- Medium relaxation time scale $\langle \hat{O}_M(t) \hat{O}_M(0) \rangle \sim e^{-t/t_M} \rightarrow \frac{1}{T}$
- Intrinsic probe time scale $t_P \sim \frac{1}{\omega_i - \omega_j} \rightarrow \frac{1}{E}$
- Probe relaxation time scale $\langle p(t) \rangle \sim e^{-t/t_{\text{rel}}} \rightarrow \frac{1}{\text{self-energy}} \sim \frac{1}{\alpha_s a_0^2 \Lambda^3} \quad \Lambda = T, E$

$$\frac{1/r \gg T \sim m_D \gg E}{t_{\text{rel}}, t_P \gg t_M} \rightarrow$$

$$\frac{d\rho_{\text{probe}}}{dt} = -i[H_{\text{probe}}, \rho_{\text{probe}}] + \sum_n \left(C_n \rho_{\text{probe}} C_n^\dagger - \frac{1}{2} \{C_n^\dagger C_n, \rho_{\text{probe}}\} \right)$$

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

Non-Relativistic QCD (NRQCD)

Caswell and Lepage (1986), Bodwin, Braaten and Lepage (1994)

$$\mathcal{L}_{NRQCD} = \mathcal{L}_g + \mathcal{L}_q + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

$$\mathcal{L}_g = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + \frac{d_2}{m_Q^2}F_{\mu\nu}^a D^2 F^{\mu\nu a} + d_g^3 \frac{1}{m_Q^2} g f_{abc} F_{\mu\nu}^a F_{\alpha}^{\mu b} F^{\nu\alpha c}$$

$$\mathcal{L}_\psi = \psi^\dagger \left(iD_0 + c_2 \frac{D^2}{2m_Q} + c_4 \frac{D^4}{8m_Q^3} + c_F g \frac{\sigma \mathbf{B}}{2m_Q} + c_D g \frac{\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D}}{8m_Q^2} + i c_S g \frac{\sigma(\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8m_Q^2} \right) \psi$$

$$\mathcal{L}_\chi = \text{c.c of } \mathcal{L}_\psi$$

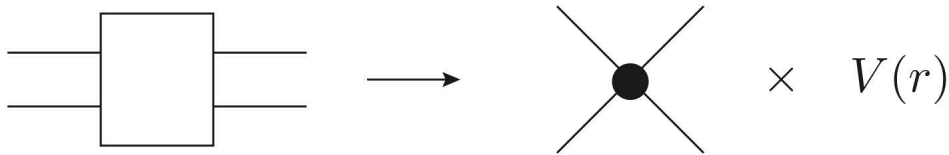
$$\mathcal{L}_{\psi\chi} = \frac{f_1(^1S_0)}{m_Q^2} \psi^\dagger \chi \chi^\dagger \psi + \frac{f_1(^3S_1)}{m_Q^2} \psi^\dagger \sigma \chi \chi^\dagger \sigma \psi + \frac{f_8(^1S_0)}{m_Q^2} \psi^\dagger T^a \chi \chi^\dagger T^a \psi + \frac{f_8(^3S_1)}{m_Q^2} \psi^\dagger T^a \sigma \chi \chi^\dagger T^a \sigma \psi$$

- **Integrating out the scale m can be done perturbatively** and is not affected by the presence of the medium since $m \gg \Lambda_{QCD}, T$.
- **Hard gluons**, with energy and momentum of order m .
- **Soft gluons**, with energy and momentum of order mv .
- **Potential gluons**, with energy of order mv^2 and momentum of order mv .
- **Ultrasoft gluons**, with both energy and momentum of order mv^2

NRQCD $\rightarrow$ Potential NRQCD (pNRQCD)

Pineda and Soto, '97; Brambilla, Pineda, Soto, and Vairo '99, '00, '03

Degrees of freedom at scale $\frac{1}{r} = mv$ are integrated out



Power counting

$$r \sim \frac{1}{mv} \quad t, R \sim \frac{1}{mv^2}, \frac{1}{\Lambda_{\text{QCD}}}$$

Gauge fields are multiple expanded

$$A(r, R, t) = A(R, t) + \mathbf{r} \cdot \nabla A(R, t) + \dots$$

Non-analytic behavior in $r \rightarrow$ matching coefficients V

- Resulting degrees of freedom are singlet and octet states (see Lagrangian on next slide).
- Allows to obtain manifestly gauge-invariant results.
- Easier connection lattice QCD.
- If $1/r \gg T$ we can use this as a starting point.
- In other cases, the matching between NRQCD and pNRQCD will be modified.

NRQCD $\rightarrow$ Potential NRQCD (pNRQCD)

Pineda and Soto, '97; Brambilla, Pineda, Soto, and Vairo '99, '00, '03

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu,a} + \text{Tr} \left\{ \textcolor{blue}{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - \textcolor{green}{V}_s \right) \textcolor{blue}{S} + \textcolor{red}{O}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - \textcolor{green}{V}_o \right) \textcolor{red}{O} \right\}$$

$$+ \textcolor{green}{V}_A \text{Tr} \left\{ \textcolor{red}{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \textcolor{blue}{S} + \textcolor{blue}{S}^\dagger \mathbf{r} \cdot g\mathbf{E} \textcolor{red}{O} \right\} \rightarrow \frac{\textcolor{red}{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \textcolor{blue}{S}}{\textcolor{red}{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \textcolor{blue}{S}}$$

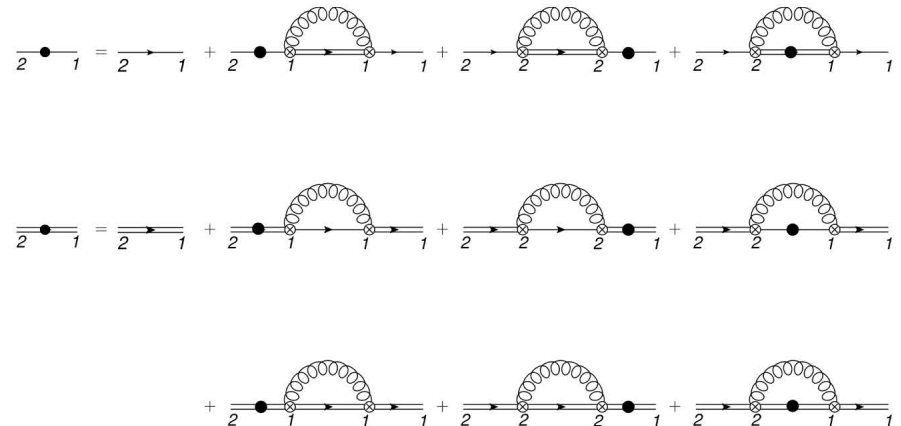
$$+ \frac{\textcolor{green}{V}_B}{2} \text{Tr} \left\{ \textcolor{red}{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \textcolor{red}{O} + \textcolor{red}{O}^\dagger \textcolor{red}{O} \mathbf{r} \cdot g\mathbf{E} \right\} \rightarrow \frac{\textcolor{red}{O}^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, \textcolor{red}{O} \}}{\textcolor{red}{O}^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, \textcolor{red}{O} \}}$$

Singlet and octet potentials

$$\textcolor{green}{V}_s(r) = -C_F \frac{\alpha_s}{r}$$

$$\textcolor{green}{V}_o(r) = \frac{\alpha_s}{2N_c r}$$

- Based on this Lagrangian, we can perform first-principles calculations.
- Right figure shows diagrams contributing to singlet and octet self-energies.
- These enter into the calculation of Lindblad/collapse/jump operators.



OQS + pNRQCD – Lindblad reorganization

$$\frac{d\rho_{\text{probe}}}{dt} = -i[H_{\text{probe}}, \rho_{\text{probe}}] + \sum_n \left(C_n \rho_{\text{probe}} C_n^\dagger - \frac{1}{2} \{C_n^\dagger C_n, \rho_{\text{probe}}\} \right)$$

- H_{probe} is Hermitian (includes singlet and octet states)
- C_n are the **collapse (or jump) operators** (connect different internal states)
- Partial and **total decay width operators** are

$$\Gamma_n = C_n^\dagger C_n \quad \Gamma = \sum_n \Gamma_n$$

- Can reorganize Lindblad equation by defining

$$H_{\text{eff}} = H_{\text{probe}} - \frac{i}{2} \Gamma$$

← **Non-Hermitian effective Hamiltonian**

$$\longrightarrow \frac{d\rho_{\text{probe}}}{dt} = -iH_{\text{eff}}\rho_{\text{probe}} + i\rho_{\text{probe}}H_{\text{eff}}^\dagger + \sum_n C_n \rho_{\text{probe}} C_n^\dagger$$

LO OQS + pNRQCD Hamiltonian and collapse operators

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

$$\frac{d\rho_{\text{probe}}}{dt} = -iH_{\text{eff}}\rho_{\text{probe}} + i\rho_{\text{probe}}H_{\text{eff}}^\dagger + \sum_n C_n \rho_{\text{probe}} C_n^\dagger$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H_{\text{probe}} = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2} \gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

mass shift

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}} r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}} r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\Gamma = \kappa r^i \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix} r^i$$

Total width $\rightarrow \text{Im}[V]$

$$H_{\text{eff}} = H_{\text{probe}} - \frac{i}{2}\Gamma$$

Six collapse operators cover

- singlet $\rightarrow$ octet,
- octet $\rightarrow$ singlet
- octet $\rightarrow$ octet

$$\gamma \equiv \frac{g^2}{6 N_c} \text{Im} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, \mathbf{0}) E^{a,i}(0, \mathbf{0}) \rangle$$

$$\kappa \equiv \frac{g^2}{6 N_c} \text{Re} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, \mathbf{0}) E^{a,i}(0, \mathbf{0}) \rangle$$

LO OQS + pNRQCD Hamiltonian and collapse operators

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$$\frac{d\rho_{\text{probe}}}{dt} = -iH_{\text{eff}}\rho_{\text{probe}} + i\rho_{\text{probe}}H_{\text{eff}}^\dagger + \sum_n C_n \rho_{\text{probe}} C_n^\dagger$$

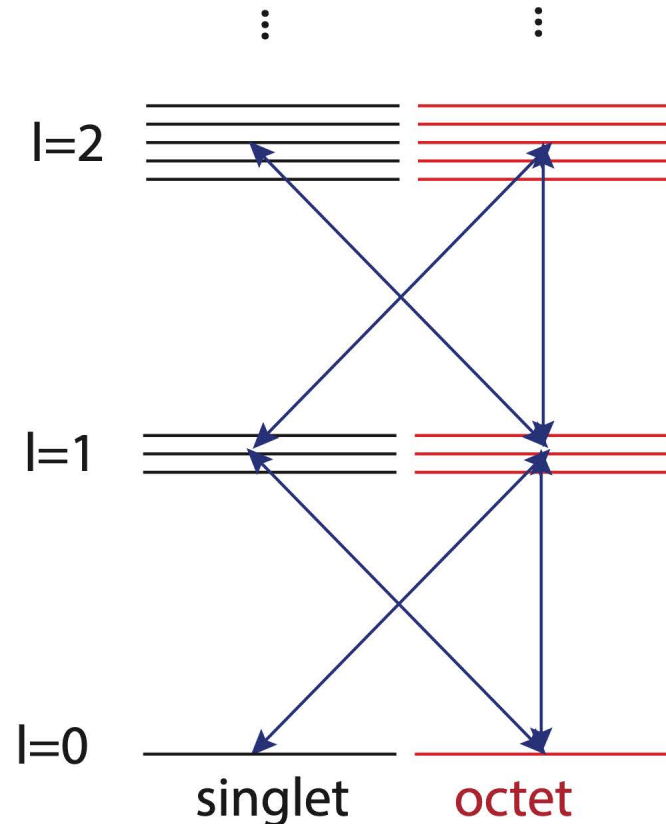
$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2 - 1}} r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2 - 1} & 0 \end{pmatrix},$$

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Six collapse operators cover

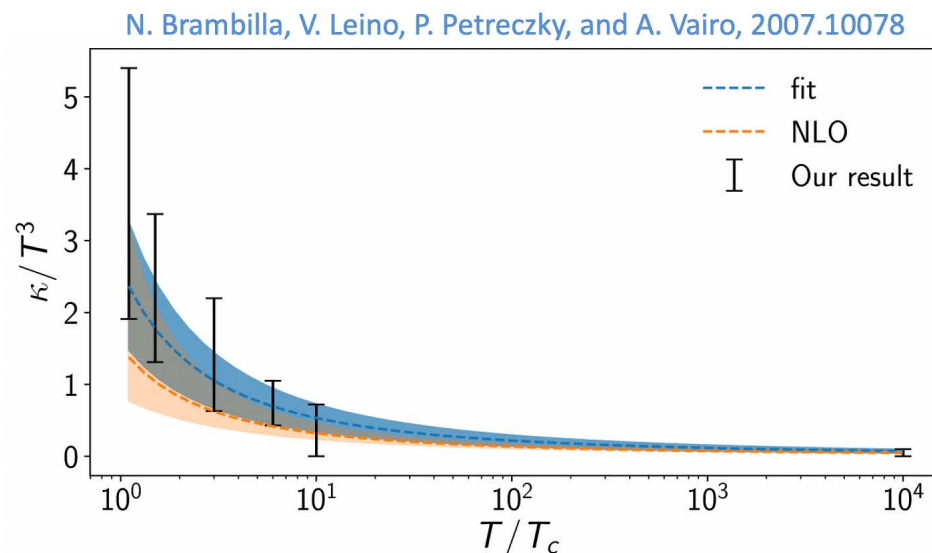
- singlet $\rightarrow$ octet,
- octet $\rightarrow$ singlet
- octet $\rightarrow$ octet



Values of $\hat{\kappa}$ and $\hat{\gamma}$ used

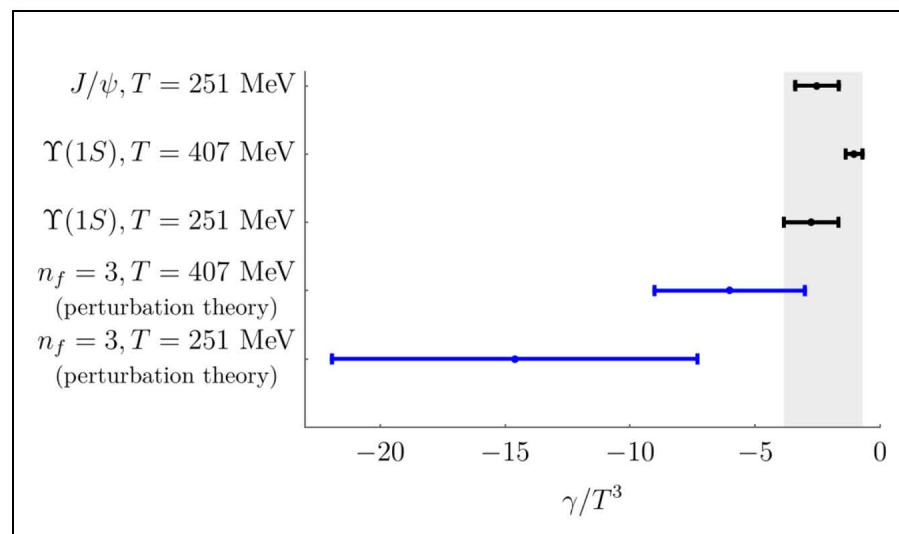
- We used NLO fits to recent lattice measurements of the heavy quark transport coefficient $\hat{\kappa} \equiv \kappa/T^3$. Note that this is related to the heavy quark diffusion constant D .

- N. Brambilla, V. Leino, P. Petreczky, and A. Vairo, 2007.10078



- The value of $\hat{\gamma} \equiv \gamma/T^3$ is less constrained, we vary it in the range $-3.5 < \hat{\gamma} < 0$.

- N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248.
- N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1711.04515.
- N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, 1903.08063.



N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, 1903.08063.

A parallelizable approach: Quantum trajectories

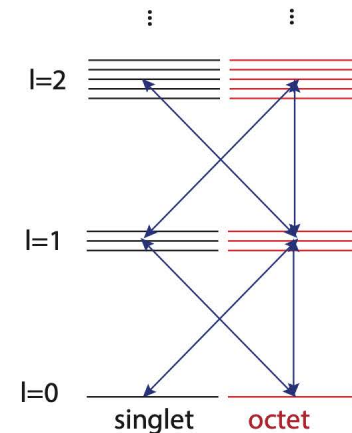
N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, 2012.01240

$$\frac{d\rho_{\text{probe}}}{dt} = -iH_{\text{eff}}\rho_{\text{probe}} + i\rho_{\text{probe}}H_{\text{eff}}^{\dagger} + \sum_n C_n \rho_{\text{probe}} C_n^{\dagger}$$

Non-unitary “no jump” evolution

Can treat this “quantum jump” term stochastically

- Can be reduced to the solution of a large set of “quantum trajectories” in which we solve a 1D Schrödinger equation with a **non-Hermitian Hamiltonian** H_{eff} , subject to **stochastic quantum jumps**.
- The evolution with the non-Hermitian H_{eff} preserves the color and angular momentum state of the system (but not norm).
- Collapse/jump operators encode transitions between different color/angular momentum states (subject to selection rules).
- For each **physical trajectory** (path through the QGP) we average over a large set of **independent quantum trajectories** → **Embarrassingly parallel**
- **Added benefit: Can describe all angular momentum states (no cutoff) .**



Going beyond leading order in E/T

N. Brambilla, M.-A. Escobedo, A. Islam, M.S., A. Tiwari, A. Vairo, and P. Vander Griend, 2205.10289

- Results on previous slides were obtained by truncating at leading order (LO) in E/T (binding energy over temperature). This was extended to NLO in the reference above.

- By decomposing states into radial and angular components, for an isotropic system we can rewrite the operators as acting only on the 1d effective wavefunction

$$u(r,t) = r R(r,t).$$

- At NLO, there are six jump operators (shown on the right).
- Based on these, one can write down the singlet and octet non-Hermitian effective Hamiltonian (see reference above for details).

$$C_{s \rightarrow o}^{\uparrow} = r - \frac{N_c \alpha_s}{8T} + \frac{1}{2MT} \left(\frac{\partial}{\partial r} - \frac{l+1}{r} \right),$$

$$C_{s \rightarrow o}^{\downarrow} = r - \frac{N_c \alpha_s}{8T} + \frac{1}{2MT} \left(\frac{\partial}{\partial r} + \frac{l}{r} \right),$$

$$C_{o \rightarrow s}^{\uparrow} = r + \frac{N_c \alpha_s}{8T} + \frac{1}{2MT} \left(\frac{\partial}{\partial r} - \frac{l+1}{r} \right),$$

$$C_{o \rightarrow s}^{\downarrow} = r + \frac{N_c \alpha_s}{8T} + \frac{1}{2MT} \left(\frac{\partial}{\partial r} + \frac{l}{r} \right),$$

$$C_{o \rightarrow o}^{\uparrow} = r + \frac{1}{2MT} \left(\frac{\partial}{\partial r} - \frac{l+1}{r} \right),$$

$$C_{o \rightarrow o}^{\downarrow} = r + \frac{1}{2MT} \left(\frac{\partial}{\partial r} + \frac{l}{r} \right).$$

Full NLO effective Hamiltonian

The effective Hamiltonian for singlet and octet evolution is defined by $H_{s,o}^{\text{eff}} = h_{s,o} + \text{Im}(\Sigma_{s,o}) - i\Gamma_{s,o}/2$ with $\Gamma_s = \sum_{i \in \{\uparrow, \downarrow\}} \Gamma_{s \rightarrow o}^i$ and $\Gamma_o = \sum_{i \in \{\uparrow, \downarrow\}} (\Gamma_{o \rightarrow s}^i + \Gamma_{o \rightarrow o}^i)$. When expressed as operators acting on the reduced wave function, the singlet effective Hamiltonian $\overline{H}_s^{\text{eff}}$ is given by

$$\text{Re}[\overline{H}_s^{\text{eff}}] = \frac{\overline{\mathcal{D}}^2}{M} - \frac{C_f \alpha_s}{r} + \frac{\hat{\gamma} T^3}{2} r^2 + \frac{\hat{\kappa} T^2}{4M} \{r, p_r\}, \quad (3.12)$$

$$\text{Im}[\overline{H}_s^{\text{eff}}] = -\frac{\hat{\kappa} T^3}{2} \left[\left(r - \frac{N_c \alpha_s}{8T} \right)^2 - \frac{3}{2MT} + \frac{\overline{\mathcal{D}}^2}{(2MT)^2} + \frac{1}{2MT} \left(\frac{N_c \alpha_s}{4T} \right) \frac{1}{r} \right], \quad (3.13)$$

where $p_r = -i\partial_r$. Similarly, the octet effective Hamiltonian $\overline{H}_o^{\text{eff}}$ is given by

$$\text{Re}[\overline{H}_o^{\text{eff}}] = \frac{\overline{\mathcal{D}}^2}{M} + \frac{1}{2N_c} \frac{\alpha_s}{r} + \frac{N_c^2 - 2}{2(N_c^2 - 1)} \left[\frac{\hat{\gamma} T^3}{2} r^2 + \frac{\hat{\kappa} T^2}{4M} \{r, p_r\} \right], \quad (3.14)$$

$$\begin{aligned} \text{Im}[\overline{H}_o^{\text{eff}}] = & -\frac{\hat{\kappa} T^3}{2(N_c^2 - 1)} \left[\left(r + \frac{N_c \alpha_s}{8T} \right)^2 - \frac{3}{2MT} + \frac{\overline{\mathcal{D}}^2}{(2MT)^2} - \frac{1}{2MT} \left(\frac{N_c \alpha_s}{4T} \right) \frac{1}{r} \right] \\ & - \frac{\hat{\kappa} T^3}{4(N_c^2 - 1)} \left[r^2 - \frac{3}{2MT} + \frac{\overline{\mathcal{D}}^2}{(2MT)^2} \right], \end{aligned} \quad (3.15)$$

where $\hat{\kappa} = \kappa/T^3$ and $\hat{\gamma} = \gamma/T^3$.

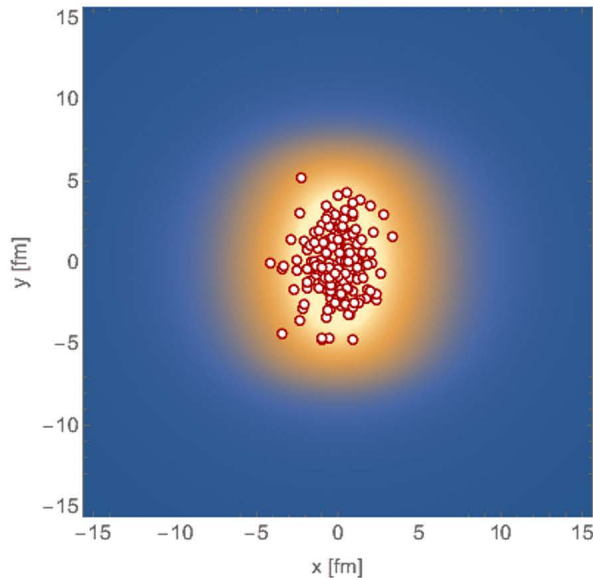
$$\overline{\mathcal{D}}^2 = -\frac{\partial^2}{\partial r^2} + \frac{l(l+1)}{r^2}$$

How can one numerically solve these equations?

$$\frac{d\rho_{\text{probe}}}{dt} = -iH_{\text{eff}}\rho_{\text{probe}} + i\rho_{\text{probe}}H_{\text{eff}}^{\dagger} + \sum_n C_n \rho_{\text{probe}} C_n^{\dagger}$$

- Each block of the density matrix in color space can be decomposed into orbital angular momentum blockwise.
- Upon truncating in angular momentum ($l \leq l_{\text{max}}$) one can reduce both the singlet and octet blocks of the reduced density matrix to size $(l_{\text{max}} + 1)^2$.
- One can then discretize the radial wavefunction ($N = \#$ of lattice points) and evolve the reduced density matrix using standard differential equation and matrix solvers gives $\sim N^2(l_{\text{max}} + 1)^2$ matrix size.
- **Need to describe bound and unbound states with highly localized initial wave function, so the box must be large and have small lattice spacing $\rightarrow$ large N and large l_{max} .**
- As N and l_{max} become large, the computation becomes very challenging.
- **Need a better/faster method which we can easily parallelize.**

Computing survival probabilities with QTraj



Survival probability

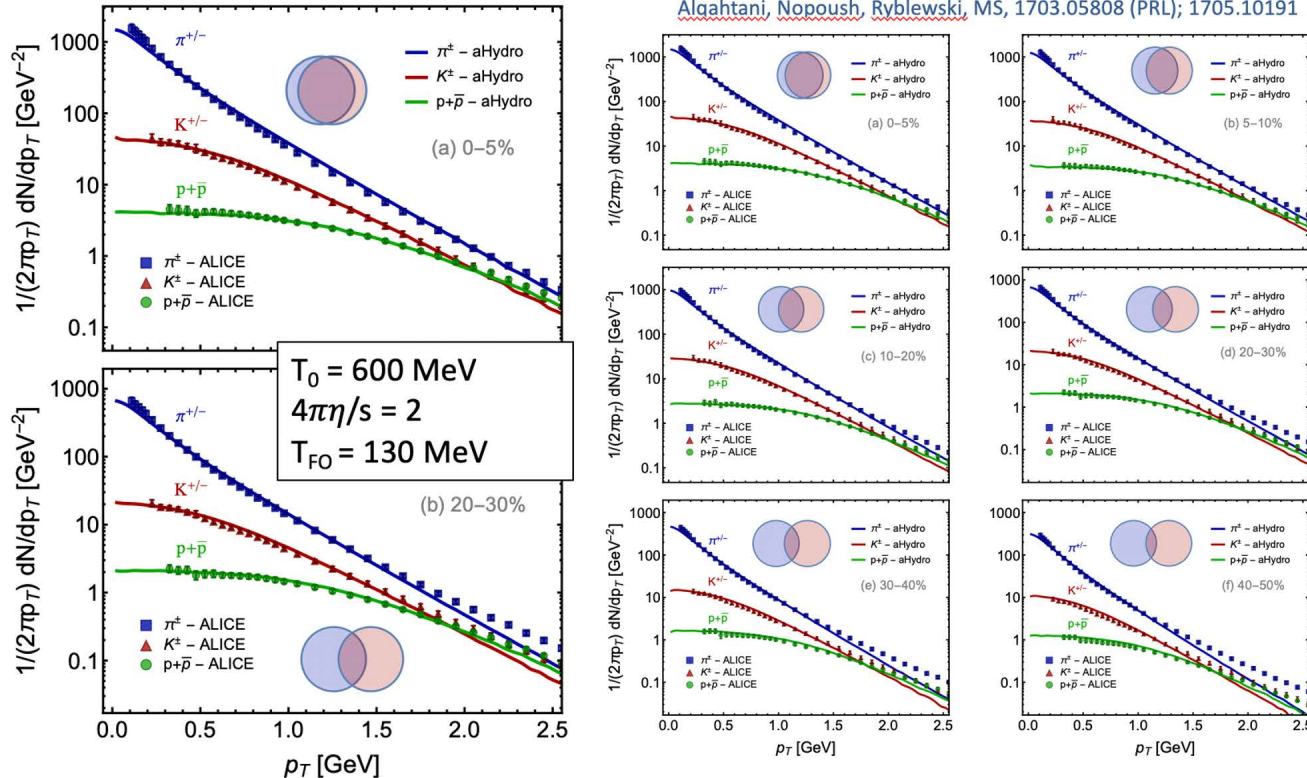
$$SP(n, l) = \frac{|\langle n, l | \psi(t_f) \rangle|^2}{|\langle n, l | \psi(t_0) \rangle|^2}$$

- Used $N = 4096$ points
- $L = 108 a_0$
- $\Delta t = 2 \times 10^{-4}$ fm

- We sampled bottomonium production points and initial COM transverse momentum using **Monte-Carlo sampling** (physical trajectories).
- Temperature evolution provided by **3+1D anisotropic hydrodynamics** (good description of identified soft hadron spectra and anisotropic flow, see backup slides for evidence).
- Along each physical trajectory, we solved the **real-time 3D Schrödinger equation with a complex potential and stochastically sampled jumps** → Lindblad equation.
- We then solved for the **survival probability** of S- and P-wave states (see box to the left).

3+1D hydrodynamical background

Identified particle spectra



M. Strickland

Data are from the ALICE collaboration data for **Pb-Pb collisions @ 2.76 TeV/nucleon**

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For 5.02 TeV, $T_0 = 630 \, \text{MeV}$ @ $t_0 = 0.25 \, \text{fm}/c$

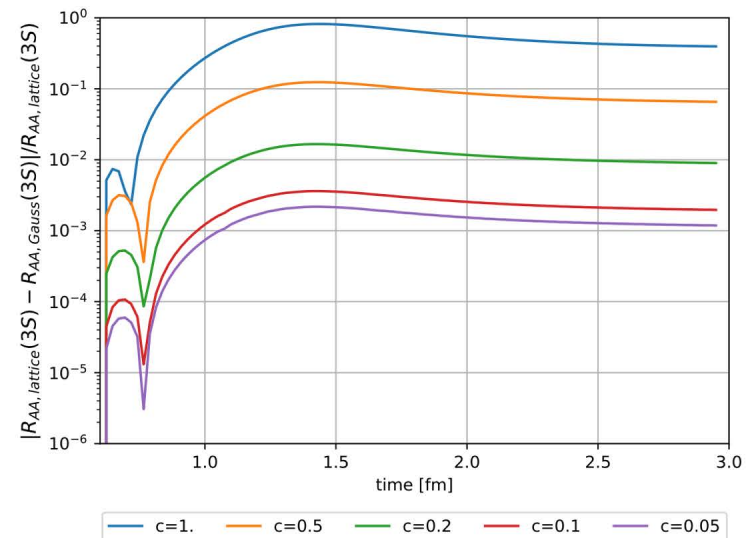
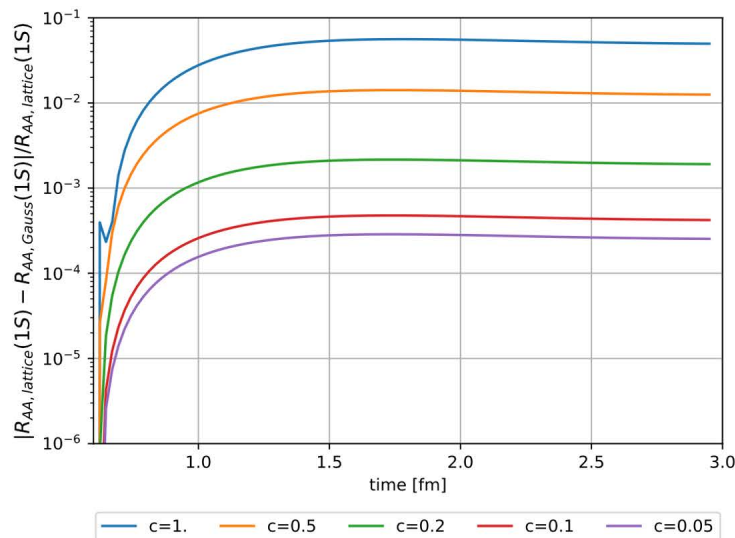
- We use a 3+1D dissipative code for the hydro background (**quasiparticle anisotropic hydrodynamics**)
- Has been tuned to RHIC and LHC heavy ion collisions
- Reproduces spectra, multiplicities, charged and identified elliptic flow of light hadrons, HBT radii, etc.

Initial bottomonium wavefunction

- We took the initial wavefunction to be given by a smeared delta function (local production due to large mass, $\Delta \sim 1/M$) of the form

$$u_\ell(r, \tau = 0) \propto r^{\ell+1} \exp(-r^2/\Delta^2)$$

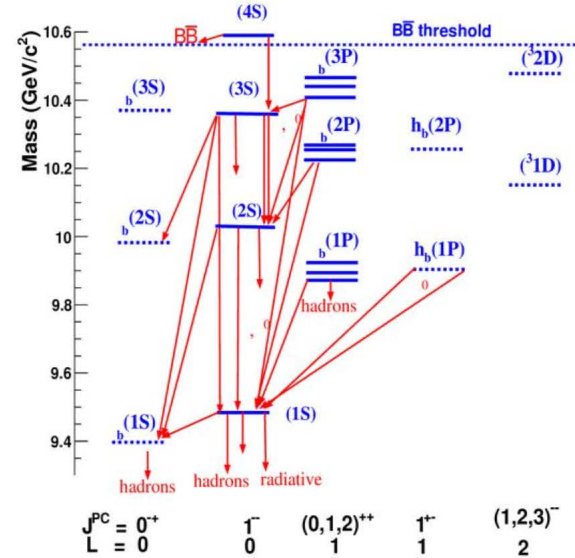
- For a given l , the **initial state is a quantum linear superposition** of the eigenstates of H.
- Includes both bound and unbound states.**
- We took $\Delta = 0.2 a_0$ which reproduces results obtained with a true delta to within 1%.



Feed-down implementation

$$\vec{N}_{\text{observed}} = F \vec{N}_{\text{direct}}$$

$$F = \begin{pmatrix} 1 & 0.2645 & 0.0194 & 0.352 & 0.18 & 0.0657 & 0.0038 & 0.1153 & 0.077 \\ 0 & 1 & 0 & 0 & 0 & 0.106 & 0.0138 & 0.181 & 0.089 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0091 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0051 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

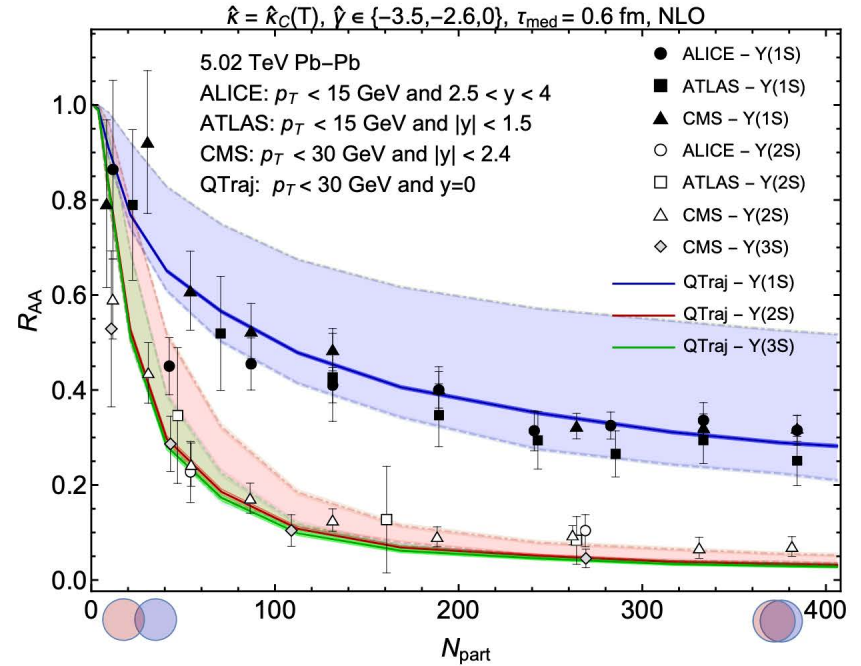
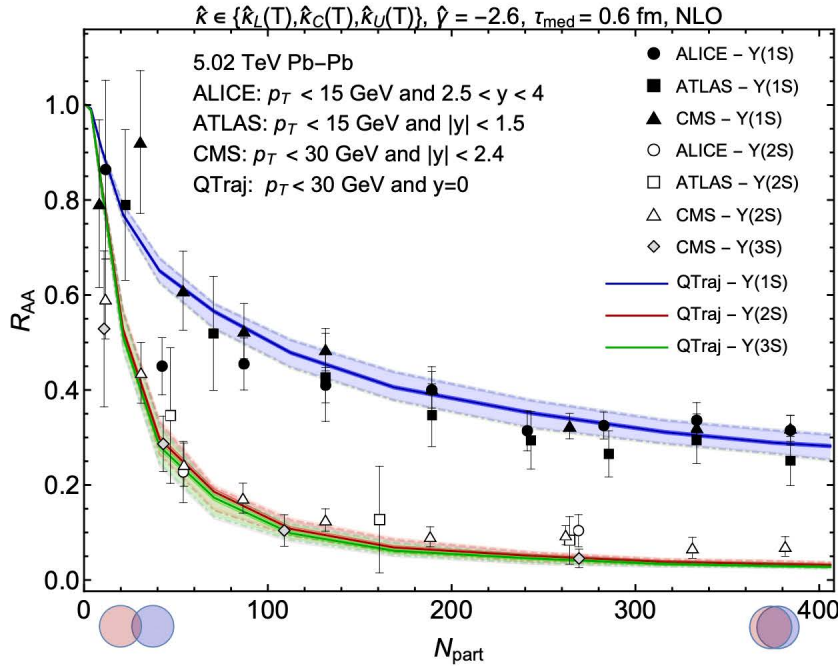


- N_{direct} corresponds to $(N_{1S}, N_{2S}, N_{1P} \times 3, N_{3S}, N_{2P} \times 3, N_{2D})^T$ where, e.g., N_{1S} is the final number of $Y(1S)$ states that can decay in the dilepton channel.
- N_{direct} can be obtained using $\langle N_{\text{bin}}(b) \rangle * \sigma_{\text{direct}} * (\text{Survival probability})$
- After feed down, we then normalize by the pp collision result scaled to AA $\rightarrow R_{AA}$.

$$R_{AA}^i(c) = \frac{(F \cdot S(c) \cdot \vec{\sigma}_{\text{direct}})^i}{\sigma_{\text{exp}}^i}$$

NLO OQS + pNRQCD predictions for R_{AA} vs N_{part} at LHC

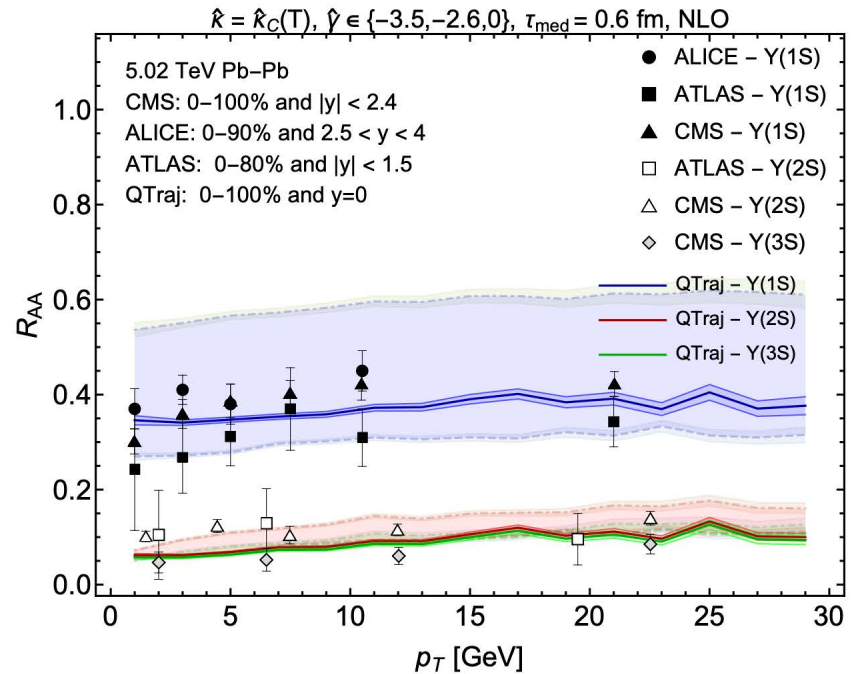
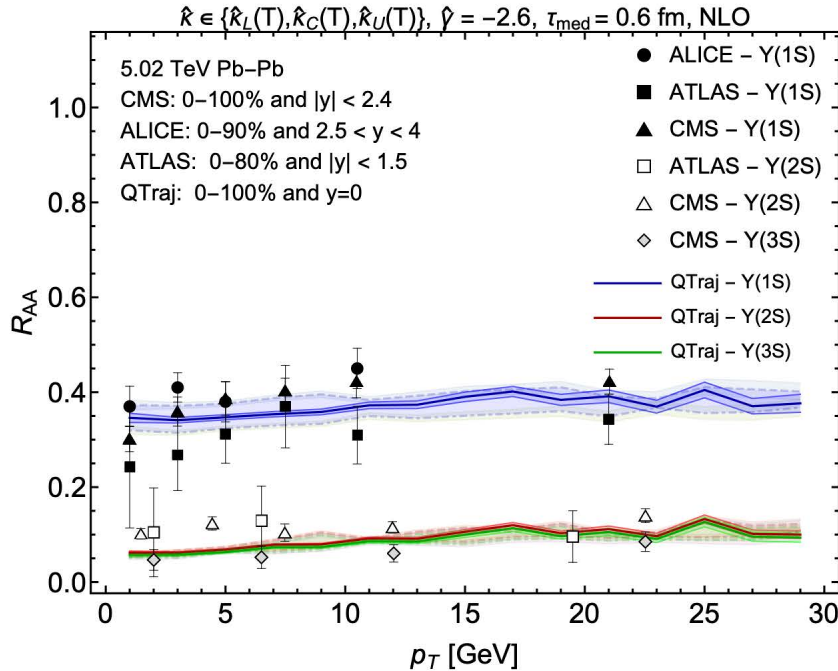
N. Brambilla, M.-A. Escobedo, A. Islam, M.S., A. Tiwari, A. Vairo, P. Vander Griend, 2205.10289



- **Left panel:** Result including feed down, when varying $\hat{K}$ over the theoretical uncertainty.
- **Right panel:** Result including feed down, when varying $\hat{Y}$ over the theoretical uncertainty
- **Note:** The NLO results above ignore jumps (H_{eff} only). The effect of jumps is expected to be small based on prior studies, but requires lots of computer time (forthcoming).
- The statistical uncertainty associated with the average over physical trajectories is on the order of the line width.

NLO R_{AA} vs transverse momentum

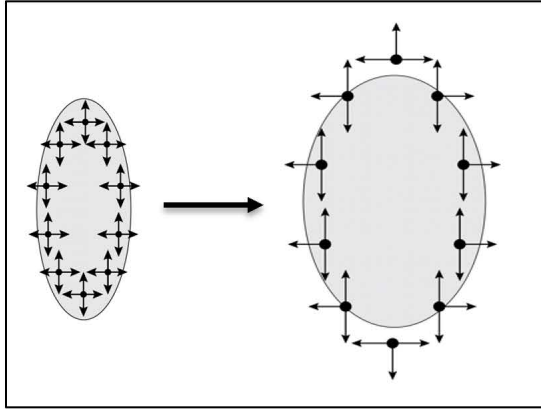
N. Brambilla, M.-A. Escobedo, A. Islam, M.S., A. Tiwari, A. Vairo, P. Vander Griend, 2205.10289



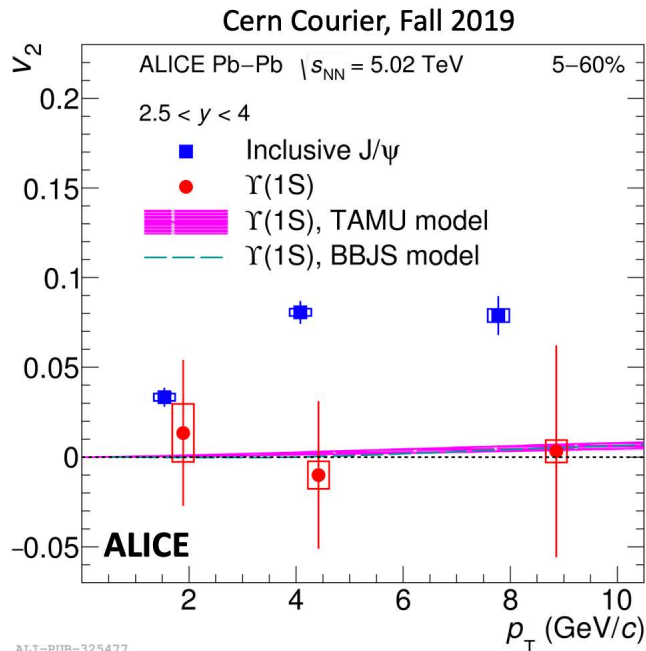
- QTraj predictions consistent with experimental observations.
- Very flat. Small decrease comes from longer time spent in the QGP as the velocity decreases.
- Once again, larger variation from uncertainty in $\hat{y}$.

Momentum-space anisotropies

4d flow tomography

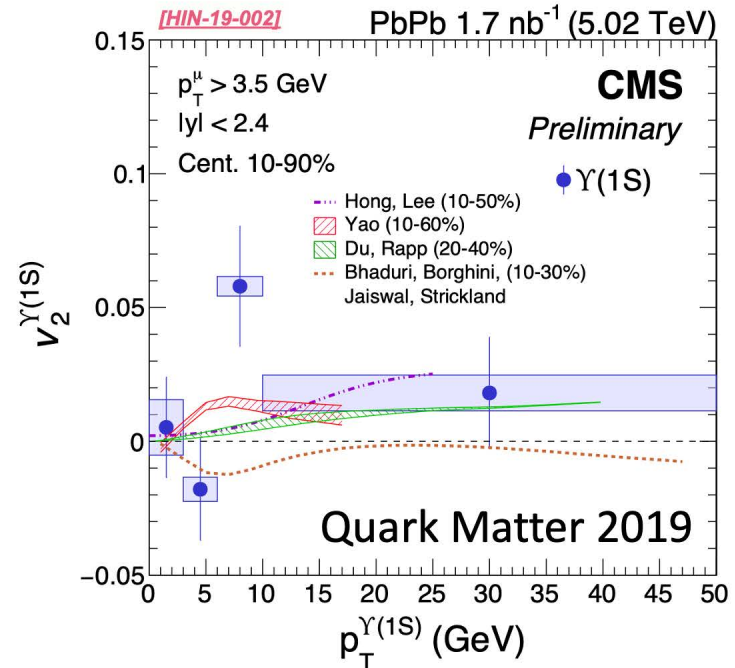


- Bottomonium probably doesn't flow in the "collective flow" sense.
- However, there can be momentum-space anisotropies induced by path-length differences in suppression along the short and long sides of the QGP.



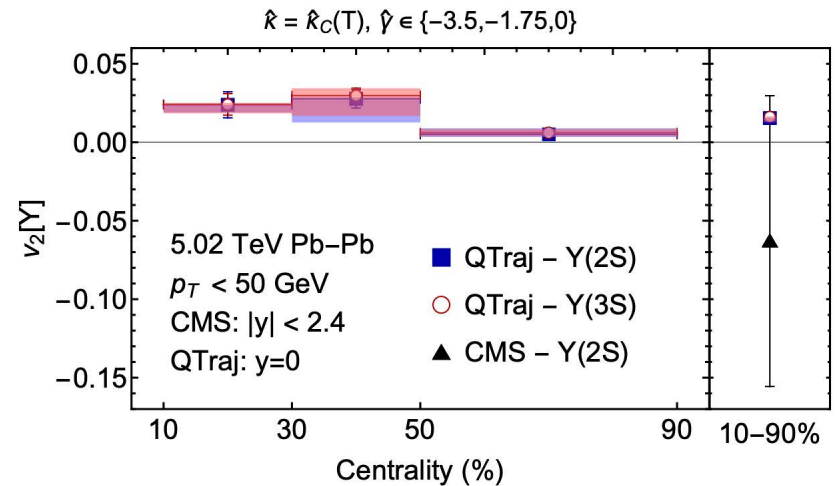
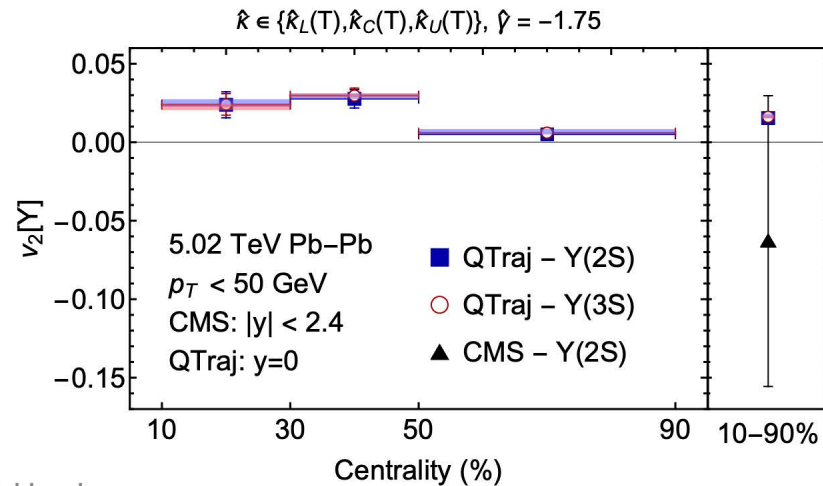
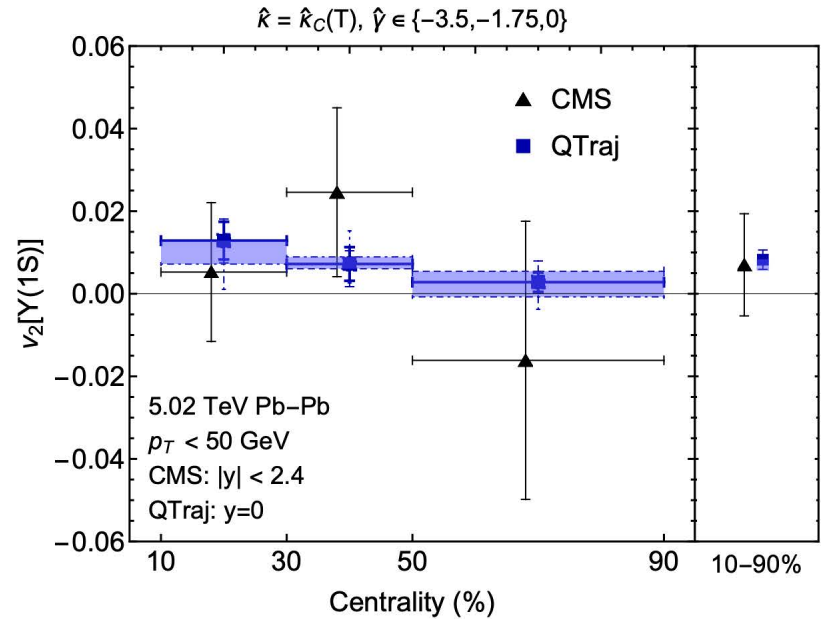
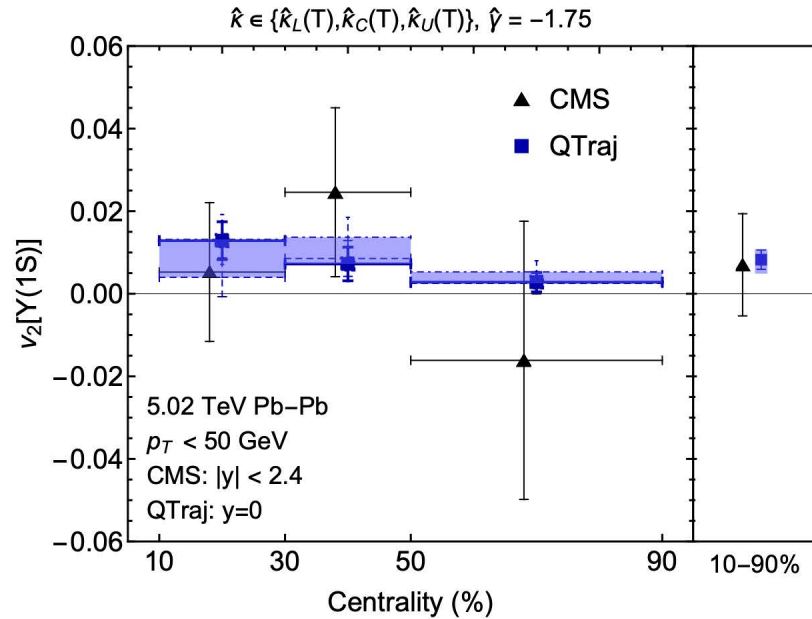
TAMU: Phys. Rev. C 96, (2017) 054901
BBJS: arXiv:1809.06235

ALICE Collaboration: arXiv:1907.03169



LO Momentum-space anisotropies at LHC

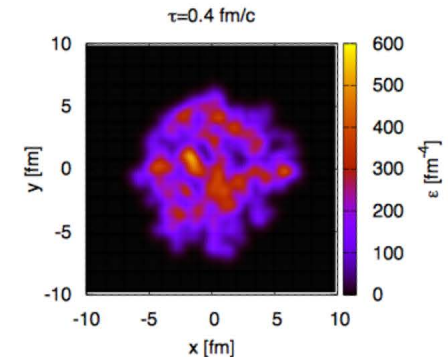
N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, 2107.06222 (Leading-Order)



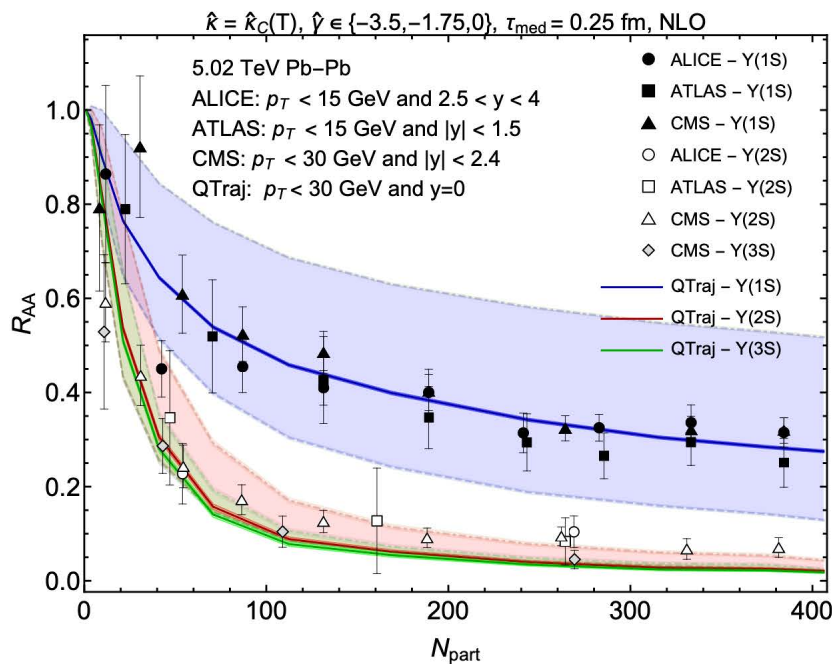
The effect of fluctuating initial conditions

H. Alalawi, J. Boyd, and M. Strickland, forthcoming

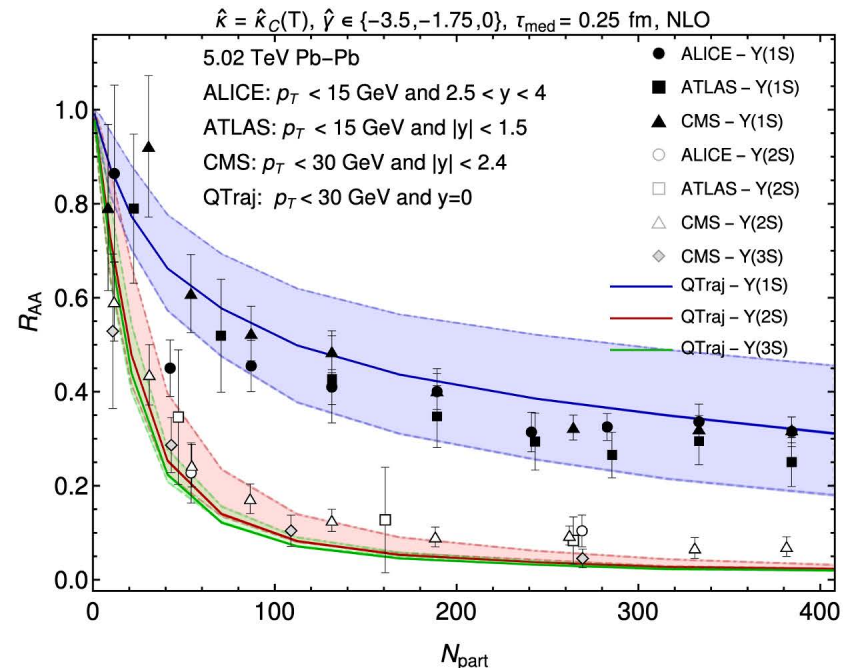
- Can also study the effect of fluctuating IC hydrodynamics.
- Using MUSIC + IPGlasma tuned to 5.02 TeV Pb-Pb collisions.
- Agrees well with aHydro results for R_{AA}



NLO + Glauber

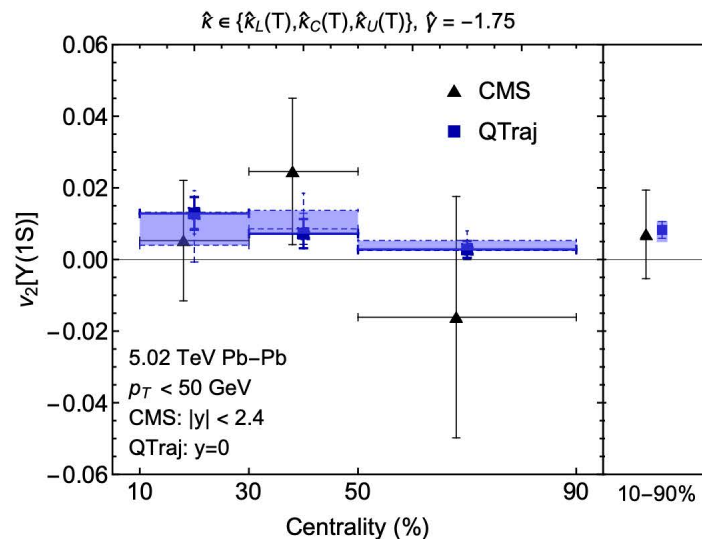
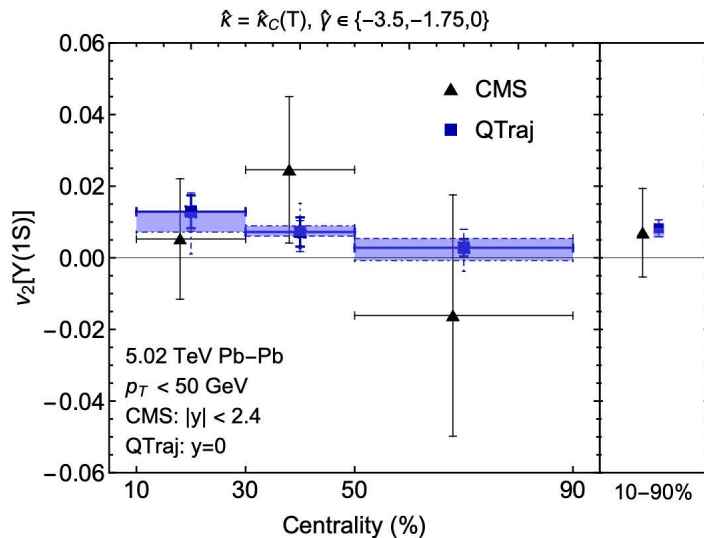


NLO + IPGlasma fluctuating ICs

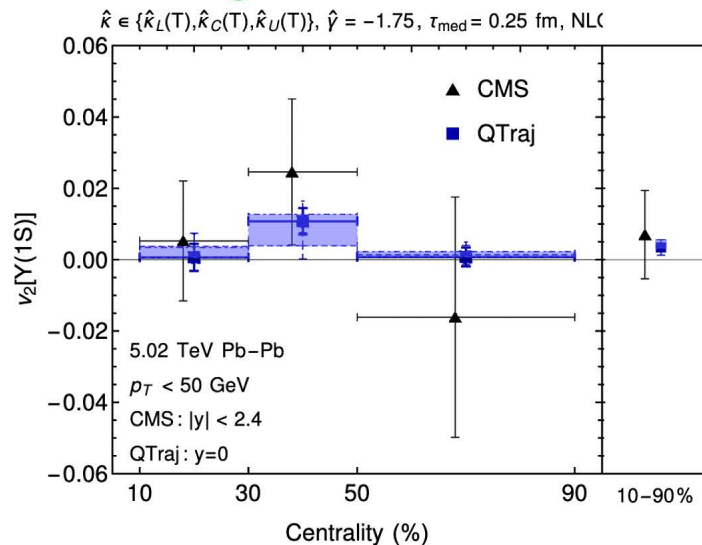
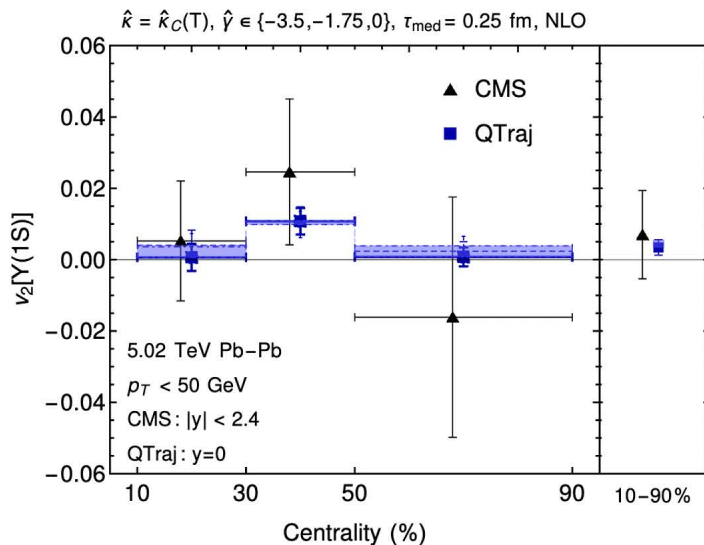


v_2 with/without fluctuating ICs

LO + Glauber



NLO + IPGlasma fluctuating ICs



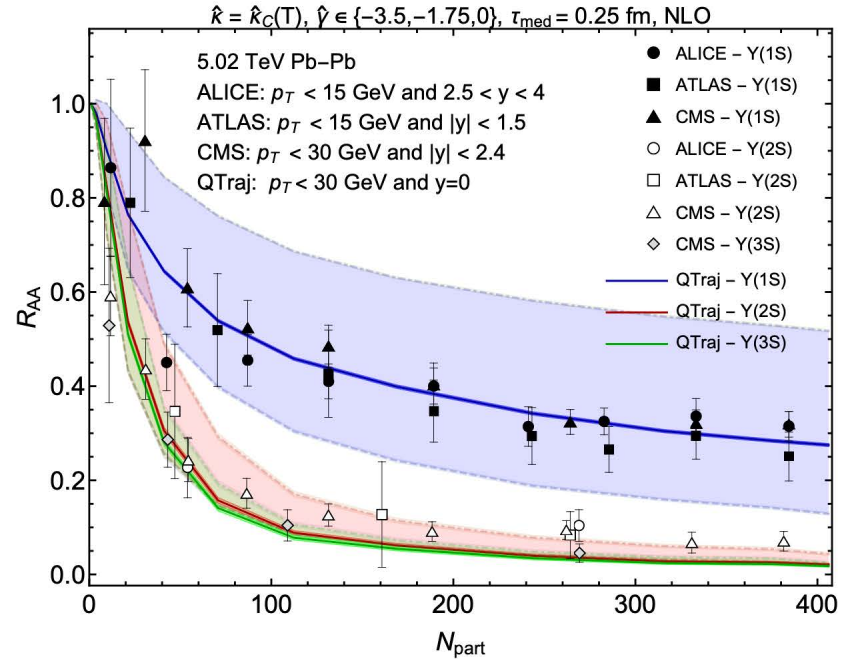
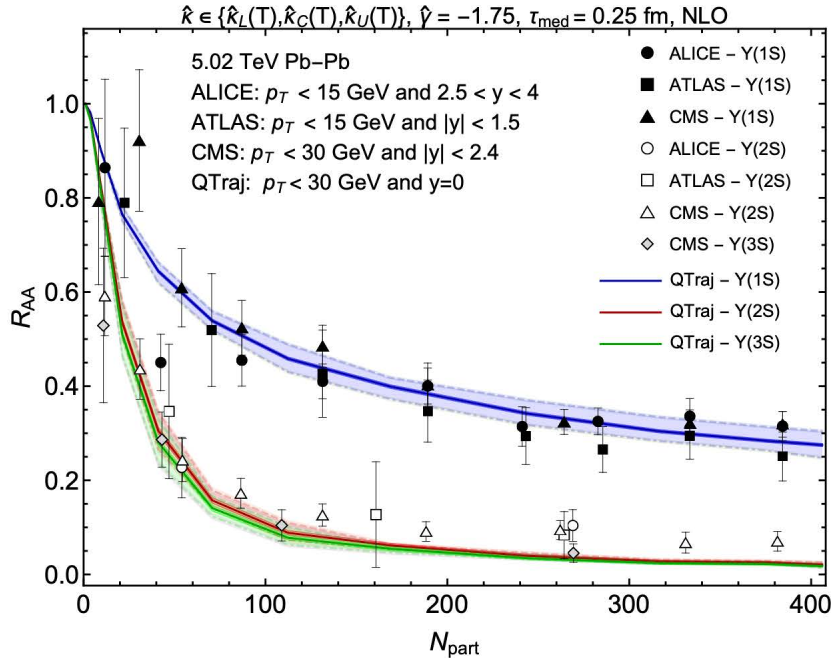
Conclusions and Outlook

- **First 3D quantum non-abelian treatment of heavy quarkonium within the OQS+pNRQCD framework.**
- Phenomenological treatment has been extended to NLO in E/T.
- Transport coefficients used were **constrained by independent lattice measurements.**
- **OQS + pNRQCD works quite well in describing bottomonium suppression and “flow” vs N_{part} and p_T seen at LHC energies.**
- Demonstrated that Upsilon[nS] R_{AA} (and their double ratios) can provide **experimental constraints on these transport coefficients.**
- The **quantum trajectory algorithm** allows us to include effect of “**quantum jumps**” between color and angular momentum states in a **computationally scalable manner**. Code (Qtraj) has been released as an open-source package.
- Could be useful in other simulations of open quantum systems evolution, such as the evolution of dark matter in the early universe.

Backup slides

NLO OQS + pNRQCD predictions for R_{AA} vs N_{part} at LHC

N. Brambilla, M.-A. Escobedo, A. Islam, M.S., A. Tiwari, A. Vairo, P. Vander Griend, 2205.10289



- Note that It is possible to use a shorter medium initialization time and obtain a similar description of the data.
- Above shows results using $\tau_{med} = 0.25 \text{ fm}$ rather than 0.6 fm
- In this case, I changed the central value of gamma plotted, but the range of variation is the same.