

# Subleading Power Discussion

Moderator: Iain Stewart

# Topics

- Lattice Limits: when does Euclidean (LaMET) work (A.V's talk)
- Choice of Basis for subleading power operators (L.G.'s talk)
- Factorization ok?: Glauber interactions @ Subleading Power
- Endpoint Divergences@ Subl.Power, impede factorization?
- Resummation: form of RGE at subleading power

# “Factorization Violation”

My Definition: The expected form for a factorization formula is invalid.

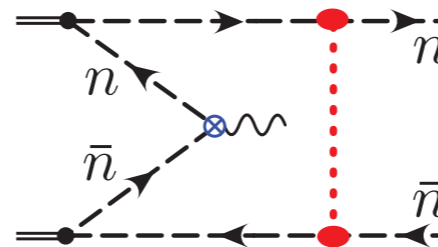
## Reasons Factorization can fail:

- **Measurement doesn't factor:** no simple factorization with universal functions. (eg. Jade jet algorithm)
- **Divergent convolutions,** not controlled by ones regulation procedures. (Requires more careful construction.)

$$\int_0^1 \frac{dx}{x^2} \phi_\pi(x, \mu)$$

- Interactions that couple other modes and spoil factorization.

**Glauber exchange**



spectator-spectator **CSS**  
cancel in proof for Drell-Yan

- **Collinear Wilson Line universality fails.**  
examples studied by Collins, Qiu, Aybat, Rogers, ...

$H_3, H_4 \simeq$  back-to-back  
 $H_1 + H_2 \rightarrow H_3 + H_4 + X$   
pT dependent

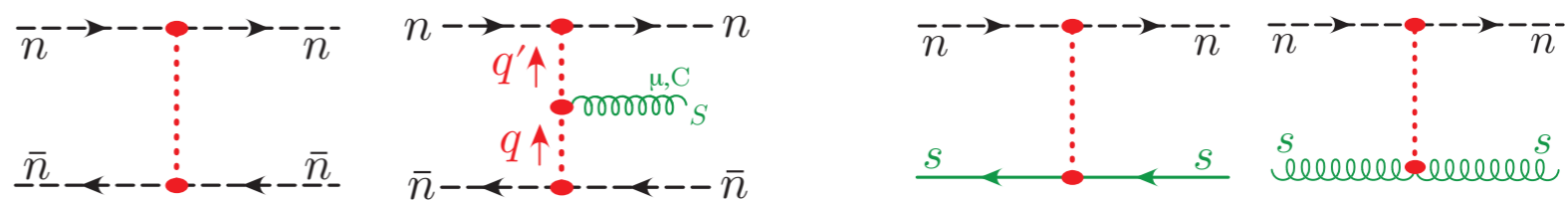
- Factorization ok?: Glauber interactions @ Subleading Power

easy to state precisely what is required in SCET

$$\begin{aligned}
 \mathcal{L}_{\text{SCET II}} = & \mathcal{L}_s^{(0)}(\psi_s, A_s) + \sum_n \mathcal{L}_n^{(0)}(\xi_n, A_n) + \mathcal{L}_{\text{hard}}(\text{field products}) \\
 & + \mathcal{L}_G^{(0)}(\psi_s, A_s, \xi_{n_i}, A_{n_i}) \\
 & + \sum_j \mathcal{L}_{\text{subl.power}}^{(j)}(\text{field products})
 \end{aligned}$$

only  $\mathcal{L}_G^{(0)}$  can break factorization, even at subleading power  
 question is simply how it interferes with subleading power operators

$$\mathcal{L}_G^{\text{II}(0)} = \sum_{n, \bar{n}} \sum_{i, j=q, g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_n \sum_{i, j=q, g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$



...

...

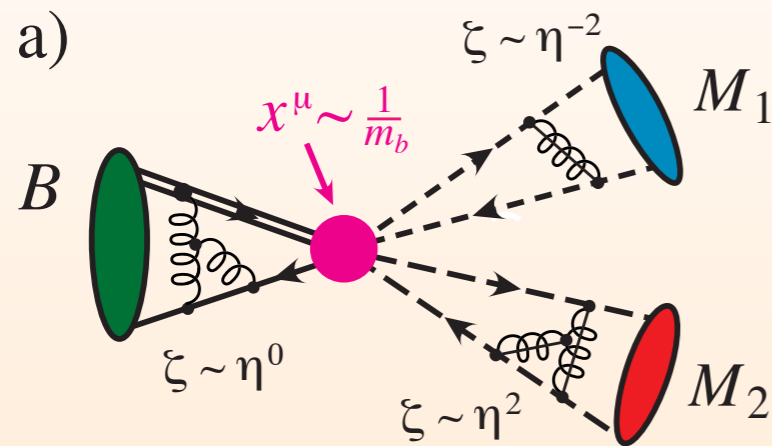
- Endpoint Divergences @ Subl.Power, do they impede factorization?

SCET literature:

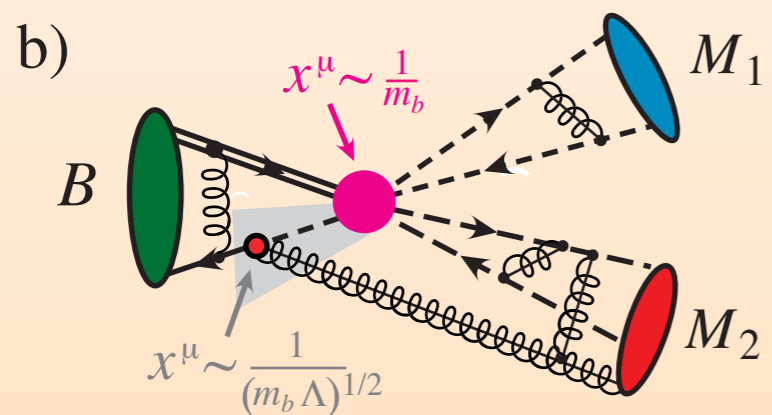
$$\int_0^1 \frac{dx}{x} \frac{\phi(x)}{x} \sim \int_0^1 \frac{dx}{x} = ?$$

- $B \rightarrow \pi \ell \nu, B \rightarrow \pi \pi$  are subleading power (we must turn convert a spectator quark from soft to collinear) & have endpoint singularities. These processes involve sums of contributions, including terms with both soft & collinear components.  
[Bauer, Pirjol, Stewart; Beneke, Feldmann; Neubert et.al. (~2001) ... ]
- Zero-bin subtractions avoid double counting between collinear and soft regions at subleading power  
[Manohar & Stewart hep-ph/0605001 ]
- Rapidity regulators; and how they work at subleading power  
[Chiu, Jain, Neill, Rothstein (2012); ..., ]
- eg. Annihilation channel for  $B \rightarrow \pi \pi$  and soft-collinear overlaps  
[Arnesen, Ligeti, Rothstein, & I.S. (2006) ]

# Annihilation is real at lowest order in $\alpha_s$ expansion

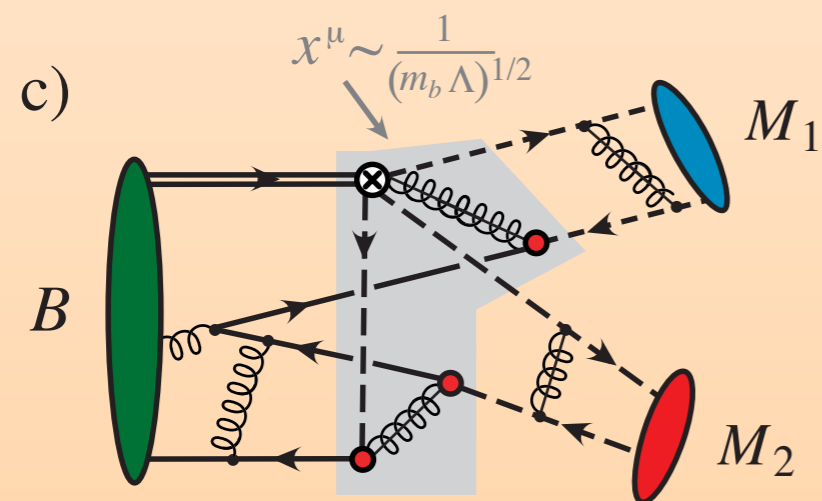


Suffers from endpoint divergences.  
But they do not introduce a phase.



Leading order, its same size as a).  
No endpoint divergences here.  
No imaginary part here either.

$$\propto \phi_B^+(k) \phi_{M_1}(y) \phi_{3M_2}(x_1, x_2) \quad \text{Arnesen et al.} \\ \text{hep-ph/0611356}$$



A soft rescattering annihilation  
contribution DOES have a strong  
phase, but is one higher order in  $\alpha_s$

**However:**

Might be large if intermediate scale  $\alpha_s$   
expansion did not converge here

Solution to endpoint divergence problem has been worked out explicitly in SCET for several (simpler) examples

see talk by Julian Strohm at SCET 2022 for a review

Factorization Theorems for processes with endpoint divergences

$$B \rightarrow \chi_{cJ} K \text{ and } h \rightarrow \gamma\gamma$$

[Beneke, Vernazza; 0810.3575]

[Liu, Mecaj, Neubert, Wang; 1912.08818, 2009.04456, 2009.06779]

## NLP Endpoint Factorization and Resummation of Off-Diagonal "Gluon" Thrust

Simplest example to look at:  
Julian Strohm SCET 2022

Joint Work with Martin Beneke, Mathias Garry, Sebastian Jaskiewicz, Robert Szafron, Leonardo Vernazza, and Jian Wang.

[arXiv:2207.14199](https://arxiv.org/abs/2207.14199)

For off-diagonal channels, the leading logarithms already exhibit non-trivial structure in contrast to diagonal channels.

[Moult, Stewart, Vita, Zhu; 1804.04665][Beneke et al.; 1809.10631]

 we referred to this as the "Soft Quark Sudakov"

- Resummation: form of RGE at subleading power

Known for many operators in SCET

[Beneke et.al. (2004); Neubert et.al. (2004), ...,  
Beneke et.al. (2017, 2018); Vladimirov, Moos, Scimemi (2021) ]

Factorization theorems are sums of factorized terms, and RGEs in general are not multiplicative, but can mix into new operators eg. thrust, threshold resummation  $\delta(k - \dots) \rightarrow \theta(k - \dots)$

[Moult, I.S., Vita, Zhu (arXiv:1804.04665), Beneke et.al. (arXiv:2205.04479), ... ]

In general at subleading power we get a combination of cusp anomalous dimension terms (dble. logs), convolutions from the hard region (DGLAP), and convolutions from the soft region

Harder to solve in general