Hydrodynamics of fluctuations

Stochastic

Random hydro variables: $\breve{\psi}$

$$\partial_t \breve{\psi} = -\nabla \cdot \Big(\text{Flux}[\breve{\psi}] + \text{ Noise} \Big)$$

$+$ fewer variables and eqs.

$-$ cutoff dependence

Deterministic

$\psi \equiv \langle \breve{\psi} \rangle$, $G \equiv \langle \breve{\psi}\breve{\psi} \rangle$, etc.

$$\partial_t \psi = -\nabla \cdot \text{Flux}[\psi, G];$$

$$\partial_t G = \mathsf{L}[G; \psi].$$

$-$ more variables and eqs.

$+$ no cutoff dependence (renormalization)

Deterministic approach to non-Gaussian fluctuations

- *Infinite* hierarchy of coupled equations *An et al 2009.10742 PRL*

  for hydrodynamic correlators $H_n \equiv \langle \underbrace{\delta\breve{\psi} \ldots \delta\breve{\psi}}_{n} \rangle^{\text{connected}}$:

$$\partial_t H_n = -n\Gamma(H_n - \bar{H}_n[\psi, H, \ldots, H^{n-1}]) + \mathcal{O}(\varepsilon^n);$$

  To leading order, the equations are iterative and “linear”.

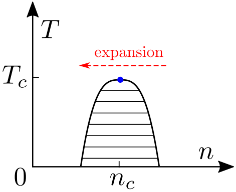
- Generalized Wigner transform:

$$H(x_1, \ldots, x_n) \to W(x; \boldsymbol{q}_1, \ldots, \boldsymbol{q}_n)$$

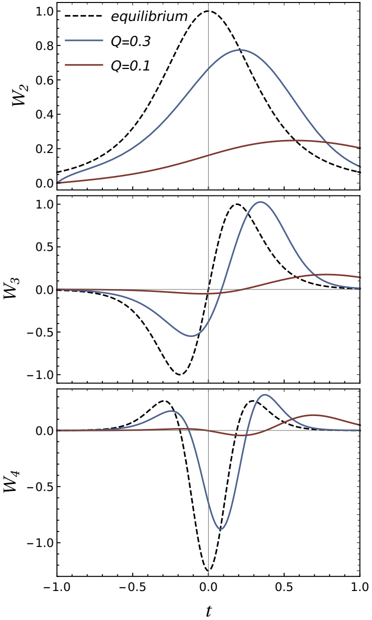
  W_n’s quantify magnitude and non-gaussianity of fluctuation harmonics with wave-vectors $\boldsymbol{q}_i$.

Example: expansion through a critical region

An et al 2009.10742, PRL



- Two main features:
  - Lag, ”memory”.
  - Smaller Q – slower evolution. Conservation laws.



Freezeout of fluctuations

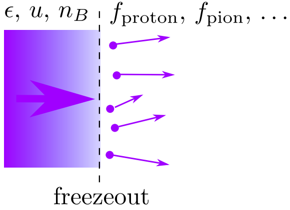
Pradeep, MS, 2211.09142, PRL

- Freezeout: mapping of correlators of hydrodynamic fluctuations ($\psi = \epsilon, n_B, u$)

  $\langle \delta\psi \ldots \delta\psi \rangle = H_n(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n)$

  to particle correlators

  $\langle \delta f \ldots \delta f \rangle = G_n(\boldsymbol{x}_1, \boldsymbol{p}_1, \ldots, \boldsymbol{x}_n, \boldsymbol{p}_n)$.



- There is a mapping from H_n to G_n which:
  - Satisfies *all* conservation laws, and
  - Maximizes entropy.