Critical slowing down and bulk viscosity in binary neutron star mergers

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arxiv:2603.13474

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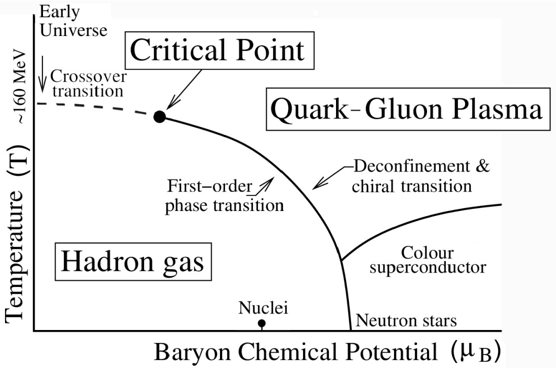
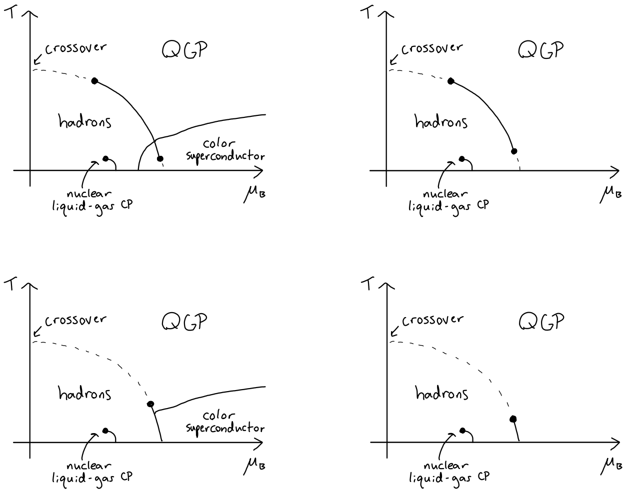
Phase diagram propaganda



Also consistent with our current knowledge:



see e.g. Hatsuda et. al. arXiv:hep-ph/0605018

Can this affect NS mergers?

- In a finite relaxation time $\tau \sim 1$ ms, the correlation length can grow to

$$\xi \lesssim \xi_0 \left(\frac{\tau}{\xi_0}\right)^{1/z} \sim 10 \text{ nm}$$

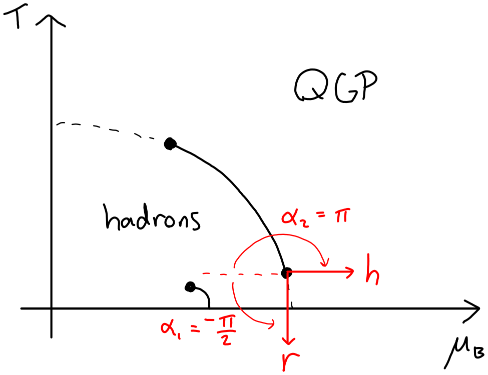
c.f. Stephanov, Rajagopal, Shuryak arXiv:hep-ph/9903292

- The bulk viscosity scales as

$$\zeta \sim \xi^{2.8}$$

Is this enhancement enough to compete with the background electroweak bulk viscosity?

Ising Model Mapping



$$\frac{T - T_c}{\mu_c} = -\rho w r$$
$$\frac{\mu - \mu_c}{\mu_c} = w h$$

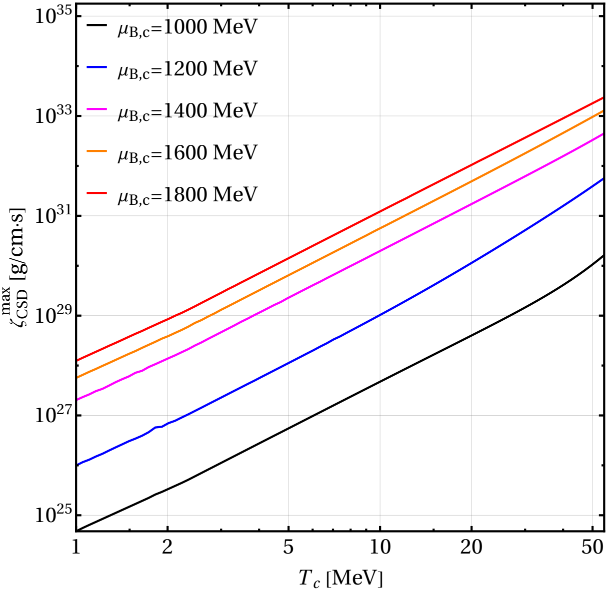
Parotto et. al. arXiv:1805.05249

Expression for Bulk Viscosity

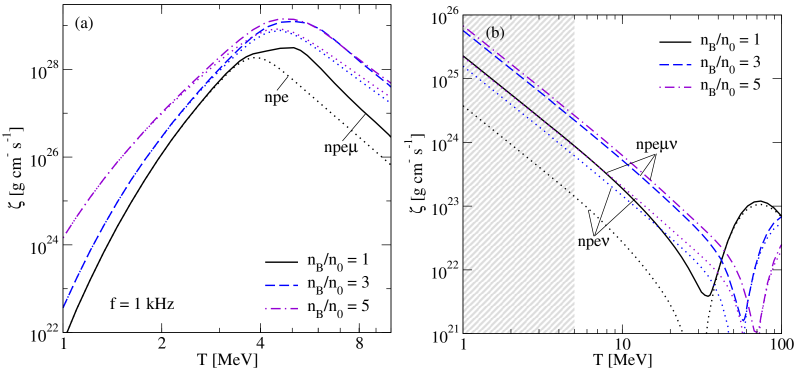
$$\zeta_{\mathrm{CSD}}(\omega) = \frac{3}{\pi}(\underbrace{\rho w}_{\text{mapping}}\ \gamma_{\pm}^{R})^2(\tau_0^2\xi_0^3)\overbrace{(e+P)^2}^{\text{enthalpy}}\ \underbrace{I_K(\omega)}_{\substack{\text{frequency-}\\\text{dependent}\\\text{piece}}}\ \overbrace{\eta_0}^{\substack{\text{shear}\\\text{viscosity}}}\left(\frac{\xi}{\xi_0}\right)^{z-\alpha/\nu}$$

Martinez, Schäfer, Skokov arXiv:1906.11306

For this order-of-magnitude calculation, let's use an excluded-volume hadron resonance gas model (EV-HRG) to calculate $e + P$ and η_0.


$\mu_{\mathrm{B,c}}$=1000 MeV
$\mu_{\mathrm{B,c}}$=1200 MeV
$\mu_{\mathrm{B,c}}$=1400 MeV
$\mu_{\mathrm{B,c}}$=1600 MeV
$\mu_{\mathrm{B,c}}$=1800 MeV
$\zeta^{\mathrm{max}}_{\mathrm{CSD}}$ [g/cm·s]
10^{35}
10^{33}
10^{31}
10^{29}
10^{27}
10^{25}
1
2
5
10
20
50
T_c [MeV]

Electroweak bulk viscosity for comparison



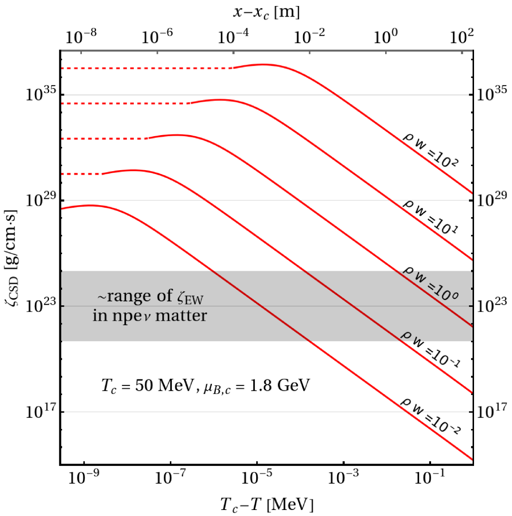
Alford, Harutyunyan, Sedrakian arXiv:2209.04717

The catch: this effect falls off quickly

Moving away from the critical point, ξ scales as

$$\xi = \xi_0(\rho w)^\nu \left(\frac{\Delta T}{\mu_{B,c}}\right)^{-\nu} .$$

Still, $\zeta_{\text{CSD}} \gtrsim \zeta_{\text{EW}}$ is possible at typical coarse-graining scales!



Summary

- For a low-temperature QCD critical point, substantial critical slowing down is possible in NS mergers, $\xi/\xi_0 \sim 10^6$.
- The bulk viscosity due to the critical slow mode could compete with the electroweak bulk viscosity!
- Indicative of other promising observables? For example: enhanced scattering of neutrinos.
- Please also ask me about hydro-like behavior near far-from-equilibrium attractors in heavy ion collisions!

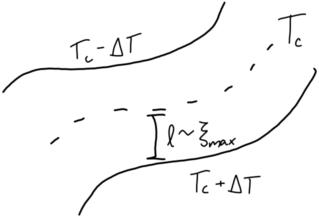
Backup Slides

Finite Temperature Gradients

- In an infinite equilibrated medium, a small chunk of matter of size ℓ near T_c has $\xi \lesssim \ell$.
- Suppose a BNS merger has a temperature gradient $|\nabla T|$.

- At distance ℓ from matter at T_c, we can use $\xi \sim r^{-\nu}$ to predict

$$\xi \sim \xi_0 \left(\frac{\Delta T}{\mu_c}\right)^{-\nu} \text{ where } \Delta T = \ell \nabla T \,.$$



- Approximate $\xi_{\max}$ by solving $\xi = \ell$:

$$\xi \lesssim \xi_0 \left(|\nabla T| \frac{\xi_0}{\mu_c}\right)^{-\nu/(1+\nu)} \sim 100 \text{ nm}$$

for $\mu_c \sim 1.2$ GeV and $|\nabla T| = 5$ MeV/km

Expression for Bulk Viscosity

$$\zeta_{\mathrm{CSD}}(\omega) = \left(\frac{\rho w \gamma_{\pm}^{R}}{\pi}\right)^2 T^3(\tau_0^2 \xi_0^3) \left(\frac{e+P}{T}\right)^2 \frac{I_K(\omega)}{\Gamma_\xi} \left(\frac{\xi}{\xi_0}\right)^{-\alpha/\nu}$$

where

$$I_K(\omega) = \mathrm{Re} \int_0^\infty dy \, \frac{y^2}{(1+y^{2-\eta})^2} \frac{1}{K(y) - i\omega/(2\Gamma_\xi)}$$

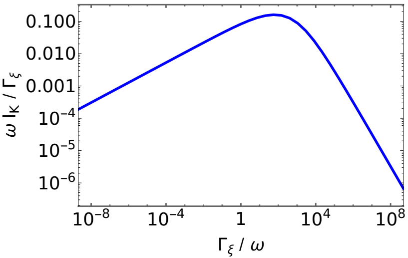
$$K(y) = \frac{3}{4}\left[1 + y^2 + (y^3 - 1/y)\arctan(y)\right]$$

$$\Gamma_\xi = \frac{T\xi^{-z}}{6\pi\eta_0}$$

Martinez, Schäfer, Skokov arXiv:1906.11306

Resonant structure of bulk viscosity

Bulk viscosity occurs when compression/expansion drives a system out of local equilibrium, and is largest when the driving frequency ω is comparable to the relaxation rate Γ.



At finite driving frequency, ζ has a peak at finite ξ.

At higher ξ than this peak value, ζ becomes less sensitive to ξ.

Which scaling regime are we in?

$$\Gamma_\xi = \frac{T\xi^{-z}}{3\pi\eta_0}$$

The least favorable scenario for resonance is a large T_c and small η_0. Taking

$$T_c = 100 \text{ MeV}, \ \frac{\eta_0}{s} = \frac{1}{4\pi}, \ \omega = 1 \text{ kHz}$$

and assuming $s \sim 1/\xi_0^3$, we have

$$\frac{\Gamma_\xi}{\omega} \sim 10^3$$

$\Rightarrow$ Although $\zeta \sim \xi^{z-\alpha/\nu}$ far from T_c, our maximum bulk viscosity will be relatively insensitive to the precise values of $\xi_{\max}, \eta_0$.

Hadron Resonance Gas Model

We will treat the matter as a gas of interacting hadrons.

$$\frac{n}{T^3} = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty \frac{dp\, p^2}{(-1)^{B_i - 1} + e^{\sqrt{p^2 + \tilde{m}_i^2} - \tilde{\mu}_i}}$$

Excluded Volume: Associate a volume v with each hadron to introduce a repulsive interaction.

$$\frac{p_v}{T} = n e^{-v p_v / T} \Rightarrow p_v = \frac{T}{v} W(vn)$$

Entropy and baryon density immediately follow:

$$\frac{n_B}{T^3} = \frac{\partial (p_v / T^4)}{\partial (\mu_B / T)} \qquad s = \frac{\partial p_v}{\partial T}$$

Rischke et al Z. Phys. C - Particles and Fields 51, 485–489 (1991)

Hadron Resonance Gas Model

The shear viscosity is

$$\eta_0 = \frac{5}{64\sqrt{8}}\frac{T}{r^2}\frac{1}{n}\sum_i n_i \frac{\int_0^\infty dp\, p^3 e^{-\sqrt{p^2+\tilde{m}_i{}^2}+\tilde{\mu}_i}}{\int_0^\infty dp\, p^2 e^{-\sqrt{p^2+\tilde{m}_i^2}+\tilde{\mu}_i}}$$

where $v = 4 \cdot \frac{4}{3}\pi r^3$.

Hard core radius r (and by extension v) is a free parameter, but comparison to lattice at $\mu_B = 0$ suggests $r \sim .1 - .25$ fm. We will take $r \sim .1$ fm.