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Form Factor and Model Dependence in Neutrino-Nucleus Cross Sections

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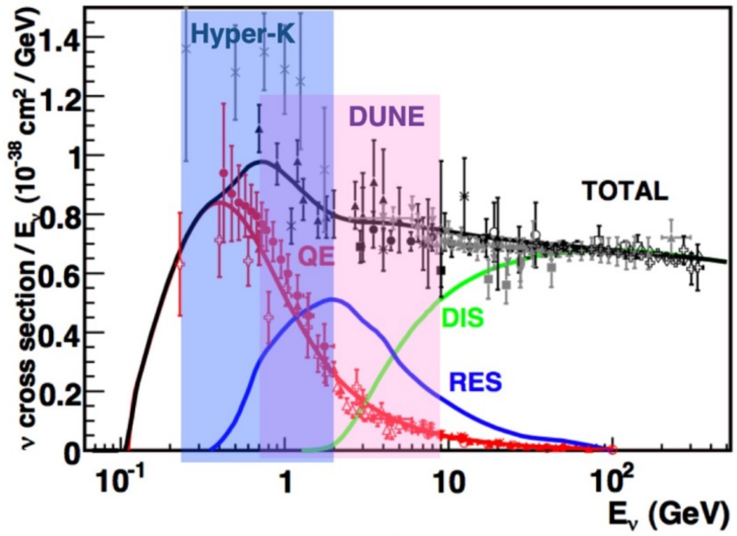
INT 2023

Based on arXiv 2210.02455 [hep-ph]

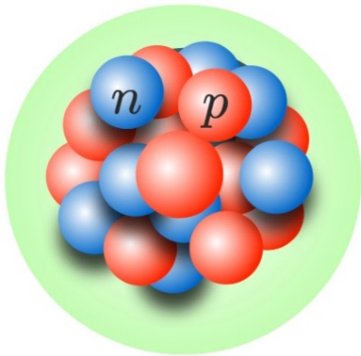
J.A. Formaggio and G.P. Zeller, Rev. Mod. Phys. 84 (201

Introduction



- Next generation of oscillation experiments require cross sections with
  - **unprecedented accuracy**
  - **robust theory uncertainty estimates**

- Theory uncertainties come from
  - Single and few nucleon form factors
  - Approximations from solving many body problem



- Estimation of these errors
  - Requires consistent formalism capable of including all interaction mechanisms
  - Disentanglement of the effects single nucleon form factors from nuclear effects

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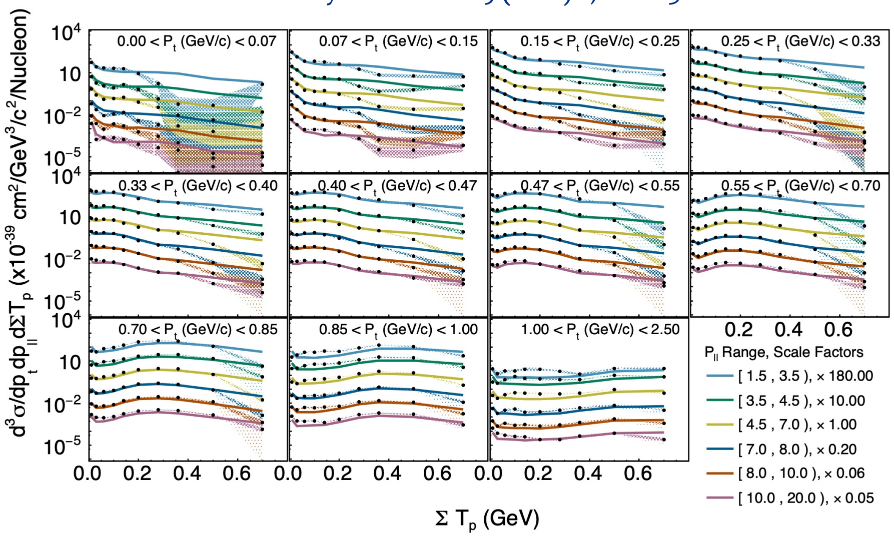
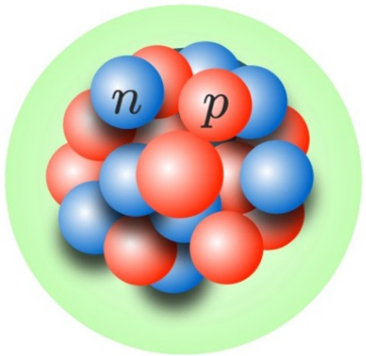
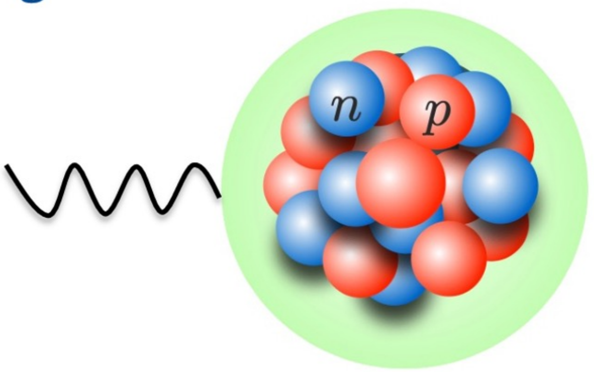
MINERvA Phys.Rev.Lett. 129 (2022) 2, 021803





Many Body solution to Neutrino-Nucleus Scattering

- Neutrino-nucleus scattering described by leptonic and nuclear response tensors
  $$\sigma \sim L_{\mu\nu} R^{\mu\nu}$$
- Nuclear response to probe (weak boson) contains all information on the structure of nucleus



$$R^{\mu\nu} = \sum_f \langle 0|J^{\mu\dagger}|f\rangle\langle f|J^\nu|0\rangle\delta(E_0 + \omega - E_f)$$

$$J^\mu = \sum_i j_i^\mu + \sum_{j>i} j_{ij}^\mu \longleftrightarrow H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

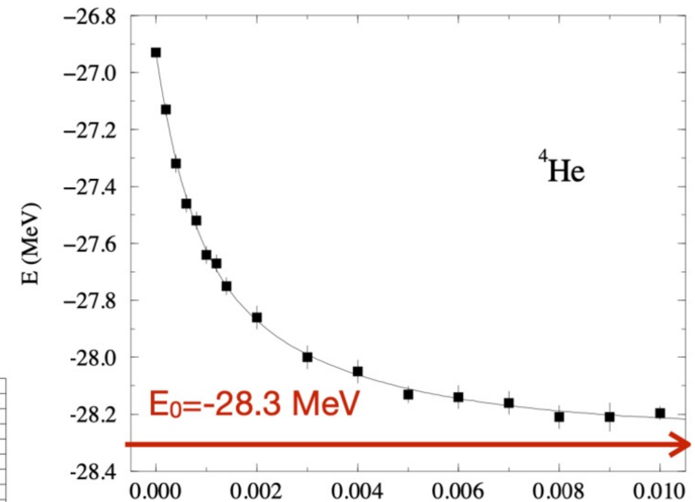
- Realistic Hamiltonian provided by AV18 an IL7 potentials
  - Fit to a wide range of nn and pn scattering data
- Look at two many body methods which share the same underlying nuclear dynamics
  - Very different approximations

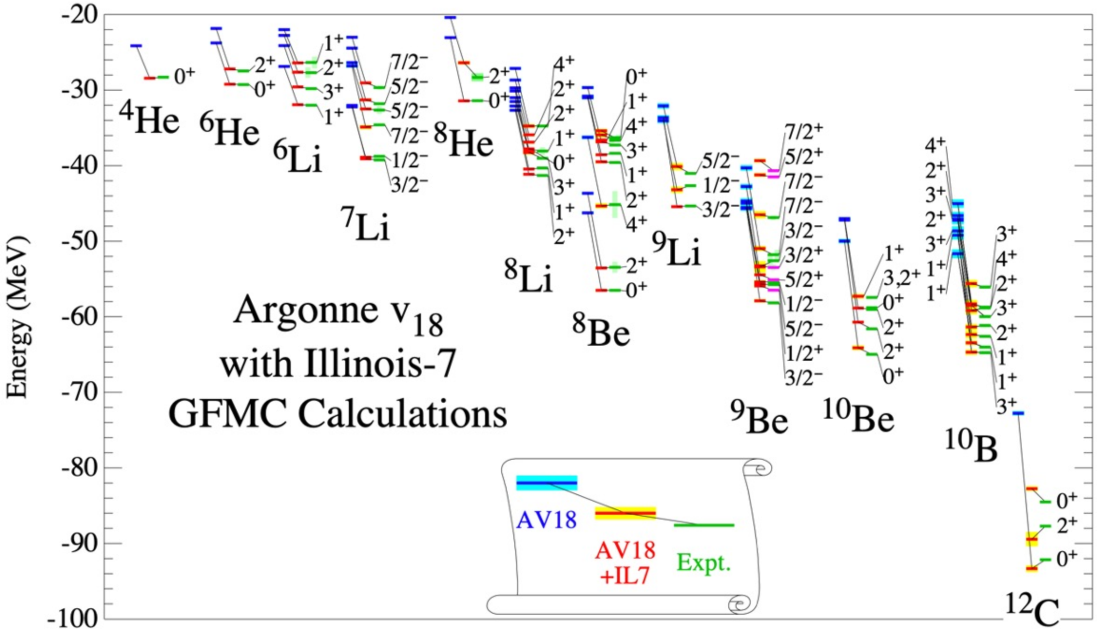
Cross Sections: Greens Function Monte Carlo (GFMC)

- Ground state solved via variational principle and imaginary time evolution

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \quad e^{-(H-E_0)\tau}|\Psi_T\rangle \rightarrow |\Psi_0\rangle$$

B. Pudliner et al., PRC **56**, 1720 (1997)





Carlson et al., Rev. Mod. Phys. **87**, 1067

- Able to compute the ground state (and excited state) energies of light nuclei with 1% precision

Cross Sections: Greens Function Monte Carlo (GFMC)

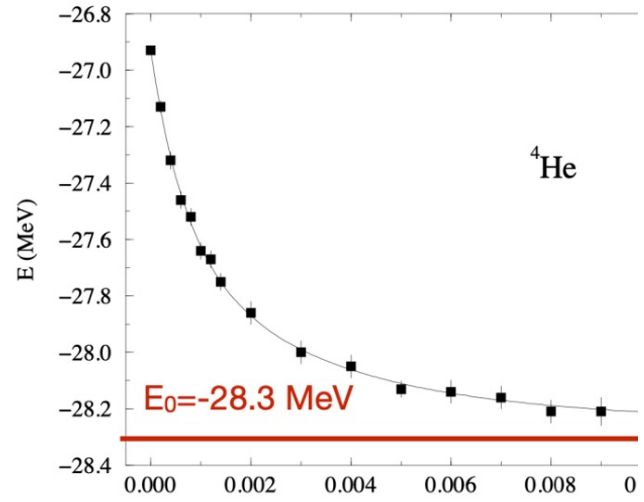
- Ground state solved via variational principle and imaginary time evolution

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \quad e^{-(H-E_0)\tau}|\Psi_T\rangle \rightarrow |\Psi_0\rangle$$

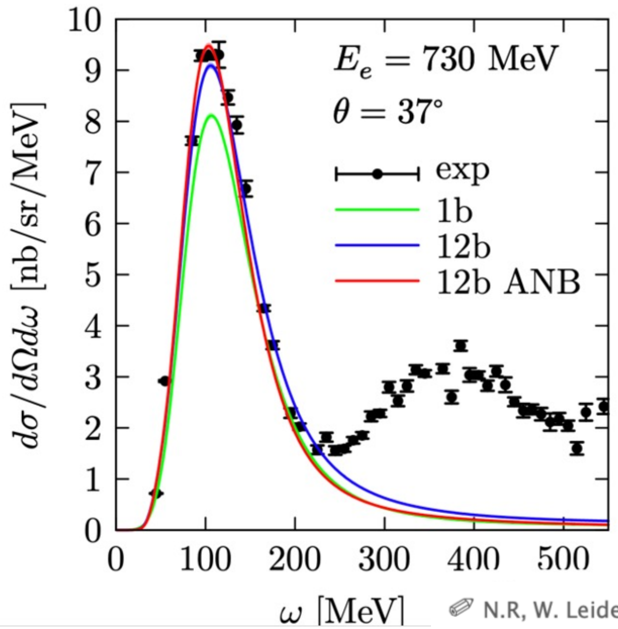
- Compute the Euclidean response, imaginary time evolve

$$E_{\alpha\beta}(\mathbf{q},\tau) = \int_{\omega_{\rm th}}^{\infty} d\omega e^{-\omega\tau} R_{\alpha\beta}(\mathbf{q},\omega)$$

B. Pudliner et al., PRC **56**, 1720 (1997



- Inversion needed to obtain response function



N.R, W. Leidemann, et al PRC 97 (2018) no.5, 055501

- Fully retains many body correlations in initial and final state
- Validated via electron scattering
- Number of approximations
  - Non-relativistic
  - Static delta

$$J^\mu = \boxed{\sum_i j_i^\mu} + \sum_{j>i} j_{ij}^\mu$$

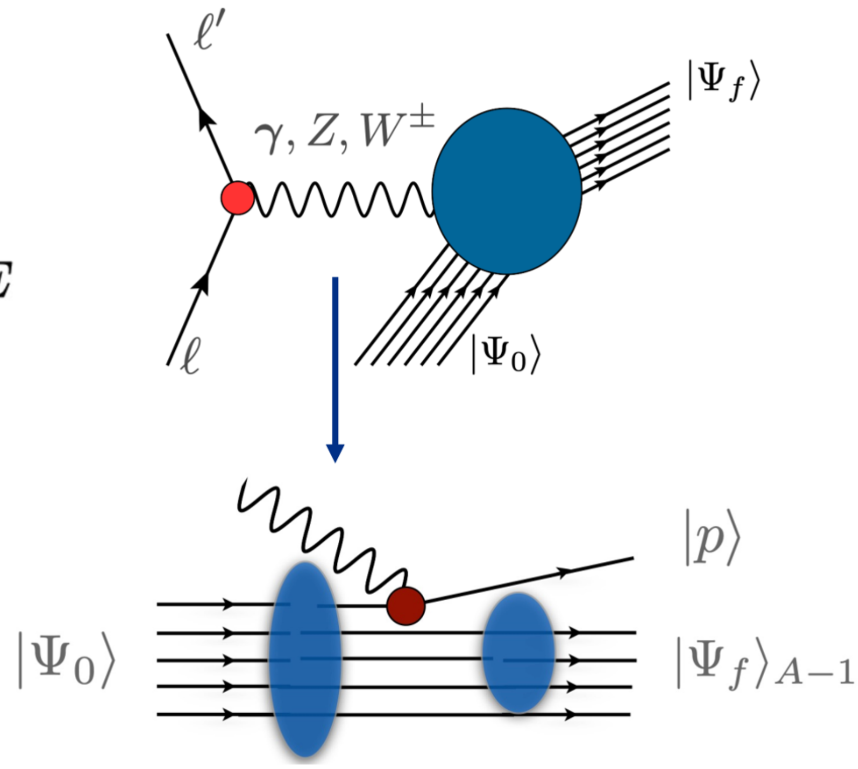
Cross Sections: Spectral function approach (SF)

- For sufficient |**q**|, scattering factorizes
  - Incoherent sum of scattering with individual nucleons

- Single nucleon knockout (QE)

$$|f\rangle = |\mathbf{p}'\rangle \otimes |\Psi_f^{A-1}, \mathbf{p}_{A-1}\rangle$$

$$d\sigma = \int \boxed{(d\sigma)}_{nucleon} \boxed{P(\mathbf{p}, E)} d^3k dE$$



- Ingredients boil down to
  - Single nucleon cross section
  - Hole spectral function

Cross Sections: MEC Calculation

$$J^\mu = \sum_i j_i^\mu + \boxed{\sum_{j>i} j_{ij}^\mu}$$

- Two nucleon knockout $|\psi_f^A\rangle \rightarrow |pp'\rangle_a \otimes |\psi_f^{A-2}\rangle$

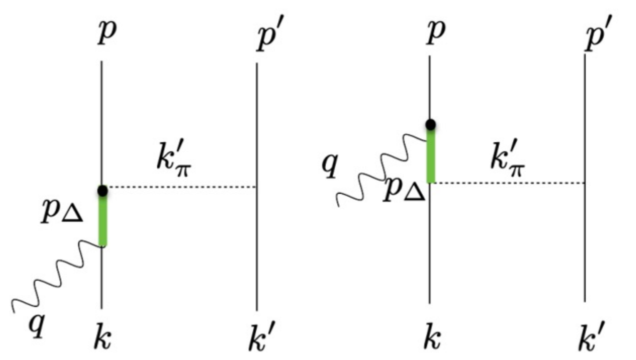
$$R_{2\mathrm{b}}^{\mu\nu}(\mathbf{q},\omega) = \frac{V}{2}\int dE \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}\frac{m_N^4}{e(\mathbf{k})e(\mathbf{k}')e(\mathbf{p})e(\mathbf{p}')}$$
$$\times \boxed{P_h \quad (\mathbf{k},\mathbf{k}',E)} \sum_{ij} \boxed{\langle k\,k'|j_{ij}^{\mu\,\dagger}|p\,p'\rangle}_a \langle p\,p'|j_{ij}^{\nu}|k\,k'\rangle$$
$$\times\, \delta(\omega - E + 2m_N - e(\mathbf{p}) - e(\mathbf{p}'))\,.$$

- Two body current

$$j_{\mathrm{CC}}^\mu = (j_{\mathrm{pif}}^\mu)_{\mathrm{CC}} + (j_{\mathrm{sea}}^\mu)_{\mathrm{CC}} + (j_{\mathrm{pole}}^\mu)_{\mathrm{CC}} + \boxed{(j_\Delta^\mu)_{\mathrm{CC}}}$$

  - Δ current gives dominant contribution and is highly model dependent

- Two body spectral function



QMC Spectral Function

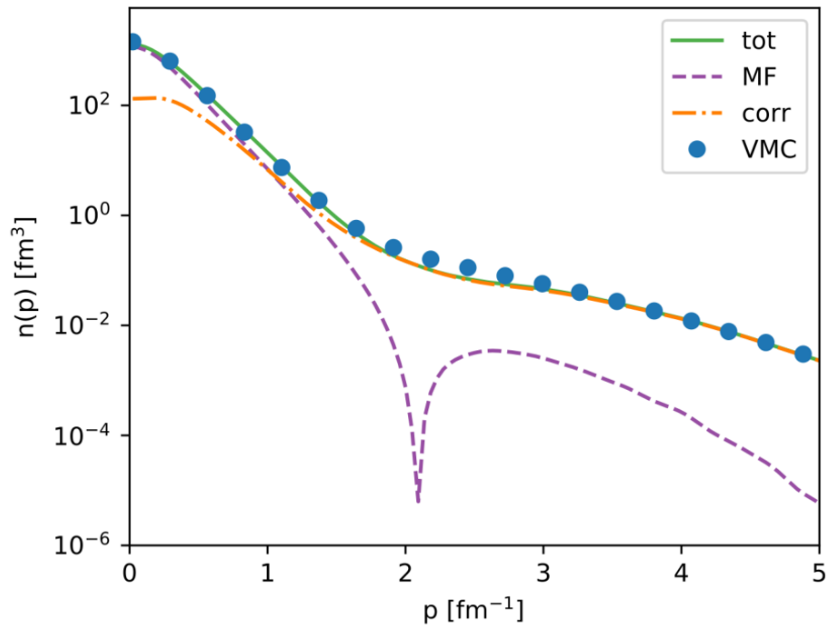
- One nucleon spectral function

$$P_{p,n}(\mathbf{k},E) = \sum_n |\langle\Psi_0^A|[|k\rangle|\Psi_n^{A-1}\rangle]|^2 \times \delta(E + E_0^A - E_n^{A-1})$$
$$= P^{MF}(\mathbf{k},E) + P^{\mathrm{corr}}(\mathbf{k},E)$$

  - Mean Field (A-1 bound states)
  - Correlation component from continuum
  - Momentum space overlaps obtained from VMC overlaps
    - Same Hamiltonian as GFMC!

- Two nucleon spectral function
  - Only mean field contribution

$$P^{\mathrm{MF}}_{\tau_k,\tau'_k}(\mathbf{k},\mathbf{k}',E) = n_{\tau_k,\tau_{k'}}(\mathbf{k},\mathbf{k}')$$
$$\times\, \delta\Big(E - B_0 + \bar{B}_{A-2} - \frac{\mathbf{K}^2}{2m_{A-2}}\Big)$$



N.B.

Different calculations of SF available from different experiments, QMC applicable for comparison with GFMC

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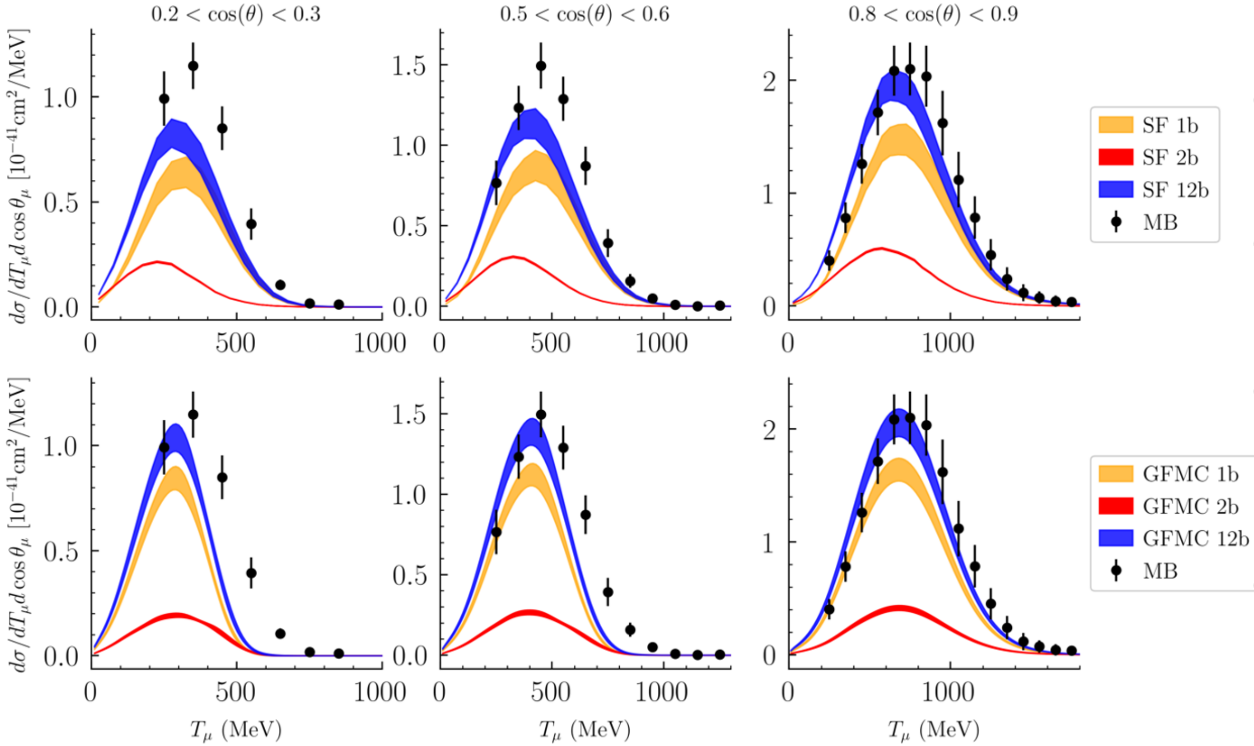
MiniBooNE – 1 and 2 Body Breakdown



- Separate 1 Body and 2 Body contributions
- SF and GFMC show deficit for small cos θ
- Model dependent pion subtraction at small T_μ
- GFMC non-relativistic nature means disagreements at large Q^2
- SF and GFMC 2 Body peaks shifted b/c of interference effects

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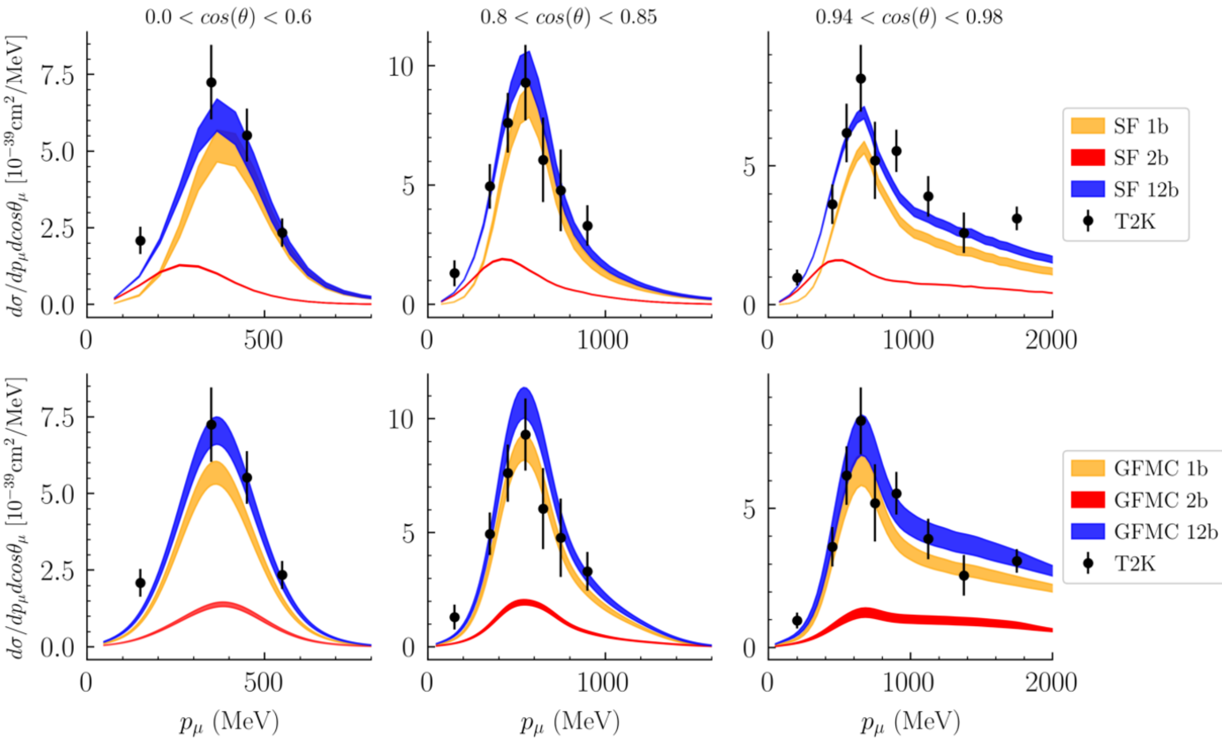
T2K – 1 and 2 Body Breakdown



- GMFC and SF provide excellent agreement
- T2K flux peaks at lower energies
- SF and GFMC 2 Body peaks shifted b/c of interference effects

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SF vs GFMC predictions

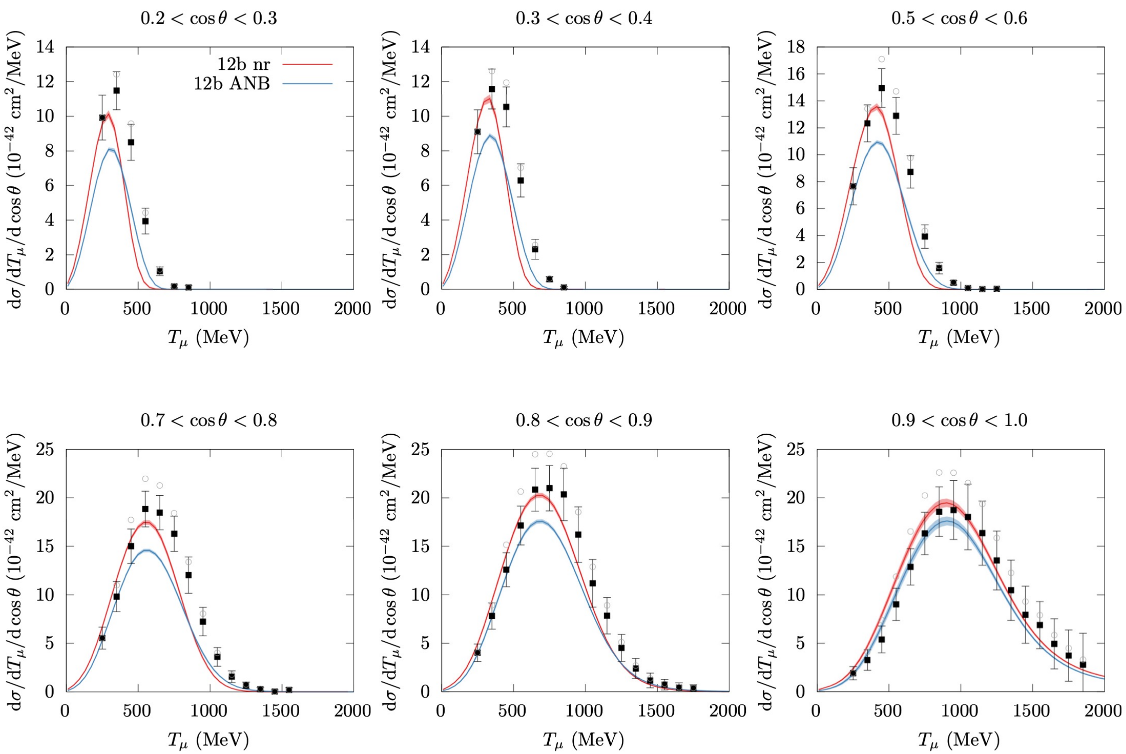
MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$	$0.5 < \cos\theta_\mu < 0.6$	$0.8 < \cos\theta_\mu < 0.9$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	22.8	20.3	5.6

T2K	$0.0 < \cos\theta_\mu < 0.6$	$0.80 < \cos\theta_\mu < 0.85$	$0.94 < \cos\theta_\mu < 0.98$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	13.4	7.3	10.0

- Differences due to:
  - GFMC
    - Non-relativistic nature of GFMC
    - Static treatment of Δ propagator
  - SF
    - No FSI in factorization scheme
    - Lack of 1-2 body interference
  - First attempt at uncertainty due to factorization approach

Improvements to GFMC and SF

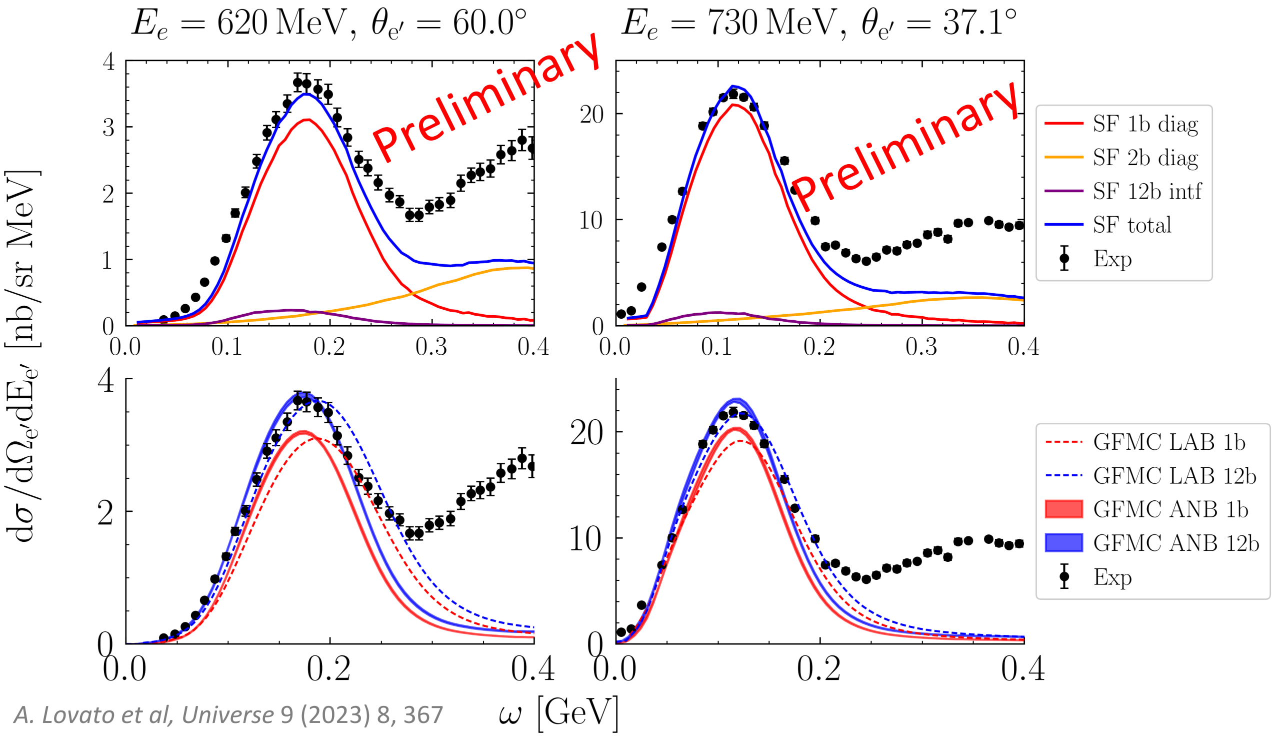
- GFMC kinematics is non-relativistic
- Compute responses in frame that minimizes initial and final nucleon momentum
  - ANB Frame shown to reduce relativistic effects



A. Nikolakopoulos et al, 2304.11772 [nucl-th]

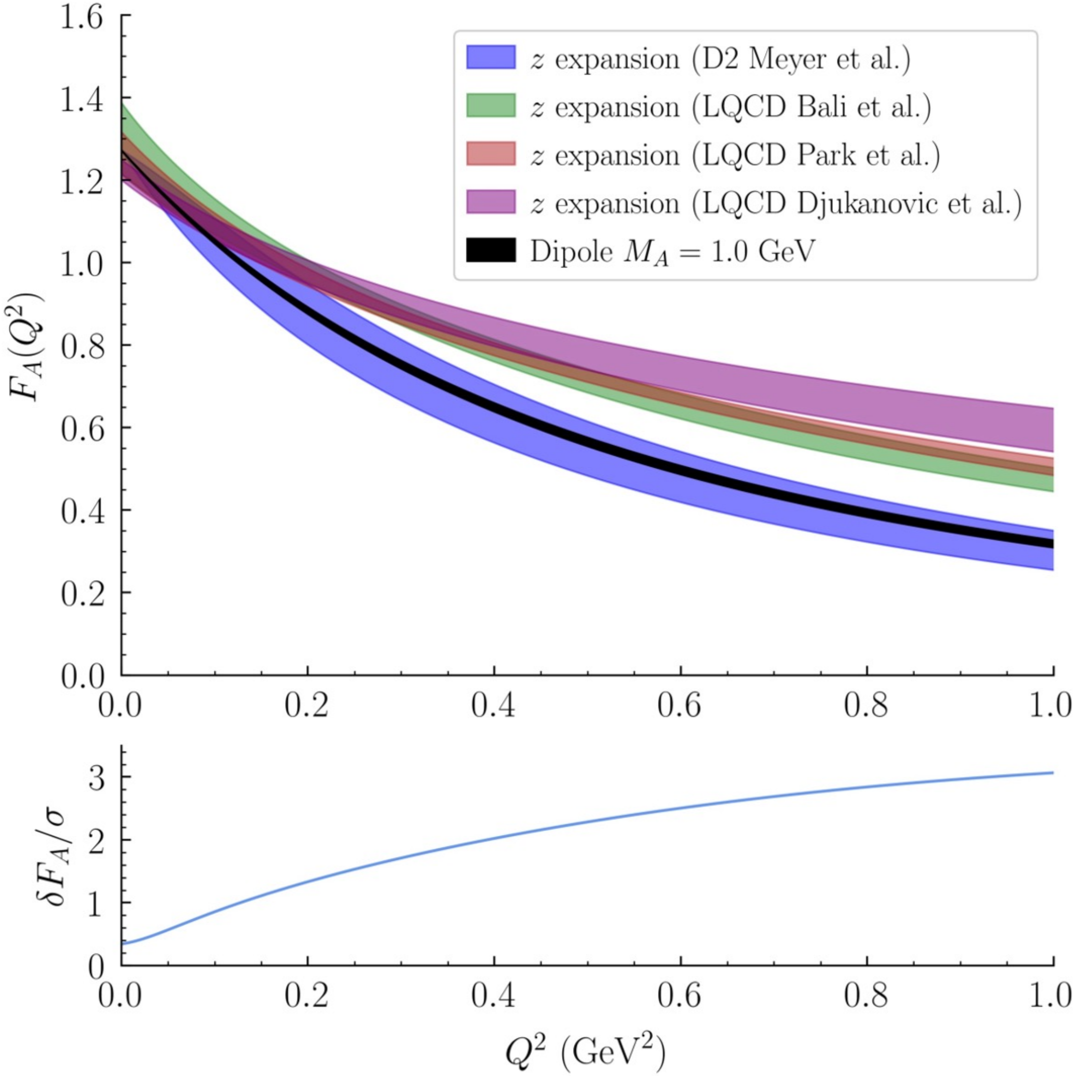
Improvements to GFMC and SF

- SF missing interference effects between 1 and 2 body currents
- Explicit in GFMC calculations (dominant effect of 2 body currents)
- Preliminary work on extending the SF formalism to include these interference effects



A. Lovato et al, Universe 9 (2023) 8, 367

Axial Form Factor

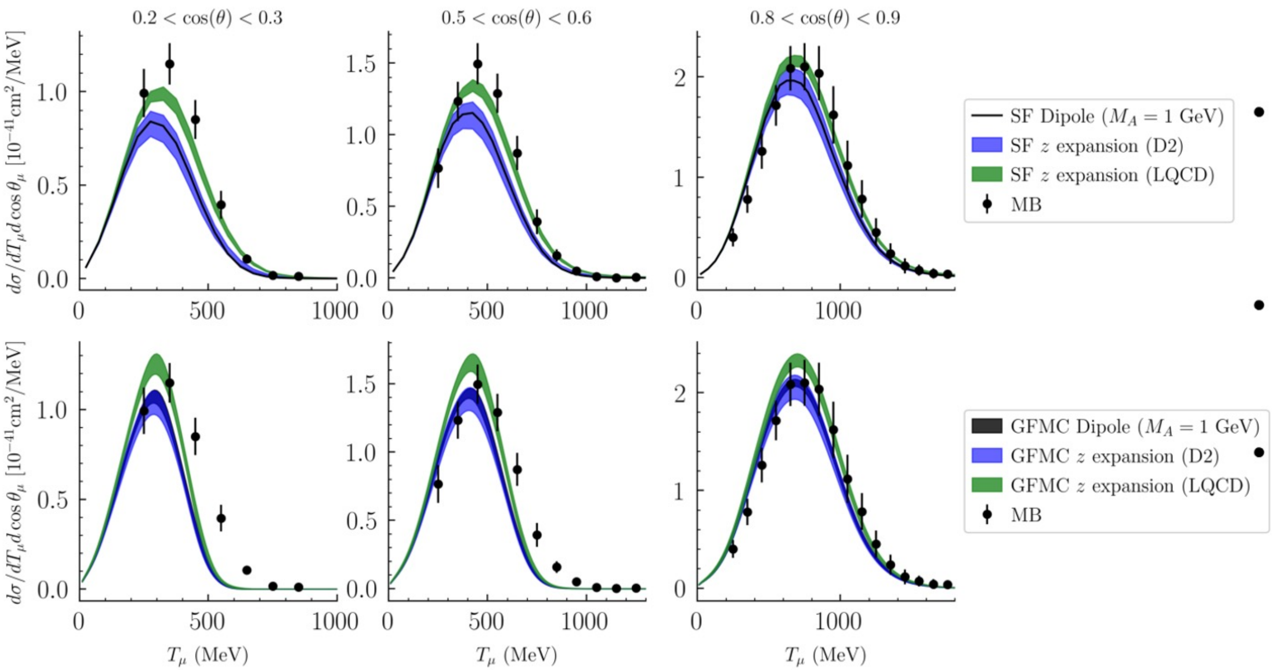


$$z(Q^2) = \frac{\sqrt{t_c + Q^2} - \sqrt{t_c - t_0}}{\sqrt{t_c + Q^2} + \sqrt{t_c - t_0}}$$

$$F_A(Q^2) = \sum_{k=0}^{\infty} a_k\, z(Q^2)^k \approx \sum_{k=0}^{k_{\max}} a_k\, z(Q^2)^k$$

- Dipole parameterization severely underestimates uncertainty
- Meyer et al. D2 z expansion gives similar CV but larger errors
- LQCD Bali and Park et al. z expansion give much larger normalization at $Q^2 > 0.3$ GeV^2

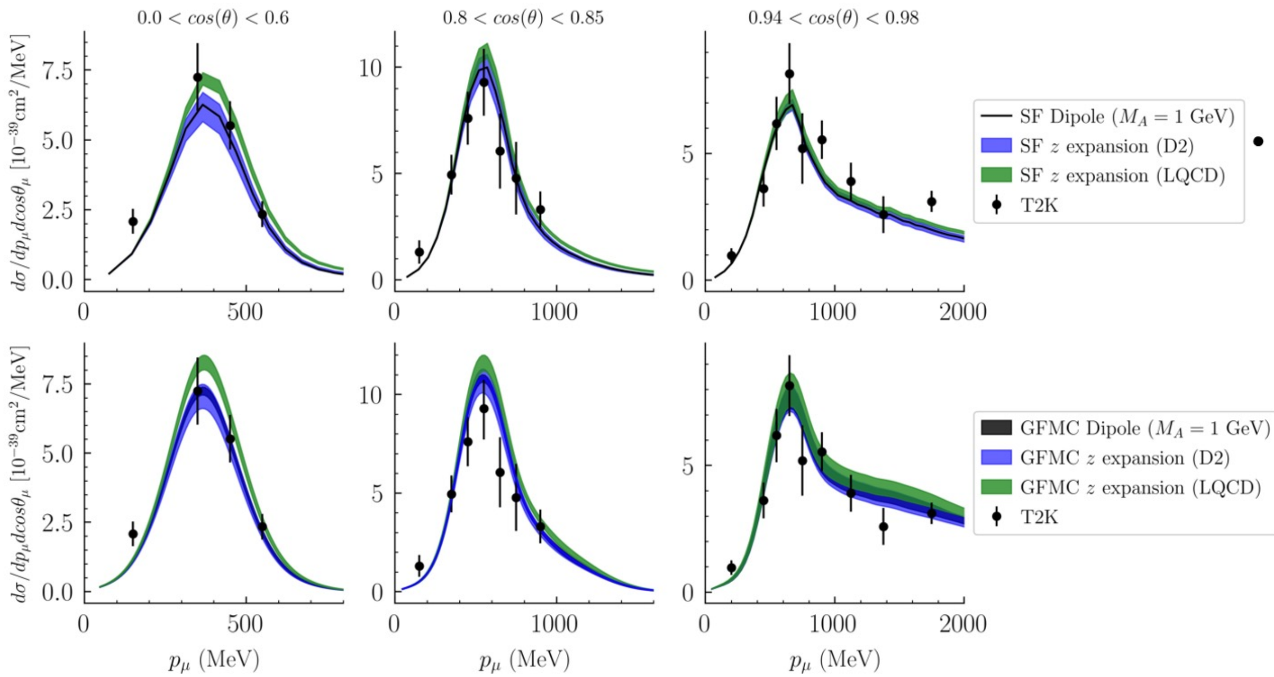
MiniBooNE – Form Factor Breakdown



- Dipole vs. LQCD z expansion vs. D2 z expansion
- Universal 10-20% increase in normalization with LQCD z expansion
- SF agreement better with LQCD z expansion
- GFMC disagreement regardless of form factor

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$	$0.5 < \cos\theta_\mu < 0.6$	$0.8 < \cos\theta_\mu < 0.9$
SF Difference in $d\sigma_{\rm peak}$ (%)	16.3	17.1	9.3
GFMC Difference in $d\sigma_{\rm peak}$ (%)	18.6	17.1	12.2

T2K – Form Factor Breakdown



- T2K comparison fairly independent of parameterization
- Mostly due to T2K's lower beam energy and thus Q^2 where form factors agree

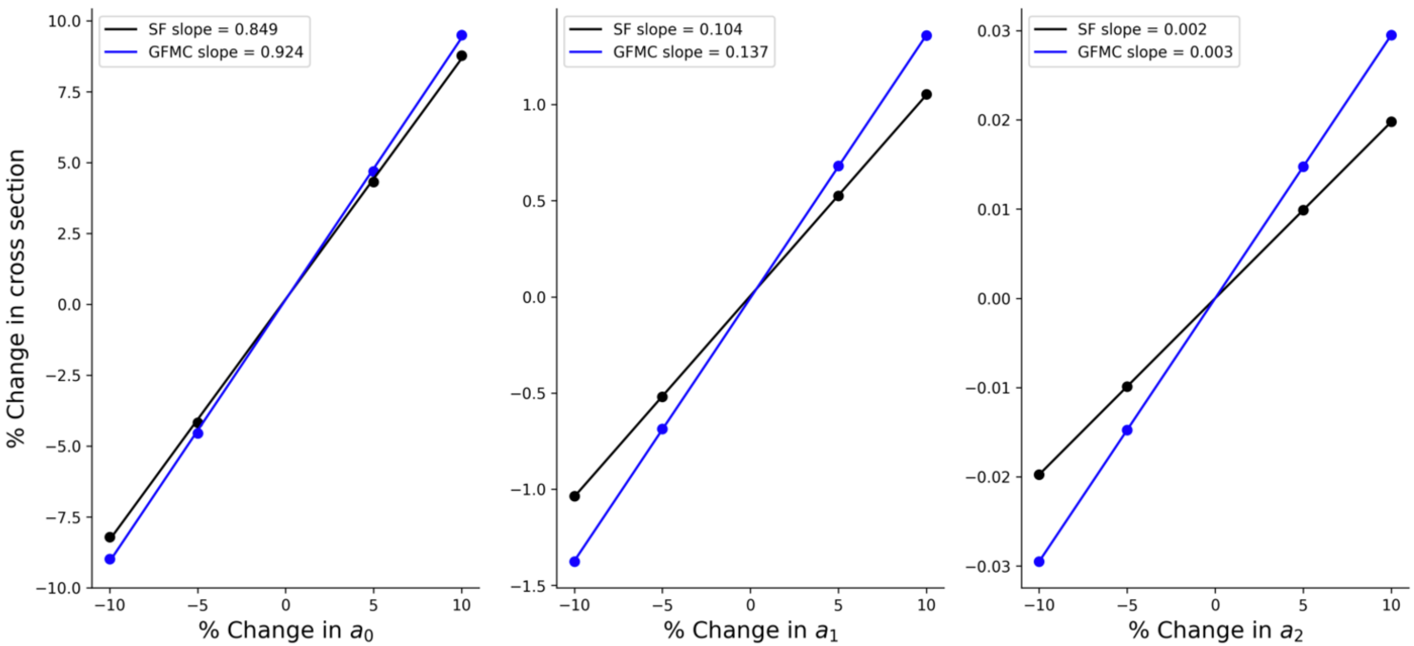
T2K	$0.0 < \cos\theta_\mu < 0.6$	$0.80 < \cos\theta_\mu < 0.85$	$0.94 < \cos\theta_\mu < 0.98$
SF difference in $d\sigma_{\text{peak}}$ (%)	15.3	8.2	3.3
GFMC difference in $d\sigma_{\text{peak}}$ (%)	15.8	8.0	4.6

Form Factor Uncertainty Analysis

LQCD precision target

- Numerically estimate $\delta \frac{d\sigma_{peak}}{d\cos\theta dT_\mu}$, LQCD precision targets
  - Change in peak cross section with respect to change in z expansion parameters a_k

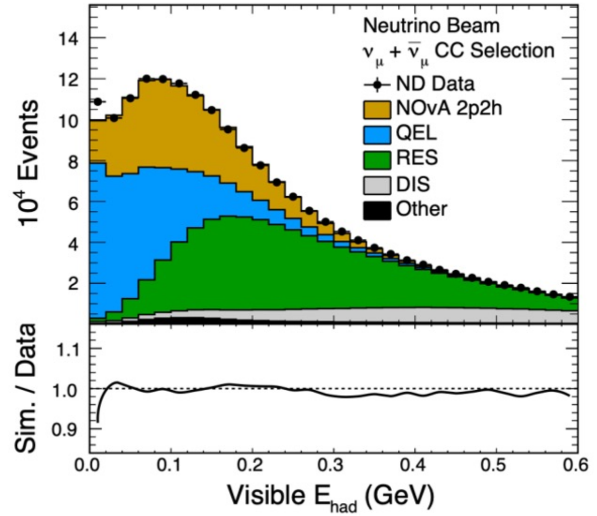
MiniBooNE $0.5 < \cos\theta < 0.6$



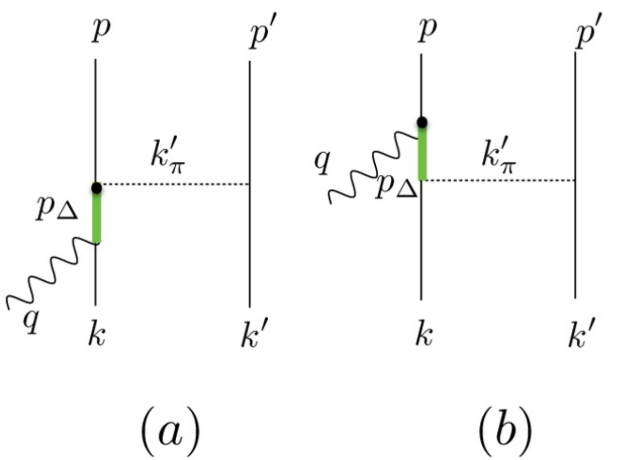
  - 1% (10%) effect on a_0 (a_1) gives 1% effect on peak of flux folded cross section (effect decreases at forward angles)

Model dependence of MEC calculation in SF

- Many experiments tune MEC e.g. Nova, MINERvA, MicroBooNE, etc.
- Unconstrained due to N → Δ transition form factors
- Investigate necessary precision on Δ parameters needed for future oscillation analysis



Eur.Phys.J.C 80 (2020)



$$C_5^A = \frac{1.2}{(1 - q^2/M_{A\Delta})^2} \times \frac{1}{1 - q^2/(3M_{A\Delta})^2)}$$

- Leading axial coupling of N → Δ

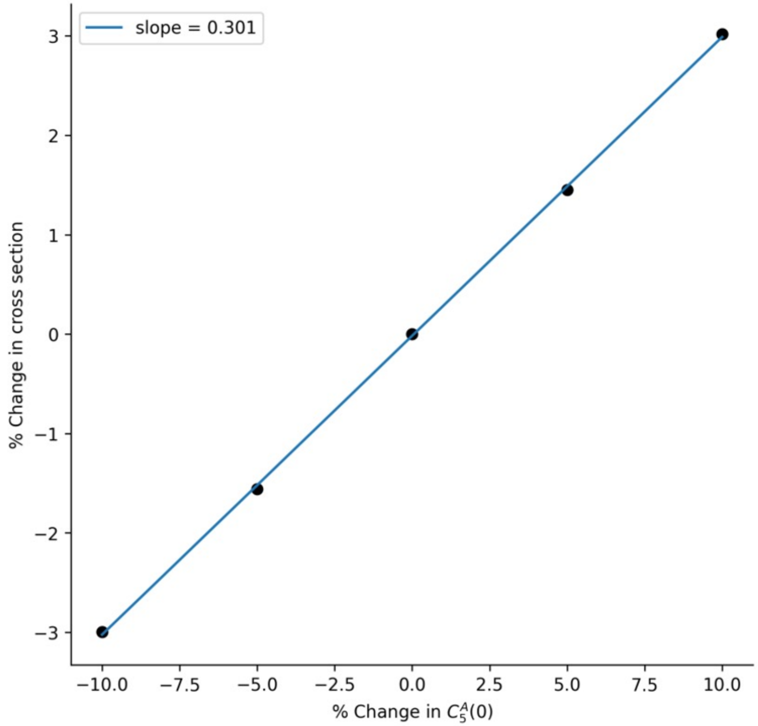
$$\Gamma(p_\Delta) = \frac{(4f_{\pi N\Delta})^2}{12\pi m_\pi^2}\frac{|\mathbf{d}|^3}{\sqrt{s}}(m_N + E_d)R(\mathbf{r}^2) \quad R(\mathbf{r}^2) = \left(\frac{\Lambda_R^2}{\Lambda_R^2 - \mathbf{r}^2}\right)$$

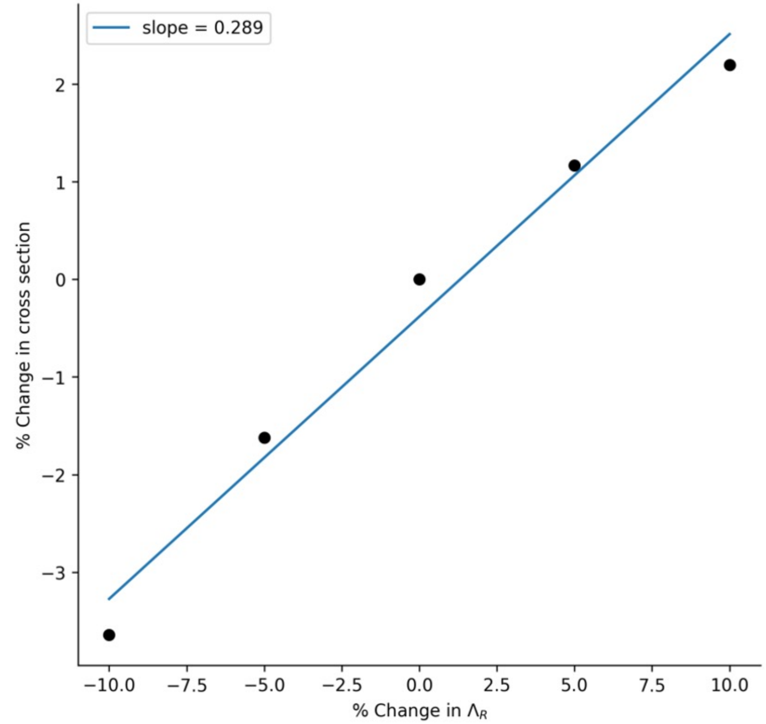
- Renormalizes Δ self energy

Model dependence of MEC calculation in SF

- Numerically estimate $\delta \frac{d\sigma_{peak}}{d\cos\theta dT_\mu}$
  - Change in peak cross section with respect to change in MEC parameter

Need O(%) level precision! LQCD target





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$$\Gamma(p_\Delta) = \frac{(4 f_{\pi N \Delta})^2}{12\pi m_\pi^2} \frac{|\mathbf{d}|^3}{\sqrt{s}} (m_N + E_d) R(\mathbf{r}^2) \quad R(\mathbf{r}^2) = \left(\frac{\Lambda_R^2}{\Lambda_R^2 - \mathbf{r}^2}\right)$$

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Conclusion

- Comparison of two different many body methods which share he same underlying nuclear dynamics

- First quantitative look at affect of factorization vs. exact calculations
  - Several improvements to SF and GFMC calculations to make this more precise

- Changes in the axial form factor have similar effects on SF and GFMC
  - Use to set precision goals for LQCD

- N$\rightarrow\Delta$ transition form factors highly uncertain
  - Large affects on 2 body currents
  - Need much more precision (extraction from experiment or LQCD)