About Radiative Corrections

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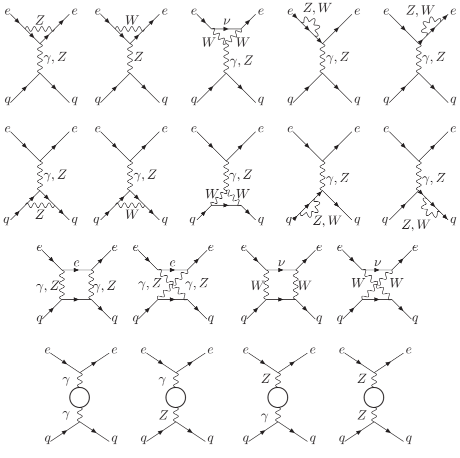
INT Workshop on PVDIS and EW Physics
at JLab 12 GeV and Beyond

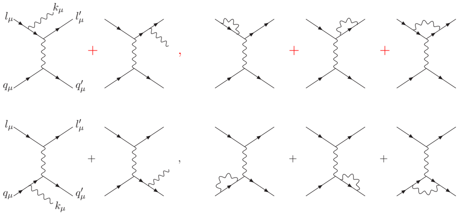
- Some general remarks
- Classification of radiative corrections
- Leptonic corrections (kinematic shift)
- Towards higher-order corrections

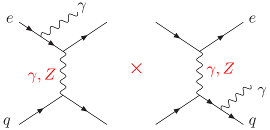
- "Radiative" corrections
  = effects from (unobserved) radiation
- radiative "Corrections"
  = corrections relative to a prediction
  in perturbation theory
- "Corrections" = a not so interesting part ?
  to be removed ??

- Soft radiation:
  $d\sigma \propto \frac{dE_\gamma}{E_\gamma}$
  Infrared divergence: $E_\gamma \to 0$
- Cancelled by virtual-photon corrections (loops)
  Bloch, Nordsieck, Kinoshita, Lee, Nauenberg
  need Loops and Legs
- In the Standard Model:
  Include complete set of n-loop corrections:
  Radiative corrections: complete corrections of order α^n
  (in practice: $n = 1$, $n = 2$ (?))

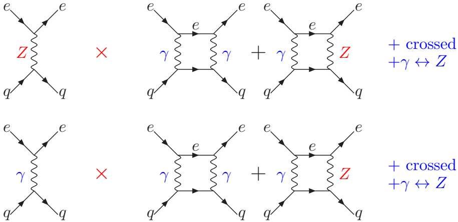


DIS: high Q^2
lepton-quark scattering





To cancel IR divergences,
Add real radiation:
lepton-quark interference
and loops:
box graphs



DJANGOH: for inclusive DIS

- Complete QED and electroweak corrections at $O(\alpha)$ for NC and CC scattering, polarized lepton, polarized nucleon

QCD-based event generation, valid at large Q^2: parton model

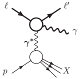
- Interface to LEPTO, JETSET. Jets, parton showers, hadronic final state. SOPHIA for low-mass hadronic final states

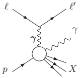
Fortran code. Well tested.

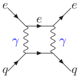
Code and manual for version 4.6.19:

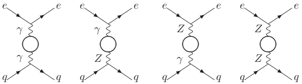
https://github.com/spiesber/DJANGOH

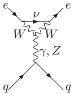
Version for variable beam energy by N. Pierre, A. Bressan: TDJANGOH

- QED corrections (one extra photon: real or virtual)
  - Leptonic 

Mo, Tsai, ...
  - Quarkonic 

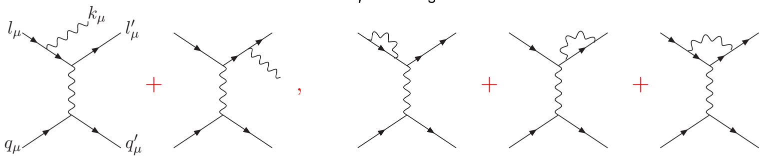
absorbed in PDFs
  - Interference 

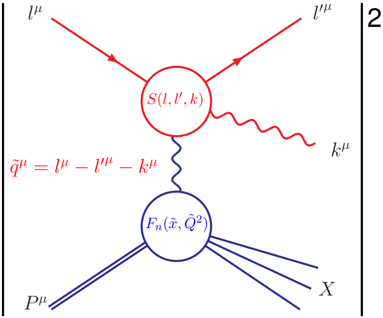
box graphs
- Self-energy insertions 

effective couplings
- Purely weak corrections: extra virtual Z, W, etc 

Feynman diagrams for leptonic radiation at $O(\alpha)$ (NC)

for *eq* scattering:





radiative leptonic tensor $S_{\mu\nu}(l, l', k)$ is

- gauge invariant
- infrared finite
- universal

(includes Born + loops: $\delta^{(4)}(k^\mu)$)

Observed cross section:

Convolution of cross section $\otimes$ radiator functions

$$\mathrm{d}\sigma^{\mathrm{obs}}(P,q) = \int \frac{d^3k}{2k^0} \sum_n R_n(l,l',k)\,\mathrm{d}\hat{\sigma}_n^{(0)}(P,q-k)$$

Shifted kinematics

observed momentum transfer $Q^2 = -(l-l')^2$,
➜ shifted momentum transfer $\tilde{Q}^2 = -(l-l'-k)^2$
observed Bjorken $x = Q^2/2P\cdot(l-l')$
➜ shifted Bjorken $\tilde{x} = \tilde{Q}^2/2P\cdot(l-l'-k)$
Use $Q^2 = xyS$ ➜ $\tilde{Q}^2 = \tilde{x}\tilde{y}S$

$$\mathrm{d}\sigma^{\mathrm{obs}}(x,Q^2) = \int_x^1 d\tilde{x} \int_0^y d\tilde{y} \sum_n R_n(x,\tilde{x};y,\tilde{y})\,\mathrm{d}\hat{\sigma}_n^{(0)}(\tilde{x},\tilde{Q}^2)$$

$\mathrm{d}\hat{\sigma}_n^{(0)}$ = theory prediction for cross section without radiation
(sometimes called “true” — a misnomer)

with partial fractioning, write: $R_n(l, l', k) = \frac{J}{k \cdot l} + \frac{F}{k \cdot l'} + \frac{C}{\tilde{Q}^2} + \dots$

- initial state radiation, $k \cdot l$ small for $\sphericalangle(\mathrm{e_{in}}, \gamma) \to 0$
- final state radiation, $k \cdot l'$ small for $\sphericalangle(\mathrm{e_{out}}, \gamma) \to 0$
- Compton peak, $\tilde{Q}^2$ small for $p_T(\mathrm{e_{out}}) \simeq p_T(\gamma)$

ISR, FSR: narrow peaks, width $\simeq \sqrt{m_l/E_l}$: collinear or mass singularities

upon angular integration: large logarithm $\propto \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \simeq 10\,\%$

Note: additional large logarithms from experimental cuts $\propto \log \frac{\Delta E}{E_{max}}$

For high precision: have to keep non-logarithmic terms

Leptonic radiation leads to kinematic shift:

$$Q^2 = -(l - l')^2 \quad \longrightarrow \quad \tilde{Q}^2 = -(l - l' - k)^2$$

$$x = \frac{Q^2}{2P(l - l')} \quad \longrightarrow \quad \tilde{x} = \frac{\tilde{Q}^2}{2P(l - l' - k)}$$

$$y = \frac{P(l - l')}{Pl} \quad \longrightarrow \quad \tilde{y} = \frac{P(l - l' - k)}{Pl}$$

$$\tilde{x} \geq x, \qquad \tilde{y} \leq y$$

... but not only: also non-radiative part is corrected!

PDF fits: x, Q^2 coverage

arXiv:1912.10053

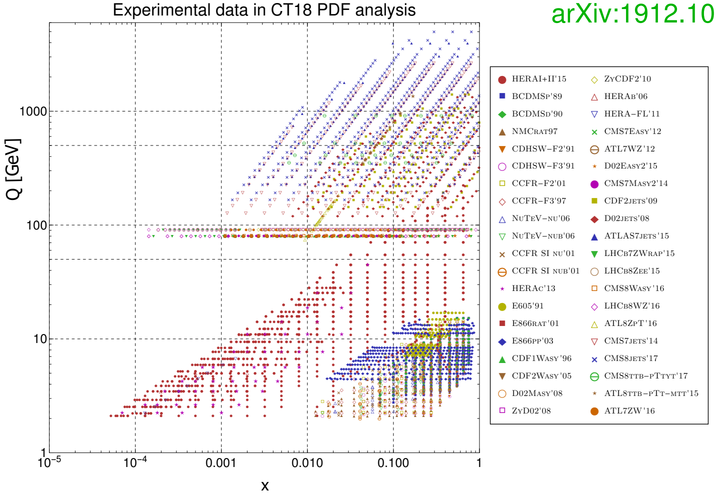
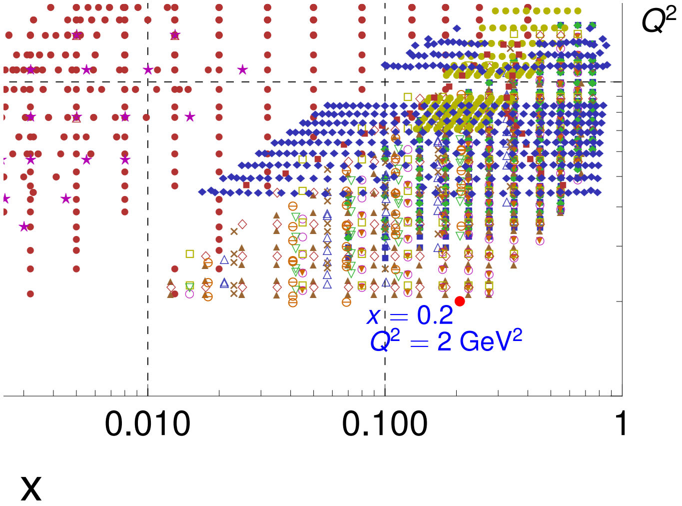


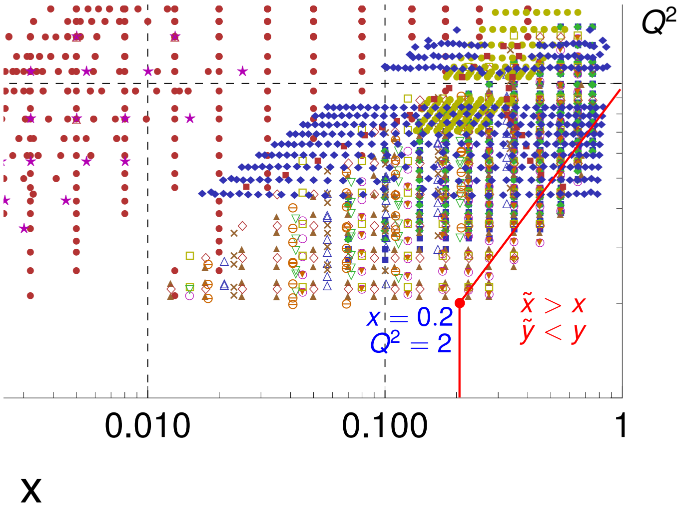
FIG. 1: The CT18 data set, represented in a space of partonic (x, Q), based on Born-level kinematical matchings, $(x, Q) = (x_B, Q)$, in DIS, *etc.*. The matching conventions used here are described in Ref. [20]. Also shown are the ATLAS 7 TeV W/Z production data (ID=248), labeled ATL7WZ'12, fitted in CT18Z.

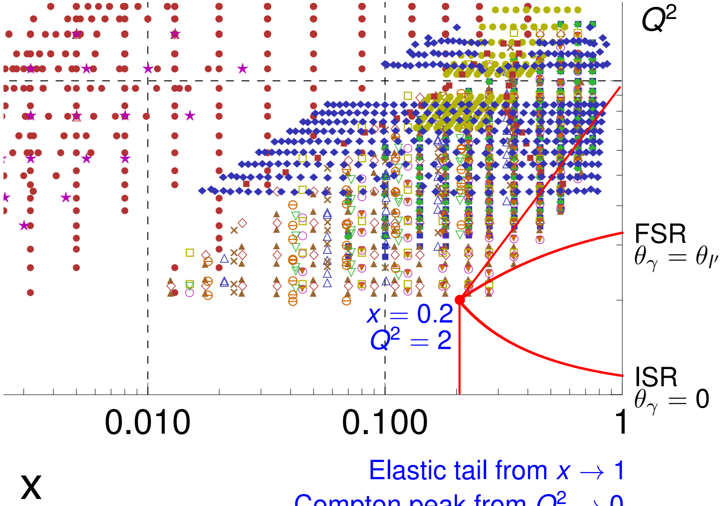
Low-Q^2 data for F_1, F_2 excluded from PDF fits

Radiative tails



Radiative tails





Elastic tail from $x \to 1$

Compton peak from $Q^2 \to 0$

- Radiative corrections are sensitive to non-DIS structure functions
- Relative uncertainty can be small
  at nominal x, Q^2
  but much larger for radiative corrections
- Use existing SF data at larger x, lower Q^2
  (not in PDFs)
  Measure cross section at larger x, lower Q^2
- Compare calculations with PDF and SF input
  to obtain a reliable uncertainty estimate
  i.e. include HT, TMC, F_L

- Relate structure functions and PDFs:

$$F_2 = 2x \sum_f q_f(x, Q^2) \quad + \quad \frac{\alpha_s}{\pi} H_2[q_f(x, Q^2), g(x, Q^2)] \quad + \ldots$$

$$F_L = 0 \quad + \quad \frac{\alpha_s}{\pi} H_L[q_f(x, Q^2), g(x, Q^2)] \quad + \ldots$$

  Systematic expansion in α_s^n
  (no new parameters)

- No large logs from mixed QED-QCD corrections at $O(\alpha_{em}\alpha_s)$
- Higher twist and target mass corrections: HT + TMC
  Systematic expansion in $(\Lambda/Q)^n$ (equivalently: $(M_N/Q)^n$)
  Need new (less well-known) structure functions

- Estimate from squared 1-loop corrections, naive way:
  - $\alpha^2 \simeq 0.5 \times 10^{-4}$
  - $\left(\frac{\alpha}{\pi} \ln(Q^2/m_e^2)\right)^2 \simeq 0.2 \times 10^{-3}$ (from QED)
  - $(\alpha \ln(Q^2/M_Z^2))^2 \simeq 3 \times 10^{-4}$ (from electroweak)

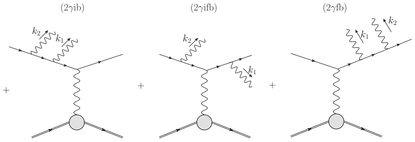
- $\sigma \propto |M^{(0)} + M^{(1)} + M^{(2)} + \ldots|^2$

  first-order correction: $2\,\mathrm{Re}\, M^{(0)*} M^{(1)}$

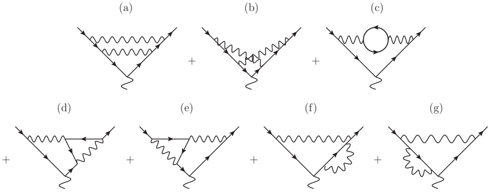
  second-order correction: $2\,\mathrm{Re}\, M^{(0)*} M^{(2)} + |M^{(1)}|^2$

  Aleksejevs et al, for Moller: $|M^{(1)}|^2 \simeq$ few per cent

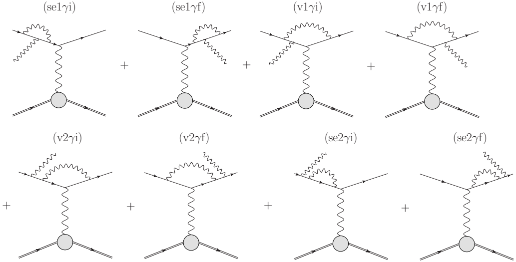
Second-order corrections: QED, elastic ep



2-photon radiation



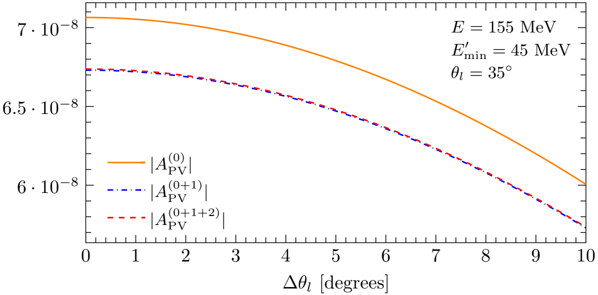
2-loop



1-loop corrected 1-photon radiation

Bucoveanu, HS

Banerjee et al



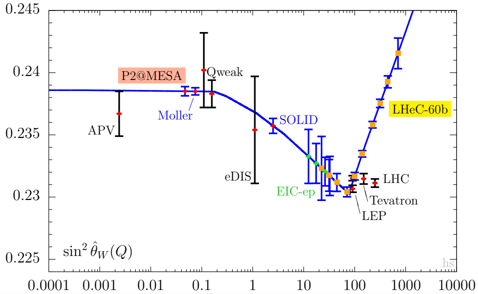
Bucoveanu, HS

POLARES

QED does not violate parity symmetry,
but large correction due to shift of Q^2
A kinematic effect: 1γ radiation has it all

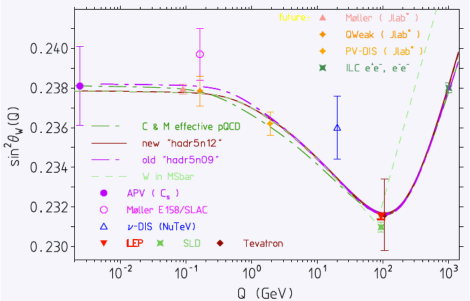
➜ No large $O(\alpha^2)$ corrections for A_{PV}

- Renormalization = fix the free parameters of the theory
- Renormalization scheme: choice of the free parameters and how they are fixed
- In the electroweak SM: choice of the weak coupling:
  - On-shell $\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$
  - $\overline{\text{MS}}$ scheme $\sin^2\hat{\theta}_W(\mu)$, e.g. PDG
  - effective $\sin^2\theta_{W,eff}(Q^2)$, e.g. by Czarnecki&Marciano



$\overline{\text{MS}}$ definition $\sin^2\hat\theta(\mu^2)$

Resummed by RGE



Effective coupling

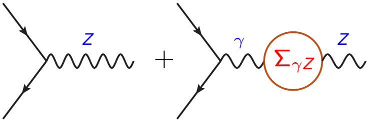
appropriate for Moller scattering

Czarnecki, Marciano

Jegerlehner 2012

differences $\simeq 1-2$ permille

The underlying 1-loop contribution:



➜ $\overline{\text{MS}}$ definition $\sin^2\hat{\theta}(\mu^2)$

Logarithmic contributions

Resummed by RGE

Erler, Ferro-Hernández

$\Delta\sin^2\hat{\theta}(0) \simeq 5\cdot 10^{-5}$

➜ Effective coupling

Including non-log terms

Czarnecki, Marciano, Jegerlehner

2012 obsolete prescription for hadronic contribution

Always need matching to the full calculation

i.e. add terms not absorbed into the running $\sin^2\theta_W$

Expect differences at higher order, e.g. in $g_V^e g_V^q$

Estimate higher-order corrections from $\overline{\text{MS}}$ – on-shell (to be done)

- Uncertainties at $O(\alpha)$ from PDFs and SF
  Analysis scheme including auxiliary measurements
  at larger x, smaller Q^2, different E_{beam}
- Uncertainties for 2-boson exchange:
  applicability of parton model? (not in SF)
- 2-loop calculation for eq scattering
  in the $\overline{\text{MS}}$ scheme
  feasible, but to be done

Extra Slides

- Data analysis: compare $\sigma_{\rm exp} = \sigma_{\rm theory}$
- Theory prediction: $\sigma_{\rm theory} = \sigma^{(0)}_{\rm theory} + \sigma^{(1)}_{\rm theory} + \ldots$

  $\sigma_{\rm exp} = \sigma_{\rm theory} = \sigma_{\rm theory}$(many parameters)

  $\sigma_{\rm theory}(\underbrace{\alpha, G_F, \sin^2\theta_W, \ldots}_{\text{electroweak}}, \underbrace{\alpha_s, \text{PDFs}, \ldots}_{\text{strong / hadron structure}})$

  To be solved for unknowns

  - exploit hierarchy of dependencies
  - decide about an order of parameters
  - use input from other experiments
    for well-measured parameters, α, G_F, etc

- Parameter dependence of higher-order corrections:

  may be helpful: $\sigma^{(0)}_{\rm theory}[\text{parameter1}] + \sigma^{(1)}_{\rm theory}[\text{parameter1}] = \sigma_{\rm exp}$

  may be a disturbance: $\sigma^{(0)}_{\rm theory}[\text{parameter2}] = \sigma_{\rm exp} - \sigma^{(1)}_{\rm theory}$

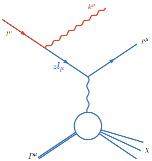
- SM at 1-loop including full x and Q^2 dependence
- Effective couplings at zero energy / zero momentum:

  $C_1^f = g_{AV}^{ef} = 2g_A^e g_V^f$ + corrections
  $C_2^f = g_{VA}^{ef} = 2g_V^e g_A^f$ + corrections

  Marciano, Sirlin 1983

  not suitable for DIS, i.e. $Q^2 \neq 0$
- Can include RGE logarithms: see Erler, Su
  i.e. some logarithmic Q^2 dependence, but not all

Collinear approximation (peaking approximation)



e.g., for initial-state radiation: assume $k^\mu = (1 - z) l^\mu$
➜ Radiator function

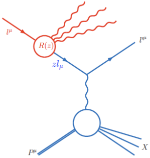
$$R_{\text{ISR}} = \frac{\alpha}{2\pi} \frac{1 + z^2}{1 - z} \log \frac{Q^2}{m_e^2}$$

$(+\delta(1 - z)$ from loops $\to$ $+$-distribution $1/(1 - z)_+)$

$$\mathrm{d}\sigma_{\text{ISR}} = \int \frac{\mathrm{d}z}{z} R_{\text{ISR}}(z)\, \mathrm{d}\sigma_{\text{Born}}(l^\mu \to z l^\mu)$$

(similar for final-state radiation)

Can be extended to include multi-photon emission:

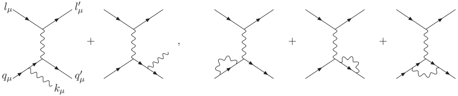


$$R^{(2)}_{\text{ISR}}(z) = \int_z^1 \frac{\mathrm{d}z'}{z'} R^{(1)}_{\text{ISR}}(z') R^{(1)}_{\text{ISR}}(z/z') + \ldots$$

Solution of evolution equations like DGLAP
Known at $O(\alpha^2)$ (complete) and partially at $O(\alpha^3)$

Quarkonic radiation

at large Q^2: DIS, parton model
emission of photons
like emission of gluons



infrared divergences (soft photons / gluons) cancel with loops,
collinear emission leads to mass-singular large log's $\frac{\alpha}{2\pi} \log m_q^2$,

➜ factorization: absorb collinear divergences into PDFs

$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m_q^2)) q_f(x)$$

$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m_q^2)) q_f(x) = \sum_f d\hat{\sigma}_f \hat{q}_f(x, Q^2)$$

➜ renormalized PDFs

$$\hat{q}_f(x, Q^2) = (1 + \delta_f(Q^2; m_q^2)) q_f(x)$$

➜ modified scaling violations

well-known in QCD, $\overline{\text{MS}}$ factorization