

Constraints on the dense nuclear matter EOS from Au+Au collisions in the BES FXT range

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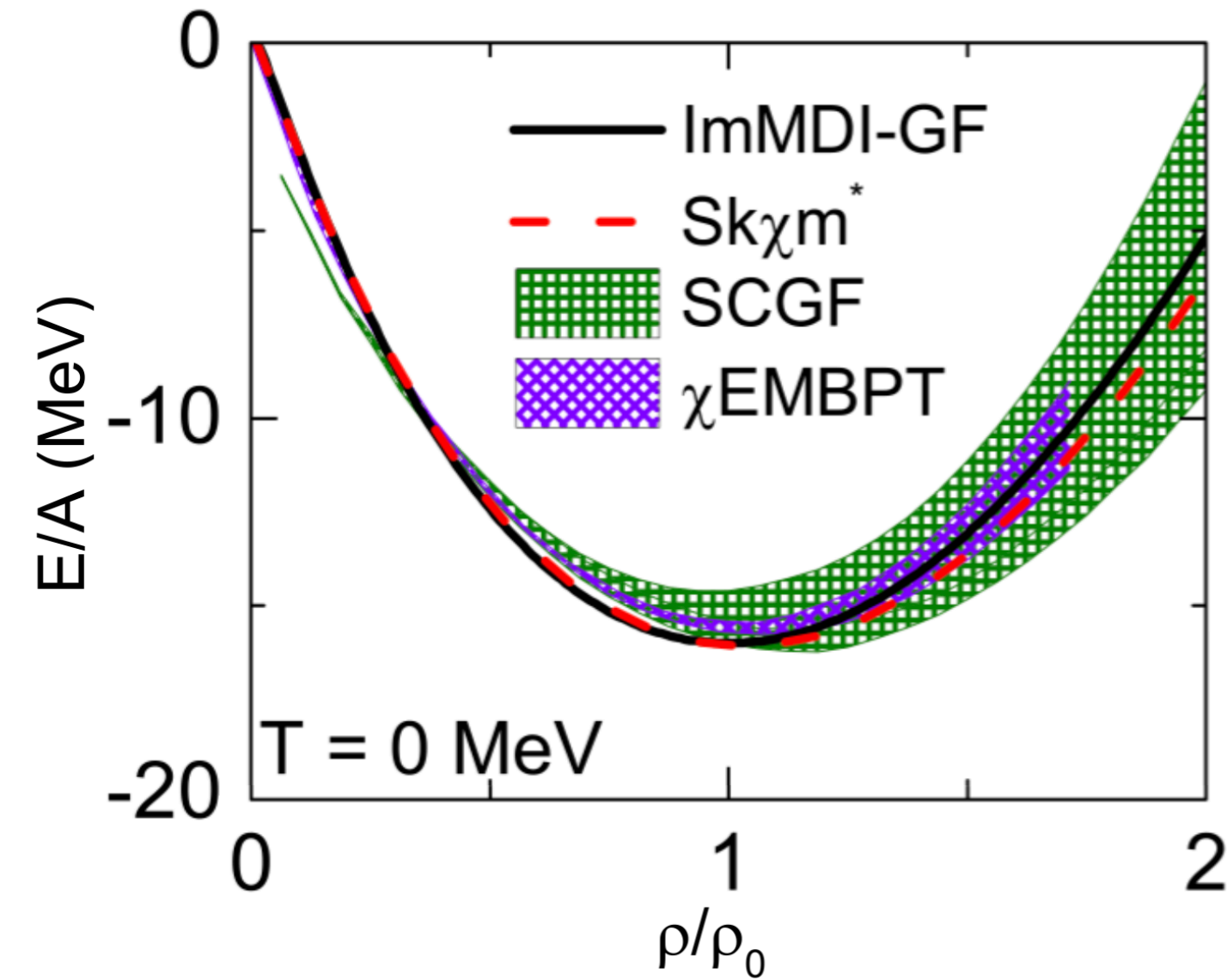


INT WORKSHOP INT-20R-1C

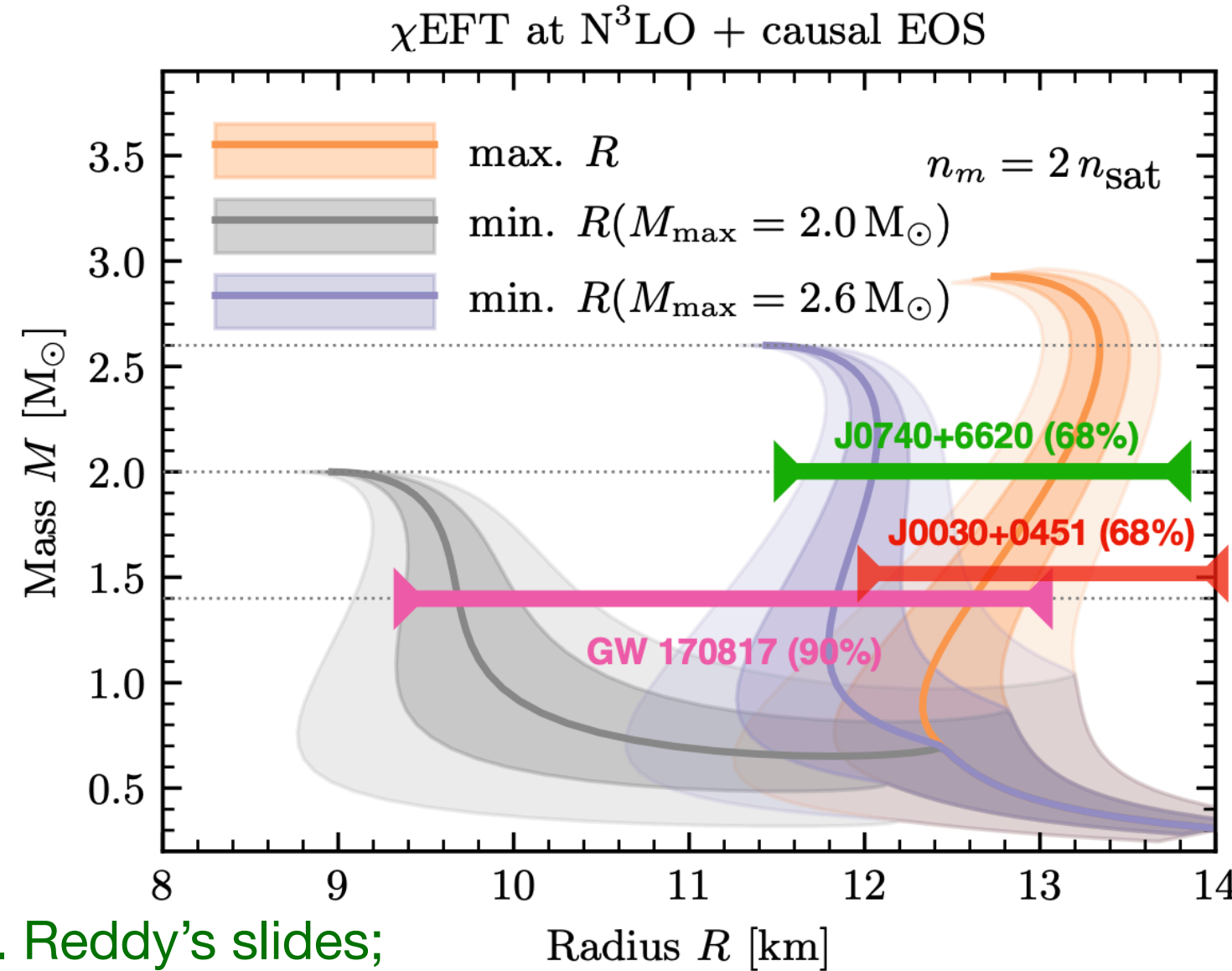
Chirality and Criticality: Novel Phenomena in Heavy-Ion Collisions

August 21, 2023 - August 25, 2023

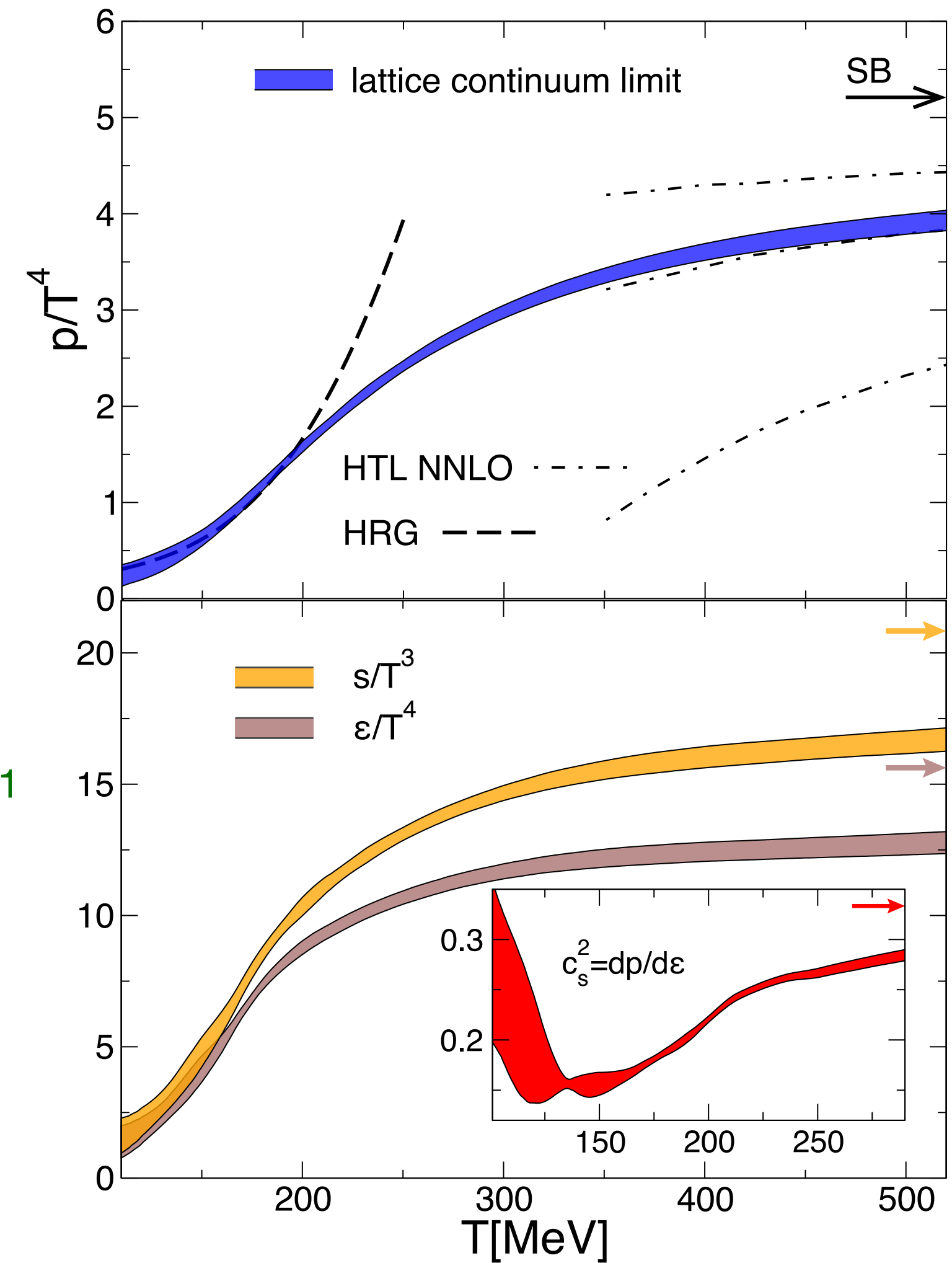
The EOS = key to understanding fundamental properties of QCD matter



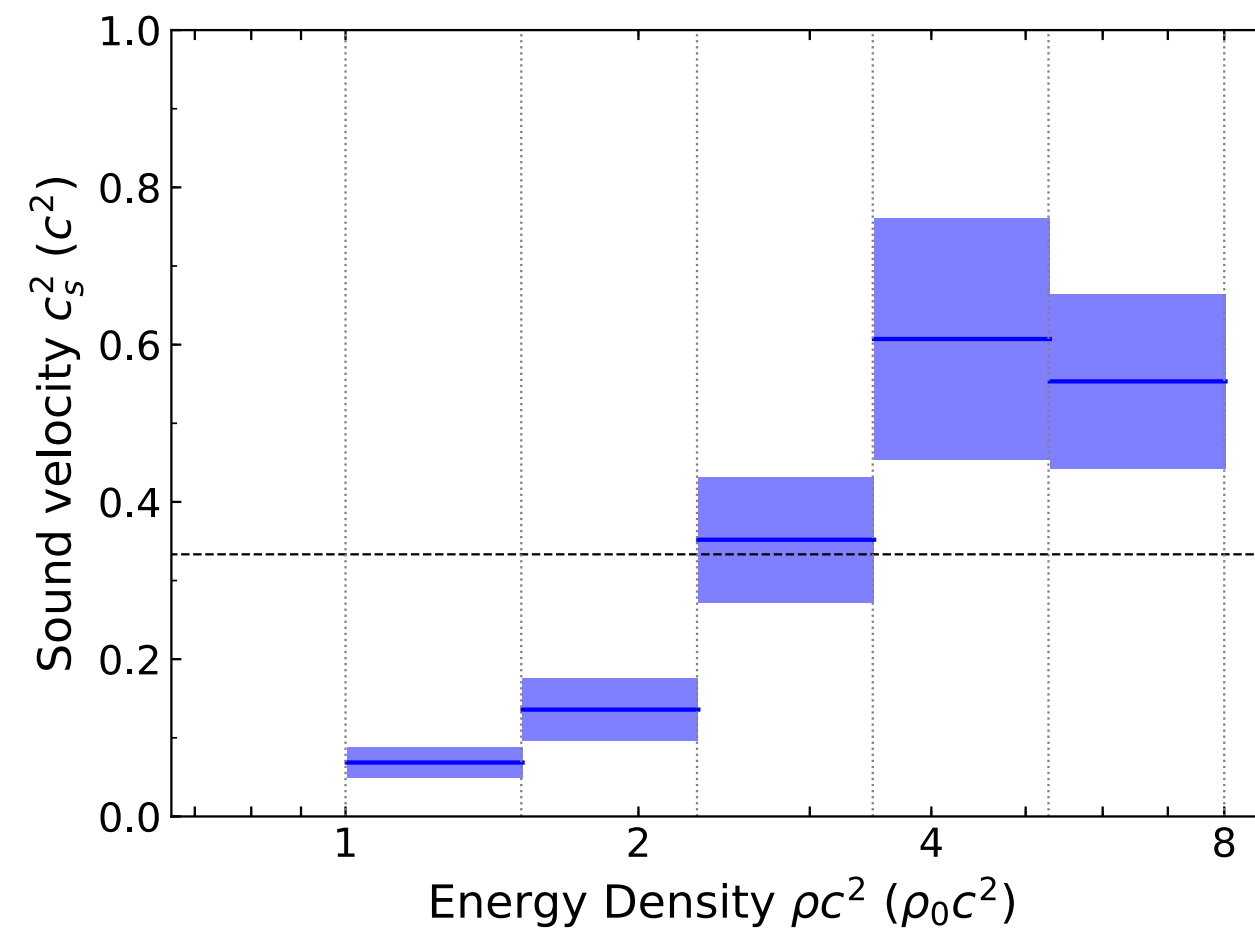
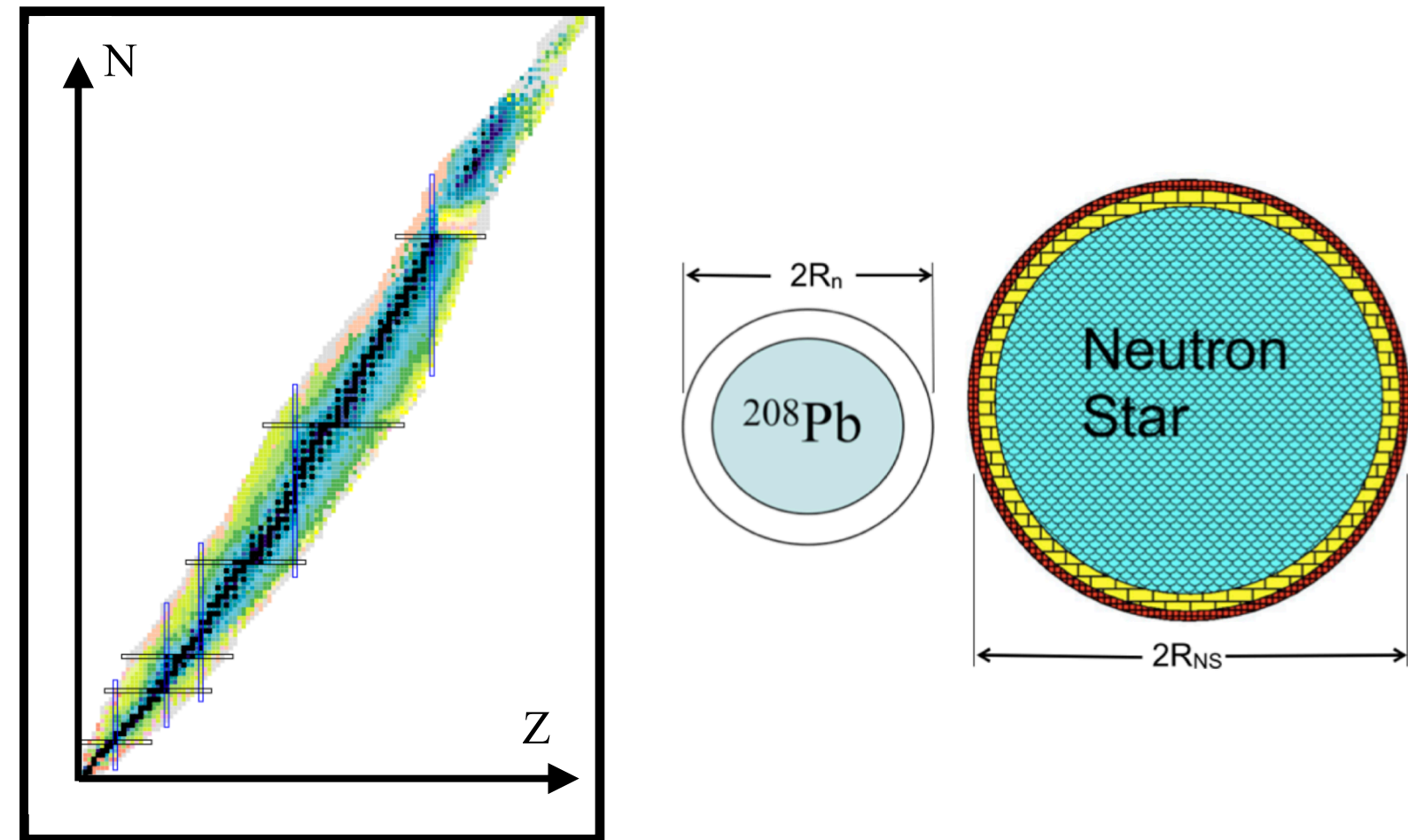
J. Xu, A. Carbone, Z. Zhang, C.-M. Ko, Phys. Rev. C **100**, 2, 024618 (2019) arXiv:1904.09669



* from S. Reddy's slides;
M-R results: C. Drischler, S. Han, J. M. Lattimer, M. Prakash, S. Reddy, T. Zhao, Phys. Rev. C **103** 4, 045808 (2021), arXiv:2009.06441



S. Borsanyi, Z. Fodor, C. Hoelbling, S. Katz, S. Krieg, K. K. Szabo, Phys. Lett. B **730** 99–104 (2014) arXiv:1309.5258



Y. Fujimoto, K. Fukushima, K. Murase, Phys. Rev. D **101**, 5, 054016 (2020), arXiv:1903.03400

The EOS is a common effort within the nuclear physics community

A. Sorensen *et al.*, arXiv:2301.13253

Dense Nuclear Matter Equation of State from Heavy-Ion Collisions *

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Endorsing authors:

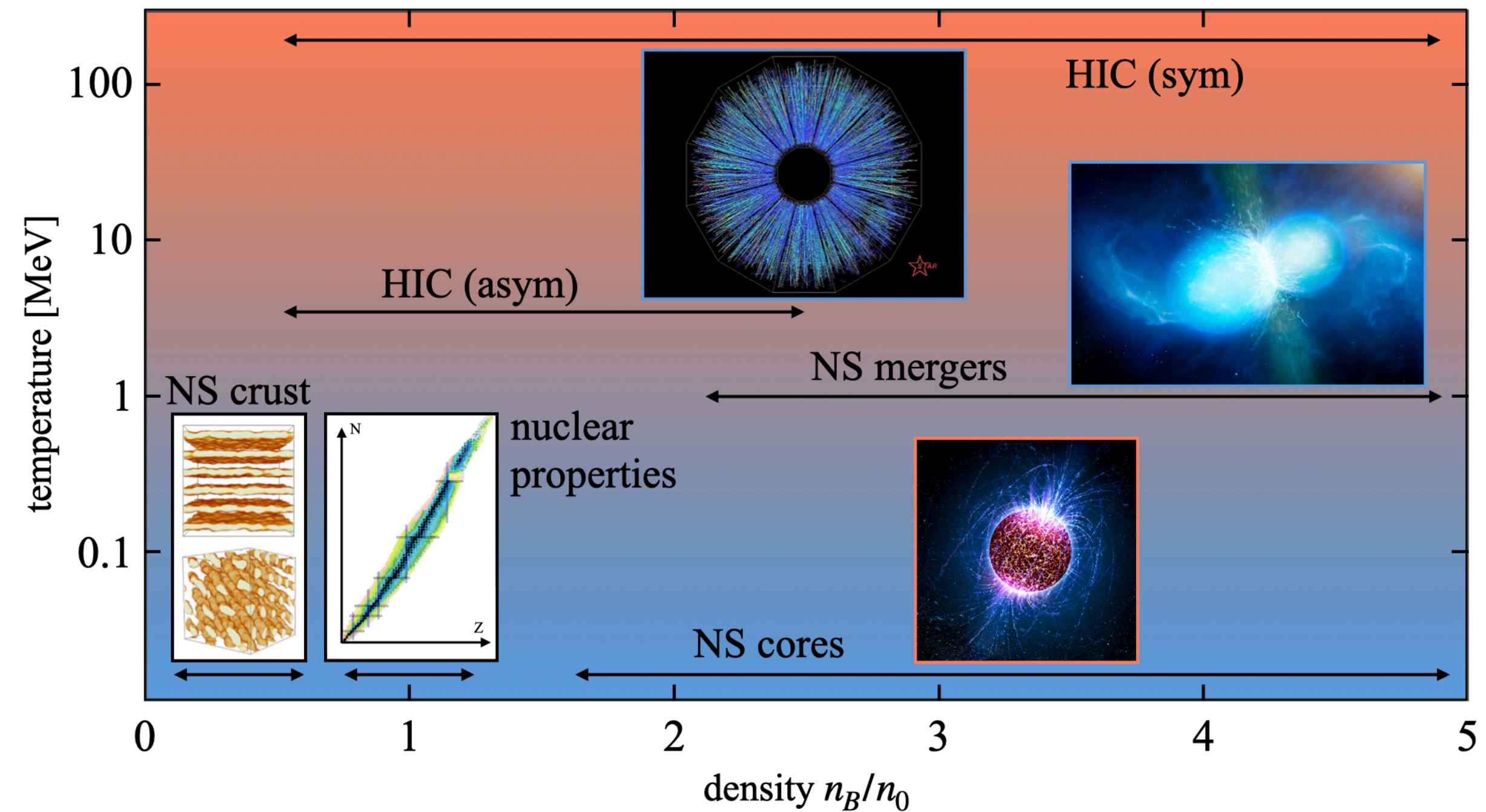
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Hot QCD

Cold QCD

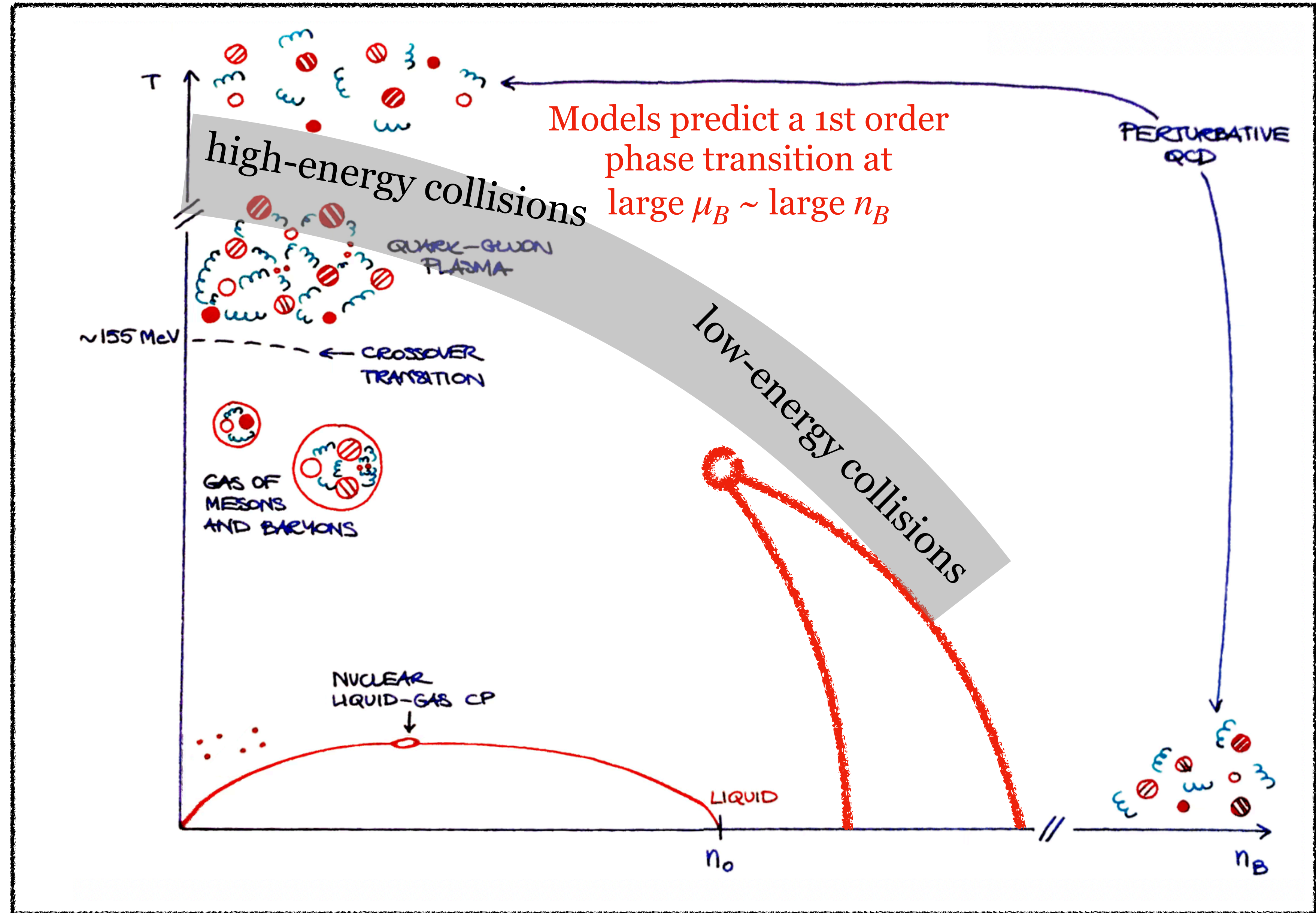
Nucl. reactions + astro

Fundamental sym.

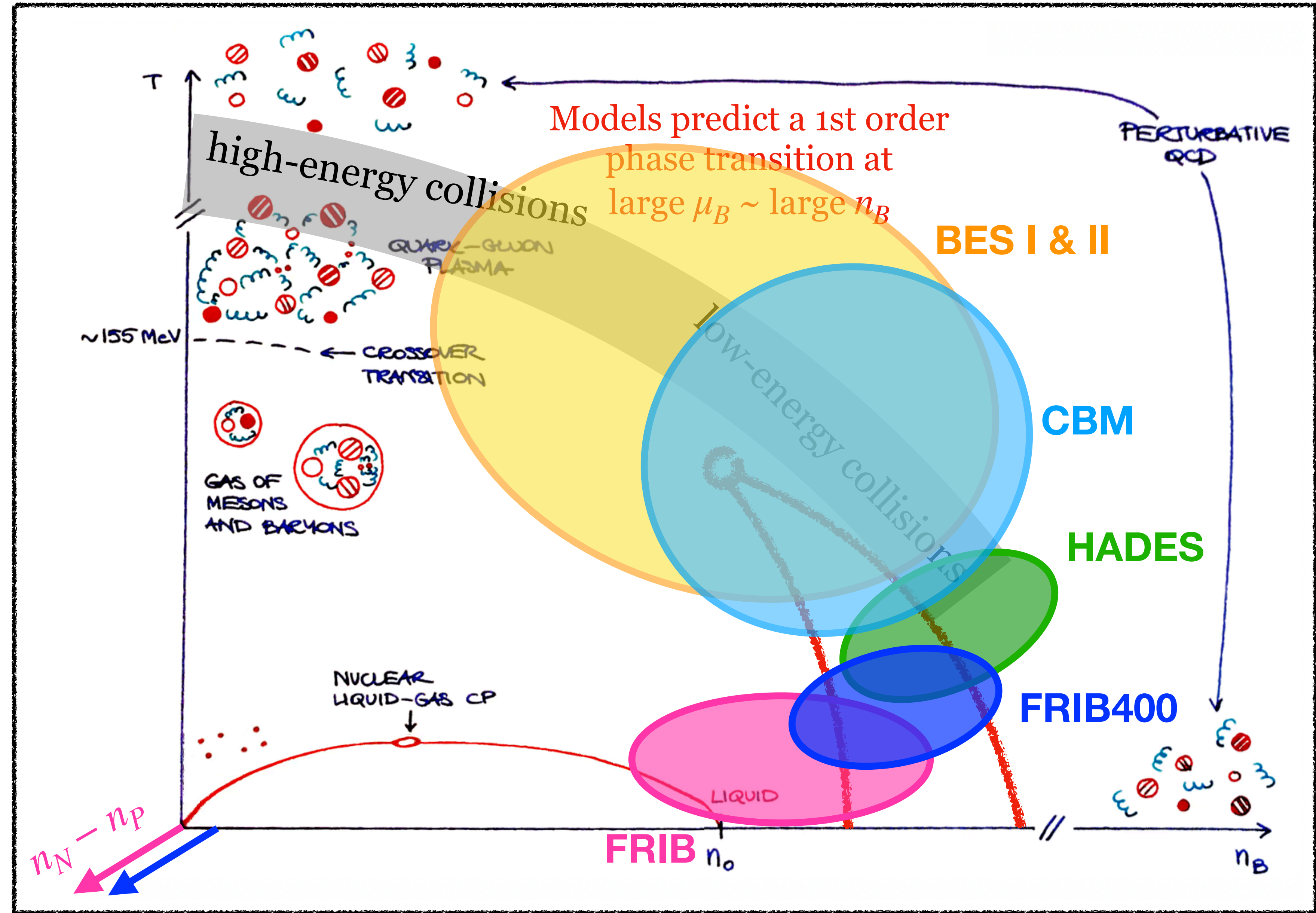


arXiv:2301.13253v2 [nucl-th] 25 Feb 2023

The QCD phase diagram: great interest in behavior at high n_B

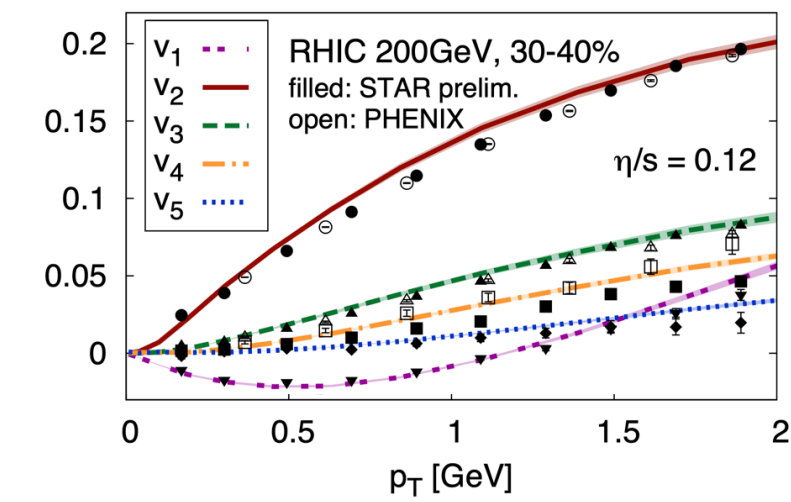


The QCD phase diagram: great interest in behavior at high n_B



The QCD phase diagram: great interest in behavior at high n_B

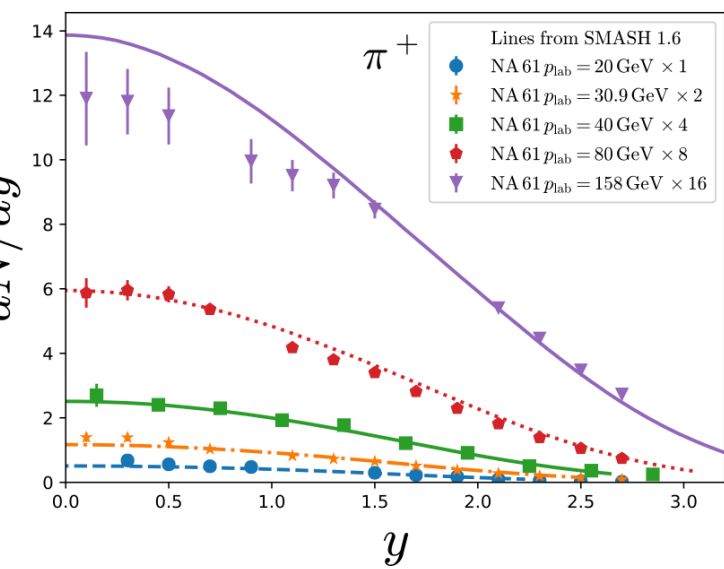
Relativistic viscous hydrodynamic simulations with LQCD EOS: amazing agreement with data from high-energy collisions



C. Gale, S. Jeon, B. Schenke, P. Tribedy, R. Venugopalan, Phys. Rev. Lett. **110** (2013) 1, 012302, arXiv:1209.6330

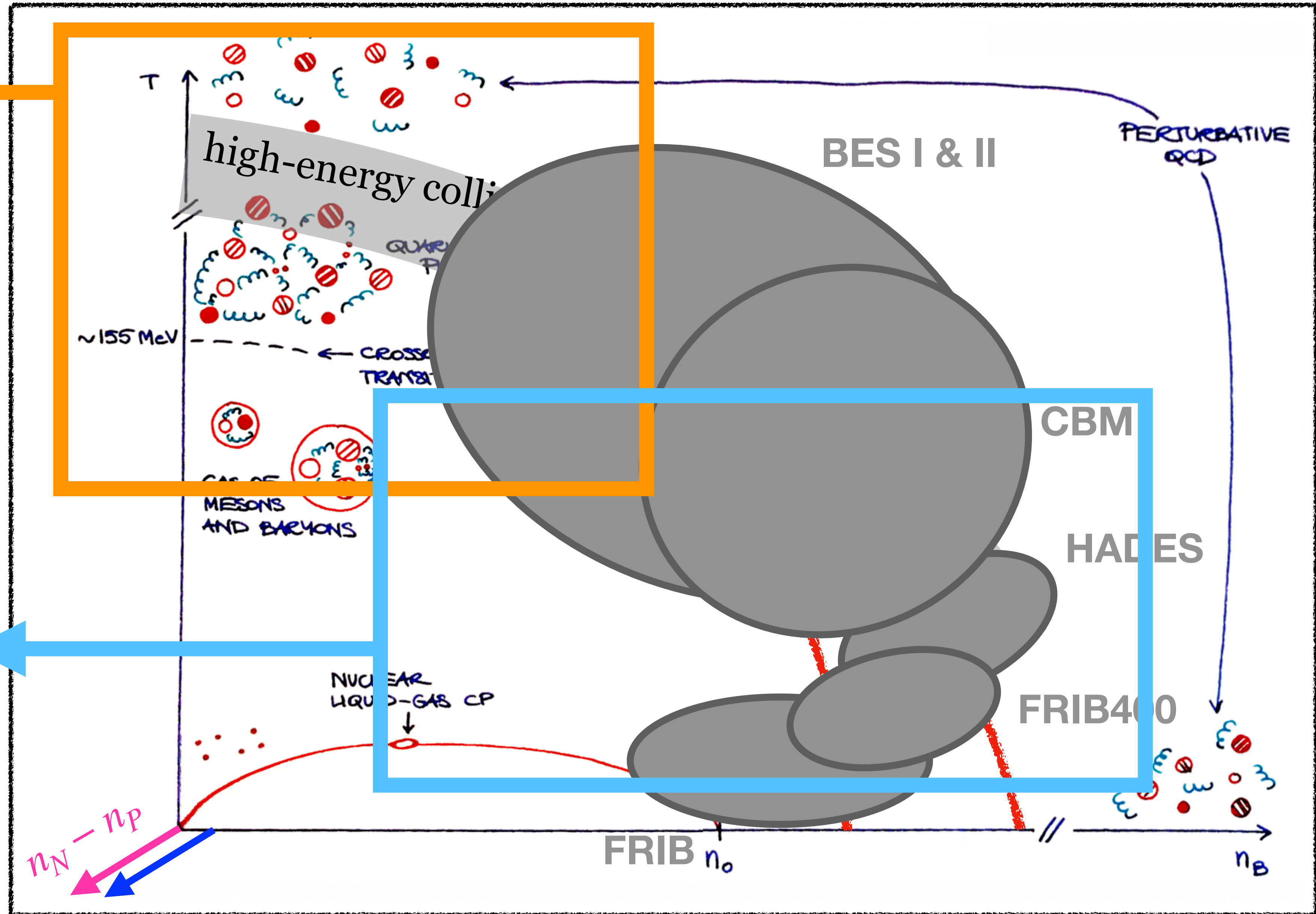
fast equilibration = hydro applies

Hadronic transport simulations:



systems out of equilibrium = microscopic approach needed

J. Mohs, S. Ryu, H. Elfner, J. Phys. G **47** (2020) 6, 065101 arXiv:1909.05586



Transport model simulations of heavy-ion collisions

- Boltzmann-Uehling-Uhlenbeck (BUU)-type codes:

- solve coupled Boltzmann equations

$$\forall i : \quad \frac{\partial f_i}{\partial t} + \frac{d\mathbf{x}_i}{dt} \frac{\partial f_i}{\partial \mathbf{x}_i} + \frac{d\mathbf{p}_i}{dt} \frac{\partial f_i}{\partial \mathbf{p}_i} = I_{\text{coll}}^{(i)}$$

with the method of test particles: the distribution is *oversampled* with a *large* number of discrete test-particles, which are evolved according to the single-particle EOMs (test particles probe the evolution in the phase space)

- forces from gradients of single-particle energies (mean-fields: needs a robust density calculation!)

- collision term based on measured cross-sections for scatterings and decays

- Quantum Molecular Dynamics (QMD)-type codes

- solve molecular dynamics problem (evolve nucleons according to their EOMs)

- forces: in principle distance-dependent particle-particle interactions, in practice: often mean-fields!

- collisions based on measured cross-sections for scatterings and decays

Transport model simulations of heavy-ion collisions

- Boltzmann-Uehling-Uhlenbeck (BUU)-type codes:

- solve coupled Boltzmann equations

$$\forall i : \quad \frac{\partial f_i}{\partial t} + \frac{d\mathbf{x}_i}{dt} \frac{\partial f_i}{\partial \mathbf{x}_i} + \frac{d\mathbf{p}_i}{dt} \frac{\partial f_i}{\partial \mathbf{p}_i} = I_{\text{coll}}^{(i)}$$

with the method of
test-particles,
(test particles)

- forces from gradients

- collision term

Transport *automatically* includes:

- non-equilibrium evolution, including triggered by probing unstable regions of the phase diagram
- effects due to the interplay between participants and spectators
- baryon, strangeness, charge transport/diffusion

number of discrete

per calculation!)

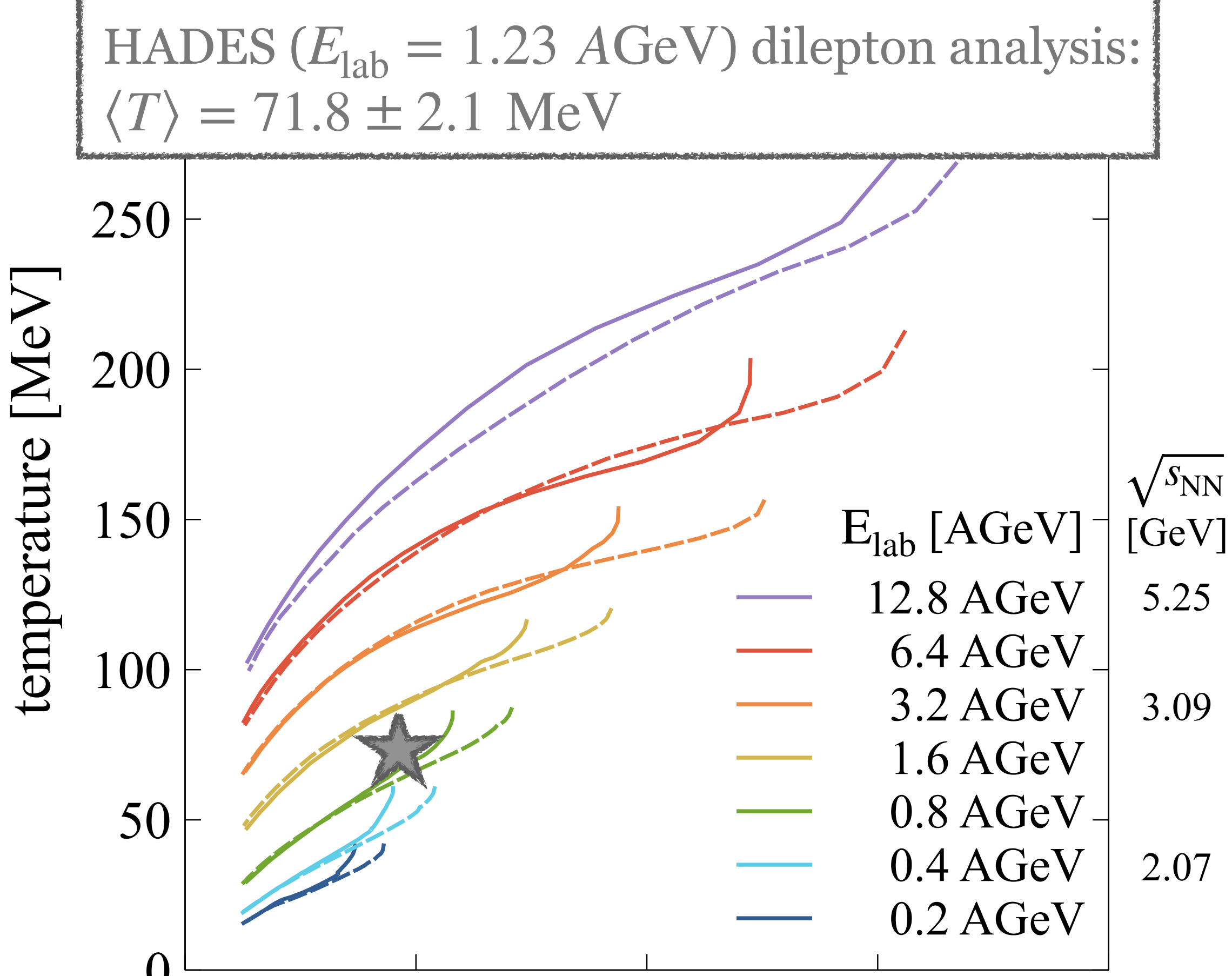
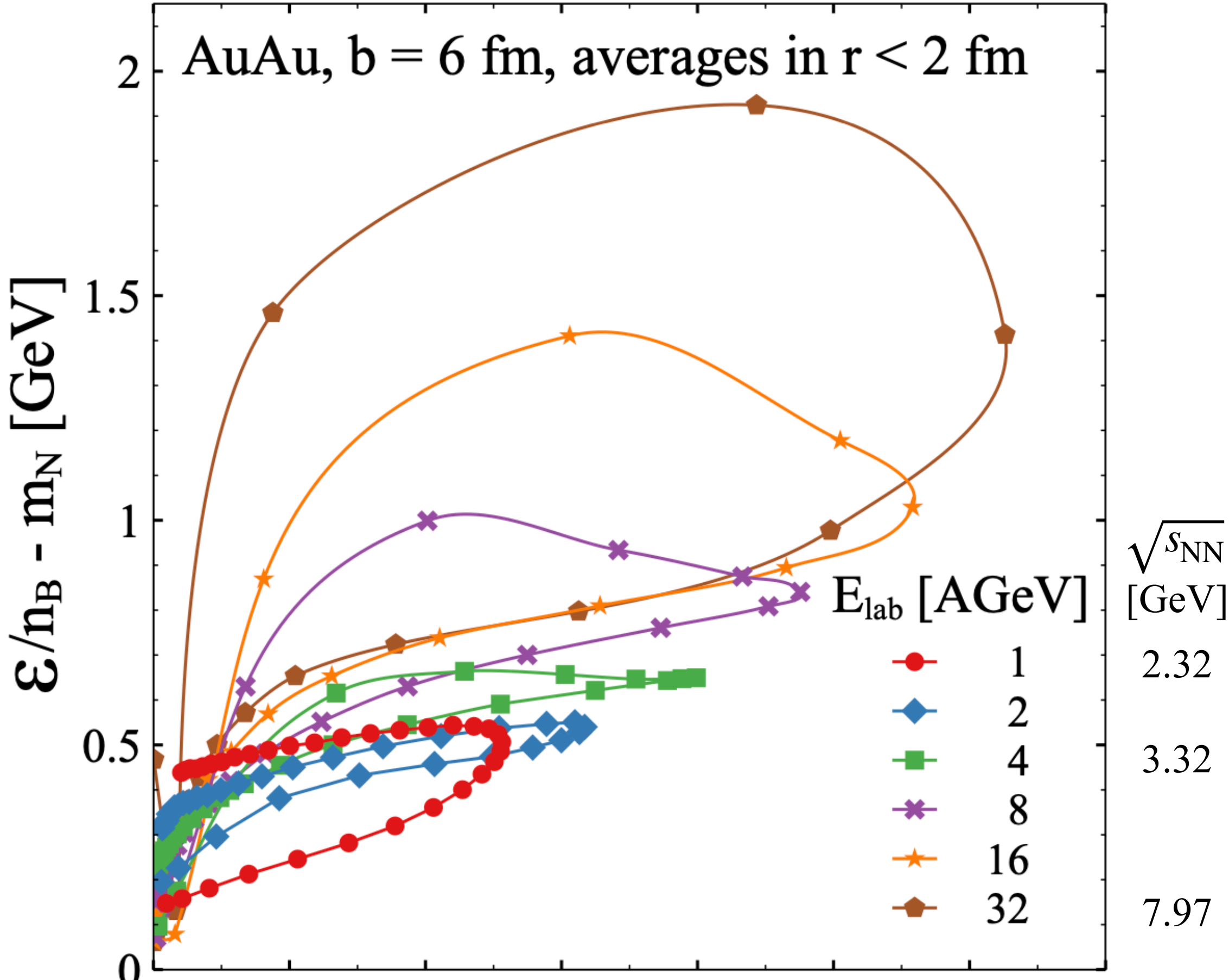
- Quantum Molecular Dynamics

- solve molecular dynamics

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- collisions based on measured cross-sections for scatterings and decays

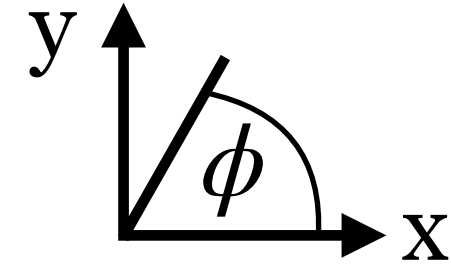
Intermediate-energy heavy-ion collisions probe wide ranges of density and temperature



- HICs = the only means to probe densities away from n_0 in controlled terrestrial experiments
- Hadronic transport is necessary to interpret the results: BES FXT, HADES, CBM, FRIB, FRIB400

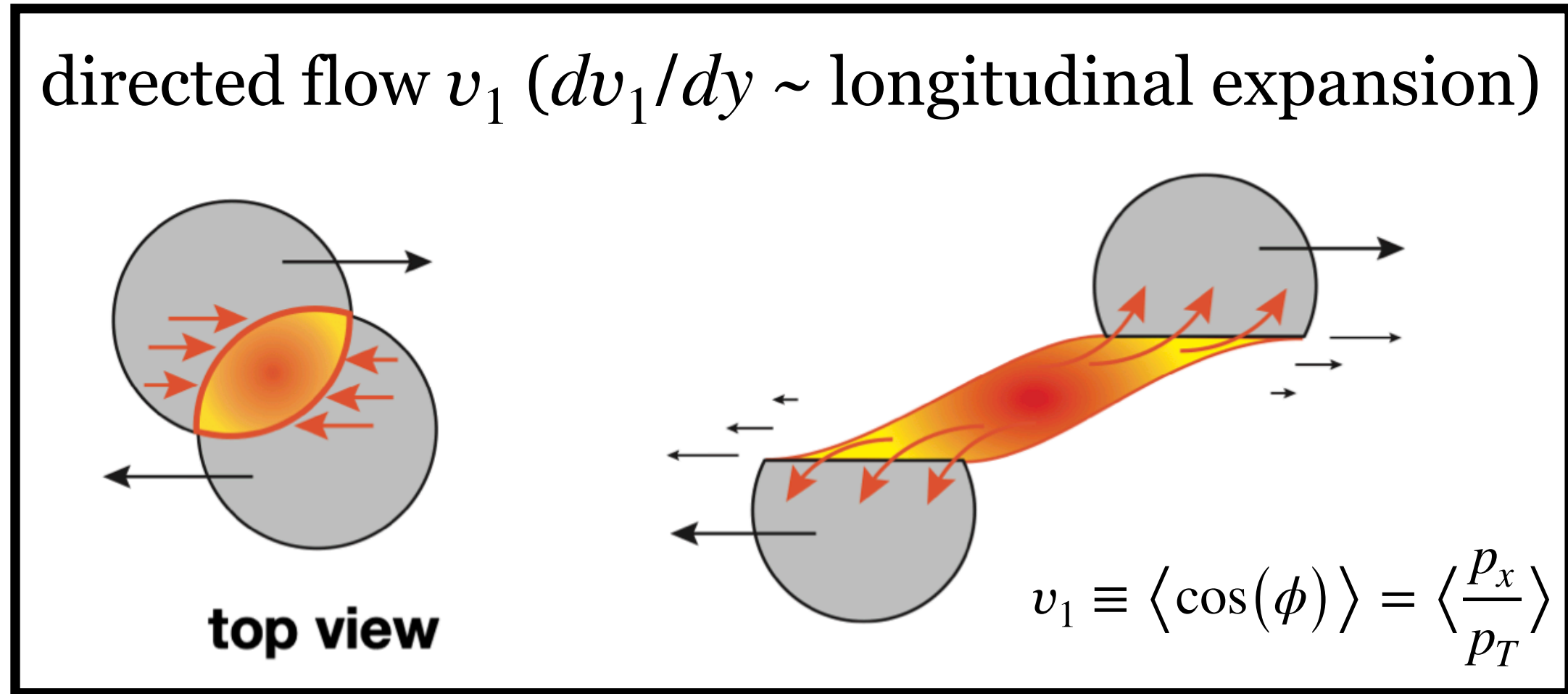
EOS from flow observables in heavy-ion collisions

Flow $v_n \equiv \langle \cos(n\phi) \rangle$



J. Adamczewski-Musch *et al.* (HADES),
arXiv:2208.02740

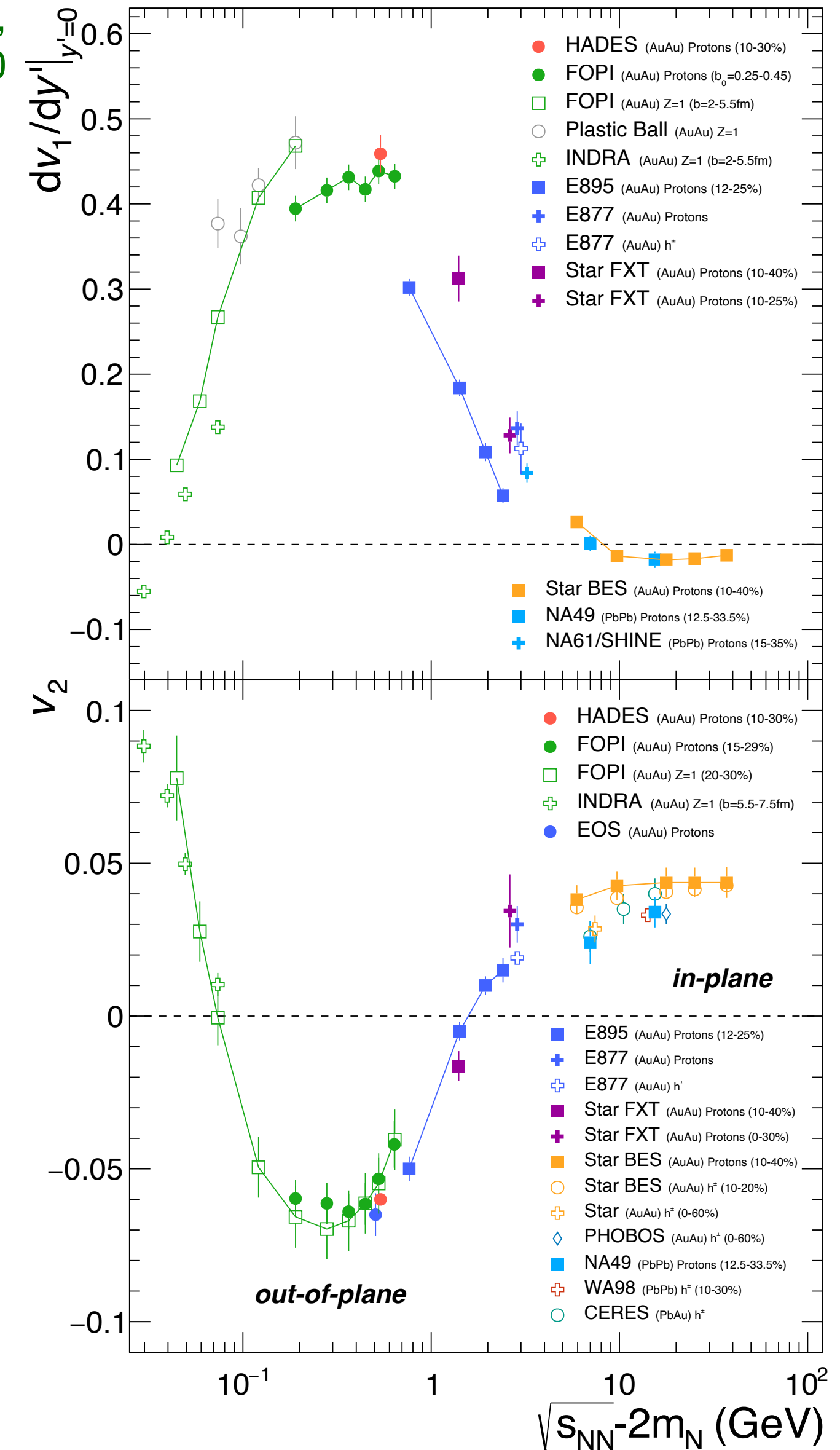
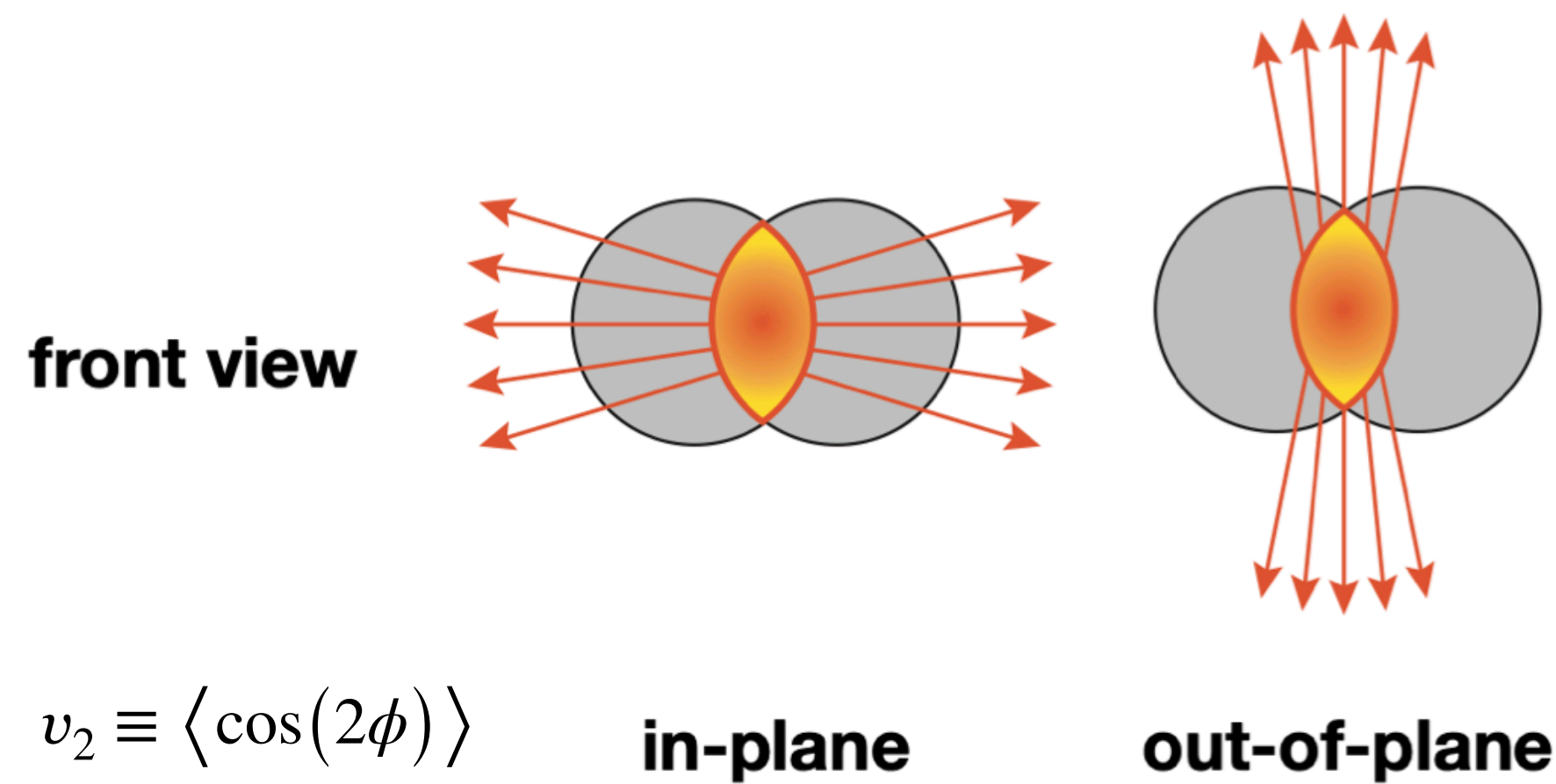
directed flow v_1 ($dv_1/dy \sim$ longitudinal expansion)



These observables
are extremely
sensitive to the EOS

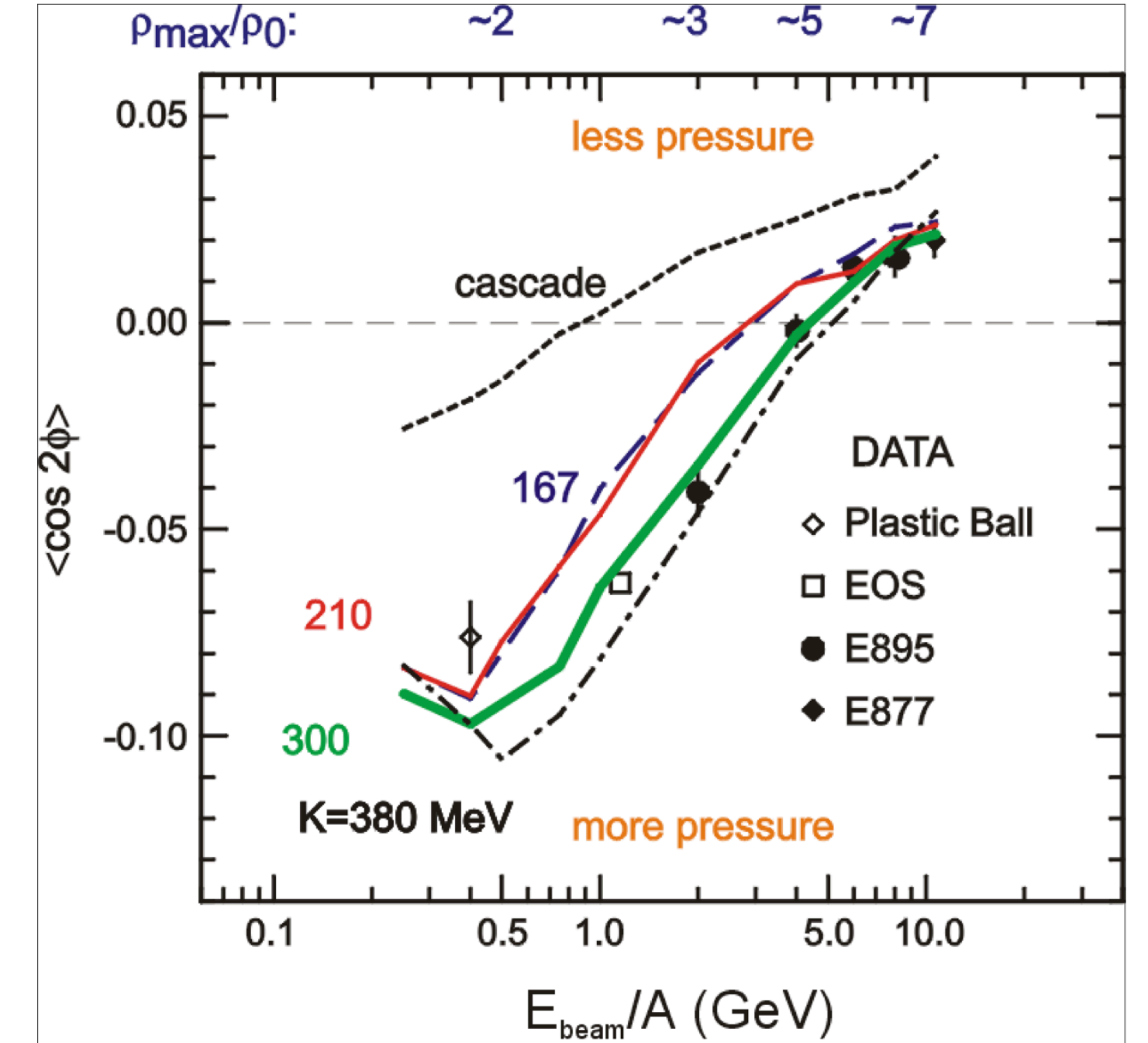
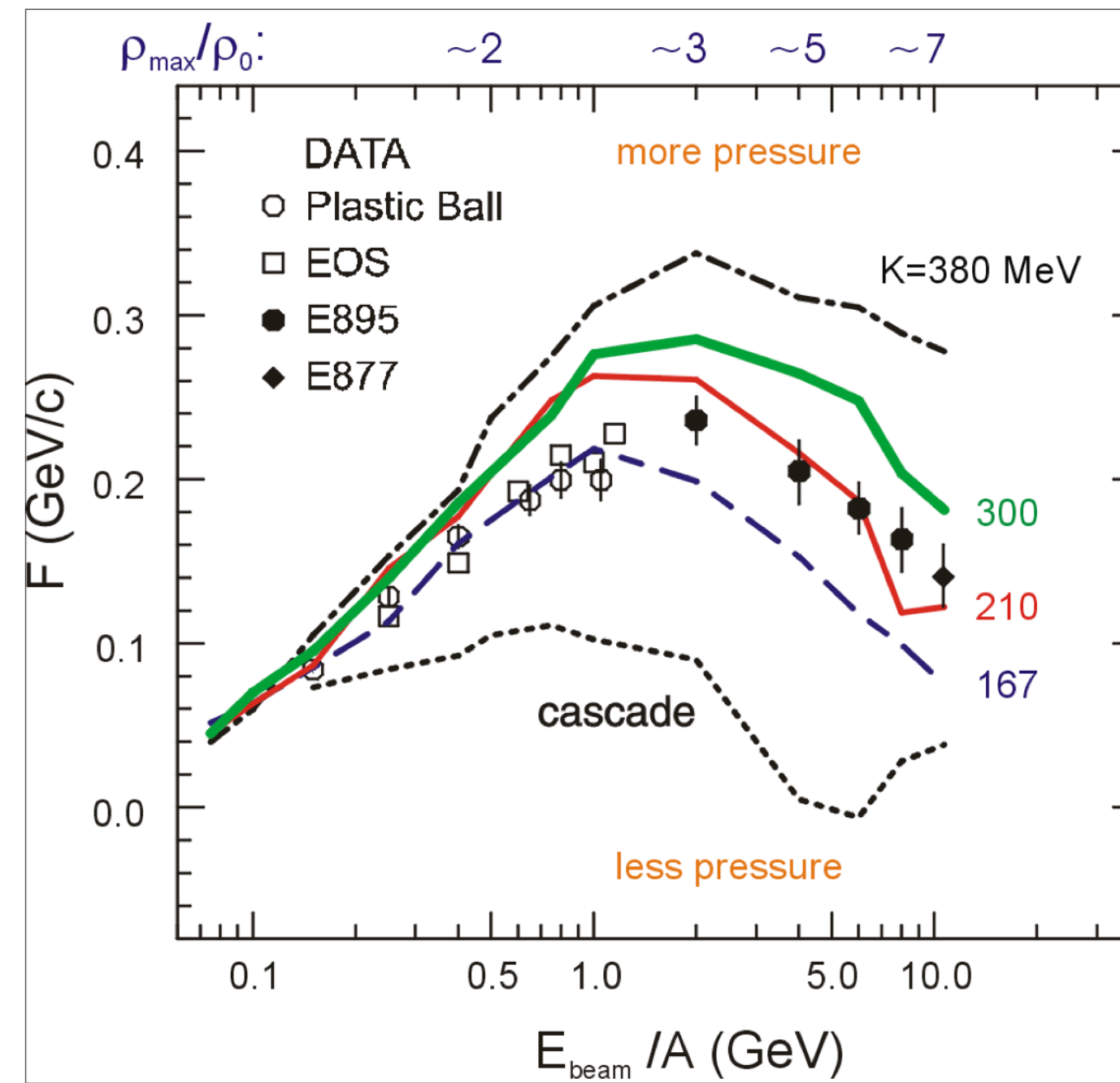
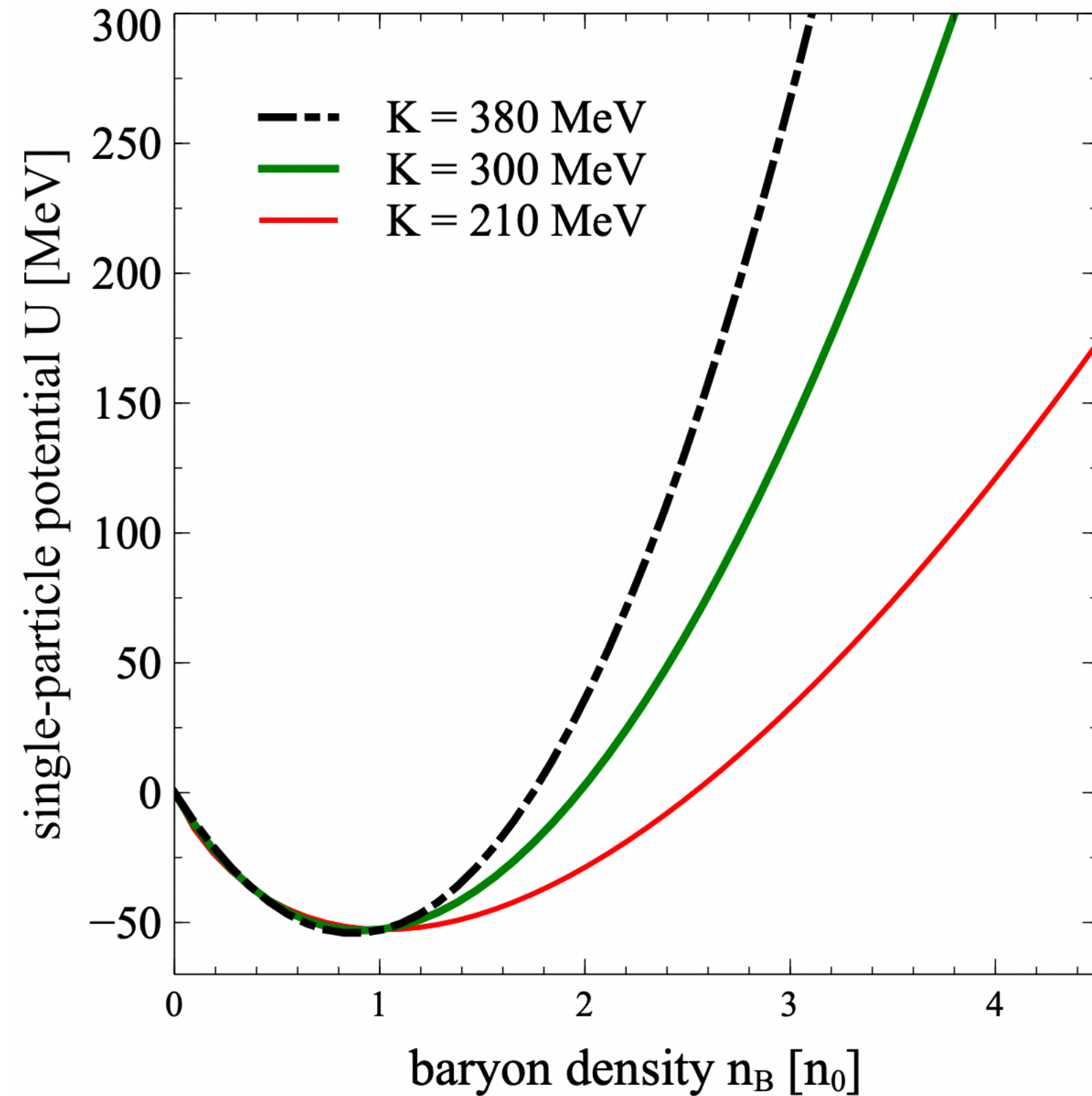
illustrations from a presentation
by B. Kardan (HADES)

elliptic flow v_2 ($v_2(y \approx 0) \sim$ midrapidity)



Standard way of modeling the EOS: Skyrme potential

The most common form of the EOS is the “Skyrme potential”: $U(n_B) = A \left(\frac{n_B}{n_0} \right) + B \left(\frac{n_B}{n_0} \right)^\tau$



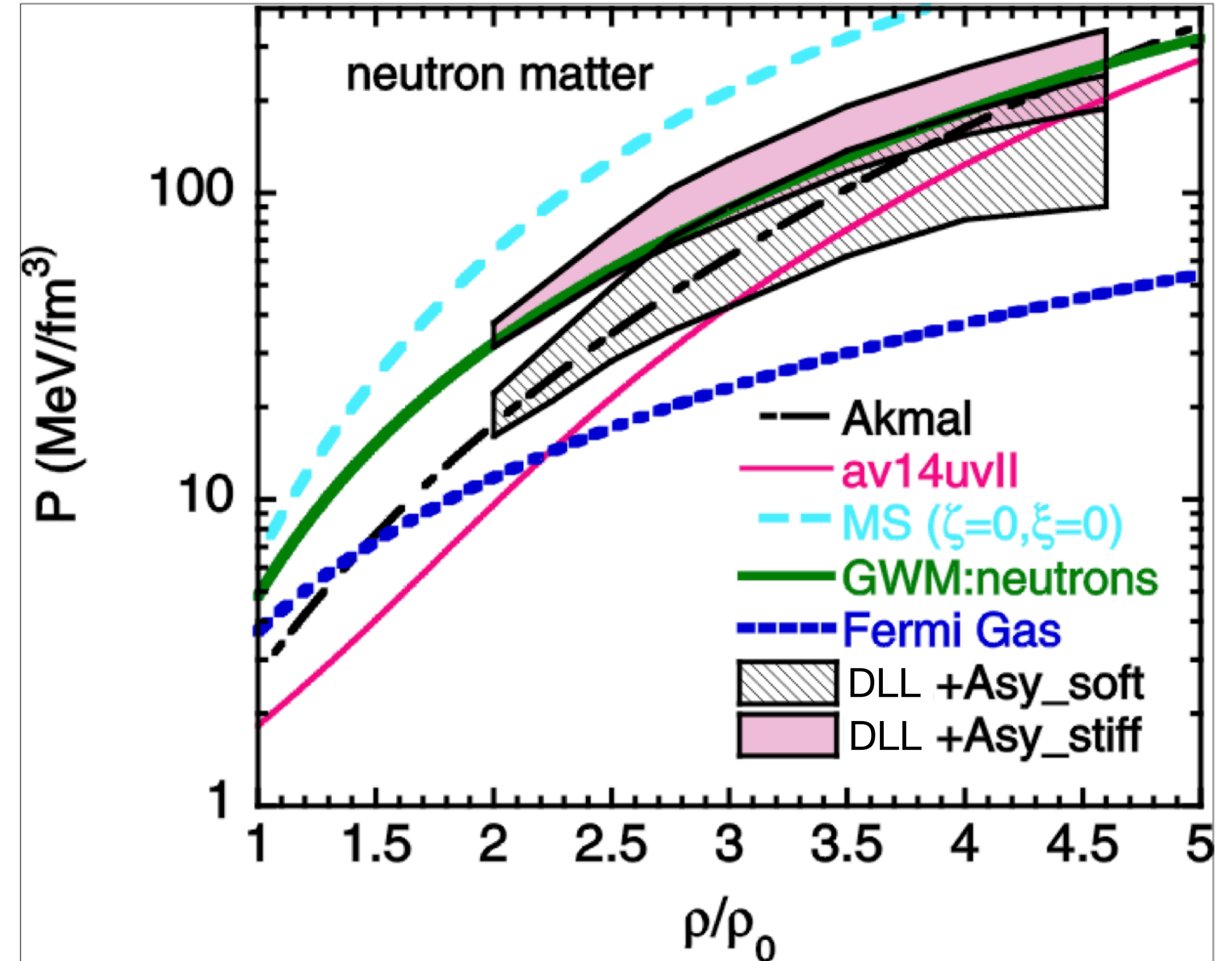
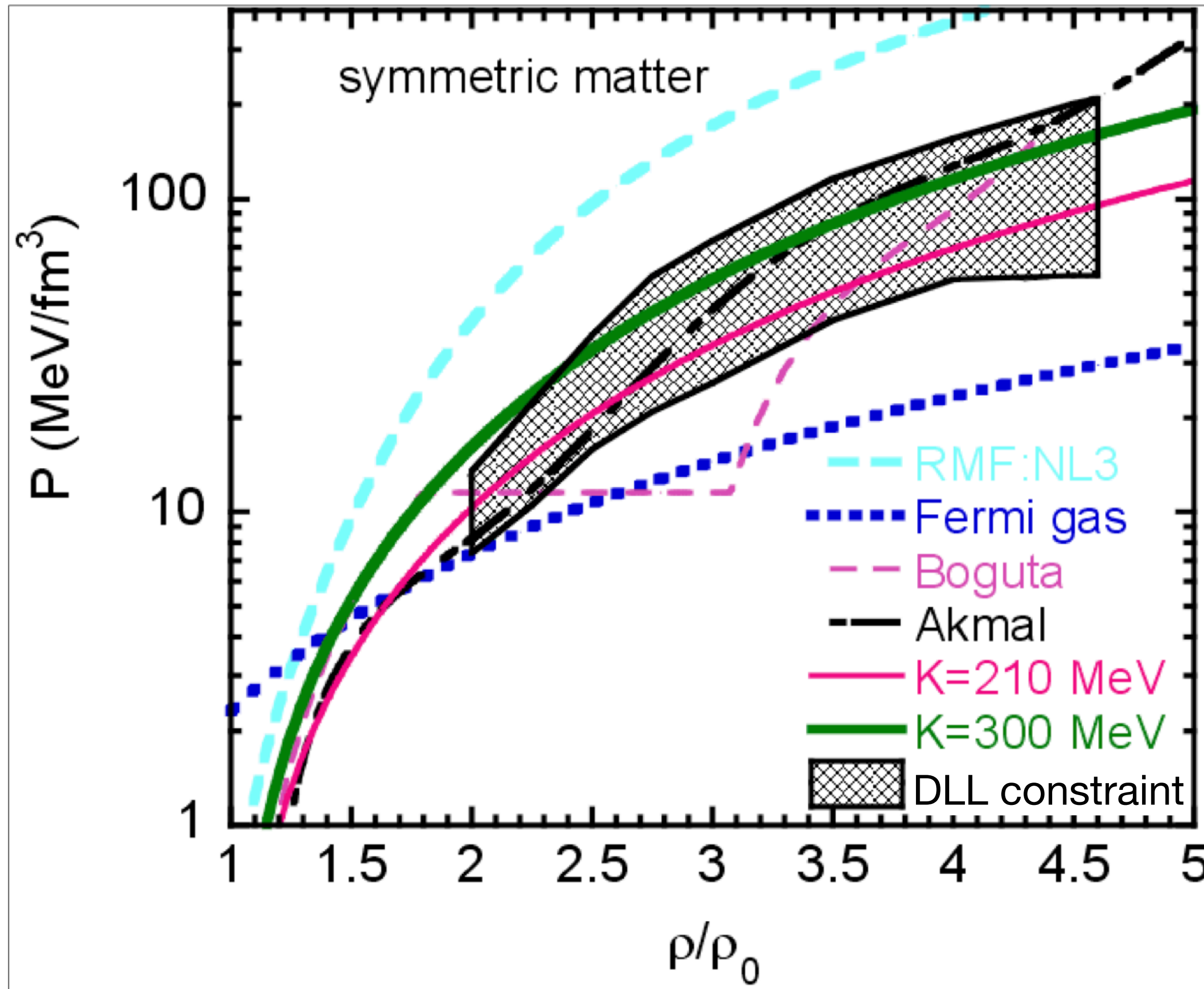
$$F = \left. \frac{d\langle p_x/A \rangle}{d(y/y_{\text{cm}})} \right|_{y/y_{\text{cm}}=1}$$

P. Danielewicz, R. Lacey, W. G. Lynch,
Science **298**, 1592–1596 (2002), arXiv:nucl-th/0208016

Standard way of modeling the EOS: Skyrme potential

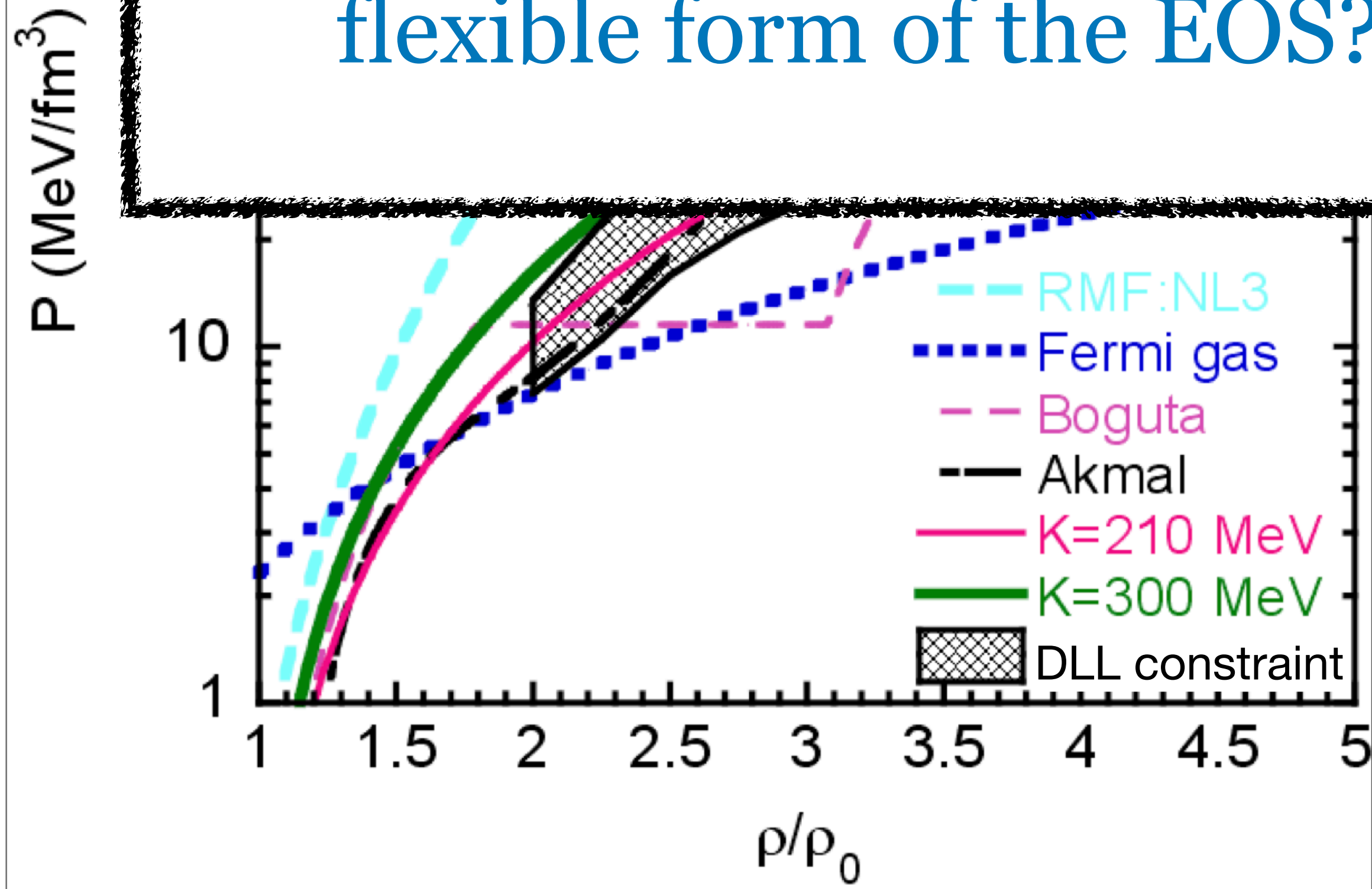
P. Danielewicz, R. Lacey, W. G. Lynch,
Science **298**, 1592–1596 (2002), arXiv:nucl-th/0208016

“the heavy-ion constraint”

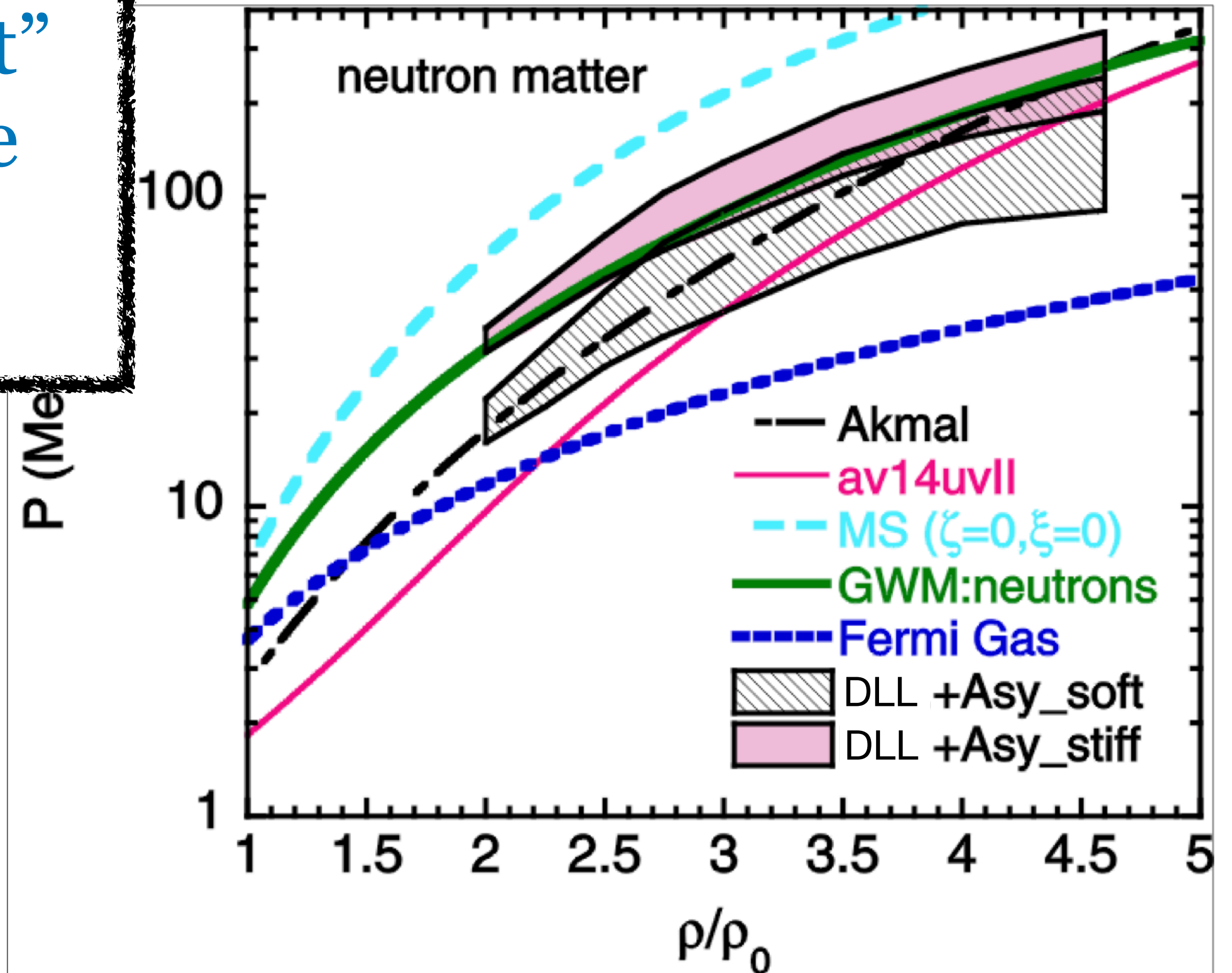


Standard way of modeling the EOS: Skyrme potential

Can the “heavy-ion constraint” be improved by using a more flexible form of the EOS?



“the heavy-ion constraint”



Relativistic vector density functional (VDF) model

A. Sorensen, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

inspired by relativistic Landau Fermi-liquid theory: G. Baym, S. A. Chin, Nucl. Phys. A **262**, 527 (1976)

$$j_\mu j^\mu = n_B^2$$

$$\epsilon_{\text{kin}} = \sqrt{\left(\vec{p} - \sum_{i=1}^N C_i (j_\mu j^\mu)^{\frac{b_i}{2}-1} \vec{j}\right)^2 + m^2}$$

1) Postulate the energy density of the system:

$$\mathcal{E}_N = \mathcal{E}_N[f_{\mathbf{p}}] = g \int \frac{d^3p}{(2\pi)^3} \epsilon_{\text{kin}} f_{\mathbf{p}} + \sum_{i=1}^N C_i (j_\mu j^\mu)^{\frac{b_i}{2}-1} \left[j^0 j^0 - g^{00} \left(\frac{b_i-1}{b_i} \right) j_\lambda j^\lambda \right] \leftarrow \text{Lorentz covariant}$$

$$\mathcal{E}_N \Big|_{\text{rest frame}} = g \int \frac{d^3p}{(2\pi)^3} \sqrt{\vec{p}^2 + m^2} f_{\mathbf{p}} + \sum_{i=1}^N \frac{C_i}{b_i} n_B^{b_i} \leftarrow \text{mean-field interactions parameterized by } C_i \text{ and } b_i$$

thermodynamic consistency!

2) Quasiparticle energy:

$$\epsilon_{\mathbf{p}} \equiv \frac{\delta \mathcal{E}[f_{\mathbf{p}}]}{\delta f_{\mathbf{p}}} = \epsilon_{\text{kin}} + \sum_{i=1}^N C_i (j_\mu j^\mu)^{\frac{b_i}{2}-1} j^0$$

3) Get EOMs:

$$\frac{dx^i}{dt} \equiv -\frac{\partial \epsilon_{\mathbf{p}}}{\partial p_i}, \quad \frac{dp^i}{dt} \equiv \frac{\partial \epsilon_{\mathbf{p}}}{\partial x_i} \leftarrow \text{input to transport code; use in Boltzmann eq. to obtain } T^{\mu\nu}$$

4) Use $T^{\mu\nu}$ to get the pressure:

$$P_N = \frac{1}{3} \sum_k T^{kk} \Big|_{\text{rest frame}} = g \int \frac{d^3p}{(2\pi)^3} T \ln \left[1 + e^{-\beta(\epsilon_{\mathbf{p}} - \mu_B)} \right] + \sum_{i=1}^N C_i \frac{b_i-1}{b_i} n_B^{b_i}$$

VDF model: two 1st order phase transitions

A. Sorensen, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

Systems with two 1st order phase transitions: nuclear and “quark/hadron”, or “QGP-like”

- degrees of freedom: nucleons
- “QGP-like” PT: “more dense” matter coexists with “less dense” matter
- minimal model: 4 interactions terms = 8 parameters to fix:

$$P = g \int \frac{d^3p}{(2\pi)^3} T \ln \left[1 + e^{-\beta(\varepsilon_p - \mu_B)} \right] + \sum_{i=1}^{N=4} C_i \frac{b_i - 1}{b_i} n_B^{b_i}$$

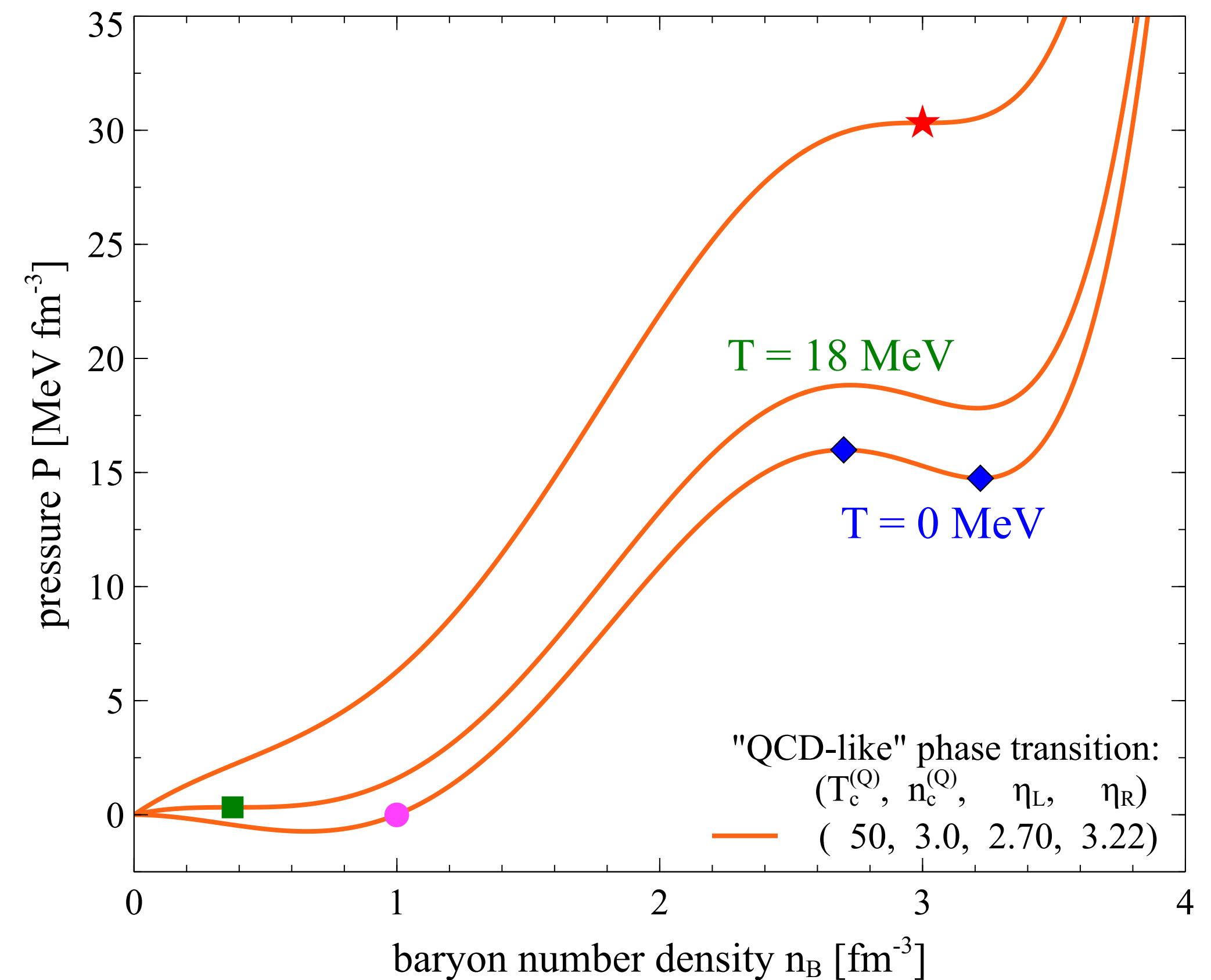
C_i and b_i are fitted to reproduce:

$n_0 = 0.160 \text{ fm}^{-3}$, $E_B = -16.3 \text{ MeV}$

$T_c^{(N)} = 18 \text{ MeV}$, $n_c^{(N)} = 0.375 n_0$

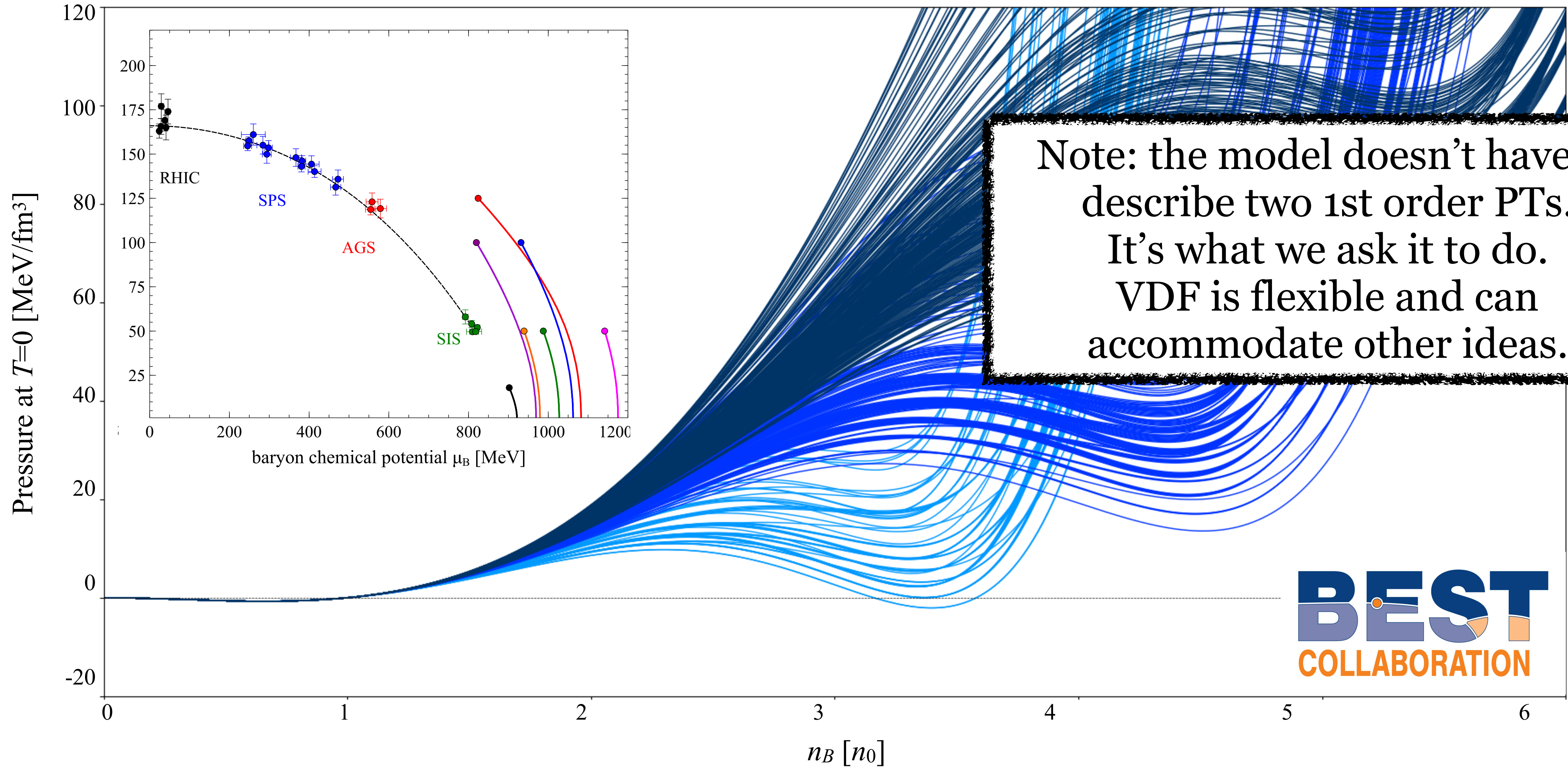
$T_c^{(Q)} = ?$, $n_c^{(Q)} = ?$

$\eta_L = ?$, $\eta_R = ?$



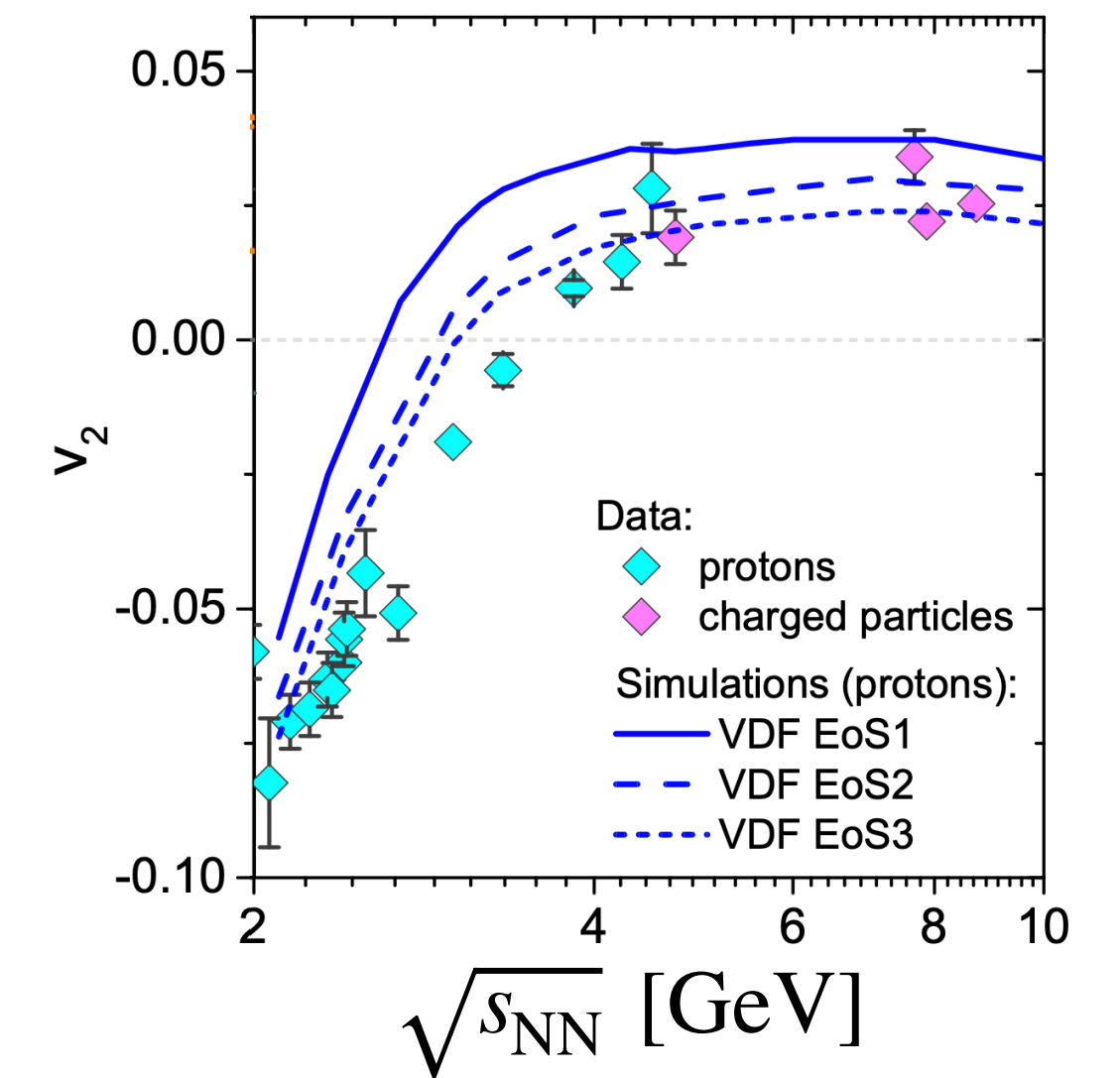
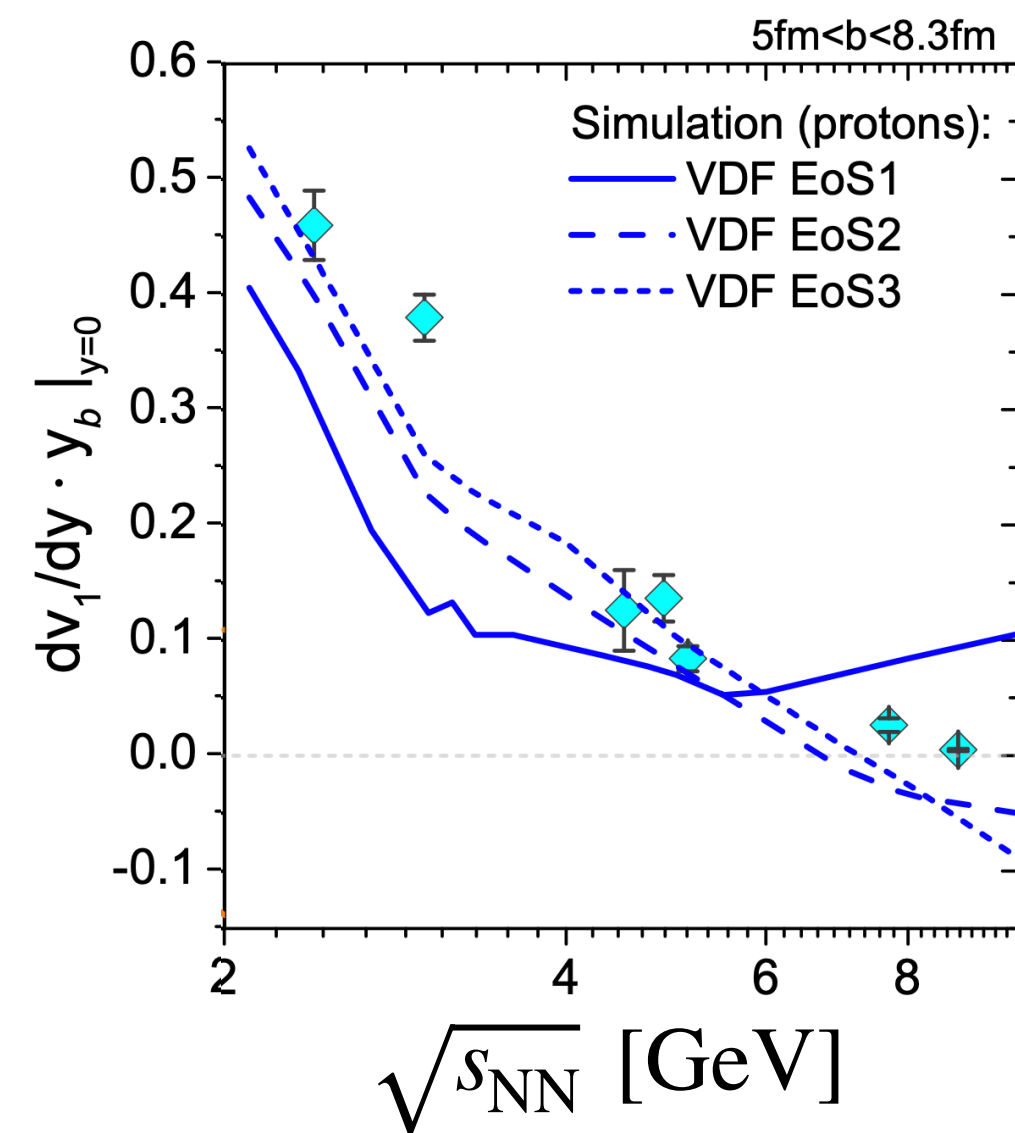
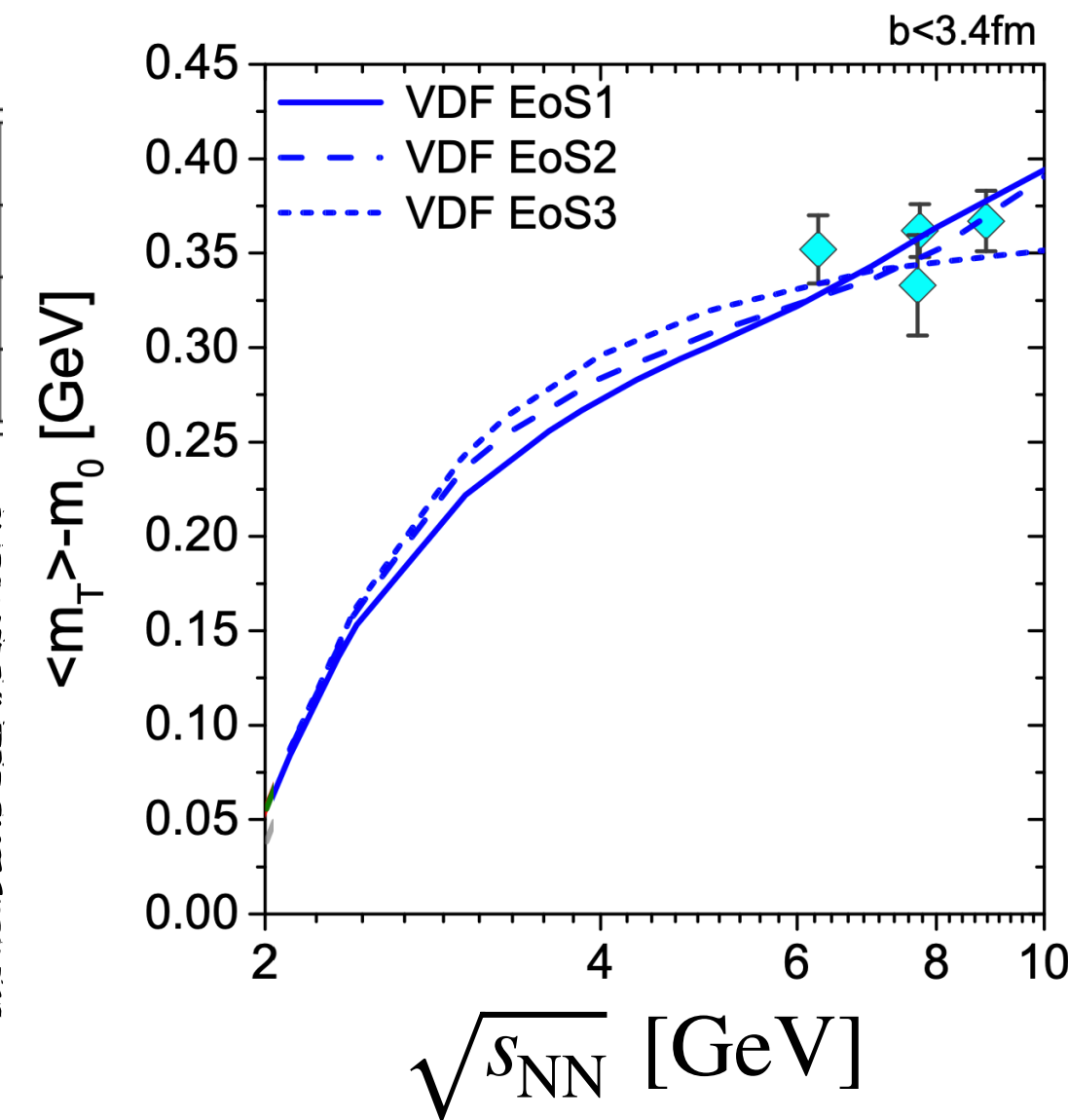
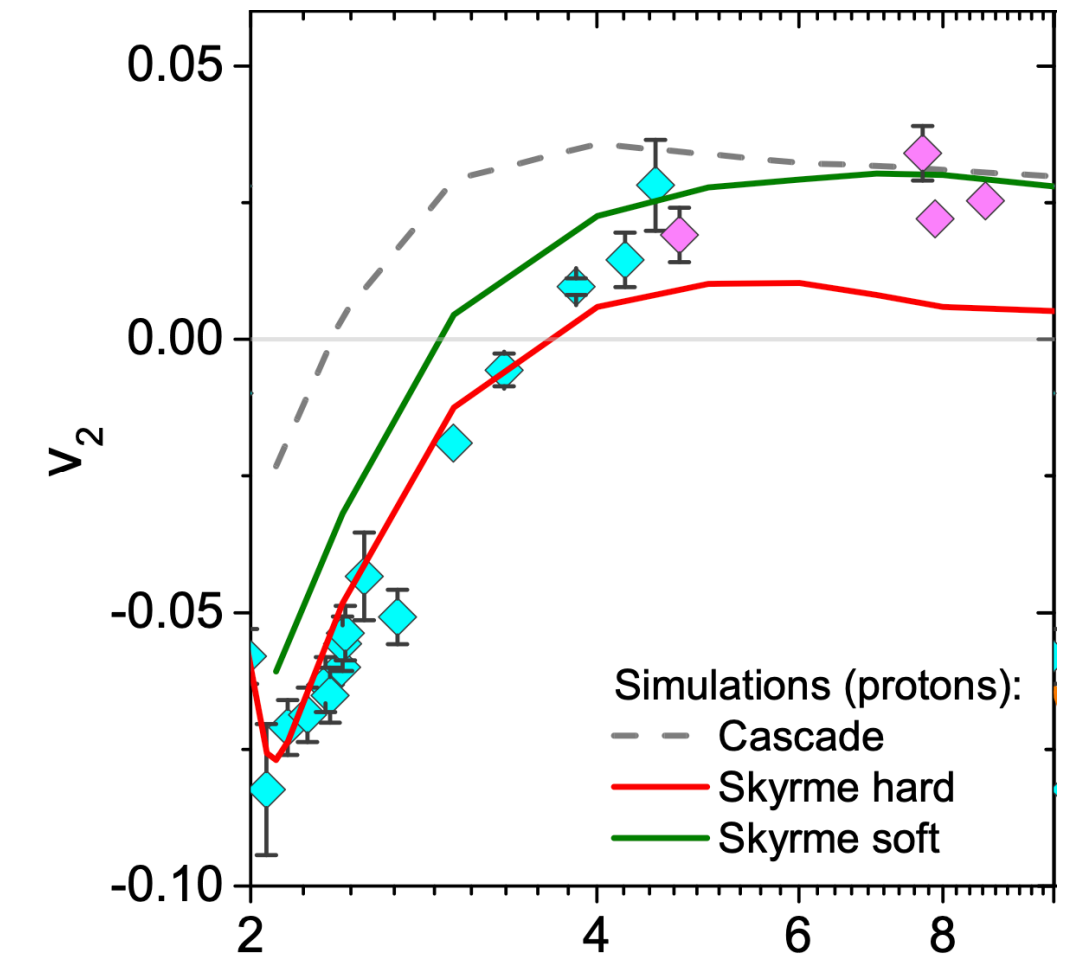
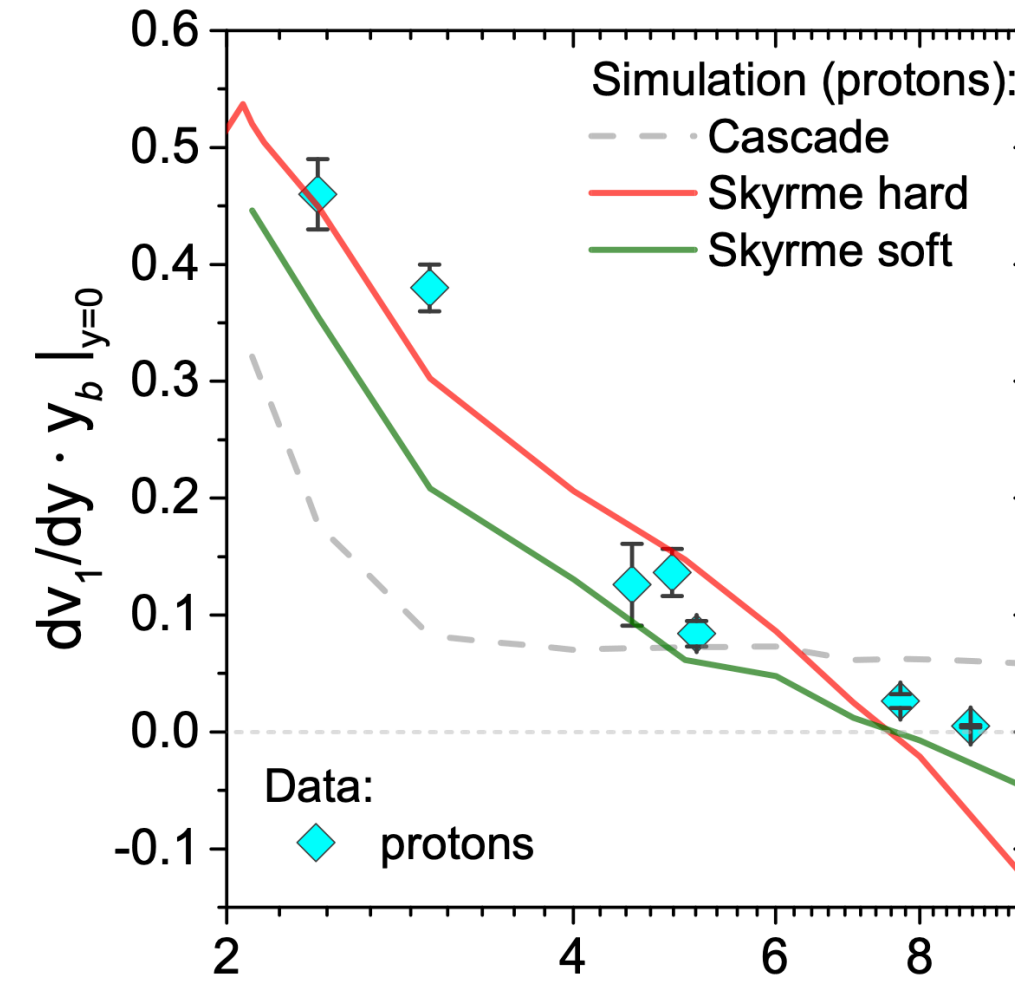
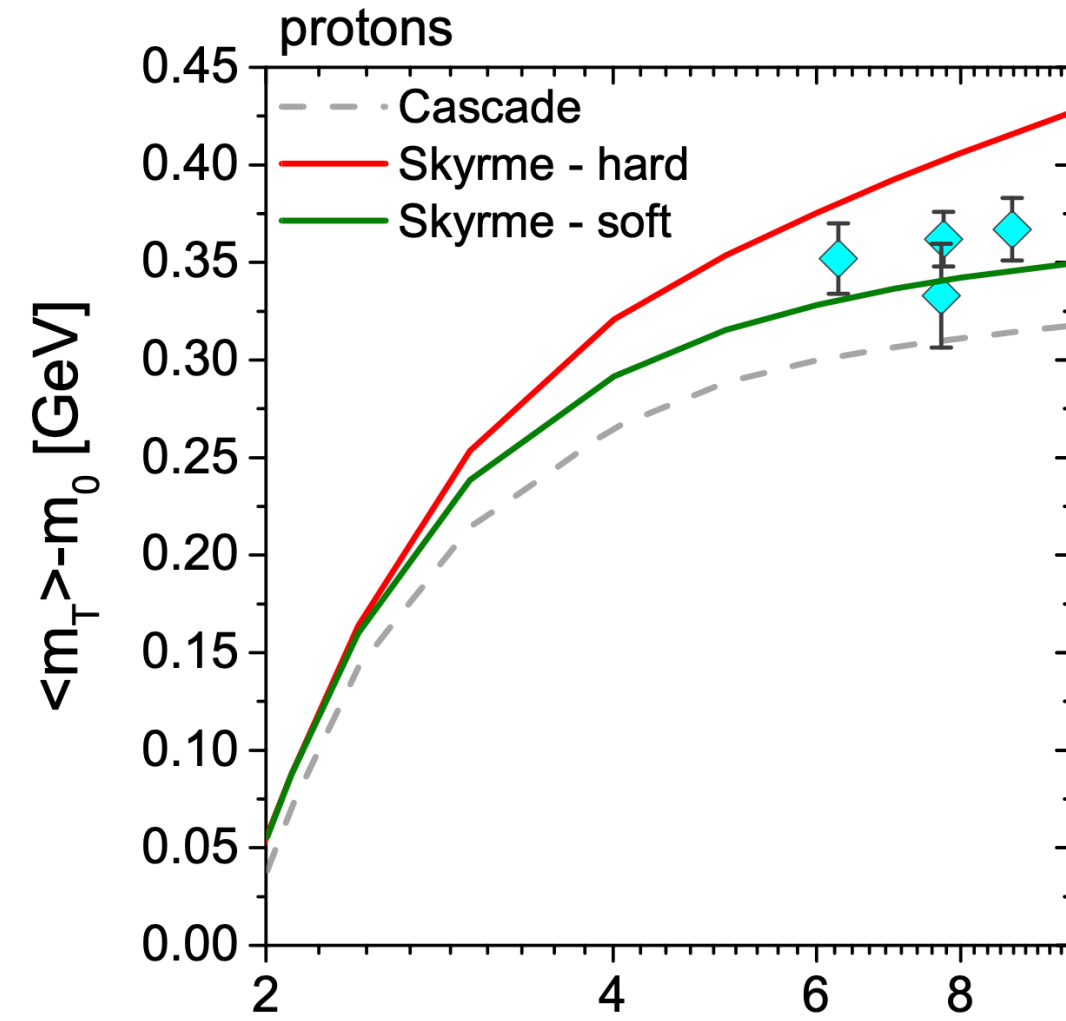
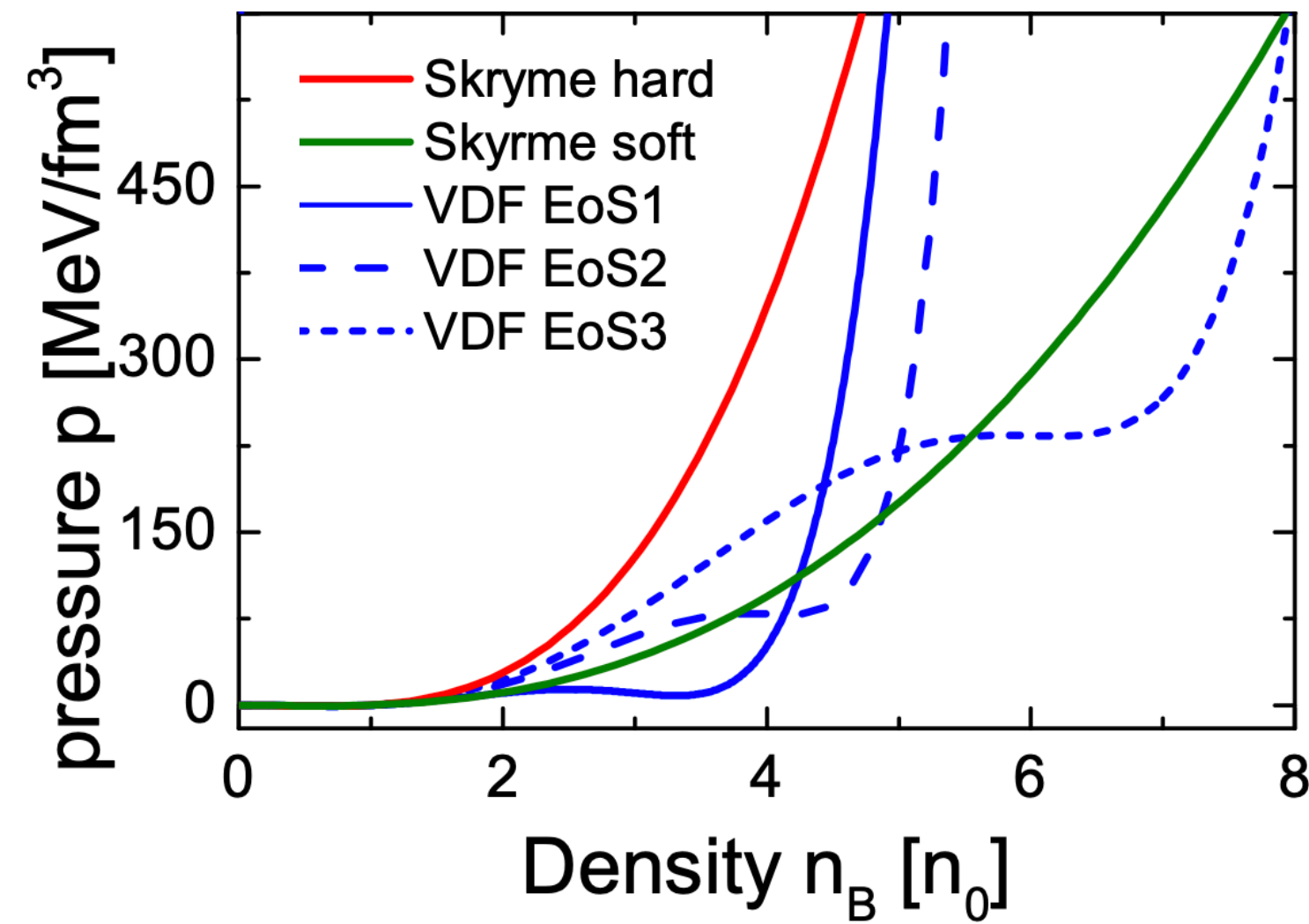
VDF model: two 1st order phase transitions

A. Sorensen, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635



Results from UrQMD with (non-relativistic) VDF

J. Steinheimer, A. Motornenko, **A. Sorensen**, Y. Nara, V. Koch,
M. Bleicher, Eur. Phys. J. C **82**, 10, 911 (2022) arXiv:2208.12091



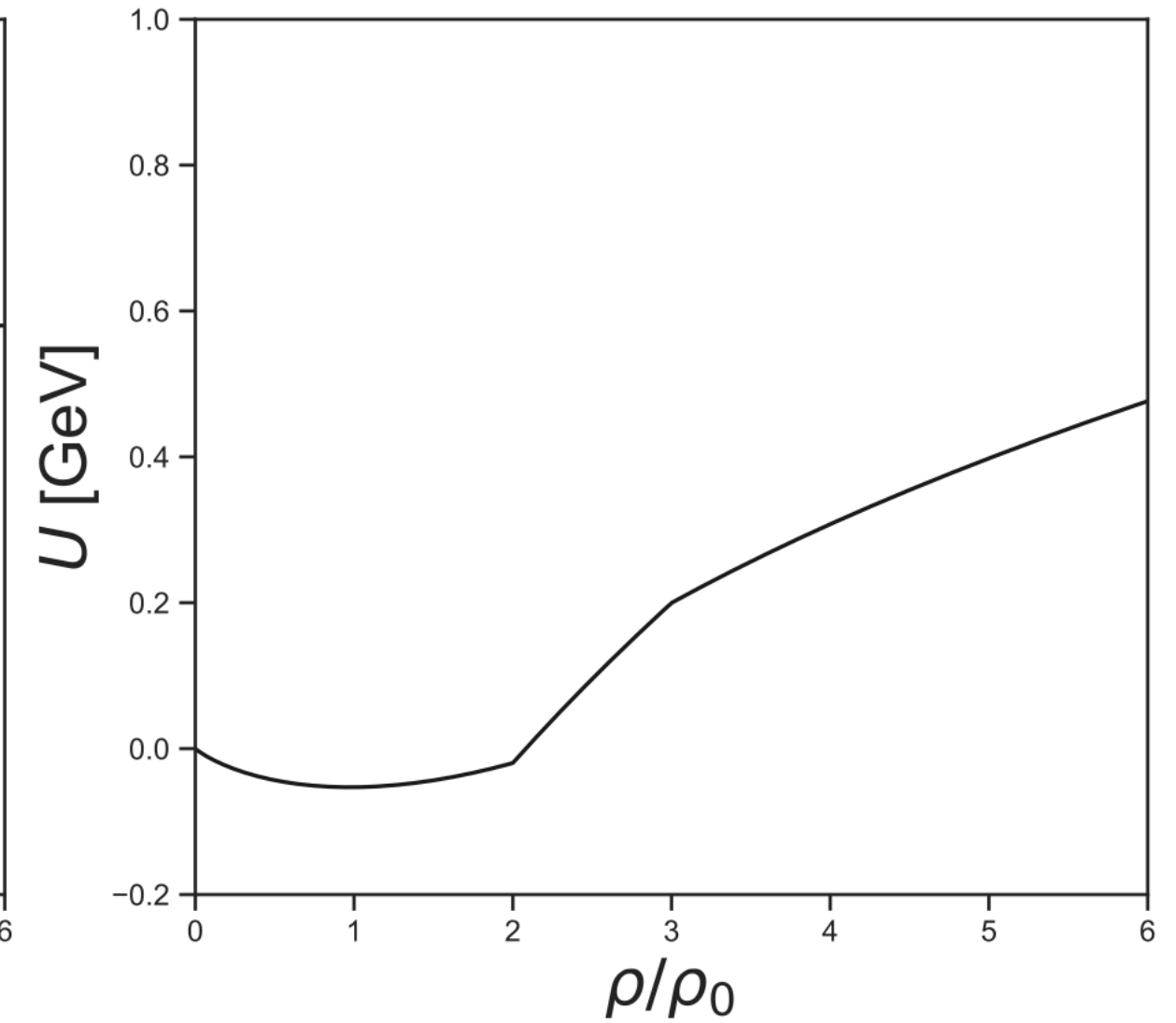
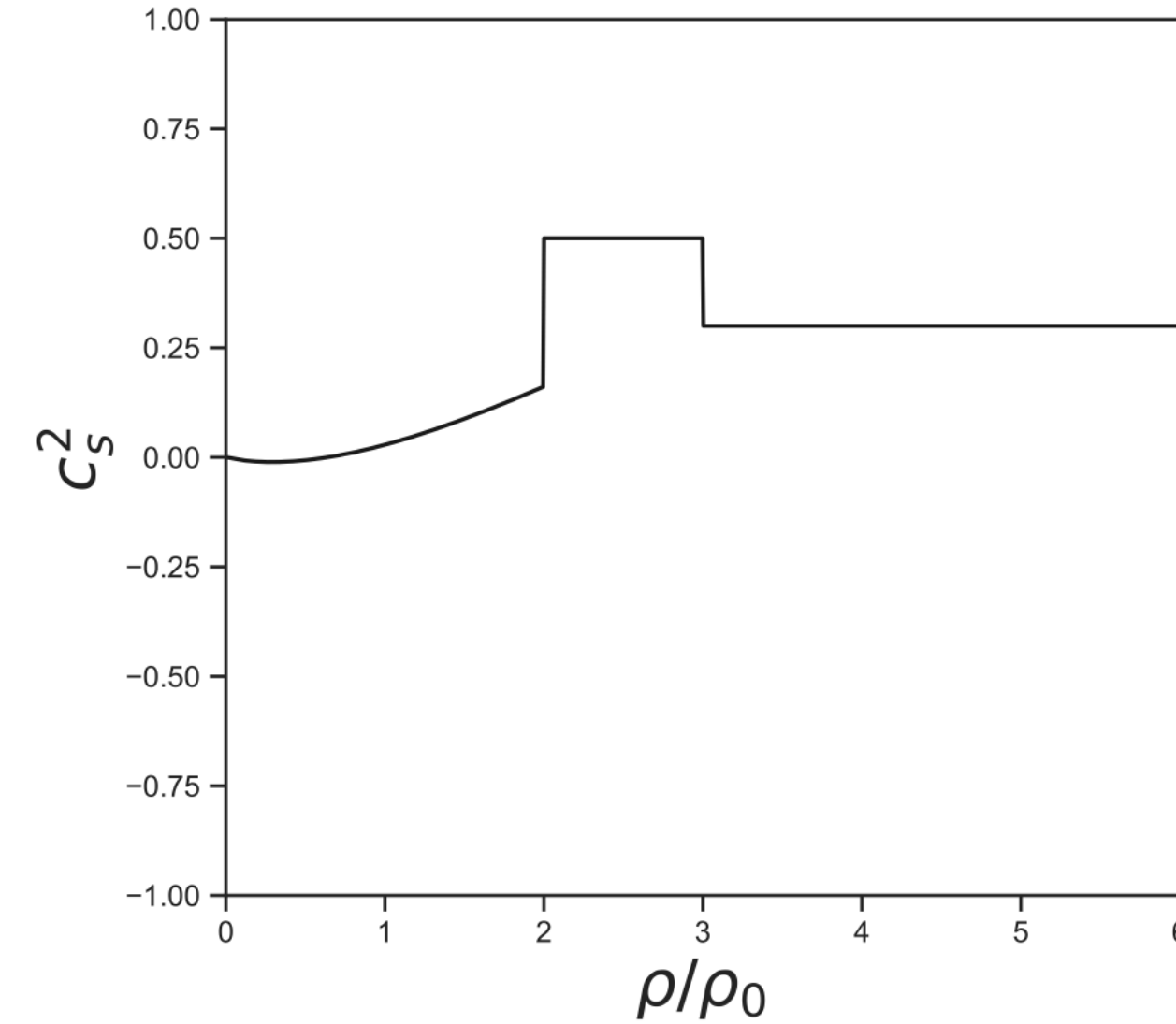
EoS	$T_c^{(N)}$ [MeV]	$n_c^{(Q)}$ [n_0]	$T_c^{(Q)}$ [MeV]	K_0 [MeV]
VDF1	18	3.0	100	261
VDF2	18	4.0	50	279
VDF3	22	6.0	50	356

Very soft EOS at $n_B \in (2,3)n_0$
not supported in VDF+UrQMD

Better suited for detailed studies: piecewise parametrization of c_s^2

Piecewise parametrization of $c_s^2(n_B)$:

$$c_s^2(n_B) = \begin{cases} c_s^2(\text{Skyrme}), & n_B < n_1 = 2n_0 \\ c_1^2, & n_1 < n_B < n_2 \\ c_2^2, & n_2 < n_B < n_3 \\ \dots & \\ c_m^2, & n_m < n_B \end{cases}$$



Single-particle potential $U(n_B) = \alpha(n_B)n_B$:

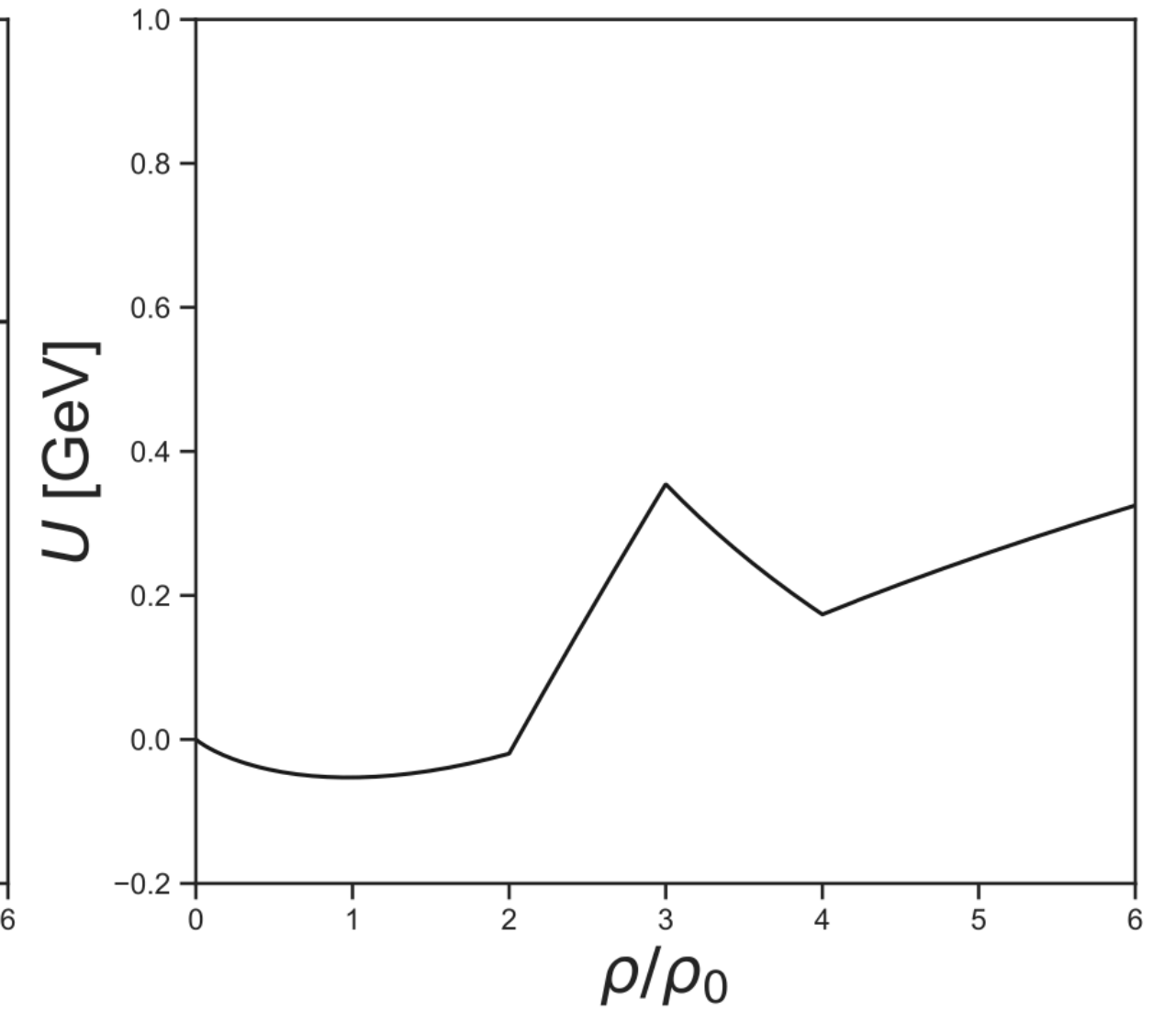
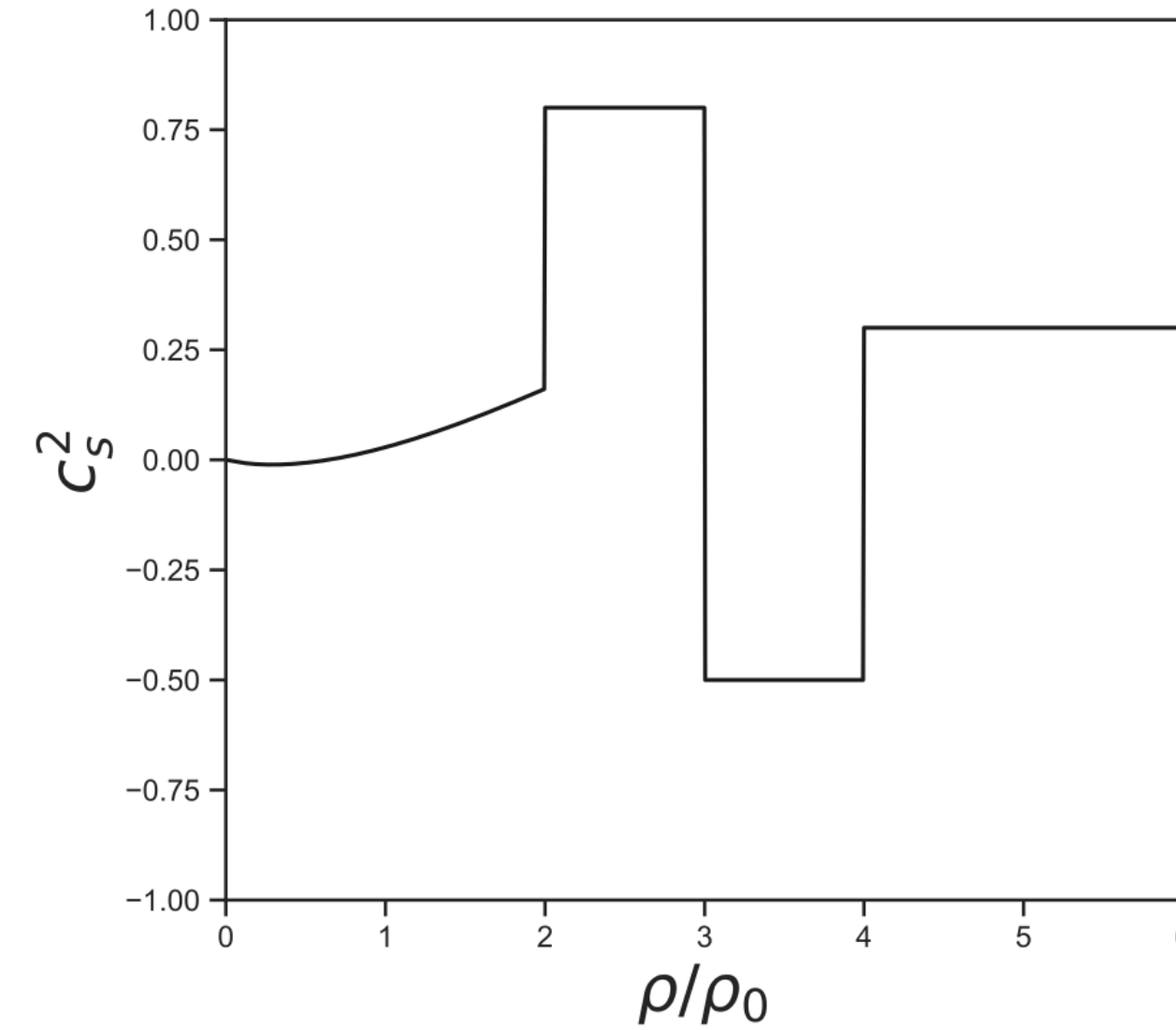
$$U(n_B) = \begin{cases} U_{\text{Sk}}(n_B), & n_B < n_1 = 2n_0 \\ \left[U_{\text{Sk}}(n_1) + \mu^*(\rho_1) \right] \left(\frac{\rho}{n_1} \right)^{c_1^2} - \mu^*(n_B), & n_1 < n_B < n_2 \\ \left[U_{\text{Sk}}(n_1) + \mu^*(n_1) \right] \left(\frac{n_B}{n_k} \right)^{c_k^2} \prod_{i=2}^k \left(\frac{n_i}{n_{i-1}} \right)^{c_{i-1}^2} - \mu^*(n_B), & n_k < n_B < n_{k+1} \end{cases}$$

Gradients of $U(n_B)$ enter the EOMs!

Better suited for detailed studies: piecewise parametrization of c_s^2

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$$c_s^2(n_B) = \begin{cases} c_s^2(\text{Skyrme}), & n_B < n_1 = 2n_0 \\ c_1^2, & n_1 < n_B < n_2 \\ c_2^2, & n_2 < n_B < n_3 \\ \dots & \\ c_m^2, & n_m < n_B \end{cases}$$



Single-particle potential $U(n_B) = \alpha(n_B)n_B$:

$$U(n_B) = \begin{cases} U_{\text{Sk}}(n_B), & n_B < n_1 = 2n_0 \\ \left[U_{\text{Sk}}(n_1) + \mu^*(\rho_1) \right] \left(\frac{\rho}{n_1} \right)^{c_1^2} - \mu^*(n_B), & n_1 < n_B < n_2 \\ \left[U_{\text{Sk}}(n_1) + \mu^*(n_1) \right] \left(\frac{n_B}{n_k} \right)^{c_k^2} \prod_{i=2}^k \left(\frac{n_i}{n_{i-1}} \right)^{c_{i-1}^2} - \mu^*(n_B), & n_k < n_B < n_{k+1} \end{cases}$$

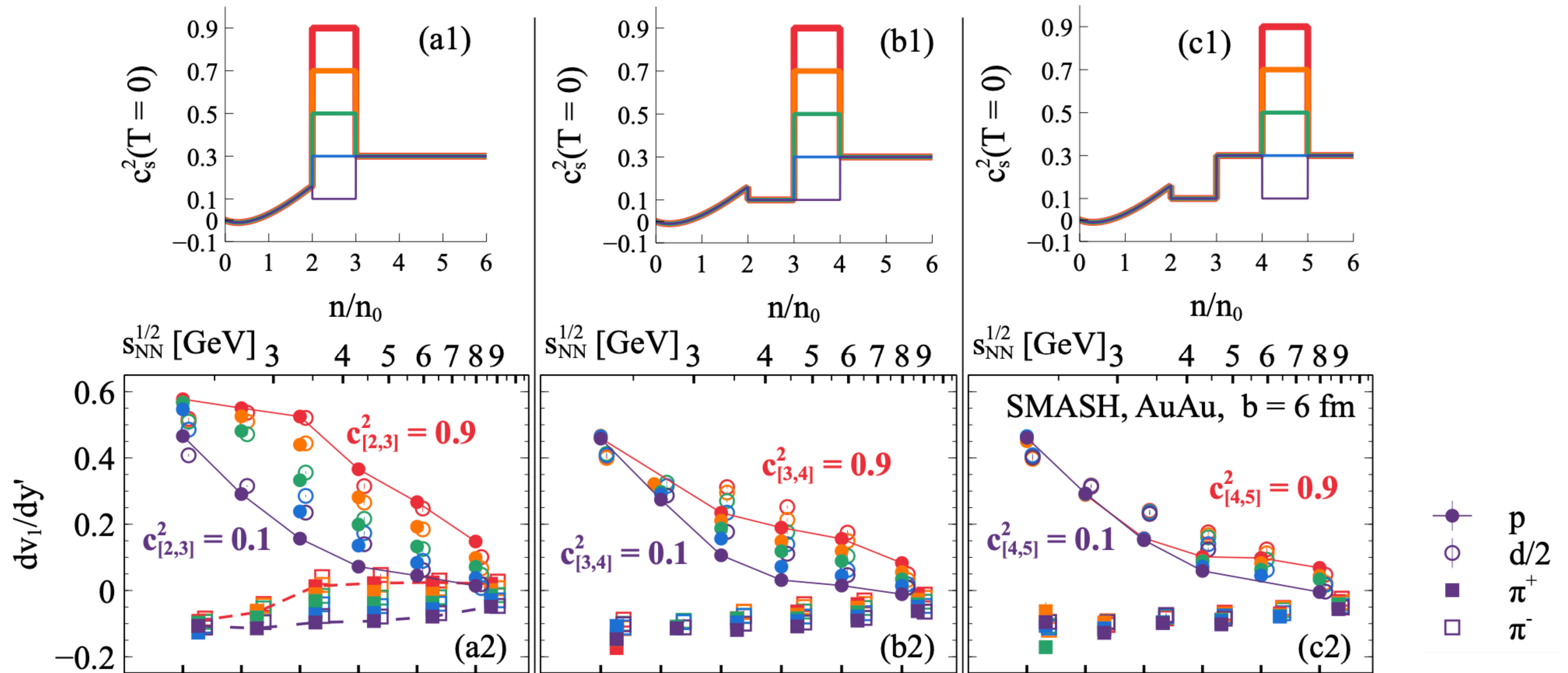
Gradients of $U(n_B)$ enter the EOMs!

Hadronic transport with c_s^2 -parametrized mean-fields

D. Oliinychenko, A. Sorensen, V. Koch, L. McLerran,
arXiv:2208.11996

Generalized VDF (n_B -dependent interaction coefficients):

mean-field potential piecewise parametrized by (constant) values of c_s^2 for $n_i < n_B < n_j$

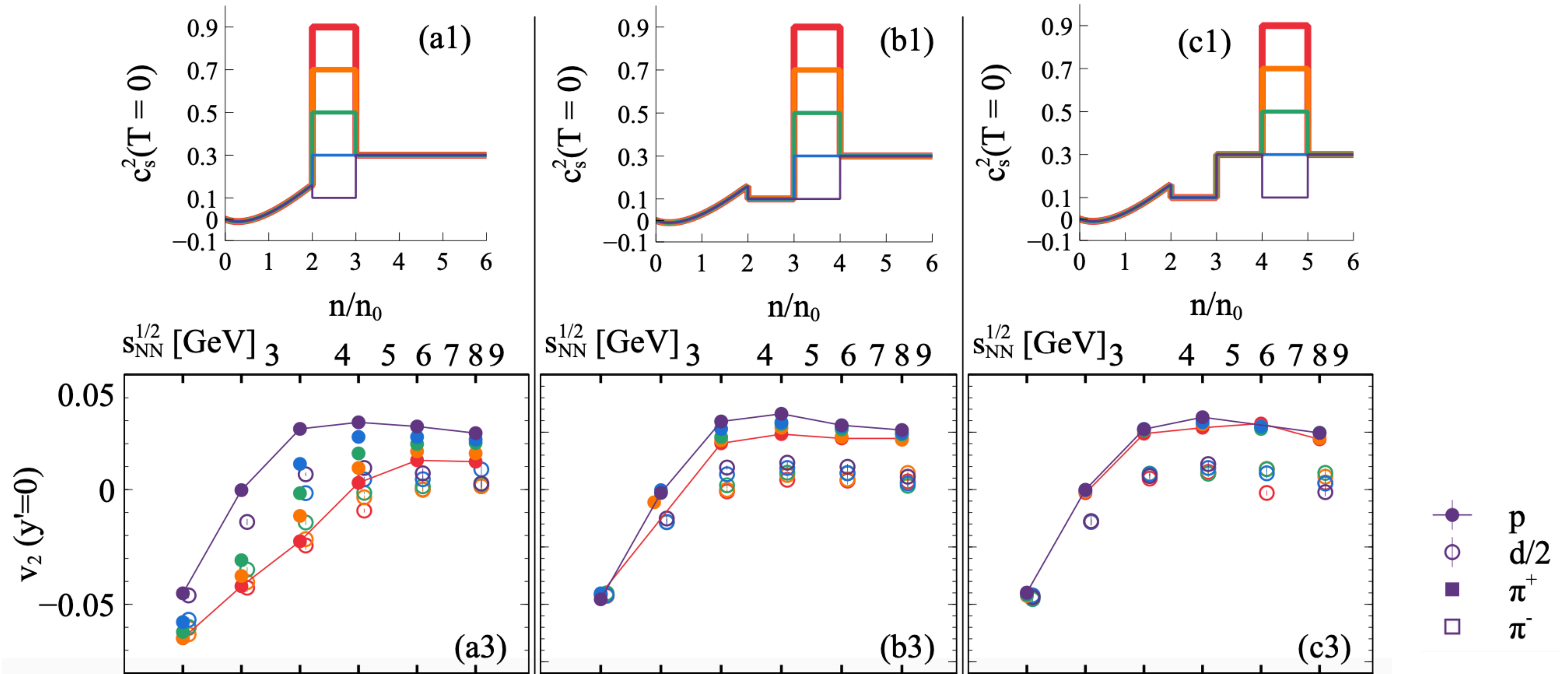


Hadronic transport with c_s^2 -parametrized mean-fields

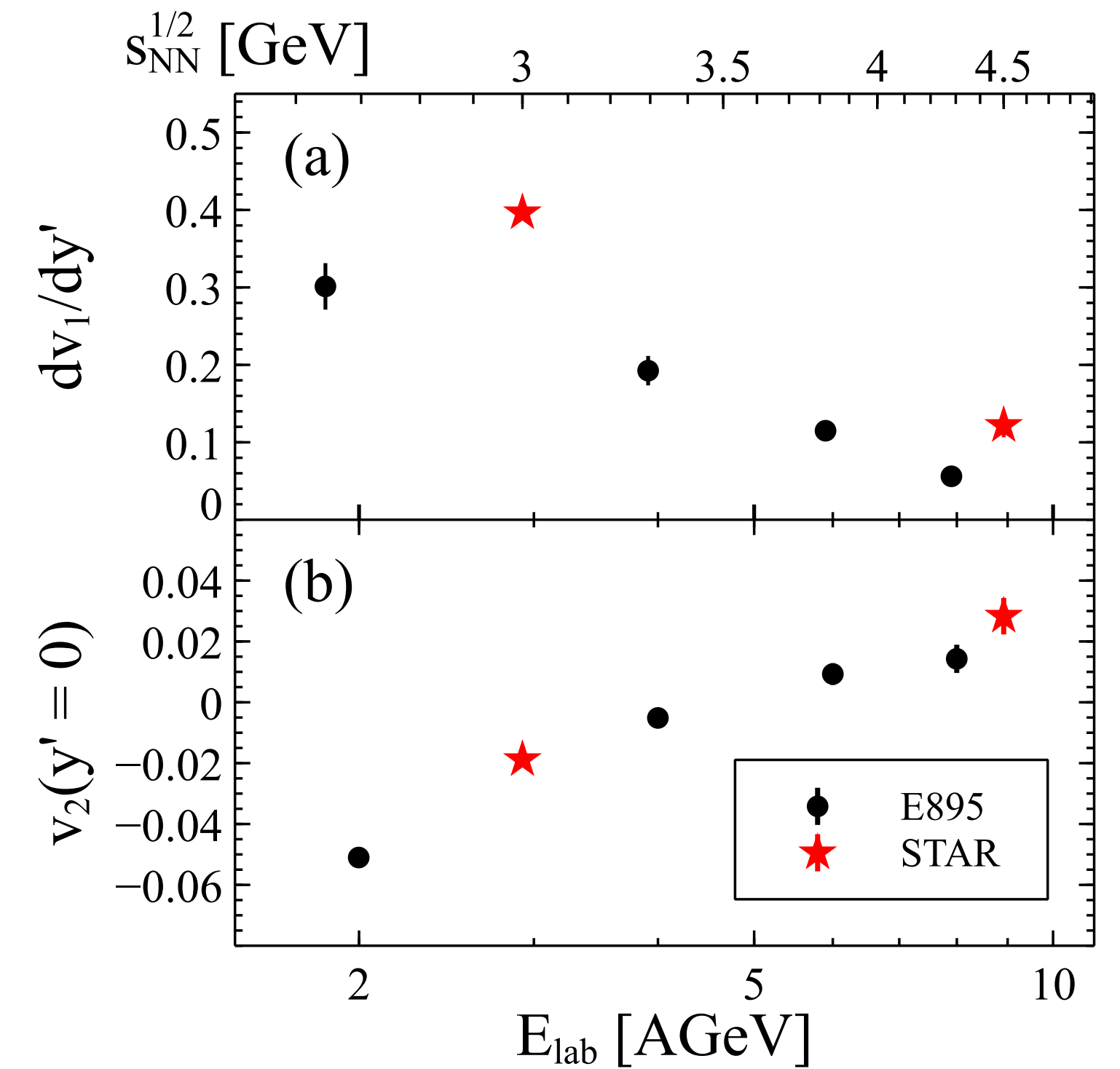
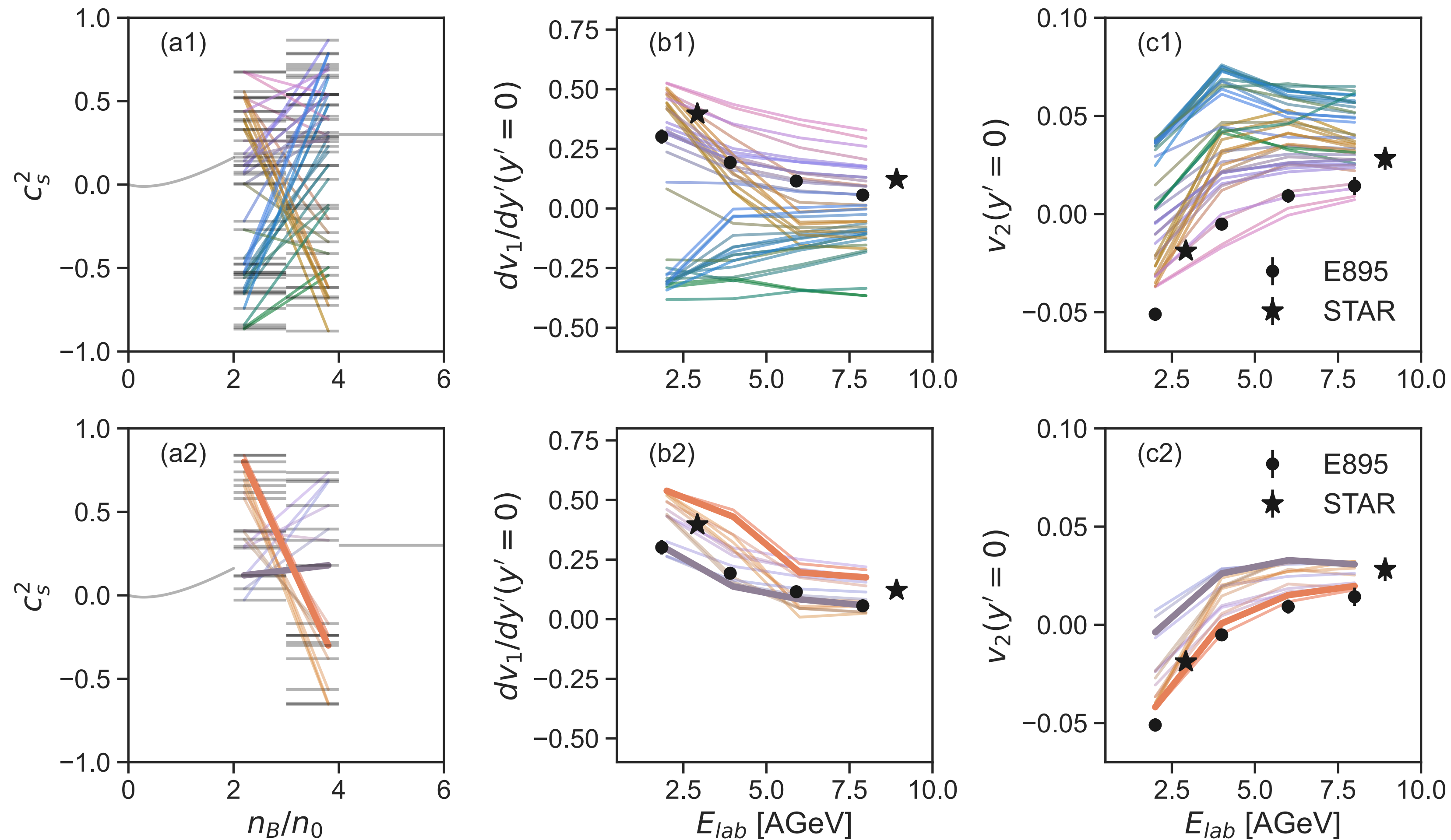
D. Oliinychenko, A. Sorensen, V. Koch, L. McLerran,
arXiv:2208.11996

Generalized VDF (n_B -dependent interaction coefficients):

mean-field potential piecewise parametrized by (constant) values of c_s^2 for $n_i < n_B < n_j$

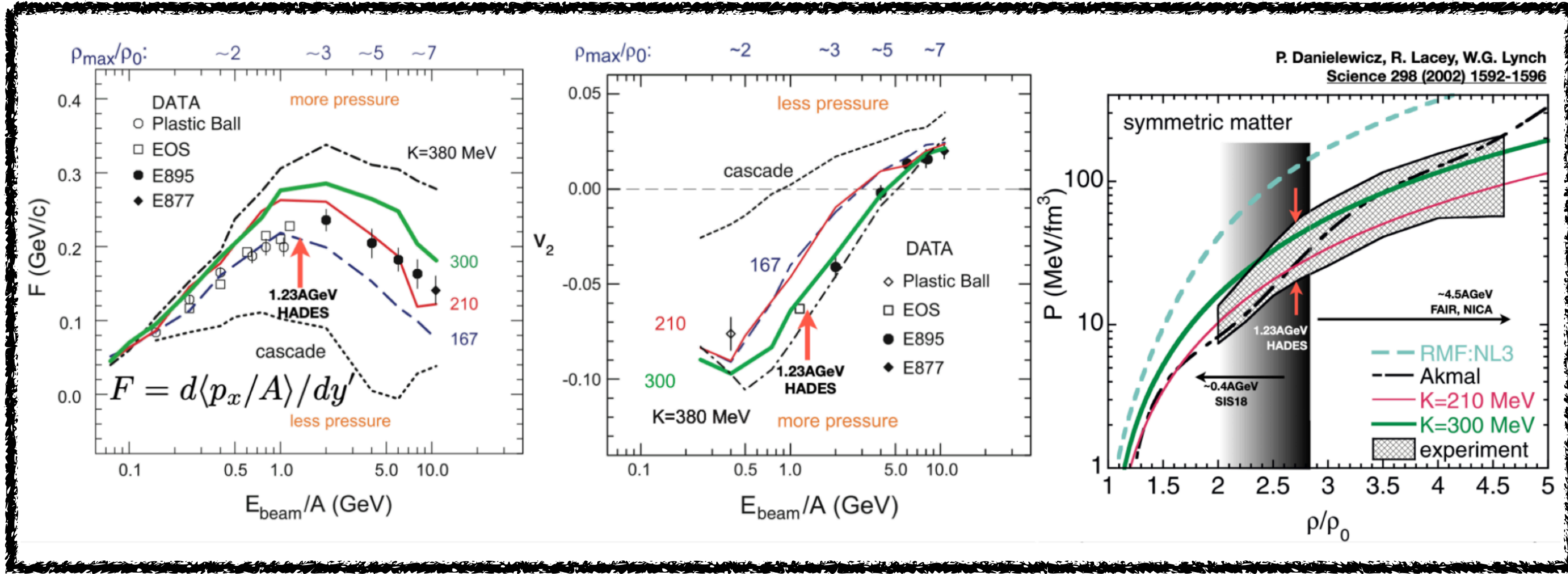


STAR and E895 data cannot be simultaneously described



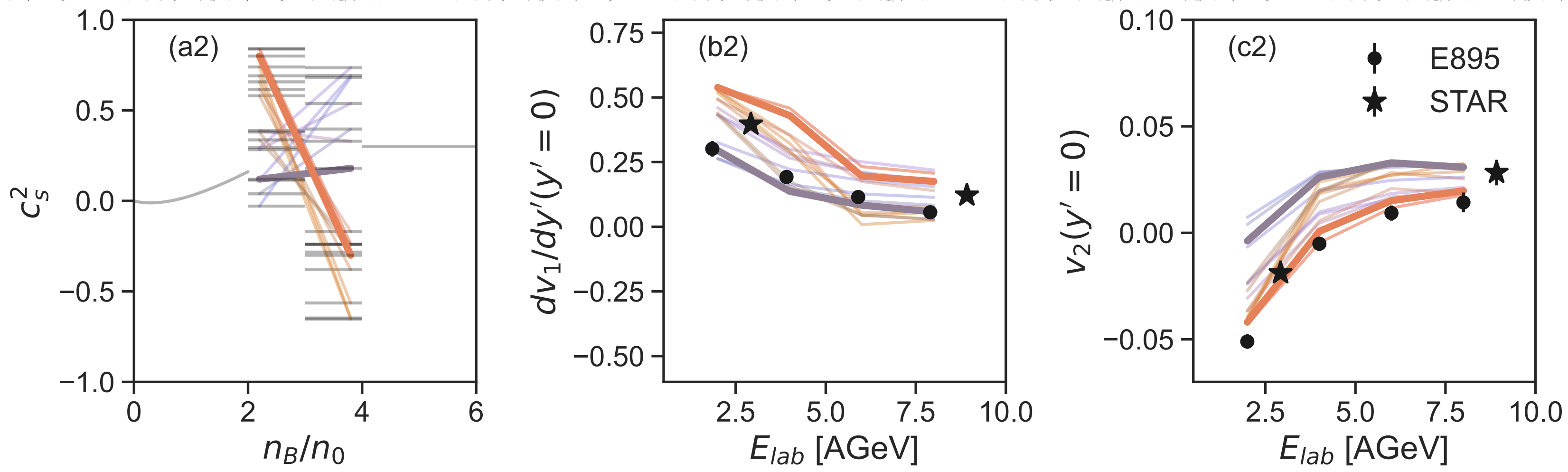
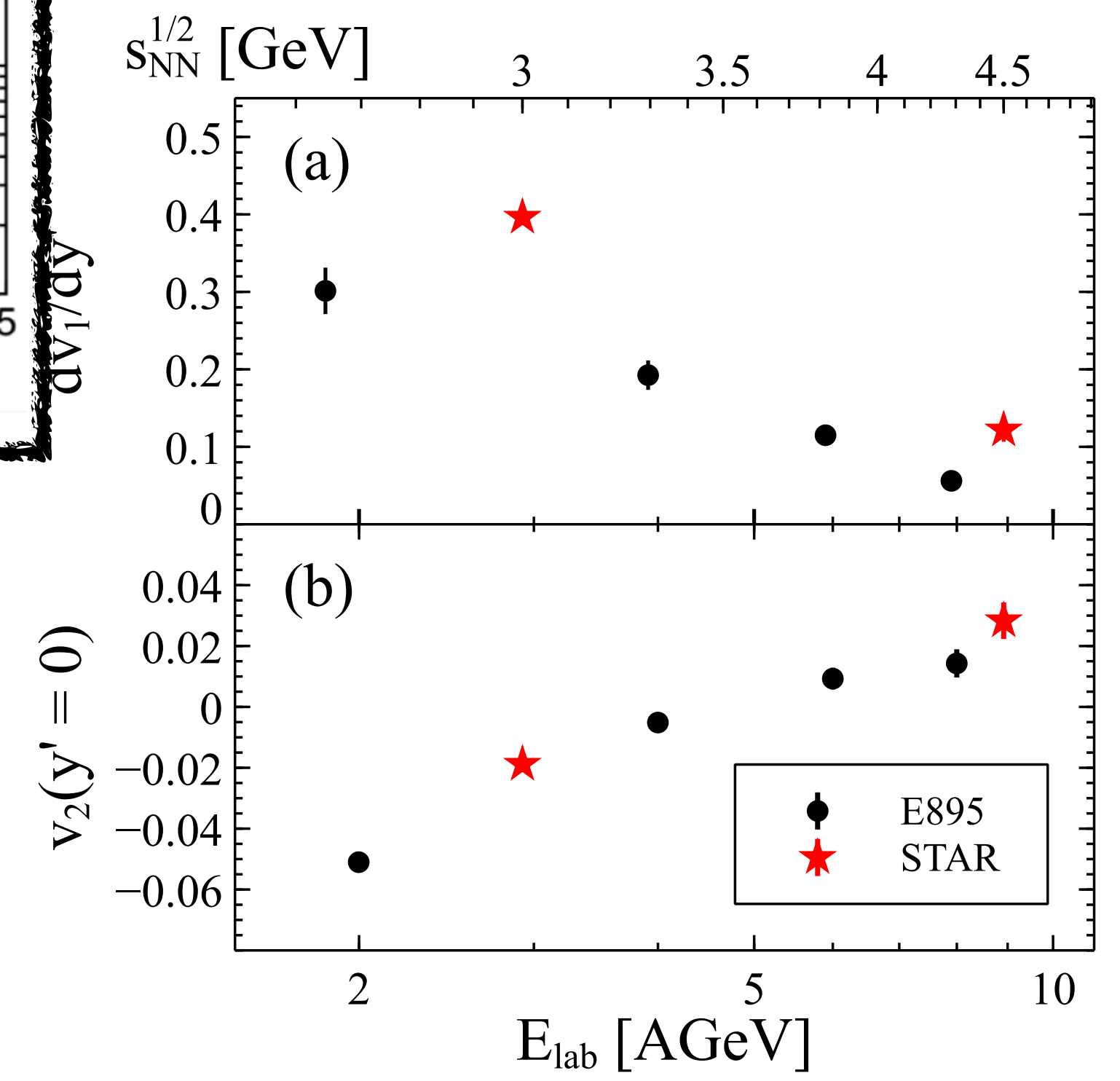
tension between the data sets

STAR and E895 data cannot be simultaneously described



Same problem as in the DLL constraint!

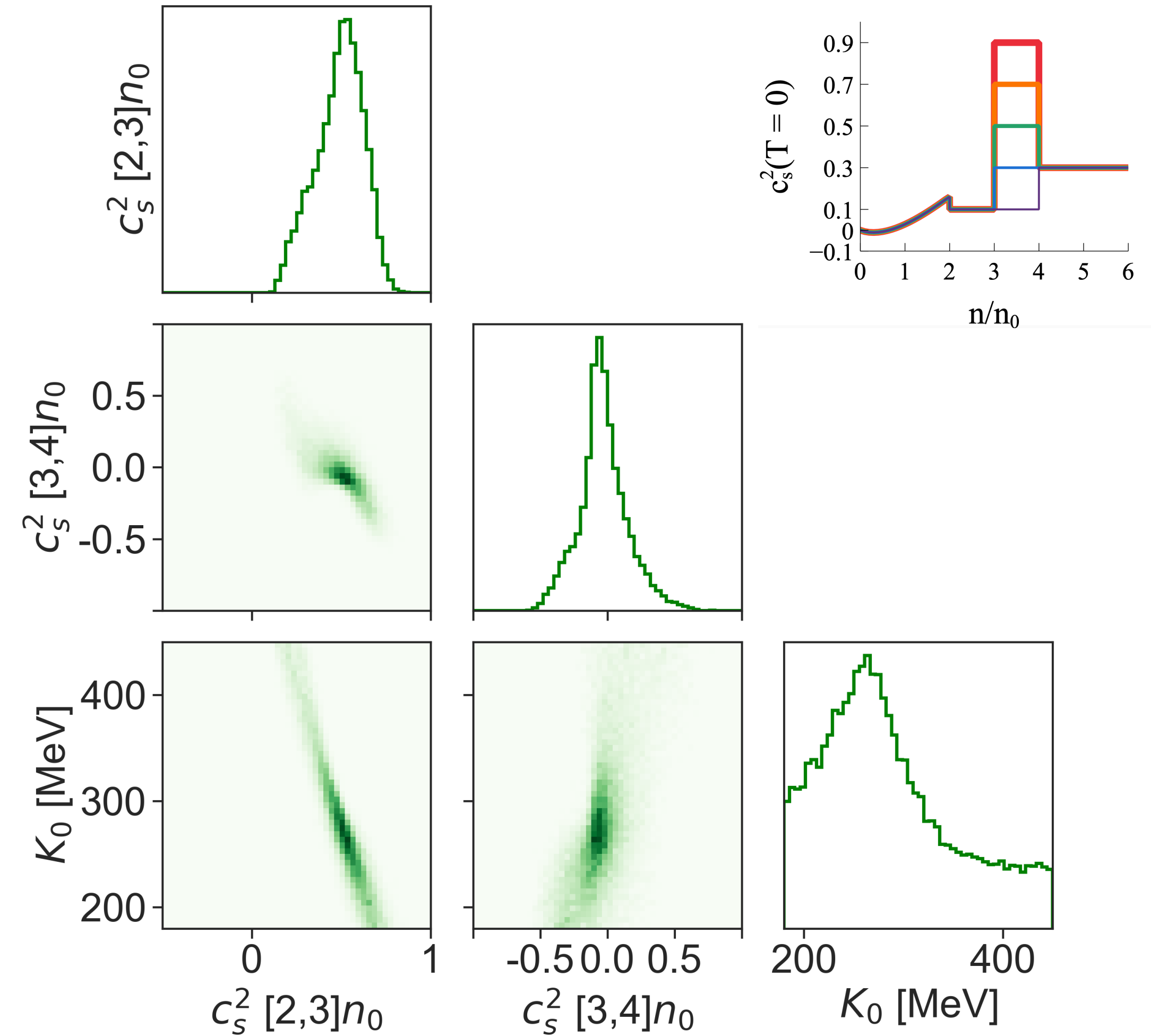
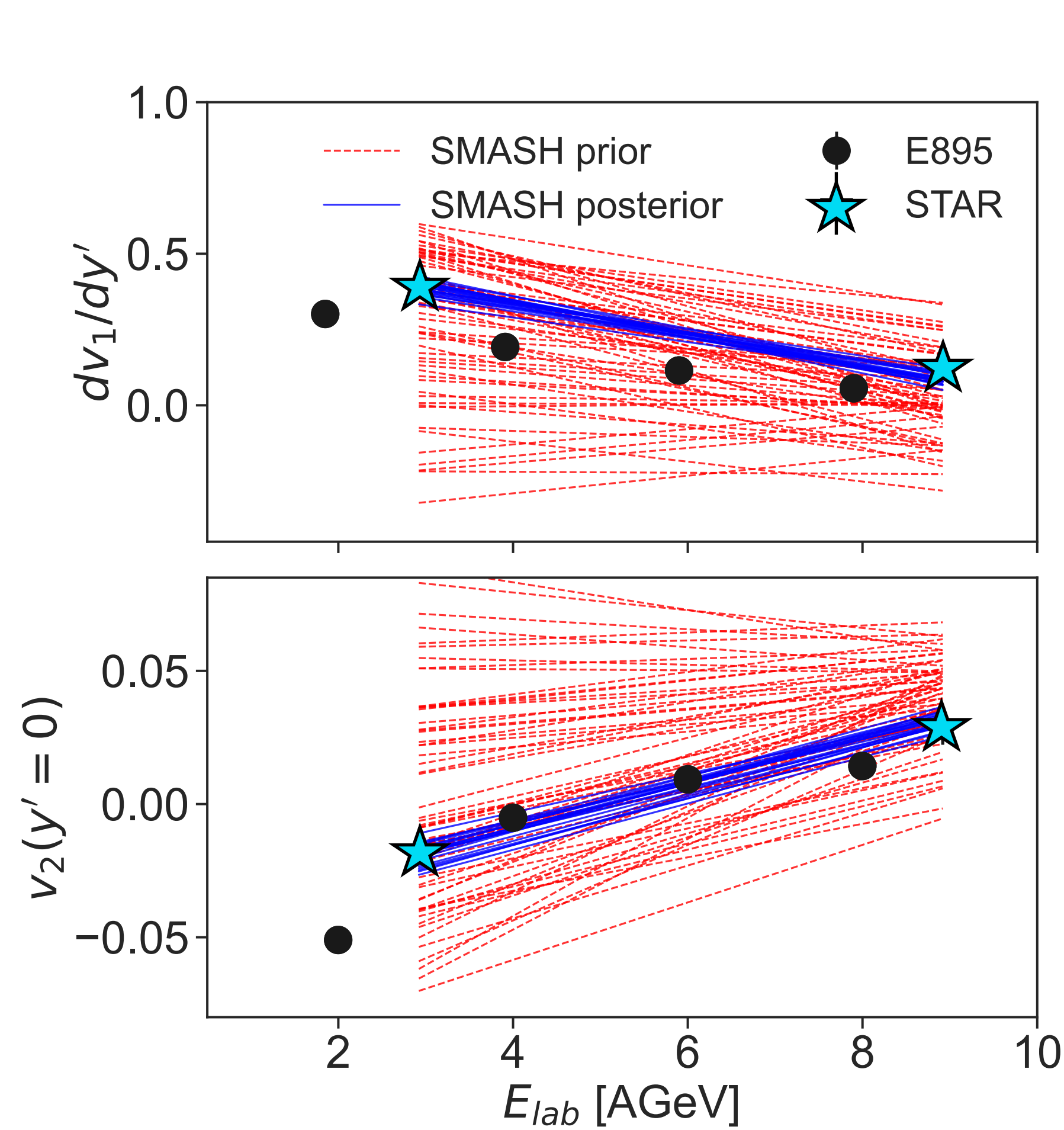
Danielewicz, Lacey, Lynch, Science 298, 1592-1596 (2002)



tension between the data sets

D. Oliinychenko, A. Sorensen, V. Koch, L. McLerran, arXiv:2208.11996

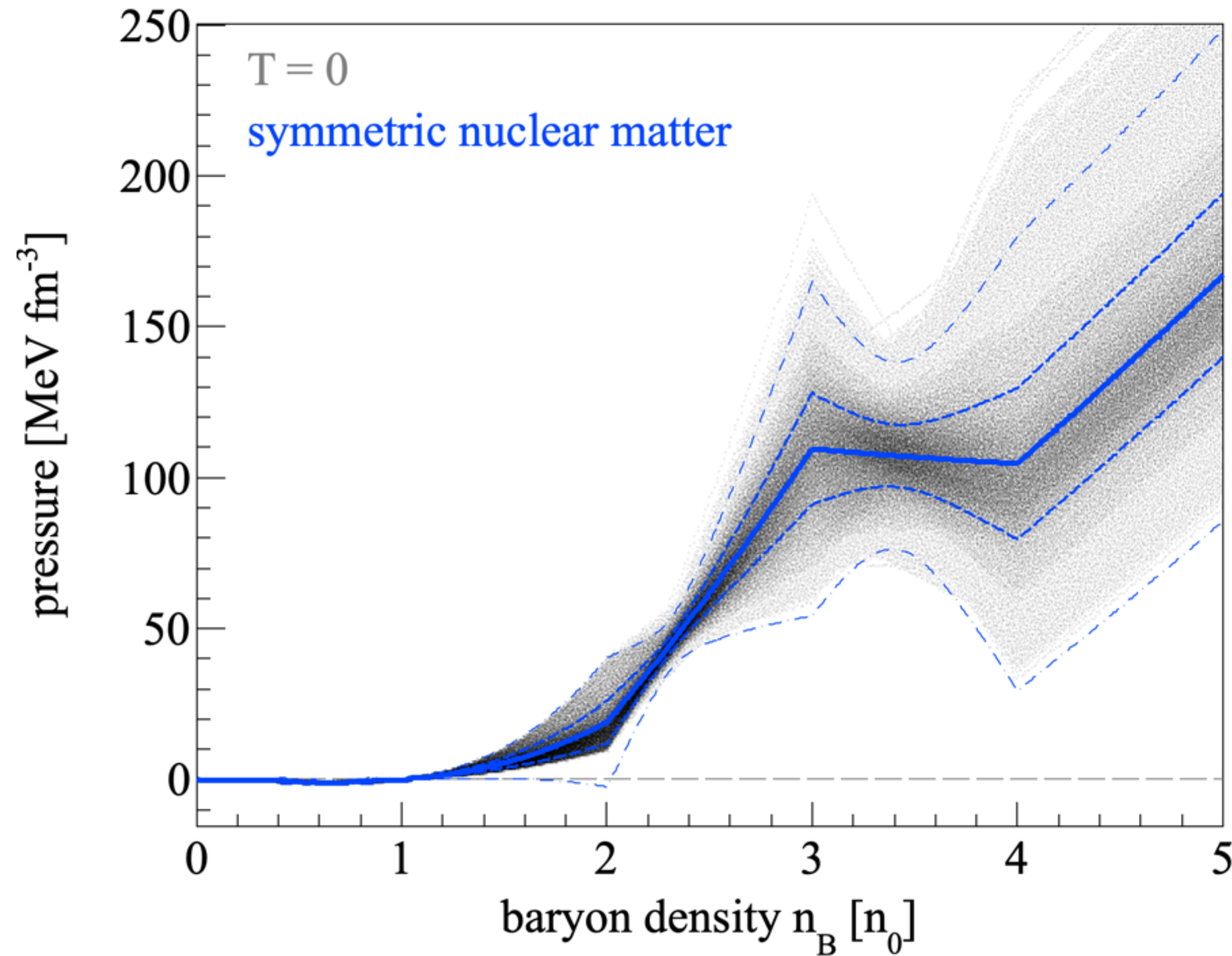
Bayesian analysis of STAR flow data with varying K_0 , $c_{[2,3]n_0}^2$, $c_{[3,4]n_0}^2$



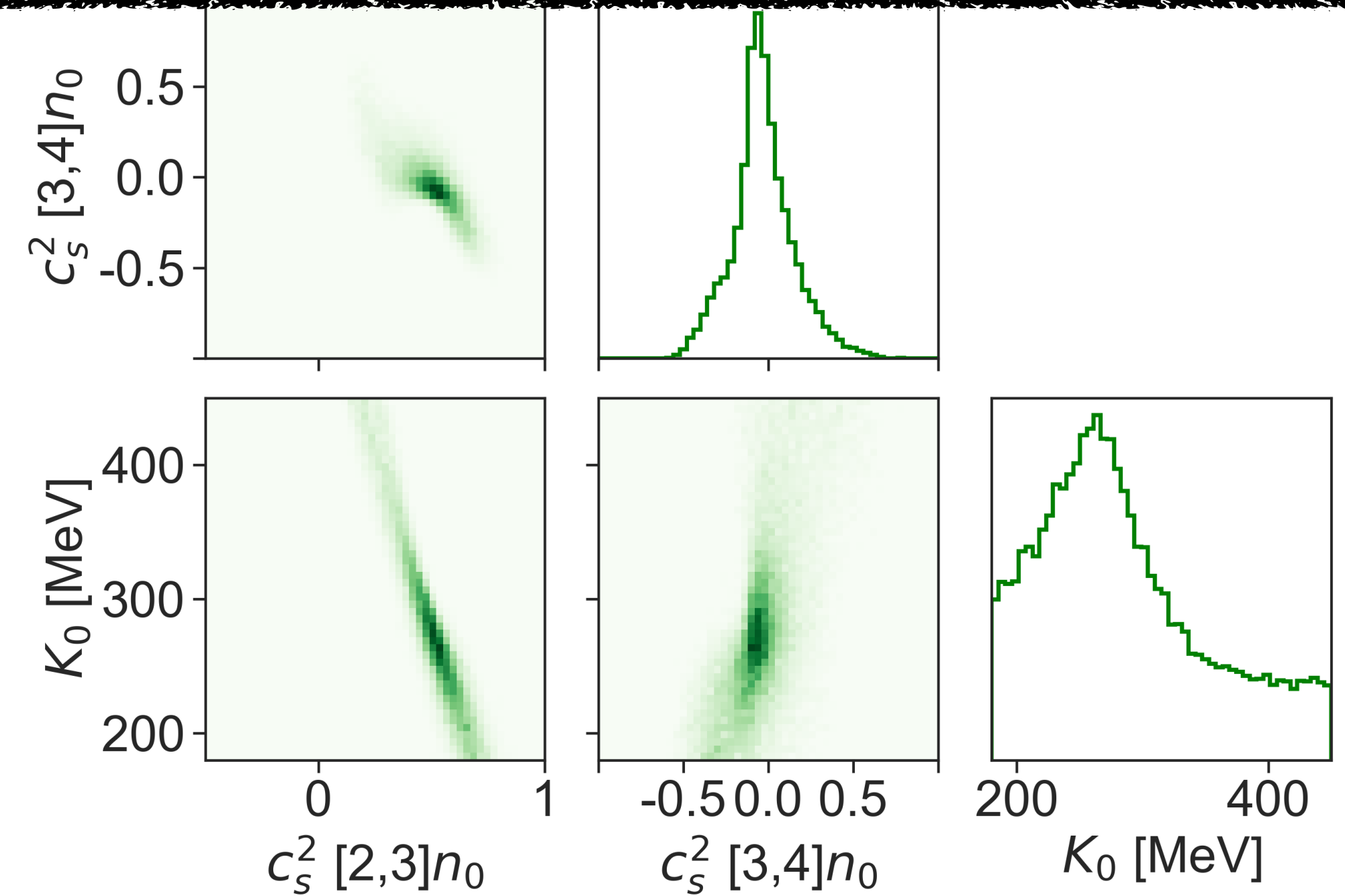
The maximum a posteriori probability (MAP) parameters are
 $K_0 = 285 \pm 67$ MeV, $c_{[2,3]n_0}^2 = 0.49 \pm 0.13$, $c_{[3,4]n_0}^2 = -0.03 \pm 0.15$

D. Oliinychenko, A. Sorensen, V. Koch, L. McLerran,
 arXiv:2208.11996

Bayesian analysis of STAR flow data with varying K_0 , $c_{[2,3]n_0}^2$, $c_{[3,4]n_0}^2$



The constrained EOS is very stiff at $n_B \in (2,3)n_0$ and very soft at $n_B \in (3,4)n_0$!



The maximum a posteriori probability (MAP) parameters are
 $K_0 = 285 \pm 67$ MeV, $c_{[2,3]n_0}^2 = 0.49 \pm 0.13$, $c_{[3,4]n_0}^2 = -0.03 \pm 0.15$

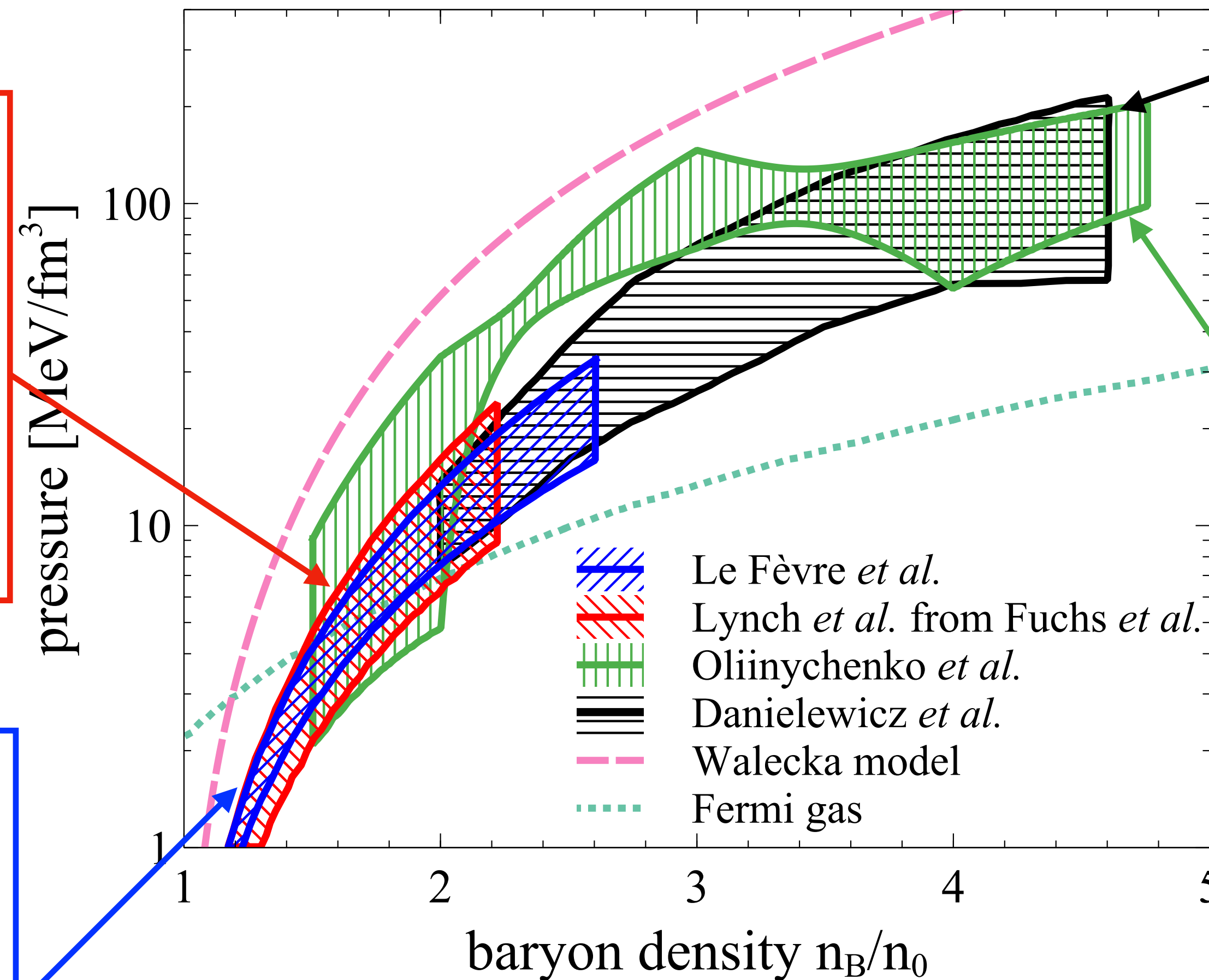
D. Oliinychenko, A. Sorensen, V. Koch, L. McLerran,
arXiv:2208.11996

EOS of symmetric nuclear matter: selected results

Symmetric nuclear matter

197Au+197Au & 12C+12C @ < 1.5 GeV/u
 ($\sqrt{s_{NN}} < 2.5$ GeV)
 observables: subthreshold kaon production (KaoS)
 model used: QMD w/ nucleons, Δ , $N^*(1440)$, pions, kaons;
 EOS parametrized by K_0 ;
 kaon potentials, momentum dependence
 C. Fuchs *et al.*, Prog. Part. Nucl. Phys. **53**, 113–124 (2004) arXiv:nucl-th/0312052

197Au+197Au @ 0.4–1.5 GeV/u
 ($\sqrt{s_{NN}} = 2.07 - 2.52$ GeV)
 observables: proton flow (FOPI)
 model used: isospin QMD (IQMD) w/ nucleons, Δ , $N^*(1440)$, deuterons, tritons;
 EOS parametrized by K_0 ;
 momentum dependence
 A. Le Fèvre, Y. Leifels, W. Reisdorf, J. Aichelin, C. Hartnack, Nucl. Phys. A **945**, 112 (2016), arXiv:1501.05246



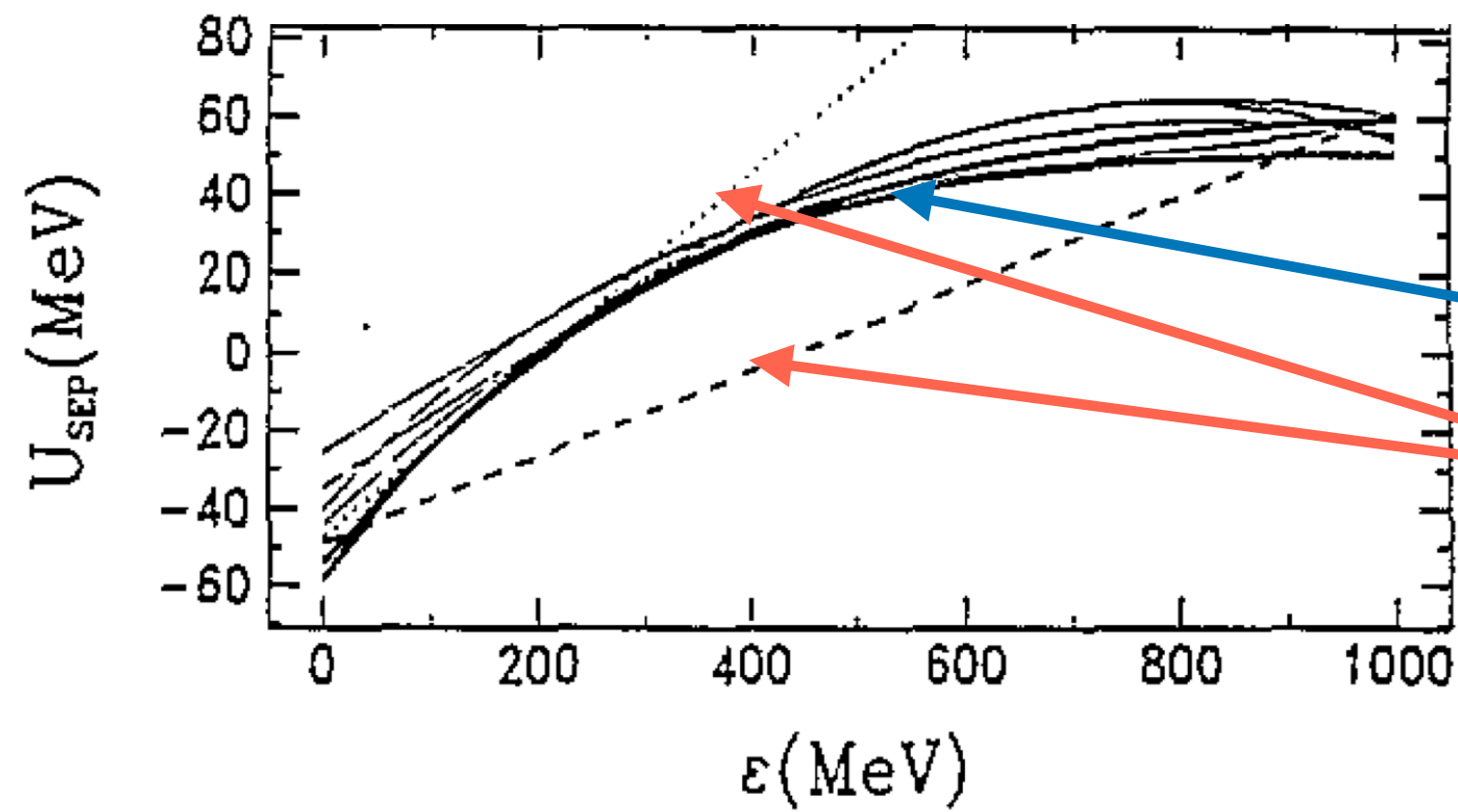
A. Sorensen *et al.*, arXiv:2301.13253

197Au+197Au @ 0.15–10 GeV/u
 ($\sqrt{s_{NN}} = 1.95 - 4.72$ GeV)
 observables: proton flow (Plastic Ball, EOS, E877, E895)
 model used: pBUU w/ nucleons, Δ , $N^*(1440)$, pions;
 EOS parametrized by K_0 ;
 momentum dependence
 Danielewicz, Lacey, Lynch, Science **298**, 1592–1596 (2002)

197Au+197Au @ 2.9–9 GeV/u
 ($\sqrt{s_{NN}} = 3 - 4.5$ GeV)
 observables: proton flow (STAR)
 model used: SMASH w/ over 120 hadronic species, including deuterons;
 relativistic EOS parametrized independently in different density regions;
NO momentum dependence
 D. Oliinychenko, AS, V. Koch, L. McLerran, arXiv:2208.11996

Momentum-dependent mean-fields are a necessary component

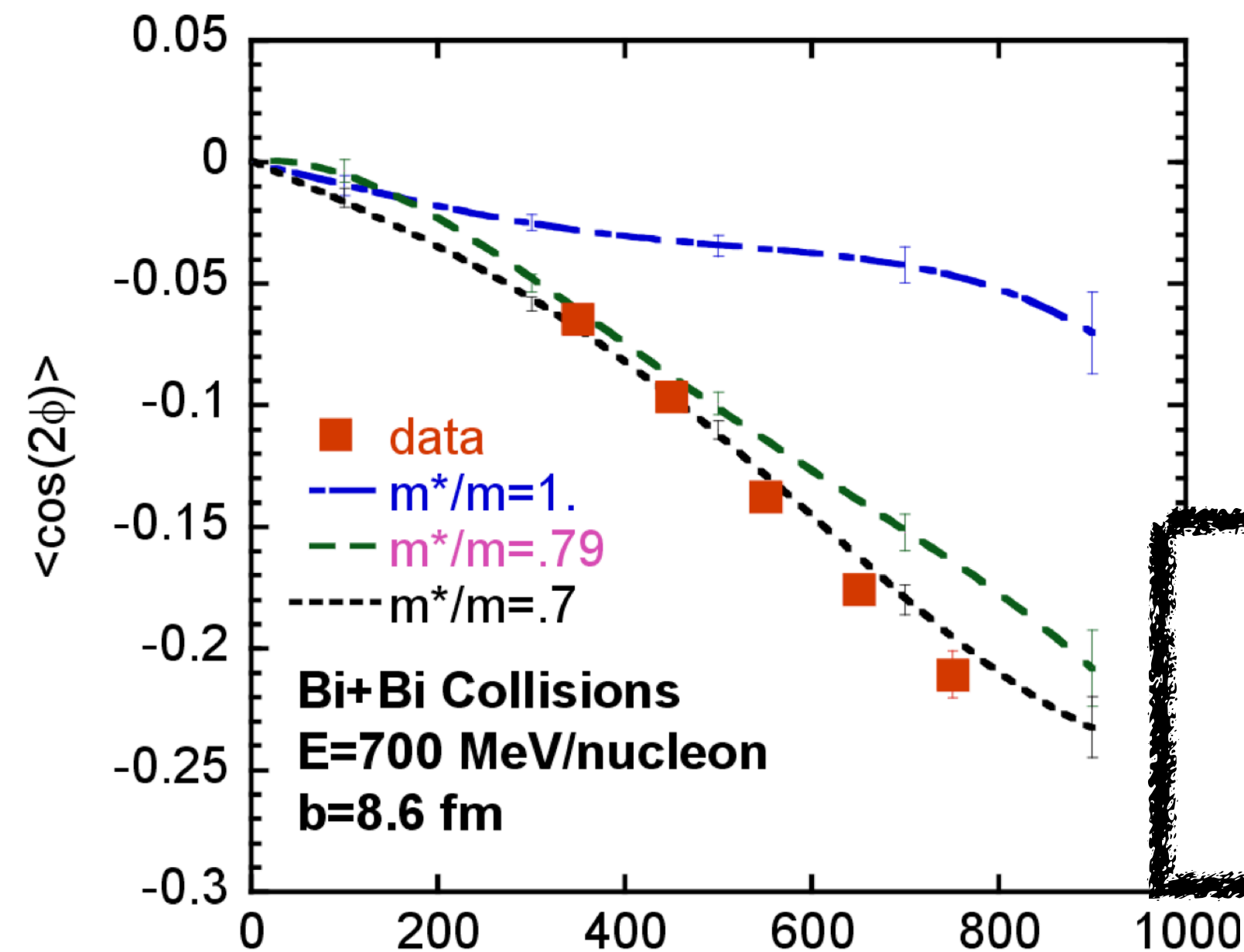
Measured in scattering experiments:



B. Blaettel, V. Koch, U. Mosel,
Rept. Prog. Phys. **56**,1–62 (1993)

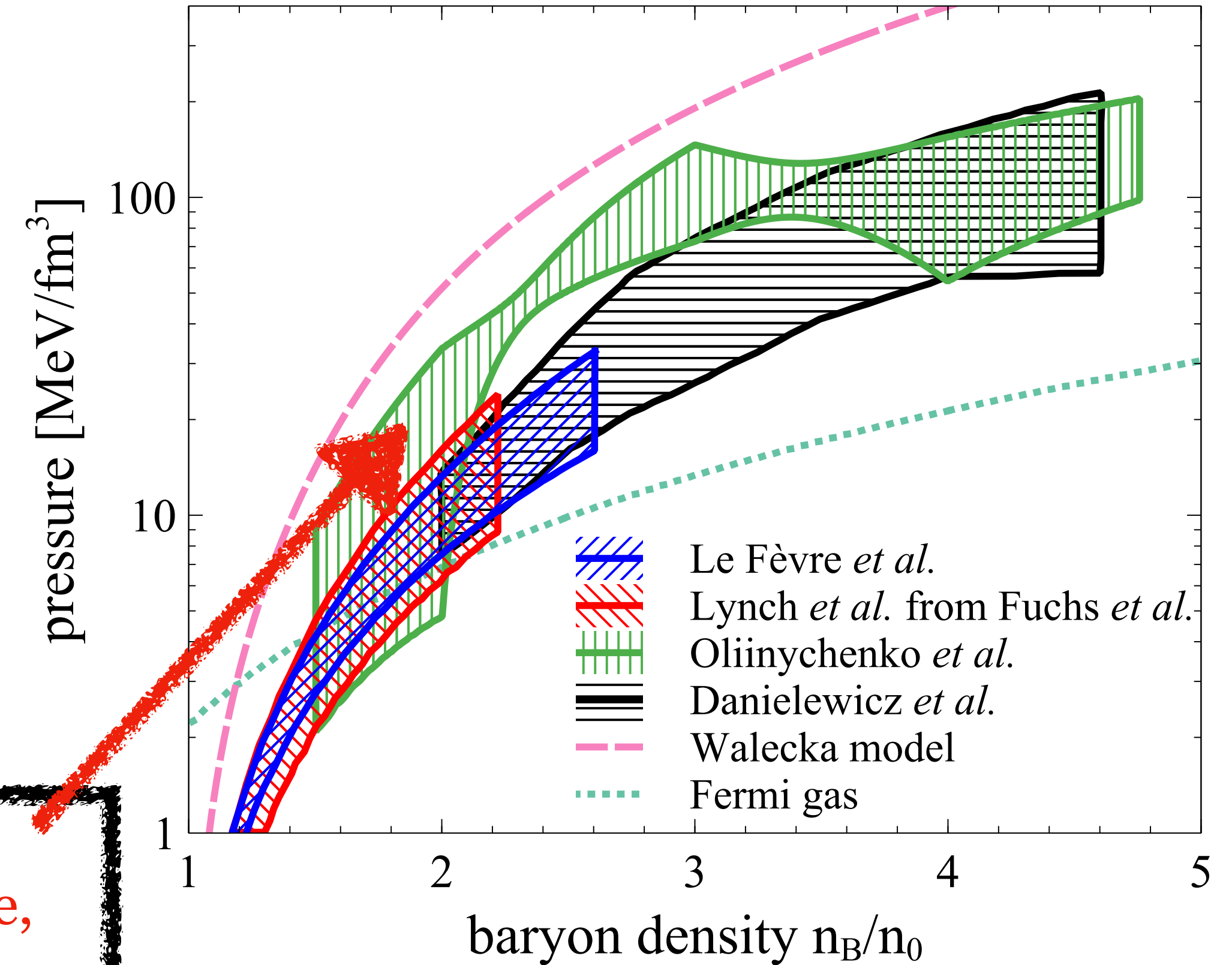
fits to data

parametrizations of
the Walecka model



Affects the p_T -dependence
of the elliptic flow

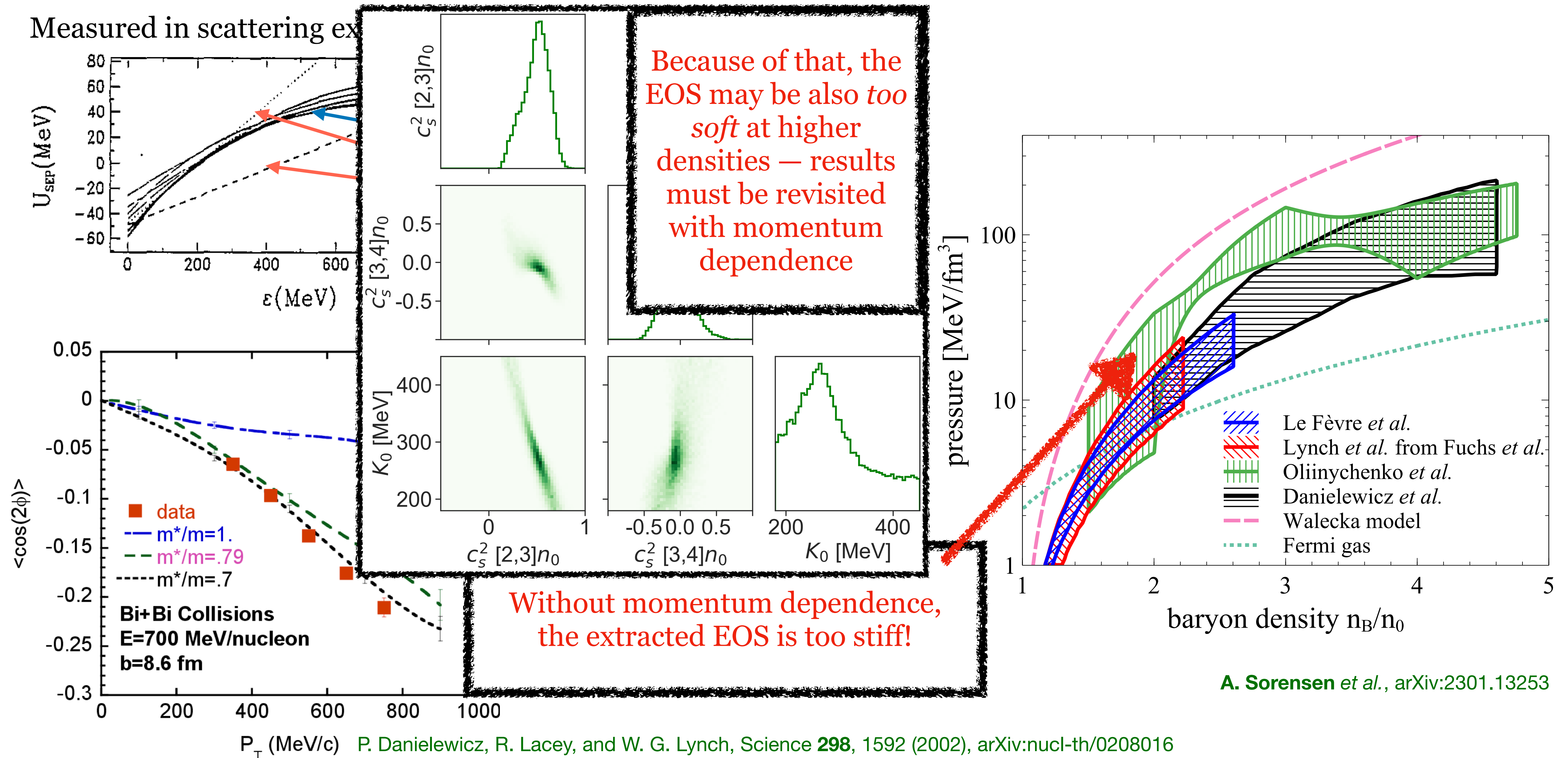
Without momentum dependence,
the extracted EOS is too stiff!



A. Sorensen *et al.*, arXiv:2301.13253

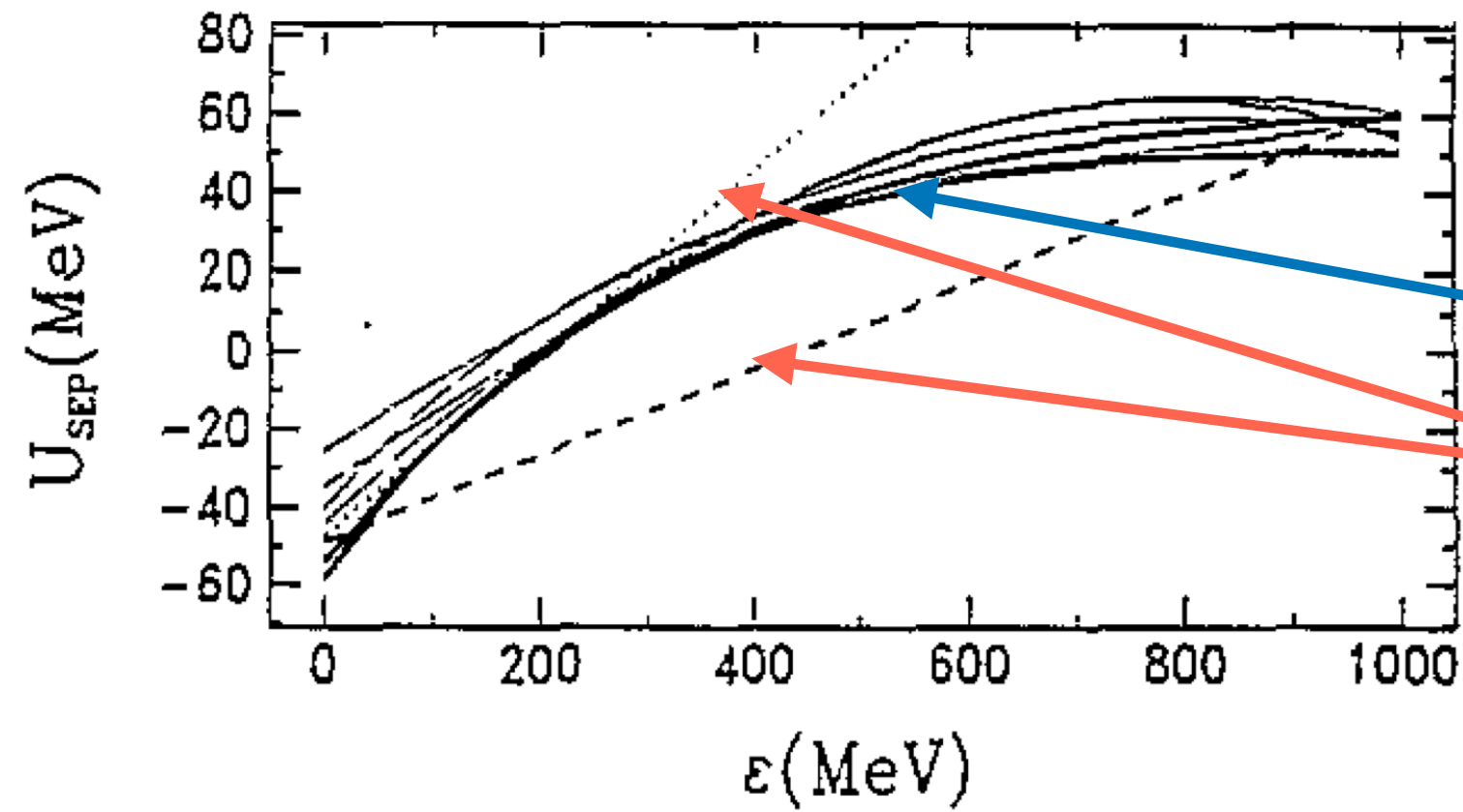
P_T (MeV/c) P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002), arXiv:nucl-th/0208016

Momentum-dependent mean-fields are a necessary component



Work in progress: Flexible momentum-dependent mean-fields

Measured in scattering experiments:



B. Blaettel, V. Koch, U. Mosel,
Rept. Prog. Phys. **56**,1–62 (1993)

fits to data

parametrizations of
the Walecka model

Solution:
vector+scalar density functional model (VSDF)

Challenge: scalar fields are costly to compute

VDF model:

$$\mathcal{E}_N = g \int \frac{d^3p}{(2\pi)^3} \epsilon_{\text{kin}} f_{\mathbf{p}} + \sum_{i=1}^N A_k^0 j_0 - g^{00} \sum_{i=1}^N \left(\frac{b_i - 1}{b_i} \right) A_k^\lambda j_\lambda$$

$$A_k^\mu = C_k (j_\lambda j^\lambda)^{\frac{b_k}{2} - 1} j^\mu, \quad j_\mu j^\mu = n_B^2, \quad j^\mu = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu - A^\mu}{\epsilon_{\text{kin}}^*} f_{\mathbf{p}}$$

VSDF model:

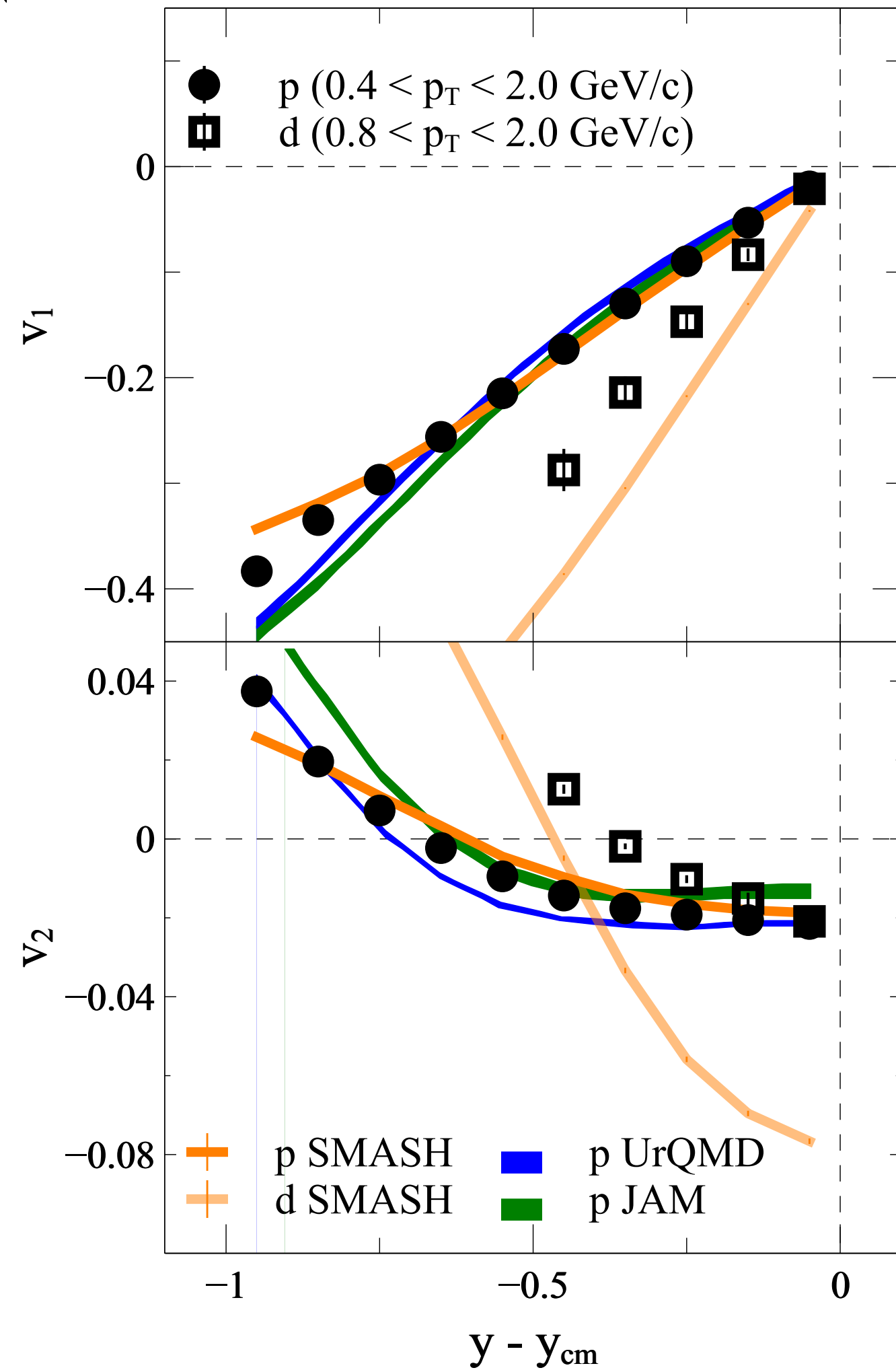
$$\mathcal{E}_{N, M} = g \int \frac{d^3p}{(2\pi)^3} \epsilon_{\text{kin}}^* f_{\mathbf{p}} + \sum_{i=1}^N A_k^0 j_0 - g^{00} \sum_{i=1}^N \left(\frac{b_i - 1}{b_i} \right) A_k^\lambda j_\lambda + g^{00} \sum_{m=1}^M G_m \left(\frac{d_m - 1}{d_m} \right) n_s^{d_m}$$

A. Sorensen, "Density Functional Equation of State and Its Application to the Phenomenology of Heavy-Ion Collisions," arXiv:2109.08105, Sorensen:2021zxd

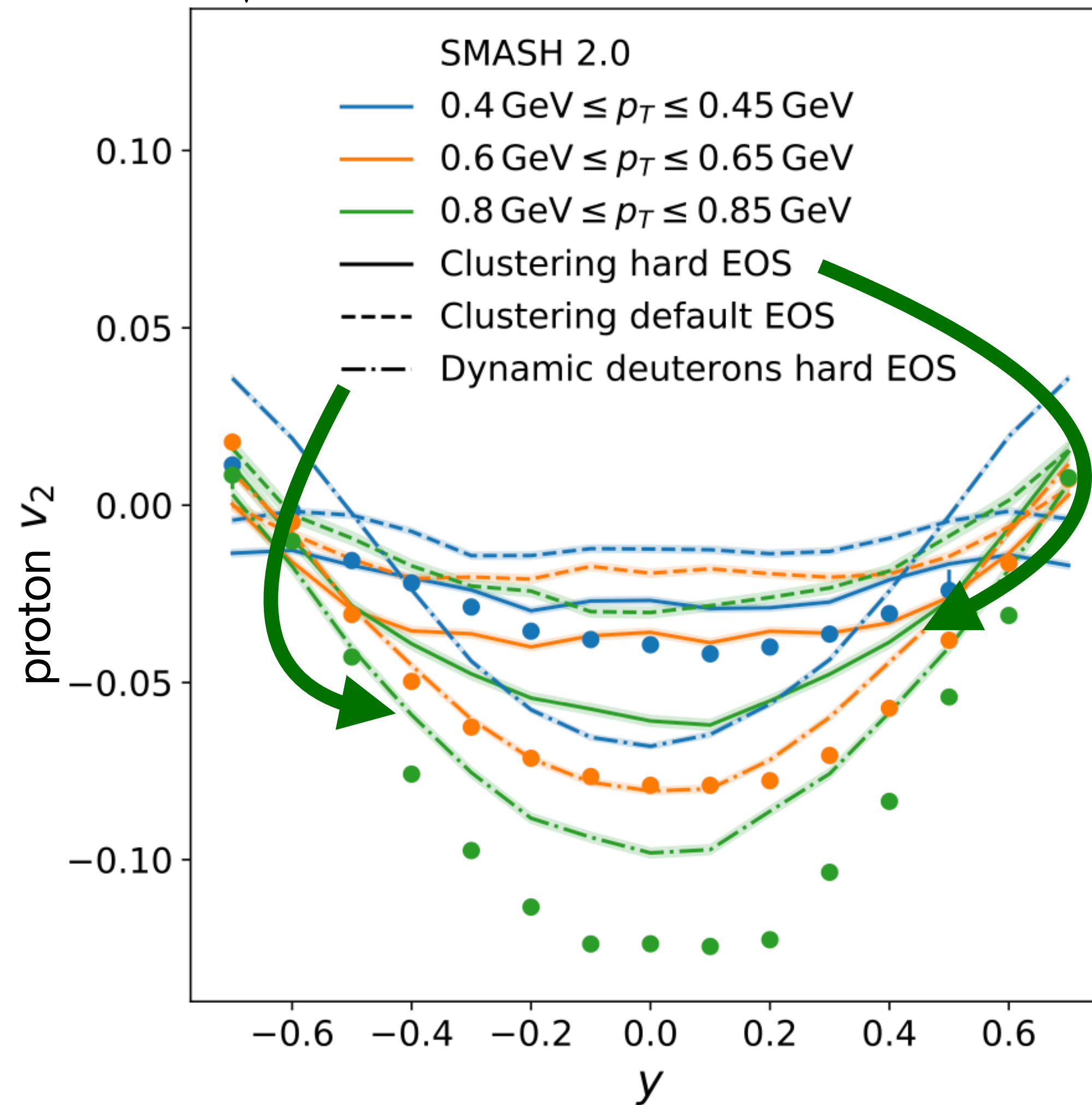
$$m^* = m_0 - \sum_{m=1}^M G_m n_s^{d_m - 1} \quad n_s = g \int \frac{d^3p}{(2\pi)^3} \frac{m^*}{\epsilon_{\text{kin}}^*} f_{\mathbf{p}}$$

Describing proton flow is not enough

$\sqrt{s_{NN}} = 3 \text{ GeV}$



$\sqrt{s_{NN}} = 2.4 \text{ GeV}$



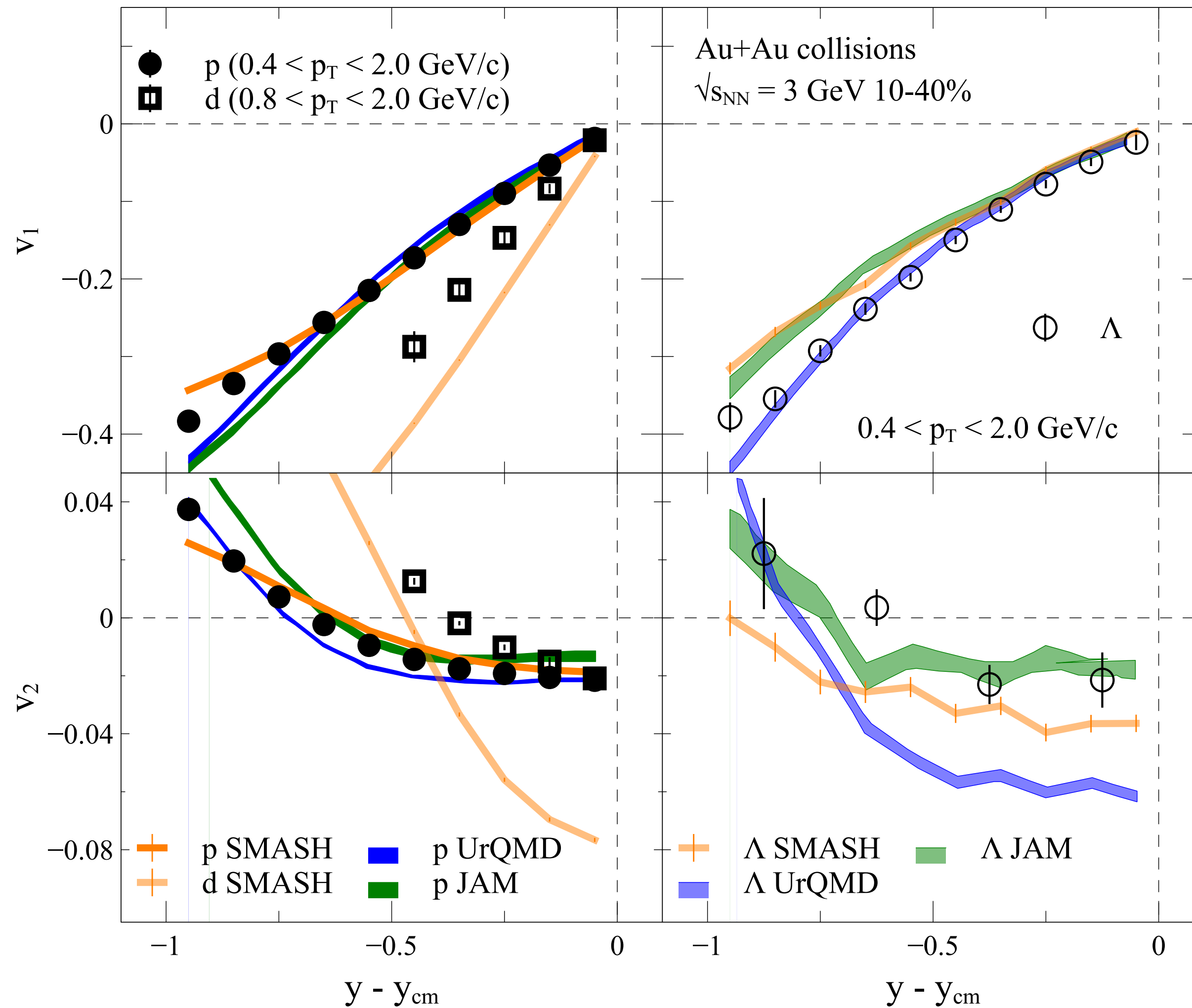
Realistic description of light cluster production needed:

- coalescence: doesn't take into account the dynamic role of light clusters throughout the evolution
- nucleon/pion catalysis: consider as separate degrees of freedom (pBUU, SMASH), produced through N or π collisions
- the Holy Grail: dynamical production through potentials

STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908
 D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996
A. Sorensen et al., arXiv:2301.13253

J. Mohs, M. Ege, H. Elfner, M. Mayer,
 Phys. Rev. C **105** 3, 034906 (2022),
 arXiv:2012.11454

Describing proton flow is not enough



Strange baryons are not well described

- the results may depend on:

- nucleon-hyperon and hyperon-hyperon interactions
- in-medium modifications of interactions

Models of interactions exists and could be tested; interactions could be based on those obtained within first-principle calculations (e.g., HALQCD collaboration

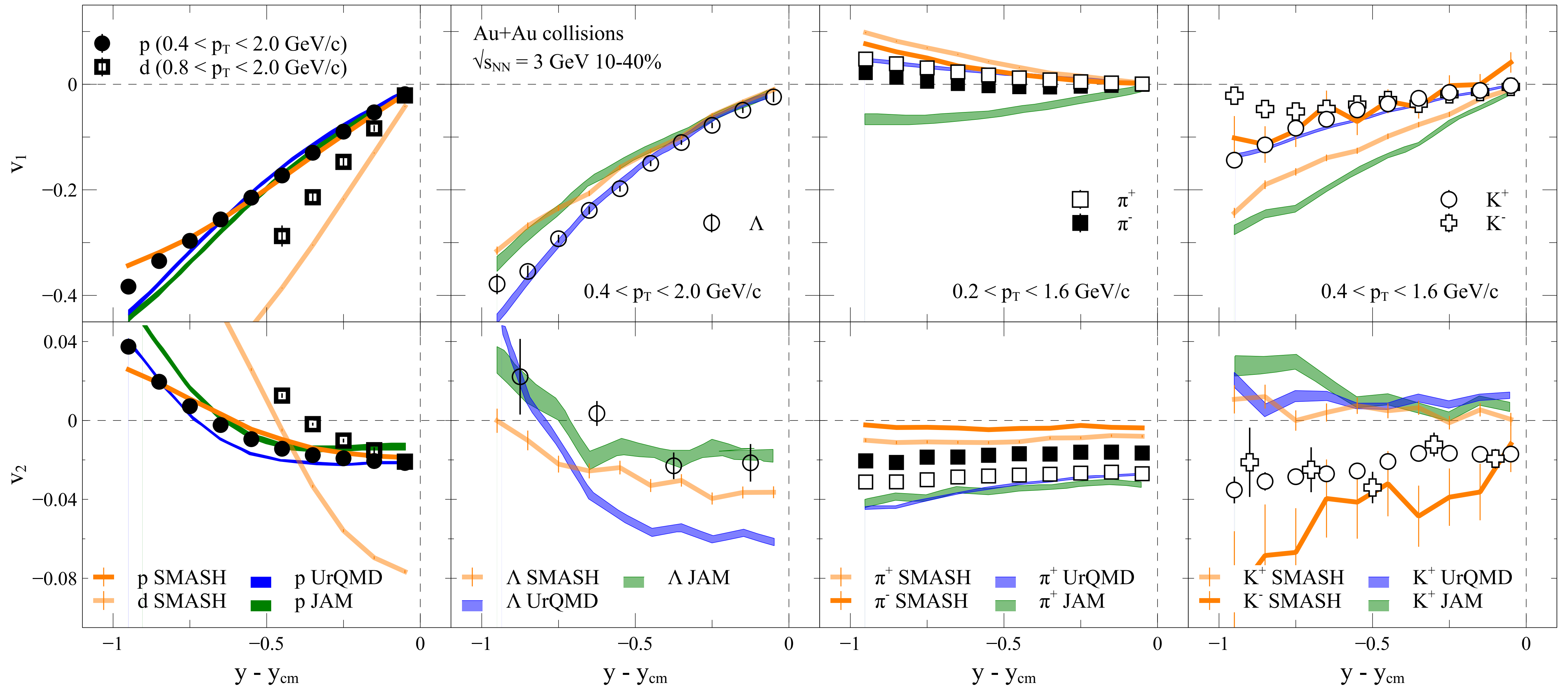
(HAL QCD, Nucl. Phys. A **998** 121737 (2020), arXiv:1912.08630)

STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

A. Sorensen et al., arXiv:2301.13253

Describing proton flow is not enough



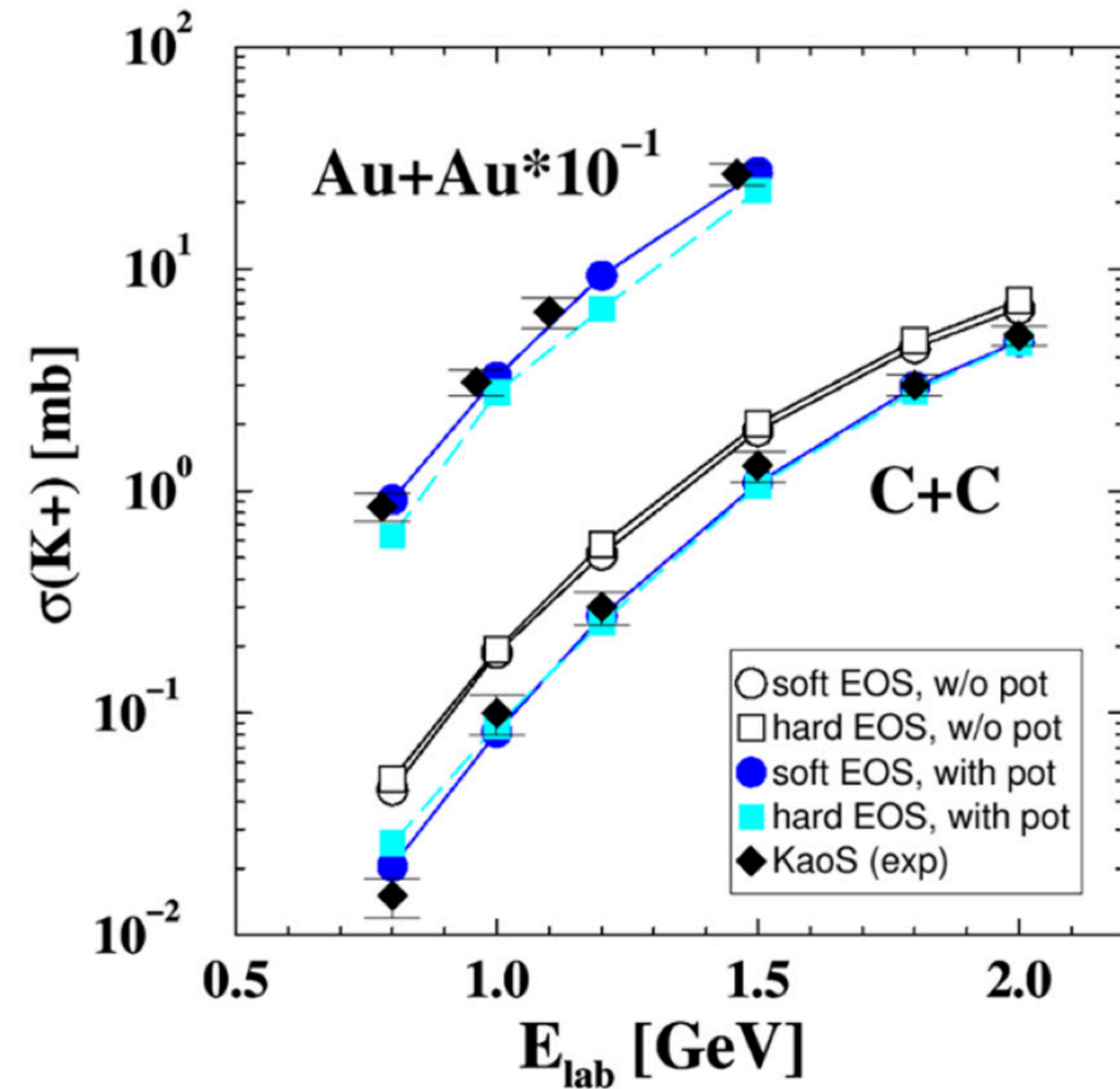
STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

A. Sorensen et al., arXiv:2301.13253

Describing proton flow is not enough

Pions and kaons NOT described!
 Not very surprising: UrQMD, JAM, and SMASH don't have mean-fields for mesons

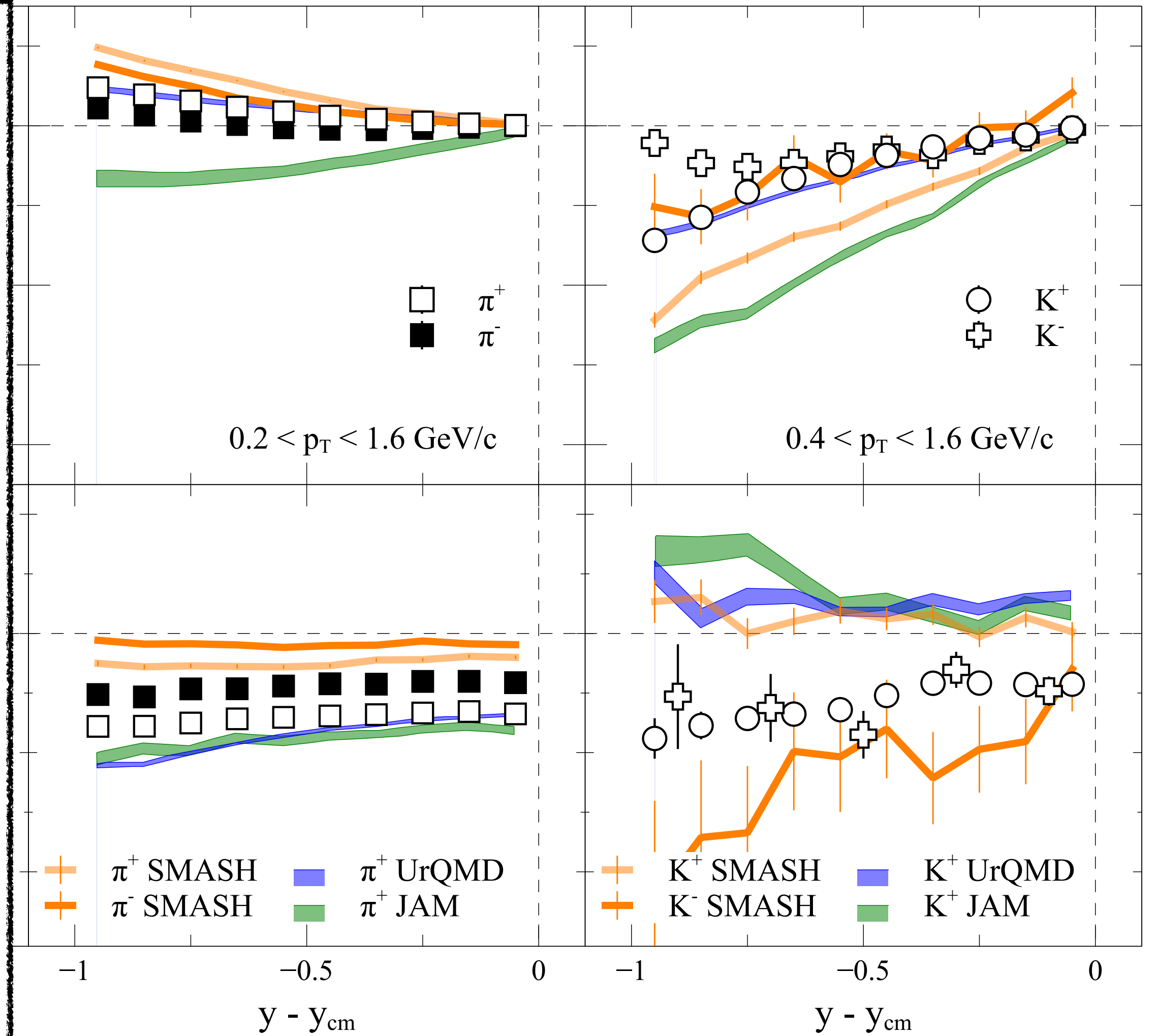


C. Fuchs, PoS CPOD07 060 (2007) arXiv:0711.3367

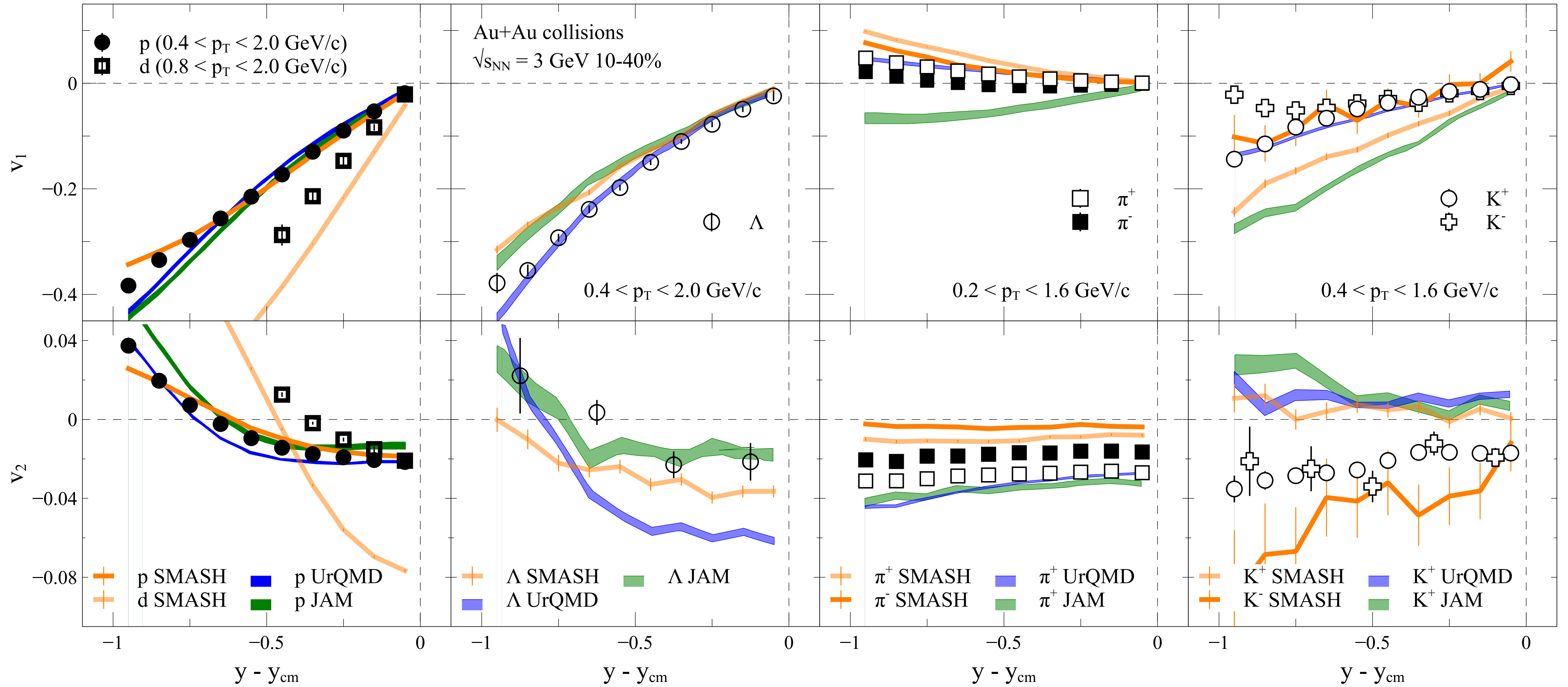
STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

A. Sorensen et al., arXiv:2301.13253



Describing proton flow is not enough



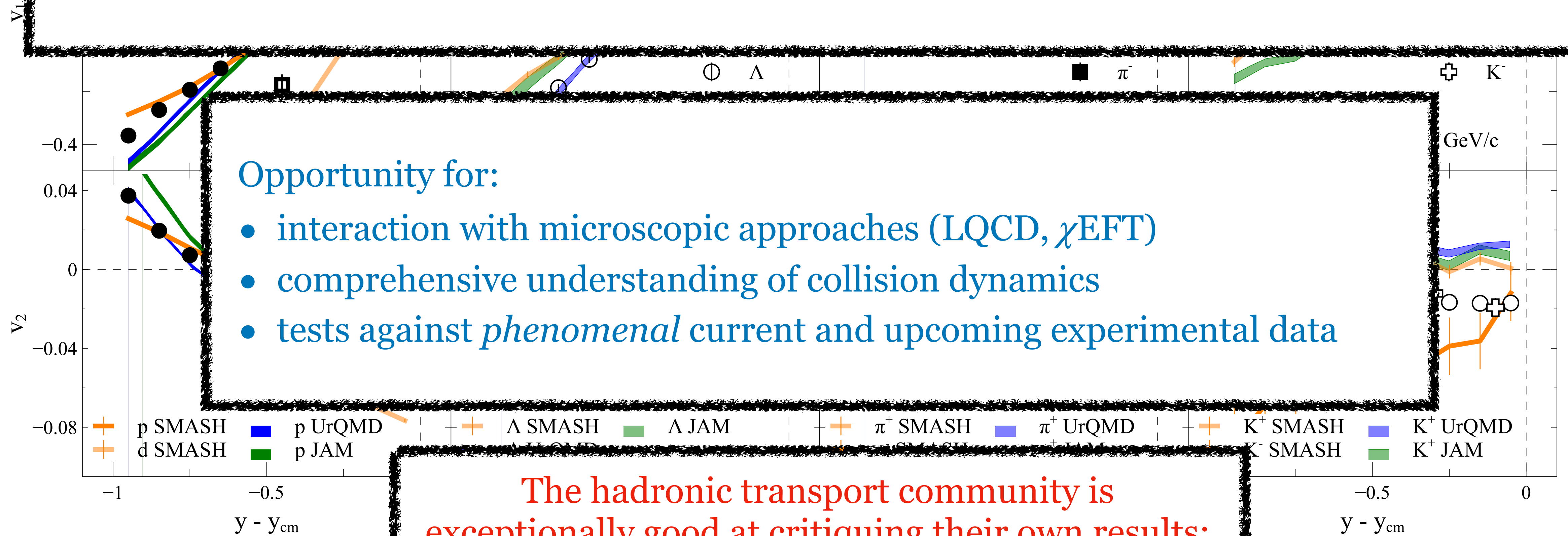
STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

A. Sorensen et al., arXiv:2301.13253

Describing proton flow is not enough

In a comprehensive Bayesian analysis, all relevant observables should be understood



STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2201.04111
 D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran
A. Sorensen et al., arXiv:2301.13253

Better modeling is necessary

Ideas to explore:

- threshold effects,
- light cluster production,
- neutron-proton effective mass splitting, ...

Strong efforts by the
Transport Model Evaluation
Project (TMEP) collaboration
to identify code-dependencies
and best model practices!

#1
Transport model comparison studies of intermediate-energy heavy-ion collisions
TMEP Collaboration · Hermann Wolter (Munich U.) et al. (Feb 14, 2022)
Published in: *Prog.Part.Nucl.Phys.* 125 (2022) 103962 · e-Print: [2202.06672](#) [nucl-th]
pdf DOI cite claim reference search 31 citations

#2
Comparison of heavy-ion transport simulations: Mean-field dynamics in a box
TMEP Collaboration · Maria Colonna (INFN, LNS) et al. (Jun 23, 2021)
Published in: *Phys.Rev.C* 104 (2021) 2, 024603 · e-Print: [2106.12287](#) [nucl-th]
pdf DOI cite claim reference search 29 citations

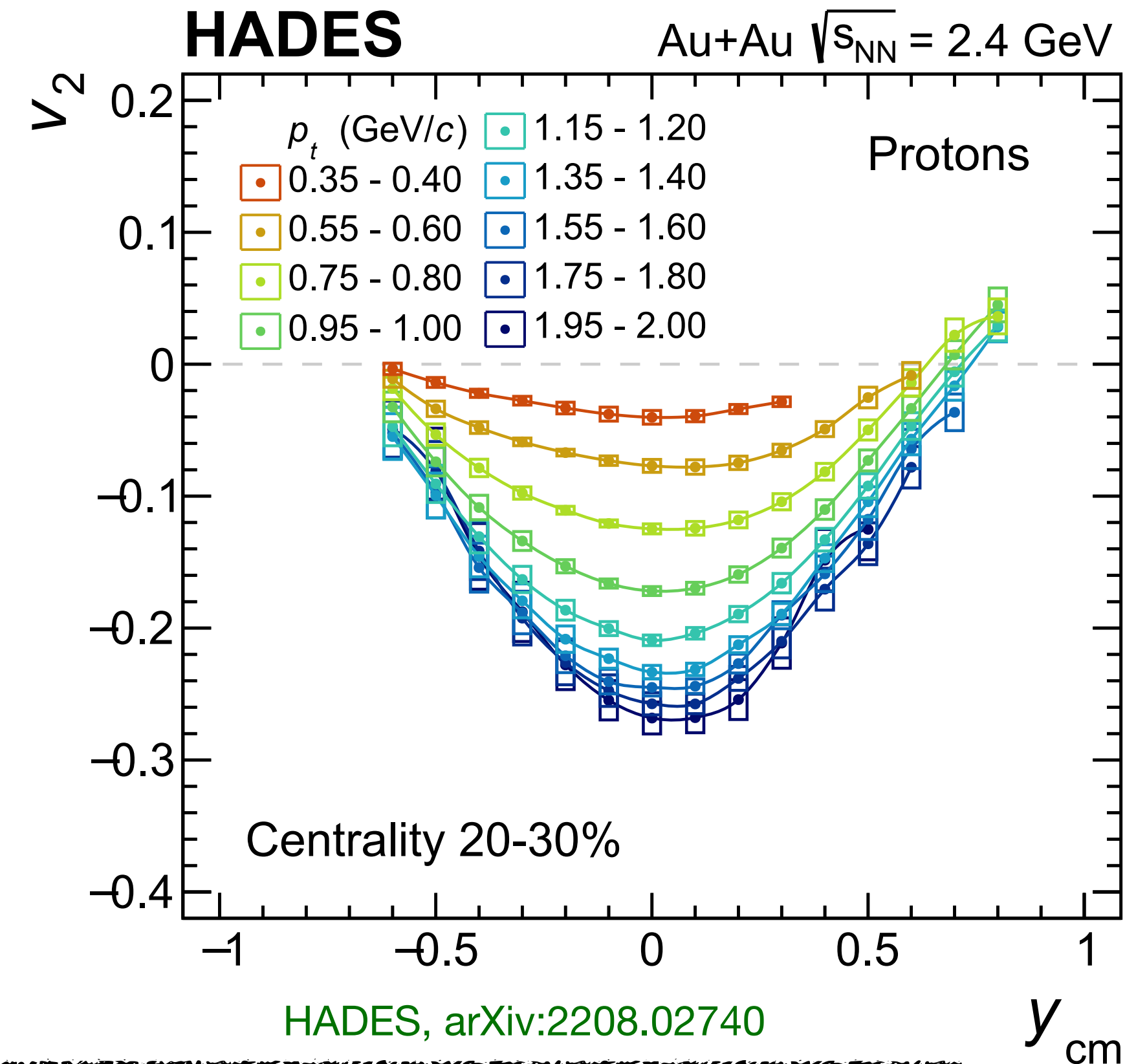
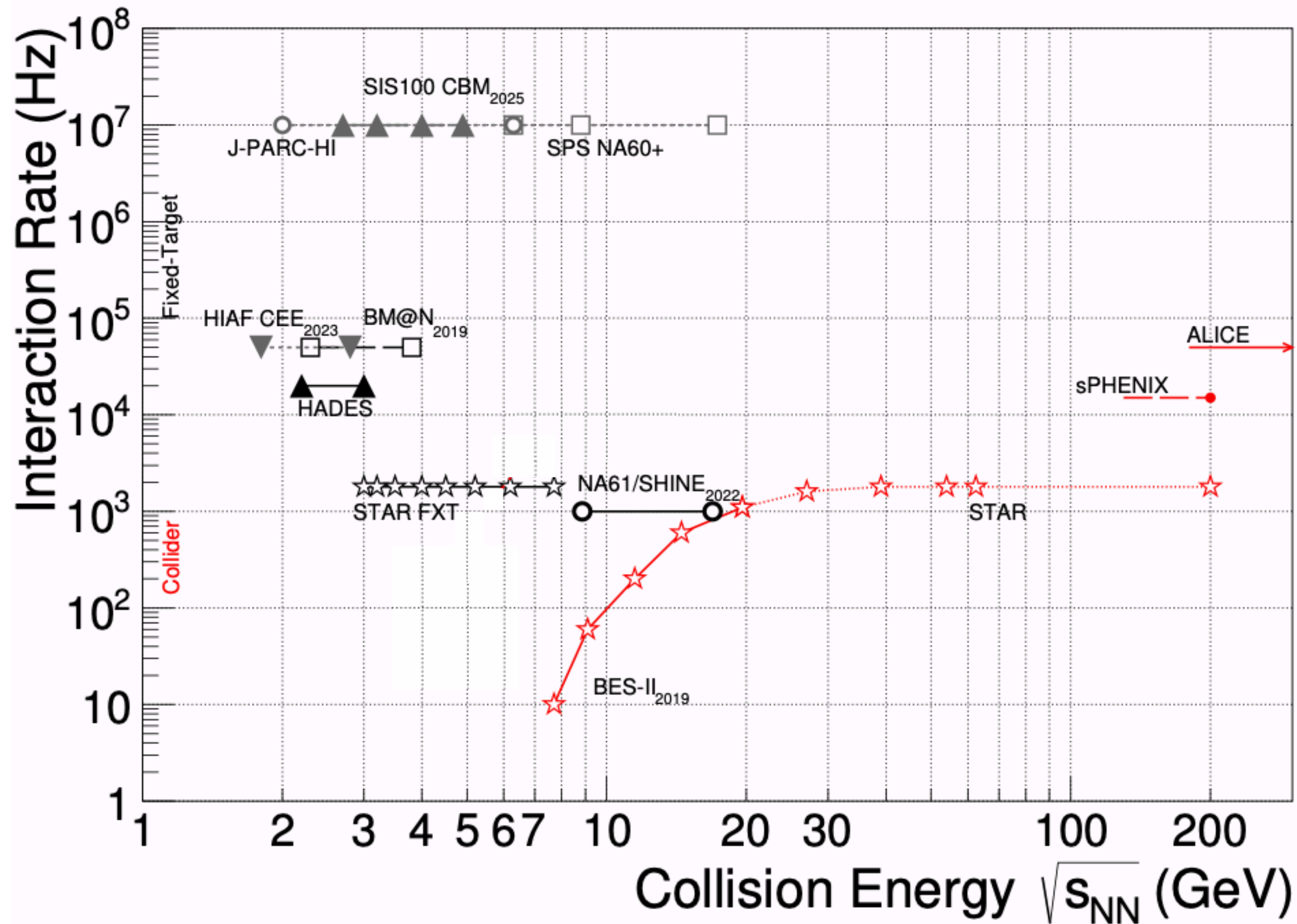
#3
Symmetry energy investigation with pion production from Sn+Sn systems
SpiRIT and TMEP Collaborations · G. Jhang et al. (Dec 13, 2020)
Published in: *Phys.Lett.B* 813 (2021) 136016 · e-Print: [2012.06976](#) [nucl-ex]
pdf DOI cite claim reference search 34 citations

#4
Comparison of heavy-ion transport simulations: Collision integral with pions and Δ resonances in a box
TMEP Collaboration · Akira Ono (Tohoku U.) et al. (Apr 5, 2019)
Published in: *Phys.Rev.C* 100 (2019) 4, 044617 · e-Print: [1904.02888](#) [nucl-th]
pdf DOI cite claim reference search 59 citations

#5
Comparison of heavy-ion transport simulations: Collision integral in a box
TMEP Collaboration · Ying-Xun Zhang (Beijing, Inst. Atomic Energy and Guangxi Normal U.) et al. (Nov 16, 2017)
Published in: *Phys.Rev.C* 97 (2018) 3, 034625 · e-Print: [1711.05950](#) [nucl-th]
pdf DOI cite claim reference search 103 citations

#6
Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV: Comparison of heavy-ion transport codes under controlled conditions
TMEP Collaboration · Jun Xu (SINAP, Shanghai) et al. (Mar 26, 2016)
Published in: *Phys.Rev.C* 93 (2016) 4, 044609 · e-Print: [1603.08149](#) [nucl-th]

Precision era of heavy-ion collisions



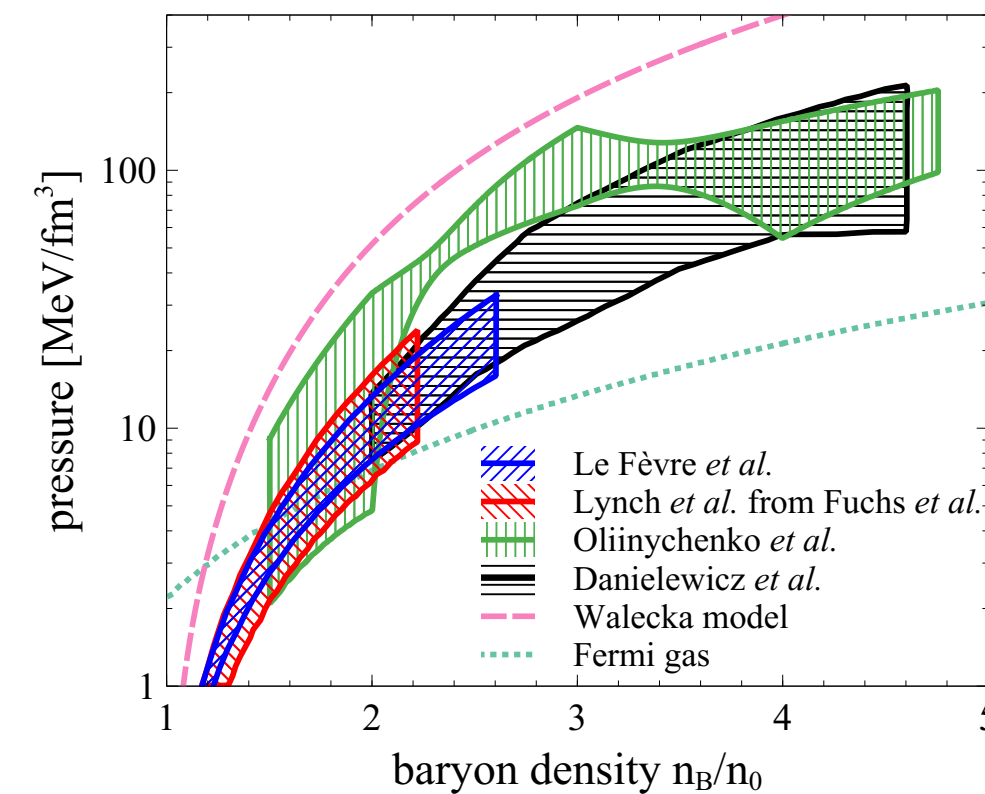
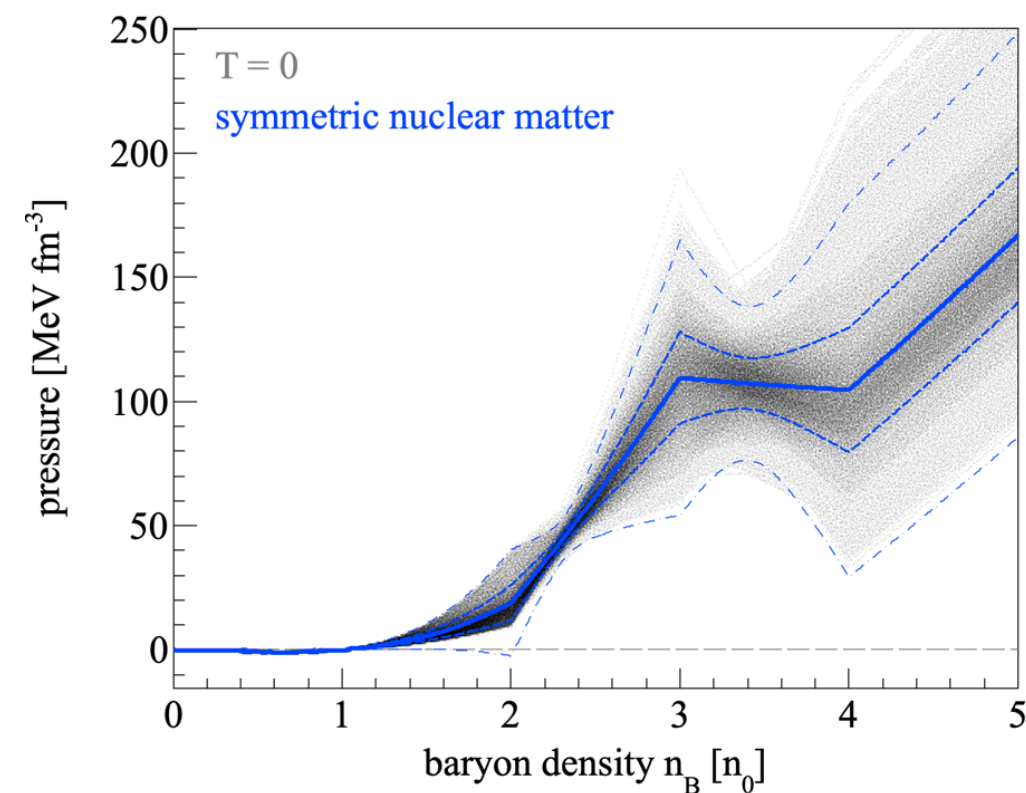
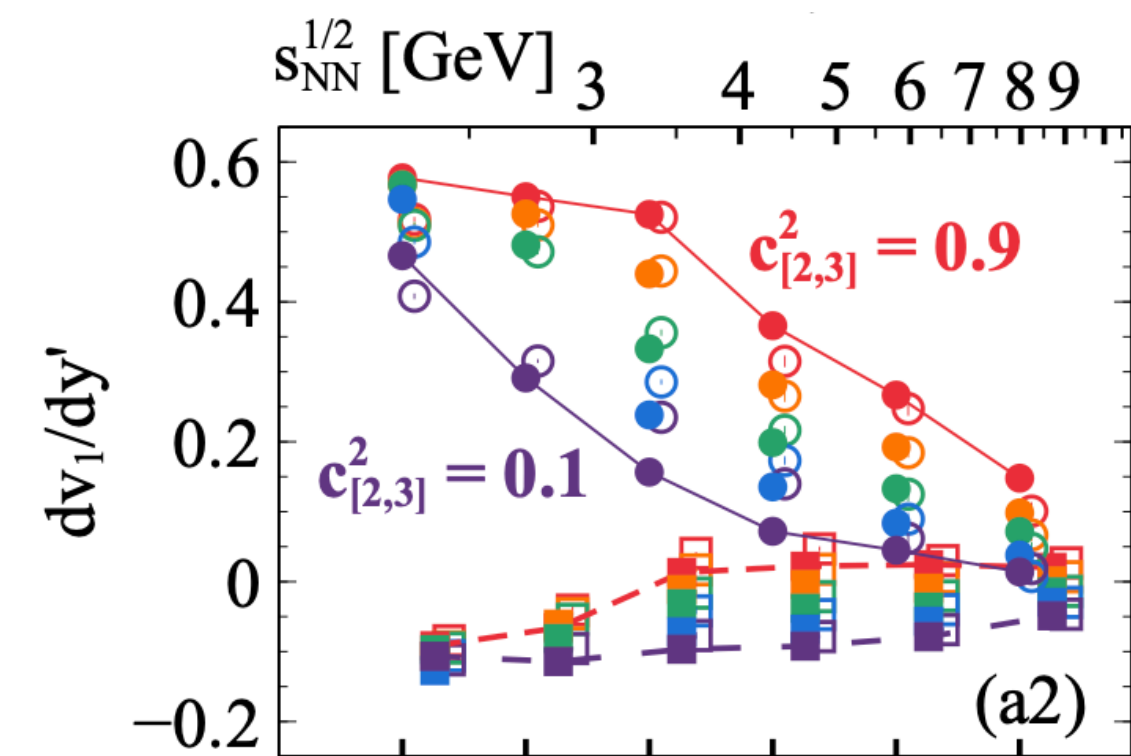
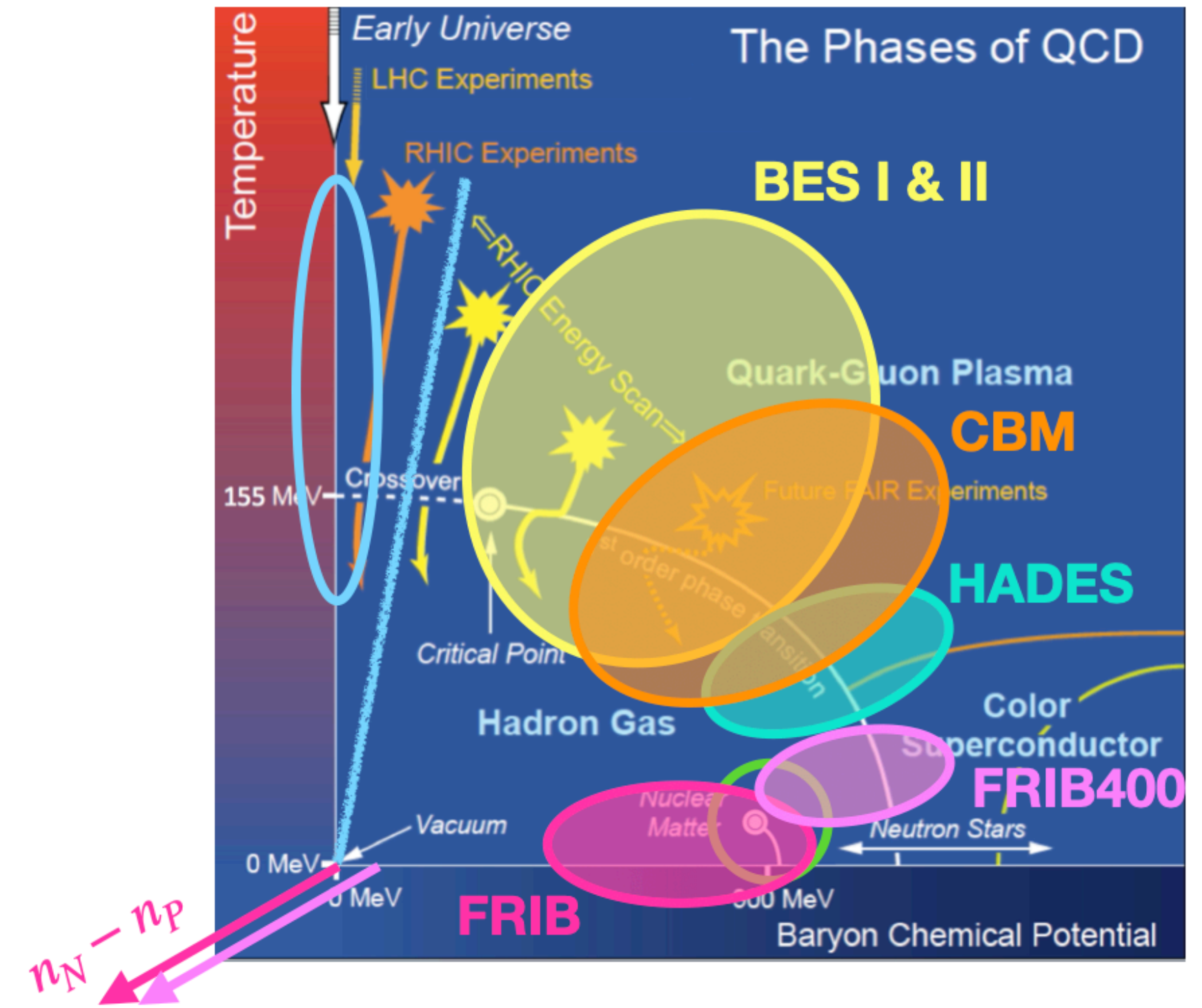
**Precision experiments
NEED precision simulations**

Very high-quality, high-statistics data are imminent from BES FXT, HADES, CBM, FRIB, FRIB400: perhaps observables are now available which were previously inaccessible?

Summary

What's different, new, exciting about *now*?

- **New analyses, new understanding:** e.g., triangular flow, quark number scaling, cumulants
- **New detectors, new data:** unprecedented measurements, from ultra-precise triple-differential flow observables to hyperon-hyperon interactions
- **New computing capabilities:** large-scale simulations possible with state-of-the-art, benchmarked hadronic transport codes
- **New approach to constraining the EOS:** Bayesian analyses using flexible parametrizations of the EOS



Thank you for your attention