The universal hill shape and ridge structure of entropy per baryon number near the QCD critical point

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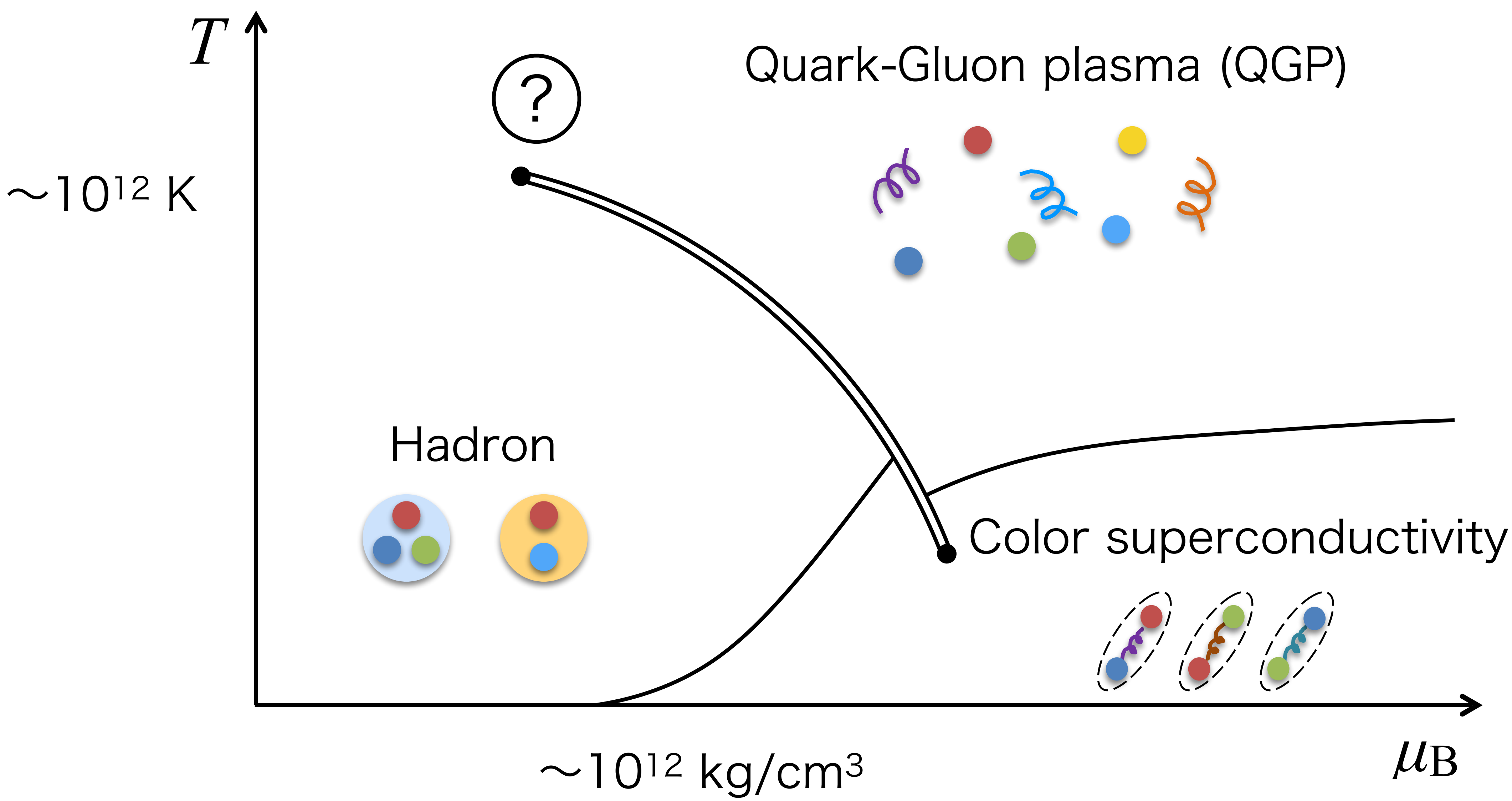
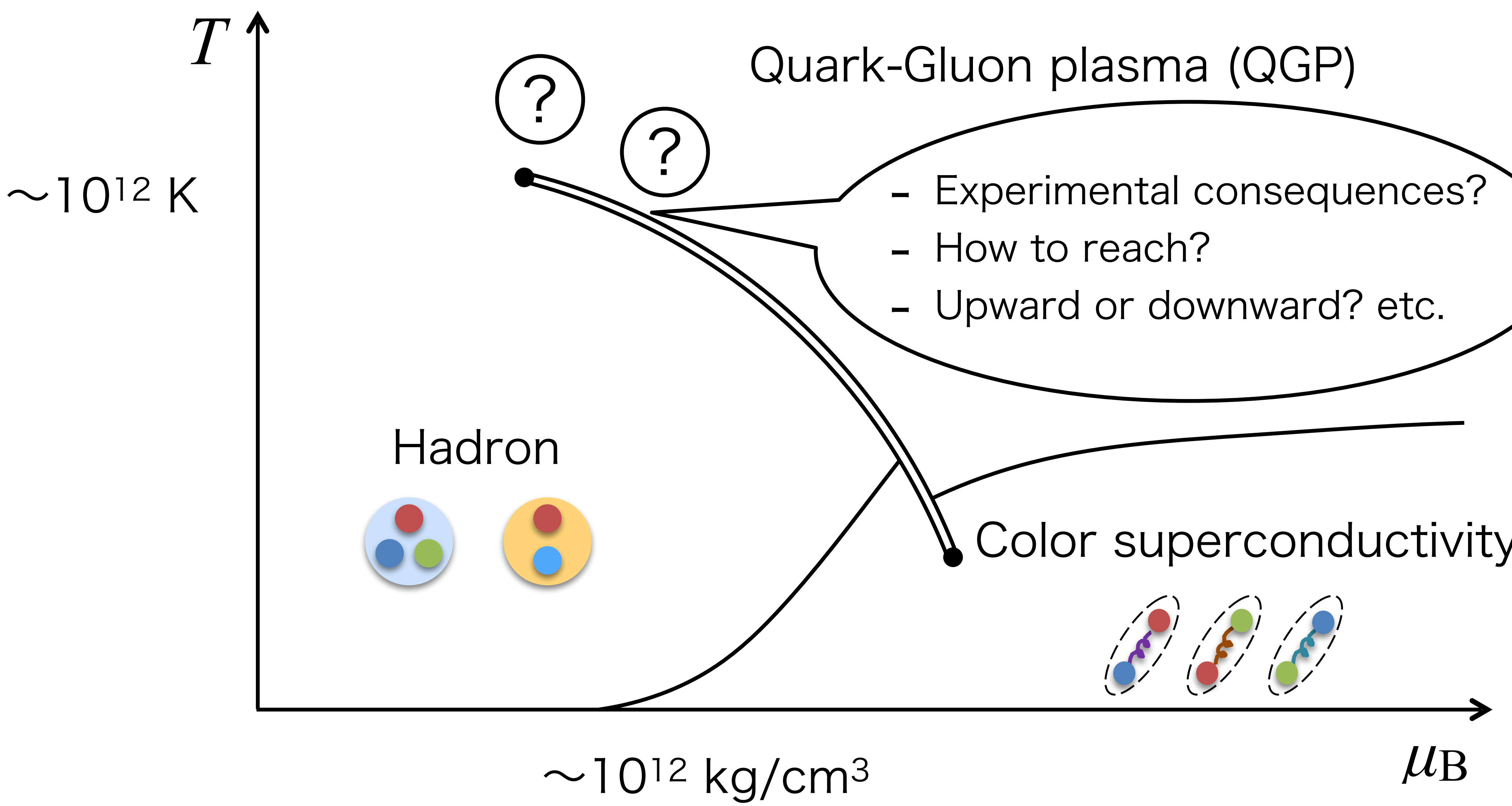
QCD phase diagram



QCD phase diagram



Setup

- Ideal hydrodynamics: 0th-order approximation in heavy-ion collisions
- Entropy and baryon number conservation in boost invariance:

$$s\tau = s_{\rm ini}\tau_{\rm ini}\,, \quad n\tau = n_{\rm ini}\tau_{\rm ini} \;\longrightarrow\; \hat{s} \equiv \frac{s}{n} = \frac{s_{\rm ini}}{n_{\rm ini}}$$

$\longrightarrow$ Time evolution = $\hat{s}$ contours on the (μ, T) plane

- The universality of critical phenomena

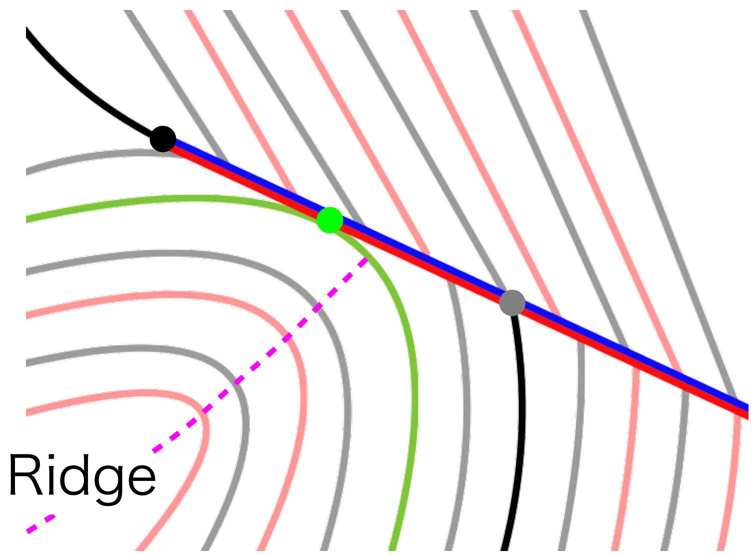
$\longrightarrow$ EOS of QCD ≃ 3D Ising mode

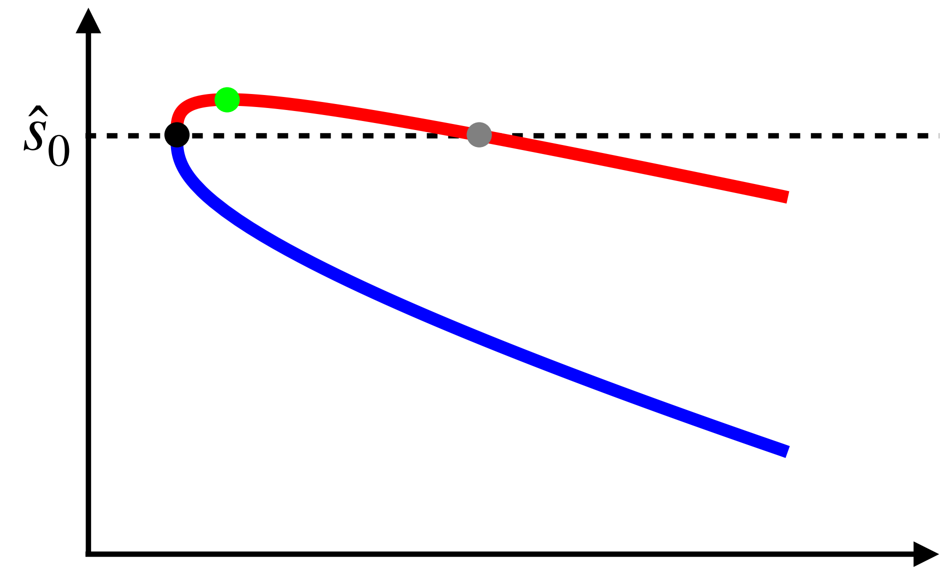
$\longrightarrow$ Universal properties on the $\hat{s}$ contour

Ridge and the hill-shape

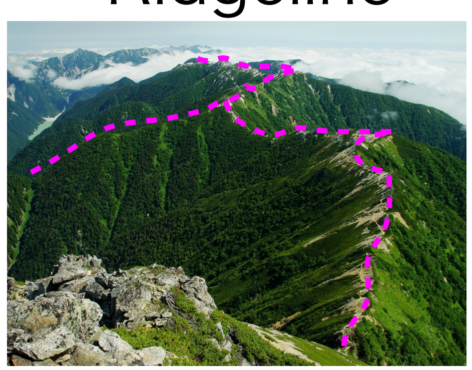
M. Pradeep, M. Stephanov, NS, and H. Yee (in preparation)

$\hat{s}(\mu, T) = \mathrm{const}\,.$





Ridgeline



Wikipedia

Inevitable non-monotonic hill shape:

- Critical degeneracy: $\hat{s} \simeq \pm (T - T_c)^\beta$
- Third law of thermodynamics: $\hat{s}(T=0) = 0$

Outline

Mapping $(h, r) \leftrightarrow (\mu, T)$

- Pressure $P(h, r)$ with relevant parameters (h, r) given by RG

R. Guida and J. Zinn-Justin (1997)

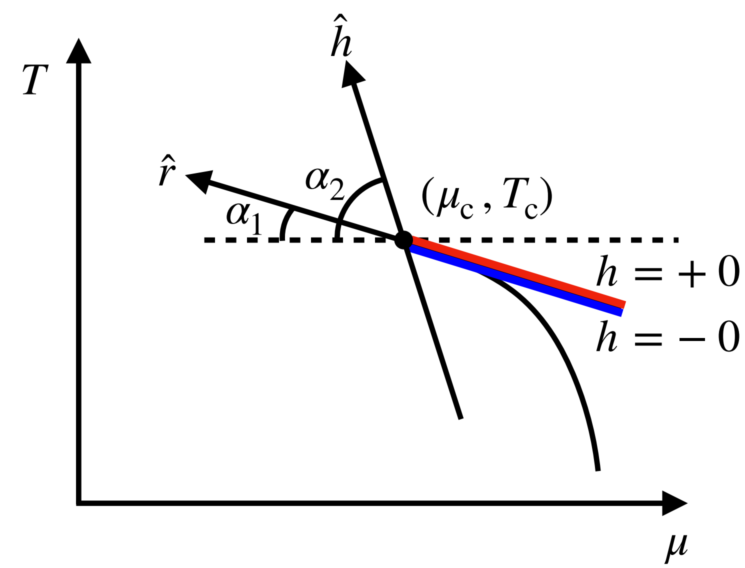
- Near the QCD critical point:

C. Nonaka, M. Asakawa (2005), Parotto et al. (2020)

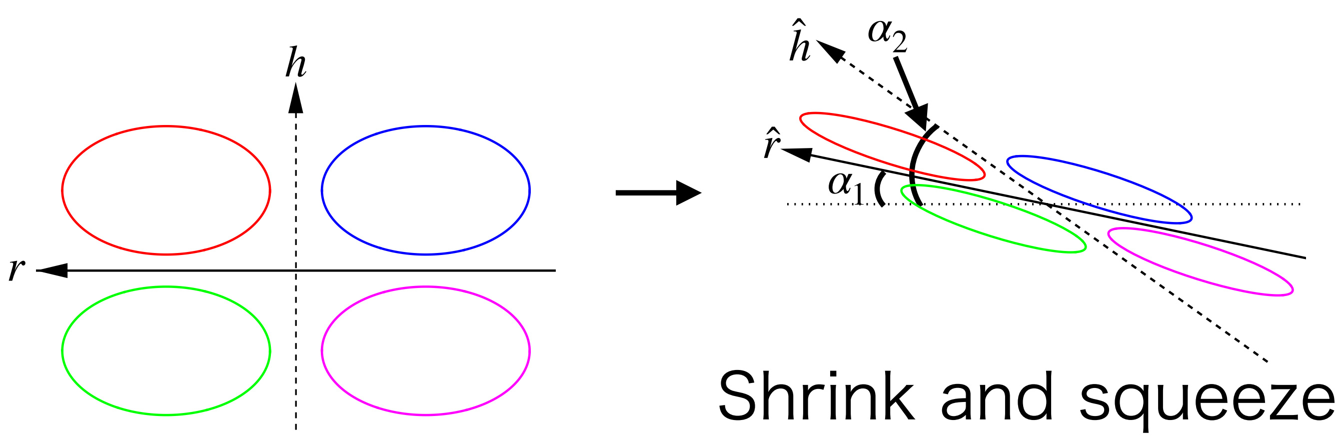
$$\frac{\mu - \mu_c}{T_c} = -w(r\rho \cos\alpha_1 + h\cos\alpha_2)$$

$$\frac{T - T_c}{T_c} = w(r\rho \sin\alpha_1 + h\sin\alpha_2)$$

$$\longrightarrow \quad h(\mu, T),\, r(\mu, T)$$



- α_2 : Shrink/squeeze ratio



Entropy/baryon near the critical point

- Thermodynamic quantities:

$$s \equiv \partial_T p + s_0 = m\partial_T h + \sigma\partial_T r + s_0 \quad (\because \text{ Chain rule})$$

$$n \equiv \partial_\mu p + n_0 = m\partial_\mu h + \sigma\partial_T r + n_0$$

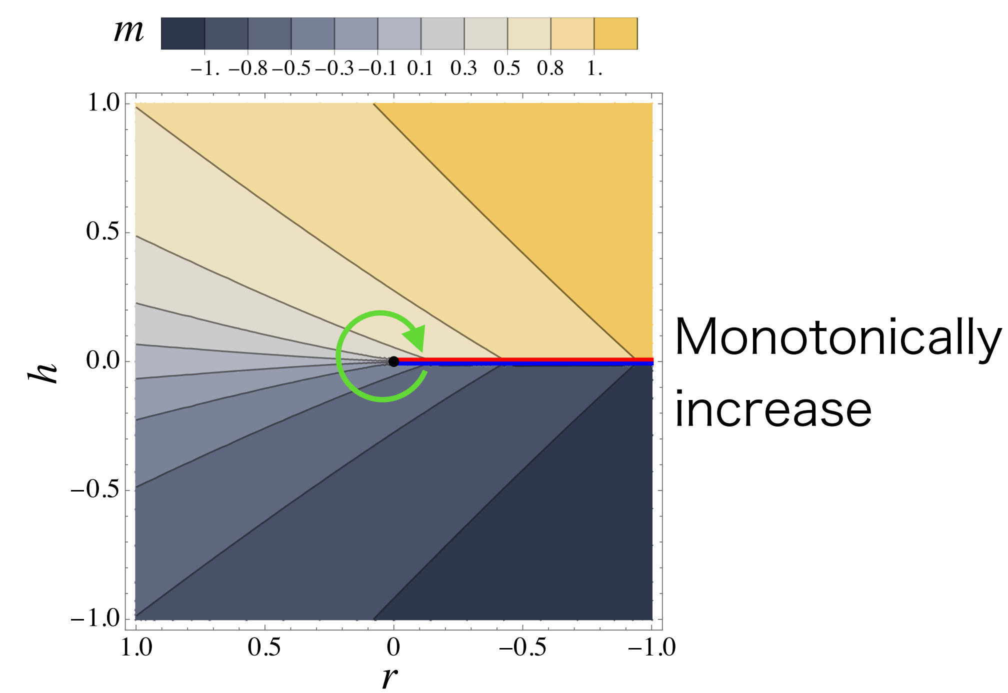
- Relevant parameter derivatives:
  - Order parameter: $m = \partial_h p$
  - Ising entropy: $\sigma = \partial_r p$

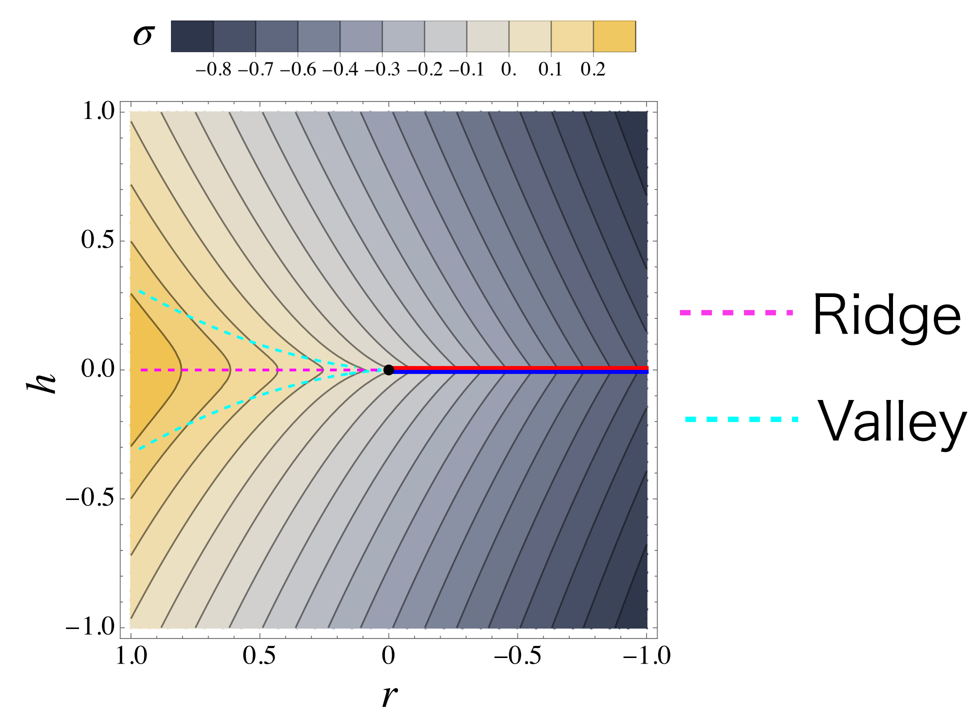
P. Schofield (1969) R. Guida and J. Zinn-Justin (1997)

- Entropy/baryon number:

$$\hat{s} \simeq \hat{s}_0 + (\partial_m \hat{s})_0 m + (\partial_\sigma \hat{s})_0 \sigma$$

$$(\mu, T) \xrightarrow{\text{Map}} (h, r) \xrightarrow{\text{Universal EOS}} m, \sigma \longrightarrow \hat{s}$$





Along the first-order line

M. Pradeep, M. Stephanov, NS, and H. Yee (in preparation)

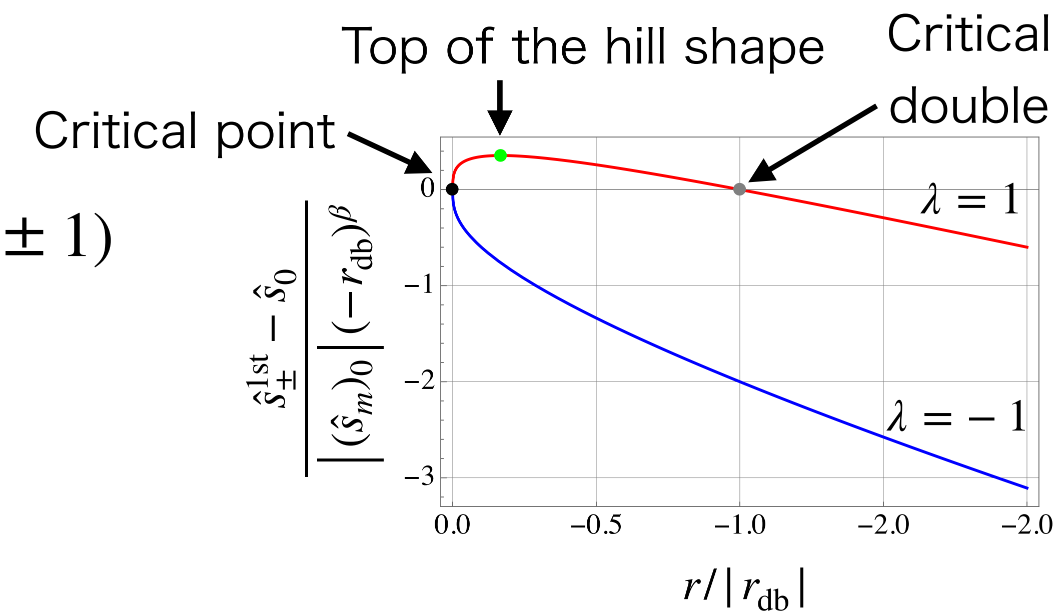
- Set $h = \pm 0\,,\ r \leq 0$

$$\hat{s}_\lambda^{1\mathrm{st}} = \hat{s}_0 + \lambda(\partial_m \hat{s})_0(-r)^\beta + (\partial_\sigma \hat{s})_0 C(-r)^{1-\alpha} \quad (\lambda = \pm 1)$$

Competition between $m \propto (-r)^\beta$ and $\sigma \propto (-r)^{1-\alpha}$

$\left(\beta = 0.326\,,\ \alpha = 0.11\right)$

⟶ non-monotonic behavior



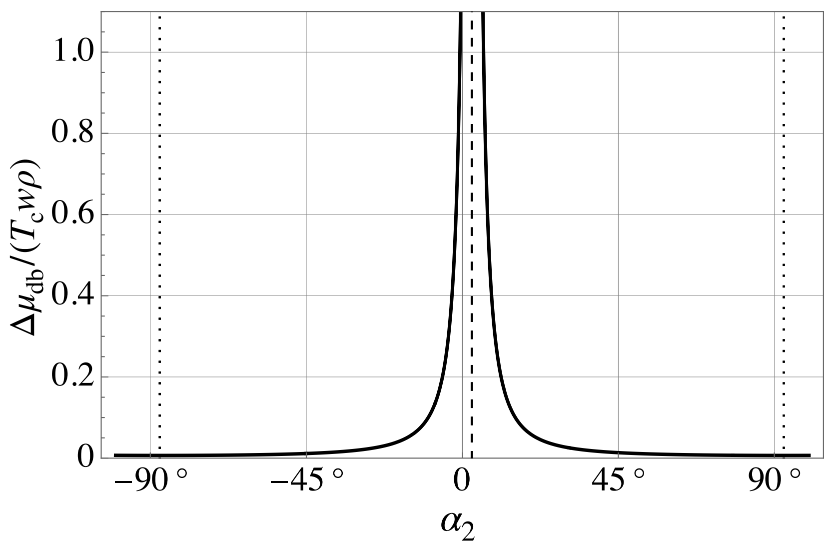
- The critical double: $(-r_{\mathrm{db}})^{1-\alpha-\beta} = \left|\dfrac{(\partial_m \hat{s})_0}{C(\partial_\sigma \hat{s})_0}\right| \propto \left|\dfrac{\cos\alpha_1 - \hat{s}_0 \sin\alpha_1}{\cos\alpha_2 - \hat{s}_0 \sin\alpha_2}\right|$

- Non-monotonic hill side: $\lambda = \lambda_{\mathrm{hill}}^{(r,h)} = \mathrm{sgn}\,(\partial_m \hat{s})_0$ ($(\partial_\sigma \hat{s})_0 > 0$ for down concavity)

(μ, T) plane

M. Pradeep, M. Stephanov, NS, and H. Yee (in preparation)

- Solve $h(\mu, T) = 0\,,\ r(\mu, T) = r_{\rm db} \longrightarrow (\mu, T) = (\mu_{\rm db}\,, T_{\rm db})$



$$\mu_{\rm db} - \mu_{\rm c} \sim \begin{cases} T_{\rm c} & (\alpha_2 \simeq 0^\circ) \\ \dfrac{T_{\rm c}}{\hat{s}_0^2} & (\alpha_2 \simeq 90^\circ) \end{cases}$$

$$\left(0^\circ < \alpha_1 \ll 90^\circ\,,\ \hat{s}_0 \gg 1\,,\ \hat{s}_0\alpha_1 \sim \mathcal{O}(1)\right)$$

- Hillside index: $\lambda_{\rm hill}^{(\mu,T)} \equiv \begin{cases} 1 & \text{above} \\ -1 & \text{below} \end{cases}$ the boundary $= \lambda_{\rm hill}^{(r,h)} {\rm sgn}\,(\alpha_2 - \alpha_1) = {\rm sgn}\left(\cot\alpha_1 - \hat{s}_0\right)$

  ↖ from the mapping

$$\lambda_{\rm hill}^{(\mu,T)} = -1 \qquad (\hat{s}_0^{-1} < \alpha_1 < 90^\circ)$$

Large region for plausible parameters: $\hat{s}_0 \gg 1$

Contours: parameter sets

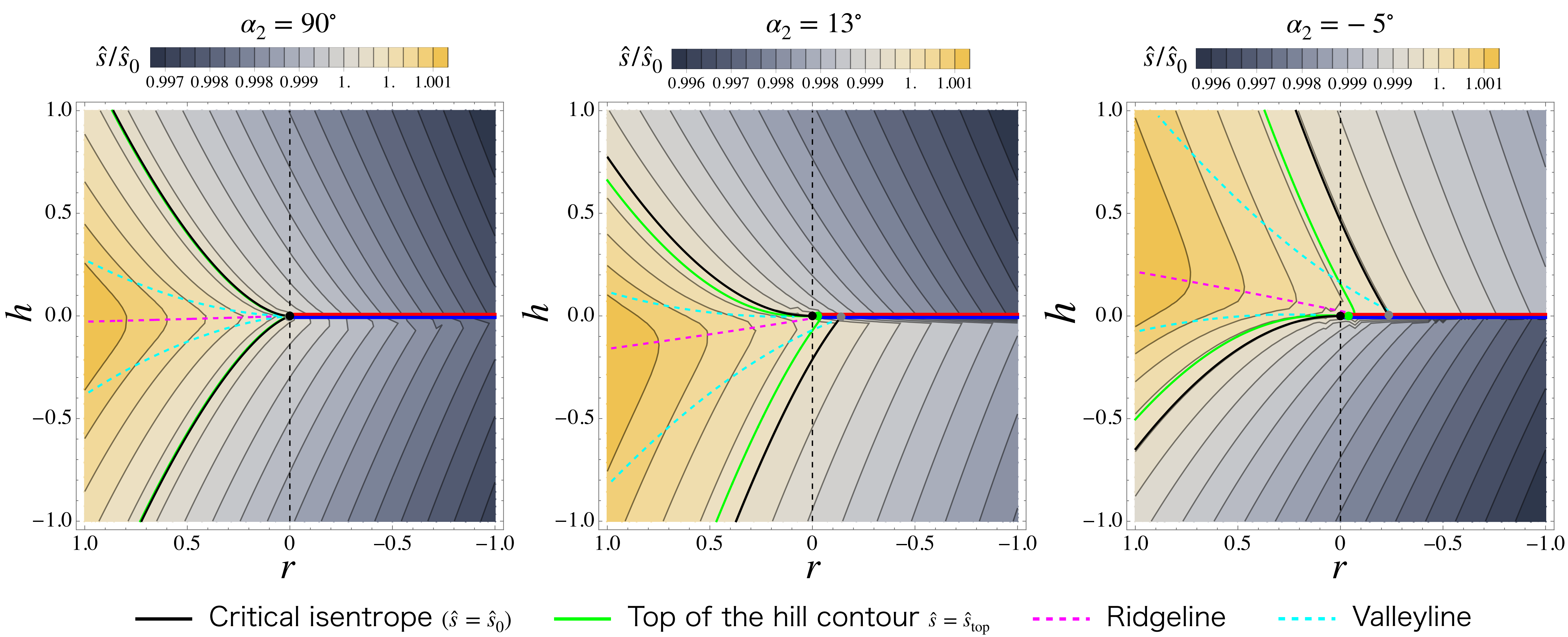
- Default: $(\mu_\mathrm{c}\,, T_\mathrm{c}\,, s_0\,, n_0\,, w\,, \rho) = (350\,\mathrm{MeV}\,,\ 143.2\,\mathrm{MeV}\,,\ 21\,,\ 1\,,\ 1\,,\ 2)$

  Parotto et al. (2020)
- Vary slope angles α_1 (the first-order line) and α_2 (shrink/stretch rate):
  - Crossover from lattice QCD $\longrightarrow$ $0 < \alpha_1 \ll 90^\circ$
  - $|\alpha_1 - \alpha_2|$ can be suppressed by m_q M. Pradeep and M. Stephanov (2019)
  - α_2 can be negative

α_1	3.85° $(\cot\alpha_1 < \hat{s}_0)$			1° $(\cot\alpha_1 > \hat{s}_0)$		
α_2	90°	13°	-5°	90°	13°	-13°

Contours (r, h) plane

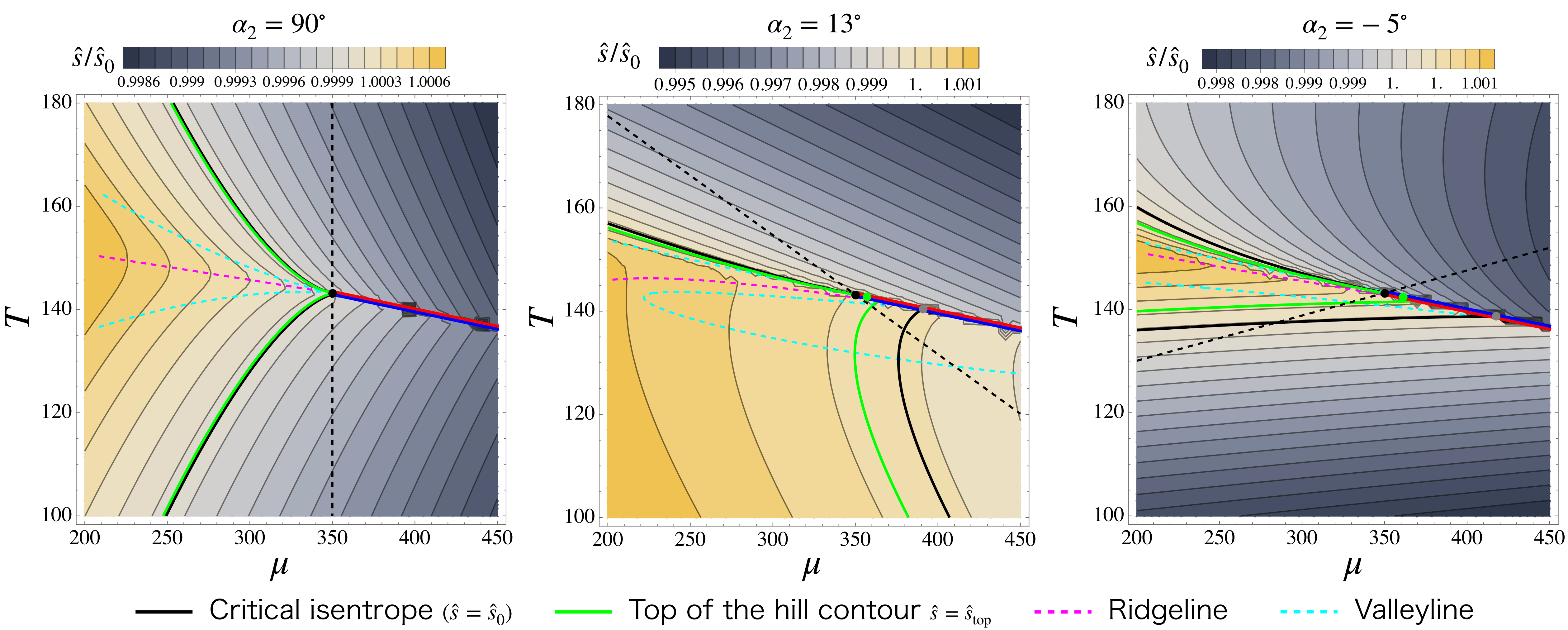
$\alpha_1 = 3.85^\circ \quad (\cot \alpha_1 < \hat{s}_0)$



- Mixing of m and σ $\longrightarrow$ The ridge(valley) lines of σ moved and curved

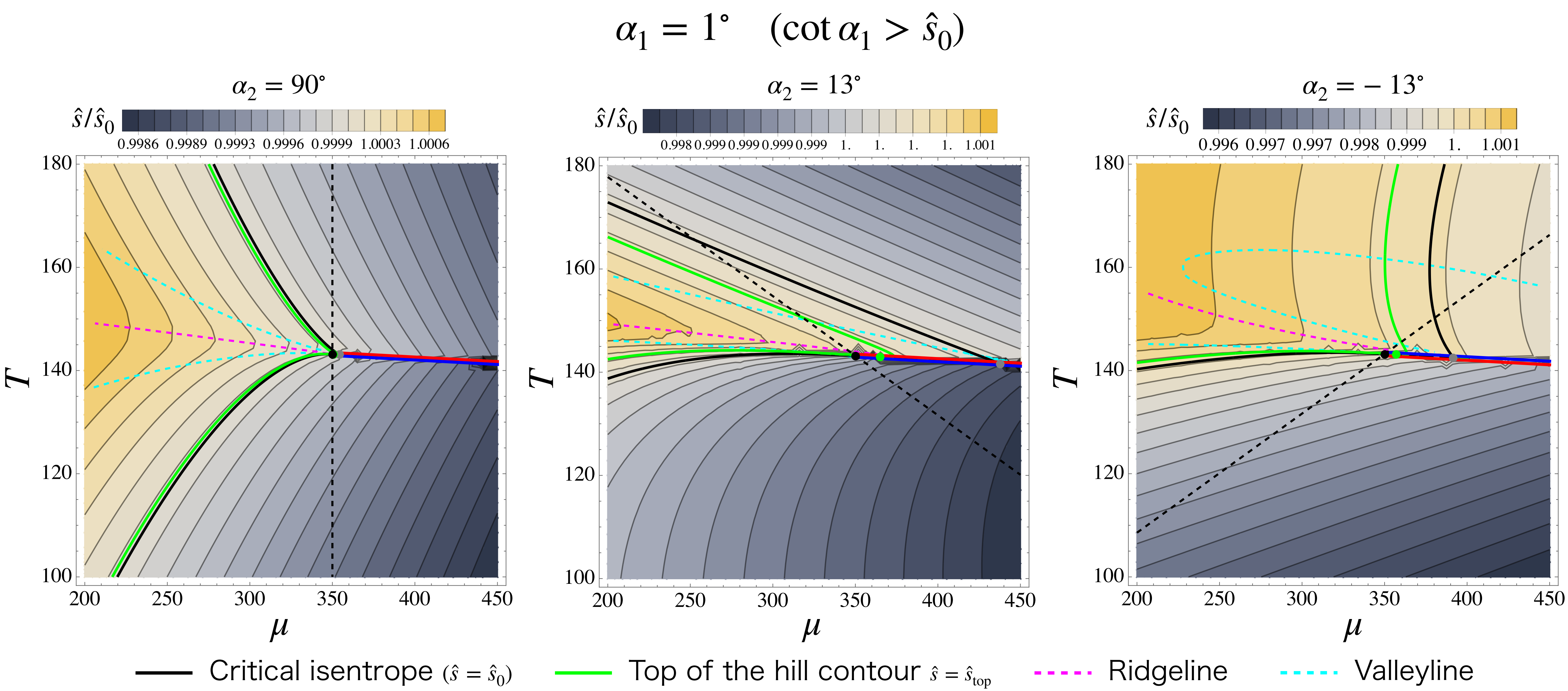
Contours (μ, T) plane

$\alpha_1 = 3.85^\circ \quad (\cot \alpha_1 < \hat{s}_0)$



- Ridgeline and valley line are robust under the shrink and squeeze of $\hat{s}(r, h)$
- $\lambda_{\text{hill}}^{(\mu,T)} = -1$

Contours (μ, T) plane



- Narrow region for $\lambda_{\text{hill}}^{(\mu,T)} = 1$: $0 < \alpha_1 < \cot^{-1}\hat{s}_0 \simeq \hat{s}_0^{-1}, (\hat{s}_0 \gg 1)$

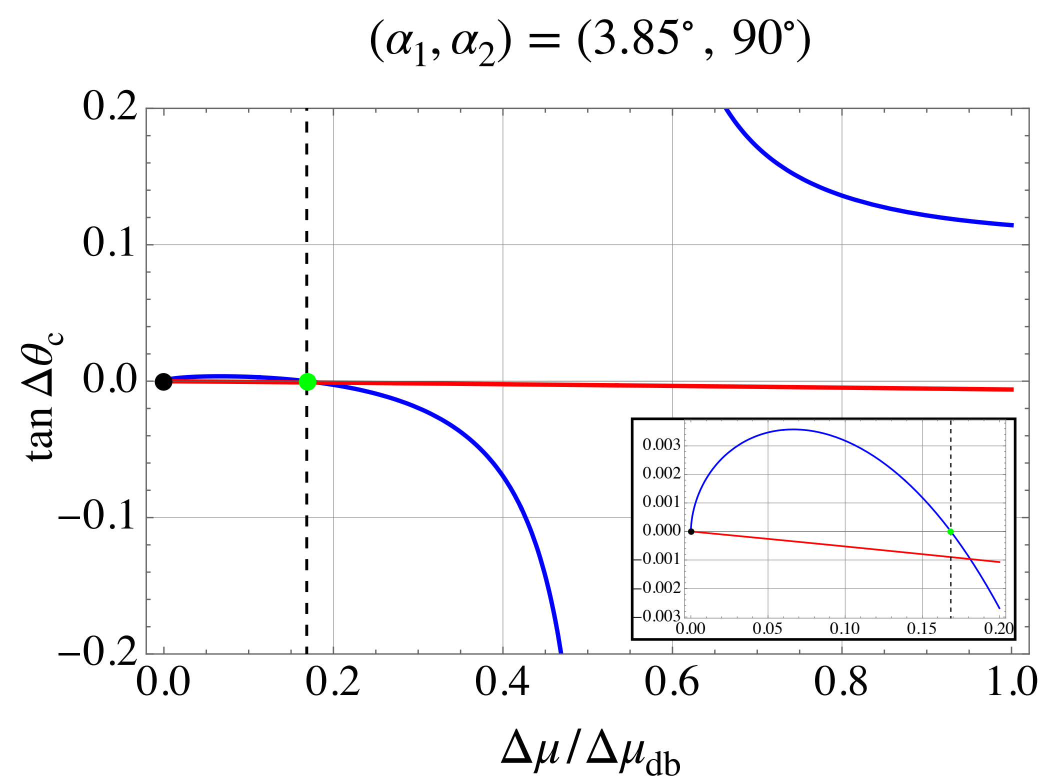
Slope formula

- The slope at an arbitral point on the contours:

$$\begin{cases} \left(\partial_\mu T\right)_{\hat{s}} = -\dfrac{\partial_\mu \hat{s}}{\partial_T \hat{s}} \\ \underset{X=(\mu,T)}{\partial_X \hat{s}} = (\partial_m \hat{s})_0 \partial_X m + (\partial_\sigma \hat{s})_0 \partial_X \sigma \end{cases}$$

Susceptibilities

- Relative slope: $\tan \Delta\theta_{\mathrm{c}} \equiv \dfrac{\left(\partial_\mu T\right)_{\hat{s}} + \tan\alpha_1}{1 - \tan\alpha_1 \left(\partial_\mu T\right)_{\hat{s}}}$

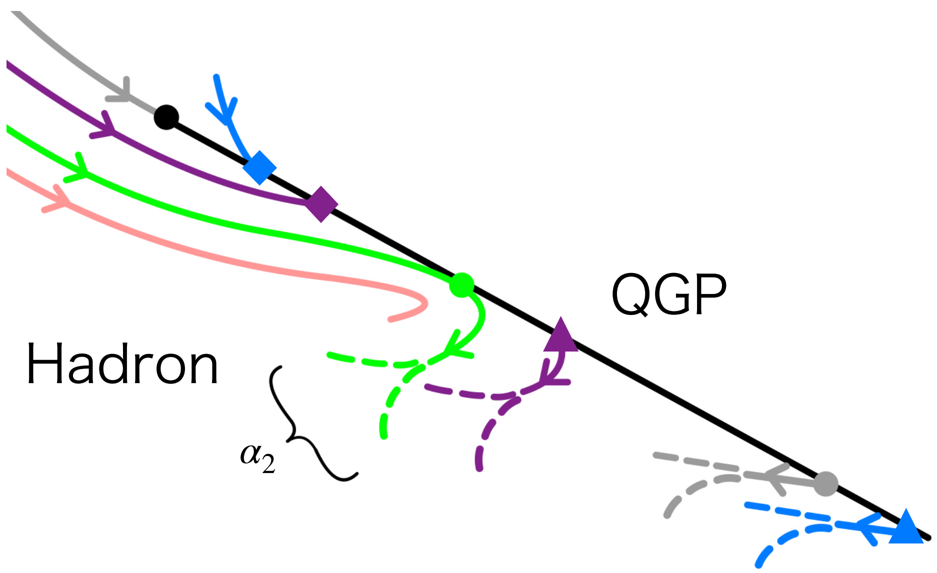


- Two branches along the first-order boundary

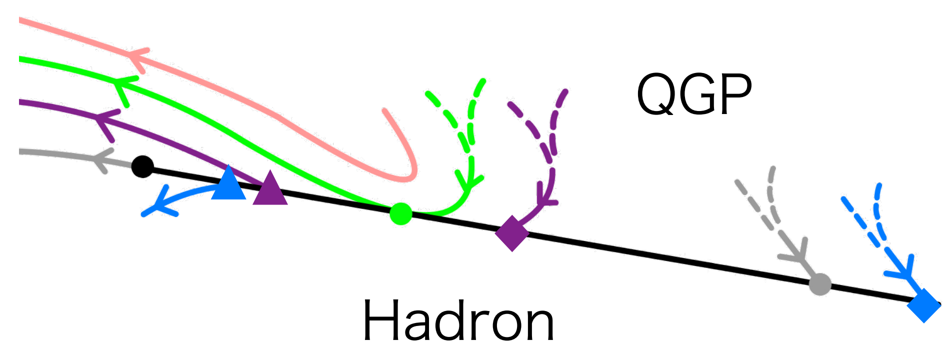
Classification of contours

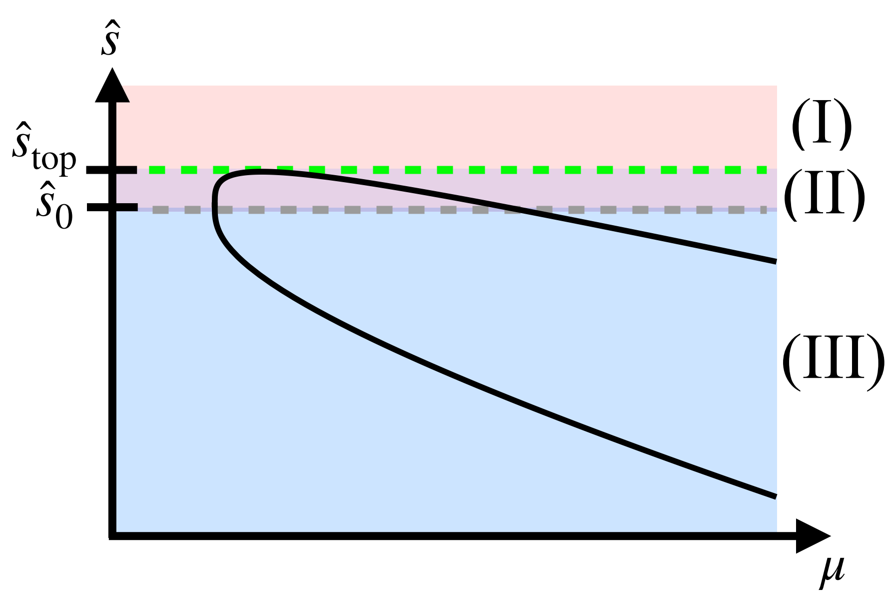
M. Pradeep, M. Stephanov, NS, and H. Yee (in preparation)

$\lambda_{\rm hill}^{(\mu,T)} = -1$



$\lambda_{\rm hill}^{(\mu,T)} = 1$





	$\lambda_{\rm hill}^{(\mu,T)} = -1$	$\lambda_{\rm hill}^{(\mu,T)} = 1$
(I)	Crossover	
(II)	Hadron → Hadron	QGP → QGP
(III)	QGP → Hadron	

Summary

M. Pradeep, M. Stephanov, NS, and H. Yee (in preparation)

- Universal properties of s/n near the QCD critical point based on the 3D Ising EOS
- Relevance to the heavy-ion collision trajectories within the ideal hydrodynamics
- Ridge structure and the hill shape along the first-order boundary
- Outlook:
    - Kinematics on the coexistence line
    - Connection to the freeze-out curve