Novel transition dynamics of the topological object in chiral soliton lattice

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Topological Phases of Matter: From Low to High Energy
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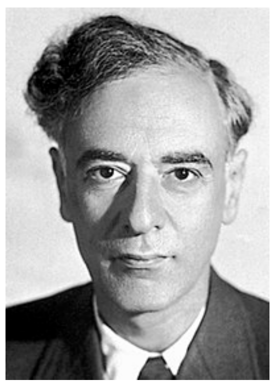
In a collaboration with Kentaro Nishimura (KEK)

Classification of transitions



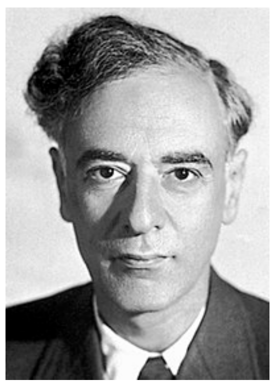
Lev Landau

Phase transition

Discontinuous → First order

Continuous → Second order

Classification of transitions



Lev Landau

P. G. de Gennes

Phase transition

Discontinuous → First order

Continuous → Second order

Trivial → Instability type

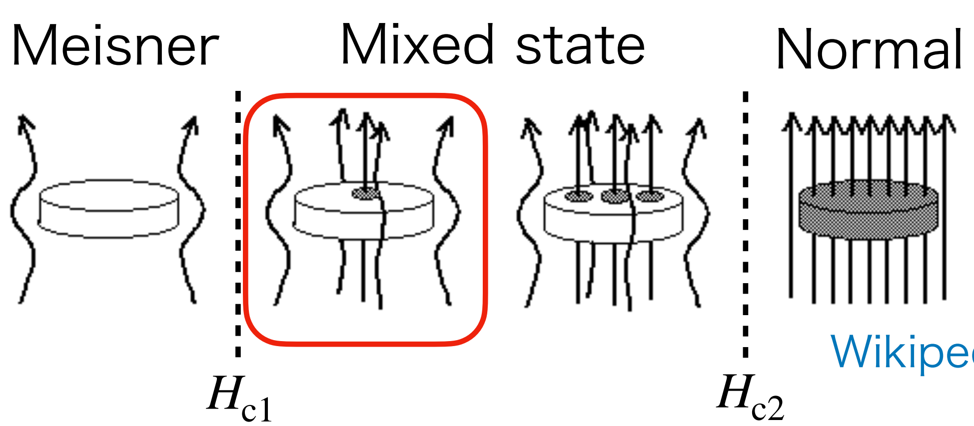
Local order parameter

Topological → Nucleation type

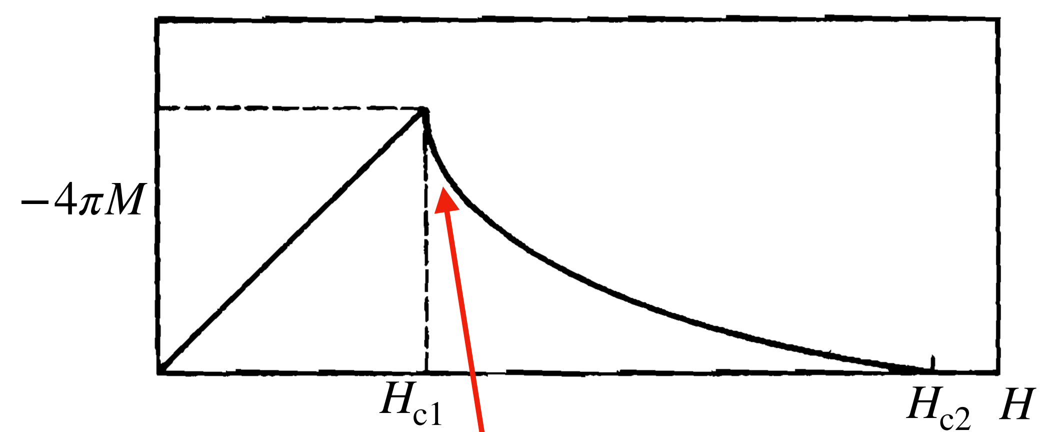
P. G. de Gennes, "Phase transition and turbulence: An introduction," in Fluctuations, Instabilities, and Phase Transitions edited by T. Riste (Springer, 1975)

Nucleation-type transition

- Order parameter with a topological constraint
- E.g., Type-II superconductors, chiral magnets, cholesteric liquid crystals, etc.



Wikipedia



$$-4\pi M \propto \log\left(\frac{1}{H - H_{c1}}\right)$$

Universal logarithmic behavior

From “Superconductivity,” Volume 2, Edited by R. D. Parks (1969)

See also I. E. Dzyaloshinsky (1964) and P. G. de Gennes and J. Prost (1993)

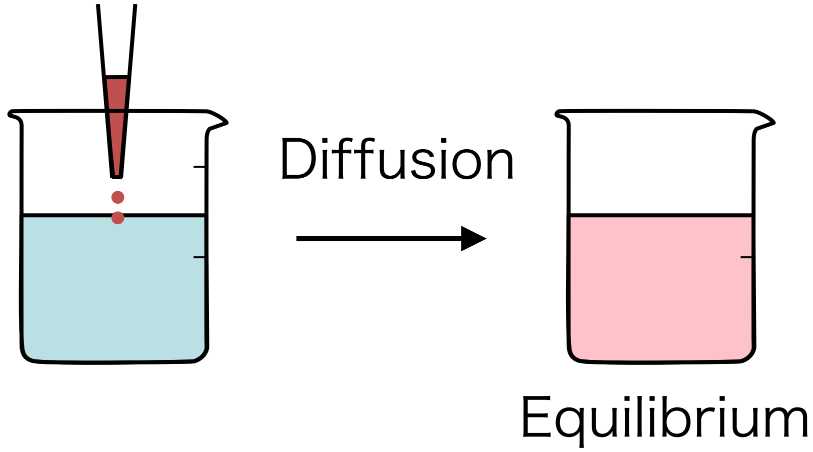
Dynamics of second-order phase transitions

- Instability-type transitions:

  Critical slowing down of hydrodynamic modes, such as relaxation, diffusion, and velocity of Nambu-Goldstone modes

  E.g., Liquid-gas critical point:

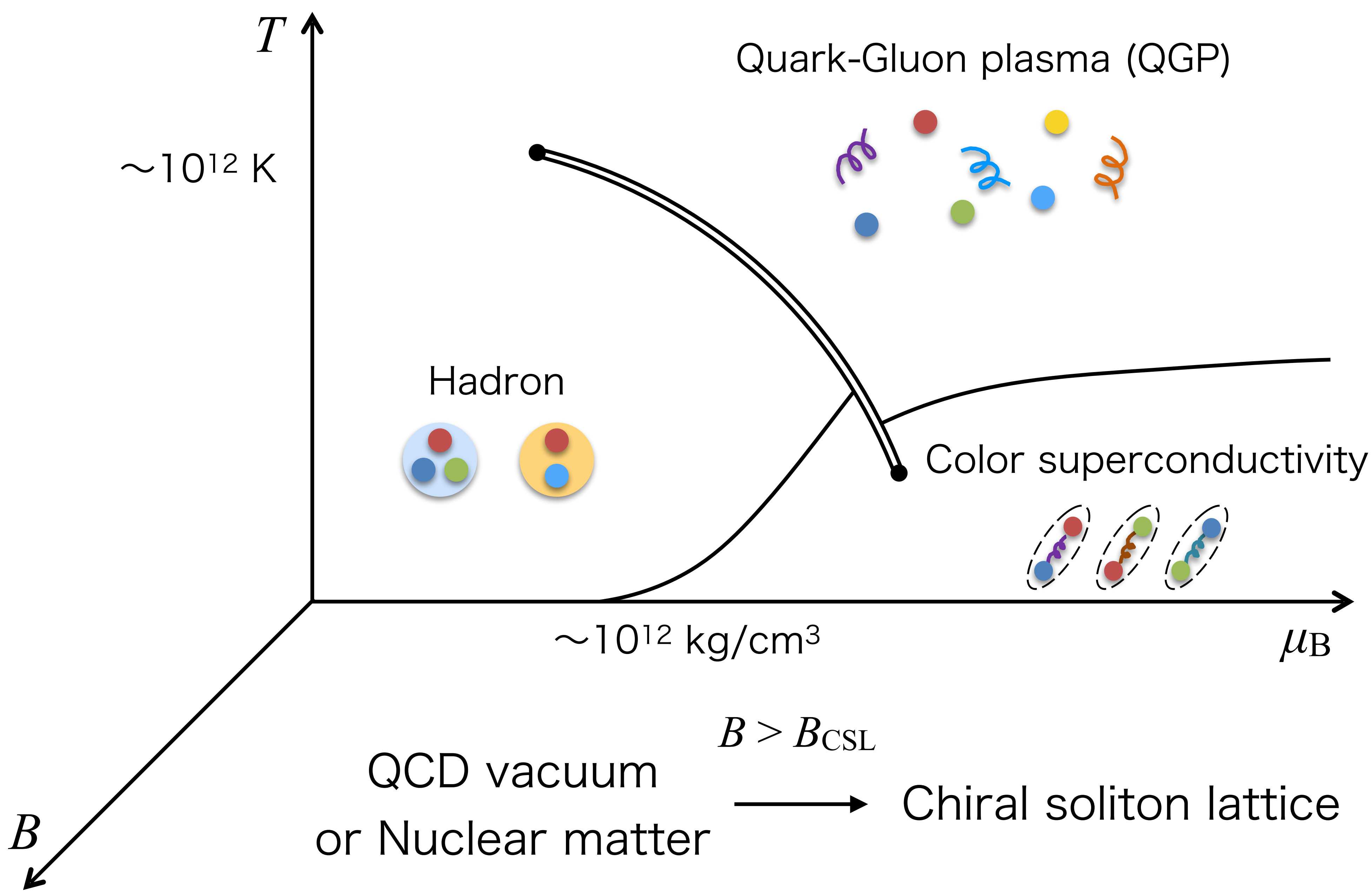
  Diffusion rate $\to 0$



- What about nucleation-type transitions?

  - The motion of topological objects, e.g., domain walls, vortices, etc.

QCD phase diagram



D. T. Son and M. A. Stephanov (2008), T. Brauner and N. Yamamoto (2017)

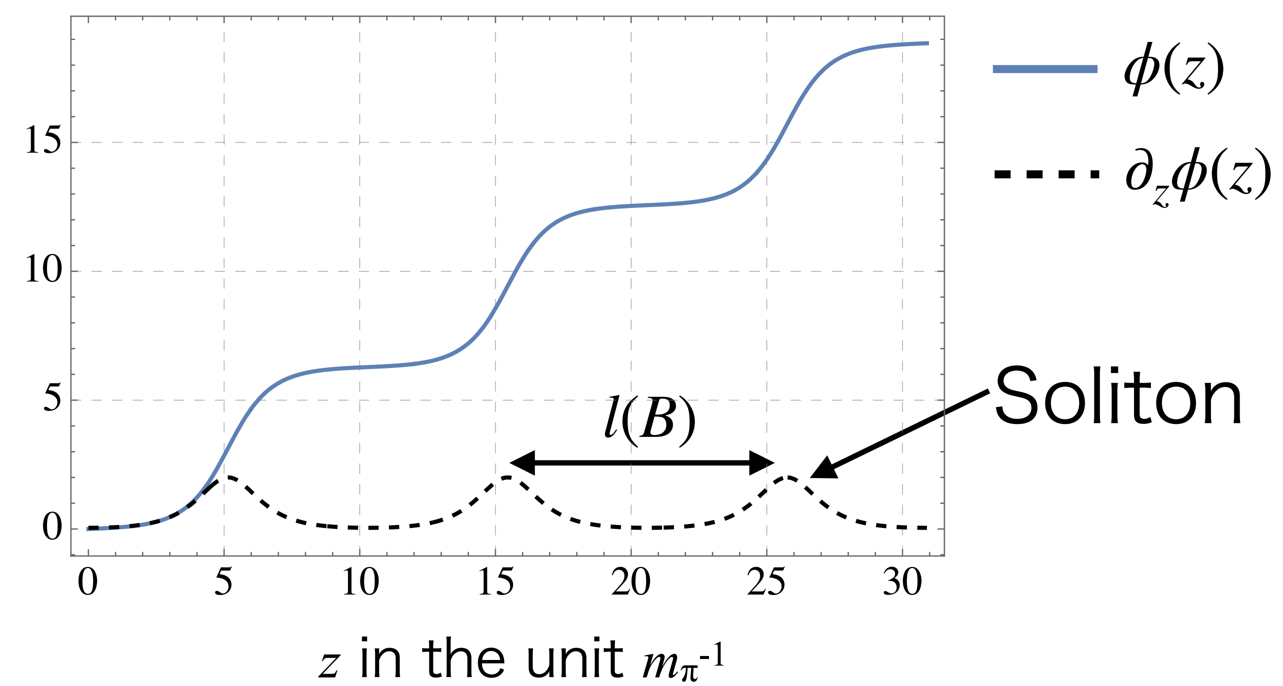
Chiral soliton lattice (CSL)

T. Brauner and N. Yamamoto (2017)

The anomaly-related ground state of QCD at finite μ_B & $B > B_{CSL}$

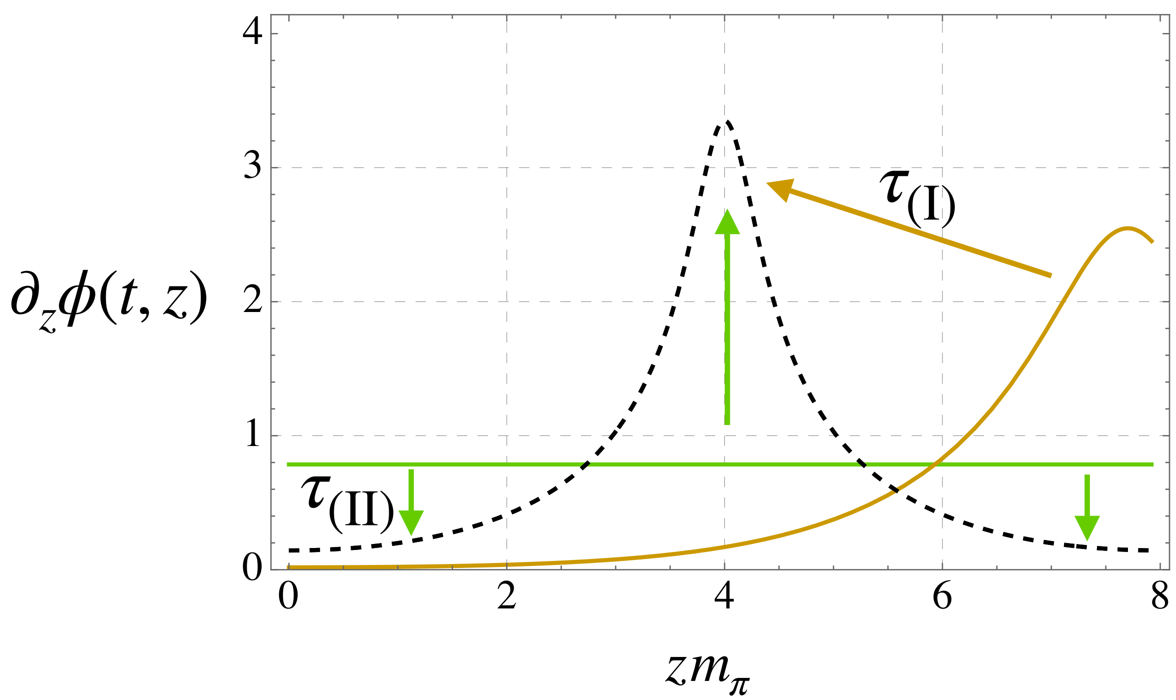
Neutral pion configuration: $\phi(z)$

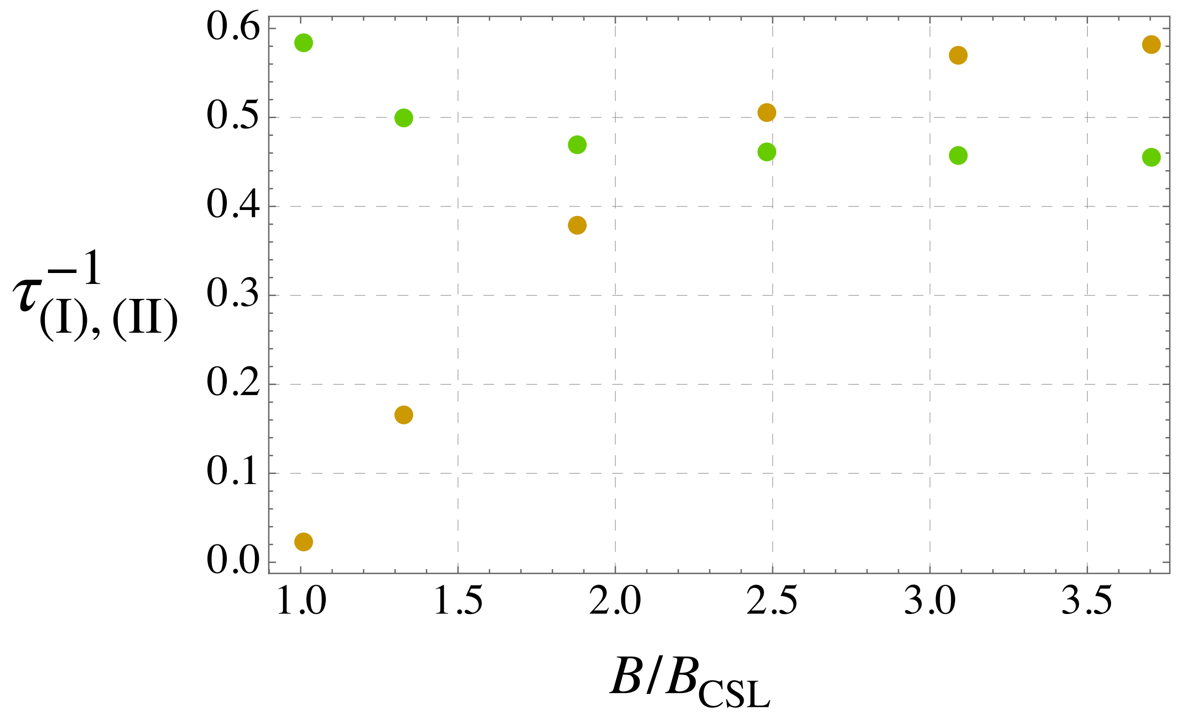
- Violating parity
- Carrying topological charges
- Breaks translational symmetry



CSL transition dynamics

NS and K. Nishimura (in preparation)





- The motion of the topological soliton slows down
- The local relaxation rate kept finite

(as $B \longrightarrow B_{\mathrm{CSL}}$)

A novel feature near the nuclear-type transition

Outline

1. Setup
2. Formulation
3. Chiral soliton lattice (CSL)
4. Dynamics near the CSL transition

Setup

- Two-flavor QCD at finite T, μ_{B}, and B
- Finite quark mass
- Hydrodynamic variables:

$\bar{q}q \sim \mathrm{e}^{2\mathrm{i}\phi t^3}$	$\rho = \bar{q}\gamma^0\gamma^5 t^3 q$
Neutral pion	Axial isospin charge

- Neglect the other axial charges, vector charges, and energy and momentum densities

Outline

Hamiltonian

- Most general Hamiltonian density at finite μ_{B} and B

$$\mathscr{H}_0 = \frac{f_\pi^2}{2}(\nabla\phi)^2 - f_\pi^2 m_\pi^2 \cos\phi$$

- Kinetic + mass

  C.f., chiral Lagrangian

$$\mathscr{H}_\rho = \frac{1}{2\chi}\rho^2$$

- χ : Axial isospin susceptibility

$$\mathscr{H}_{\mathrm{anom}} = -\frac{\mu_{\mathrm{B}}}{4\pi^2}\boldsymbol{B}\cdot\nabla\phi$$

favors inhomogeneity

- See D. Son and M. Stephanov (2008)

- "Quick derivation" from the anomaly:

$$S = \frac{1}{4\pi^2}\int \mathrm{d}x^4\,\phi\boldsymbol{E}\cdot\boldsymbol{B} \longrightarrow S = -\frac{1}{4\pi^2}\int \mathrm{d}x^4\,\phi\nabla A_0\cdot\boldsymbol{B} \sim \int \mathrm{d}x^4\,A_0\nabla\phi\cdot\boldsymbol{B}$$

Scalar potential

Low-energy dynamics

- A low-energy effective description with dissipation

Chaikin and Lubensky (1995)

See also D. T. Son (2000)

$$\partial_t \phi(\boldsymbol{x}) = \int \mathrm{d}\boldsymbol{y} \, \left[\phi(\boldsymbol{x}), \rho(\boldsymbol{y})\right] \frac{\delta H}{\delta \rho(\boldsymbol{y})} - \gamma_\phi(\nabla) \frac{\delta H}{\delta \phi(\boldsymbol{x})}$$

Reversible Dissipative

- Poisson bracket: $\left[\phi(\boldsymbol{x}), \rho(\boldsymbol{y})\right] = \delta(\boldsymbol{x} - \boldsymbol{y})$
- Derivative expansion: $\gamma_\phi(\nabla) = \kappa + \mathcal{O}(\nabla^2)$

κ ← Relaxation rate

Low-energy dynamics

NS and K. Nishimura (in preparation)

$$\partial_t \phi = \frac{\rho}{\chi} + f_\pi^2 \kappa \left(-m_\pi^2 \sin\phi + \nabla^2 \phi \right)$$

$$\partial_t \rho = f_\pi^2 \nabla^2 \phi - m_\pi^2 f_\pi^2 \sin\phi + \frac{\lambda}{\chi} \nabla^2 \rho$$

λ: Axial isospin conductivity

∵ Conservation law

- Nonlinear and dissipative generalization of previous works

D. Son (2000), D. Son and M. Stephanov (2002)

Both are essential for the relaxation process to the CSL

- Work in 1+1 d ($\boldsymbol{B} = B\boldsymbol{e}_z$)

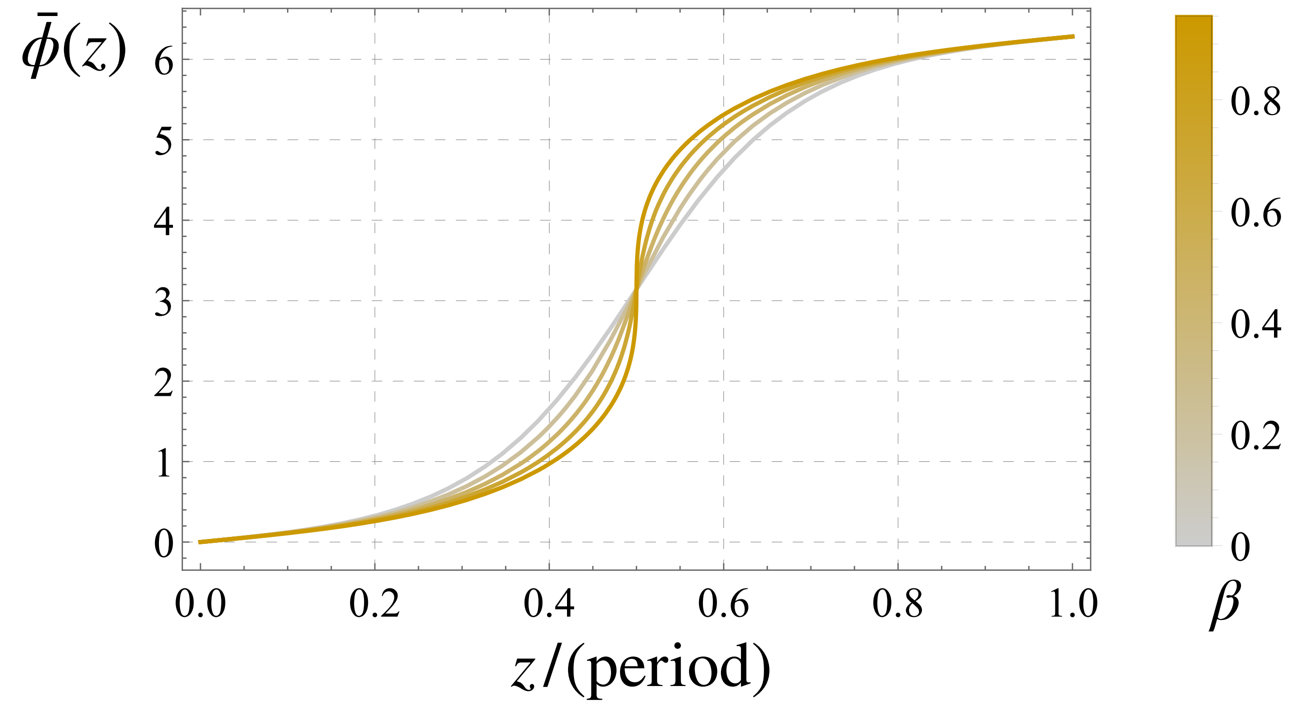
Outline

1. Setup
2. Formulation
3. **Chiral soliton lattice (CSL)**
4. Dynamics near the CSL transition

Stationary solution

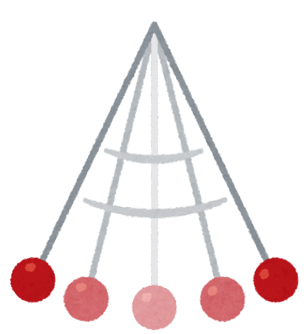
$$\partial_z^2 \bar{\phi} = m_\pi^2 \sin \bar{\phi} - \beta \partial_z^2 \sin \bar{\phi}$$

$$\beta \equiv \kappa \lambda m_\pi^2$$



NS and K. Nishimura (in preparation)

- Analytic solution at $\beta = 0$:



$$\cos \frac{\phi(zm_\pi/k)}{2} = \mathrm{sn}(zm_\pi/k, k)$$

Jacobi elliptic function Elliptic modulus

Periodic length:

$$l(k) = \frac{2kK(k)}{m_\pi}$$

$K(k)$: The complete elliptic function of the first kind

B dependence

See T. Brauner and N. Yamamoto (2017) for details

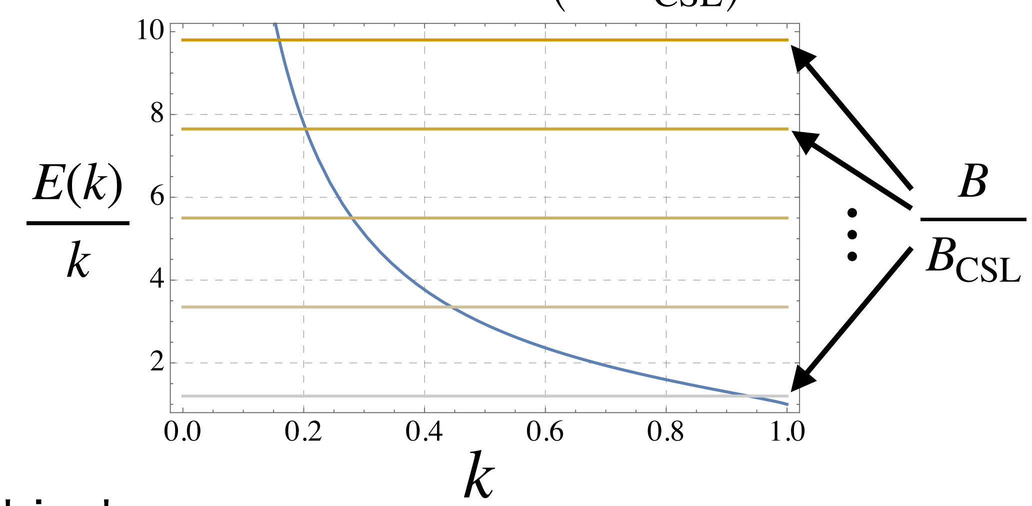
- Energy minimization condition (β=0):

$$\frac{\delta E_{\mathrm{tot}}\left[\phi = \bar{\phi}\right]}{\delta k} = 0 \quad \Leftrightarrow \quad \frac{E(k)}{k} = \frac{B}{B_{\mathrm{CSL}}}$$

Critical magnetic field: $B_{\mathrm{CSL}} = \dfrac{16\pi m_\pi f_\pi^2}{\mu}$

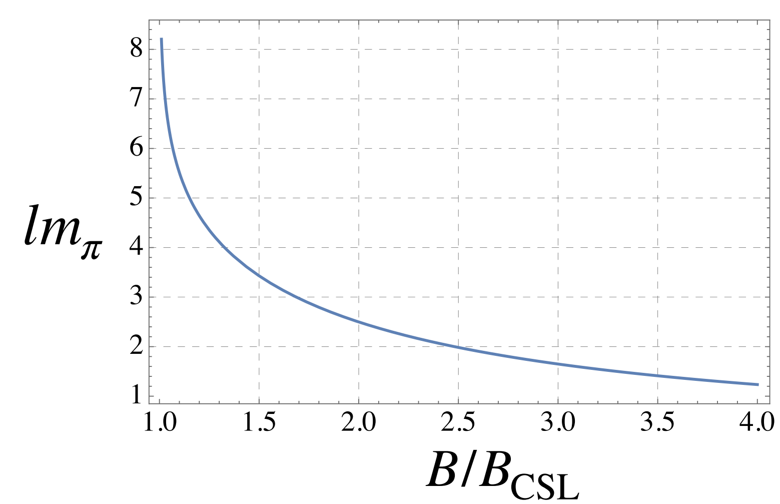
$E(k)$: The complete elliptic function of the second kind

Determine $k\left(B/B_{\mathrm{CSL}}\right)$



- Periodic length:

$$l(B) = l\left(k\left(B/B_{\mathrm{CSL}}\right)\right) \to \infty \qquad (B \to B_{\mathrm{CSL}})$$



Topological charges

D. T. Son and M. A. Stephanov (2008)
T. Brauner and N. Yamamoto (2017)

$$\mathscr{H} = \frac{f_\pi^2}{2}(\partial_z\phi)^2 - f_\pi^2 m_\pi^2 \cos\phi + \frac{1}{2\chi}\rho^2 - \frac{\mu_\mathrm{B} B}{4\pi^2}\partial_z\phi$$

Baryon number: $$N_\mathrm{B} = -\int_0^l \mathrm{d}z \frac{\partial\mathscr{H}}{\partial\mu_\mathrm{B}} = \frac{B}{4\pi^2}\int_0^l \mathrm{d}z\partial_z\phi = \frac{B}{2\pi}$$

Magnetization: $$M = -\int_0^l \mathrm{d}z \frac{\partial\mathscr{H}}{\partial B} = \frac{1}{4\pi^2}\mu\int_0^l \mathrm{d}z\partial_z\phi = \frac{\mu_\mathrm{B}}{2\pi}$$

Quantized per unit lattice

Logarithmic "critical" behavior of CSL in QCD

NS and K. Nishimura (in preparation)

- An asymptotic form of $K(k)$ and $E(k)$ for $B \simeq B_{\rm CSL}$ $(k \simeq 1)$

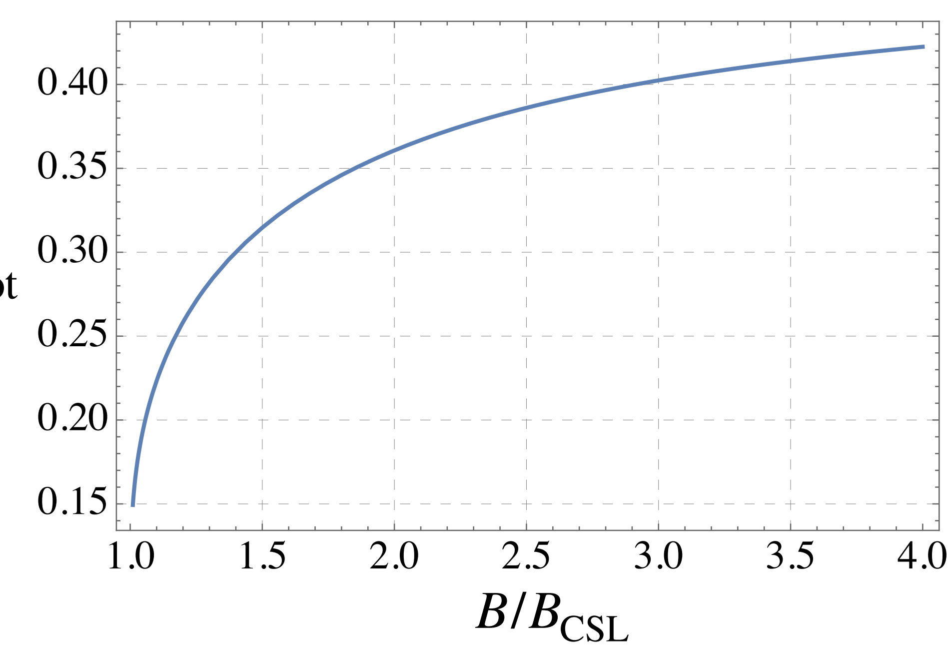
Periodic length: $l(B) \simeq \log \dfrac{8}{1 - B_{\rm CSL}/B}$

- Total baryon number and magnetization:

$$(N_{\rm B})_{\rm tot} = \frac{L}{l(B)} N_{\rm B} \sim \frac{1}{\log \frac{8}{1 - B_{\rm CSL}/B}}$$

(Same as $M_{\rm tot}$)

$\sim (N_{\rm B})_{\rm tot}$



Outline

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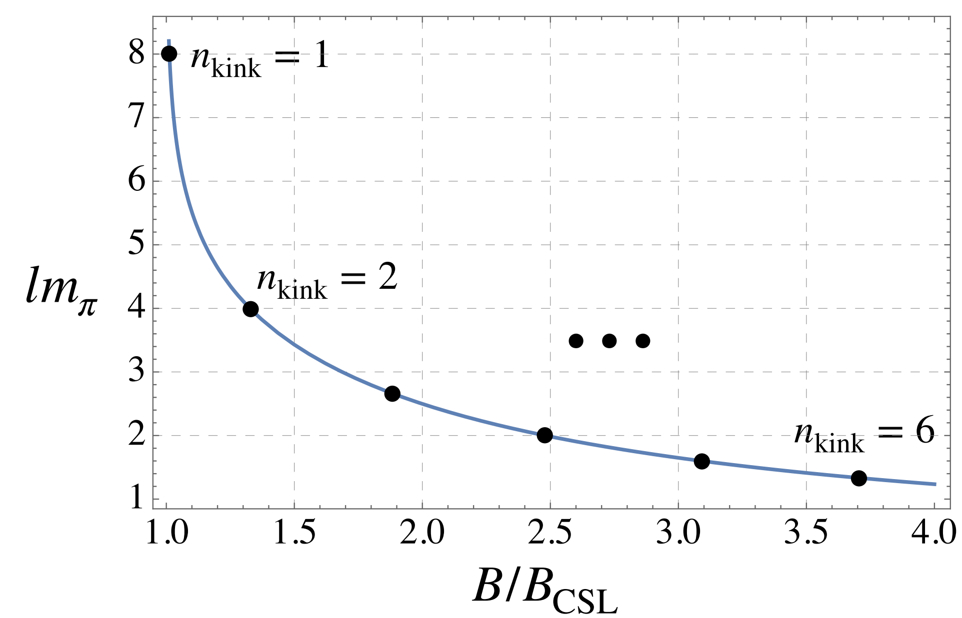
Dynamics near $B_{\rm CSL}$

- System size $L = 8\, m_\pi^{-1}$ and vary $B(L/n_{\rm kink})$
- The dissipative process to the $n_{\rm kink}$ state
- Initial and boundary conditions:



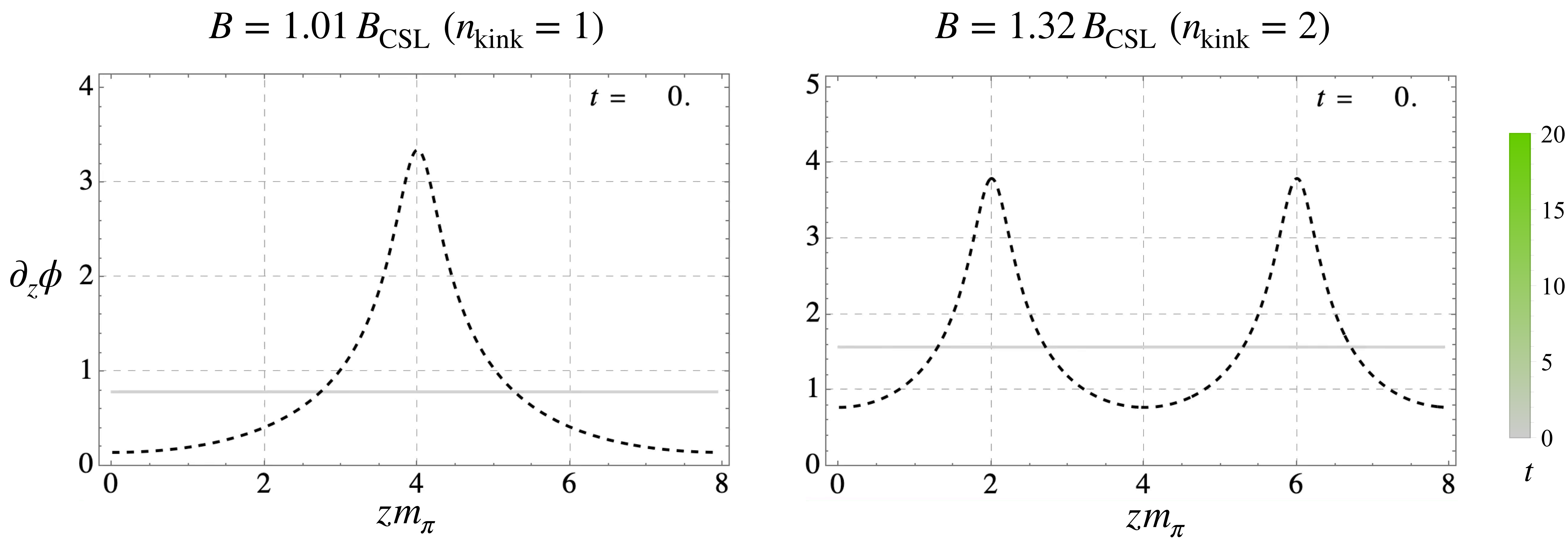
$$\text{(I)} \quad \phi(0,z) = 0\,, \quad \phi(t,L) = 2\pi n_{\rm kink}\theta(t-t_0)$$

- $B(t) = B\left(L/n_{\rm kink}\right)\theta(t-t_0)$
- The transition from the vacuum $(B=0)$ to the $n_{\rm kink}$ state at t_0

$$\text{(II)} \quad \phi(0,z) = 2n_{\rm kink}\pi z/L\,, \quad \phi(t,L) = 2n_{\rm kink}\pi$$

- An initial configuration ≠ the kink states

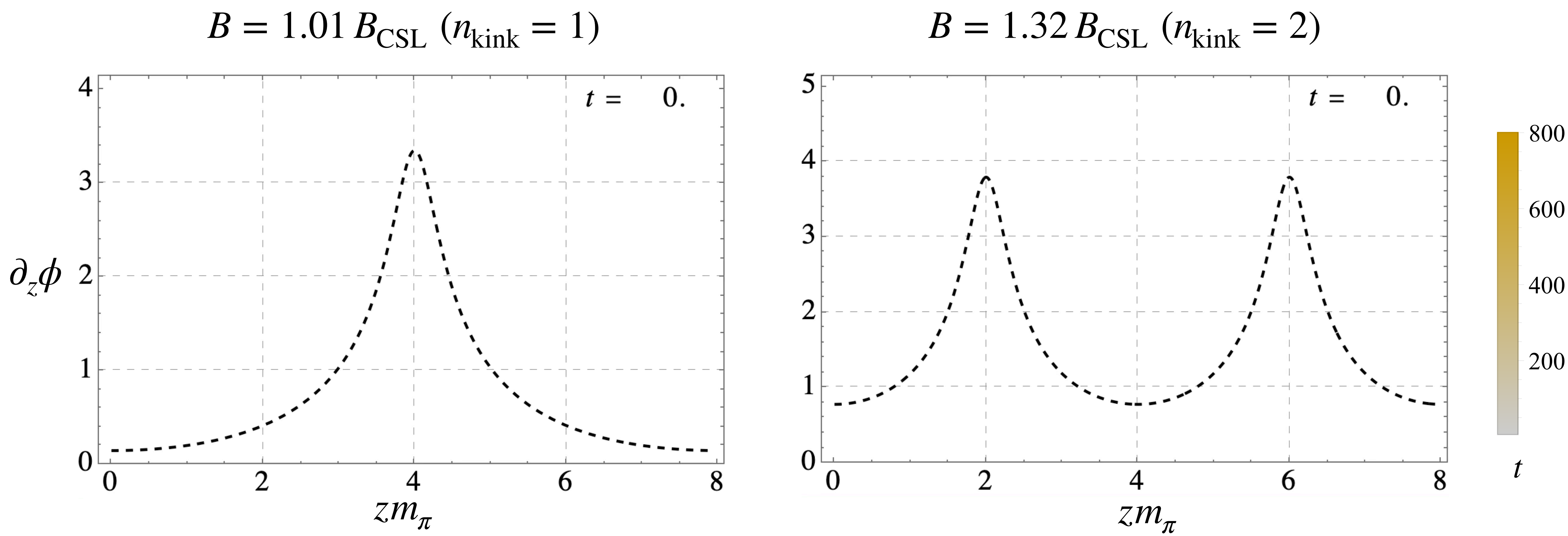
Case (II)



Movies

(time unit: $(m_\pi c)^{-1}$; c: pion velocity in the chiral limit)

Case (I)



Movies

(time unit: $(m_\pi c)^{-1}$; c: pion velocity in the chiral limit)

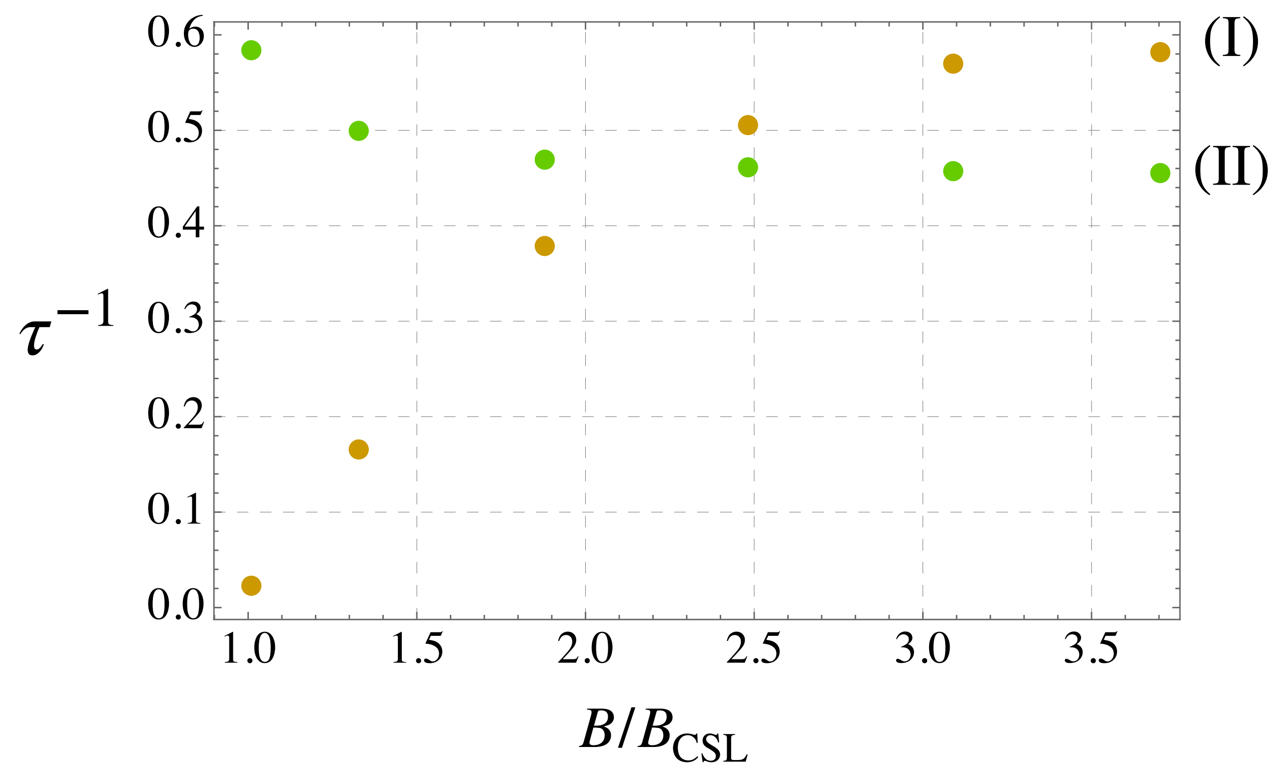
Characteristic rate

NS and K. Nishimura (in preparation)

- How fast the stationary state approaches near $B \sim B_{\rm CSL}$?
- A rate motivated by " $\propto \mathrm{e}^{-t/\tau}$ "

$$\Delta U(t) \equiv \int \mathrm{d}z \left| \phi(t,z) - \bar{\phi}(z) \right|$$

$$\longrightarrow \quad \tau \quad \text{s.t.} \quad \Delta U(\tau) = \frac{\Delta U(t_{\rm ini})}{\mathrm{e}}$$



- A qualitative difference between the B dependence of (I) and (II)

Discussion

NS and K. Nishimura (in preparation)

- As the CSL transition approaches, $B \to B_{CSL}$ (from above)

  (I) Slowing down of the soliton motion

  $\because$ Less repulsion from the other solitons

  (II) Finite local relaxation

  $\sim$ Decay of gapped excited states $(\omega \sim m_\pi)$

A novel class of second-order transitions where the motion of a topological object (soliton) **only** slows down (not the local dissipation)

Summary

NS and K. Nishimura (in preparation)

- Low-energy dynamics near the CSL transition
- The soliton motion slows down while the local relaxation keeps finite

$\longrightarrow$ Characteristic time to the ground state depends on the initial configuration, whether it forms a solitonic structure or not

- A universal feature of nuclear-type transition dynamics?