CEvNS and inelastic cross sections within coupled-cluster theory

Joanna Sobczyk

Interplay of Nuclear, Neutrino and BSM Physics at Low-Energies, 18 April 2023

Cluster of Excellence PRISMA+ Precision Physics, Fundamental Interactions and Structure of Matter

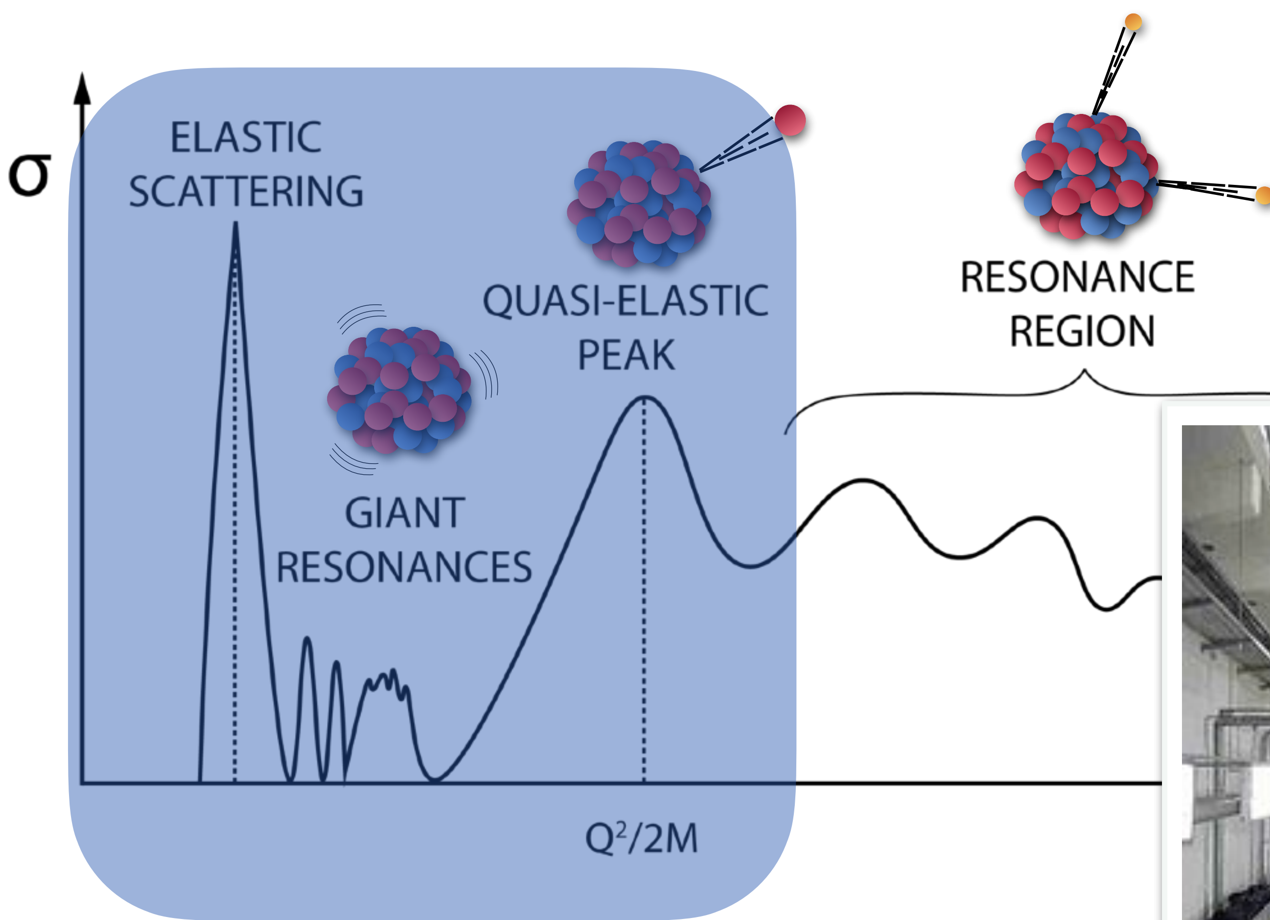
JG|U

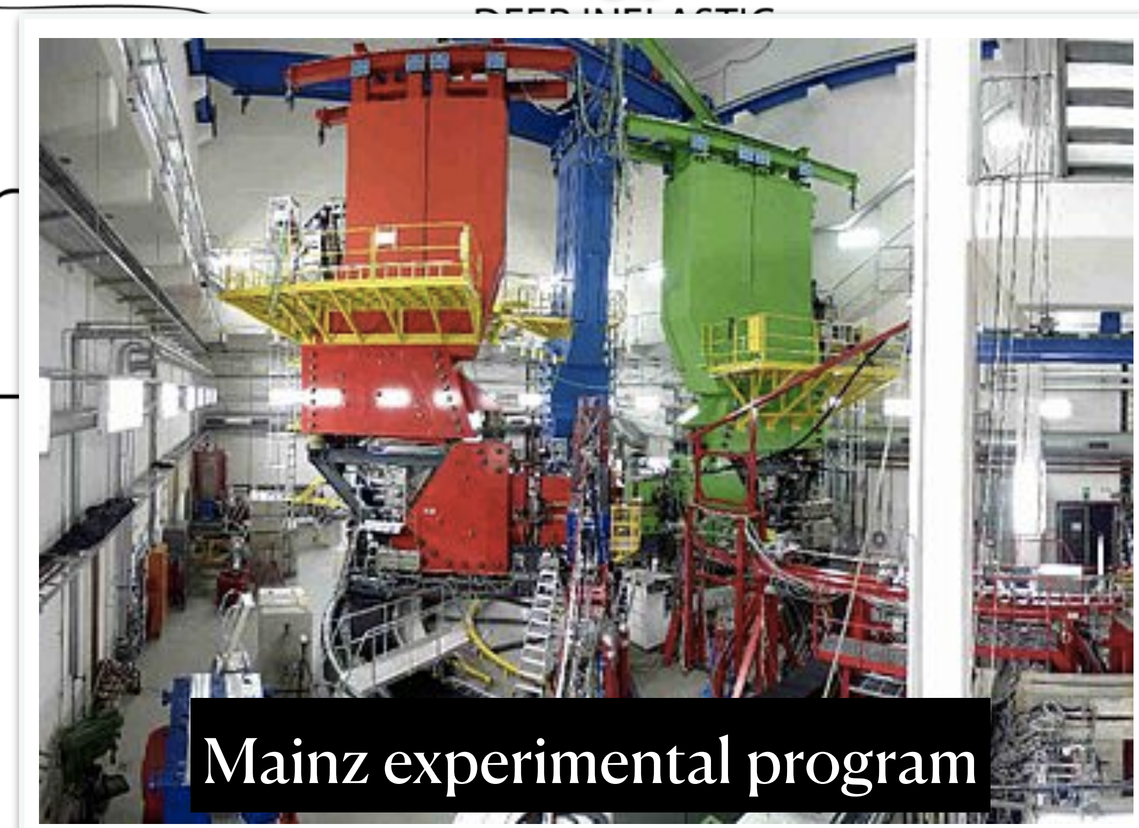
Alexander von Humboldt Stiftung/Foundation

MARIE CURIE ACTIONS

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 101026014

Introduction





Coherent elastic scattering

$\omega = 0,$
$q \lesssim 100$ MeV

Neutrino oscillation experiments

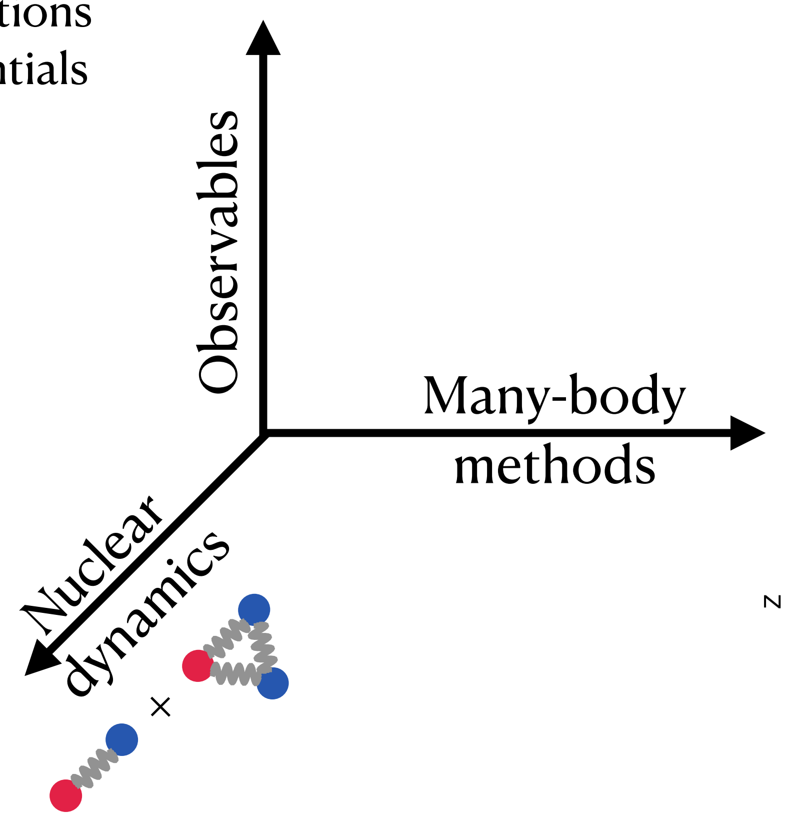
$q \sim \mathcal{O}(100\text{MeV})$

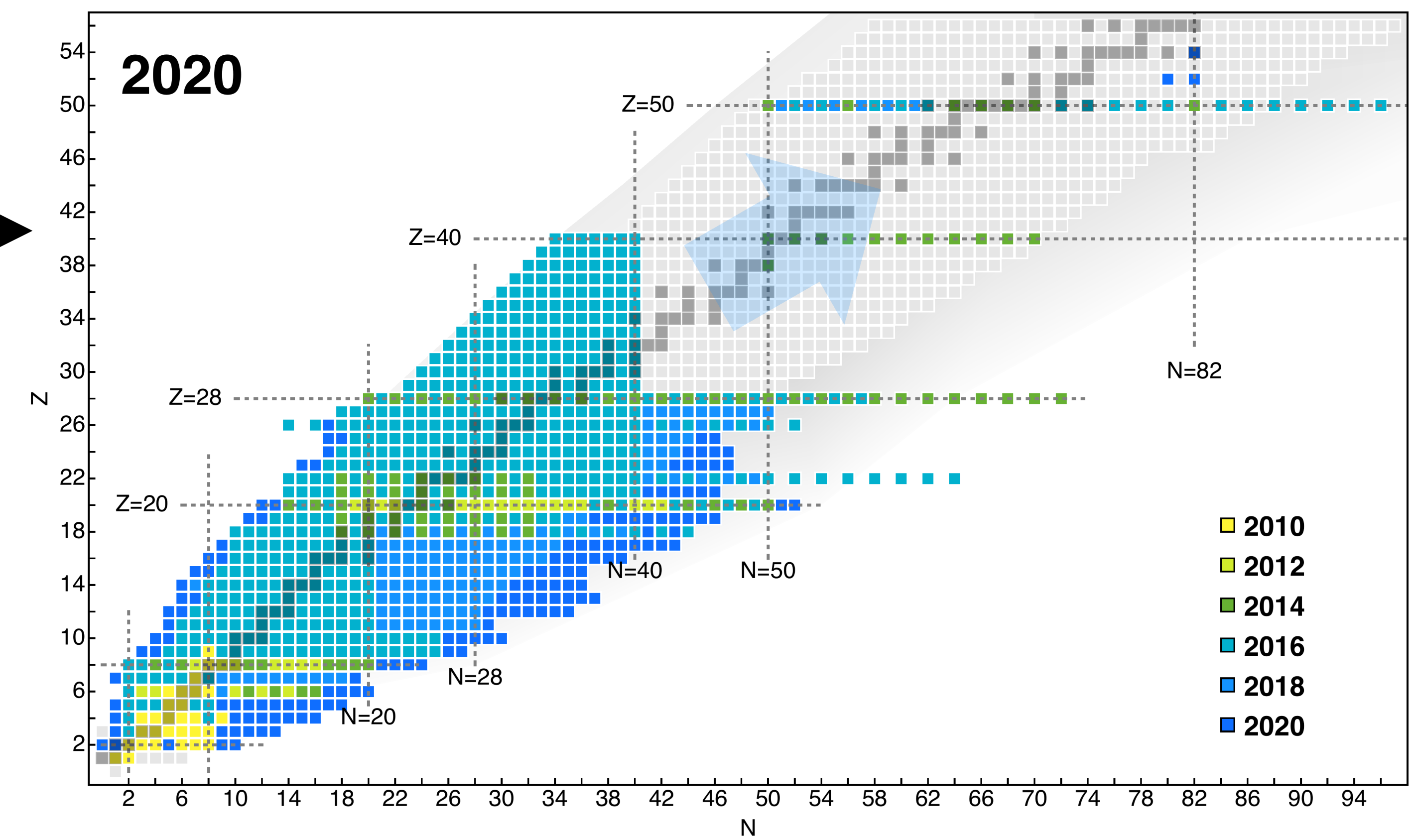
Motivation

Ab initio approach

- ➡ Nuclear responses
- ➡ Spectral functions
- ➡ Optical potentials

…





H. Hergert, *Front.in Phys.* 8 (2020) 379

➡ **Neutrinos challenge ab initio nuclear theory**

➡ **Controllable approximations within ab initial nuclear theory**

Coupled cluster method

Reference state (Hartree-Fock): $|\Psi\rangle$

Include correlations through e^{T} operator

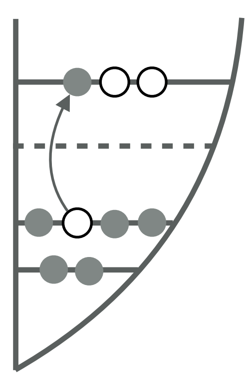
similarity transformed Hamiltonian (non-Hermitian)

$$e^{-T}\mathcal{H}e^{T}|\Psi\rangle \equiv \bar{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle$$

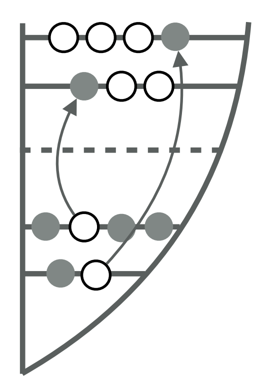
Expansion: $T = \sum t_a^i a_a^\dagger a_i + \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \dots$

←coefficients obtained through coupled cluster equations

singles



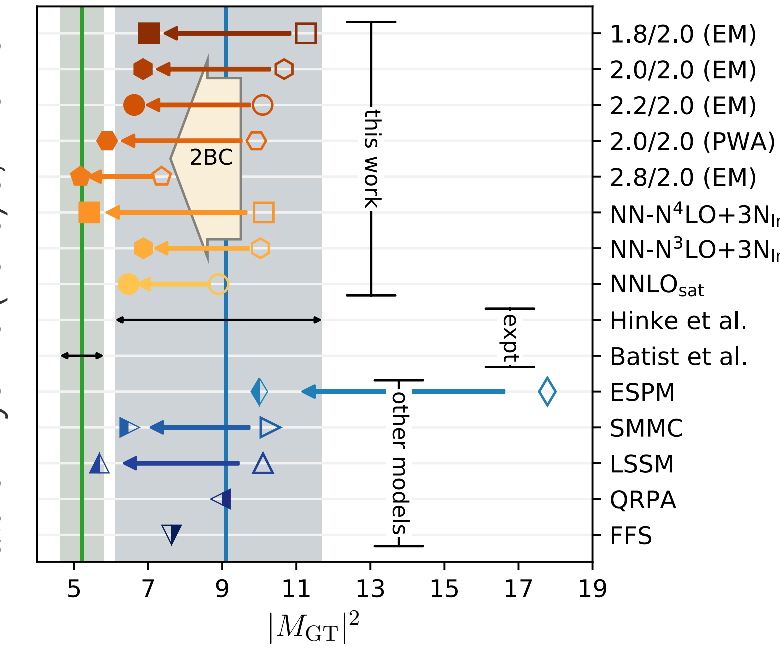
doubles



Coupled cluster method

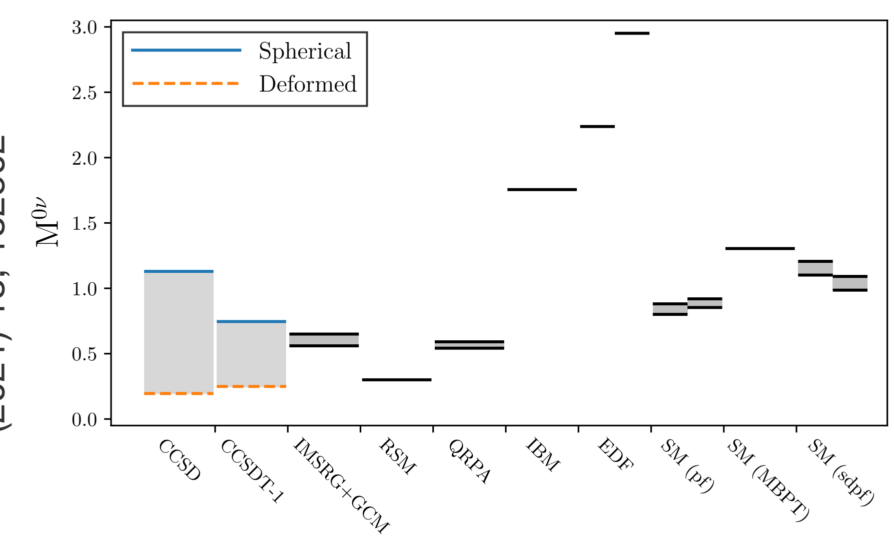
- ✓ Controlled approximation through truncation in T
- ✓ Polynomial scaling with A (predictions for ^{100}Sn, ^{208}Pb)
- ✓ Size extensive
- ✓ Works most efficiently for doubly magic nuclei

Nature Phys. 15 (2019) 5, 428-431



GT strength for β decay of ^{100}Sn

Phys.Rev.Lett. 126 (2021) 18, 182502



NME of ^{48}Ca for $0\nu\beta\beta$

Nuclear hamiltonian

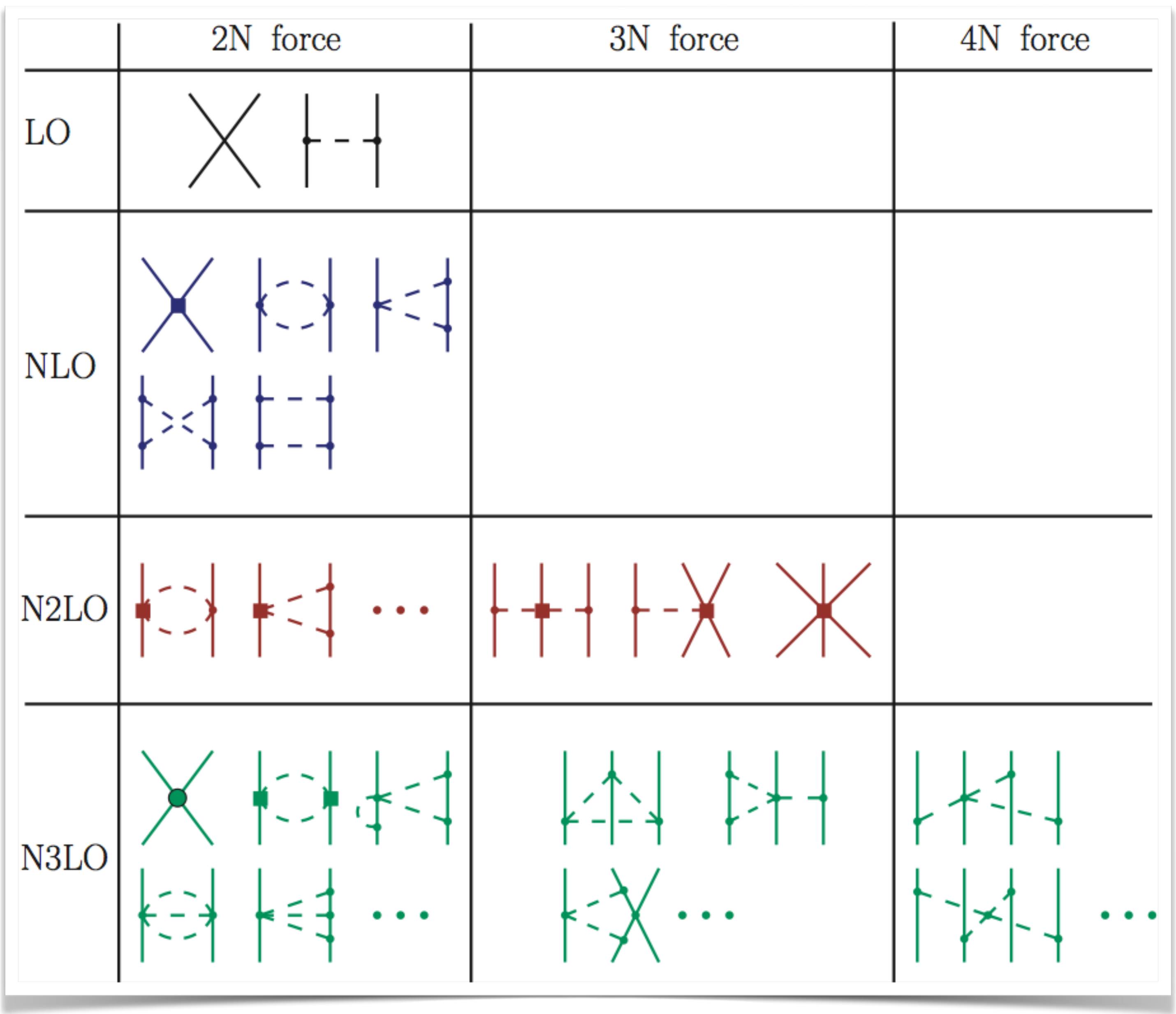
$$\mathscr{H} = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

$n = 0$

$n = 2$

$n = 3$

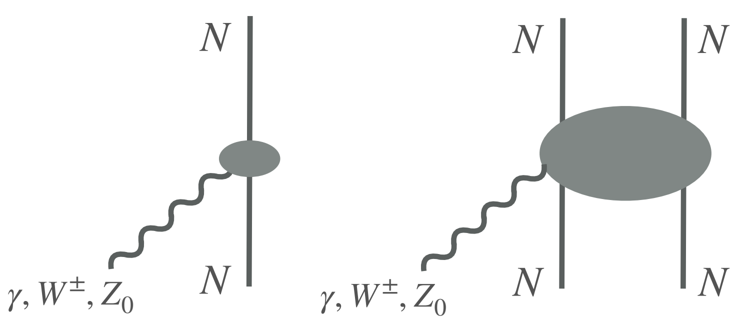
$n = 4$



- ✓ Chiral Hamiltonians exploiting chiral symmetry (QCD); $\pi, N, (\Delta)$ degrees of freedom
- ✓ counting scheme in $\left(\frac{Q}{\Lambda}\right)^n$
- ✓ low energy constants (LEC) fit to data
- ✓ uncertainty assessment

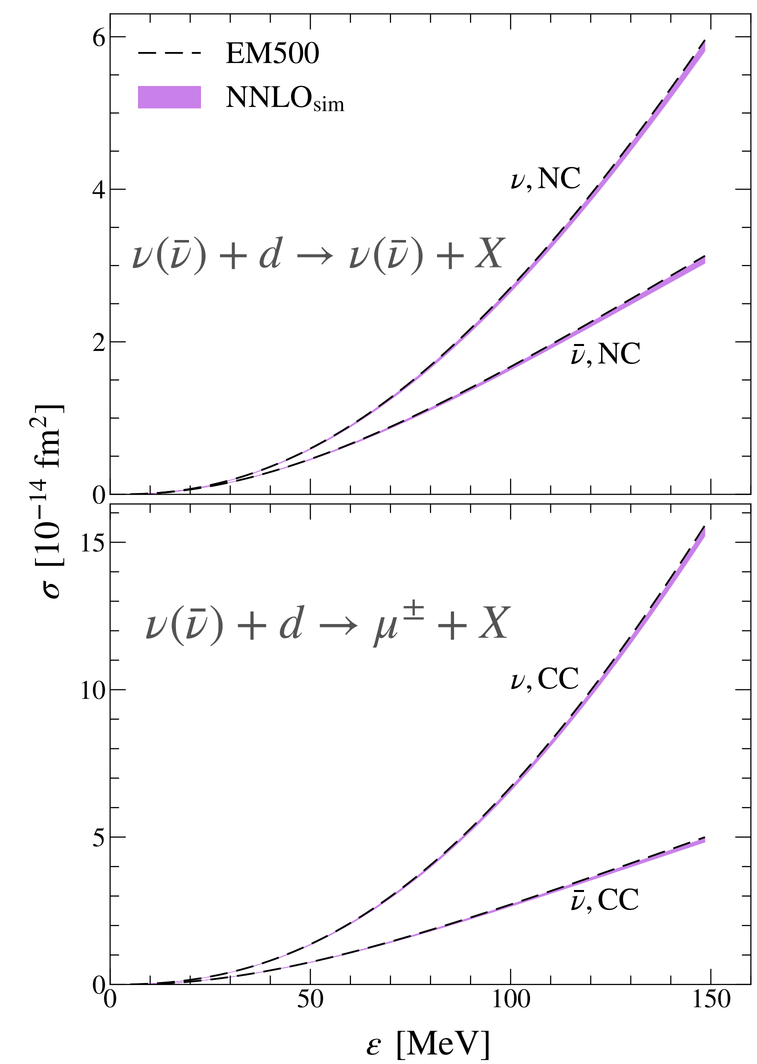
Electroweak currents

$$J = \sum_i j_i + \sum_{i<j} j_{ij} + \dots$$



known to give significant contribution for neutrino-nucleus scattering

Current decomposition into multipoles needed for various *ab initio* methods: CC, No Core Shell Model, In-Medium Similarity Renormalization Group



Multipole decomposition for 1- and 2-body EW currents

B. Acharya, S. Bacca
Phys.Rev.C 101 (2020) 1, 015505

Ab initio nuclear theory for neutrinos

Nuclear chiral Hamiltonian $\mathscr{H}\,|\,\Psi\rangle = E\,|\,\Psi\rangle$

➡ order of expansion
➡ low energy constants fit to data

Electroweak currents $J^\mu = (\rho, \vec{j})$

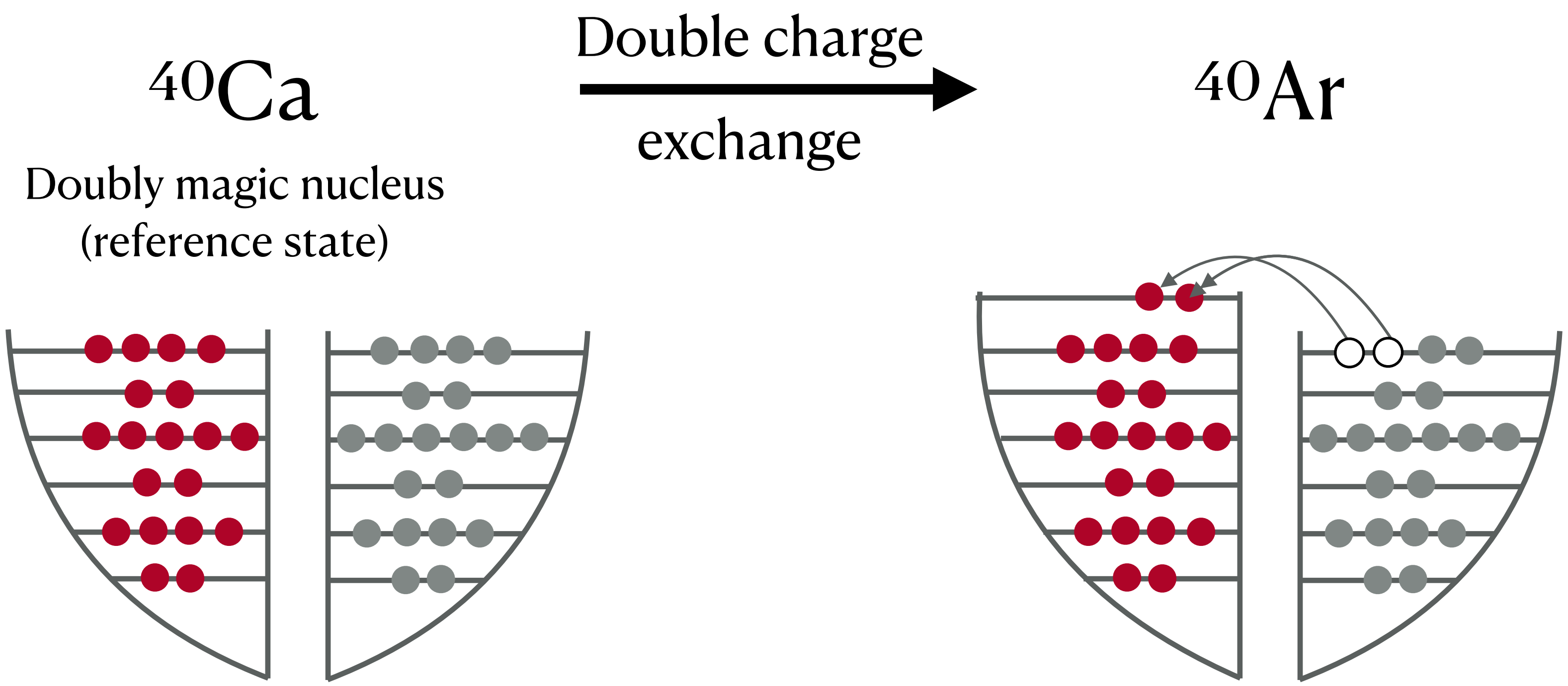
➡ order of expansion
➡ 2-body currents important

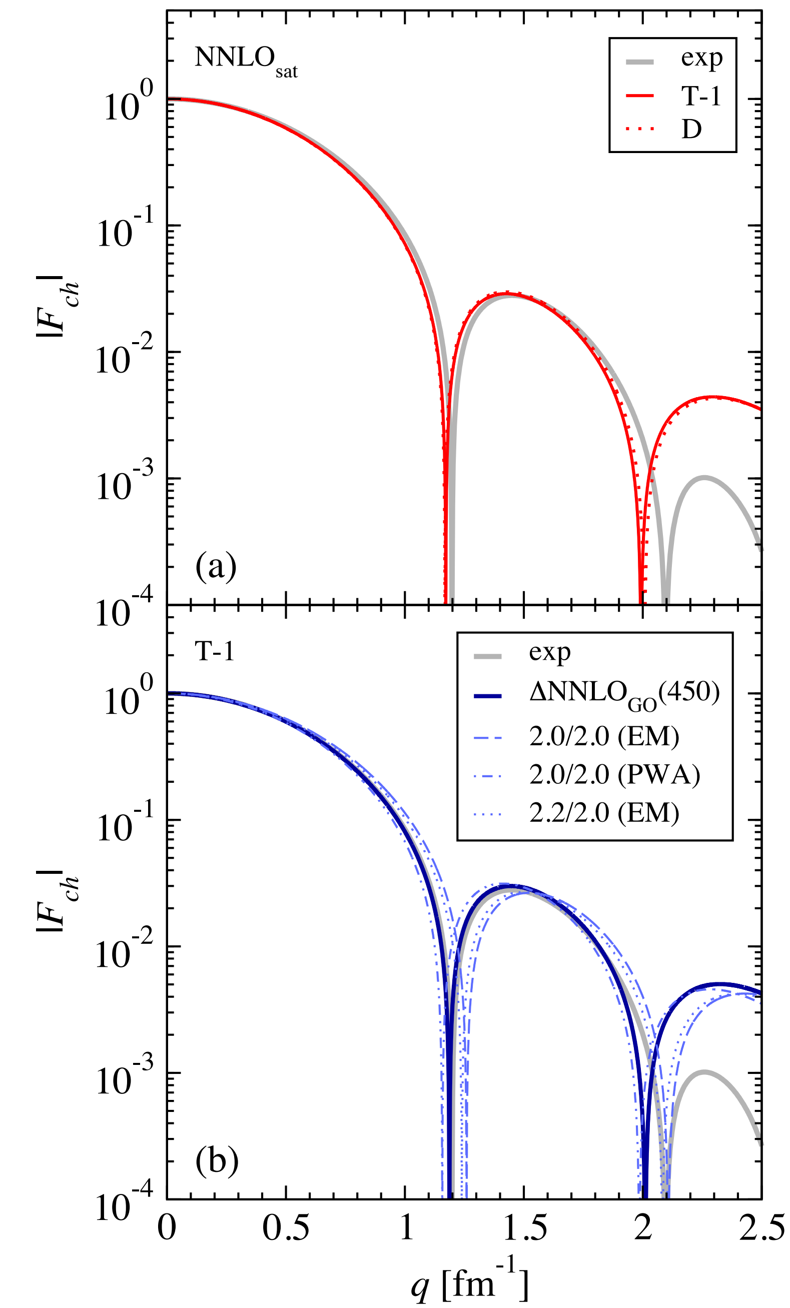
Coupled cluster method $\mathscr{A} = \langle \Psi_m \,|\, J_\mu \,|\, \Psi_n \rangle$

➡ truncation in correlations
➡ model space dependence

CEvNS

^{40}Ar within coupled cluster theory



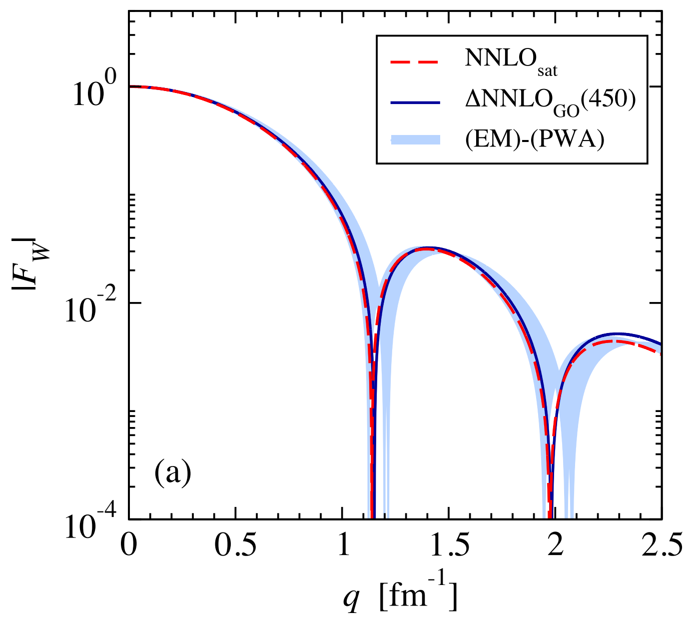


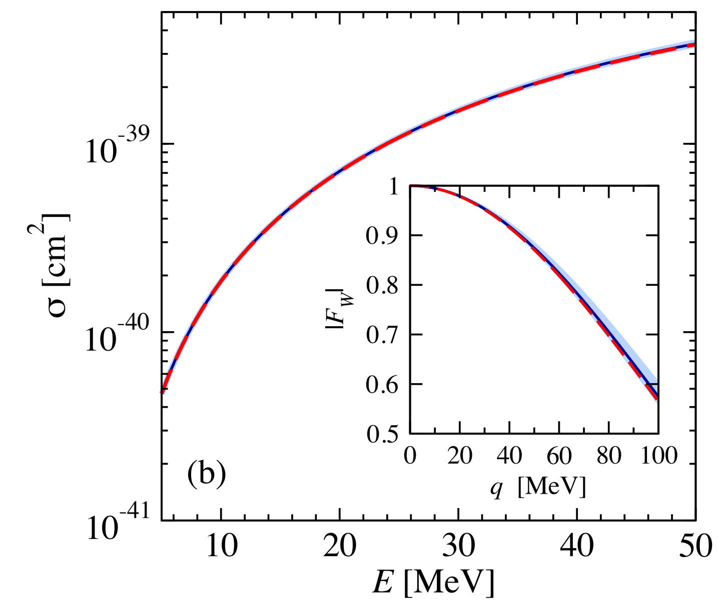
Charge form-factor

Uncertainties:

- ✓ Coupled-cluster truncations (CCSD & CCSDT1)
- ✓ Nuclear Hamiltonian dependence
- ✓ Model space

CEνNS



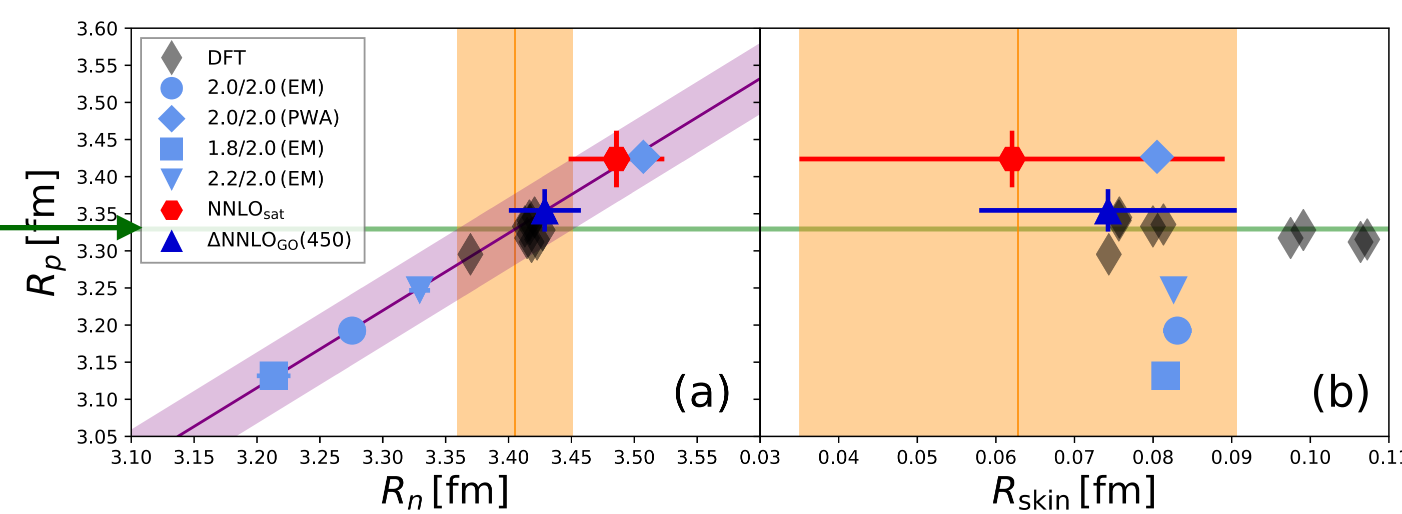


$$F_W(q^2) = \frac{1}{Q_W}\left[NF_n(q^2) - (1-4\sin^2\theta_W)ZF_p(q^2)\right]$$

$$Q_W = N - (1-4\sin^2\theta_W)Z$$

$$\frac{d\sigma}{dT}(E_\nu, T) \approx \frac{G_F^2}{4\pi}M\left[1 - \frac{MT}{2E\nu^2}Q_W^2F_W^2(q^2)\right]$$

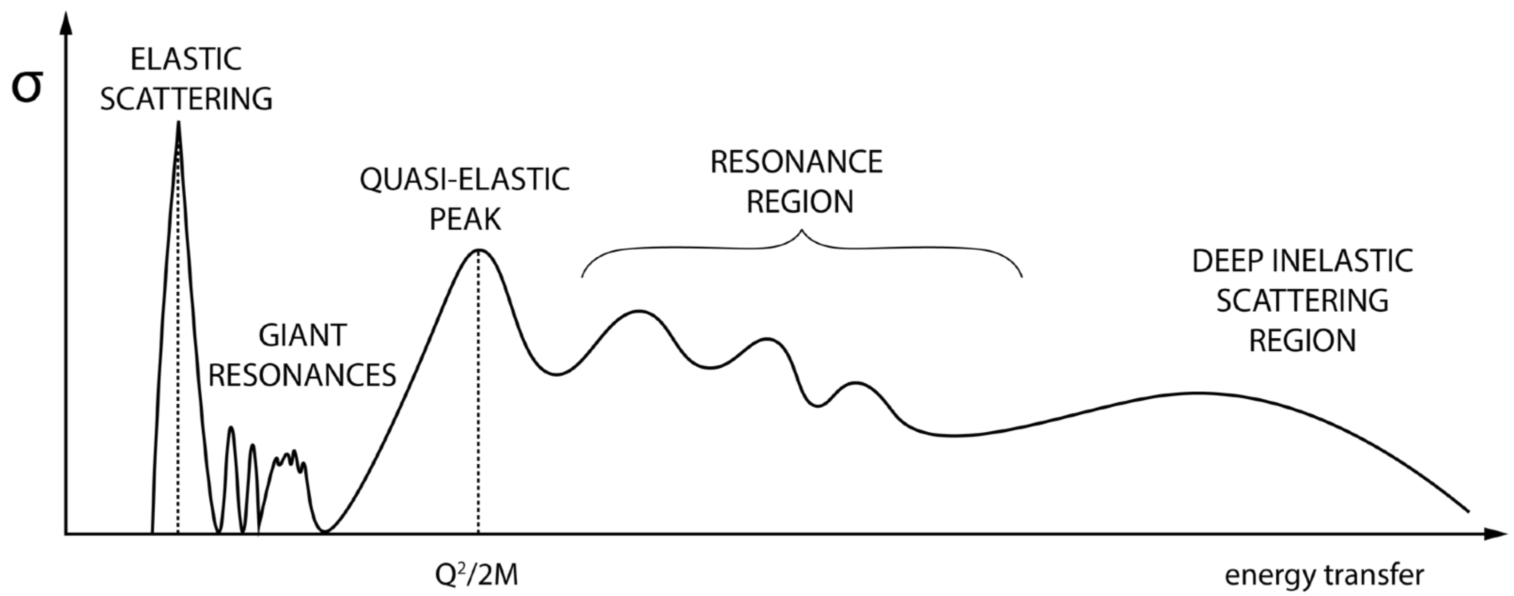
Experimental value



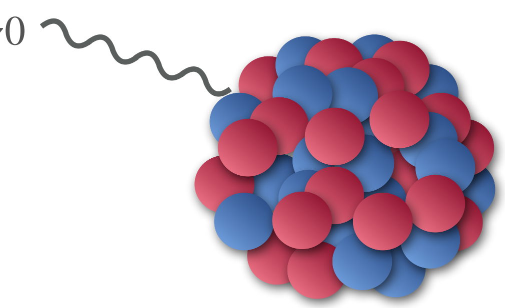
R_p, R_n correlated → less variation of neutron-skin thickness

Inelastic responses

Nuclear response



$$J_\mu = (\rho, \vec{j}) \,|\, \Psi \rangle$$



$$\sigma \propto L^{\mu\nu} R_{\mu\nu}$$

lepton tensor — nuclear responses

$$R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi \,|\, J_\mu^\dagger(q) \,|\, \Psi_f \rangle \langle \Psi_f \,|\, J_\nu(q) \,|\, \Psi \rangle \delta(E_0 + \omega - E_f)$$

Electrons for neutrinos

$$\left.\frac{d\sigma}{d\omega dq}\right|_{\nu/\bar{\nu}} = \sigma_0\left(\upsilon_{CC}R_{CC} + \upsilon_{CL}R_{CL} + \upsilon_{LL}R_{LL} + \upsilon_{T}R_{T} \pm \upsilon_{T'}R_{T'}\right)$$

$$\left.\frac{d\sigma}{d\omega dq}\right|_{e} = \sigma_M\left(\upsilon_{L}R_{L} + \upsilon_{T}R_{T}\right)$$

✓ much more precise data

✓ we can get access to R_L and R_T separately (Rosenbluth separation)

✓ experimental programs of electron scattering in JLab, MAMI, MESA

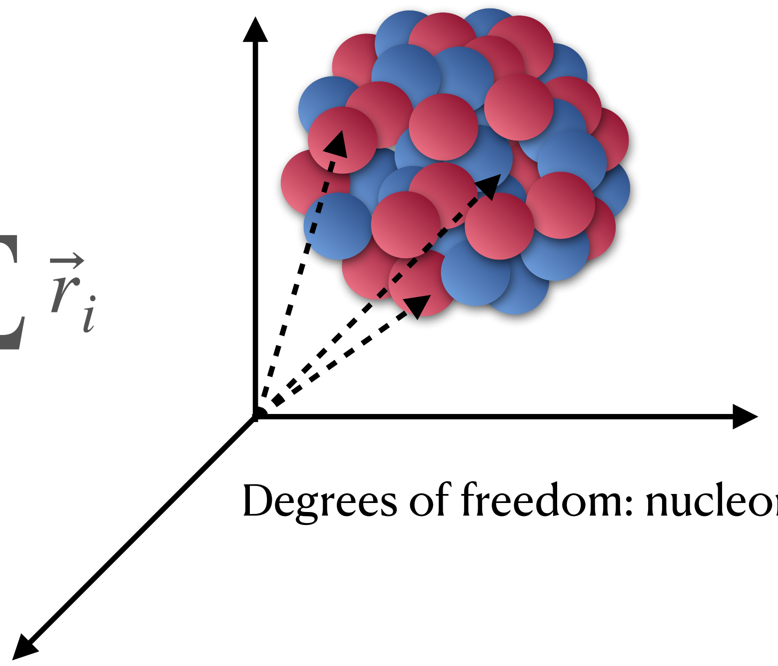
Coulomb sum rule

$$m_0(q) = \int d\omega R_L(\omega, q) = \sum_{f \neq 0} |\langle \Psi_f | \hat{\rho} | \Psi \rangle|^2 = \langle \Psi | \hat{\rho}^\dagger \hat{\rho} | \Psi \rangle - |F_{el}(q)|^2$$

easier to calculate since we do not need $|\Psi_f\rangle$

center of mass problem

$|\Psi\rangle$ has 3A coordinates $\rightarrow$ 3(A-1) intrinsic coordinates + $\vec{R} = \frac{1}{A}\sum_i^A \vec{r}_i$



With translationally non-invariant operators we may excite spurious states

Coulomb sum rule

Project out spurious states: $\hat{\rho}\,|\Psi\rangle = |\Psi_{phys}\rangle + |\Psi_{spur}\rangle$

It has been shown that to good approximation the ground state factorizes:

$$|\Psi\rangle = |\Psi_I\rangle|\Psi_{CoM}\rangle$$

center of mass wave function is a Gaussian

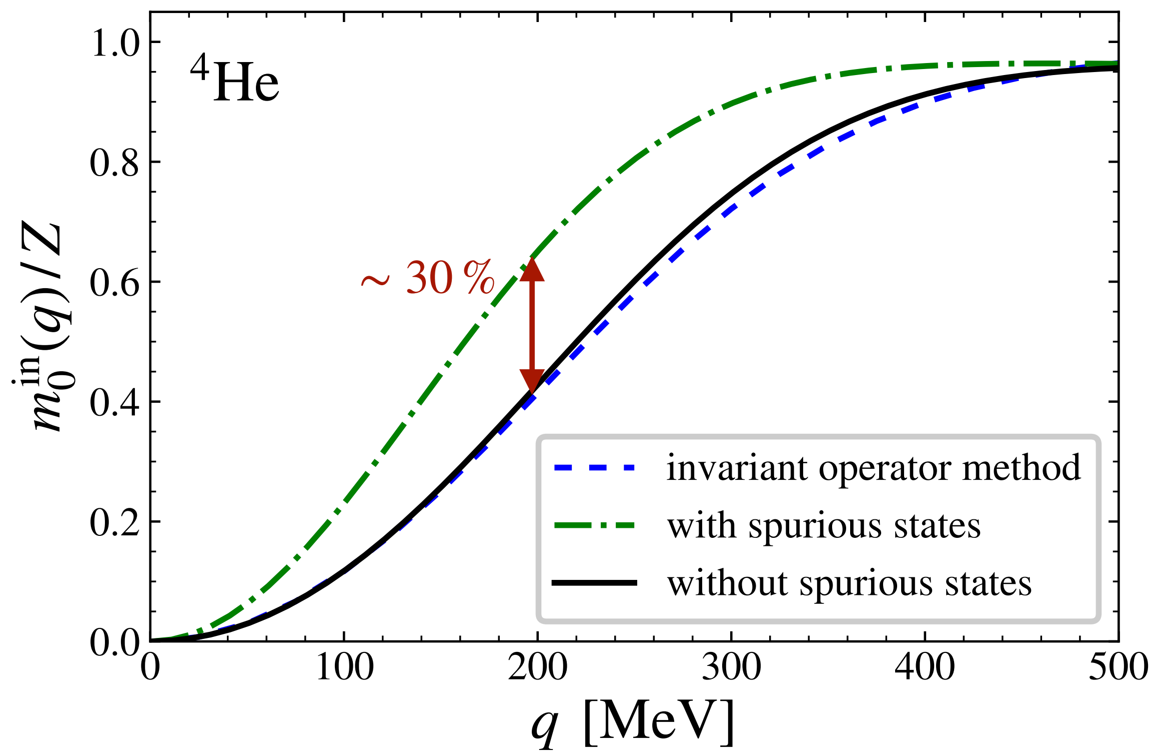
G. Hagen, T. Papenbrock, D. Dean
Phys.Rev.Lett. 103 (2009) 062503

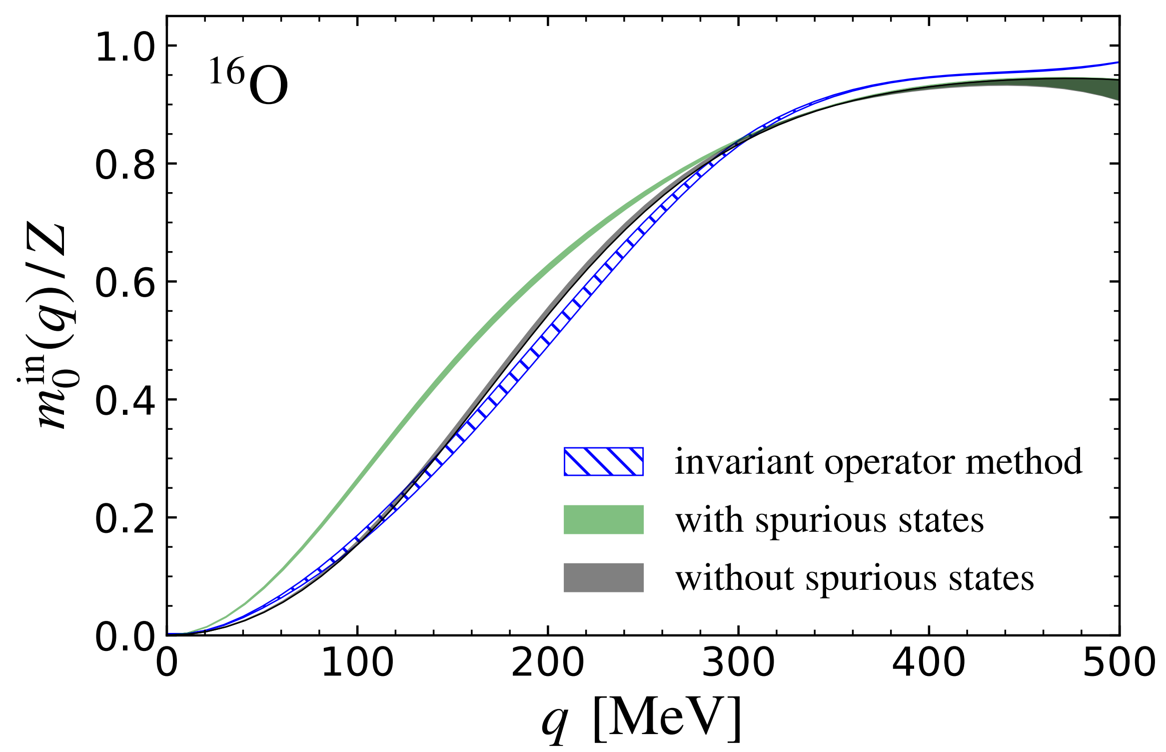
We follow a similar ansatz for the excited states:

$$\hat{\rho}\,|\Psi\rangle = |\Psi_I^{exc}\rangle|\Psi_{CoM}\rangle + |\Psi_I\rangle|\Psi_{CoM}^{exc}\rangle$$

spurious

Coulomb sum rule



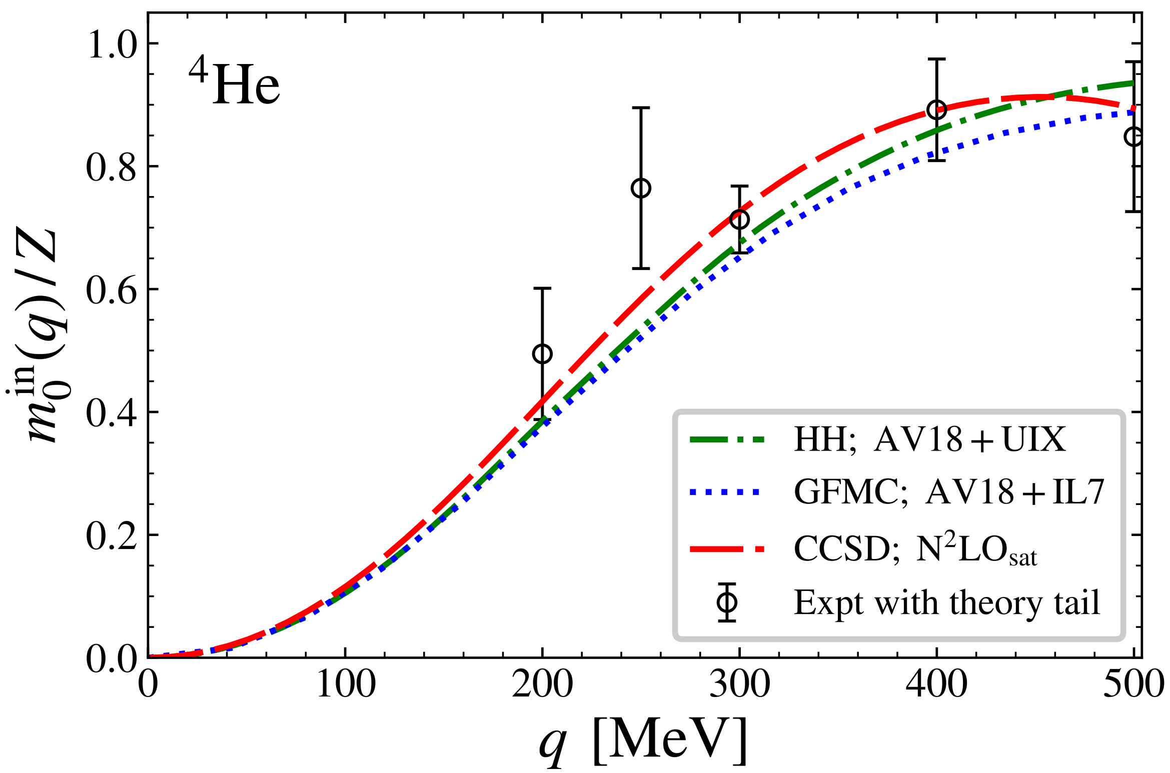


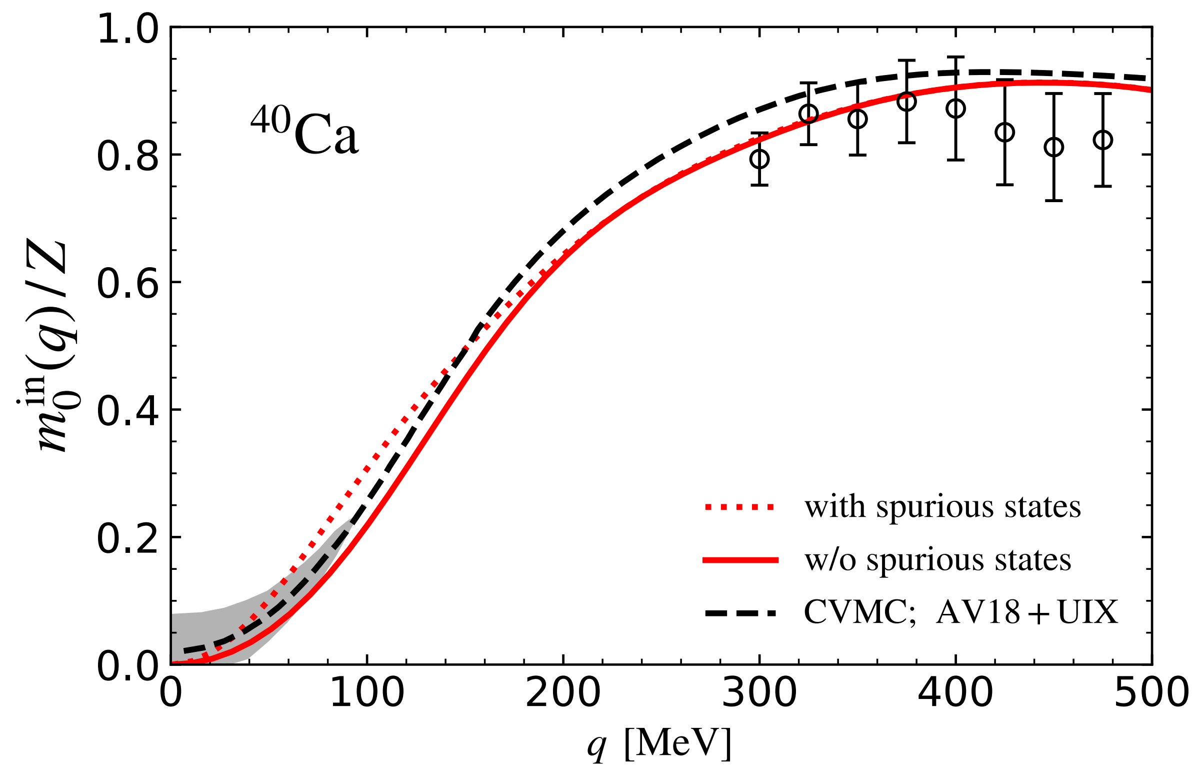
J.E.S. B. Acharya, S.Bacca, G. Hagen
Phys.Rev.C 102 (2020) 064312

CoM spurious states dominate for light nuclei

Coulomb sum rule

$$m_0(q) = \int d\omega R_L(\omega, q) = \sum_{f \neq 0} |\langle \Psi_f | \hat{\rho} | \Psi \rangle|^2 = \langle \Psi | \hat{\rho}^\dagger \hat{\rho} | \Psi \rangle - |F_{el}(q)|^2$$

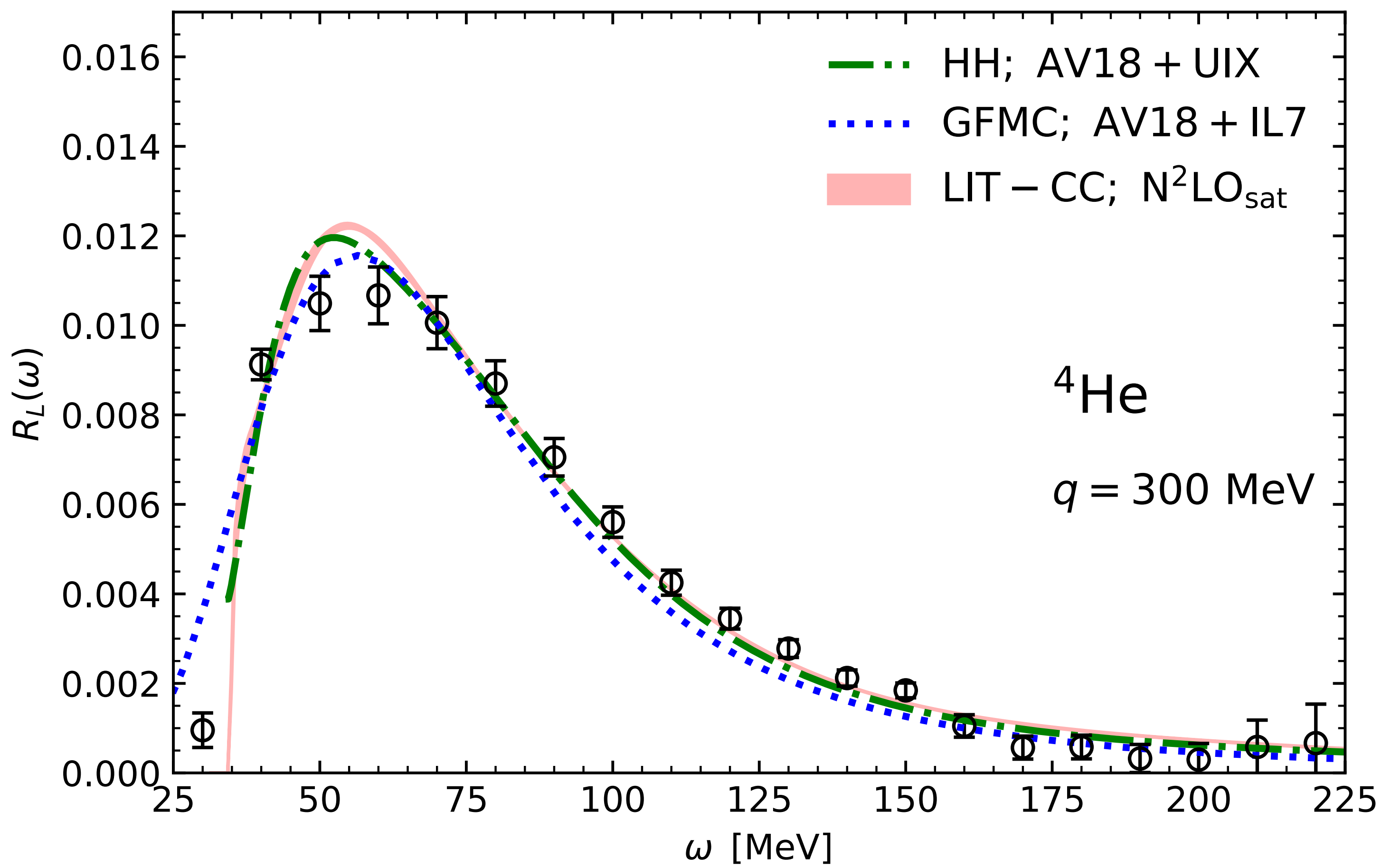




JES, B. Acharya, S.Bacca, G. Hagen Phys.Rev.C 102 (2020) 064312

PRL 127 (2021) 7, 072501 JES, B. Acharya, S. Bacca, G. Hagen

Longitudinal response



JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501

Uncertainty band: inversion procedure

$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_f \langle \Psi \,|\, J_\mu^\dagger \,|\, \Psi_f \rangle \langle \Psi_f | \, J_\nu \,|\, \Psi \rangle \delta(E_0 + \omega - E_f)$$

Lorentz Integral Transform (LIT)

$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_f \langle \Psi | J^\dagger_\mu | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

continuum spectrum

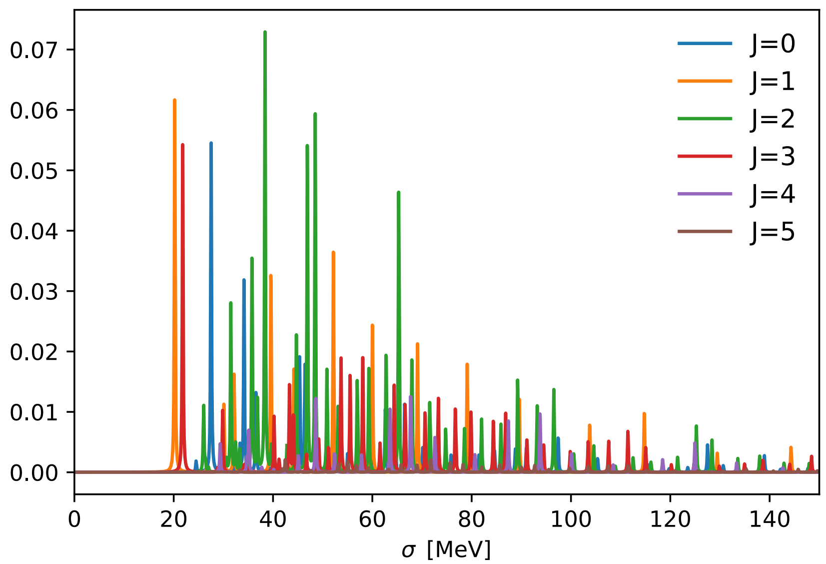
Integral transform

$$S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J^\dagger_\mu K(\mathcal{H} - E_0, \sigma) J_\nu | \Psi \rangle$$

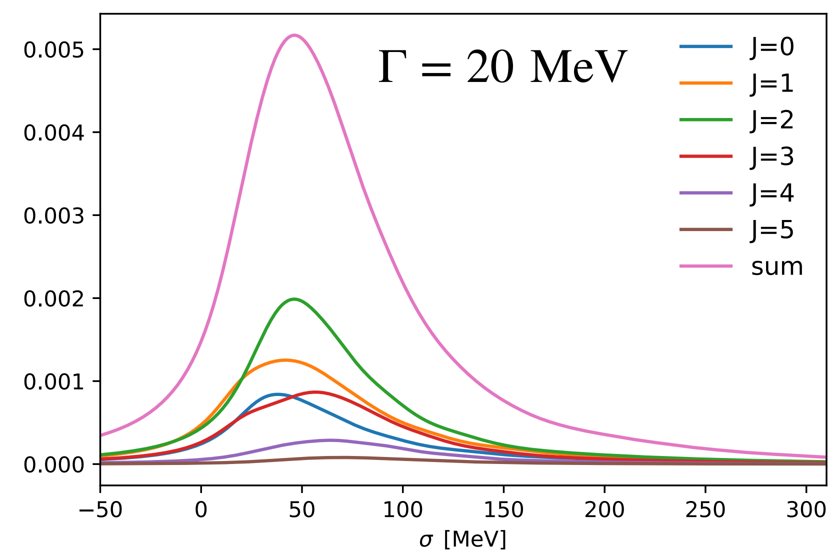
Lorentzian kernel:
$$K_\Gamma(\omega, \sigma) = \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \sigma)^2}$$

$S_{\mu\nu}$ has to be inverted to get access to $R_{\mu\nu}$

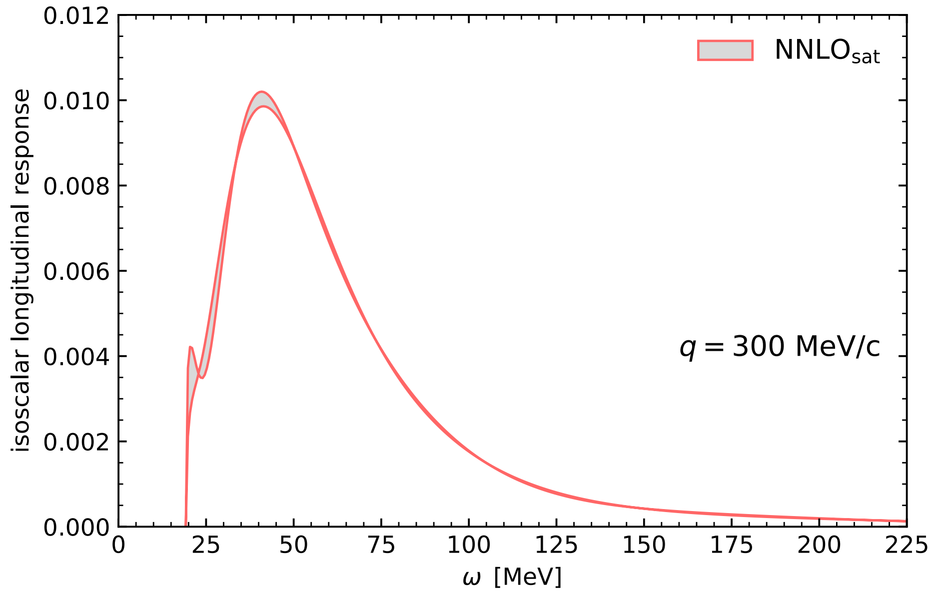
Lorentz Integral Transform



Integral transform

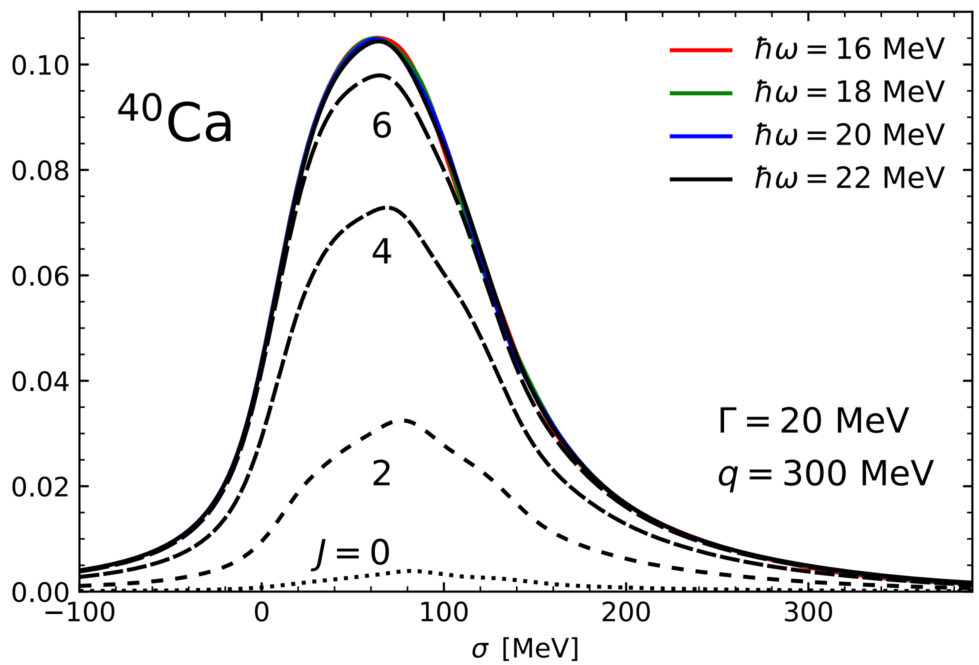


Inversion

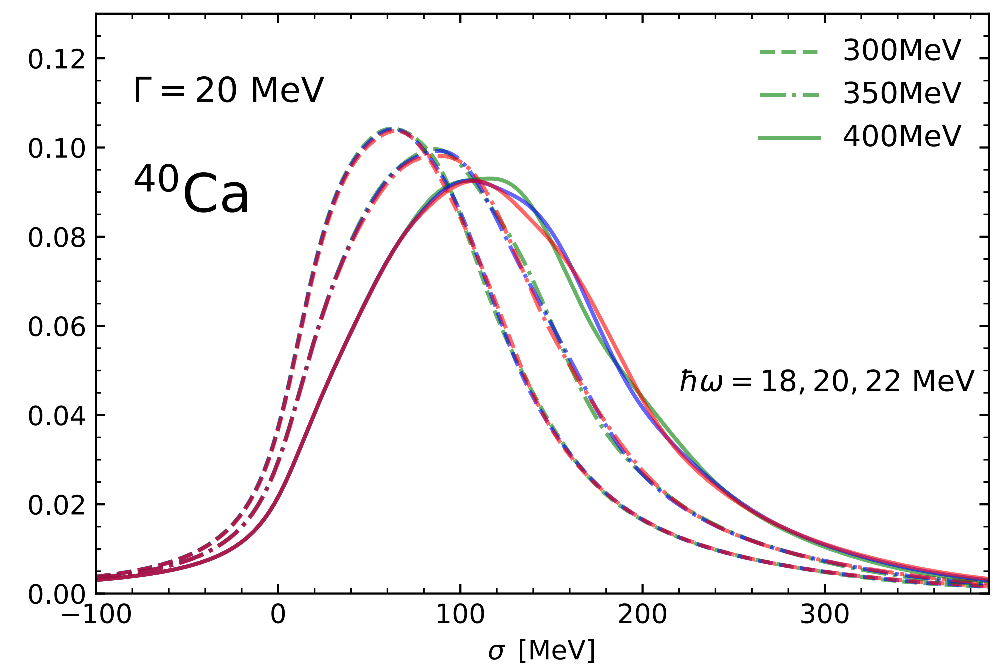


Longitudinal isoscalar response on ^{4}He at q=300 MeV

Longitudinal response ^{40}Ca

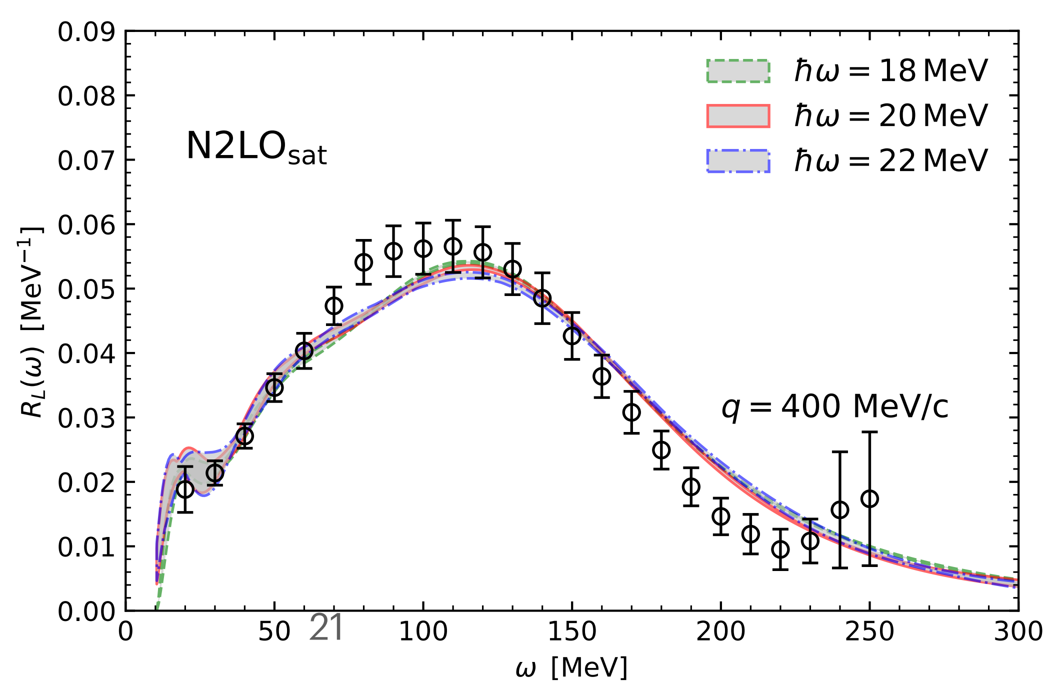


Sum over multipoles



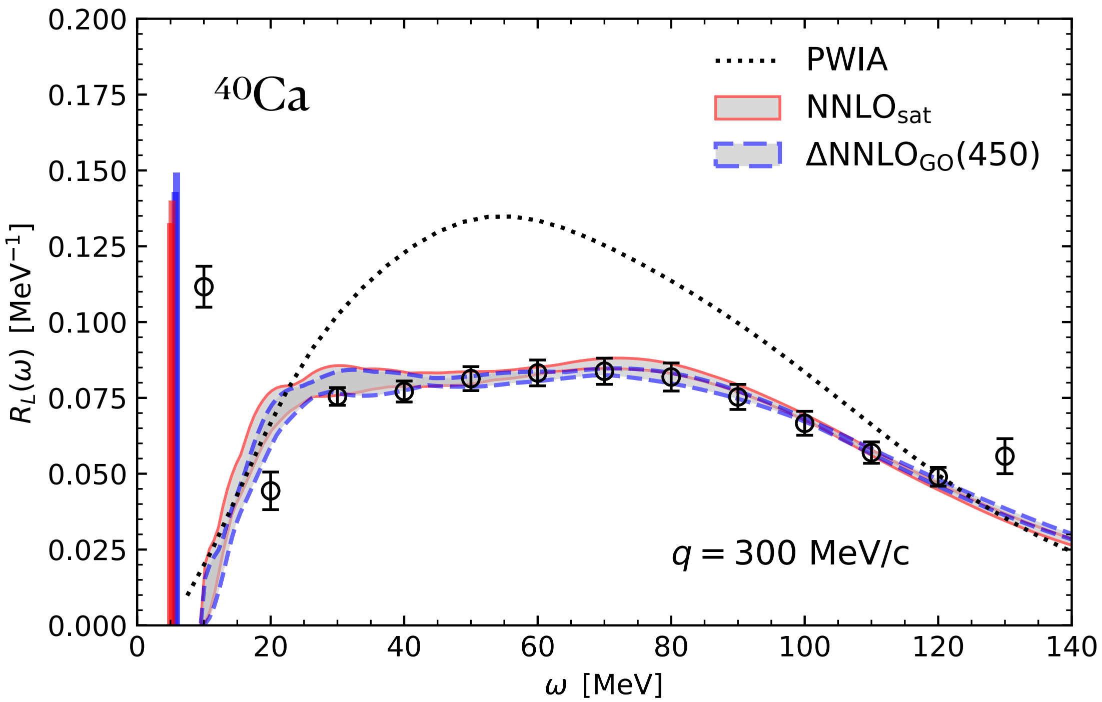
Underlying oscillator frequency

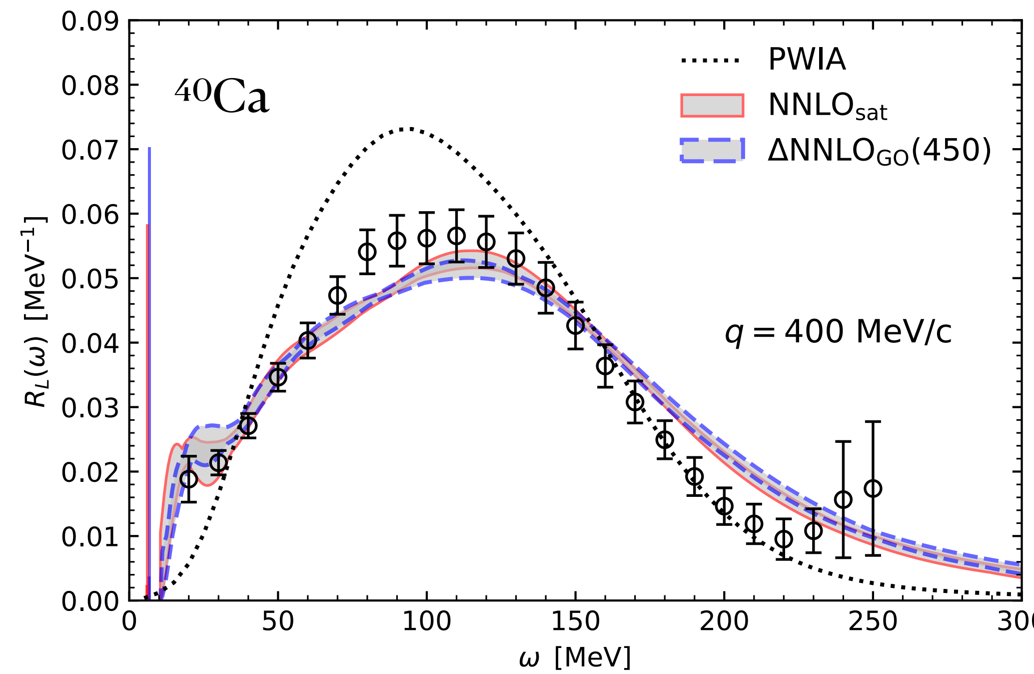
Inversion



Longitudinal response ^{40}Ca

Lorentz Integral Transform + Coupled Cluster





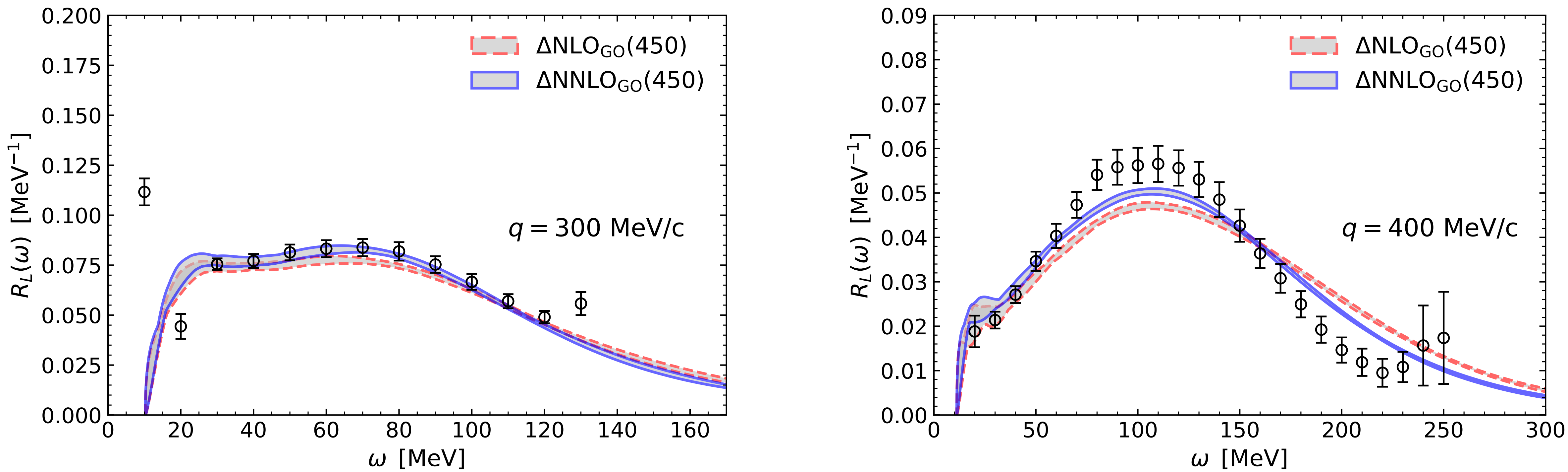
JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501

- ✓ CC singles & doubles
- ✓ varying underlying harmonic oscillator frequency
- ✓ two different chiral Hamiltonians
- ✓ *inversion procedure*

First ab-initio results for many-body system of 40 nucleons

Chiral expansion for ^{40}Ca

(Longitudinal response)

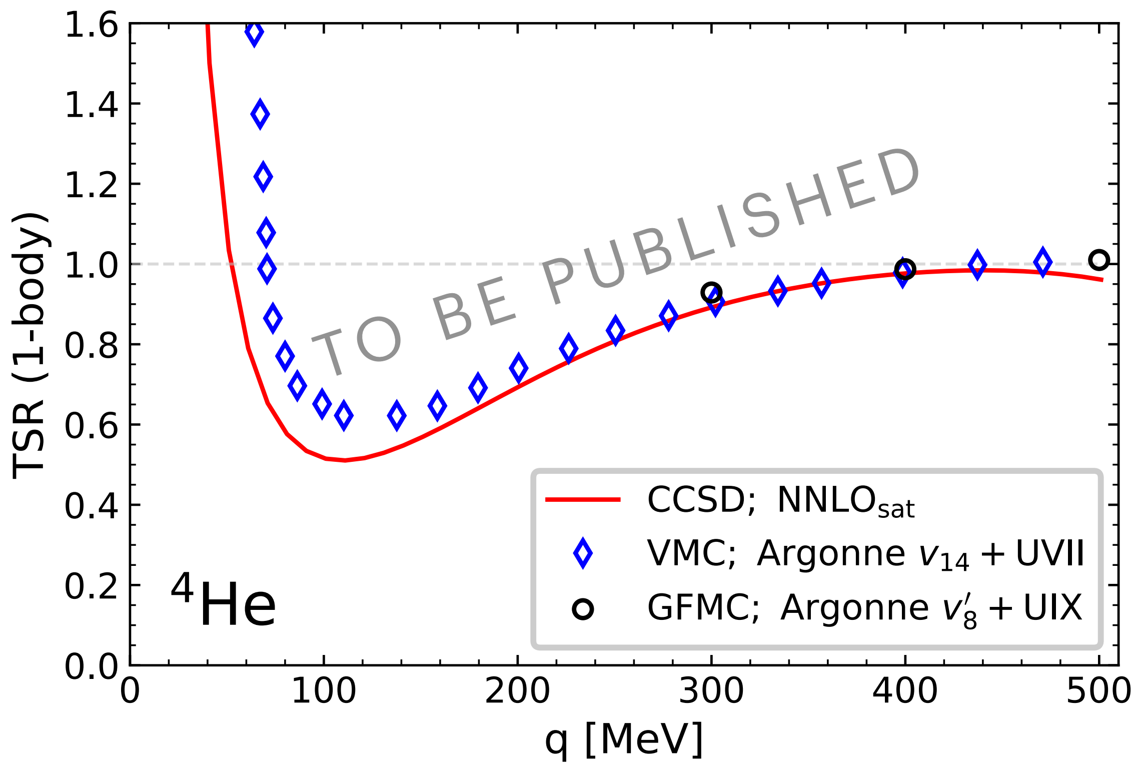


B. Acharya, S. Bacca, JES et al. arXiV 2210.04632

- ✓ Two orders of chiral expansion
- ✓ Convergence better for lower q (as expected)
- ✓ Higher order brings results closer to the data

Transverse response

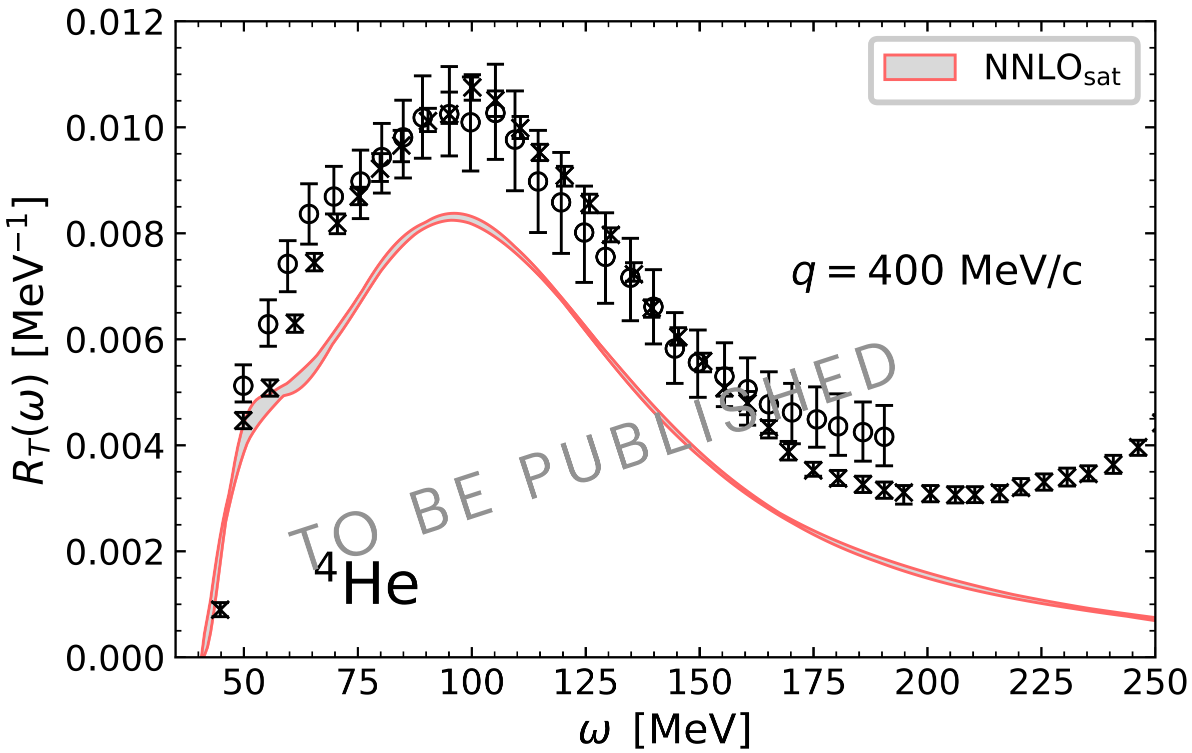
$$\mathrm{TSR}(q) = \frac{2m^2}{Z\mu_p^2 + N\mu_n^2}\frac{1}{q^2}\left(\langle\Psi|\hat{j}^\dagger\hat{j}\Psi\rangle - |\langle\Psi|\hat{j}|\Psi\rangle|^2\right)$$

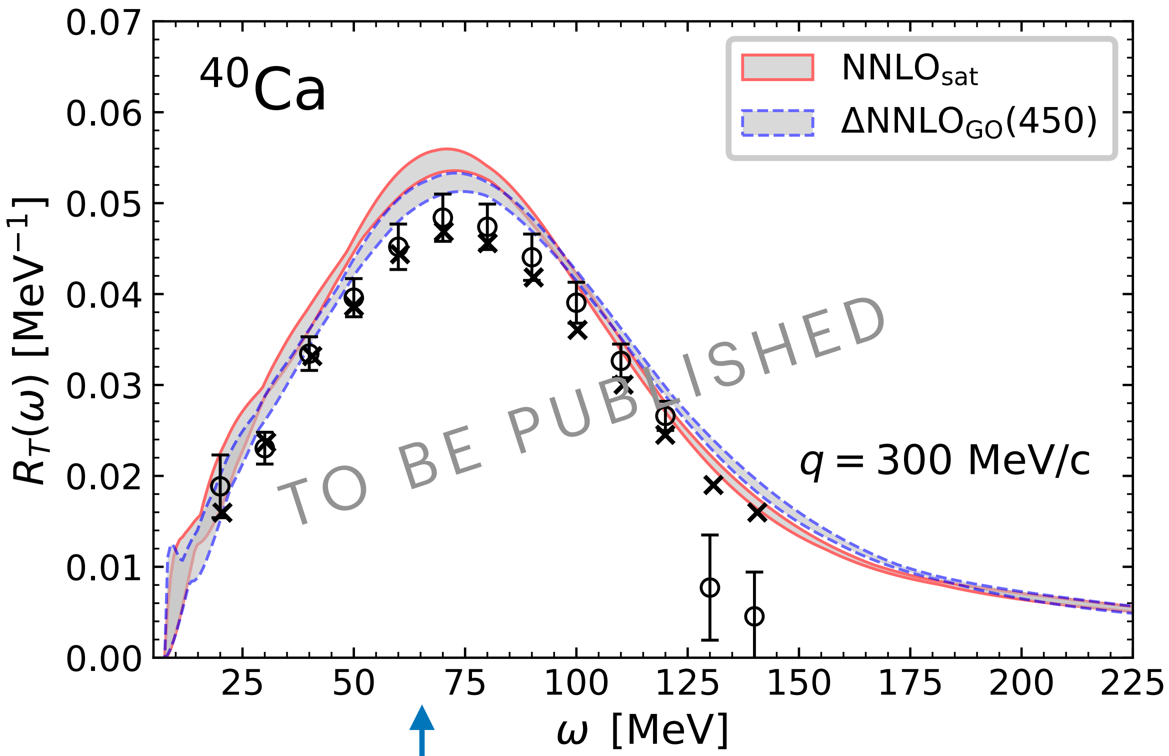


$\mathrm{TSR}(q \to 0) \propto$ kinetic energy ⟷ $\mathrm{TSR}(q \to \infty) = 1$

$$\mathbf{j}(\mathbf{q}) = \sum_i \frac{1}{2m}\epsilon_i\{\mathbf{p_i}, e^{i\mathbf{q}\mathbf{r_i}}\} - \frac{i}{2m}\mu_i\mathbf{q}\times\sigma_i e^{i\mathbf{q}\mathbf{r_i}}$$

Transverse response





- ➡ This allows to predict electron-nucleus cross-section
- ➡ Currently only 1-body current

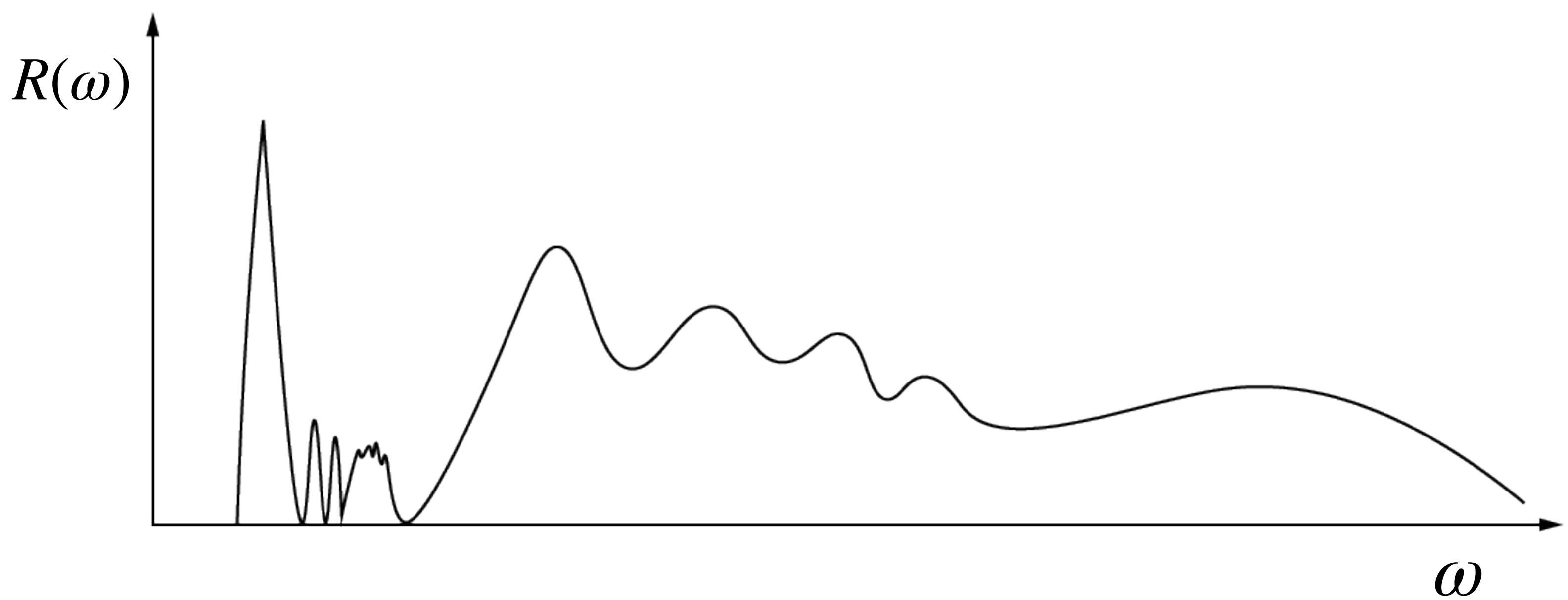
2-body currents important for 4He
→ more correlations needed?
→ 2-b currents strength depends on nucleus?

ChEK method

Chebyshev Expansion of integral Kernel

$$\Phi = \int f(\omega) R(\omega) d\omega$$

- Sum-rules
- Flux folding
- Histogram
- ...



$$\Phi \approx \tilde{\Phi} = \int f(\omega') \int K(\omega', \omega) R(\omega) d\omega d\omega'$$

expansion in Chebyshev polynomials

$$K(\omega, \sigma) = \sum_k c_k(\sigma) T_k(\omega)$$

estimated error

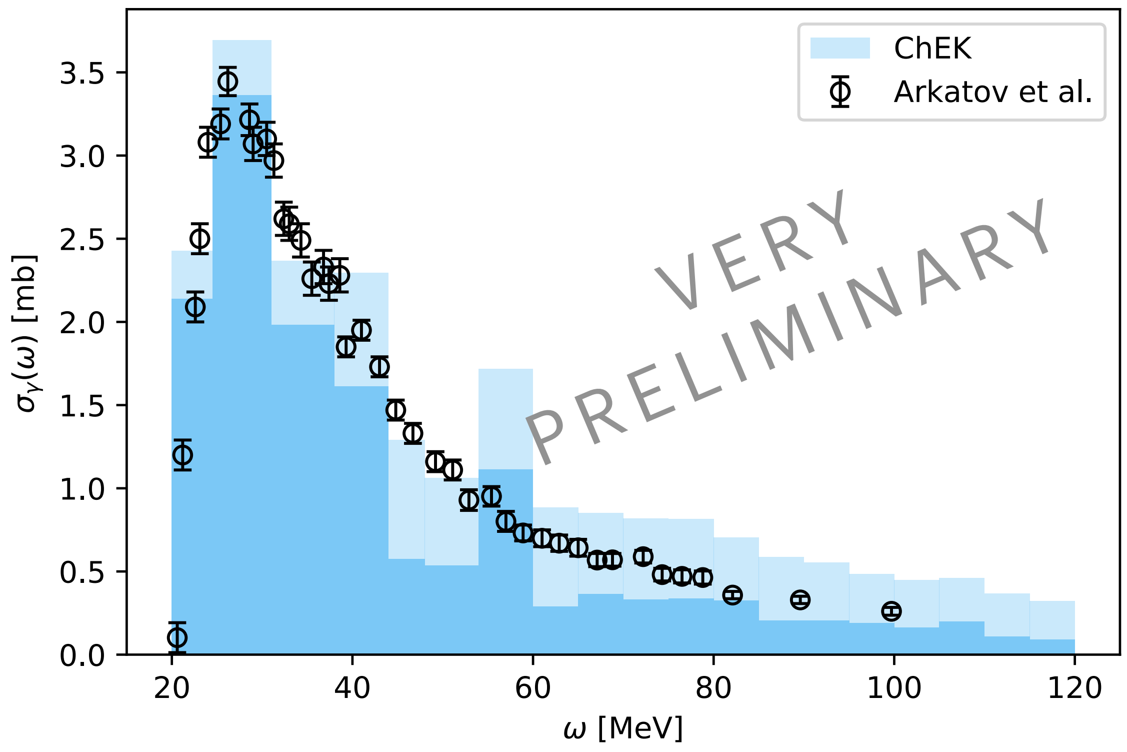
$$|\Phi - \tilde{\Phi}| < \epsilon$$

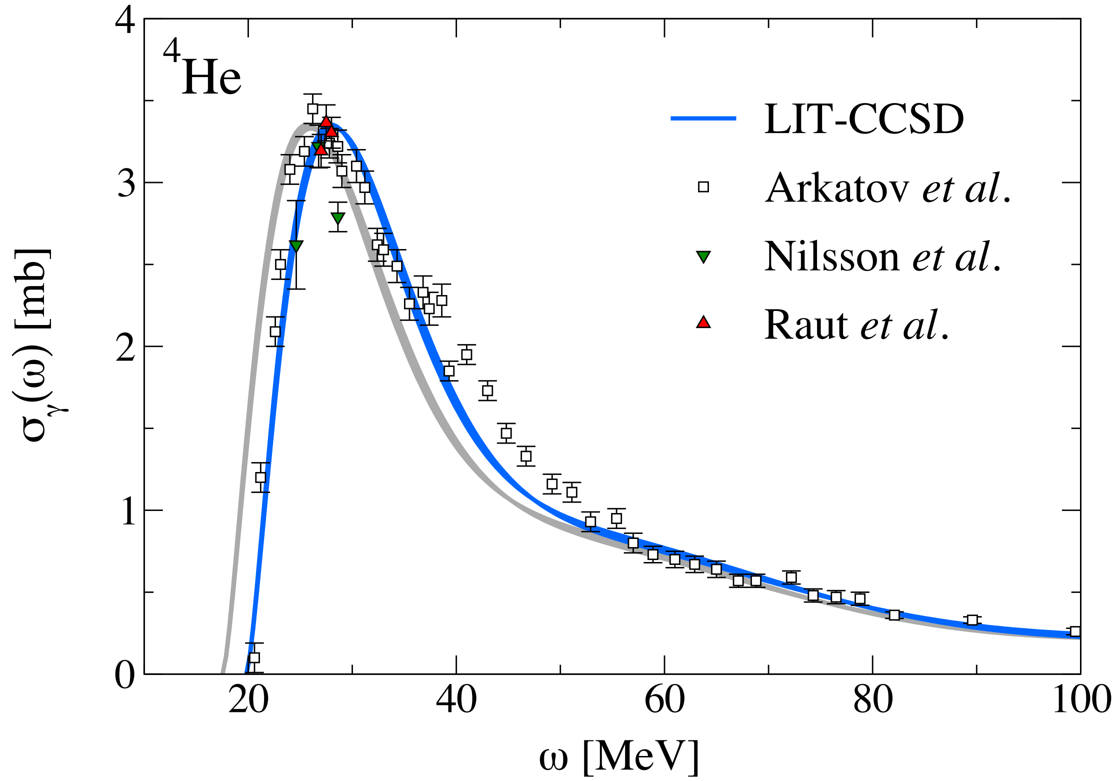
A. Roggero Phys.Rev.A 102 (2020) 2, 022409
JES, A. Roggero Phys.Rev.E 105 (2022) 055310

ChEK method

Chebyshev Expansion of integral Kernel

^{4}He photoabsorption



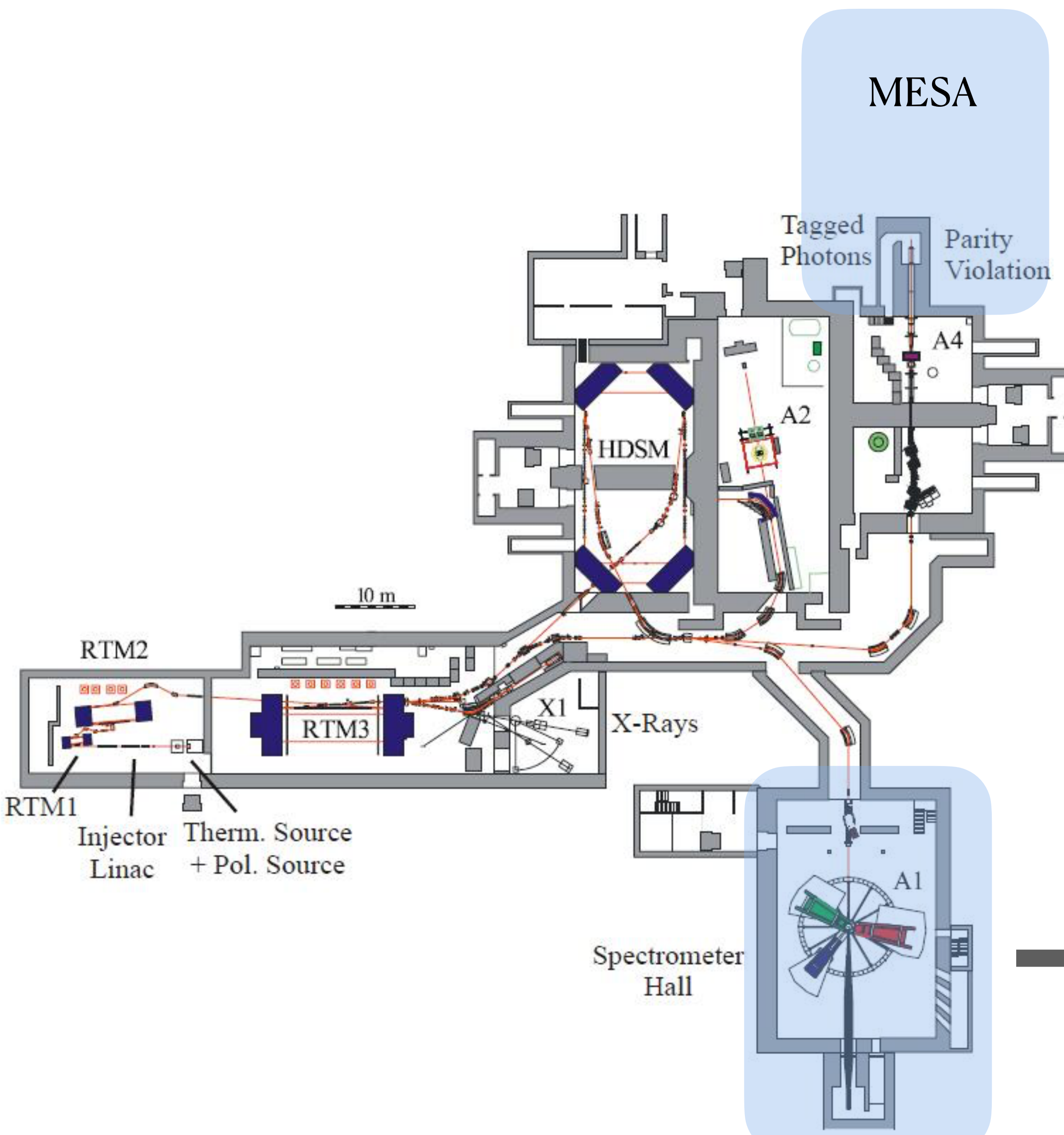


S. Bacca, N. Barnea, G. Hagen, G. Orlandini; *Phys.Rev.C* 90 (2014) 6

- ➡ No assumption about the shape of the response
- ➡ Rigorous error estimation
- ➡ Convenient when the response has a complicated structure

Experimental opportunities

MAMI — Mainz Microtron



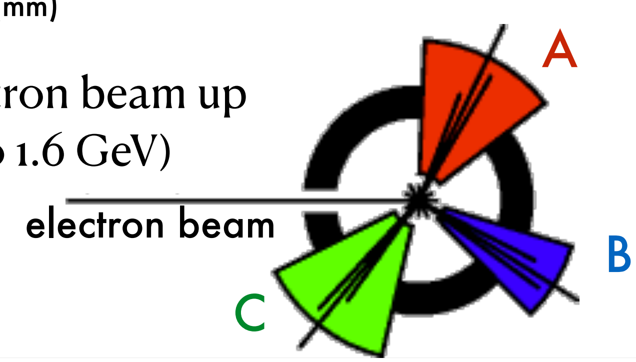
Mainz
Energy-recovery
Superconducting
Accelerator

(Electron beam up to 155 MeV)

Spectrometers

	A	B	C
Configuration	QSDD	D	QSDD
Max.Momentum (MeV)	735	870	551
Solid Angle (msr)	28	5,6	28
Mom. Resolution	10^{-4}	10^{-4}	10^{-4}
Pos. Res at Target (mm)	3-5	1	3-5

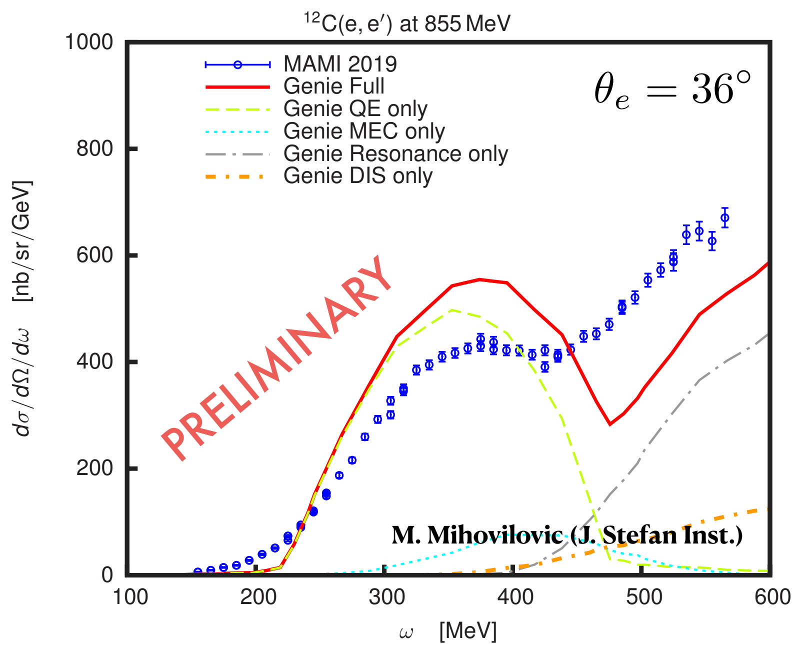
(Electron beam up to 1.6 GeV)



Experimental opportunities

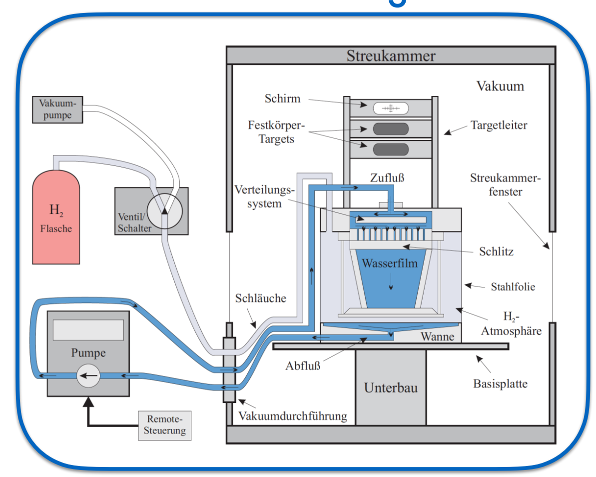
MAMI — Mainz Microtron

$q \approx 530$ MeV

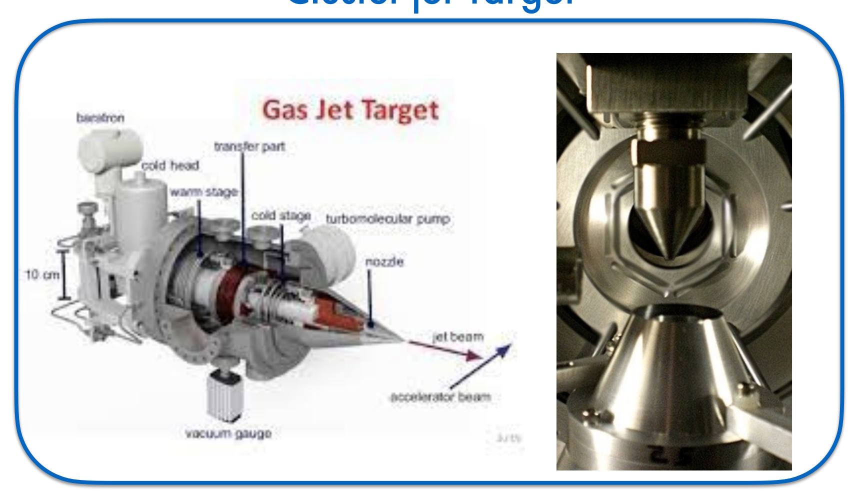


Future targets: ^{16}O, ^{40}Ar

Waterfall target



Cluster-jet Target



- Windowless target — background reduction
- Exclusive measurements possible

Outlook

$q \gtrsim 500$ MeV

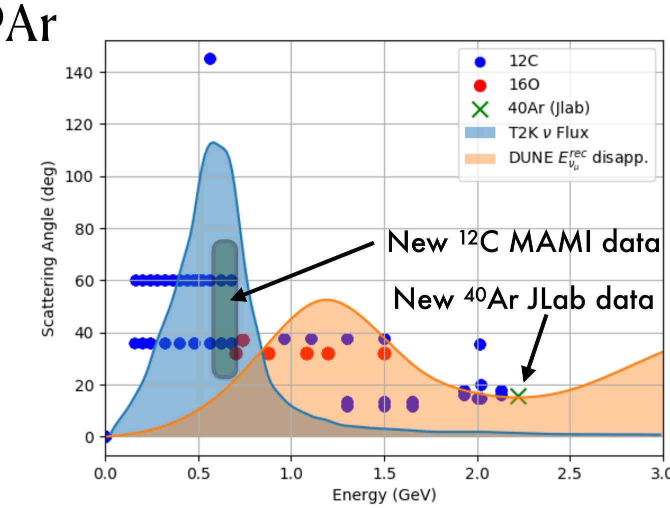
Spectral functions (within Impulse Approximation):

- *Relativistic regime*
- *Semi-inclusive processes*
- *Further steps: 2-body spectral functions, accounting for FSI*

X-section measurements @MAMI A1

Role of 2-body currents

Nuclei important for HK and DUNE: ^{16}O, ^{40}Ar



Studies of low-energy charge-current and neutral-current reactions (on ^{16}O)

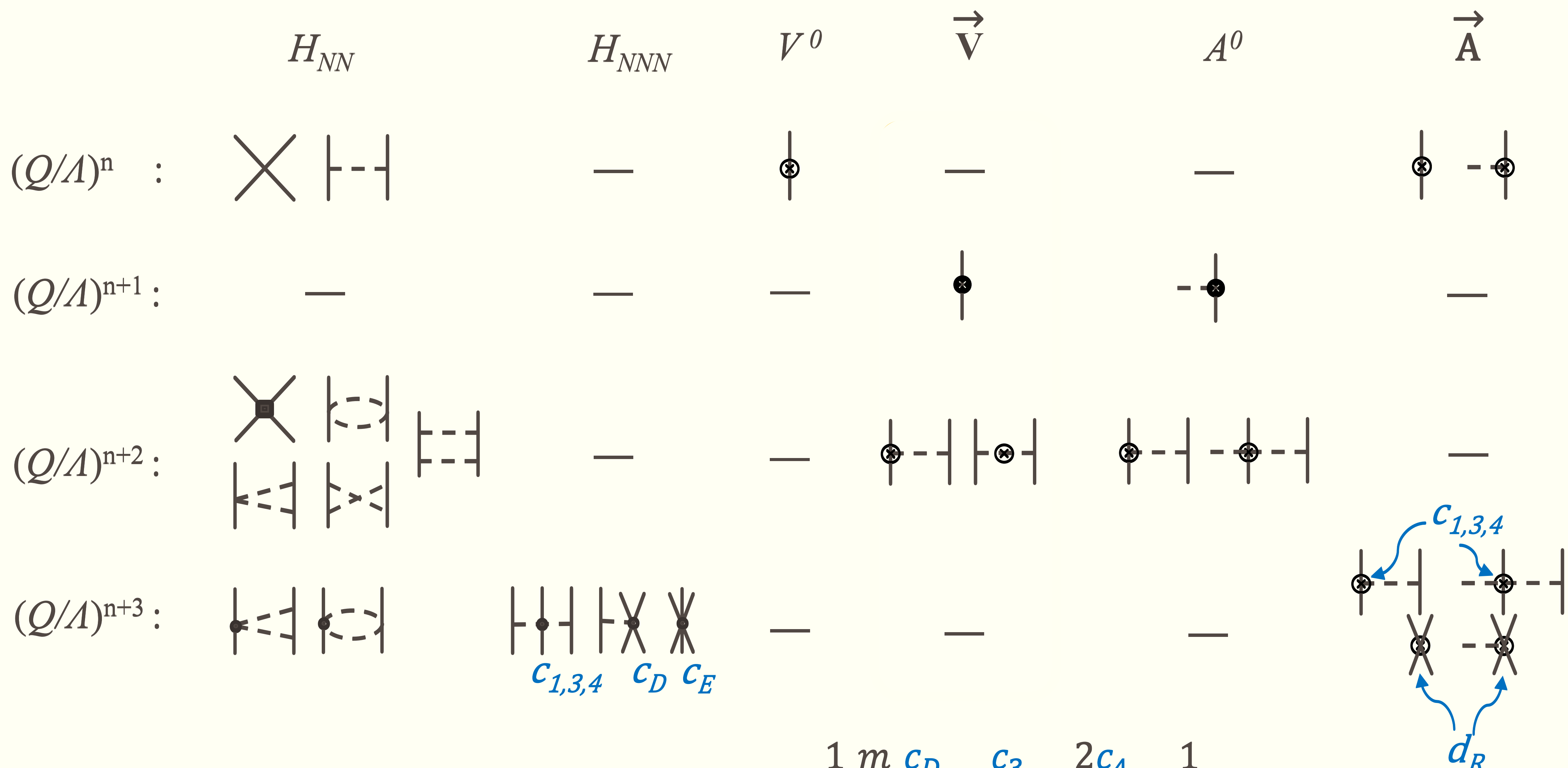
Future measurements @MESA

$q \sim \mathcal{O}(10$ MeV$)$

Thank you for attention

BACKUP

Nuclear Hamiltonian and currents



$$-\frac{1}{4}\frac{m}{g_A}\frac{c_D}{\Lambda}+\frac{c_3}{3}+\frac{2c_4}{3}+\frac{1}{6}=d_R$$

*Nucleon-structure diagrams and relativistic corrections not shown

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Details on inversion procedure

- Basis functions

$$R_L(\omega) = \sum_{i=1}^{N} c_i \omega^{n_0} e^{-\frac{\omega}{\beta_i}}$$

- Stability of the inversion procedure:
    - Vary the parameters n_0, β_i and number of basis functions N (6-9)
    - Use LITs of various width Γ (5, 10, 20 MeV)

Lorentz integral transform

$$L(\sigma) = \int \frac{R(\omega)}{(\omega-\sigma)^2+\Gamma^2} d\omega = \int \frac{R(\omega)}{(\omega+\tilde{\sigma}^*)(\omega+\tilde{\sigma})} d\omega$$

$$L(\sigma) = \int d\omega \sum_f \langle \Psi_0 | \rho^\dagger \frac{1}{\omega+\tilde{\sigma}^*} | \Psi_f \rangle \langle \Psi_f | \frac{1}{\omega+\tilde{\sigma}} \rho | \Psi_0 \rangle \delta(\omega + E_0 - E_f)$$

$$L(\sigma) = \sum_f \langle \Psi_0 | \rho^\dagger \frac{1}{E_f - E_0 + \tilde{\sigma}^*} | \Psi_f \rangle \langle \Psi_f | \frac{1}{E_f - E_0 + \tilde{\sigma}} \rho | \Psi_0 \rangle$$

$$L(\sigma) = \sum_f \langle \Psi_0 | \rho^\dagger \frac{1}{H - E_0 + \tilde{\sigma}^*} | \Psi_f \rangle \langle \Psi_f | \frac{1}{H - E_0 + \tilde{\sigma}} \rho | \Psi_0 \rangle$$

$$\langle \tilde{\Psi} | \qquad\qquad | \tilde{\Psi} \rangle$$

We need to solve

$$(H - E_0 + \tilde{\sigma}) | \tilde{\Psi} \rangle = \rho | \Psi \rangle$$

Schrodinger-like equation